Alexander von HUMBOLDT STIFTUNG

Radial Oscillations in Hybrid Stars with Slow Quark Phase Transition

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***PRD* 107 12, 123022 (2023)**

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- **Delta Baryons in NS**
- Delta Couplingss
- Equation of State
- Phase Transition
- Radial modes

⁕ Δ baryons could be present only at densities $\approx$ 8-10 ρ_0 inside the NSs.
N. K. Glendenning, Astrophys. J. 293, 470 (1985)

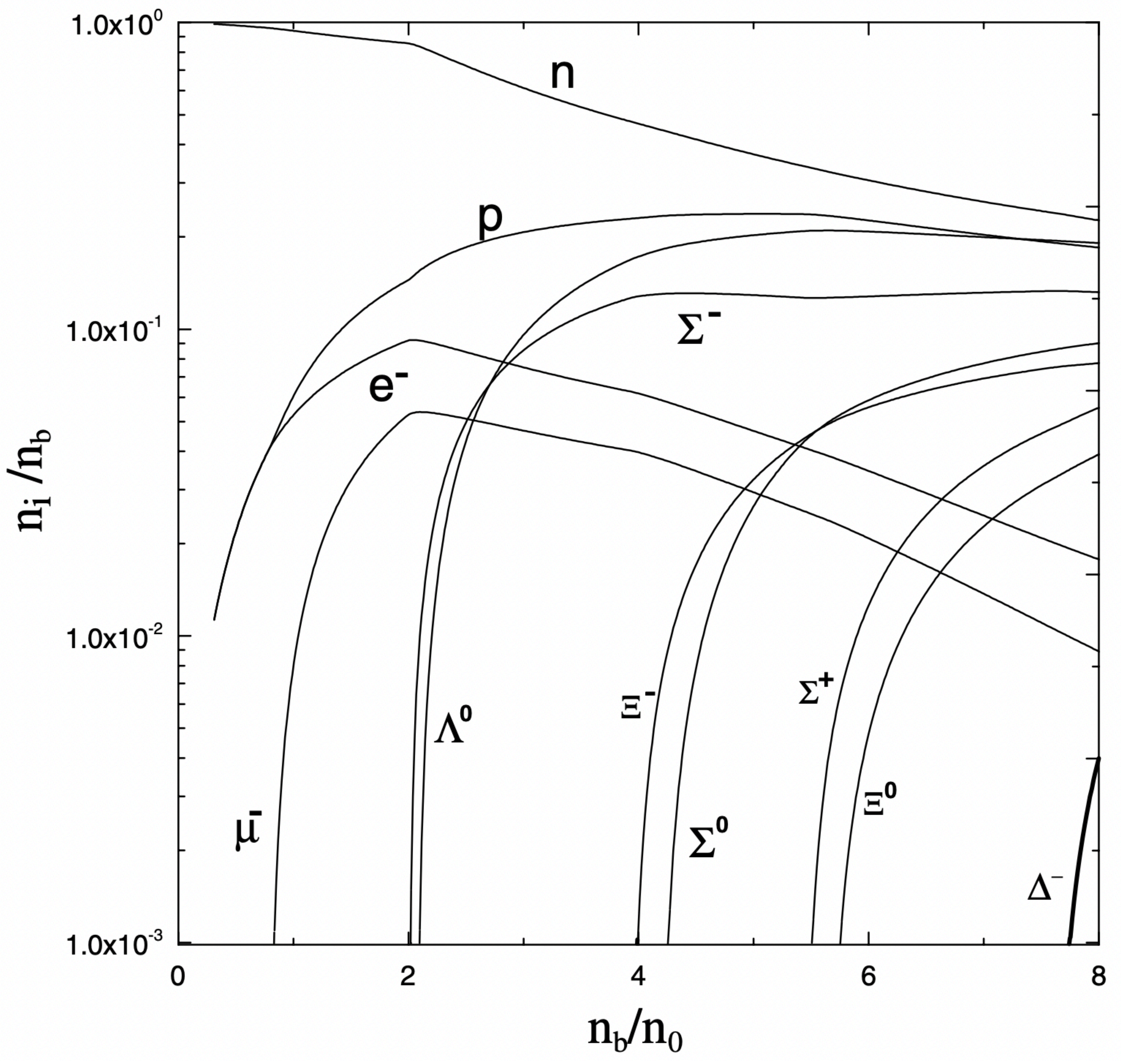
⁕ Forbidding the onset of Δ: for very repulsive coupling.

⁕ Their coupling potential for isospin-symmetric matter at saturation density is found to be **attractive** (2/3 to 1 times the potential of the nucleons)
A. Drago, PRC 90, 065809 (2014), A. R. Raduta, PLB 814, 136070 (2021).

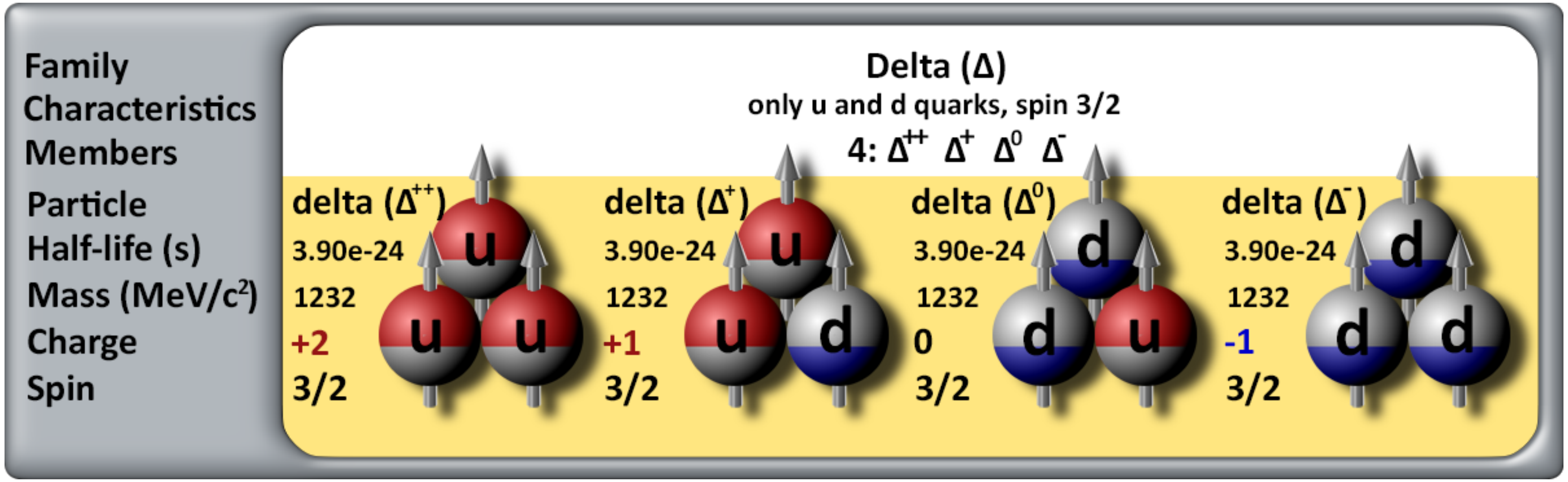
⁕ **With proper couplings**: Δ baryons might be present inside the NSs and could in fact make up a large fraction of the baryons in NS matter.
K. D. Marquez, et al., PRC 106, 055801 (2022), L. L. Lopes et al., PRD 107, 036011 (2023).

⁕ Δ baryons can exist inside NSs at almost the same density range as the hyperons ($\approx$ 2-3 ρ_0)

$$m_N < m_\Delta(1232MeV) < m_\Xi$$



OLIVEIRA, IJMPD Vol. 16, No. 02-03, pp. 175-183 (2007)



Δ- baryon couplings

Baryon coupling scheme in a unified SU(3) and SU(6) symmetry formalism

L. L. Lopes et al., PRD 107, 036011 (2023).

- Octet and Decuplet baryon couplings with mesons fixed through an unified SU(3) and SU(6) group symmetry. (S. Weissenborn, D. Chatterjee, and J. Schaffner-Bielich, Phys. Rev. C 85, 065802 (2012), J. J. Swart, Rev. Mod. Phys. 35, 916 (1963))
- Clebsch-Gordan coefficients are used to calculate all the couplings.
- Just one free parameter is left to be freely varied: α_v

$$0 \leq \left(\frac{g_{\Delta\Delta\sigma}}{g_{NN\sigma}} - \frac{g_{\Delta\Delta\omega}}{g_{NN\omega}} \right) \leq 0.2$$

K. Wehrberger, C. Bedau, and F. Beck, Nucl. Phys. A 504, 797 (1989).

K. Marquez, PRC, 106, 055801 (2022)

K. Wehrberger, C. Bedau, and F. Beck, Nucl. Phys. A 504, 797 (1989).

$$\mathscr{L}_{Yukawa} = -(g_{BBM})(\bar{\psi}_B \psi_B)M$$

Invariant under SU(3)

Partially broken $SU(6) \supset SU(3) \otimes SU(2)$

For the ω meson

$$\frac{g_{\Lambda\Lambda\omega}}{g_{NN\omega}} = \frac{4 + 2\alpha_v}{5 + 4\alpha_v}$$

$$\frac{g_{\Sigma\Sigma\omega}}{g_{NN\omega}} = \frac{8 - 2\alpha_v}{5 + 4\alpha_v}$$

$$\frac{g_{\Xi\Xi\omega}}{g_{NN\omega}} = \frac{5 - 2\alpha_v}{5 + 4\alpha_v}$$

Δ- baryon couplings

For the ϕ meson

$$\frac{g_{\Lambda\Lambda\phi}}{g_{NN\omega}} = -\sqrt{2}\left(\frac{5-2\alpha_v}{5+4\alpha_v}\right)$$

$$\frac{g_{\Sigma\Sigma\phi}}{g_{NN\omega}} = -\sqrt{2}\left(\frac{1+2\alpha_v}{5+4\alpha_v}\right)$$

$$\frac{g_{\Xi\Xi\omega}}{g_{NN\phi}} = -\sqrt{2}\left(\frac{4+2\alpha_v}{5+4\alpha_v}\right)$$

$$\frac{g_{NN\phi}}{g_{NN\omega}} = 0$$

For the ρ meson

$$\frac{g_{\Lambda\Lambda\rho}}{g_{NN\rho}} = 0$$

$$\frac{g_{\Sigma\Sigma\rho}}{g_{NN\rho}} = 2\alpha_v$$

$$\frac{g_{\Xi\Xi\rho}}{g_{NN\rho}} = -(1-2\alpha_v)$$

Couplings for baryon decuplet

$$\frac{g_{\Delta^*\Delta^*\omega}}{g_{NN\omega}} = \frac{g_{\Delta\Delta\omega}}{g_{NN\omega}} = \frac{9}{5+4\alpha_v},$$

$$\frac{g_{\Sigma^*\Sigma^*\omega}}{g_{NN\omega}} = \frac{6}{5+4\alpha_v},$$

$$\frac{g_{\Xi^*\Xi^*\omega}}{g_{NN\omega}} = \frac{3}{5+4\alpha_v},$$

$$\frac{g_{\Omega\Omega\omega}}{g_{NN\omega}} = 0,$$

$$\frac{g_{\Sigma^*\Sigma^*\phi}}{g_{NN\omega}} = \frac{-3\sqrt{2}}{5+4\alpha_v},$$

$$\frac{g_{\Xi^*\Xi^*\phi}}{g_{NN\omega}} = \frac{-6\sqrt{2}}{5+4\alpha_v},$$

$$\frac{g_{\Omega\Omega\phi}}{g_{NN\omega}} = \frac{-9\sqrt{2}}{5+4\alpha_v},$$

$$\frac{g_{\Delta^*\Delta^*\rho}}{g_{NN\rho}} = 3,$$

$$\frac{g_{\Delta\Delta\rho}}{g_{NN\rho}} = 1,$$

$$\frac{g_{\Sigma^*\Sigma^*\rho}}{g_{N,\rho}} = 2,$$

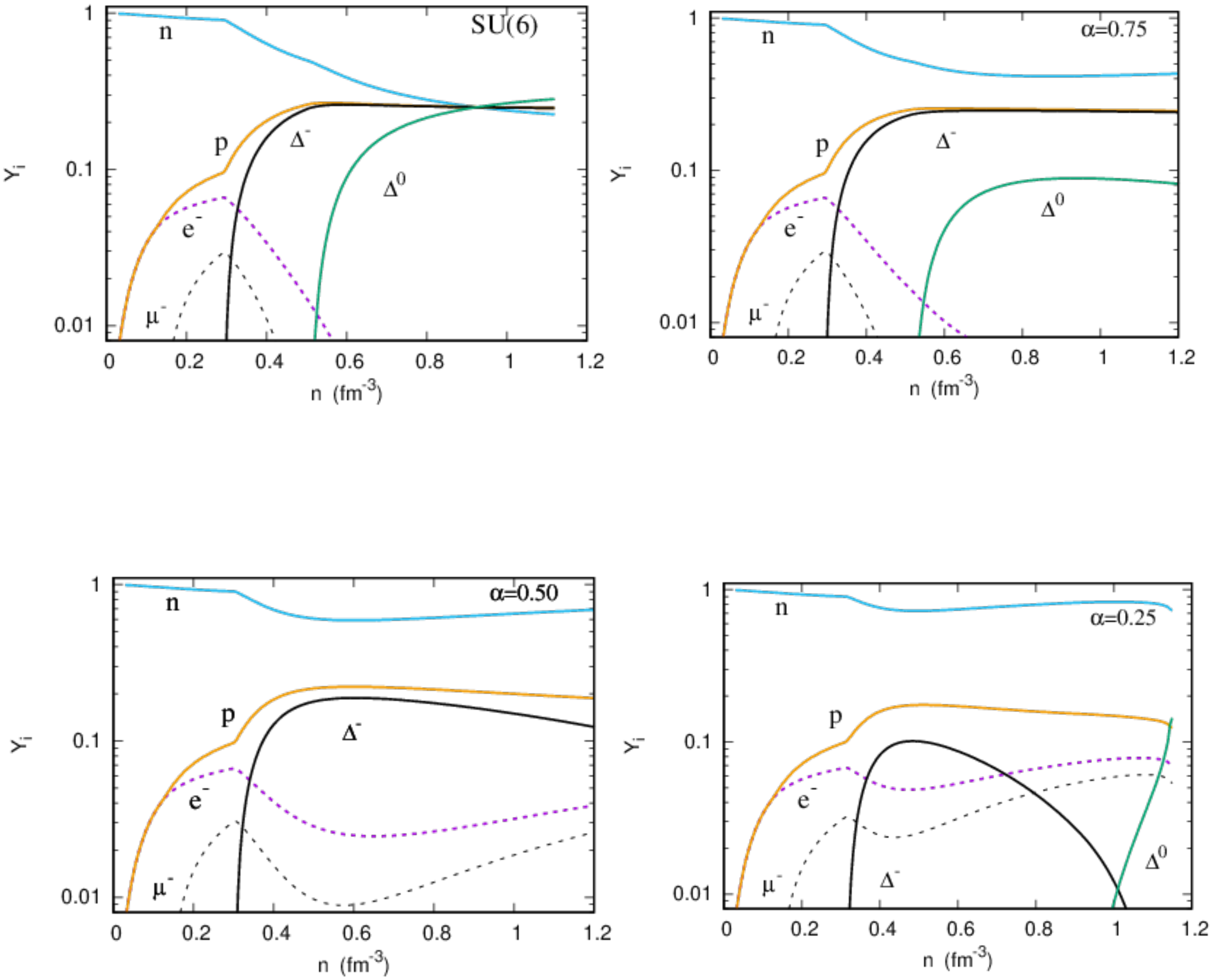
$$\frac{g_{\Xi^*\Xi^*\rho}}{g_{NN\rho}} = 1,$$

$$\frac{g_{\Omega\Omega\rho}}{g_{NN\rho}} = 0,$$

	α_v			
	1.00	0.75	0.50	0.25
$g_{\Lambda\Lambda\omega}/g_{NN\omega}$	0.667	0.687	0.714	0.75
$g_{\Sigma\Sigma\omega}/g_{NN\omega}$	0.667	0.812	1.0	1.25
$g_{\Xi\Xi\omega}/g_{NN\omega}$	0.333	0.437	0.571	0.75
$g_{\Lambda\Lambda\phi}/g_{NN\omega}$	−0.471	−0.619	−0.808	−1.06
$g_{\Sigma\Sigma\phi}/g_{NN\omega}$	−0.471	−0.441	−0.404	−0.354
$g_{\Xi\Xi\phi}/g_{NN\omega}$	−0.943	−0.972	−1.01	−1.06
$g_{\Lambda\Lambda\rho}/g_{NN\rho}$	0.0	0.0	0.0	0.0
$g_{\Sigma\Sigma\rho}/g_{NN\rho}$	2.0	1.5	1.0	0.5
$g_{\Xi\Xi\rho}/g_{NN\rho}$	1.0	0.5	0.0	−0.5
$g_{\Lambda\Lambda\sigma}/g_{NN\sigma}$	0.610	0.625	0.646	0.674
$g_{\Sigma\Sigma\sigma}/g_{NN\sigma}$	0.406	0.518	0.663	0.855
$g_{\Xi\Xi\sigma}/g_{NN\sigma}$	0.269	0.350	0.453	0.590
$g_{\Delta\Delta\omega}/g_{NN\omega}$	1.0	1.125	1.285	1.5
$g_{\Delta^*\Delta^*\omega}/g_{NN\omega}$	1.0	1.125	1.285	1.5
$g_{\Sigma^*\Sigma^*\omega}/g_{NN\omega}$	0.667	0.75	0.857	1.0
$g_{\Xi^*\Xi^*\omega}/g_{NN\omega}$	0.333	0.375	0.428	0.667
$g_{\Omega\Omega\omega}/g_{NN\omega}$	0.0	0.0	0.0	0.0
$g_{\Sigma^*\Sigma^*\phi}/g_{NN\omega}$	−0.471	−0.530	−0.606	−0.707
$g_{\Xi^*\Xi^*\phi}/g_{NN\omega}$	−0.943	−1.060	−1.212	−1.414
$g_{\Omega\Omega\phi}/g_{NN\omega}$	−1.414	−1.590	−1.818	−2.212
$g_{\Delta\Delta\rho}/g_{NN\rho}$	1.0	1.0	1.0	1.0
$g_{\Delta^*\Delta^*\rho}/g_{NN\rho}$	3.00	3.0	3.0	3.0
$g_{\Sigma^*\Sigma^*\rho}/g_{NN\rho}$	2.00	2.0	2.0	2.0
$g_{\Xi^*\Xi^*\rho}/g_{NN\rho}$	1.0	1.0	1.0	1.0
$g_{\Omega\Omega\rho}/g_{NN\rho}$	0.0	0.0	0.0	0.0
$g_{\Delta\Delta\sigma}/g_{NN\sigma}$	1.110	1.208	1.331	1.5
$g_{\Delta^*\Delta^*\sigma}/g_{NN\sigma}$	1.110	1.208	1.331	1.5
$g_{\Sigma^*\Sigma^*\sigma}/g_{NN\sigma}$	?	?	?	?
$g_{\Xi^*\Xi^*\sigma}/g_{NN\sigma}$	?	?	?	?
$g_{\Omega\Omega\sigma}/g_{NN\sigma}$	?	?	?	?

$$U_\Lambda = -28MeV, U_\Sigma = +30MeV$$

$$U_\Xi = -4MeV, U_\Delta = -98MeV$$



S

DDRMF Model

$$\mathscr{L}_{\rm RMF} = \sum_{b\in H} \bar{\psi}_b\Big[i\gamma^\mu\partial_\mu - \gamma^0(g_{\omega b}\omega_0 + g_{\phi b}\phi_0 + g_{\rho b}I_{3b}\rho_{03}) - (m_b - g_{\sigma b}\sigma_0)\Big]\psi_b$$

Spin-1/2 baryon octet

$$-\frac{i}{2}\sum_{b\epsilon\Delta}\bar{\psi}_{b\mu}\Big[\varepsilon^{\mu\nu\rho\lambda}\gamma_5\gamma_\nu\partial_\rho - \gamma^0\left(g_{\omega b}\omega_0 + g_{\rho b}I_{3b}\rho_{03}\right) - (m_b - g_{\sigma b}\sigma_0)\,\varsigma^{\mu\lambda}\Big]\psi_{b\nu}$$

Spin-3/2 baryon decuplet

Rarita-Schwinger-type int. Lag.

M. G. de Paoli, et al., J. Phys. G 40, 055007 (2013).

$$+\sum_\lambda \bar{\psi}_\lambda\left(i\gamma^\mu\partial_\mu - m_\lambda\right)\psi_\lambda - \frac{1}{2}m_\sigma^2\sigma_0^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2$$

Lepton admix + pure mesonic terms

DD couplings

$$g_{ib}(n_B) = g_{ib}(n_0)\frac{a_i + b_i(\eta + d_i)^2}{a_i + c_i(\eta + d_i)^2}$$

$$g_{\rho b}(n_B) = g_{ib}(n_0)\exp\left[-a_\rho\left(\eta - 1\right)\right]$$

$$n_b = \frac{\lambda_b}{2\pi^2}\int_0^{k_{F_b}} dk\,k^2 = \frac{\lambda_b}{6\pi^2}k_{F_b}^{\,3}$$

Baryon density

$$n_b^s = \frac{\lambda_b}{2\pi^2}\int_0^{k_{F_b}} dk\frac{k^2 m_b^*}{\sqrt{k^2 + m_b^{*2}}}$$

Scalar density

Equation of State

$$\varepsilon_B = \sum_b \frac{\gamma_b}{2\pi^2}\int_0^{k_{F_b}} dk k^2\sqrt{k^2+m_b^{*2}} + \sum_\lambda \frac{1}{\pi^2}\int_0^{k_{F_\lambda}} dk k^2\sqrt{k^2+m_\lambda^2} + \frac{m_\sigma^2}{2}\sigma_0^2 + \frac{m_\omega^2}{2}\omega_0^2 + \frac{m_\phi^2}{2}\phi_0^2 + \frac{m_\rho^2}{2}\rho_{03}^2$$

$$P = \sum_i \mu_i n_i - \epsilon + n_B\Sigma^r$$

$$\Sigma^r = \sum_b \left[\frac{\partial g_{\omega b}}{\partial n_b}\omega_0 n_b + \frac{\partial g_{\rho b}}{\partial n_b}\rho_{03}I_{3b}n_b + \frac{\partial g_{\phi b}}{\partial n_b}\phi_0 n_b - \frac{\partial g_{\sigma b}}{\partial n_b}\sigma_0 n_b^s\right]$$

Rearrangement term

$$\mu_b^* = \mu_b - g_{\omega b}\omega_0 - g_{\rho b}I_{3b}\rho_{03} - g_{\phi b}\phi_0 - \Sigma^r$$

Effective Chemical Potential

DD-ME2 parameter

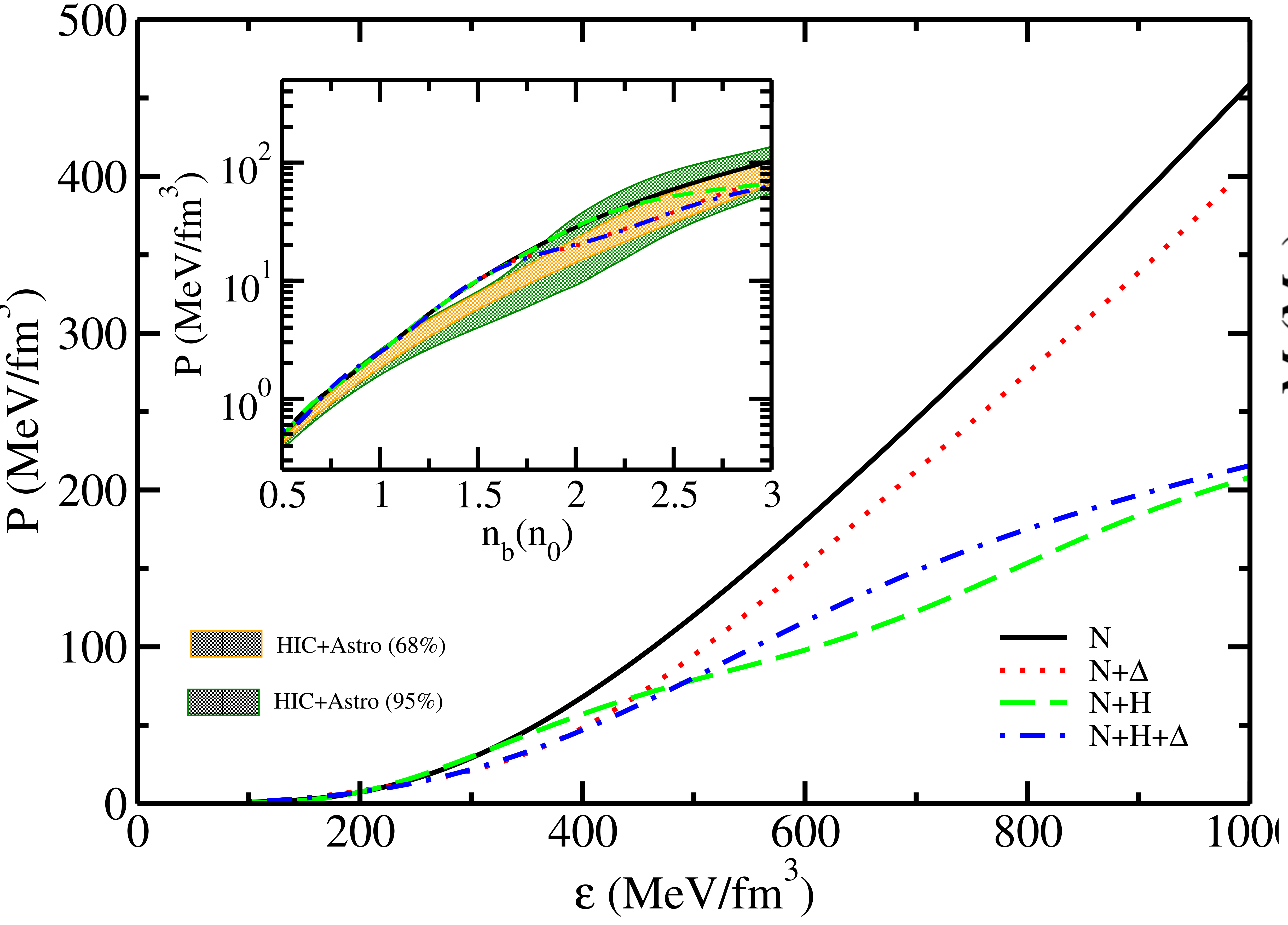
G. A. Lalazissis, *et al*., Phys. Rev. C 71, 024312 (2005).

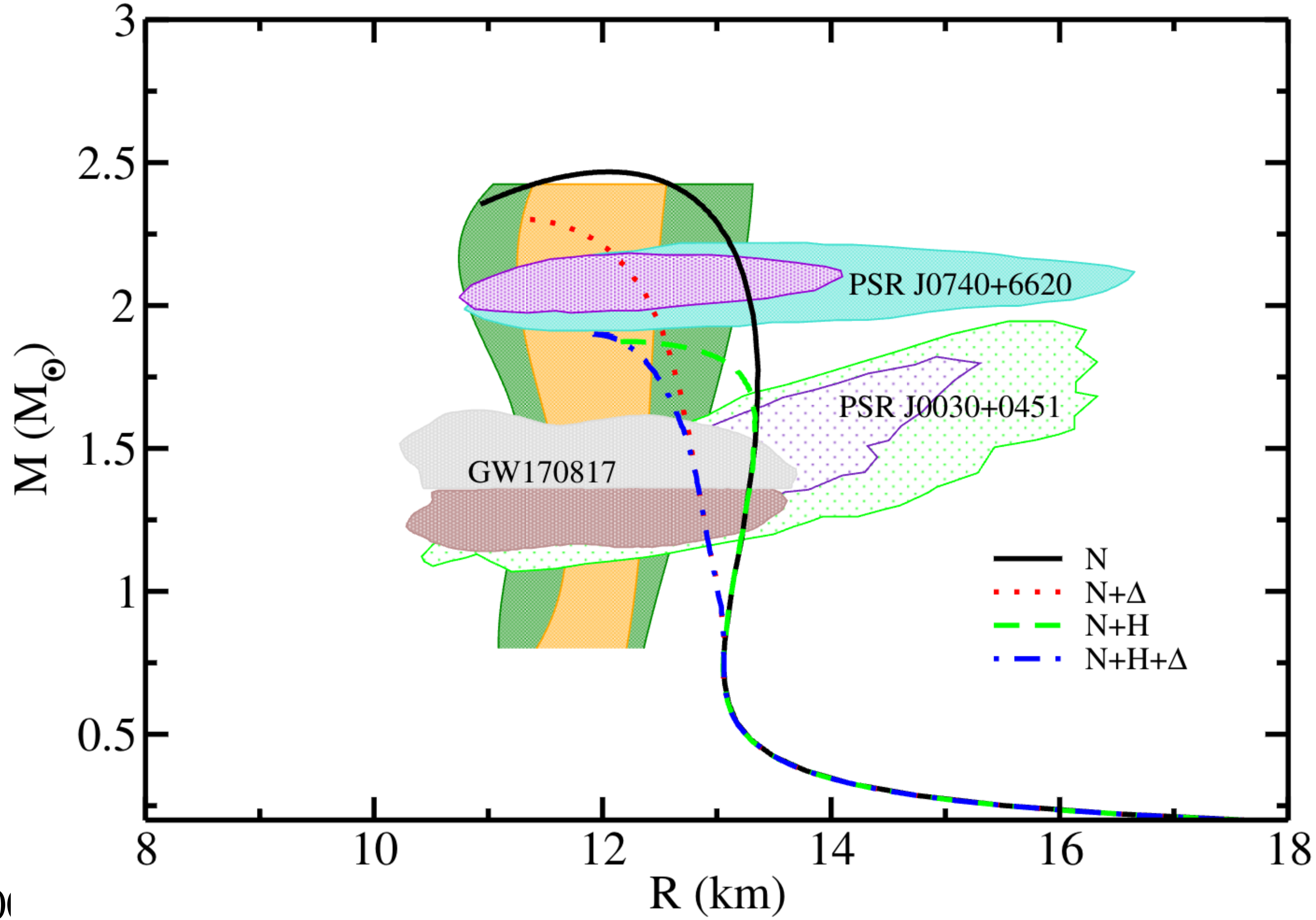
i	m_i(MeV)	a_i	b_i	c_i	d_i	$g_{iN}(n_0)$
σ	550.1238	1.3881	1.0943	1.7057	0.4421	10.5396
ω	783	1.3892	0.9240	1.4620	0.4775	13.0189
ρ	763	0.5647	—	—	—	7.3672

Quantity	Constraints [44, 49]	This model
n_0 (fm^{-3})	0.148–0.170	0.152
$-B/A$ (MeV)	15.8–16.5	16.4
K_0 (MeV)	220–260	252
S_0 (MeV)	31.2–35.0	32.3
L_0 (MeV)	38–67	51

M. Dutra, Physical Review C 90, 055203 (2014), M. Oertel , Rev. Mod. Phys. 89, 015007 (2017)

Results





M_max > 2.0 for N and N+D

All Constraints Satisfied

Stiff EoS for N and N+D

Soft EoS for N+H and N+H+D

M_max ≈ 2.0 for N+H and N+H+D

Ishfaq Rather et al. *Phys.Rev.D* 107 (2023) 12, 123022

Density-Dependent Quark mass Model

- noninteracting gas of quasiparticles with density-dependent masses.
- Overcomes the consistency between zero pressure and energy minimum
- To include quark interactions in a simple way.

G. N. Fowler, S. Raha, and R. M. Weiner, Z. Phys. C 9, 271 (1981),
S. Chakrabarty et al., Phys. Lett. B 229, 112 (1989),
S. Chakrabarty, Phys. Rev. D 43, 627 (1991)
O. G. Benvenuto and G. Lugones, Phys. Rev. D. 51, 1989 (1995)

$$m_i = m_{i0} + \frac{D}{n_B^{1/3}} + C n_B^{1/3} = m_{i0} + m_I$$

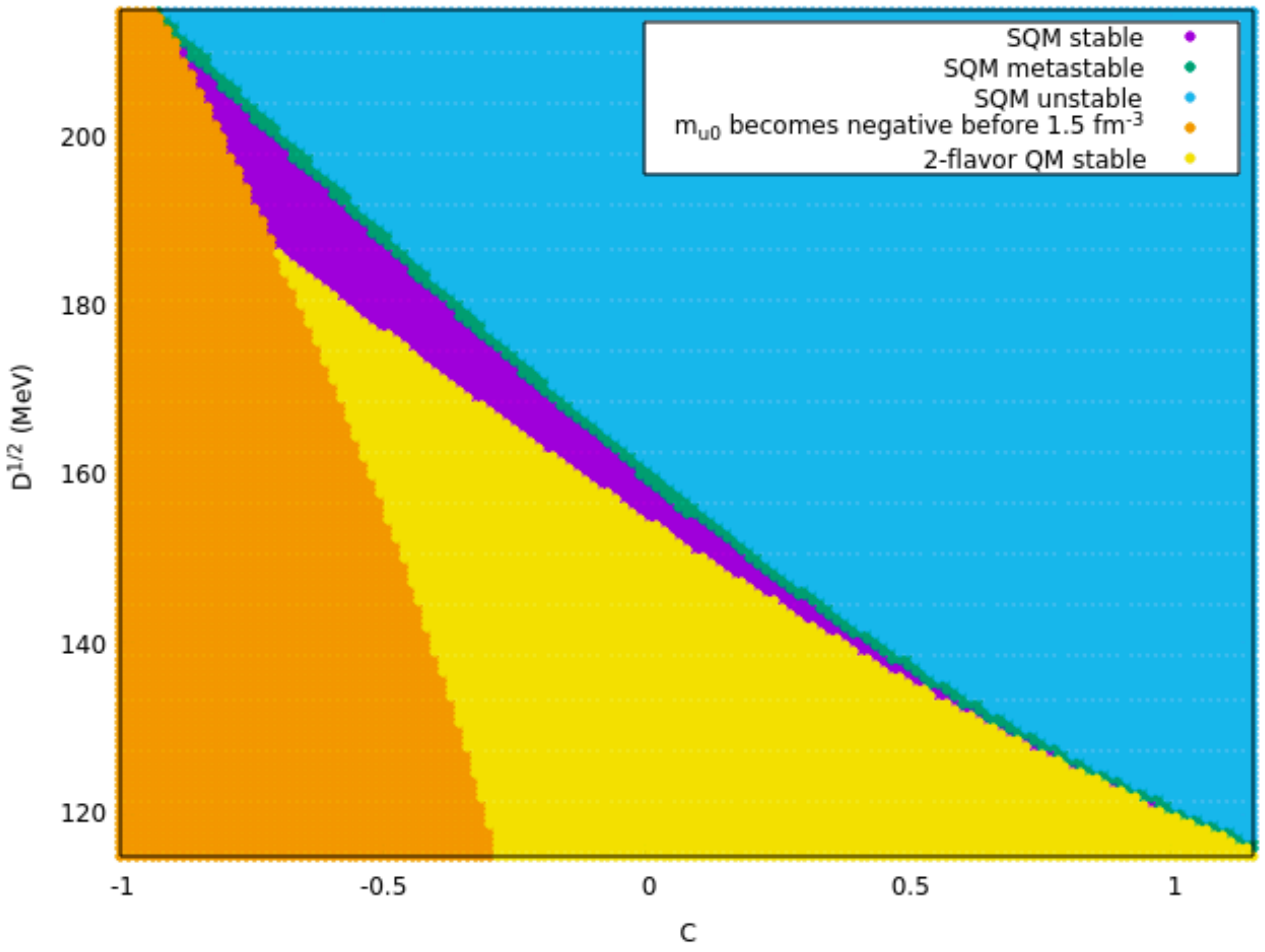
B C Backes et al . J. Phys. G: Nucl. Part. Phys. 48 (2021) 055104

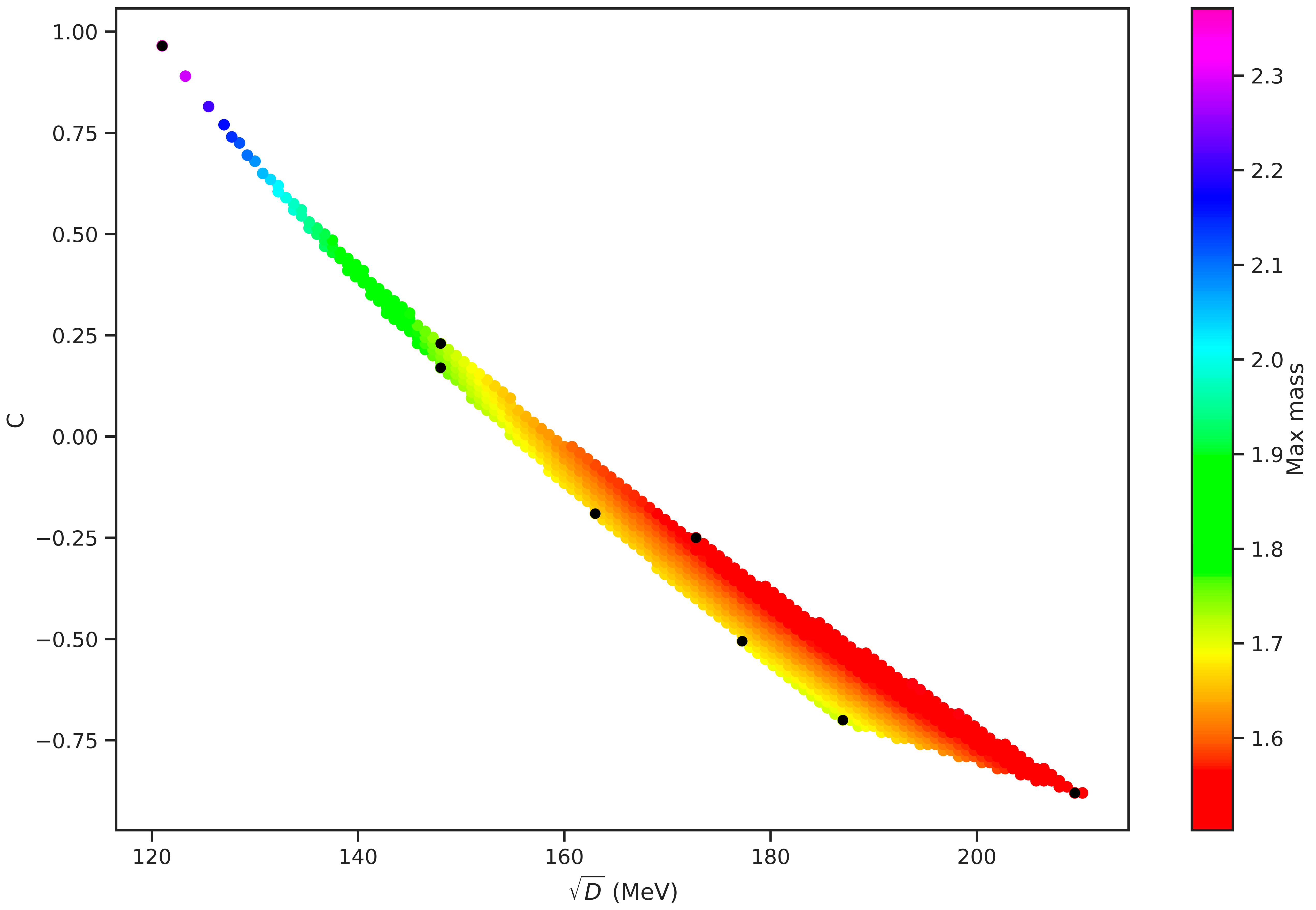
- D: **Dictates linear confinement**
- C: **Leading order perturbative interactions**
- m_{i0}: **Current quark mass**
- m_I: **DD term**

$$\mathscr{E} = \Omega_0(\{\mu_i^*\}, m_i) + \sum_i \mu_i^* n_i = \Omega_0(\{\mu_i^*\}, m_i) - \sum_i \mu_i^* \frac{\partial \Omega_0}{\partial \mu_i^*}$$

$$P = -\Omega_0 + \sum_{i,j} \frac{\partial \Omega_0}{\partial m_j} n_i \frac{\partial m_j}{\partial n_i}$$

$$\Omega_0 = -\sum_i \frac{g_i}{24\pi^2} \left[\mu_i^* \nu_i \left(\nu_i^2 - \frac{3}{2} m_i^2 \right) + \frac{3}{2} m_i^4 ln \left(\frac{\mu_i^* + \nu_i}{m_i} \right) \right],$$





B C Backes et al . J. Phys. G: Nucl. Part. Phys. 48 (2021) 055104

N

N+Δ

$C = 0.90, \sqrt{D} = 125$

N+H

N+Δ+H

$C = 0.65, \sqrt{D} = 133$

Radial Eqs. with PT

Phase Conversion

1) Conversion timescale (τ_{conv}) >> Oscillation period (τ_{osc}) → **slow phase transition**
   (fluid elements keep their nature)
2) $\tau_{conv} << \tau_{osc}$ → **Rapid phase transition**
   (fluid elements are easily converted)

p_{tr} & p^*_{tr}

Sufficiently different → **Rapid phase transition**

Close enough → **slow phase transition**

Rapid phase transition

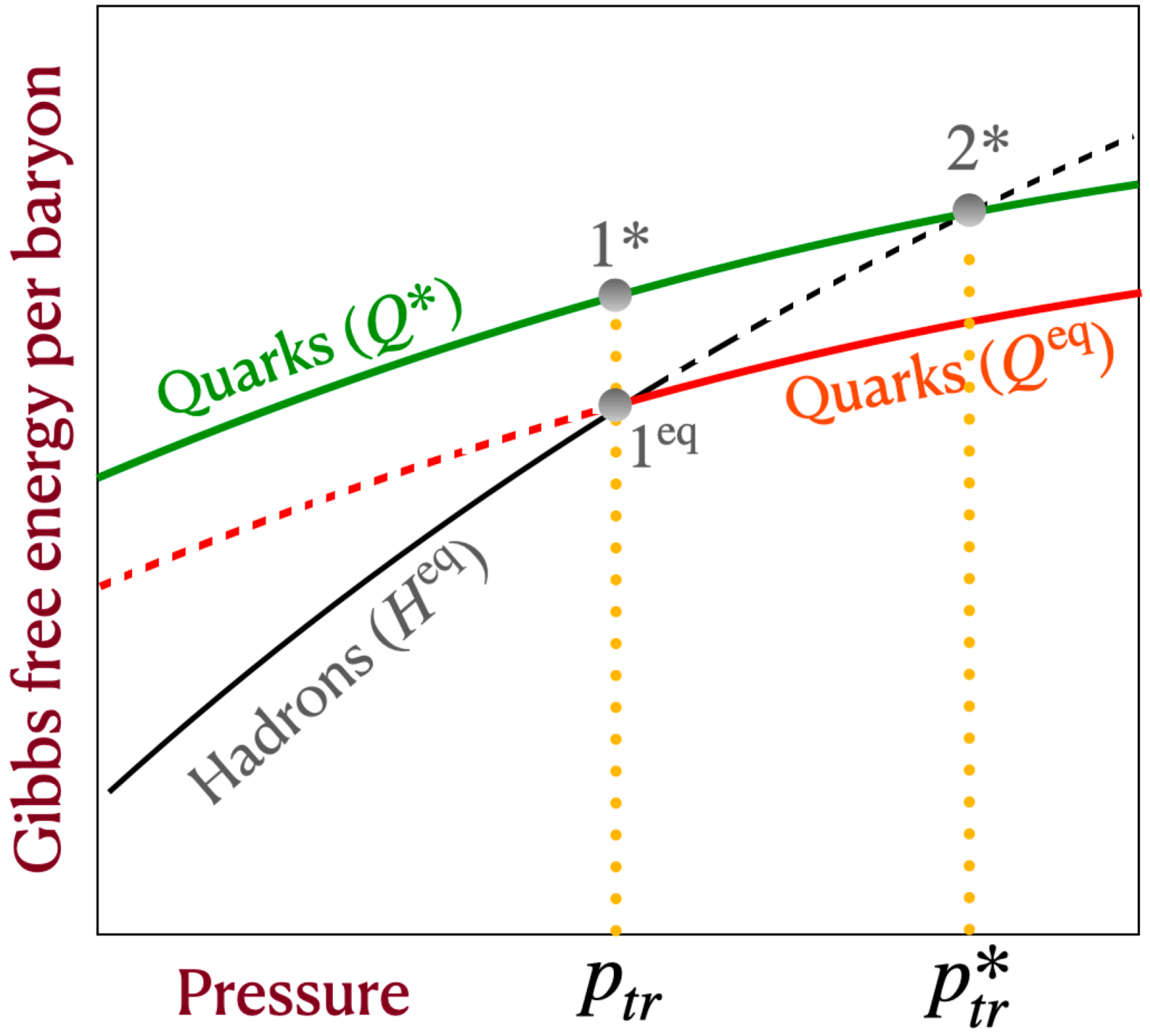
$$\frac{\partial M}{\partial \mathcal{E}_c} > 0 \implies \omega_0^2 \geq 0$$ (stable star)

$$\frac{\partial M}{\partial \mathcal{E}_c} < 0 \implies \omega_0^2 < 0$$ (unstable star)

slow phase transition

$$\frac{\partial M}{\partial \mathcal{E}_c} > 0 \implies \omega_0^2 \geq 0$$ (stable star)

$$\frac{\partial M}{\partial \mathcal{E}_c} < 0 \implies \omega_0^2 > 0$$ (stable star)



G. Lugones et al. *Universe* **2021**, *7*(12), 493

Slow-stable hybrid stars

Germán Lugones *et al* JCAP, 03 (2023) 028

J. P. Pereira, *ApJ.* 860 (2018) 12

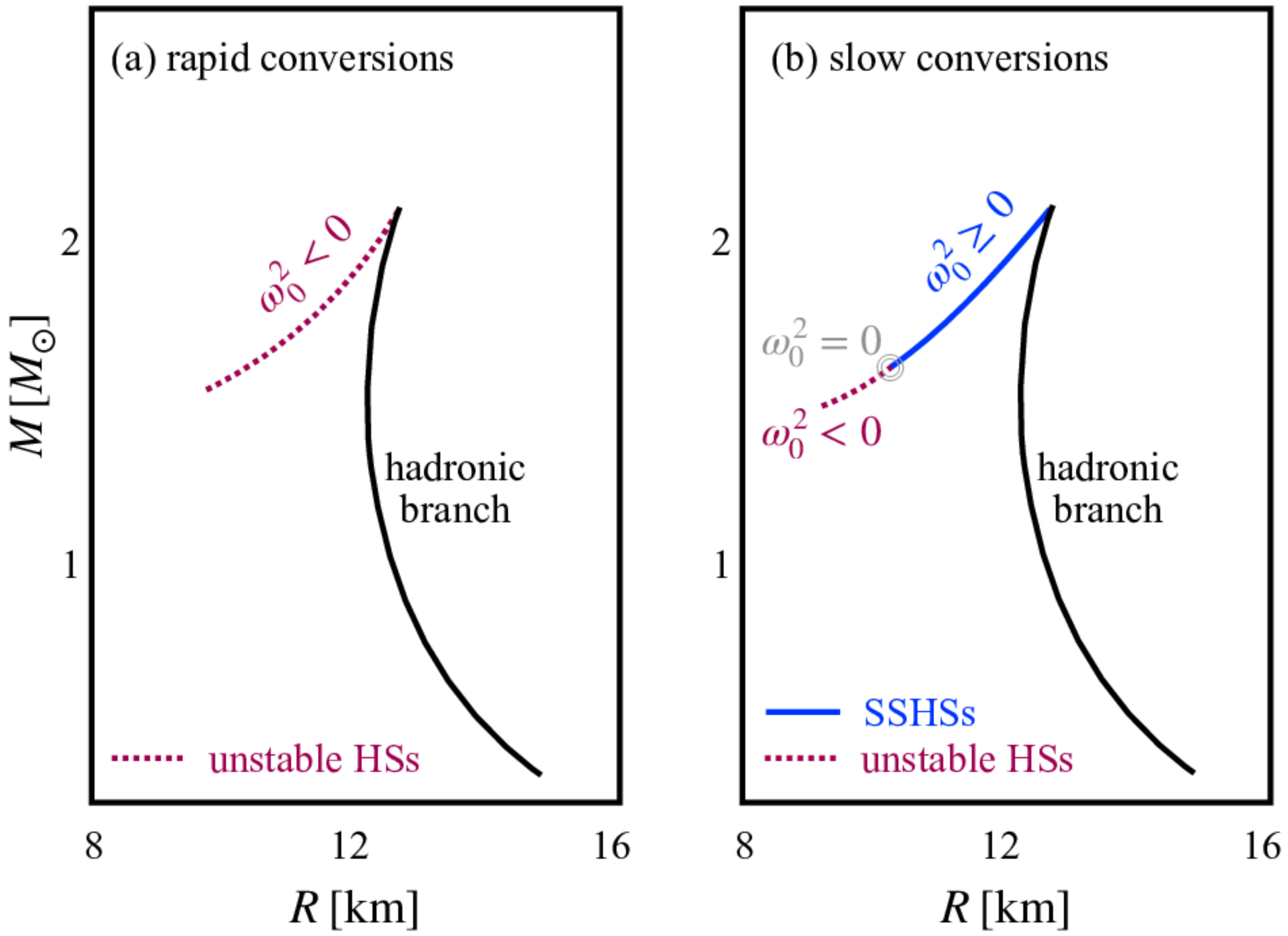
radially unstable configurations are radially stable under small perturbations

Junction conditions:

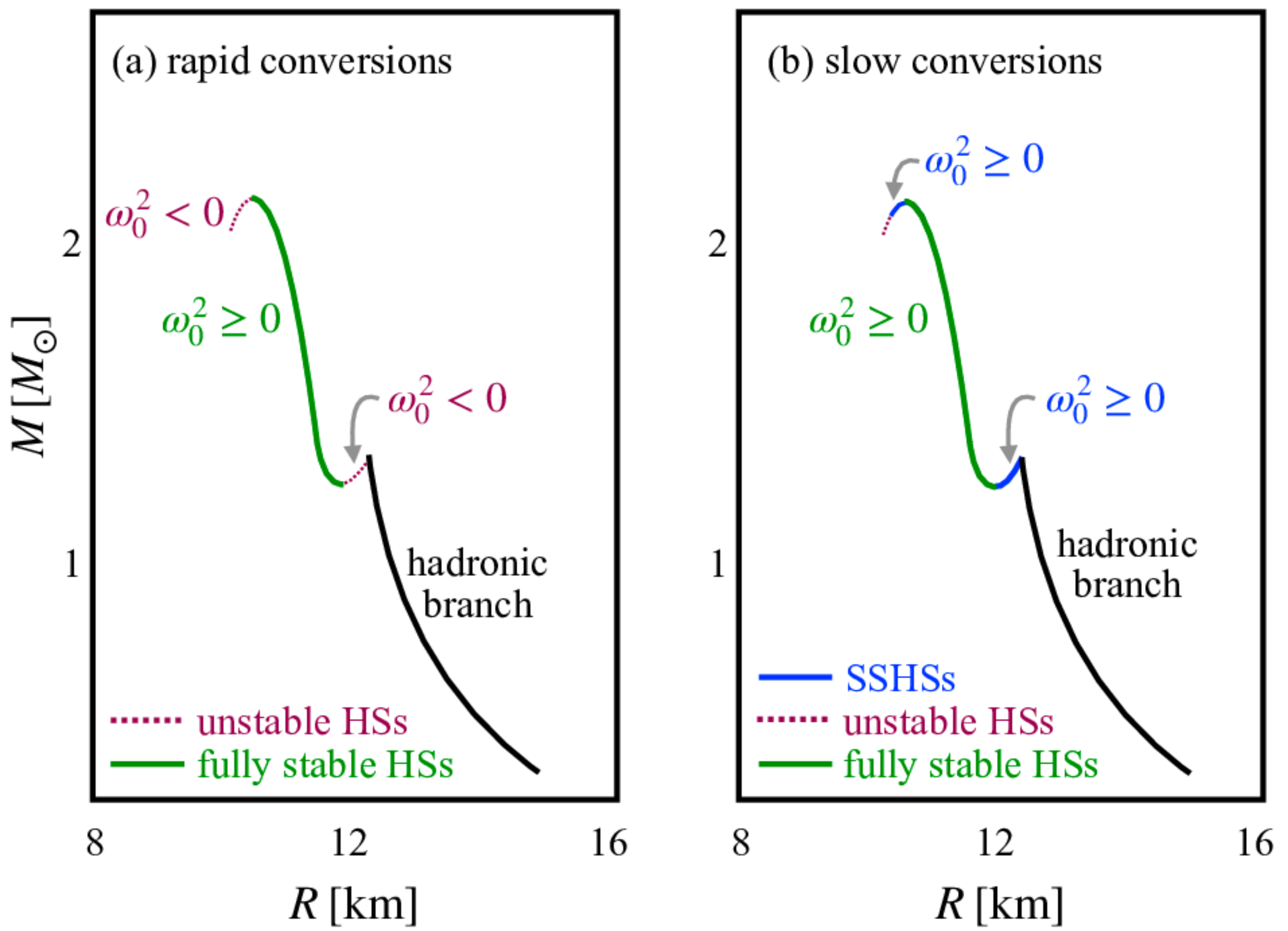
Rapid: $$[\xi]^+_- = \Delta p \left[\frac{1}{p_0'}\right]^+_- \qquad [\Delta p]^+_- = 0$$

Slow: $$[\xi]^+_- = 0 \qquad [\Delta p]^+_- = 0$$

hadron-quark transition at a high density

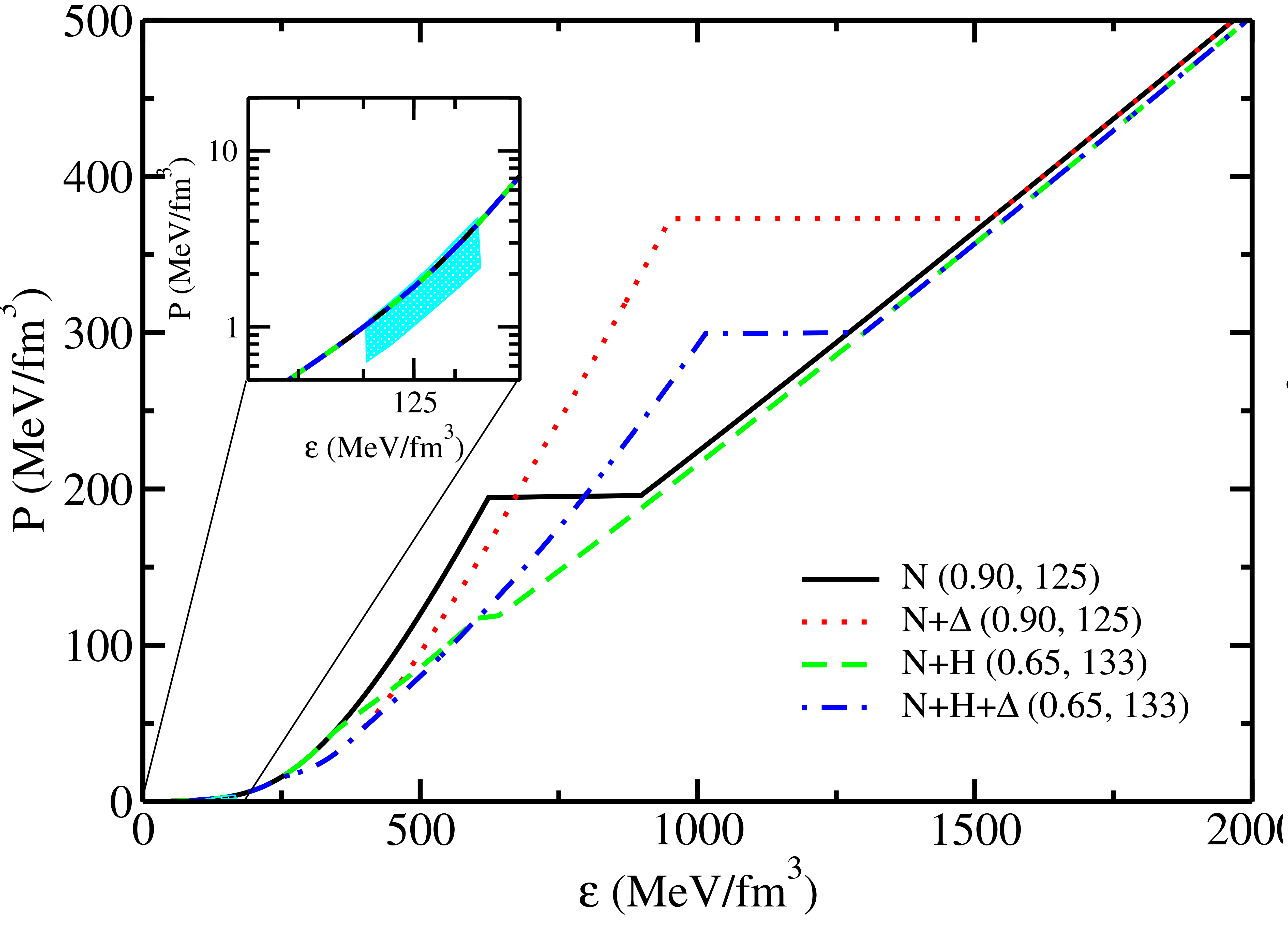


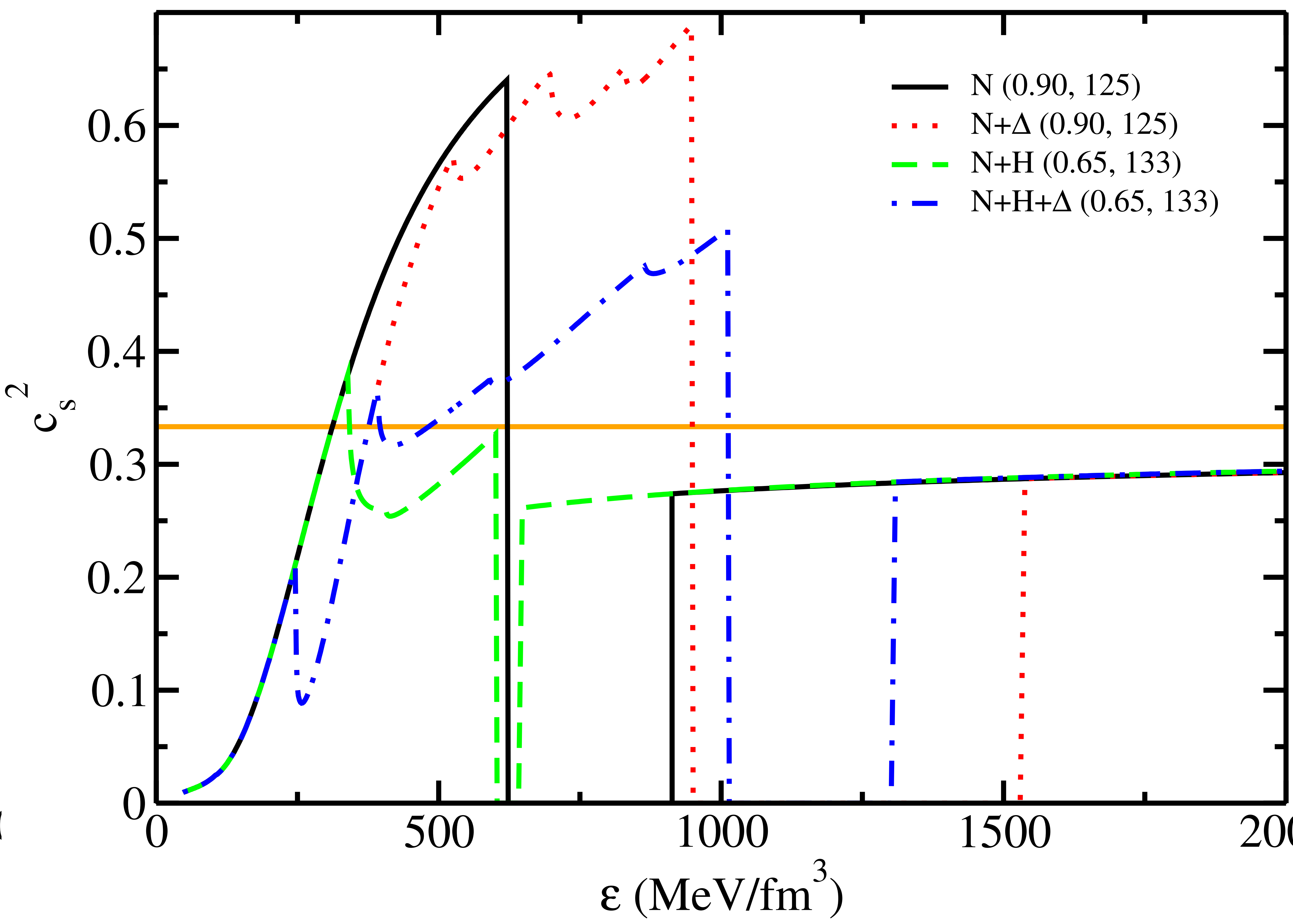
hadron-quark transition at a low density



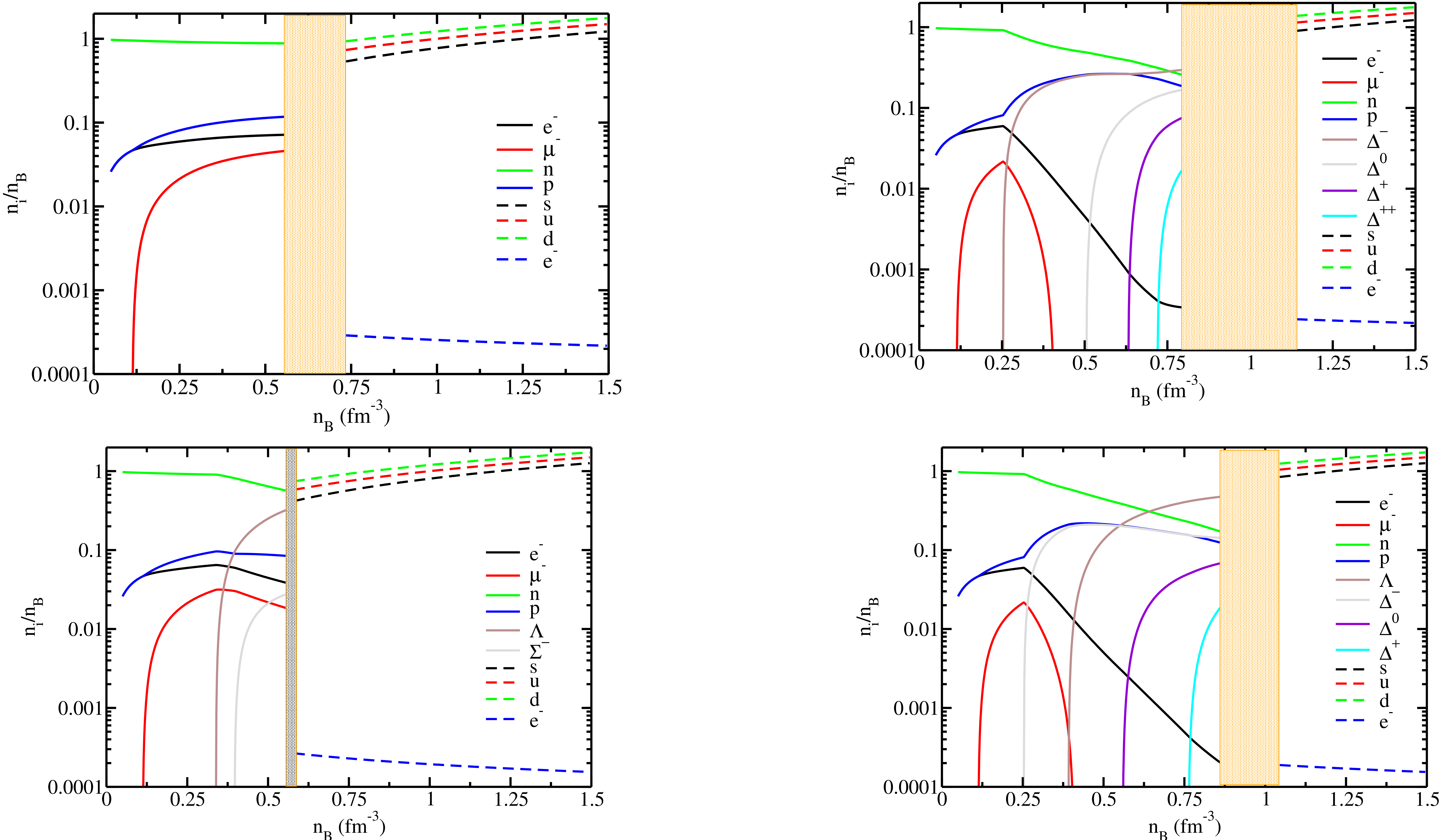
Germán Lugones *et al* JCAP, 03 (2023) 028

Results



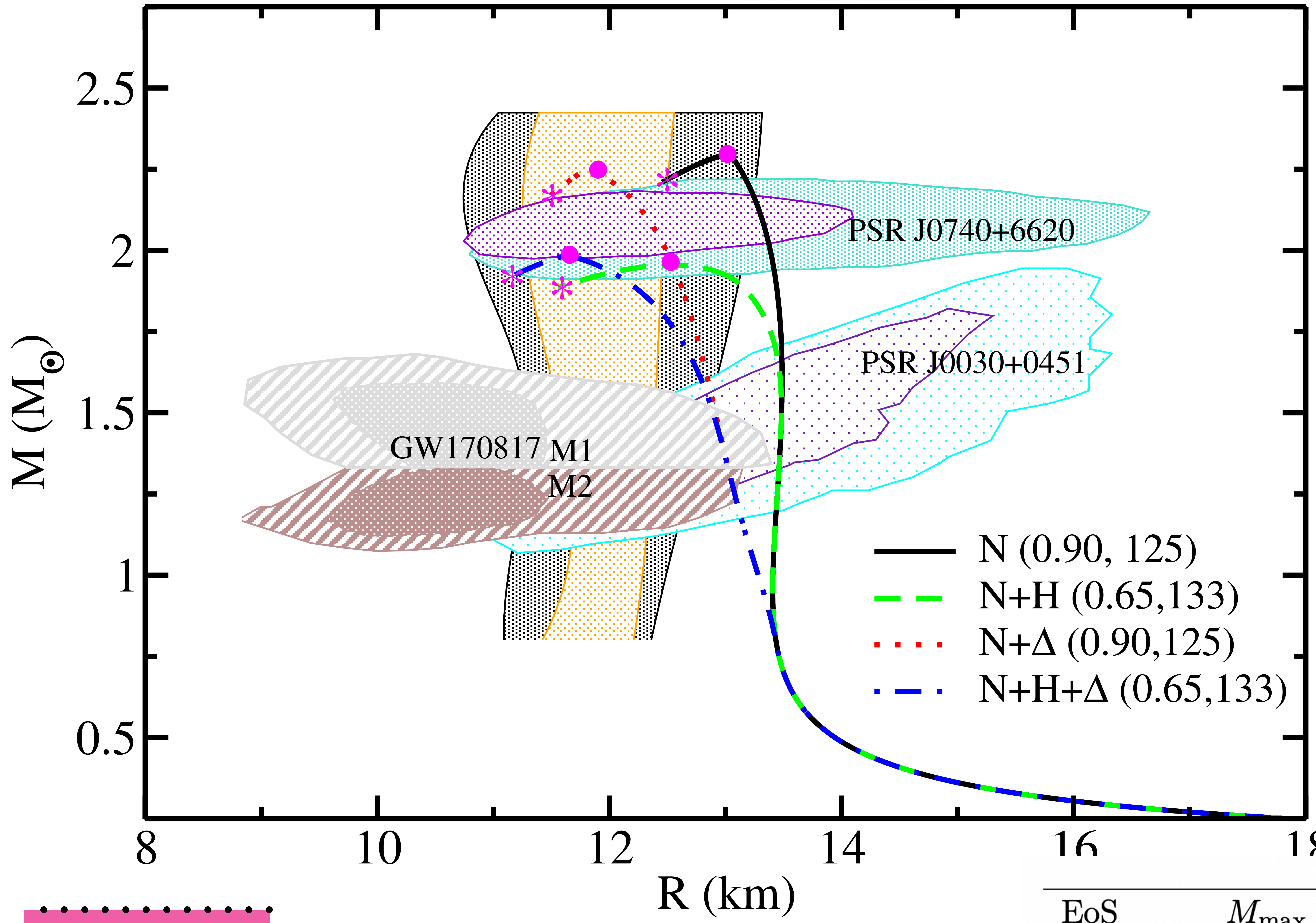


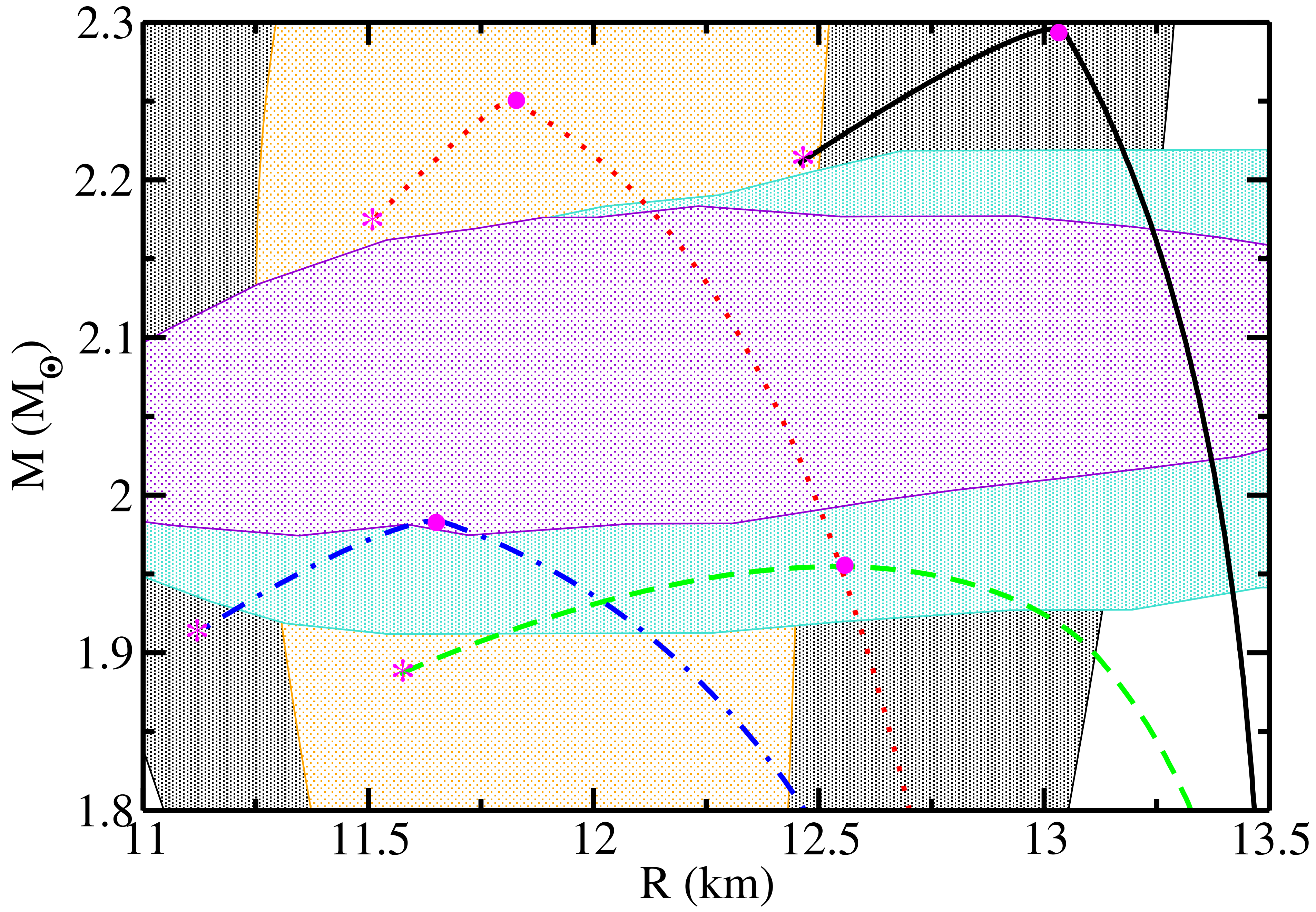
Slow Phase Transiton



n_i/n_B
n_B (fm^{-3})
0.0001
0.001
0.01
0.1
1
0
0.25
0.5
0.75
1
1.25
1.5
e^-
μ^-
n
p
s
u
d
e^-
e^-
μ^-
n
p
Δ^-
Δ^0
Δ^+
Δ^{++}
s
u
d
e^-
e^-
μ^-
n
p
Λ
Σ^-
s
u
d
e^-
e^-
μ^-
n
p
Λ
Δ^-
Δ^0
Δ^+
s
u
d
e^-

MR Profile





N+H EoS → Small Mixed phase

↓

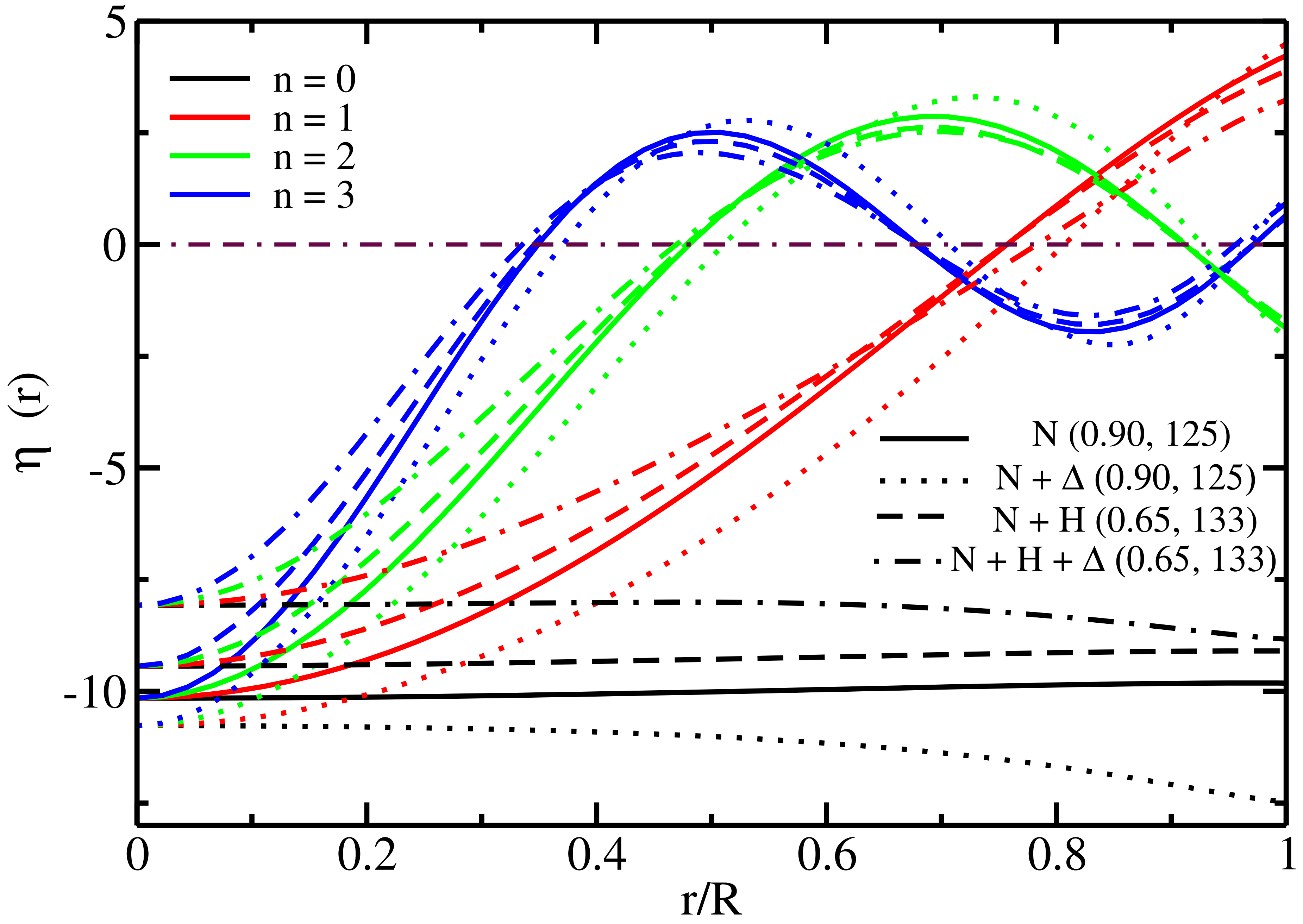
Large SSHS branch

Length of the SSHS branch

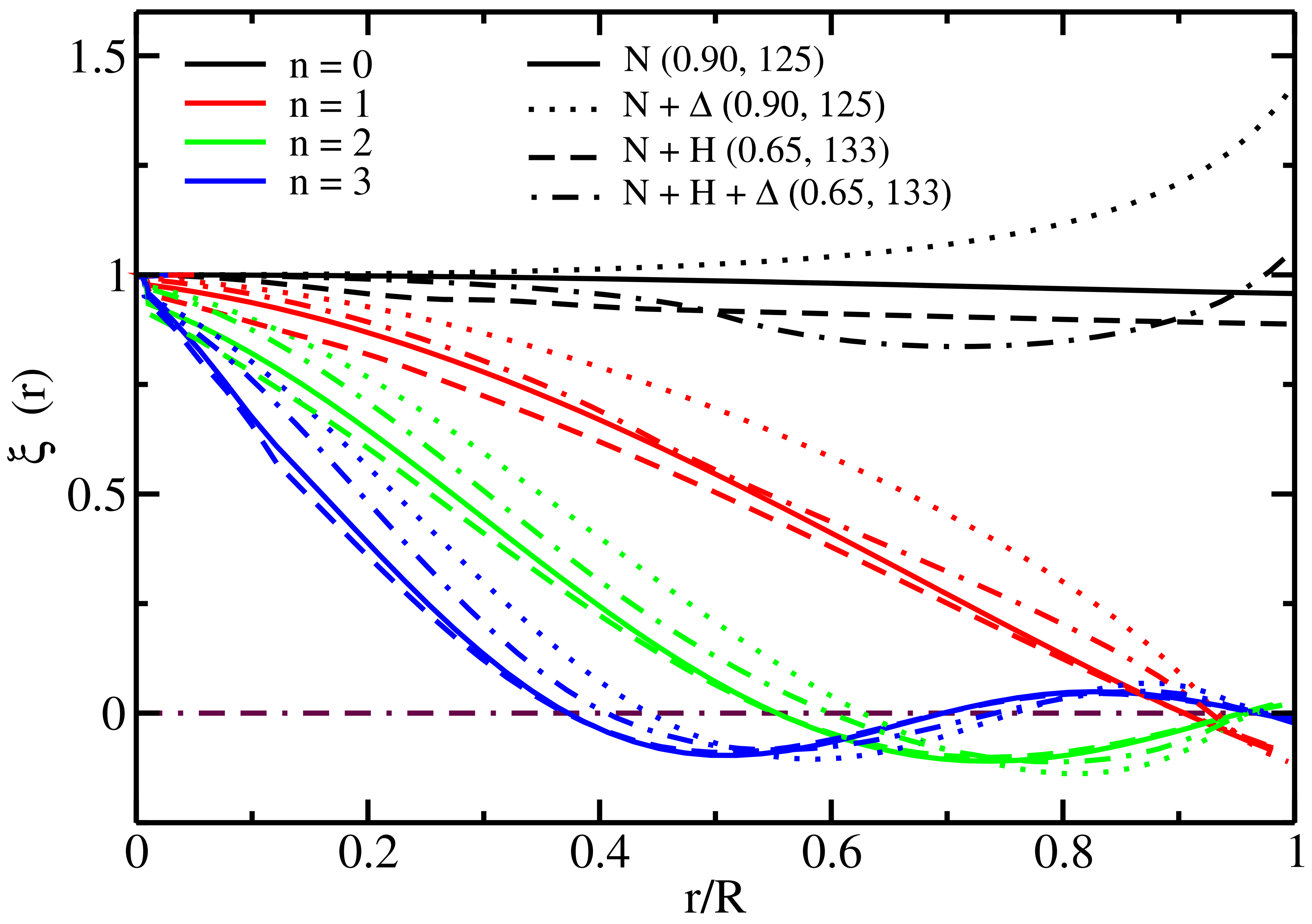
- ∝ 1/(Energy density jump between two phases).
- ∝ Stiffness of the quark EoS.

EoS	$M_{\max}$	$R_{\max}$	$\mathcal{E}_{c,\max}$	M_*	R_*	$\mathcal{E}_{c,*}$	$R_{1.4M_\odot}$	$\Lambda_{1.4M_\odot}$
N	2.30	13.03	917	2.22	12.50	1392	13.47	720.50
N+Δ	2.25	11.80	1540	2.17	11.49	2172	12.98	520.36
N+H	1.97	12.47	984	1.89	11.59	1674	13.47	720.50
N+H+Δ	1.98	11.57	1408	1.92	11.16	1948	12.97	520.36

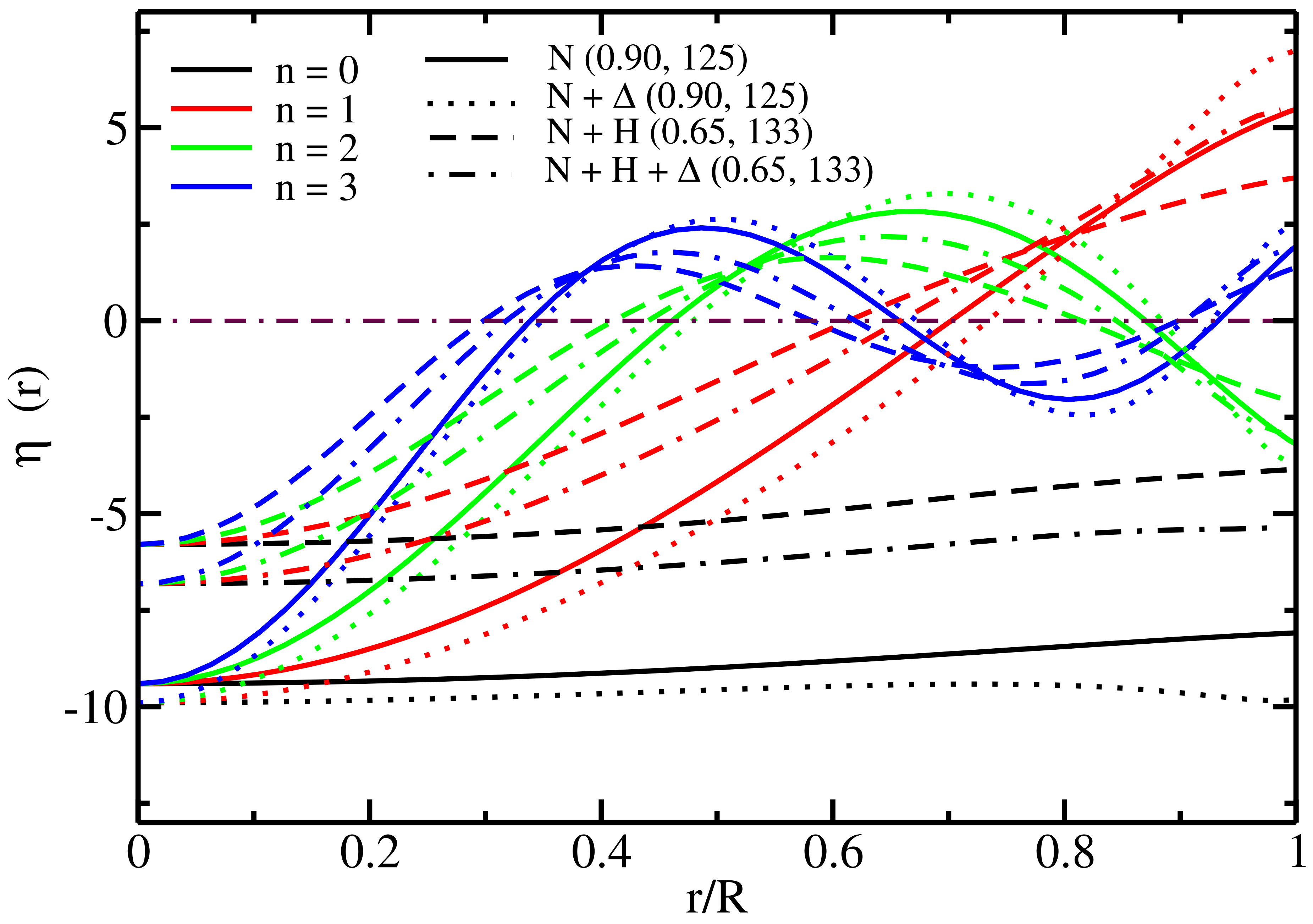
Radial profiles with PT



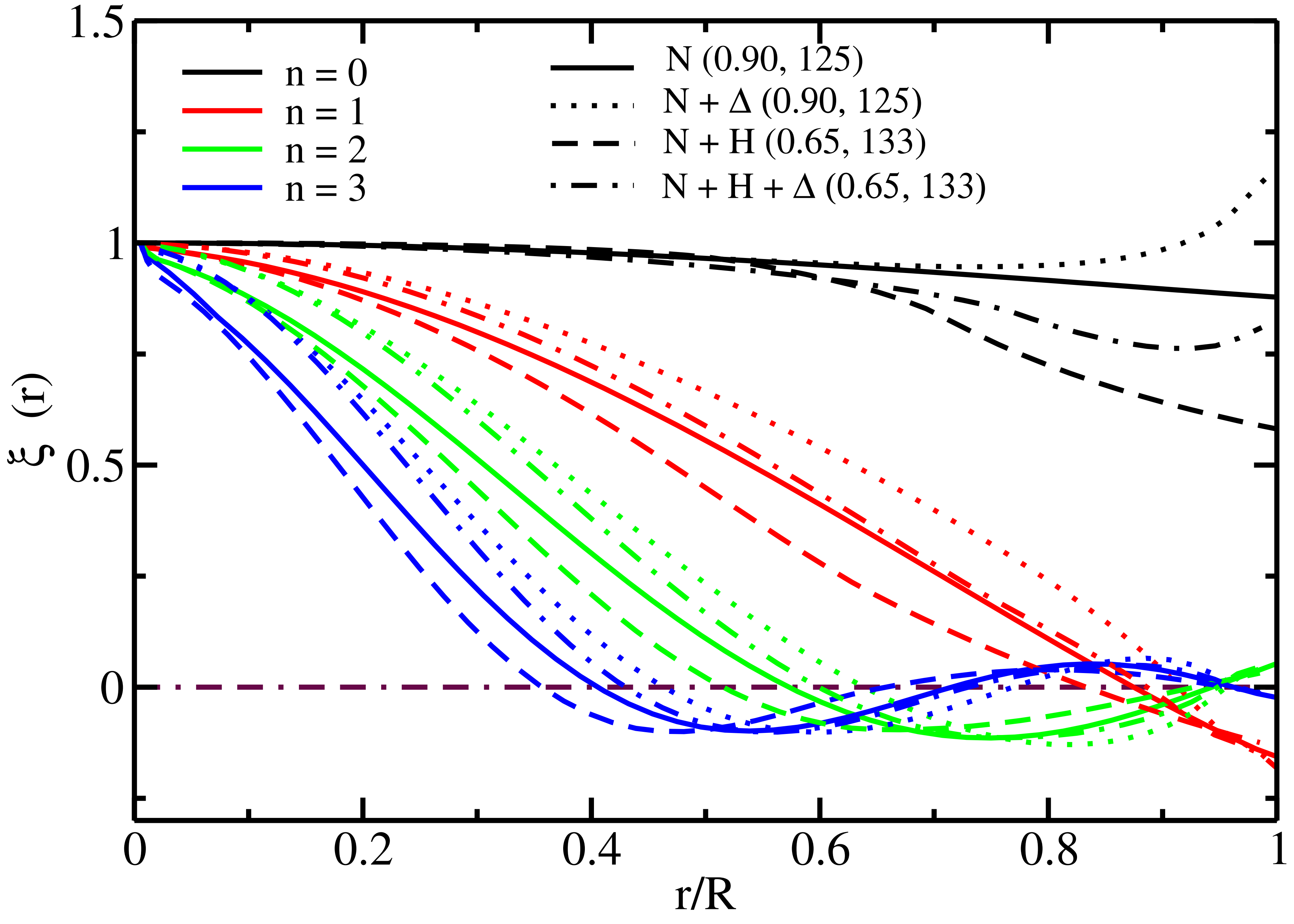
at 1.4 Mo

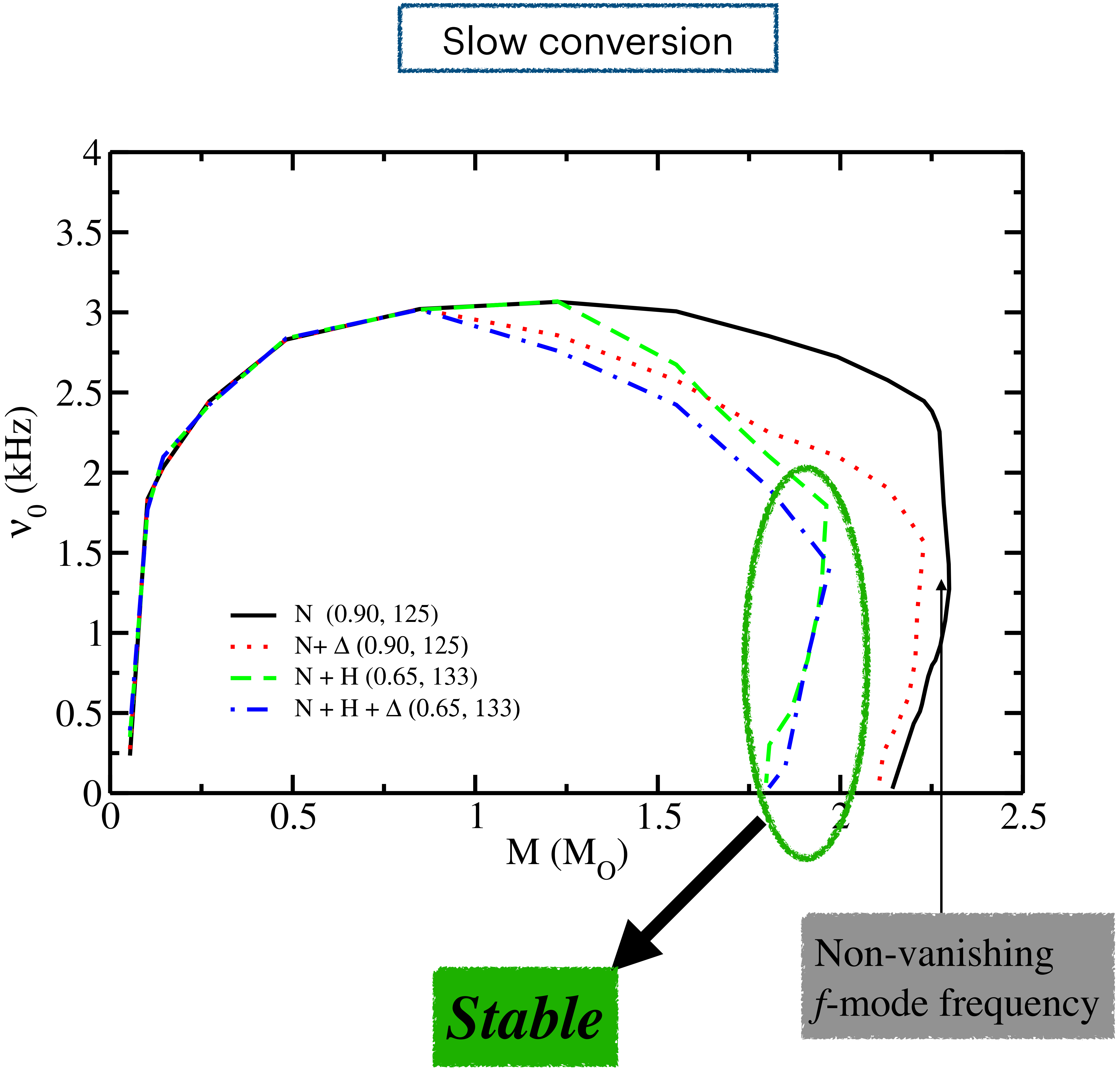


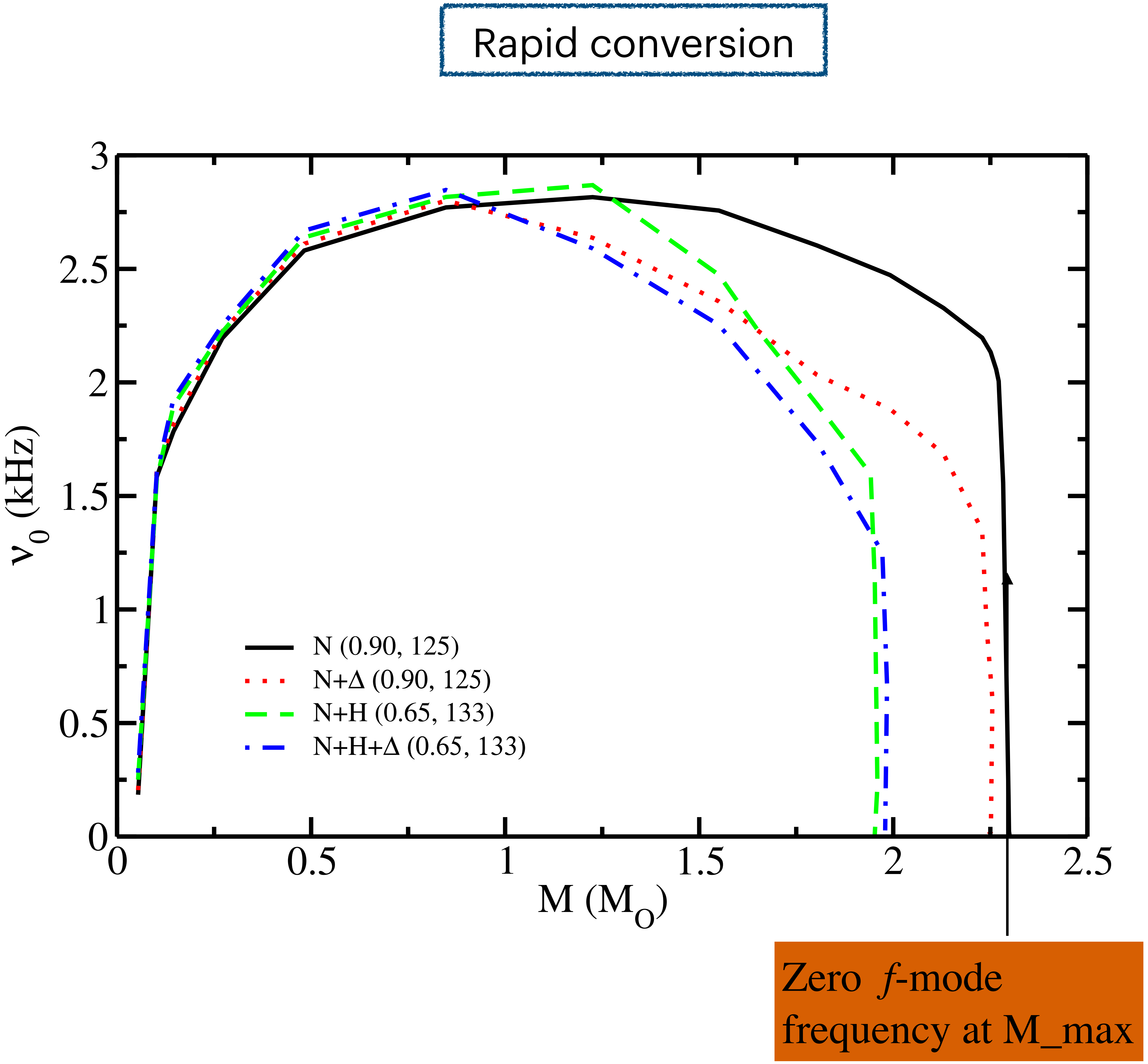
- profiles closely resemble those observed in traditionally stable NSs.
- Reduction in both the amplitude and frequency of the radial modes for SSHS.



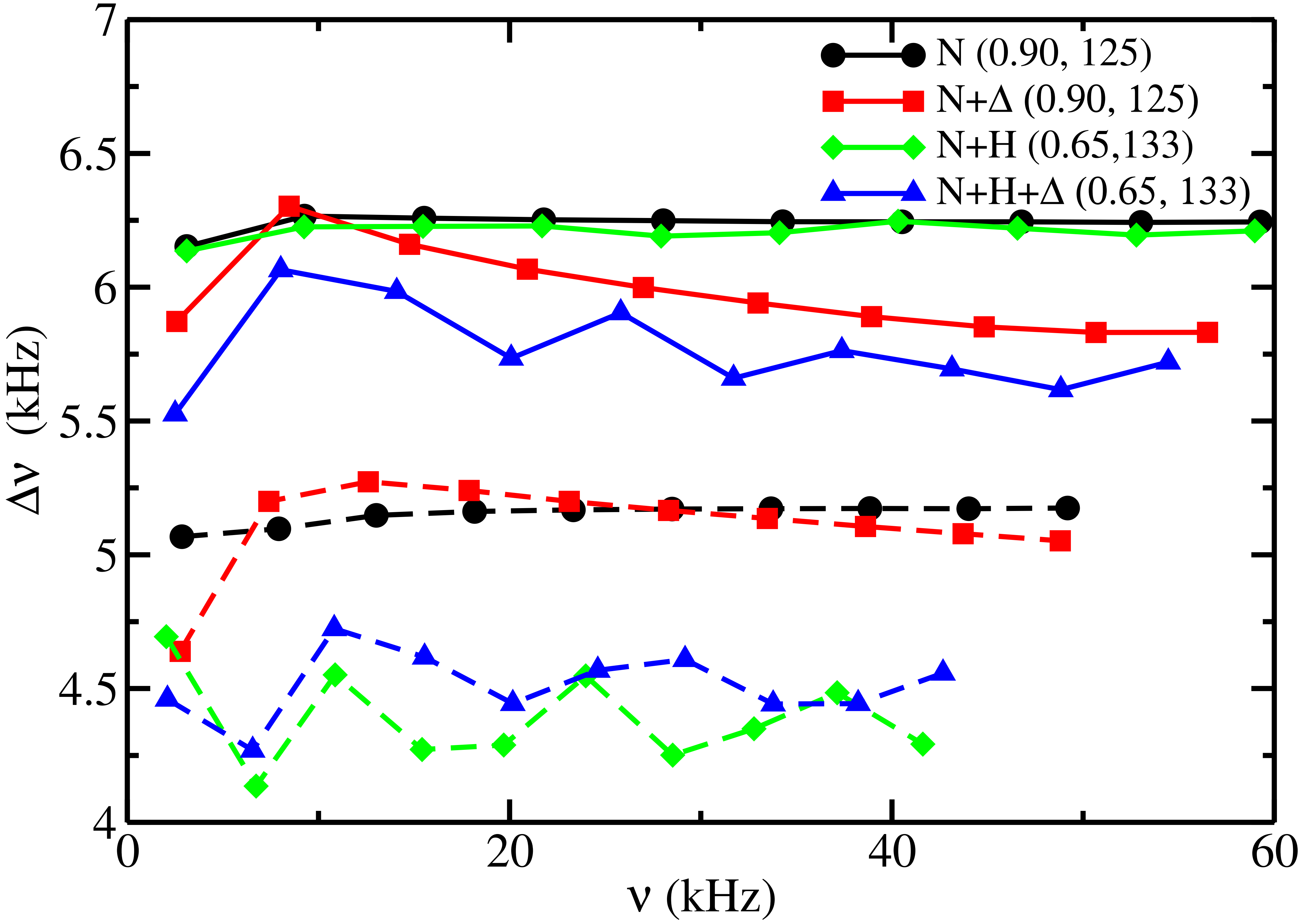
at 1.8 Mo







I. A. Rather *et al.* *JCAP* 05, 130 (2024)



Nodes	EoS			
	N	N+Δ	N+H	N+H+Δ
		$1.4\ M_\odot$		
0	3.109	2.591	3.109	2.504
1	9.262	8.463	9.244	8.030
2	15.527	14.766	15.470	14.095
3	21.786	20.927	21.697	20.080
4	28.038	26.994	27.926	25.814
5	34.287	32.993	34.117	31.719
6	40.532	38.934	40.320	37.379
7	46.776	44.824	46.567	43.141
8	53.021	50.676	52.787	48.836
9	59.263	56.507	58.982	54.453
		$1.8\ M_\odot$		
0	2.858	2.465	2.042	2.097
1	7.926	7.404	6.736	6.557
2	13.023	12.604	10.872	10.827
3	18.170	17.877	15.422	15.551
4	23.332	23.117	19.695	20.168
5	28.500	28.317	23.984	24.612
6	33.671	33.483	28.531	29.180
7	38.843	38.618	32.782	33.789
8	44.016	43.724	37.132	38.231
9	49.188	48.803	41.616	42.675

Summary

- Studied the lowest eigenfrequencies and corresponding oscillation functions of Δ-inclusive nuclear (N +Δ) and hyperonic matter (N + H + Δ).
- Radial oscillations with Δ-baryons and Phase transition to the Quark matter.
- Slow Phase transition & Slow stable hybrid stars.
- Profiles closely resemble those observed in traditionally stable NSs.
- Reduction in both the amplitude and frequency of the radial modes for SSHS.

Further work

- Studying more features of SSHSs:
- quasinormal mode frequencies
- tidal deformability of SSHSs
- g-modes
- twin hybrid stars/ triplet stars for slow phase transitions.
- Empirical relations connecting f-mode frequencies with tidal deformation.

Δ- baryon couplings

Δ BARYONS IN NEUTRON STARS

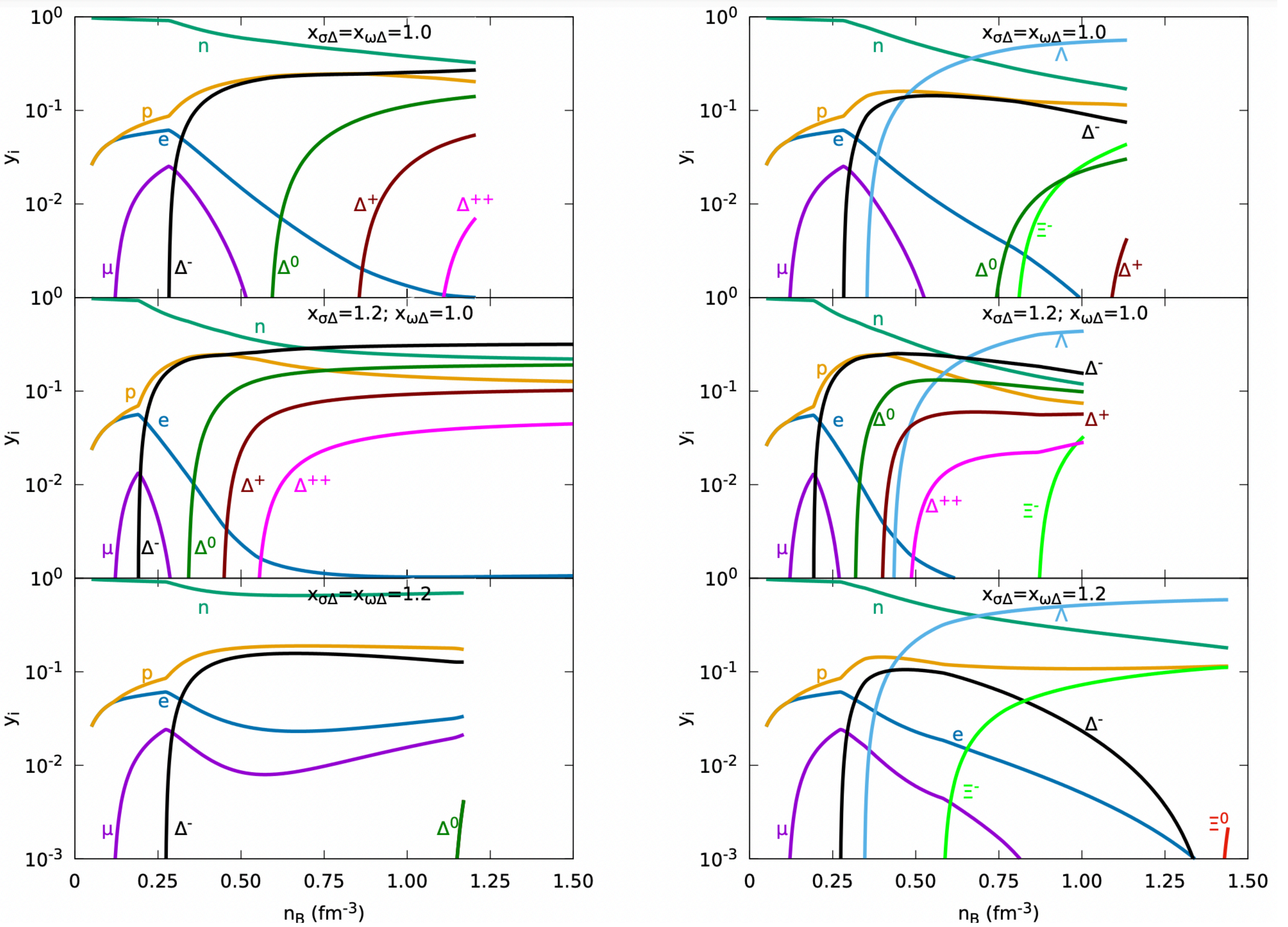
K. Marquez, PRC, 106, 055801 (2022)

- The Δ-meson couplings are obtained by experimental data.
- The hyperon-meson couplings are fitted to hypernuclear properties

The coupling constants were varied freely

$$0 \leq x_{\sigma\Delta} - x_{\omega\Delta} \leq 0.2$$

K. Wehrberger, C. Bedau, and F. Beck, Nucl. Phys. A 504, 797 (1989).



Δ- baryon couplings

$$\mathscr{L}_{Yukawa} = -g(\bar{\psi}_B \psi_B)M$$

↓

IR{8}

Yukawa coupling of the QHD is invariant under the SU(3) flavor symmetry group

↓

obtaining the SU(3) Clebsh-Gordan coefficients to keep the Yukawa-Lagrangian as an unitary singlet

↓

irreducible representation IR{1}

To preserve unitary symmetry,
the direct product $\bar{\psi}_B \otimes \psi_B$ = IR{8} when the meson eigenstate (M) belongs to IR{8}
= IR{1} when M belongs to IR{1}

$$\mathscr{L}_Y = -(gC + gC')(\bar{\psi}_B \psi_B)M$$

→ g(g′) = antisymmetric (symmetric) coupling
C(C′) = SU(3) CG coefficients

$$\mathscr{L}_Y = -g_1(\bar{\psi}_B \psi_B)M$$

$$g_{NN\rho} = -\left(-\sqrt{\frac{3}{20}}g - \sqrt{\frac{1}{12}}g'\right) \times \sqrt{\frac{1}{8}},$$

$$g_{NN\omega_8} = -\left(\sqrt{\frac{1}{20}}g - \sqrt{\frac{1}{4}}g'\right) \times \sqrt{\frac{1}{8}},$$

$$g_{\Lambda\Lambda\rho} = 0,$$

$$g_{\Lambda\Lambda\omega_8} = -\left(-\sqrt{\frac{1}{5}}g\right) \times -\sqrt{\frac{1}{8}},$$

$$g_{\Sigma\Sigma_\rho} = -\left(-\sqrt{\frac{1}{3}}g'\right) \times \sqrt{\frac{1}{8}},$$

$$g_{\Sigma\Sigma\omega_8} = -\left(-\sqrt{\frac{1}{5}}g\right) \times \sqrt{\frac{1}{8}},$$

$$g_{\Xi\Xi\rho} = -\left(-\sqrt{\frac{3}{20}}g + \sqrt{\frac{1}{12}}g'\right) \times -\sqrt{\frac{1}{8}},$$

$$g_{\Xi\Xi\omega_8} = -\left(-\sqrt{\frac{1}{20}}g - \sqrt{\frac{1}{4}}g'\right) \times -\sqrt{\frac{1}{8}},$$

$$g_{NN\phi_1} = g_{\Lambda\Lambda\phi_1} = g_{\Sigma\Sigma\phi_1} = g_{\Xi\Xi\phi_1} = g_1.$$

Fl. Stancu, Group Theory in Subnuclear Physics (Clarendon Press, Oxford, 1996).,
Th. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C 59, 21 (1999).

J. J. Swart, Rev. Mod. Phys. 35, 916 (1963)
P. McNamee and F. Chilton, Rev. Mod. Phys. 36, 1005 (1964)

$$g_8 = \frac{\sqrt{30}}{40} g + \frac{\sqrt{6}}{24} g', \quad \text{and} \quad \alpha_v = \frac{\sqrt{6}}{24} \frac{g'}{g_8},$$

J. J. Swart, Rev. Mod. Phys. 35, 916 (1963).

$$g_{NN\rho} = g_8,$$

$$g_{\Sigma\Sigma\rho} = 2 g_8 \alpha_v,$$

$$g_{\Xi\Xi\rho} = -g_8 (1 - 2\alpha_v),$$

$$g_{\Lambda\Lambda\rho} = 0,$$

$$g_{NN\omega_8} = \frac{1}{3} g_8 \sqrt{3} (4\alpha_v - 1),$$

$$g_{\Lambda\Lambda\omega_8} = -\frac{2}{3} g_8 \sqrt{3} (1 - \alpha_v),$$

$$g_{\Sigma\Sigma\omega_8} = \frac{2}{3} g_8 \sqrt{3} (1 - \alpha_v),$$

$$g_{\Xi\Xi\omega_8} = -\frac{1}{3} g_8 \sqrt{3} (1 + 2\alpha_v).$$

$$g_{NN\omega} = g_1 \cos\theta_v + g_8 \sin\theta_v \frac{1}{3} \sqrt{3} (4\alpha_v - 1),$$

$$g_{\Lambda\Lambda\omega} = g_1 \cos\theta_v - g_8 \sin\theta_v \frac{2}{3} \sqrt{3} (1 - \alpha_v),$$

$$g_{\Sigma\Sigma\omega} = g_1 \cos\theta_v + g_8 \sin\theta_v \frac{2}{3} \sqrt{3} (1 - \alpha_v),$$

$$g_{\Xi\Xi\omega} = g_1 \cos\theta_v - g_8 \sin\theta_v \frac{1}{3} \sqrt{3} (1 + 2\alpha_v).$$

SU(3) flavour symmetry,

Three free parameters: $\alpha_v, g_8/g_1, \theta_v$ → Can be fixed

↓

Yukawa LD is not only invariant under flavour SU(3) symmetry group but also the spin SU(2) symmetry group $\Longrightarrow$ $SU(6) \supset SU(3) \otimes SU(2)$

$$z = \frac{g_8}{g_1} = \frac{1}{\sqrt{6}}, \theta_v = 35.264, \alpha_v = 1.0$$

ϕ meson does not couple to the nucleon ($g_{NN\phi} = 0$);

ω meson couples to the hyperchage

ρ meson couples to the isospin.

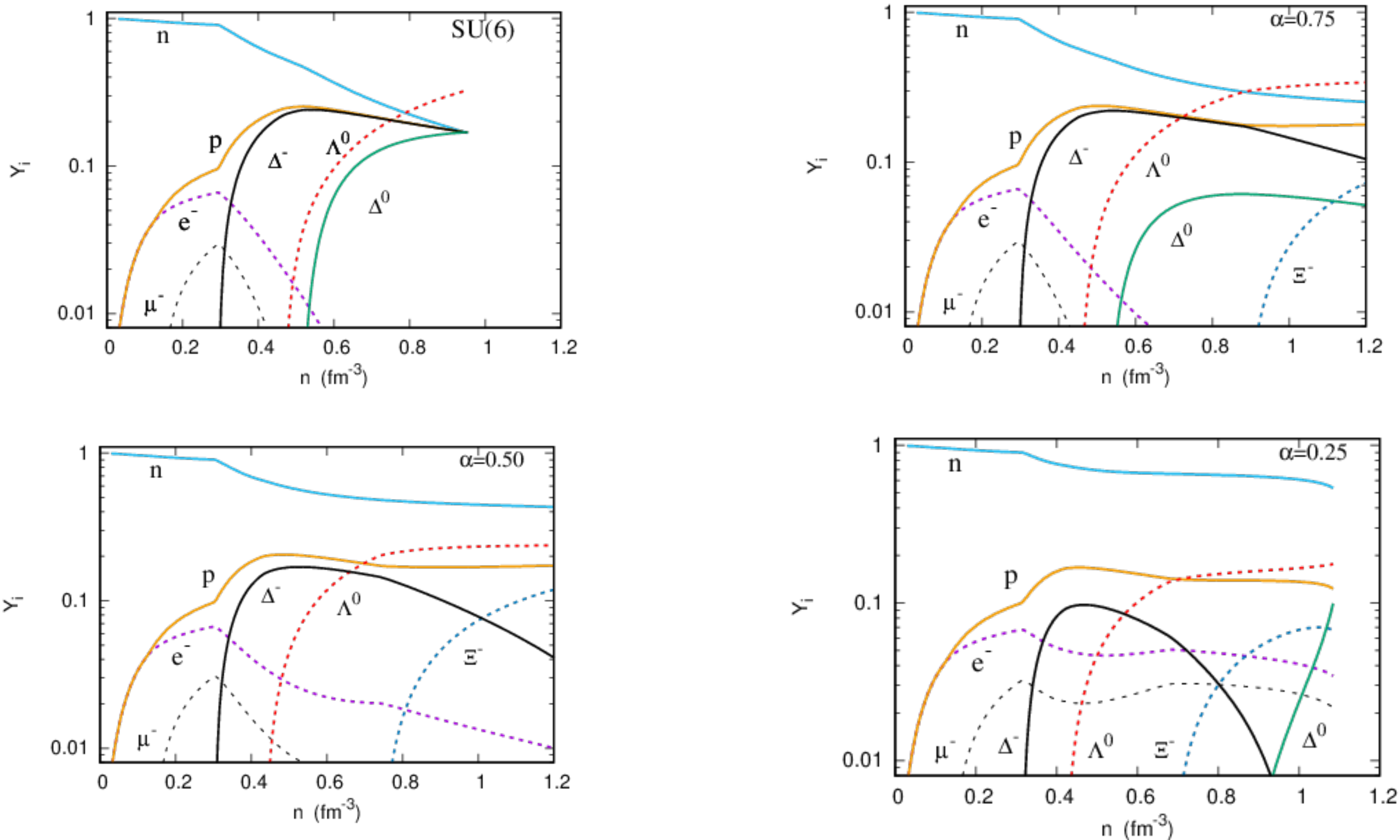
SU(3) flavor symmetry is exact, hybrid SU(6) symmetry can be partially broken

↓

Baryon-meson vector couplings obey relations

Keeping α_v as a free parameter

C. Dover and A. Gal, Prog. Part. Nucl. Phys. 12, 171 (1984)
Fl. Stancu, Group Theory in Subnuclear Physics (Clarendon Press, Oxford, 1996)
Th. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C 59, 21 (1999).
H. Lipkin and S. Meshkov, Phys. Rev. 143, 1269 (1966).
J. J. Swart, Rev. Mod. Phys. 35, 916 (196


SU(6)
α=0.75
α=0.50
α=0.25
n
p
Δ⁻
Λ⁰
Δ⁰
Ξ⁻
e⁻
μ⁻
Y_i
n (fm⁻³)
1
0.1
0.01
0
0.2
0.4
0.6
0.8
1
1.2