Institute for Theoretical Physics

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC-TR 211 Strong-interaction matter under extreme conditions

Dissociation and Regeneration of Charmonia within microscopic Langevin simulations

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Naomi Oei

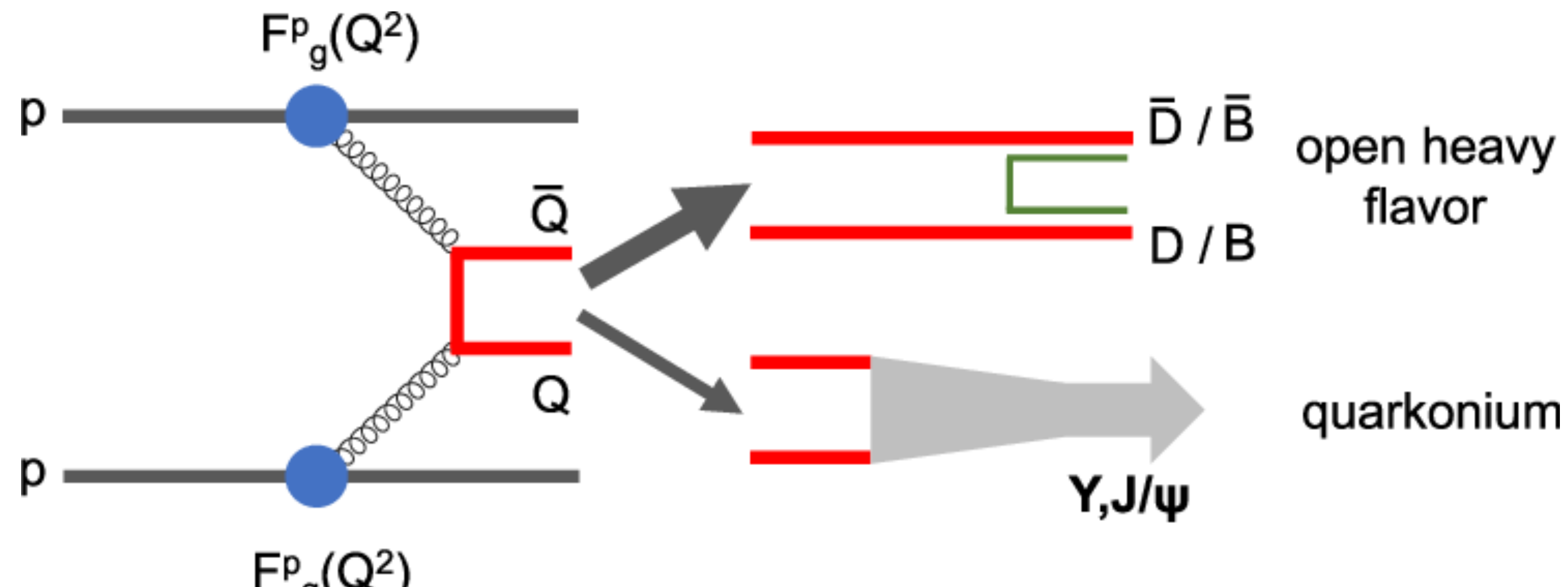
In collaboration with Juan Torres-Rincon, Hendrik van Hees and Carsten Greiner

Motivation

Heavy Quarkonia as Hard Probes

- Production in primordial hard collisions:

  ➡ Sensitivity to initial conditions



Rothkopf (2020). Heavy quarkonium in extreme conditions. Physics Reports. 858. 10.1016

- Measurement of quarkonium states:

  ➡ conclusions about QGP properties, medium interactions

- J/ψ suppression: signal for deconfinement

- Possibility of regeneration processes at higher energies

Fokker-Planck equation

- Relativistic Boltzmann equation for the phase-space distribution of the heavy quarks:

$$\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{p}}{E}\frac{\partial}{\partial x} + \boldsymbol{F}\frac{\partial}{\partial \boldsymbol{p}}\right] f_Q(t,p,x) = C[f_Q]$$

- Assumption: no mean-field effects, uniform medium

➡ Reduction to Fokker-Planck equation:

$$\frac{\partial}{\partial t} f_Q(\boldsymbol{p},t) = \frac{\partial}{\partial p_i}\left\{A_i(\boldsymbol{p}) f_Q(\boldsymbol{p},t) + \frac{\partial}{\partial p_j}[B_{ij}(\boldsymbol{p}) f_Q(\boldsymbol{p},t)]\right\}$$

- With drag and diffusion coefficients $A_i(\boldsymbol{p},T) = A(\boldsymbol{p},T)p_i$ and $B_{ij}(\boldsymbol{p},T) = B_0(\boldsymbol{p},T)\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) + B_1(\boldsymbol{p},T)\frac{p_i p_j}{p^2}$,

  ➡ Approximation: $A(\boldsymbol{p},T) \equiv \gamma(T)$, $B_0(\boldsymbol{p},T) = B_1(\boldsymbol{p},T) \equiv D(\boldsymbol{p},T)$

  ➡ Connected via fluctuation-dissipation relation: $D[E(p)] = \gamma E(p) T$

Langevin simulations

- Fokker-Planck equation realized with Langevin simulations
- Relativistic Langevin equation:

$$\frac{dp}{dt} = -\gamma p + \xi$$

- Corresponding update steps for coordinate and momentum in time interval dt:

$$dx_j = \frac{p_j}{E} dt$$

$$dp_j = -\gamma p_j dt + \sqrt{2\gamma E T dt}\,\rho_j \text{ , (for } V = 0\text{)}$$

- ρ: Gaussian-distributed white noise

Potential of the Heavy Quarks

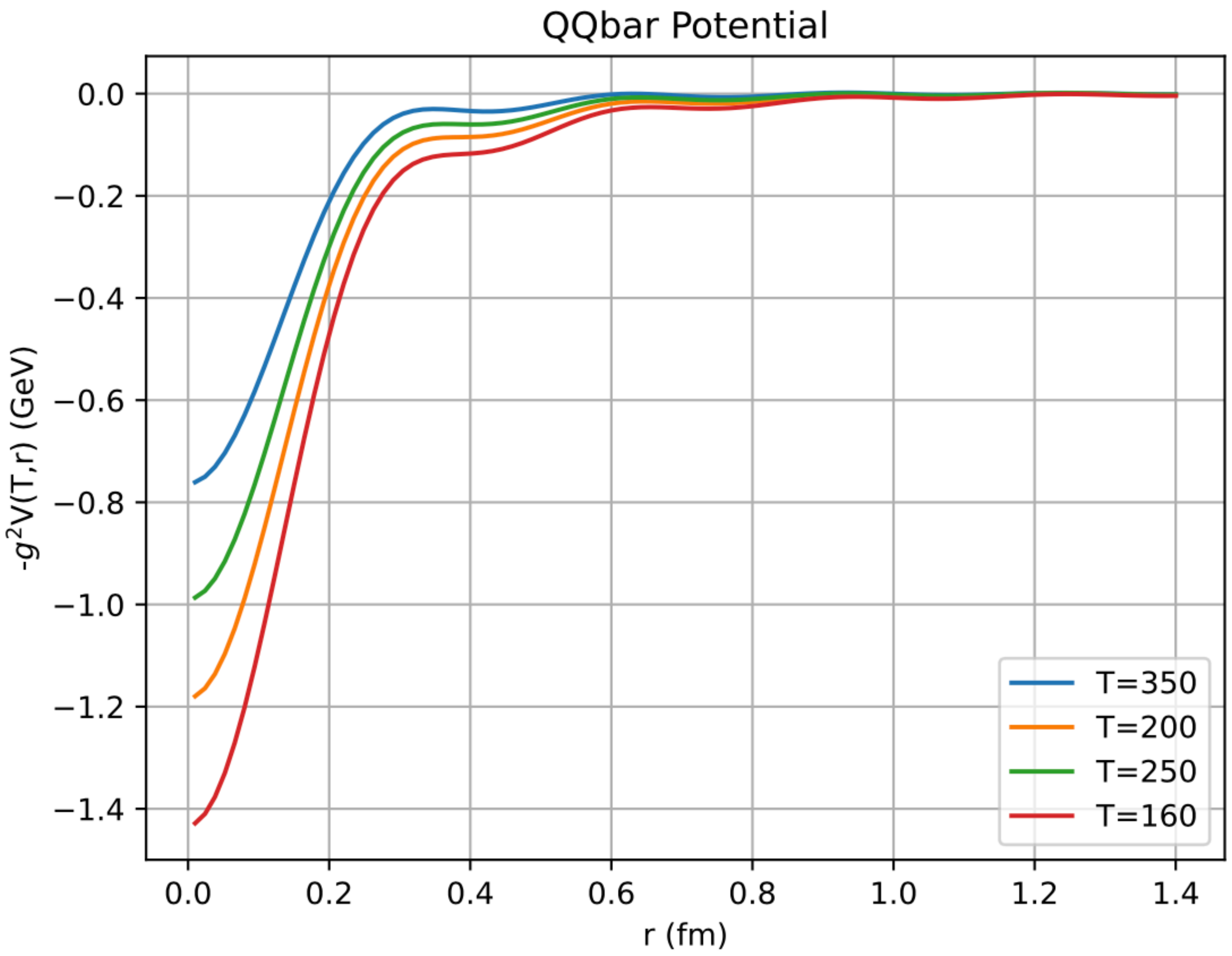
- Formalism to describe heavy quarks in Abelian plasma by Blaizot et al.
  Blaizot et al., Nucl.Phys.*A* 946 (2016) 49-88

- **Idea**: effective theory of non-relativistic HQs in plasma of relativistic particles

- Influence functional in infinite-mass limit and large time limit: interpretation as complex potential:

$$\mathscr{V}(r) = -\frac{g^2}{4\pi}m_D - \frac{g^2}{4\pi}\frac{\exp(-m_D r)}{r} - i\frac{g^2 T}{4\pi}\phi(m_D r)\ ,\ r = |\boldsymbol{r} - \bar{\boldsymbol{r}}|$$

Potential of the Heavy Quarks



- **Real part:** Screened Coulomb potential with Cut-Off $4\ GeV$ and running coupling:

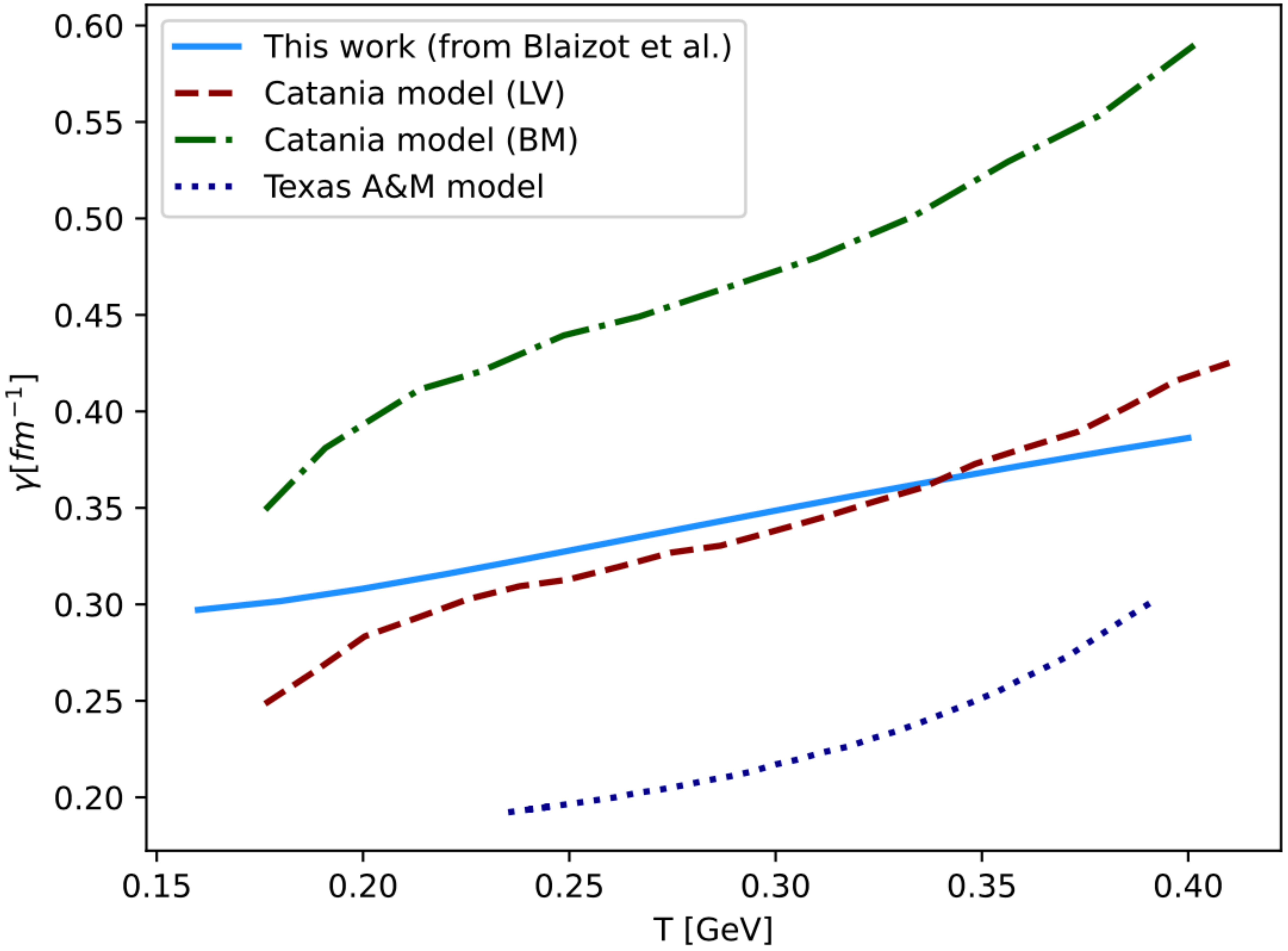
$$g^2 = 4\pi\alpha_s = \frac{4\pi\alpha_s(T_c)}{1 + C\ \ln(T/T_c)},$$

$m_c = 1.8\ GeV/c^2, T_c = 160\ MeV,$

$\alpha_s(T_c) = 0.7,\ C = 0.76$

➡ Less deeply bound states with increasing temperature

Drag Coefficient



- Drag coefficient:

Langevin:

$$\frac{dp}{dt} = -\gamma p + \xi - \nabla_j V(r)dt$$

$$dp_j = -\gamma p_j dt + \sqrt{2\gamma ETdt}\rho_j - \nabla_j V(r)dt$$

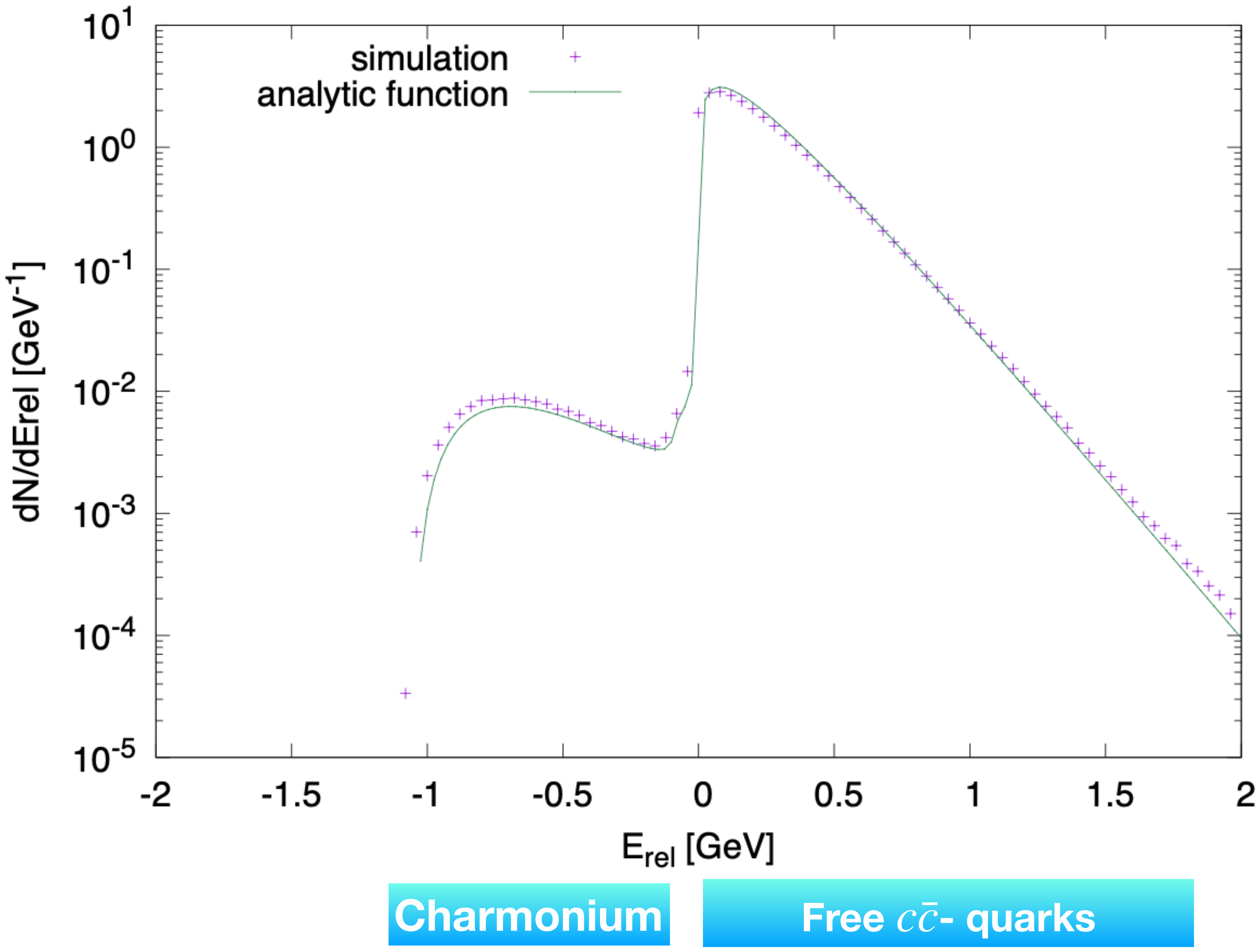
$$\gamma = \frac{m_D^2}{24\pi M}\left[\ln\left(1 + \frac{\Lambda^2}{m_D^2}\right) - \frac{\Lambda^2/m_D^2}{1+\Lambda^2/m_D^2}\right]$$

Blaizot et al., Nucl.Phys.*A* 946 (2016) 49-88

$$m_D = \sqrt{\frac{4}{3}g^2T^2}$$

Bound State Formation in Box Simulation

Energy distribution in equilibrium



- bound state if energy of charm-anticharm pair < 0
- Classical density of states:

$$\frac{dN}{dE_{rel}} = (4\pi)^2(2\mu)^{\frac{3}{2}}C\int_0^R drr^2\sqrt{E_{rel} - V(r)}\,\exp\left(-\frac{E_{rel}}{T}\right)$$

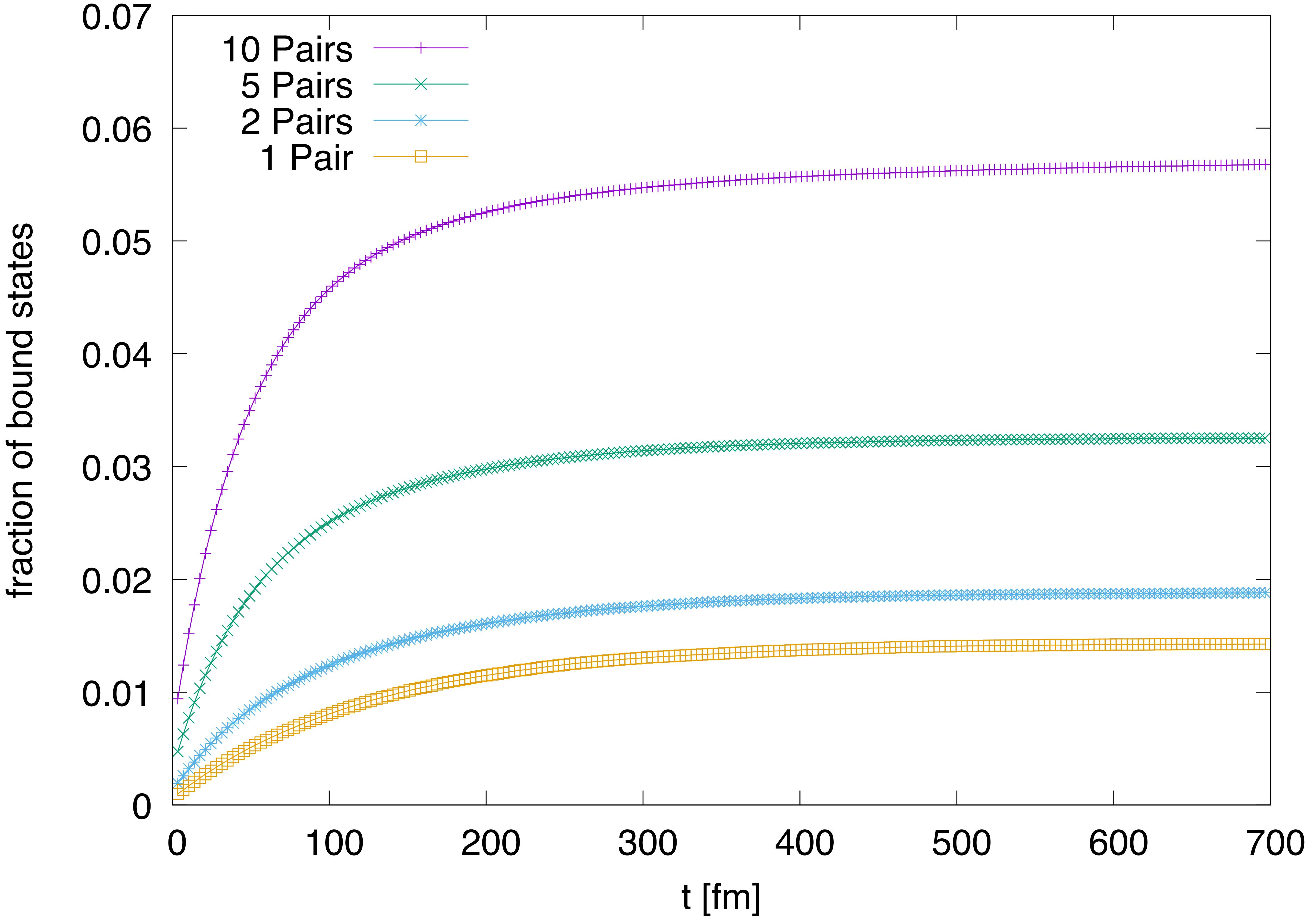
- Box simulation with 1 $c\bar{c}$-pair at $T = 160\ MeV$

➡ leads to right equilibrium density of states

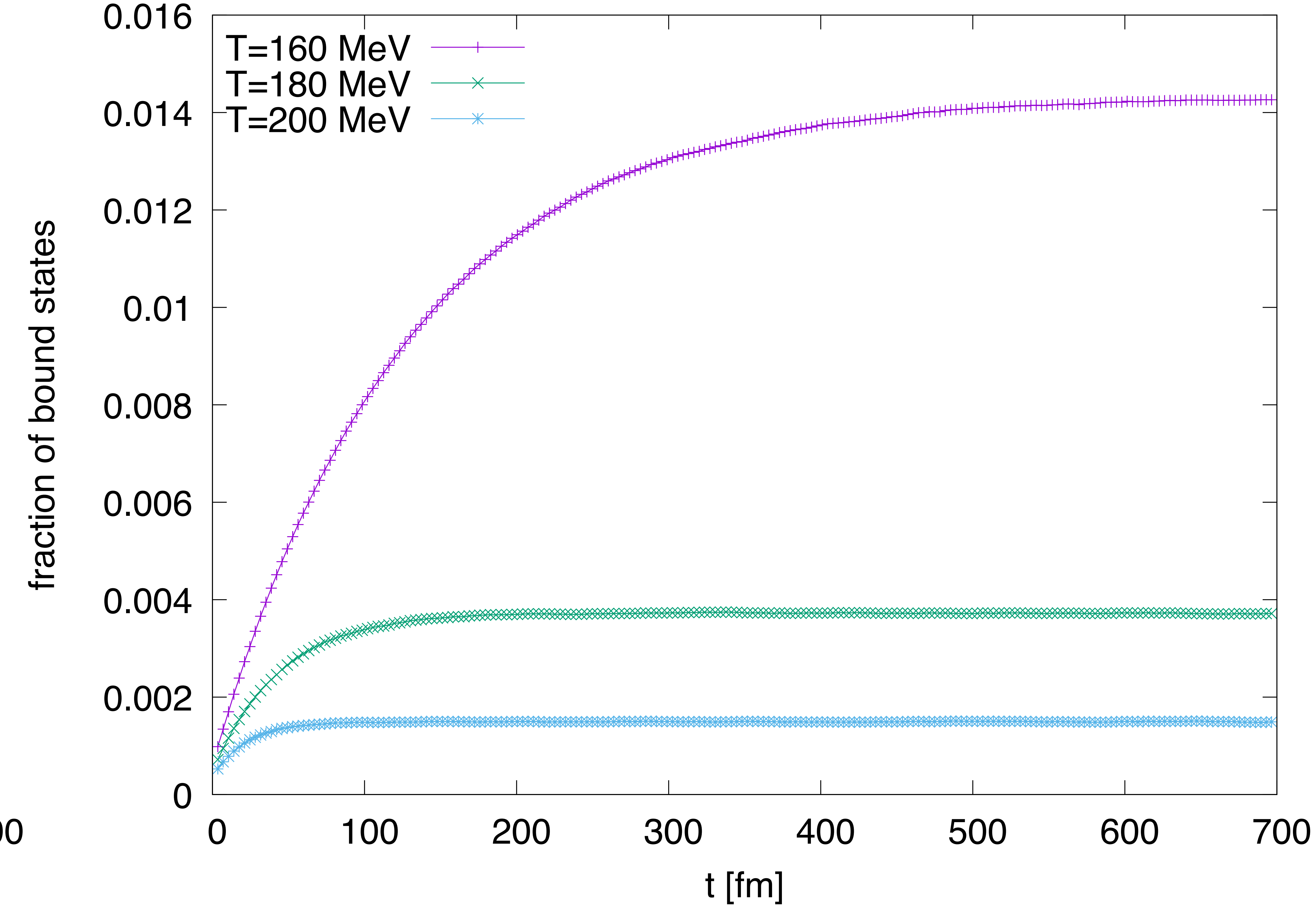
Bound State Formation in Box Simulation

Time evolution of fraction of bound states

Different initial N_{pairs}, $T = 160\ MeV$



Different Temperatures, $N_{pairs} = 1$



Comparison to Statistical Hadronization Model

Charmonium yield at $T = 160\ MeV$, Sidelength of Box: $10\ fm$

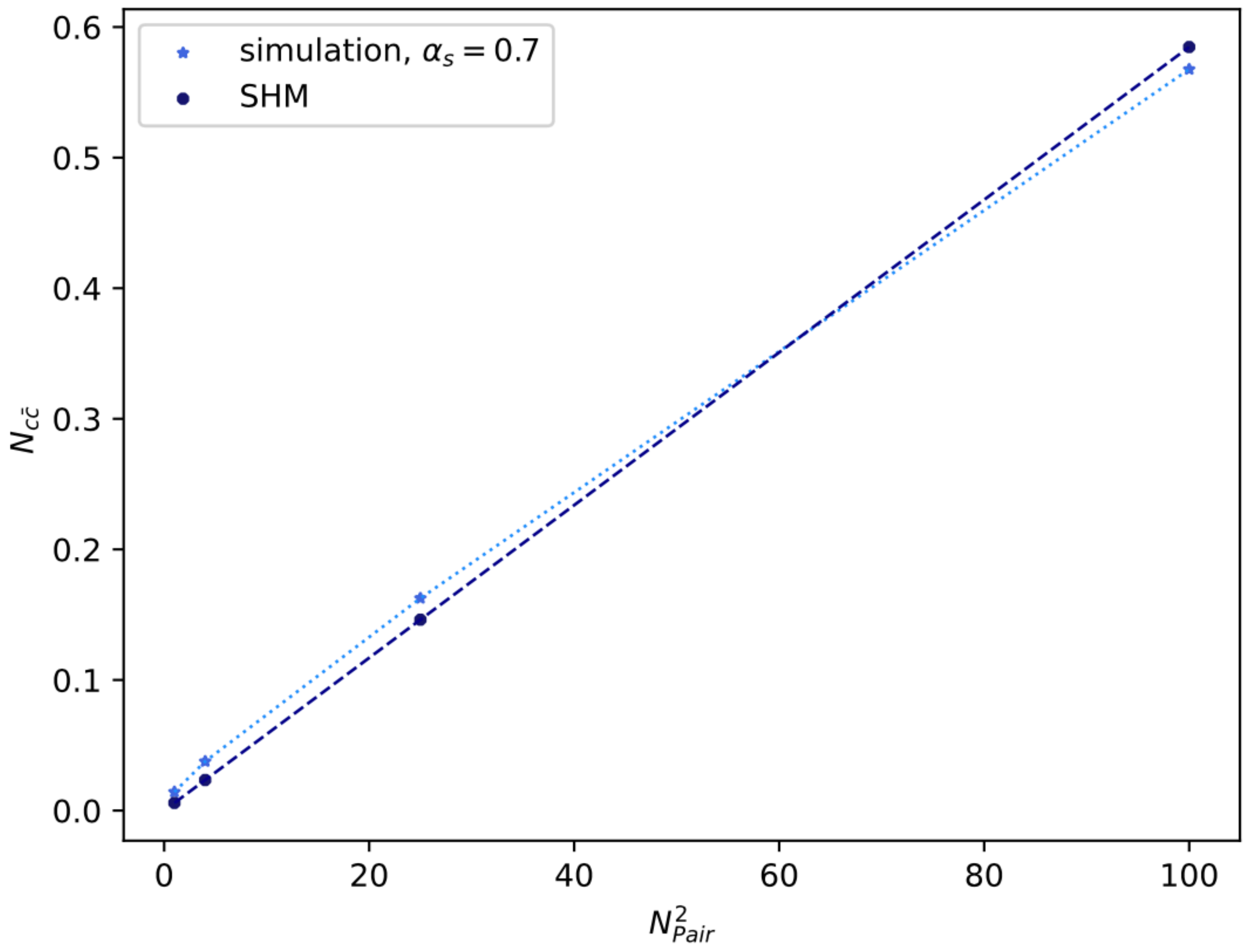
- Charmonium multiplicity according to particle number in Grand-Canonical Ensemble:

$$N_{c\bar{c}} = \sum_i \lambda_i\, d_i \frac{V}{2\pi^{3/2}} \left(m_i T\right)^{3/2} e^{-m_i T}$$

with $i = \{\eta_c, J/\psi, \psi', \chi_c\}, \lambda_i = \lambda_c^2$

➡ Charmonium yield scales with N_c^2

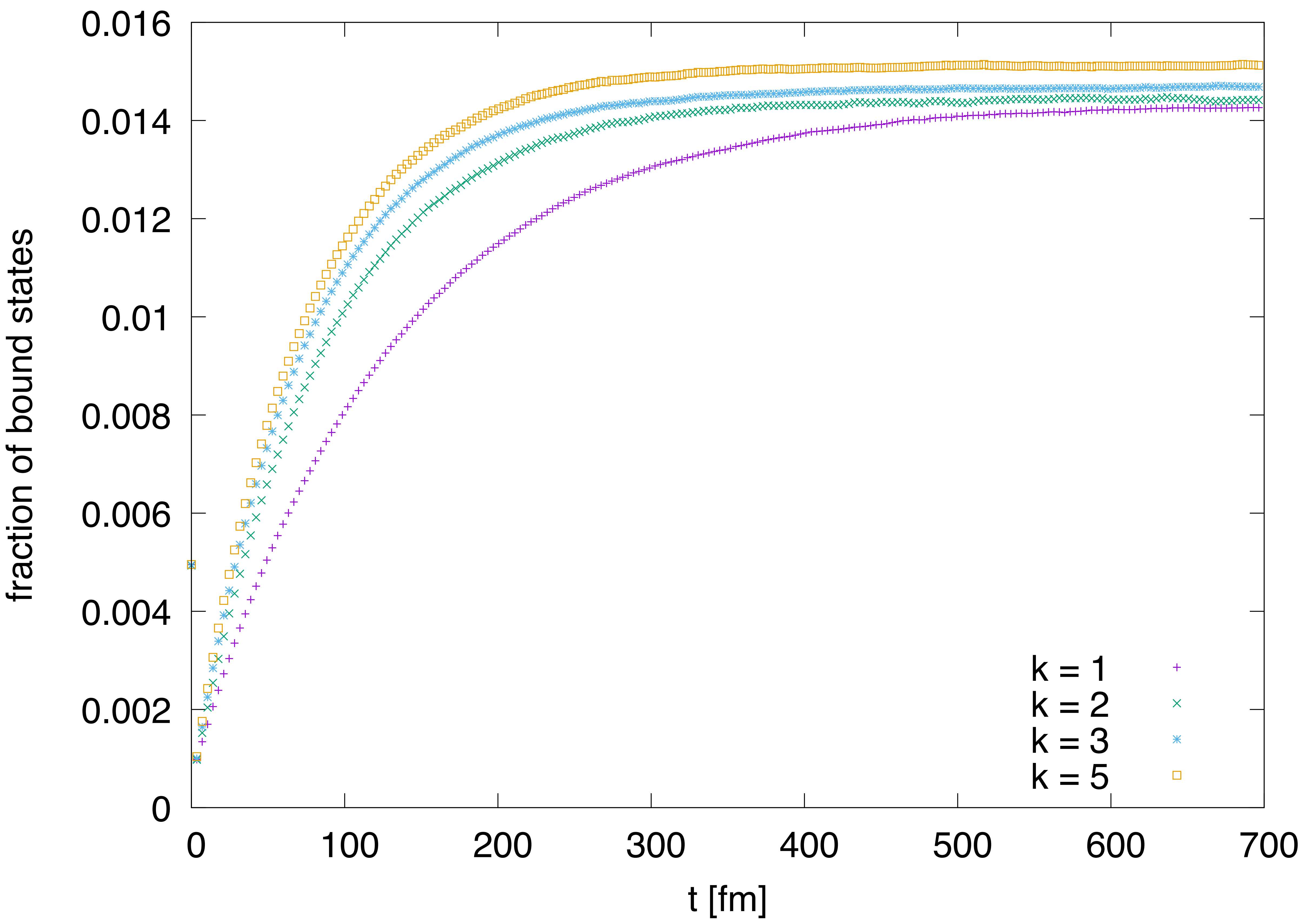
Scaling of charmonium-yield with number of pairs in the system



→ **choose** $\alpha_s(T_c) = 0.7$

Relaxation time

Equilibration for different scalings of drag coefficient γ ($T = 160\ MeV$)



- Faster equilibration for stronger drag force
- 2 intertwined mechanisms:
  1. Charm momentum relaxation towards thermal value: $\tau_{eq} = 1/\gamma$
  2. Full equilibration = time-independent number of bound states

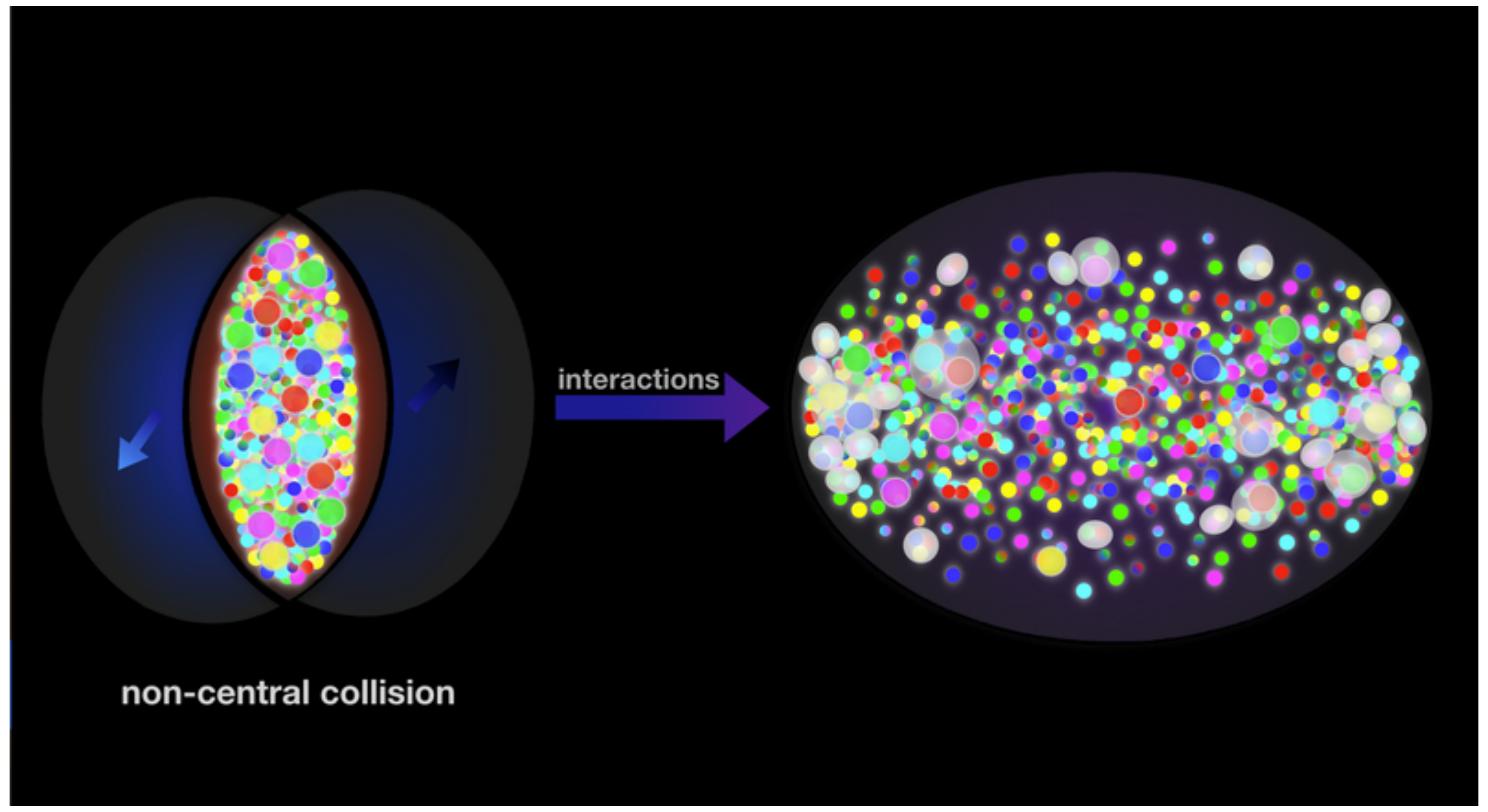
     ➡dominated by the time scales of the potential

Elliptic Fireball

- Elliptic parametrisation of transverse direction:

H. van Hees, M. He, and R. Rapp, Nuclear Physics A, (2015) Vol. 933

$$\frac{x^2}{b^2(\tau)} + \frac{y^2}{a^2(\tau)} \leq 1$$



https://irfu.cea.fr/dap/en/Phocea/Vie_des_labos/Ast/ast.php?t=fait_marquant&id_ast=4733

- Volume of medium in Fireball:

$$V(\tau) = \pi \cdot a(\tau) b(\tau) (z_0 + c\tau)\ , \quad z_0 = c\tau_0$$

with long and short semi-axes $a(\tau)$, $b(\tau)$

- semi-axes:

$$a(\tau) = a_0 + \frac{1}{a_a}\left(\sqrt{1 + a_a^2\tau^2} - 1\right),\ b(\tau) = b_0 + \frac{1}{a_b}\left(\sqrt{1 + a_b^2\tau^2} - 1\right)$$

- a_a, a_b : accelerations chosen to fit to p_T-spectra and elliptic flow of light hadrons

Elliptic Fireball

- Transverse velocity field at midrapidity with confocal elliptical coordinates:

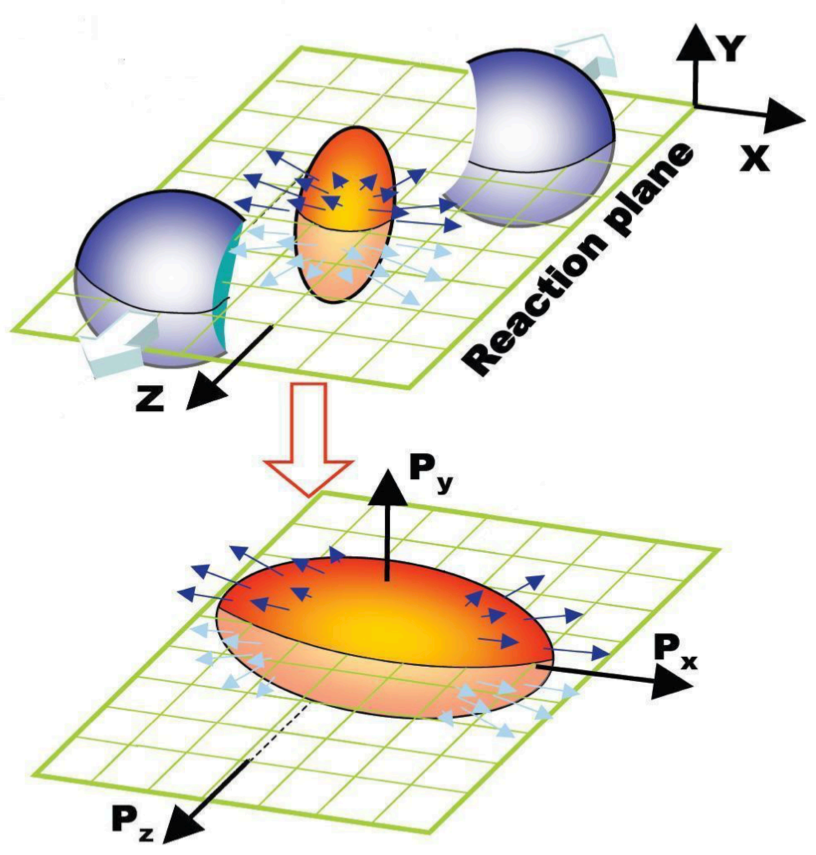
$$\boldsymbol{v}_\perp = \frac{r}{r_B}\begin{pmatrix} v_b(\tau)\cos(\nu) \\ v_a(\tau)\sin(\nu) \end{pmatrix}$$

- Extension to 3D and finite rapidity: Superimpose model with boost-invariant Bjorken flow

➡ Resulting 3D-flow field:

$$v_x = \frac{\tau}{t} v_b(\tau)\cos(\nu)\frac{r}{r_B}\,, \quad v_y = \frac{\tau}{t} v_a(\tau)\sin(\nu)\frac{r}{r_B}, \quad v_z = \tanh(\eta)$$

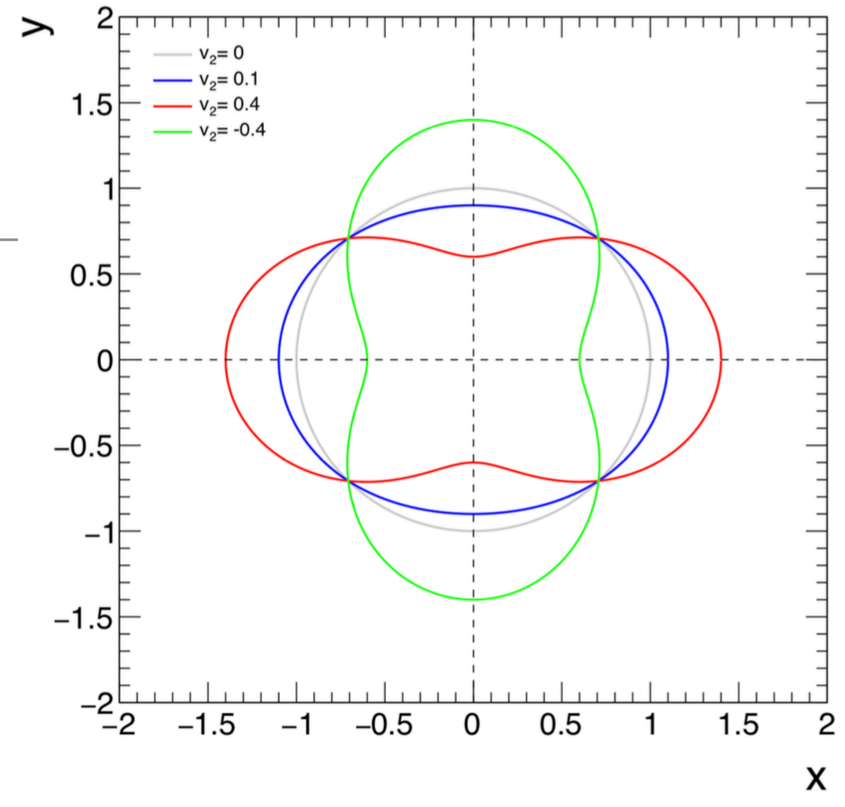
- Initial momentum distribution of heavy quarks in the fireball from PYTHIA
- Initial spatial distribution according to Glauber model



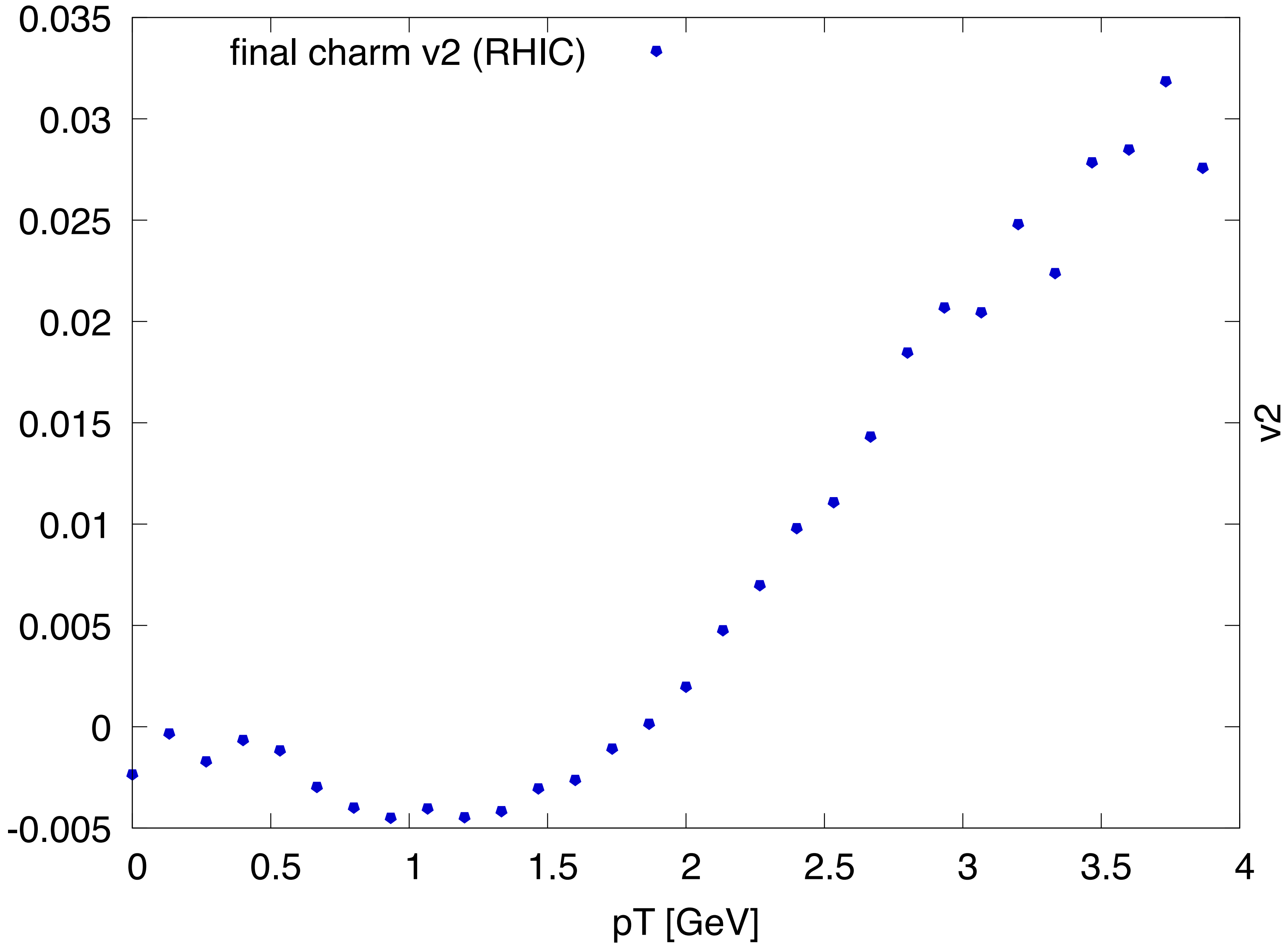
Elliptic Flow v_2: Charm Quarks

Initial momentum distribution from PYTHIA

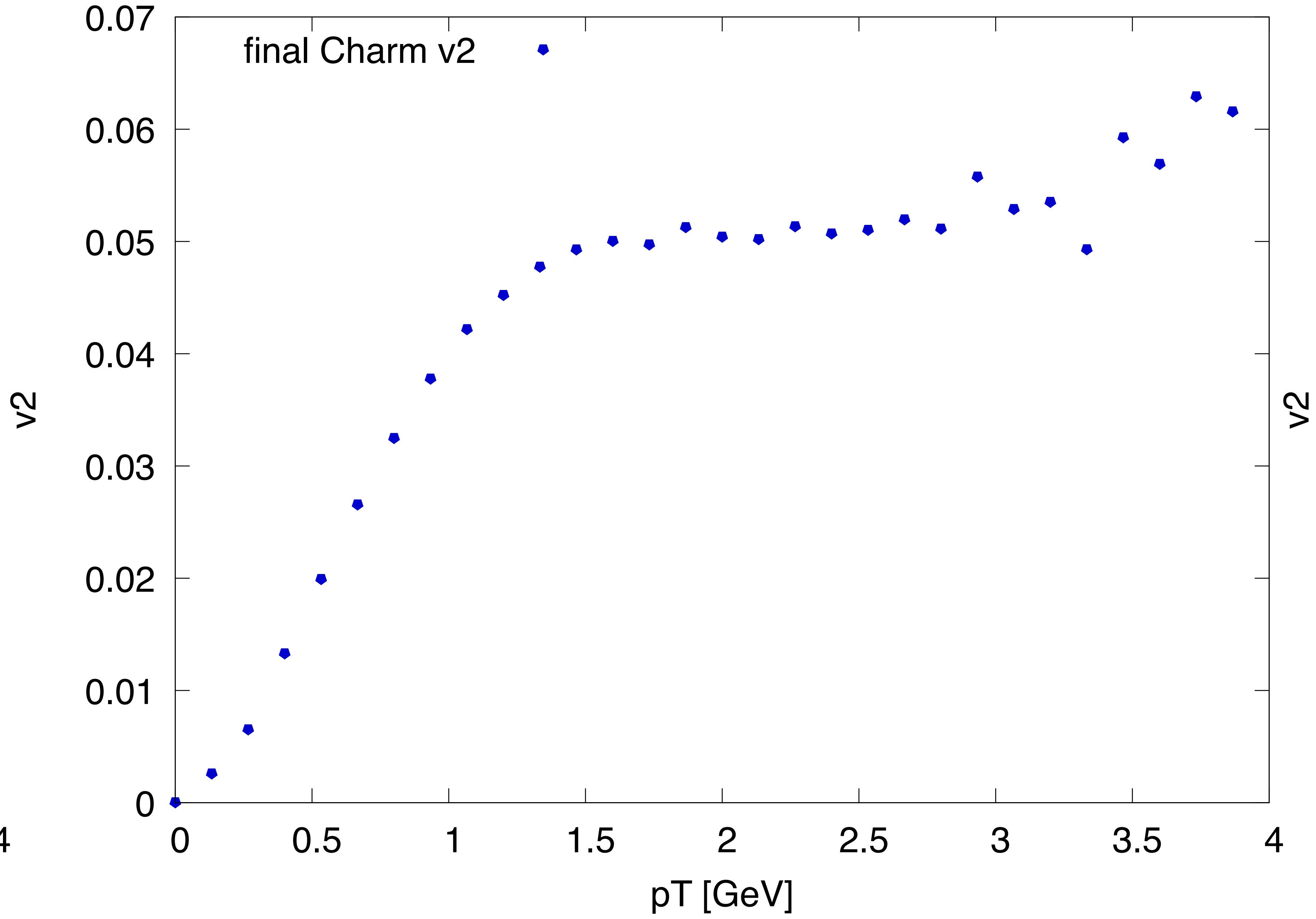
$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$



RHIC, 20-40% Centrality



LHC, 0-20% Centrality



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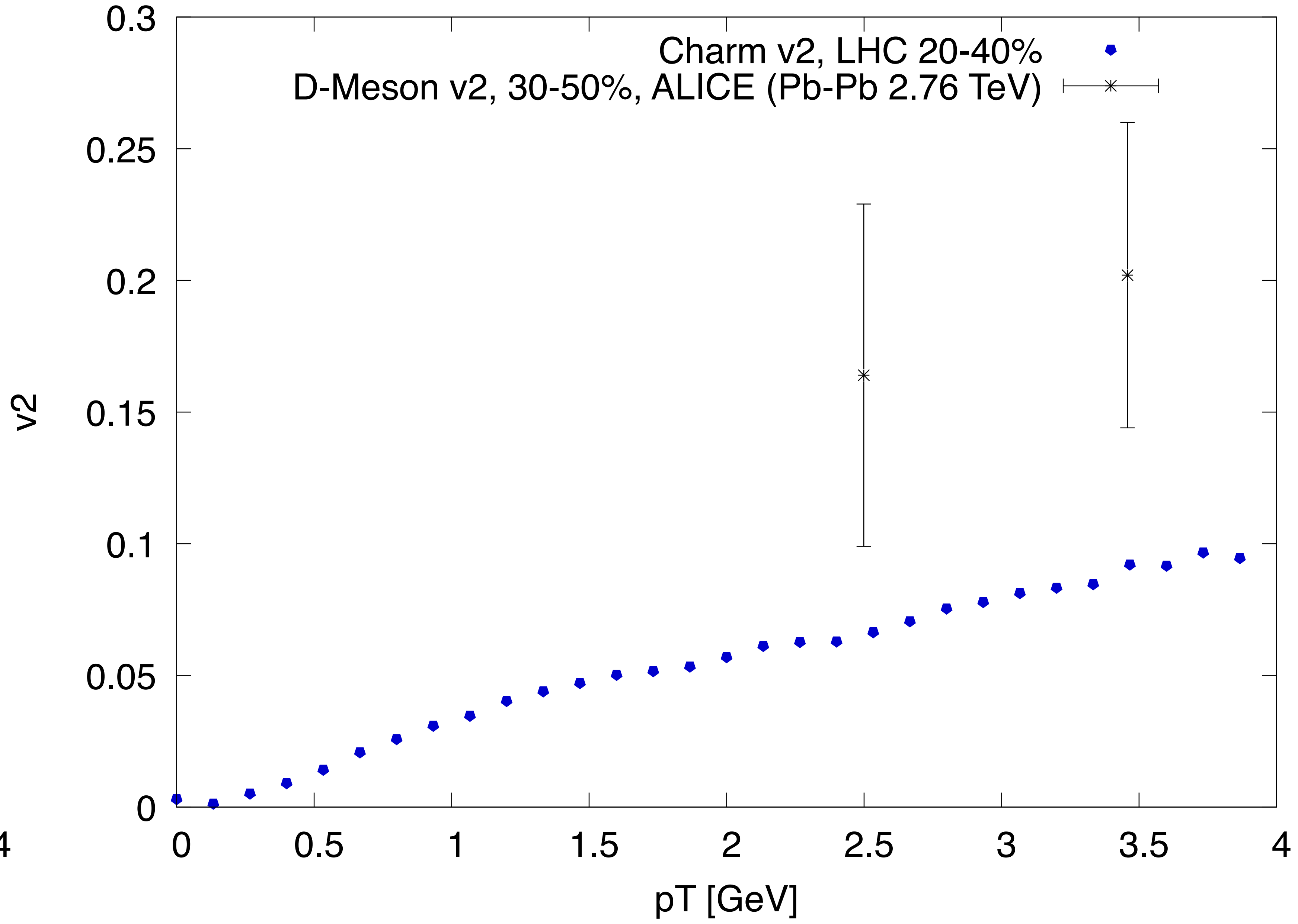
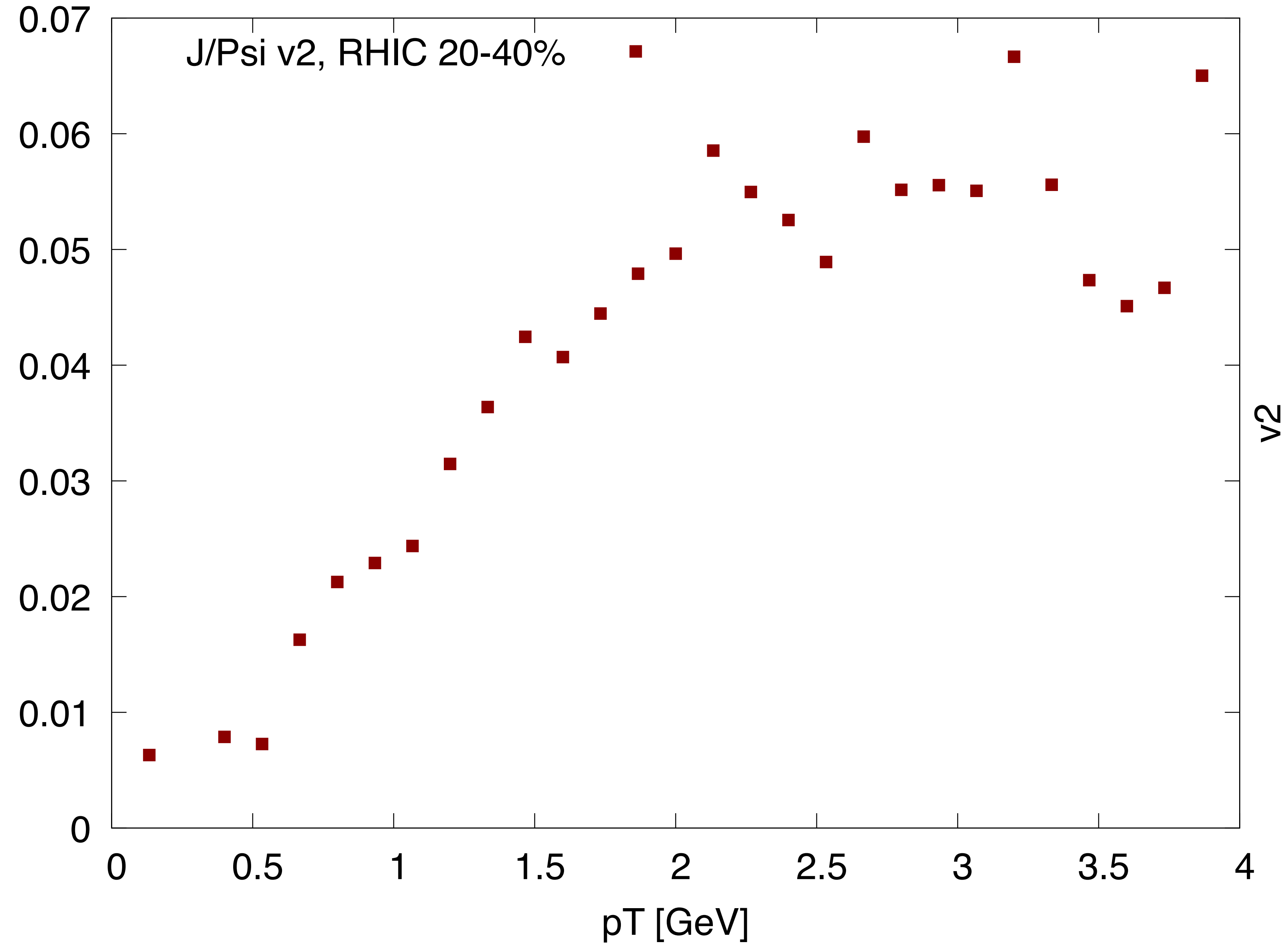


ALICE Collaboration (2013). D meson elliptic flow in non-central Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV. Phys.Rev.Lett. 111 (2013) 102301.

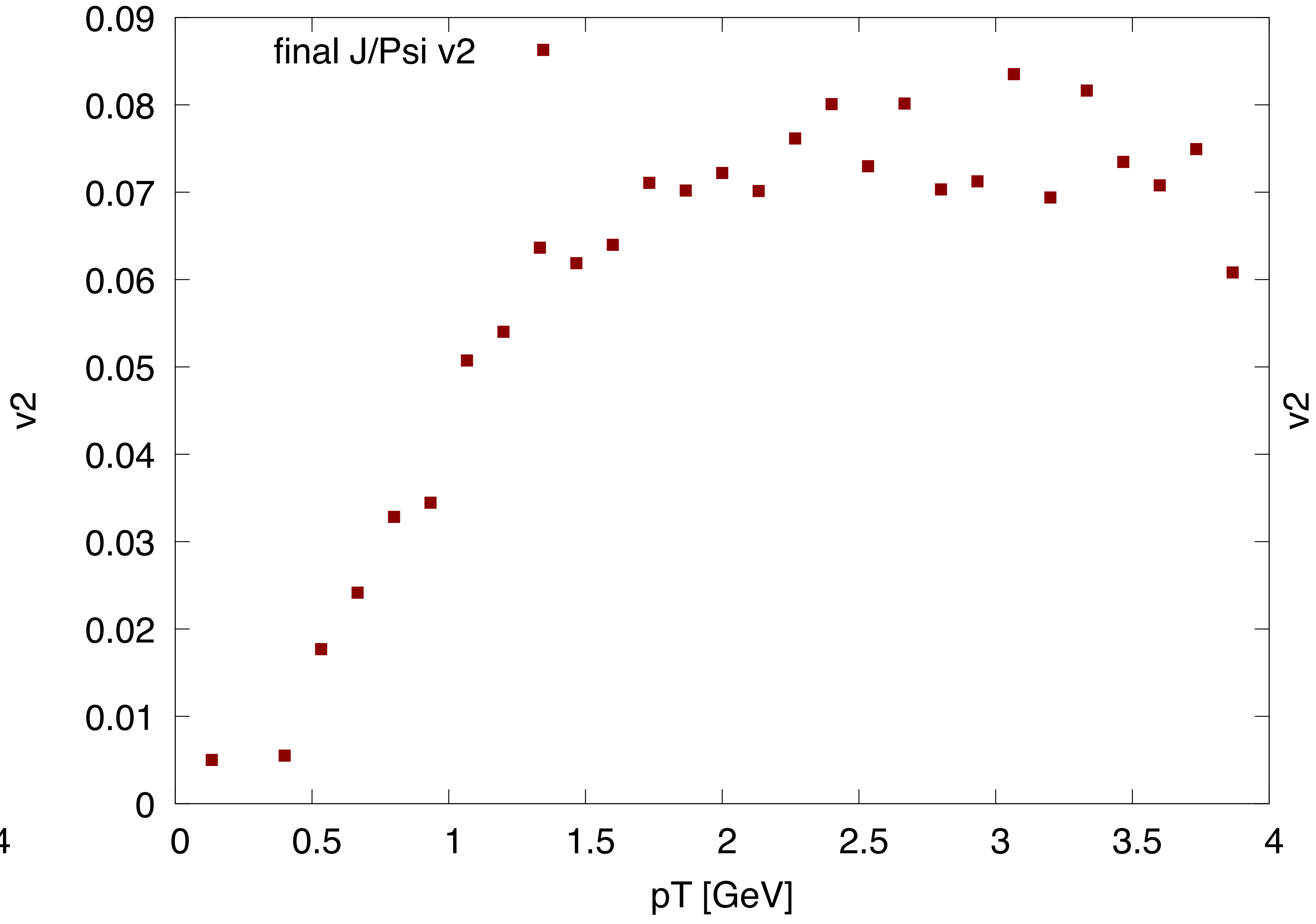
Elliptic Flow v_2: Charmonium

Initial momentum distribution from PYTHIA

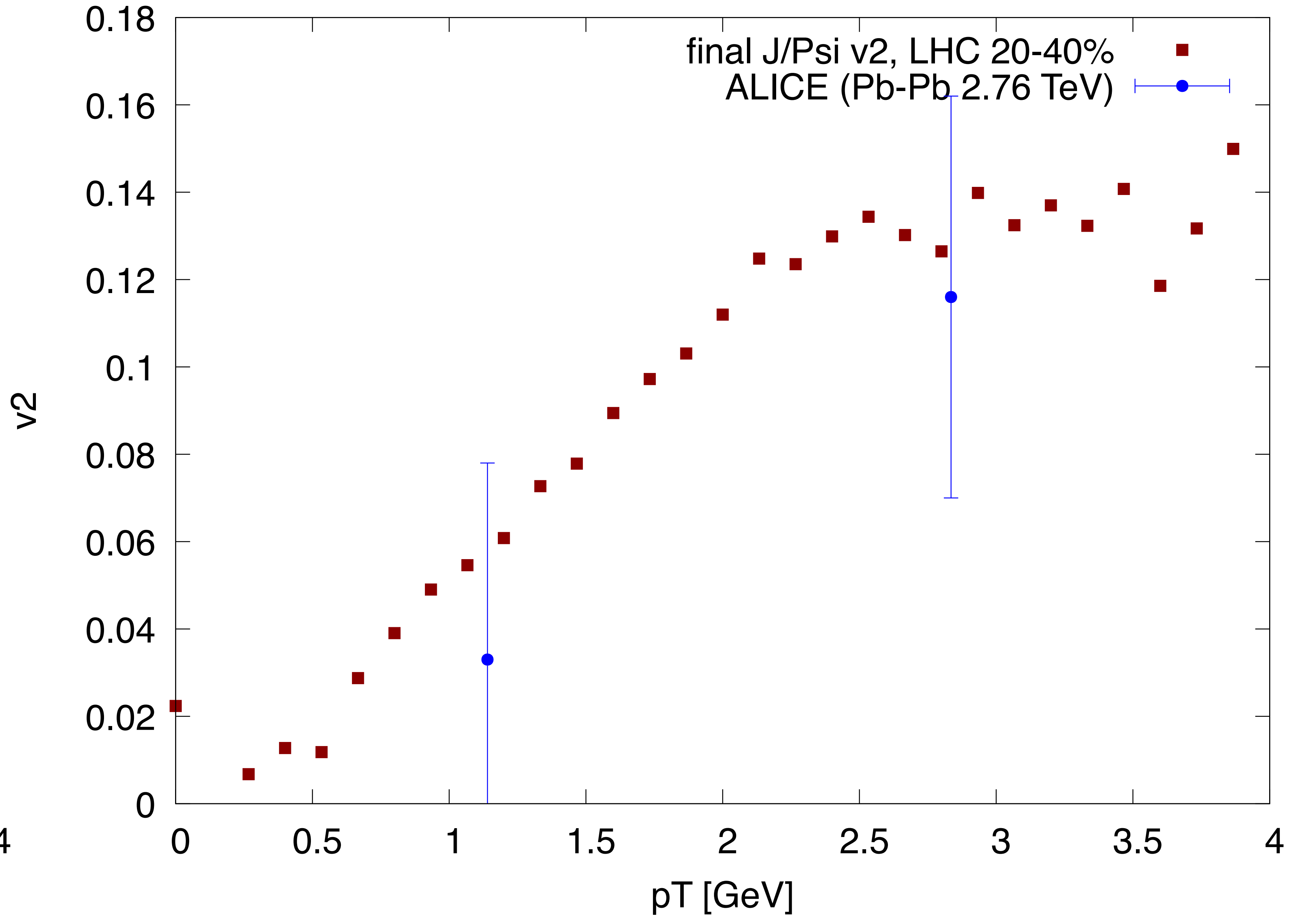
RHIC, 20-40% Centrality



LHC, 0-20% Centrality



LHC, 20-40% Centrality



➡ working on increasing the statistics

ALICE Collaboration (2013). J/Psi Elliptic Flow in Pb-Pb Collisions at $\sqrt{s_{NN}}$ = 2.76 TeV. Phys.Rev.Lett. 111 (2013) 162301.

ALICE Collaboration (2013). J/Psi Elliptic Flow in Pb-Pb Collisions at $\sqrt{s_{NN}}$ = 2.76 TeV. HEPData (collection).

Conclusions & Outlook

Summary:

- Box simulations → correct equilibrium limit, agreement with SHM
  - ➡Bound-state formation, dissociation and regeneration occurs in the expected manner
- Implementation of fireball model to describe dynamical expansion
  - ➡v_2 of charm and charmonium

Future extensions:

- Nuclear modification factor R_{AA}
- using PYTHIA:
  - ➡ include primordial charmonium
  - ➡ Expand to bottomonium sector

Backup

Langevin simulations

- First half of coordinate update step

$$\vec{r}_{c,i+\frac{1}{2}} = \vec{r}_{c,i} + \frac{\vec{p}_{c,i}}{2E_c}\Delta t$$

- Calculation of Potential for Momentum Update

$$\overrightarrow{F}(\vec{r}_{c,i+\frac{1}{2}}, \vec{r}_{\bar{c},i+\frac{1}{2}})\Delta t$$

- Boost to Medium Rest Frame

$$p_i^* = p_i - \gamma\beta_i E + (\gamma - 1)\frac{\beta_i}{\vec{\beta}^2}\vec{\beta}\vec{p}, \quad i = 1,2,3$$

Langevin simulations

- Analytic form of momentum update step:

$$dp_j = -\gamma p_j dt + \sqrt{dt} C_{jk} \rho_k$$

- Stochastic process dependent on specific choice of the momentum argument of the covariance matrix $\boldsymbol{C}_{jk}$

- Determination of momentum argument in $\boldsymbol{C}_{jk}$:

$$C_{jk} \to C_{jk}(t, \boldsymbol{x}, \boldsymbol{p} + \xi d\boldsymbol{p})$$

➡ $\xi = 0, \frac{1}{2}, 1$ for pre-point, midpoint and post-point realisation

- In this work: post-point scheme,

$$C_{jk} \to C_{jk}(t, \boldsymbol{x}, \boldsymbol{p} + d\boldsymbol{p})$$

Langevin simulations

- General dissipation-fluctuation relation between drag and diffusion coefficient in statistical equlibrium:

$$A_i(\boldsymbol{p},T) = B_{ij}(\boldsymbol{p},T)\frac{1}{T}\frac{\partial E(p)}{\partial p_j} - \frac{\partial B_{ij}(\boldsymbol{p},T)}{\partial p_j}$$

- with a diagonal approximation of the diffusion coefficient, $B_0(\boldsymbol{p},T) = B_1(\boldsymbol{p},T) \equiv D(\boldsymbol{p},T)$:

$$A(p) = \frac{1}{E(p)}\left(\frac{D[E(p)]}{T} - \frac{\partial D[E(p)]}{\partial E}\right)$$

- with

$$\Gamma(p) = \frac{1}{E(p)}\left(\frac{D[E(p)]}{T} - (1-\xi)\frac{\partial D[E(p)]}{\partial E}\right)$$

- dependent on choice of ξ
  - for post-point, $\xi = 1$: simple form of equilibrium condition:

$$D[E(p)] = \Gamma(p)E(p)T$$

Langevin simulations

- momentum update:

$$dp_j = -\gamma p_j dt + \sqrt{2dtD(|\boldsymbol{p} + \boldsymbol{dp}|)}\rho_j = -\gamma p_j dt + \sqrt{2\gamma ETdt}\rho_j$$

➡ Two-step computation:

I. Calculation of dp_j of pre-point scheme, $dp_j = -\gamma p_j dt + \sqrt{2dtD(p)}\rho_j$

II. Use result for argument $|\boldsymbol{p} + \boldsymbol{dp}|$ of D to evaluate the second part of the postpoint momentum update, $dp_j^{diff} = \sqrt{2dtD(|\boldsymbol{p} + \boldsymbol{dp}|)}\rho_j$

III. Complete momentum update: $dp_j = dp_j^{drag} + dp_j^{diff}$ with dp_j^{drag} from I.

Langevin simulations

- Boost back to computational frame
- Complete momentum update:

$$\vec{p}_{c,i+1} = \vec{p}_{c,i} + \overrightarrow{F}(\vec{r}_{c,i+\frac{1}{2}}, \vec{r}_{\bar{c},i+\frac{1}{2}})\Delta t - \gamma\vec{p}_{c,i}\Delta t + \sqrt{2ET\gamma\Delta t}\rho$$

- Second half of coordinate update step:

$$\vec{r}_{c,i+1} = \vec{r}_{c,i+\frac{1}{2}} + \frac{\vec{p}_{c,i+1}}{2E}\Delta t$$

Potential of the Heavy Quarks

- Formalism to describe heavy quarks in Abelian plasma by Blaizot et al.
- Idea: effective theory of non-relativistic HQs in plasma of relativistic particles
- Influence functional in infinite-mass limit and large time limit:

$$\Phi[\boldsymbol{Q}] \simeq g^2(t_f - t_i)\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}(1 - exp[i\boldsymbol{k}(\boldsymbol{r} - \bar{\boldsymbol{r}})]\Delta(0,\boldsymbol{k}))$$

➡ When considering equation of motion for correlator $G^{>}(t_f, \boldsymbol{Q}_f | t_i \boldsymbol{Q}_i)$ at large time: interpretation as complex potential $\mathscr{V}(\boldsymbol{r})$

Potential of the Heavy Quarks

- Real part:

$$V(\boldsymbol{r}) = -\Delta^R(0,\boldsymbol{r}) = -\int \frac{d\boldsymbol{k}}{(2\pi)^3} e^{i\boldsymbol{kr}} \Delta^R(\omega = 0,\boldsymbol{k})$$

- Imaginary part:

$$W(\boldsymbol{r}) = -\Delta^<(0,\boldsymbol{r}) = -\int \frac{d\boldsymbol{k}}{(2\pi)^3} e^{i\boldsymbol{kr}} \Delta^<(0,\boldsymbol{k})$$

with the propagator $\Delta(0,\boldsymbol{r}) = \Delta^R(0,\boldsymbol{r}) + i\Delta^<(0,\boldsymbol{r})$

➡ $$\mathscr{V}(\boldsymbol{r}) = -g^2\left[V(\boldsymbol{r}) - V_{ren}(0)\right] - ig^2\left[W(\boldsymbol{r}) - W(0)\right]$$

Heavy Quarks in Abelian Plasma

- Complex potential for $c\bar{c}$-pair after evaluation of integrals:

$$\mathscr{V}(r) = -\frac{g^2}{4\pi}m_D - \frac{g^2}{4\pi}\frac{\exp(-m_D r)}{r} - i\frac{g^2 T}{4\pi}\phi(m_D r)$$

With $\phi(x) = 2\int_0^\infty dz \frac{z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$

- Drag and diffusion coefficients derived from potential from the second derivative of W:

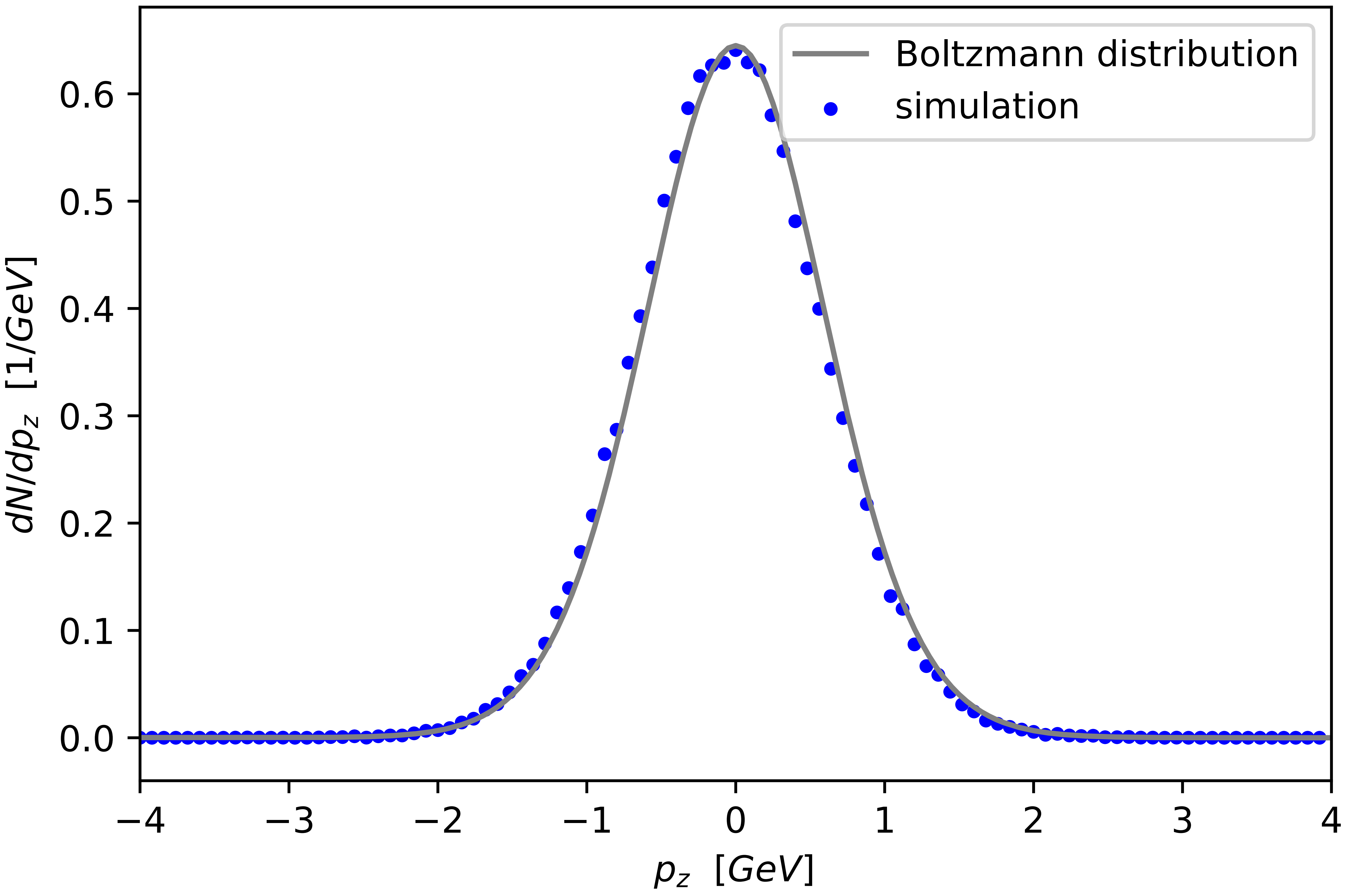
$$\mathscr{H}_{\alpha\beta}(\boldsymbol{s}) = \frac{\partial^2 W(\boldsymbol{s})}{\partial r_\alpha \partial r_\beta},$$

and using $g^2\mathscr{H}(0)_{\alpha\beta} = 2MT\gamma\delta_{\alpha\beta}$

Blaizot et al., Nucl.Phys.*A* 946 (2016) 49-88

Testing the Model

Equilibrium Conditions in Box Calculations



- Single $c\bar{c}$-pair in box calculation with $T = 180\ MeV$ and $m_c = 1.8\ GeV/c^2$
- Momentum distribution in equilibrium limit (Boltzmann-Jüttner):

$$f_{eq}(\boldsymbol{p}) \propto \exp\left[-\frac{E(\boldsymbol{p})}{T}\right]$$

Elliptic Fireball

Parametrisation of hadronic freeze-out

- differential momentum spectrum of a particle:

$$\frac{dN}{p_T dp_T d\phi_p dy} = \frac{2g}{(2\pi)^3}\tau_f \; m_T \; e^{\frac{\mu}{T_f}} \int r dr \int d\phi_s K_1(m_T, T, \beta_T) e^{\frac{p_T}{T_f \sinh(\rho(r,\phi_s)} \cos(\phi_p - \phi_b))}$$

- T_f: freeze-out temperature, ϕ_b: azimuthal angle of the boost, K_1: Bessel function
- transverse rapidity $\rho(r, \phi_s)$: function of radius r and spatial azimuthal angle ϕ_s
- Elliptic flow:

$$v_2(p_T) = \frac{\int_0^{2\pi} d\phi_p \cos(2\phi_p) \frac{dN}{p_T dp_T d\phi_p dy}}{\int_0^{2\pi} d\phi_p \frac{dN}{p_T dp_T d\phi_p dy}}$$

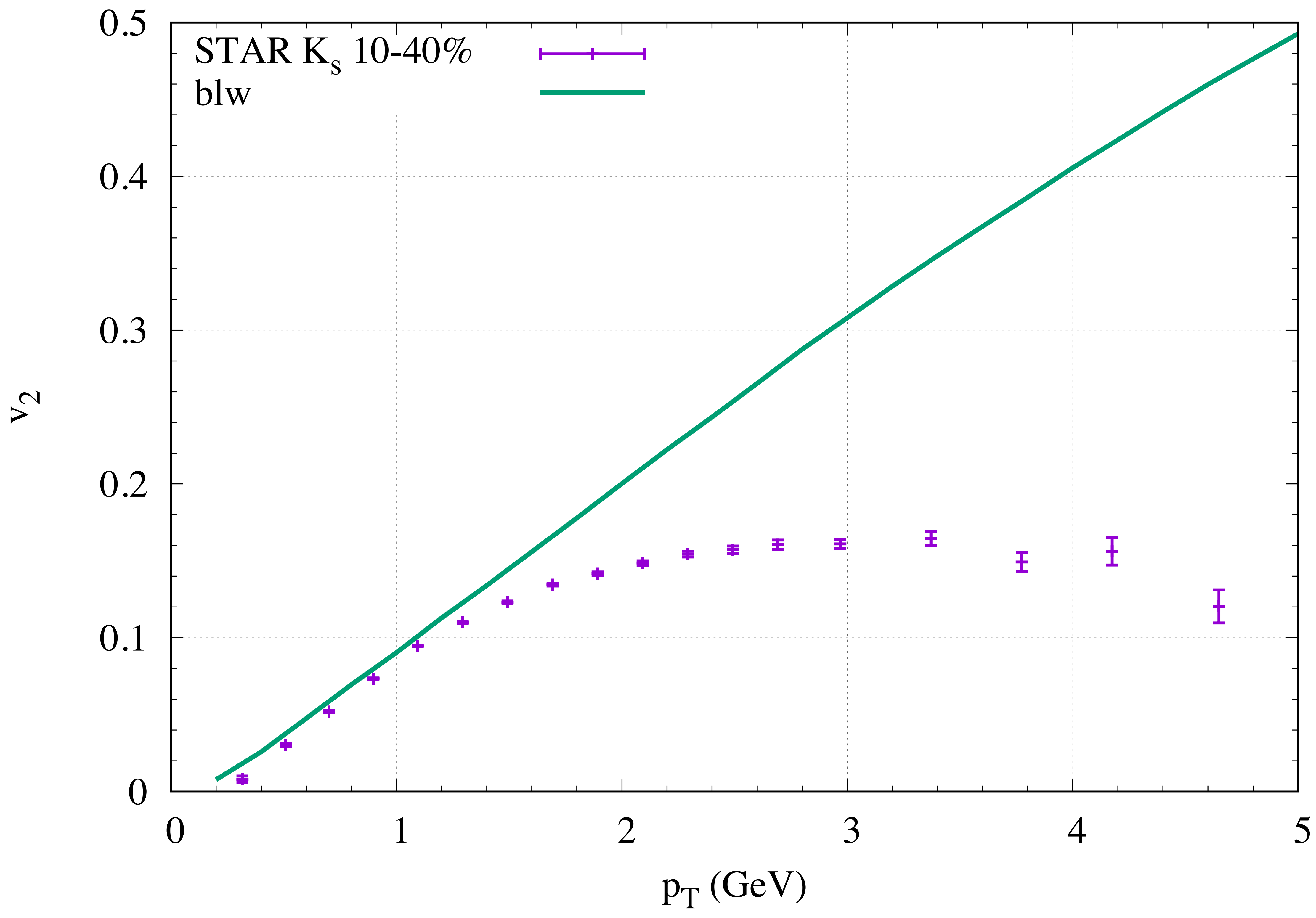
Retière, Lisa, Physical Review C, 70 (2004)

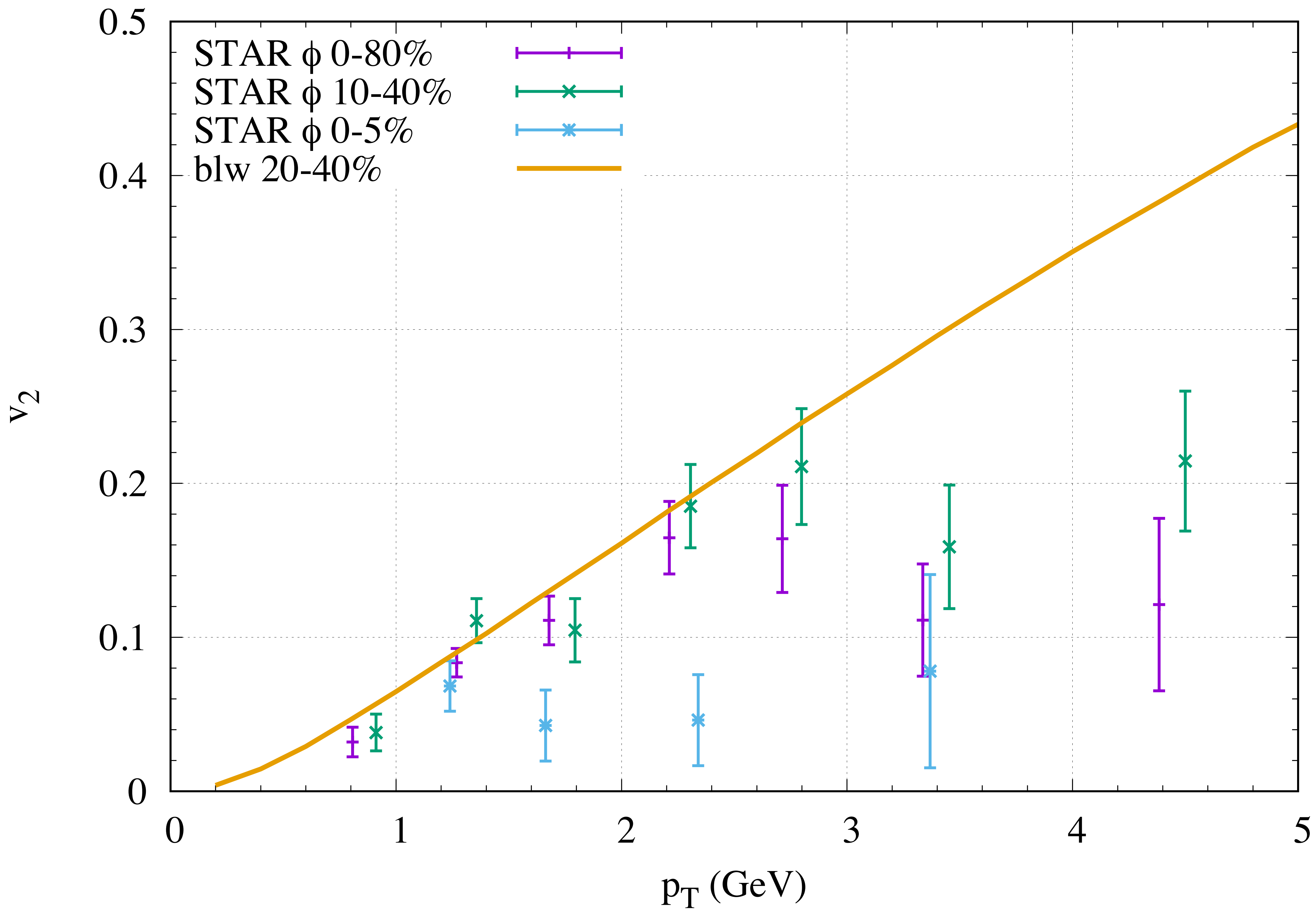
He, Fries, Rapp, Physical Review C, 82 (2010)

Parametrization of the Fireball

RHIC (20-40%), v_2

- Choice of parameters in fireball model by fitting results to experimental data
- Elliptic flow v_2 of K_S and ϕ from STAR

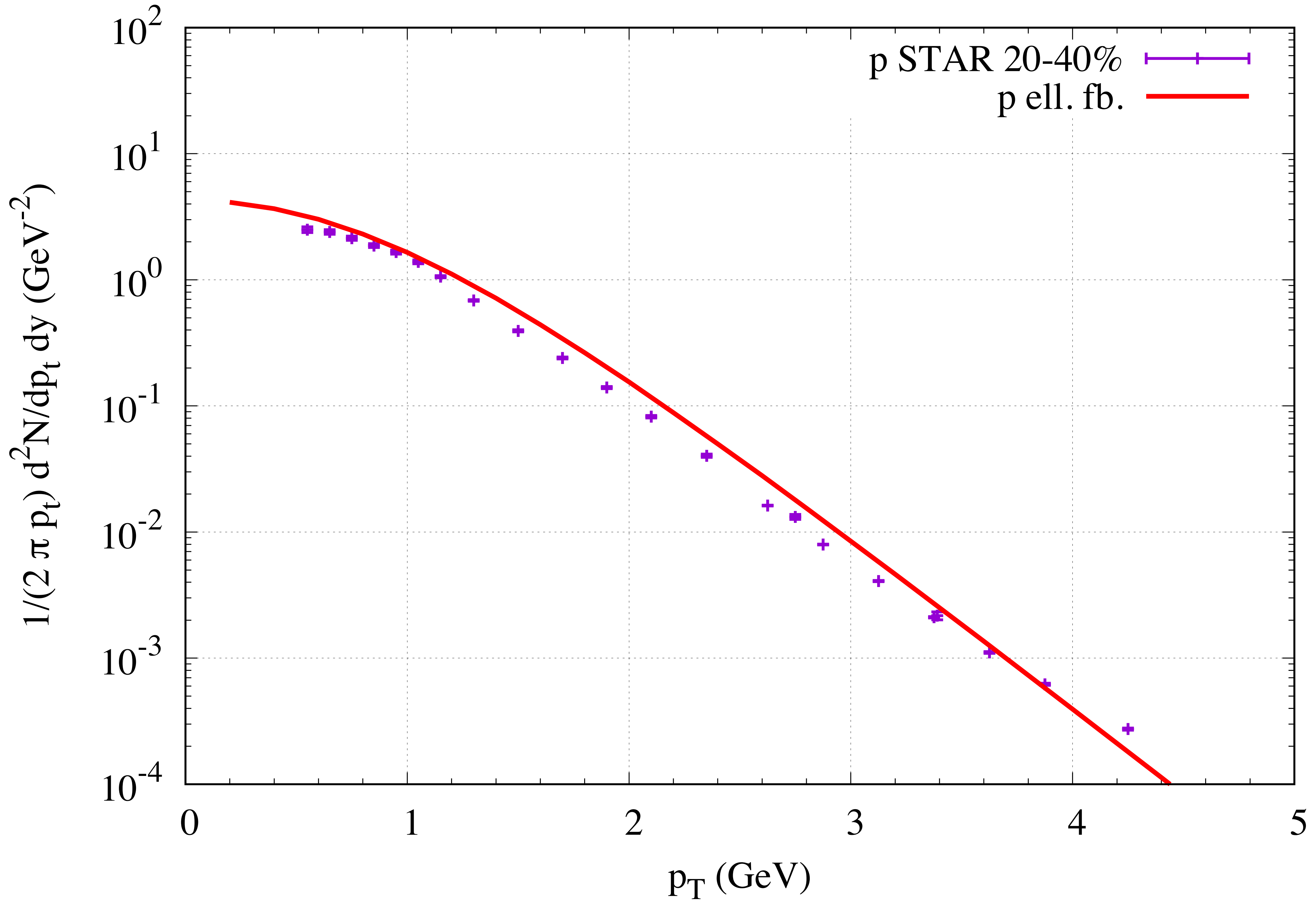


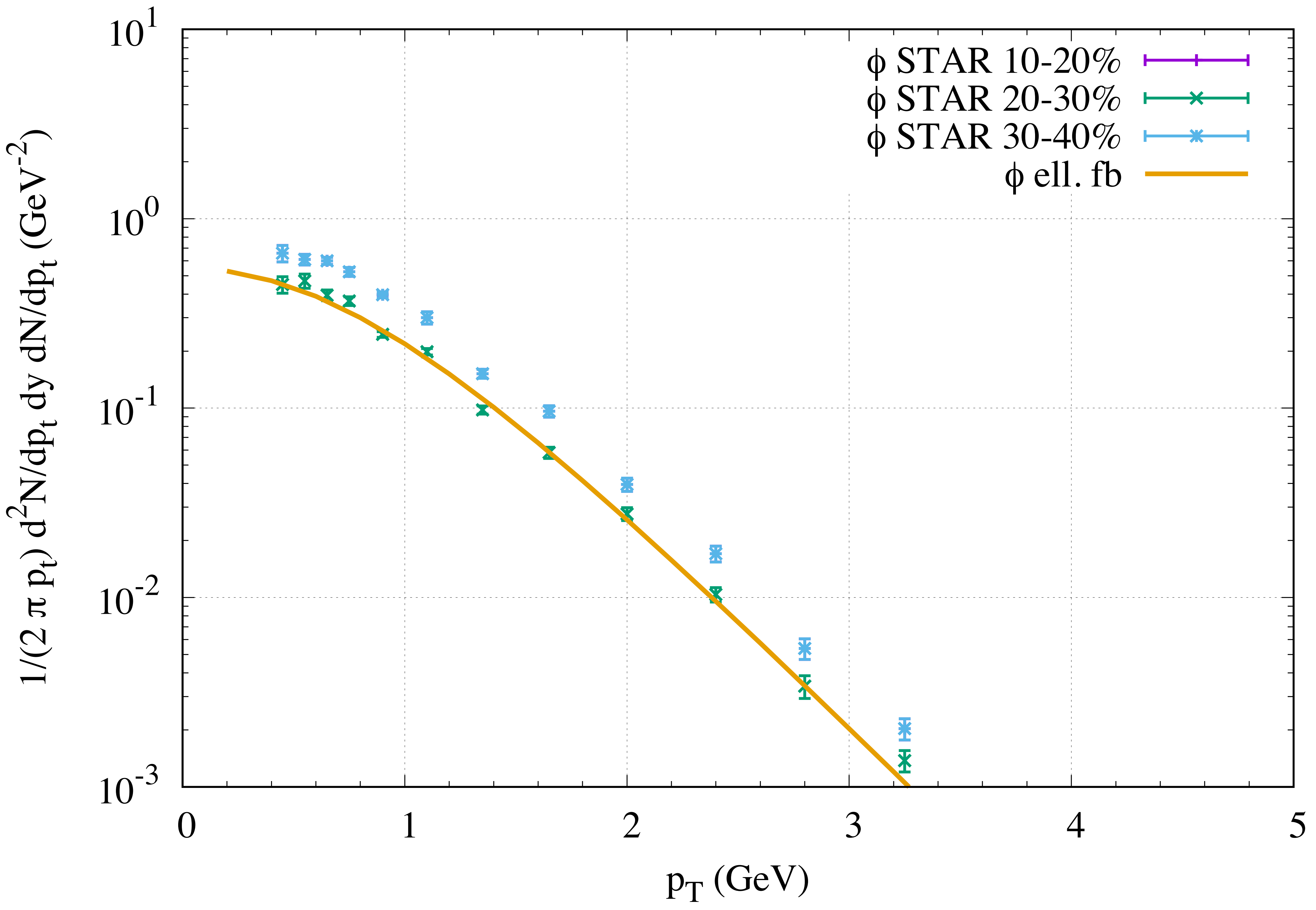


Parametrization of the Fireball

RHIC (20-40%), p_T

- Choice of parameters in fireball model by fitting results to experimental data
- p_T -spectra of p and ϕ from STAR

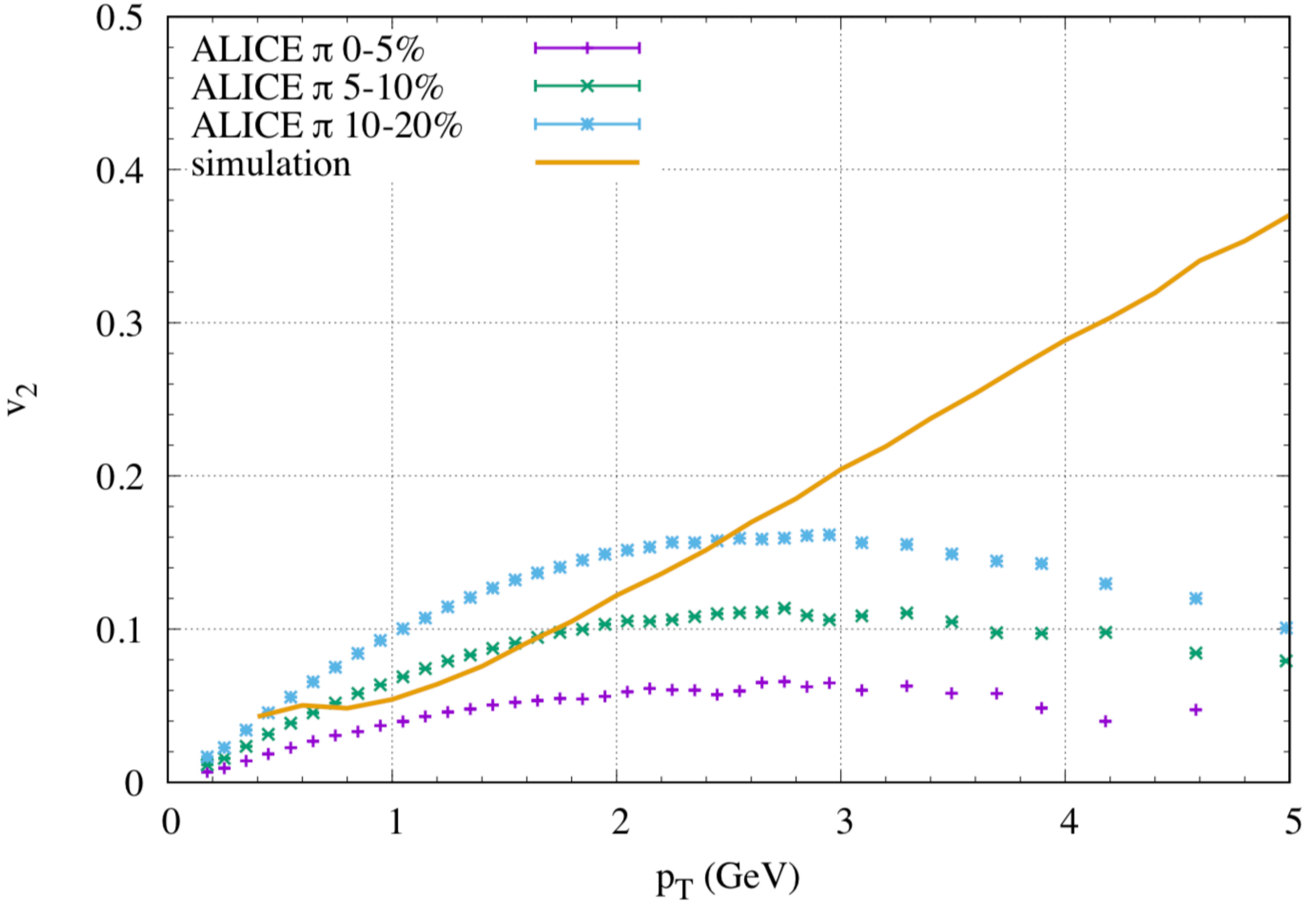


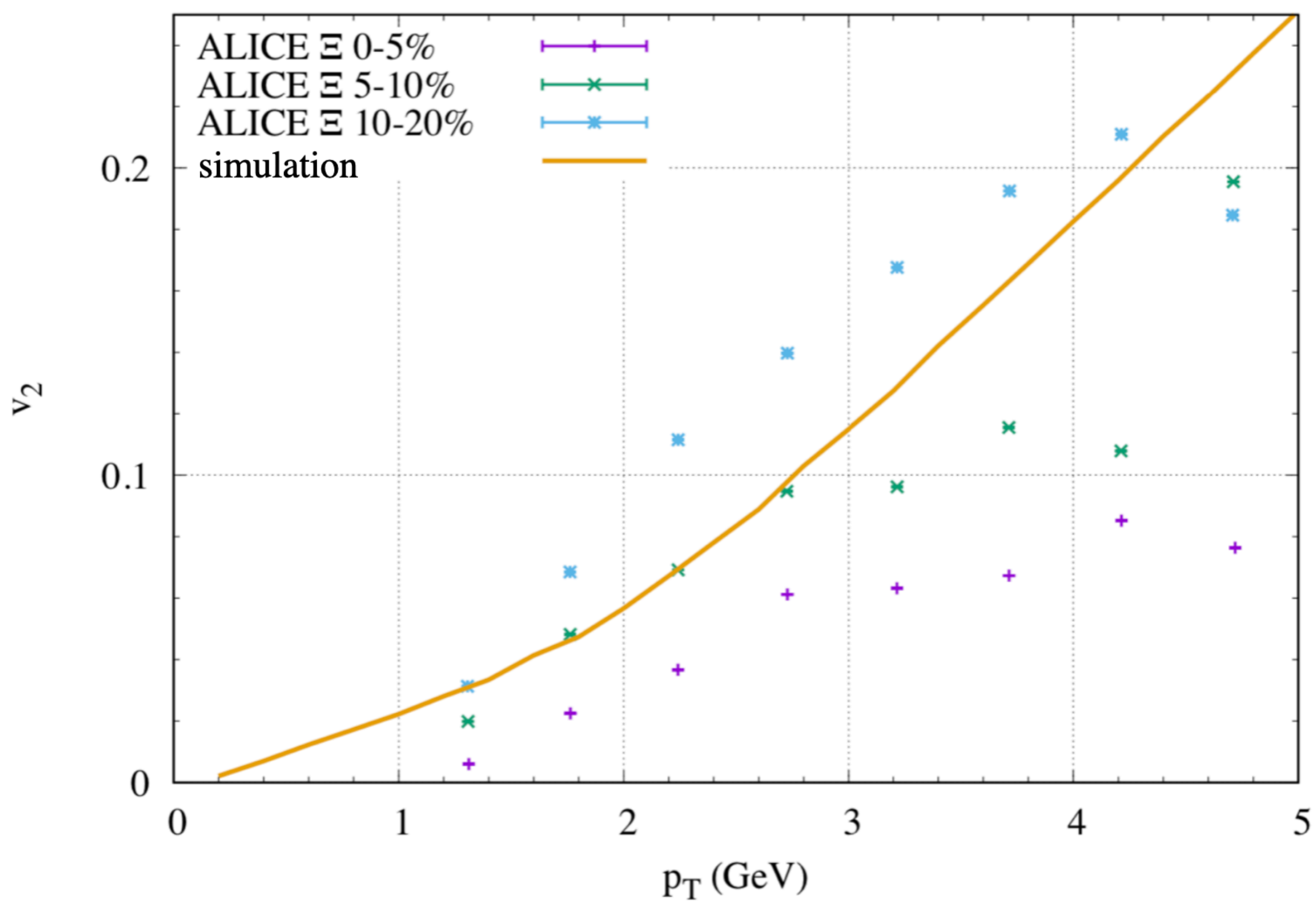


Parametrization of the Fireball

LHC (0-20%), v_2

- Comparison of elliptic flow spectra from simulation to data from ϕ and Ξ from ALICE

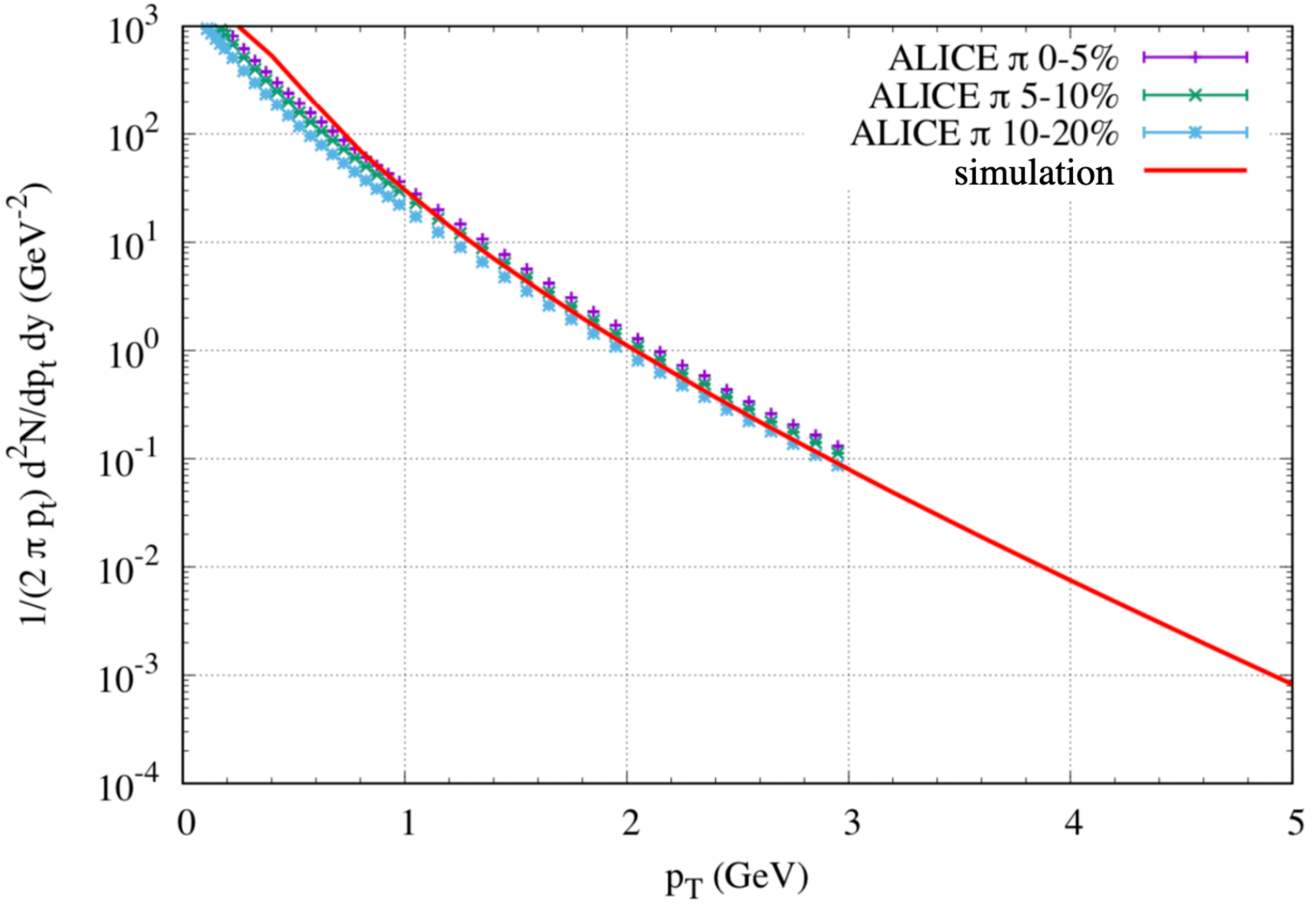


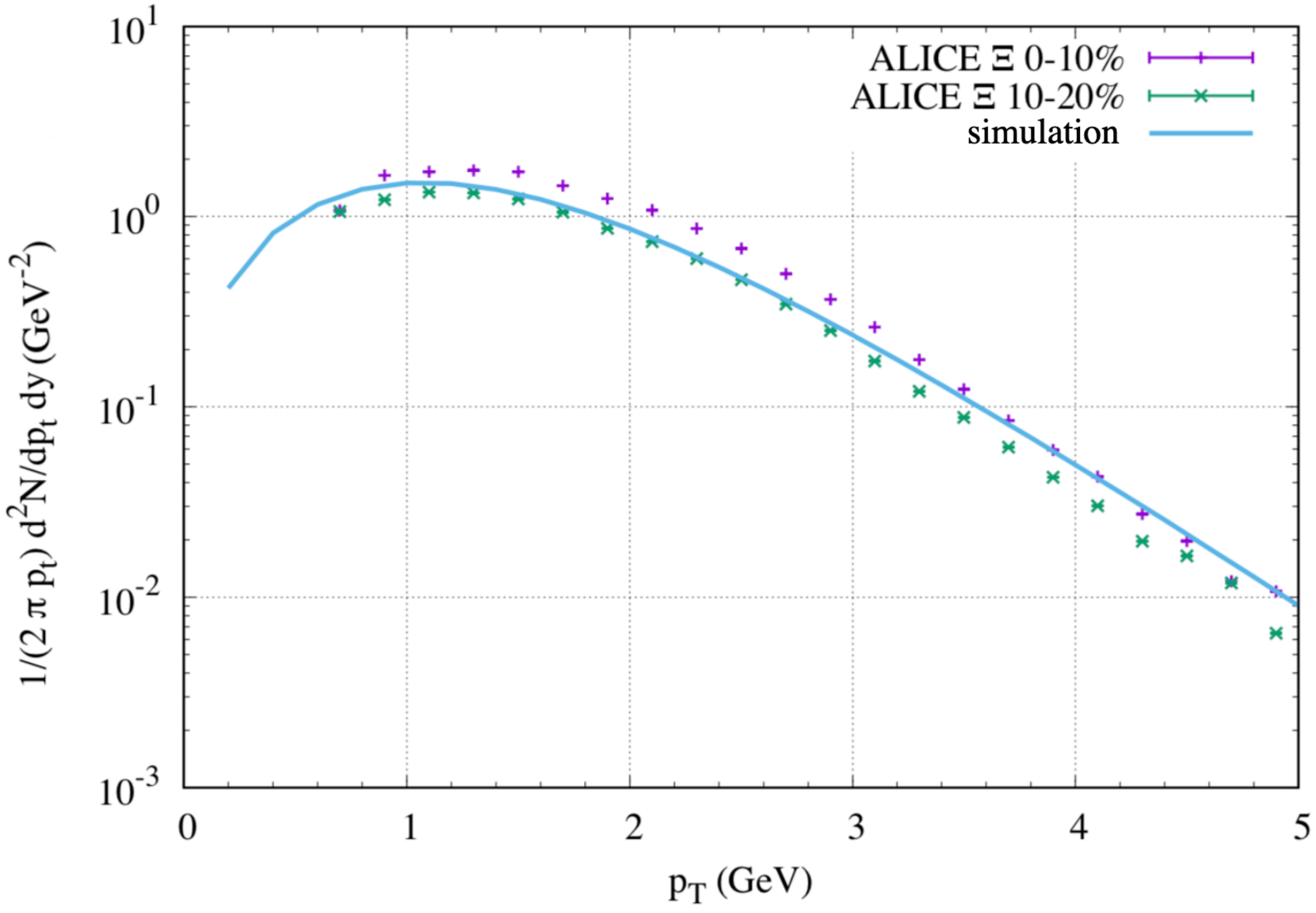


Parametrization of the Fireball

LHC (0-20%), p_T

- Comparison of p_T - spectra from simulation to data from π and Ξ from ALICE





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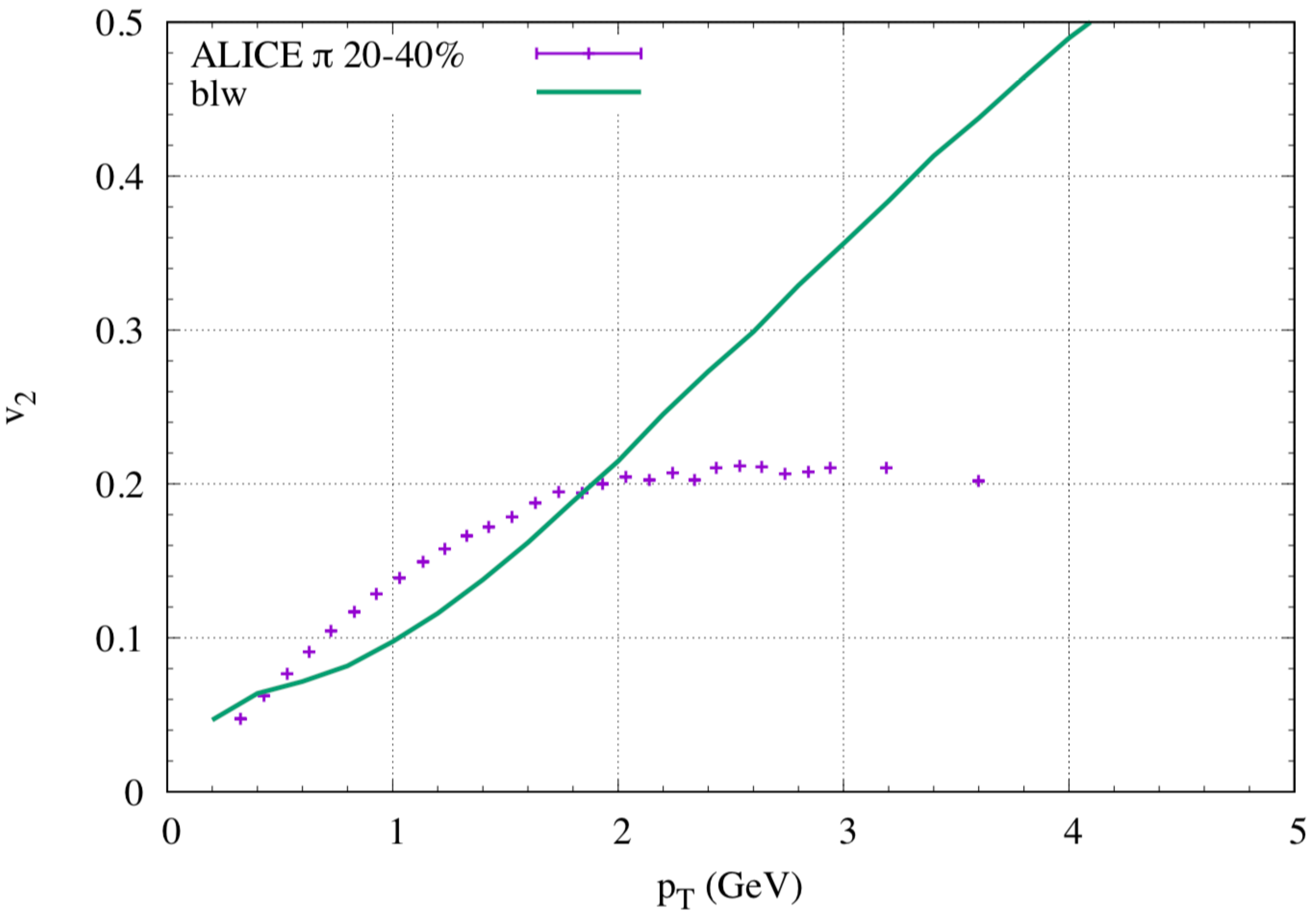
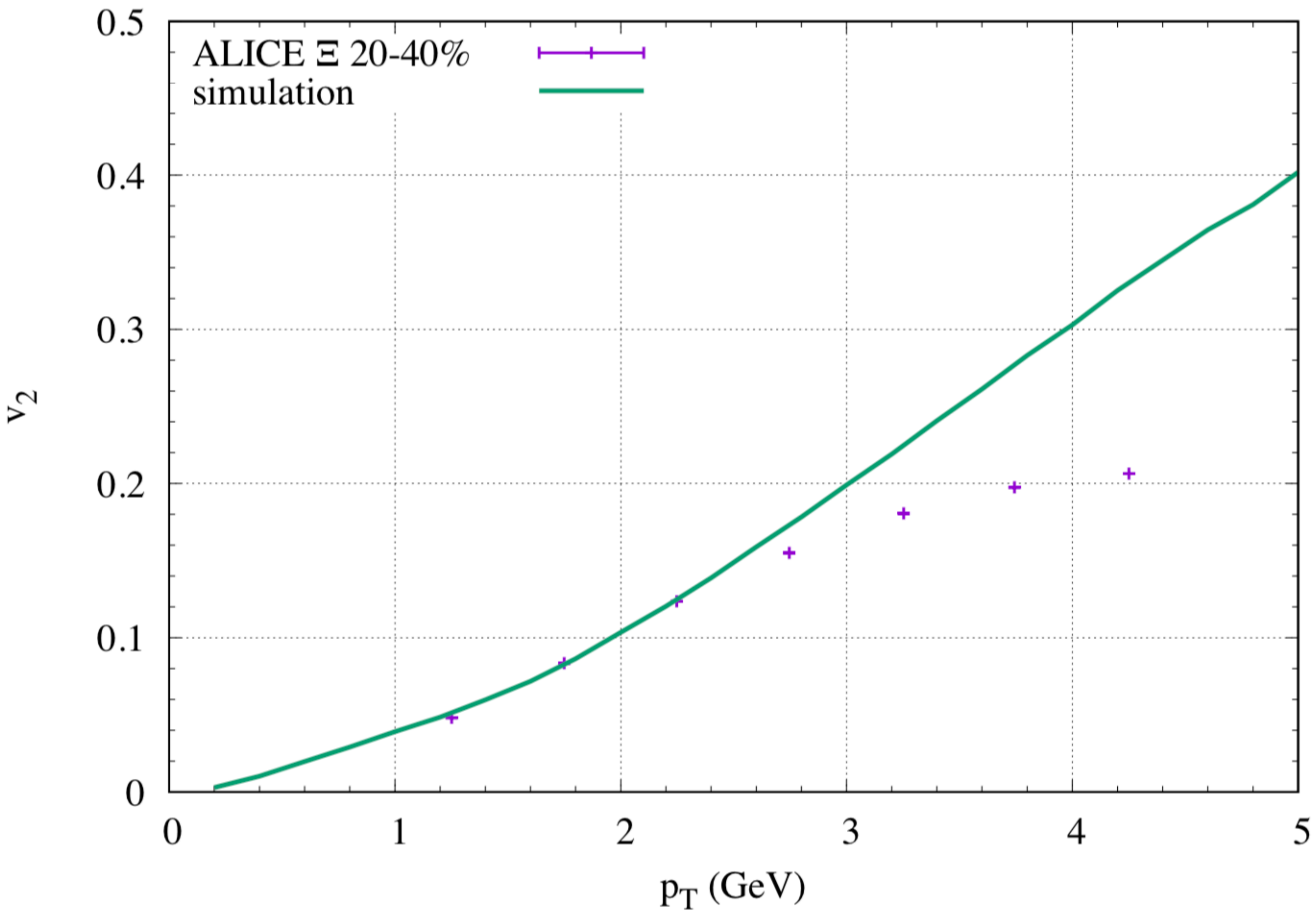
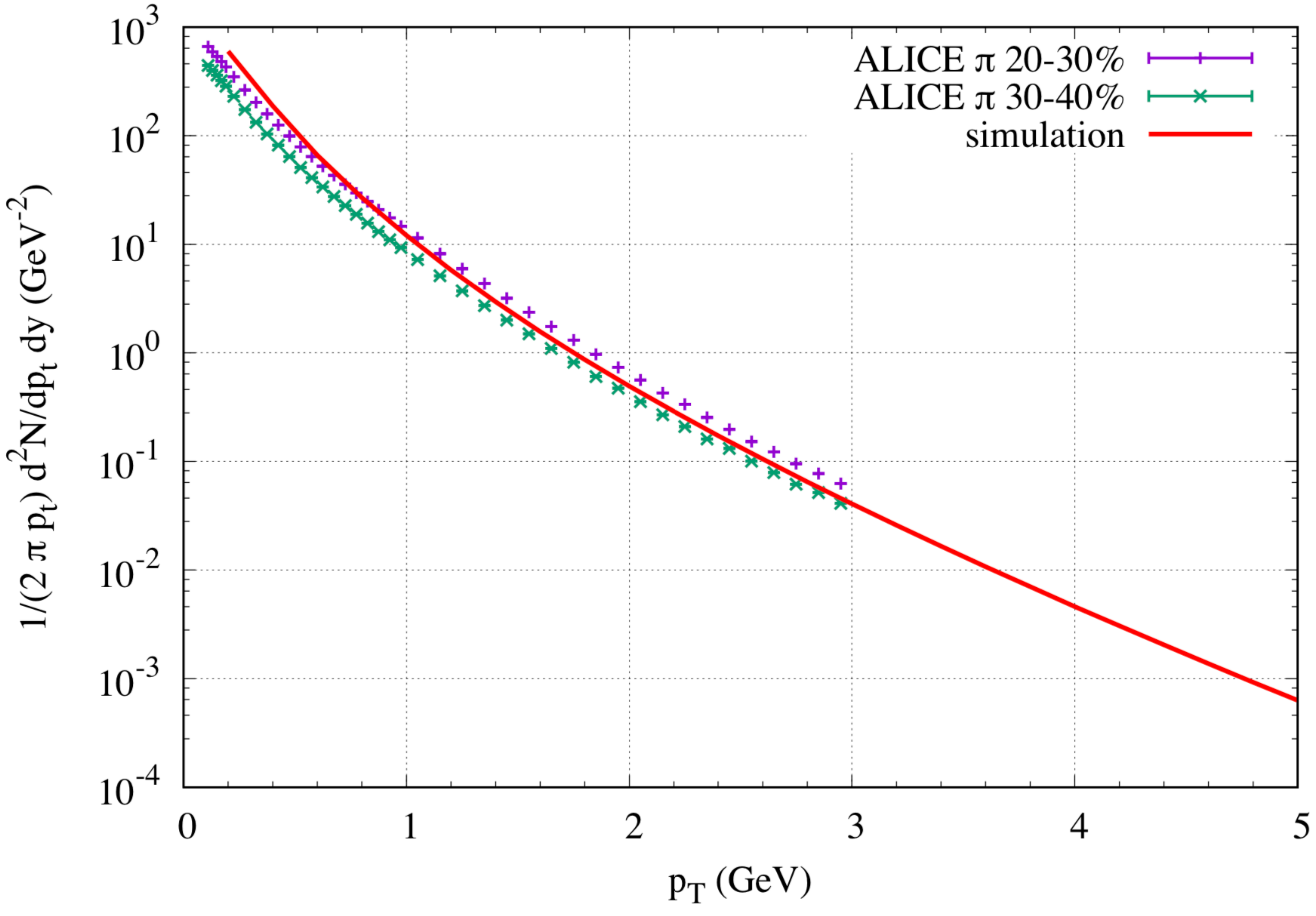


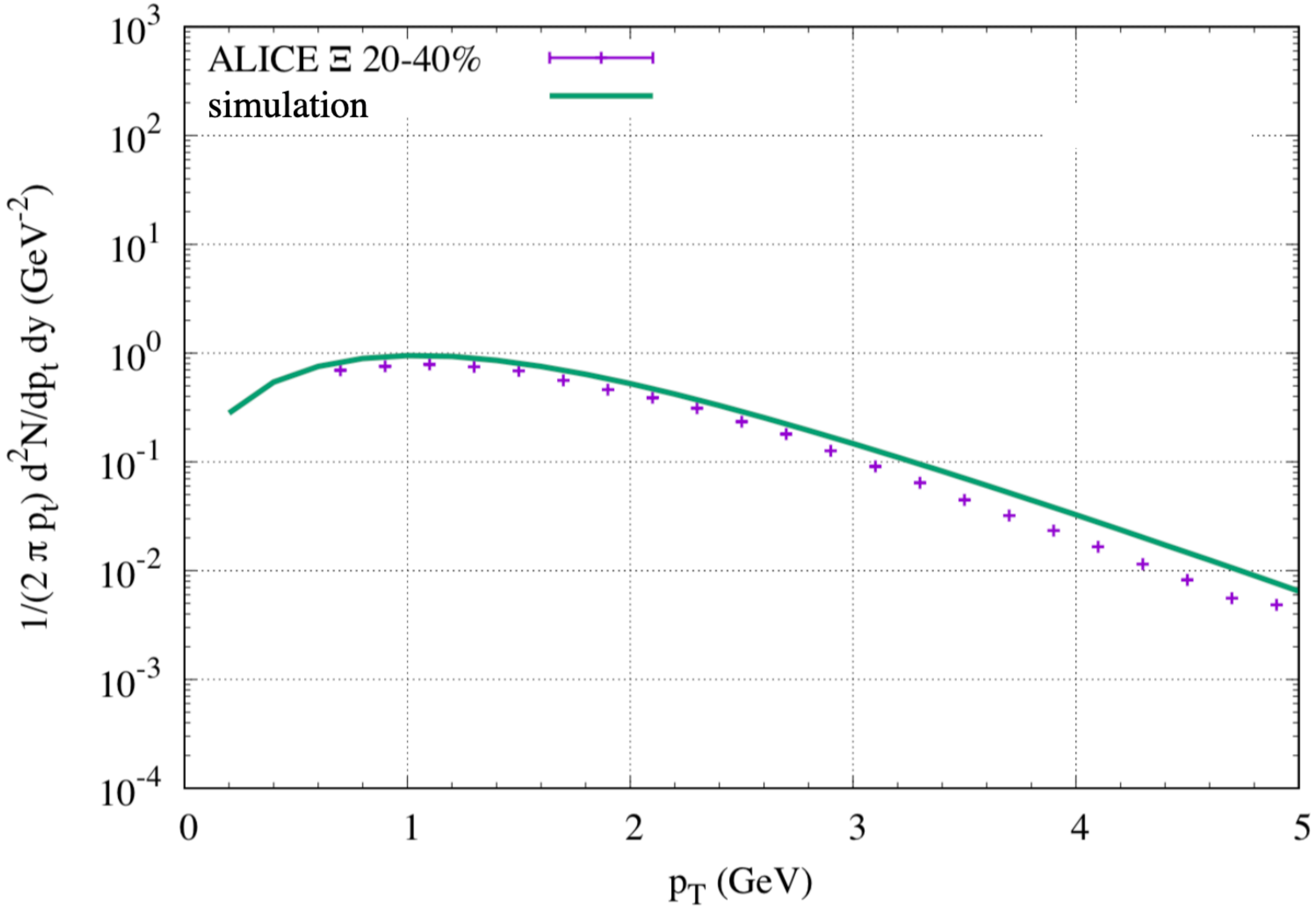


Parametrization of the Fireball

LHC (20-40%), p_T

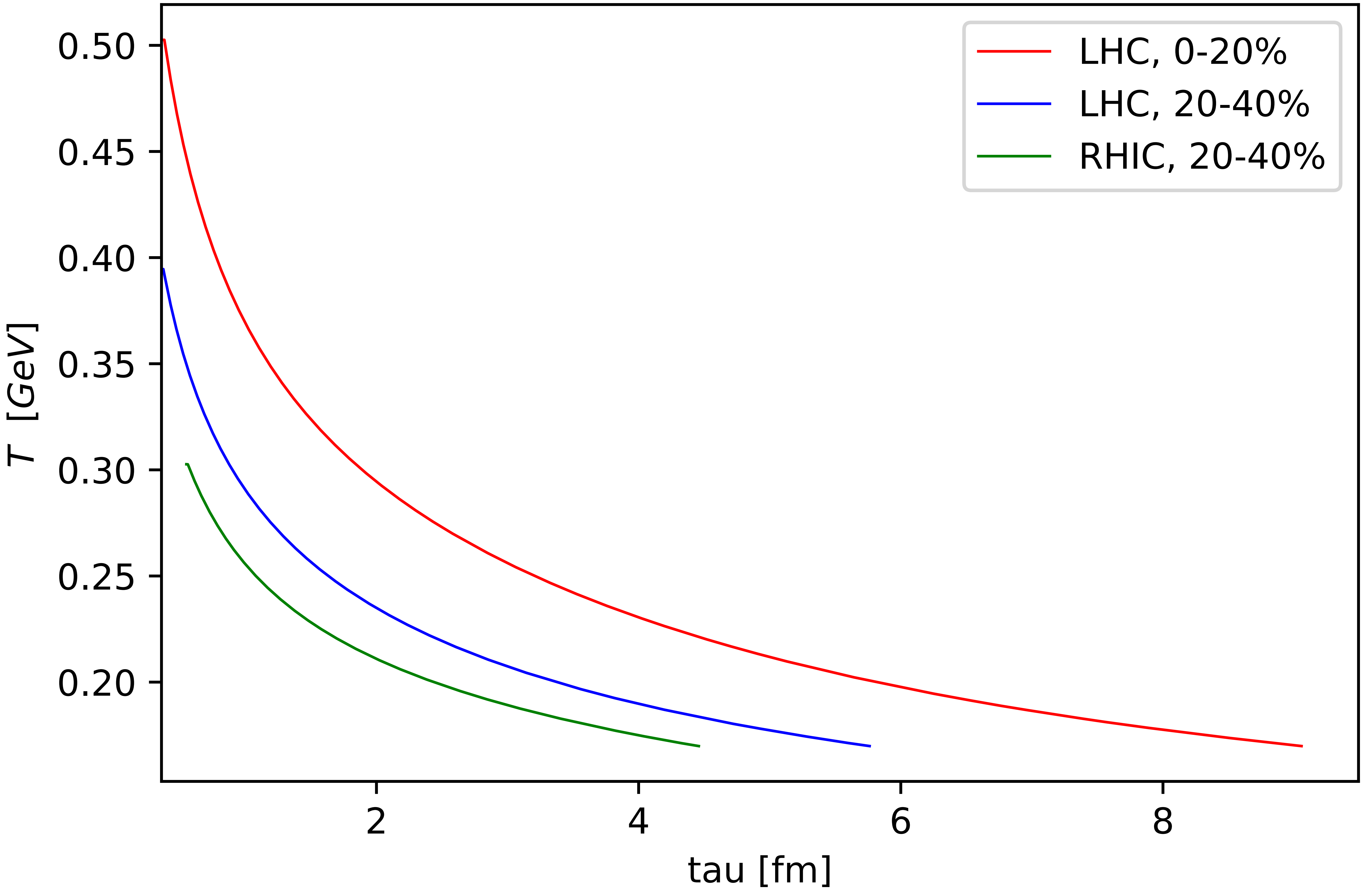
- Comparison of p_T - spectra from simulation to data from π and Ξ from ALICE





Testing the Model

Temperature of the Fireball



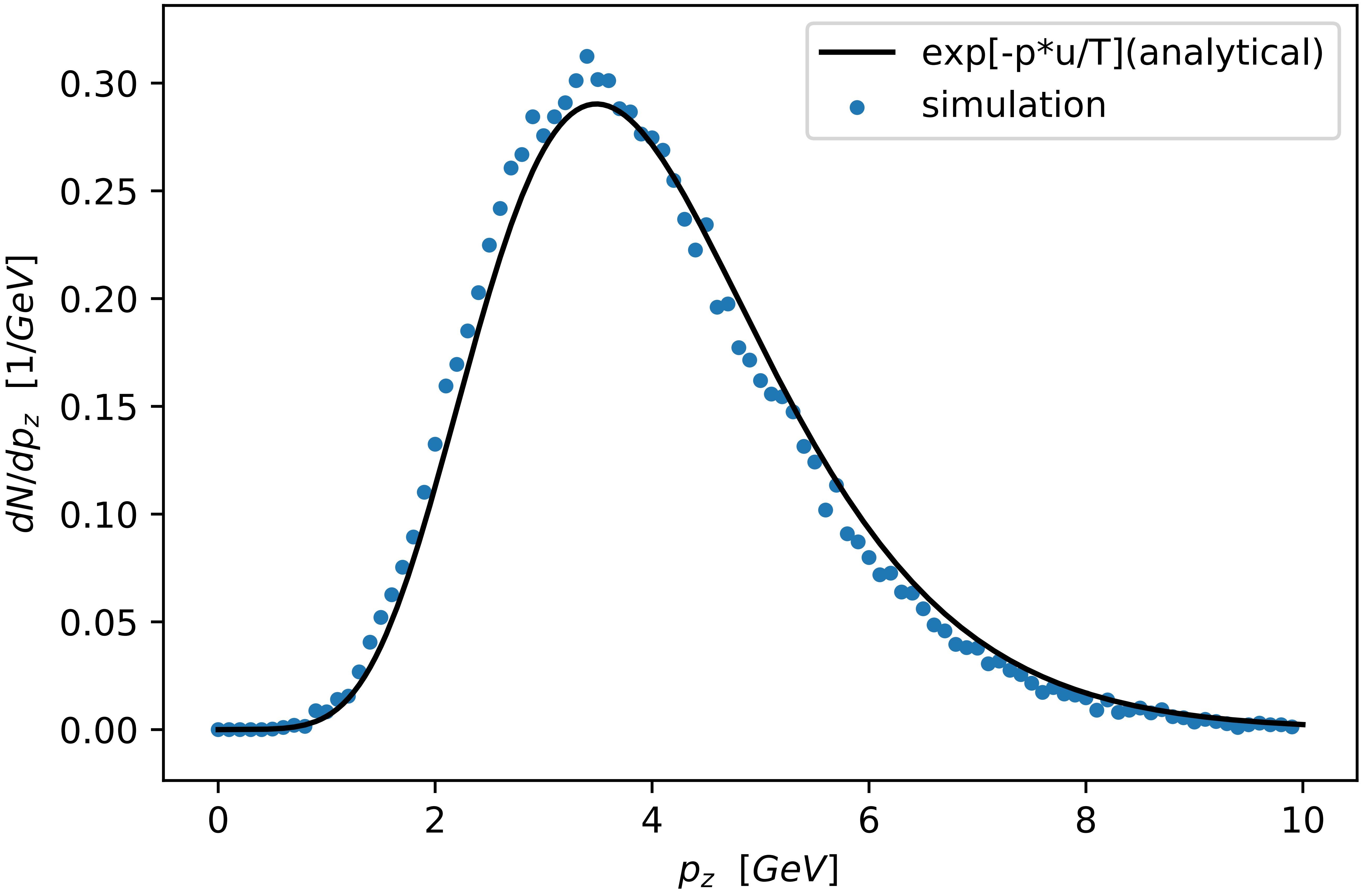
- Sequential freeze-out
- $T_{ch} = 160\ MeV$
- Isentropic expansion towards kinetic freeze-out

➡ Extrapolation to temperature in QGP-phase

- Exponential decrease of T until T_{ch}

Lorentz Boost to Moving Medium

p_z distribution



- Single $c\bar{c}$-pair in box calculation with $T = 180\ MeV$ and $m_c = 1.5\ GeV/c^2$
- constant flow-field $\boldsymbol{v} = (0, 0, 0.9)$
- Boltzmann-Jüttner distribution:

$$f_{eq}(\boldsymbol{p}) \propto \exp\left(-\frac{E(\boldsymbol{p})}{T}\right)$$

Relative energy of a $c\bar{c}$-pair

Energy distribution in equilibrium

- Relative energy of $c\bar{c}$-pair:

$$E_{rel} = E_c + E_{\bar{c}} + V(|\boldsymbol{r}_c - \boldsymbol{r}_{\bar{c}}|) - E_{tot}$$

$$= \sqrt{m_c^2 + \boldsymbol{p_c}^2} + \sqrt{m_{\bar{c}}^2 + \boldsymbol{p}_{\bar{c}}{}^2} + V(r, T) - \sqrt{(m_c + m_{\bar{c}})^2 + (\boldsymbol{p}_c + \boldsymbol{p}_{\bar{c}})^2}$$

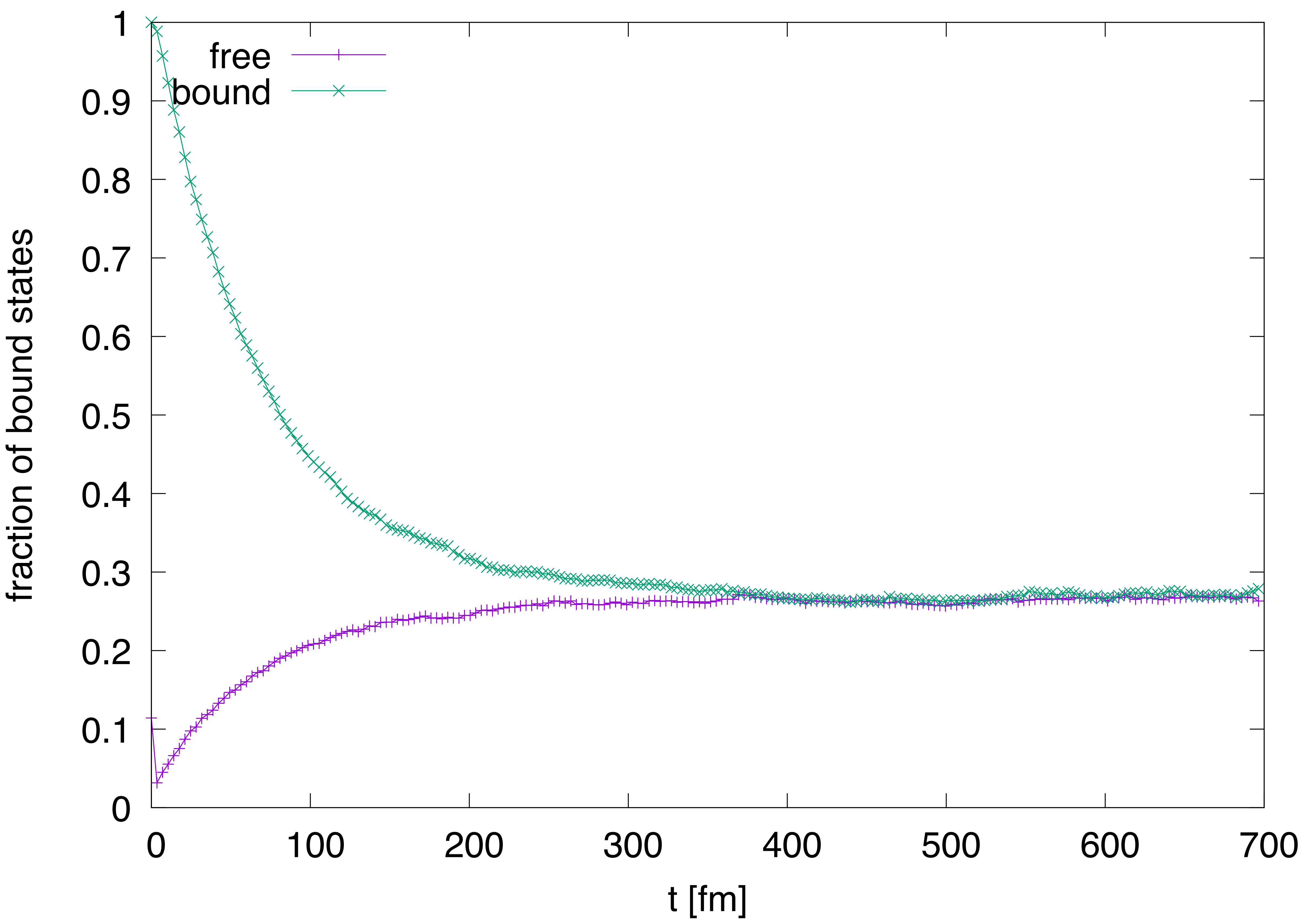
- In com-system ($(\boldsymbol{p}_c + \boldsymbol{p}_{\bar{c}}) = 0$) equivalent to

$$E_{rel} = m_{0,cms} + V(r, T) - (m_c + m_{\bar{c}})$$

With $p^{\mu}_{tot}\ p_{\mu,tot} = (E^c + E^{\bar{c}})^2 - (\boldsymbol{p_c} + \boldsymbol{p}_{\bar{c}})^2 = m^2_{0,cms}$

Thermalization of bound-state yield

Detailed Balance



- Single $c\bar{c}$-pair in box calculation:
    1. Initialisation as separate quarks
    2. Initialisation as bound state

➡ In the long-time limit the same equilibrium is reached

Input number $N_{c\bar{c}}$ for simulation

$$N_{c\bar{c}} = T_{AA}(b)\ \sigma c\bar{c} \quad (1)$$

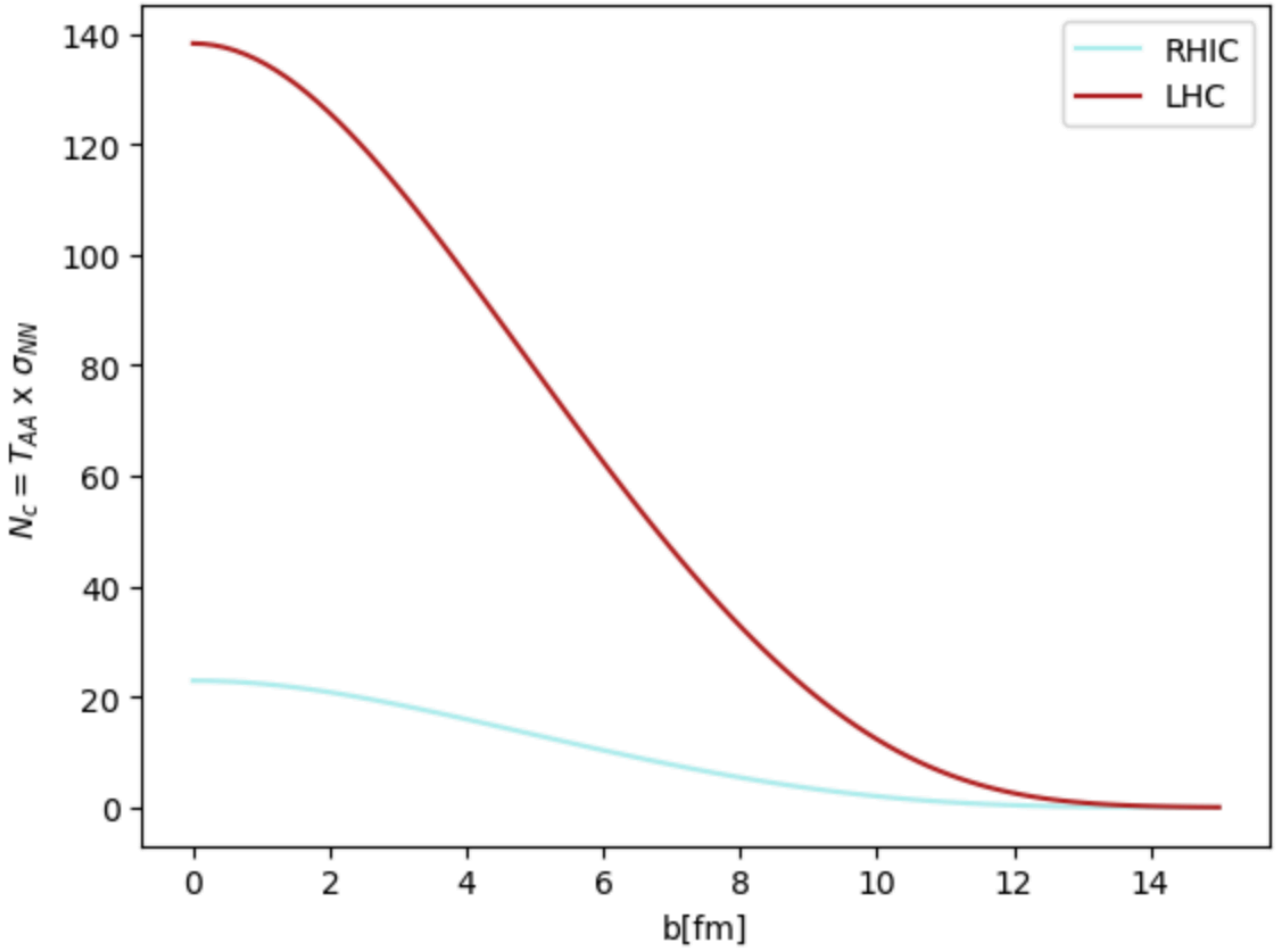
- Overlap function $T_{AA}(b) = \int_{-\infty}^{\infty} dz \rho_A(z)$ with

$$\rho_A(r) = \frac{\rho_0}{1 + \exp\left(\frac{r - r_0}{a}\right)}$$

➡ Run PYTHIA with number of pairs according to (1)

➡ Number of produced charm quarks:

RHIC, 20-40%	LHC, 0-20%	LHC, 20-40%
~ 5	~ 104	~ 39



Comparison to Grand-Canonical Ensemble

Statistical Hadronization Model, $T = 160\ MeV$

- Particle number in Grand-Canonical Ensemble

$$N = T\frac{\partial\ lnZ}{\partial\mu}, \text{ with } ln\ Z = a\sum_{\alpha}\left(1 + ae^{-(E_\alpha-\mu)/T}\right)$$

- non-relativistic classical limit, approximation of small particle numbers:

$$N = dV\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}}\lambda e^{-m/T} \quad \text{with the fugacity } \lambda = e^{\mu/T}$$

- $J/\psi \rightleftharpoons c + \bar{c}$: $\mu_{J/\psi} = 2\mu_c \longrightarrow \lambda_{J/\psi} = \lambda_c^2$

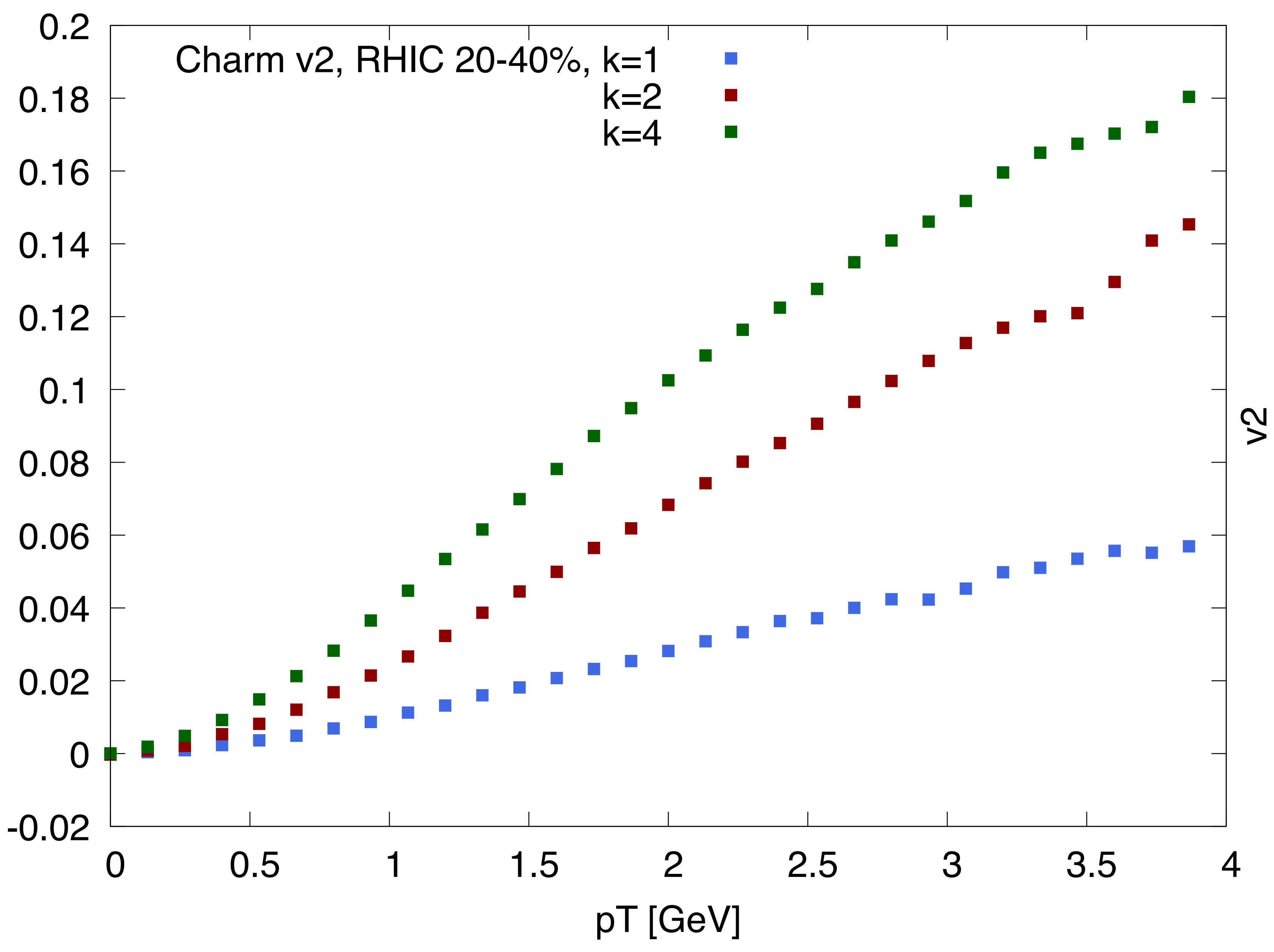
➡ J/ψ Multiplicity:

$$N_{J/\psi} = \lambda_c^2\ d_{J/\psi}\frac{V}{2\pi^{3/2}}\ \left(m_{J/\psi}T\right)^{3/2}\ \exp(-m_{J/\psi}T), \text{ with } d_{J/\psi} = 3,\ m_{J/\psi} = \langle 2m_c + E_{bin}\rangle$$

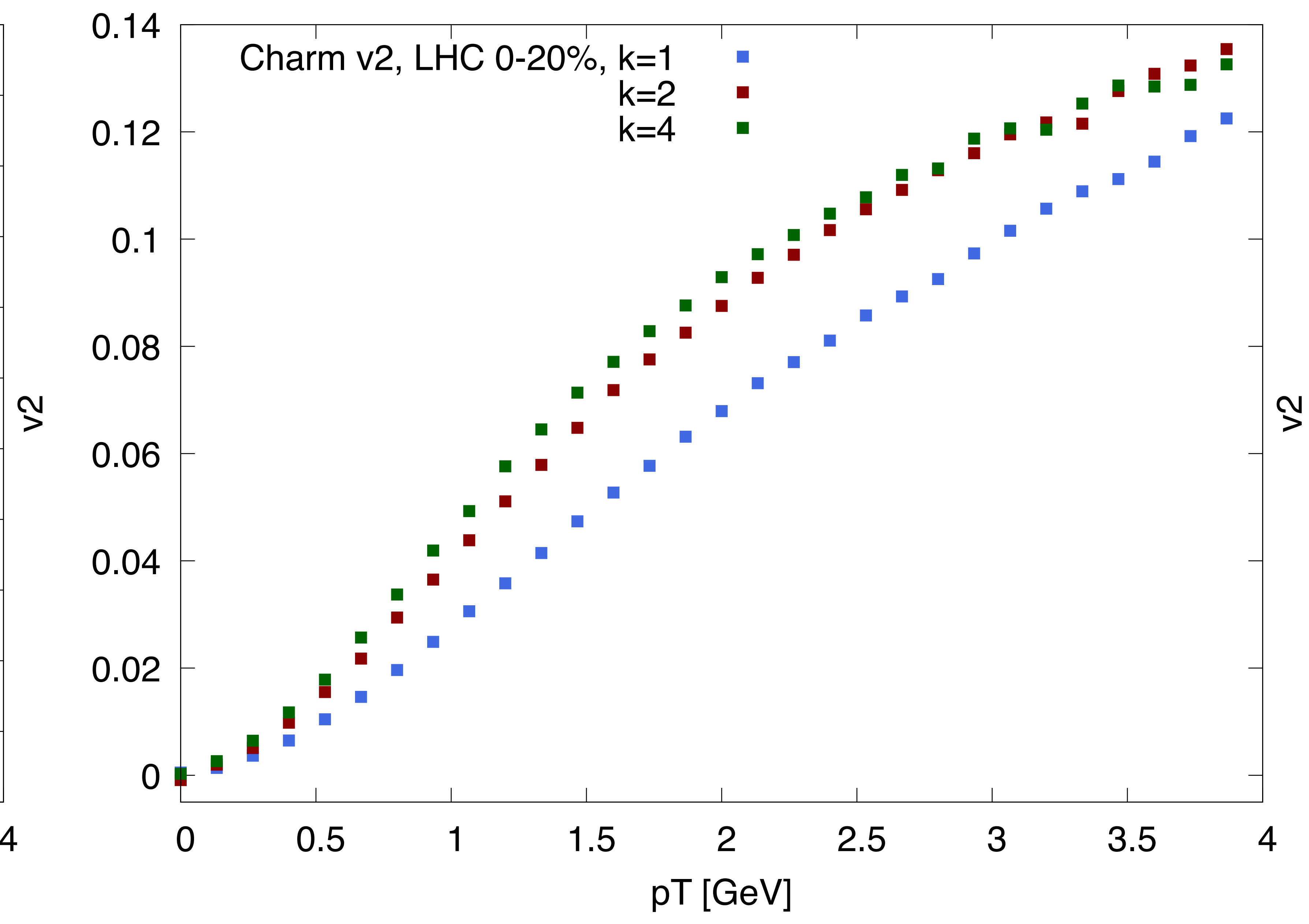
Elliptic Flow v_2: Charm Quarks, 5 Pairs

Initial momentum distribution given by parametrization to fit charm-quark spectra from PYTHIA

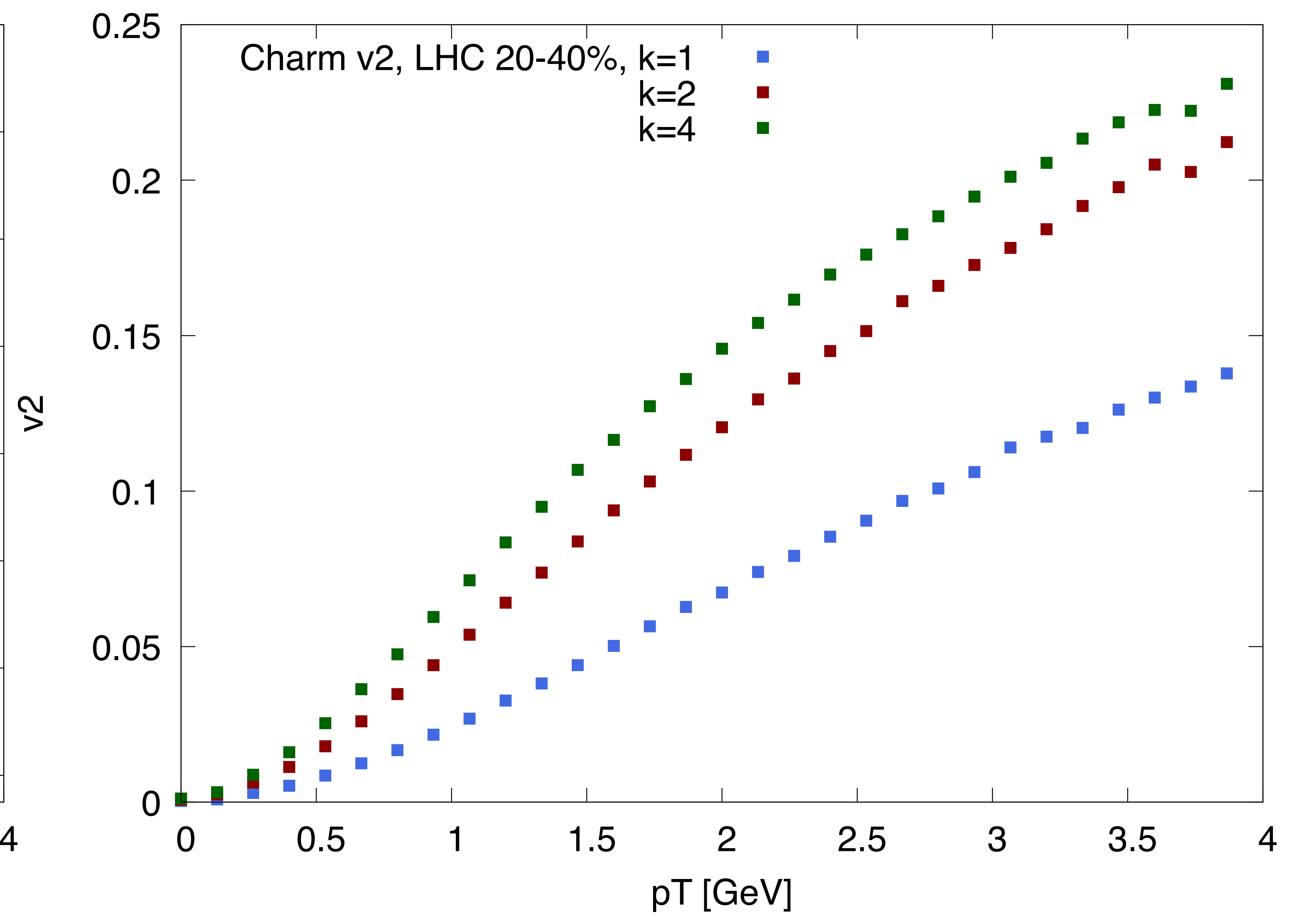
RHIC, 20-40% Centrality



LHC, 0-20% Centrality



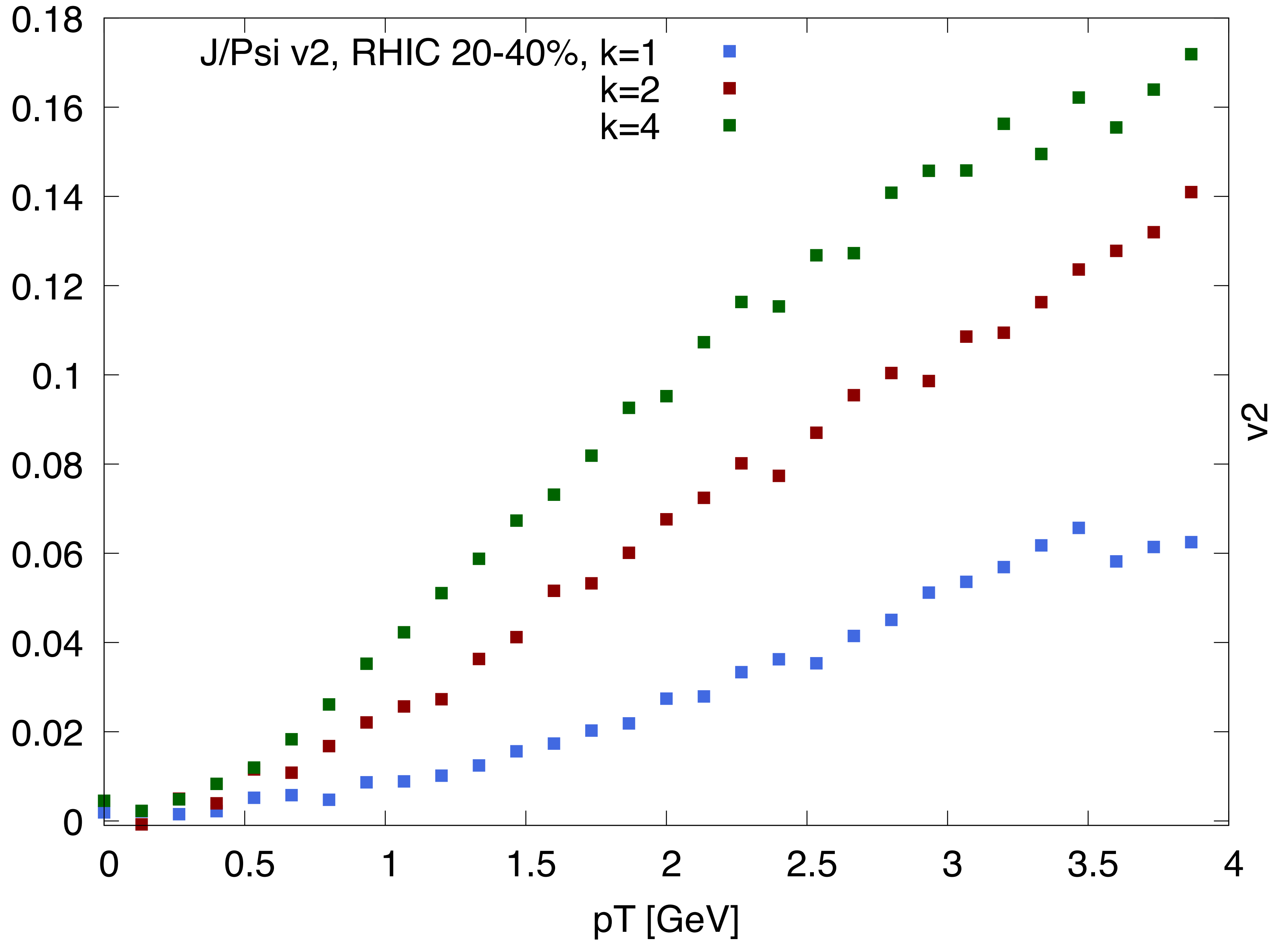
LHC, 20-40% Centrality



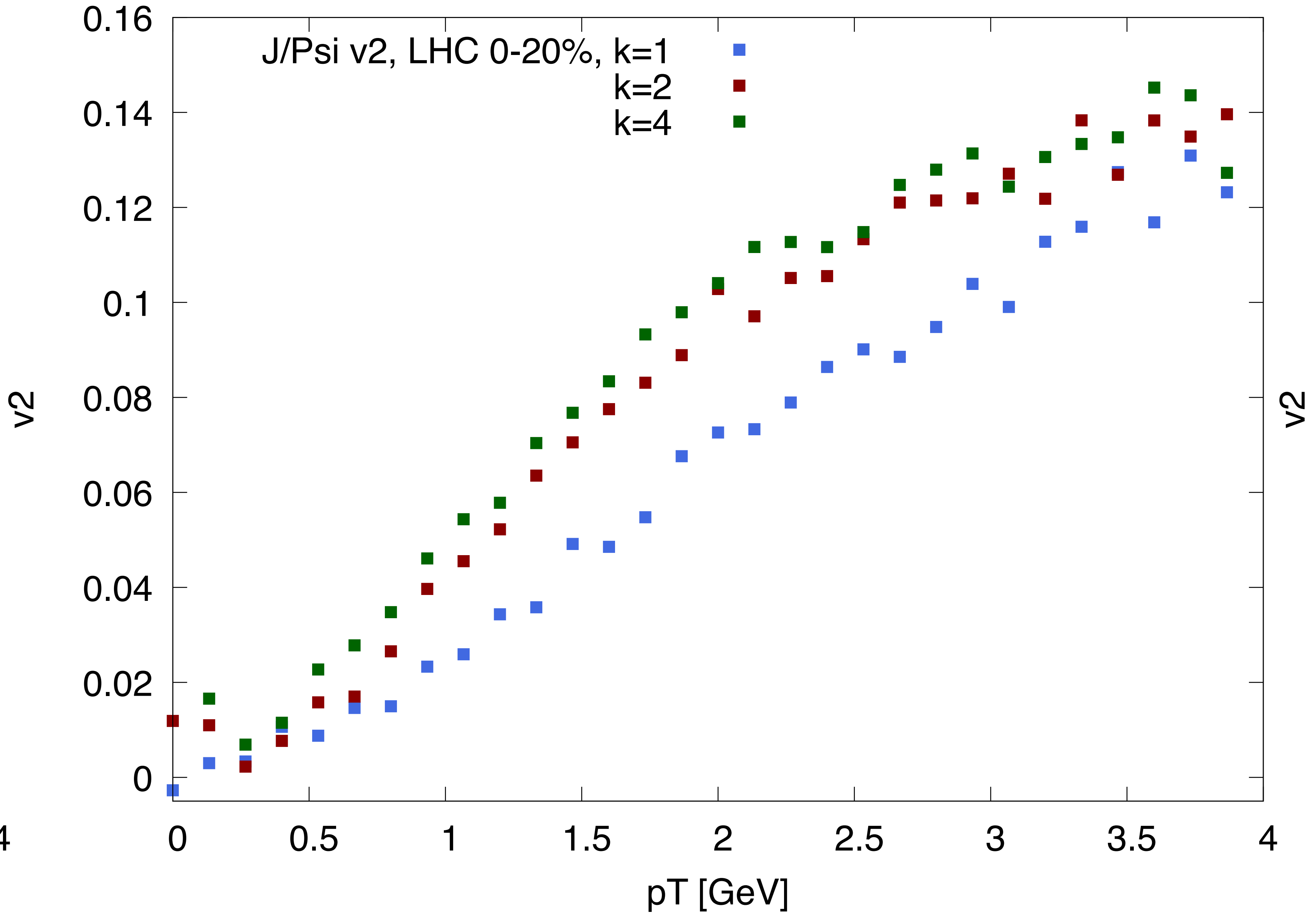
Elliptic Flow v_2 for different scalings of the drag coefficient

J/ψ, RHIC & LHC

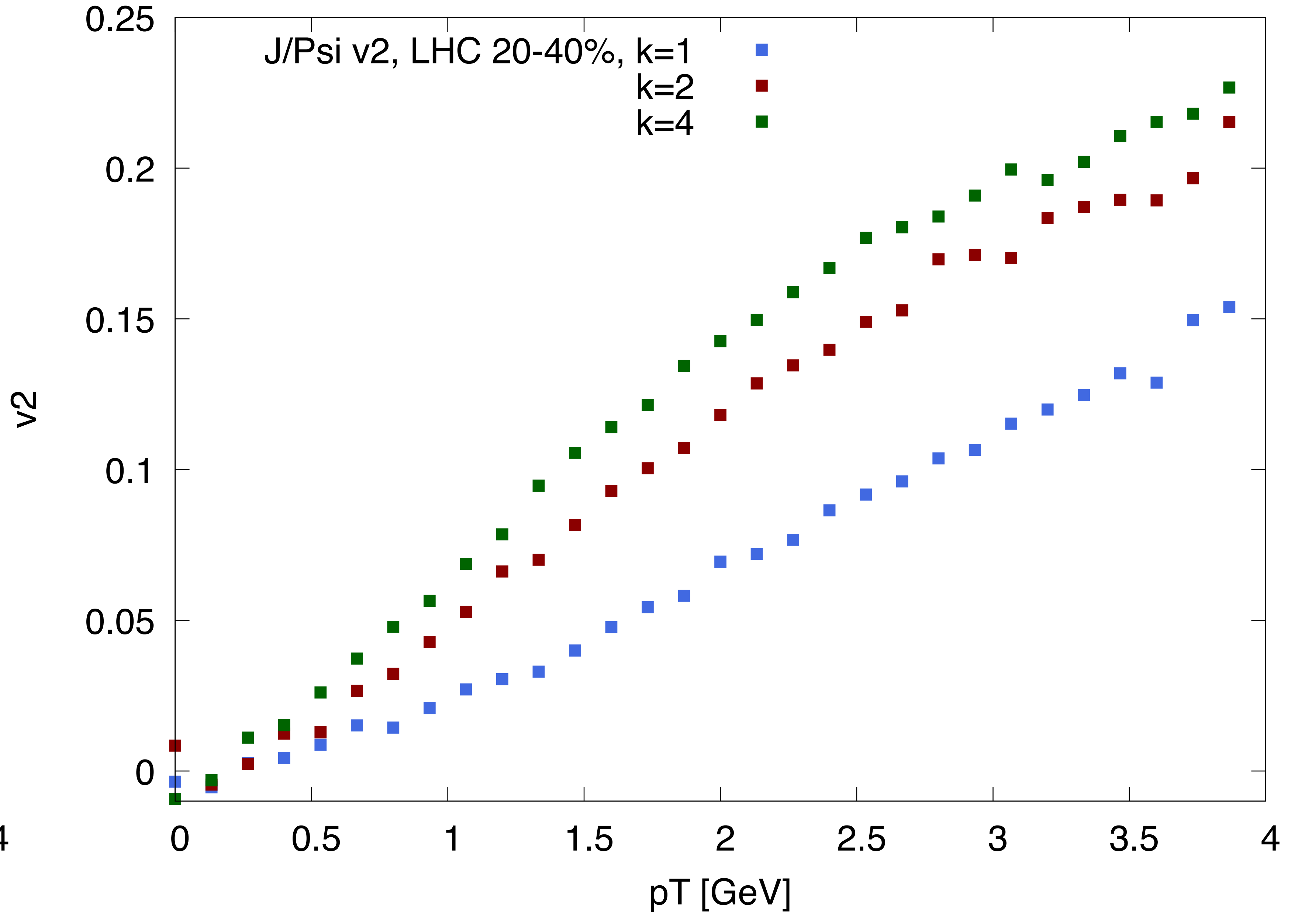
RHIC, 20-40% Centrality



LHC, 0-20% Centrality

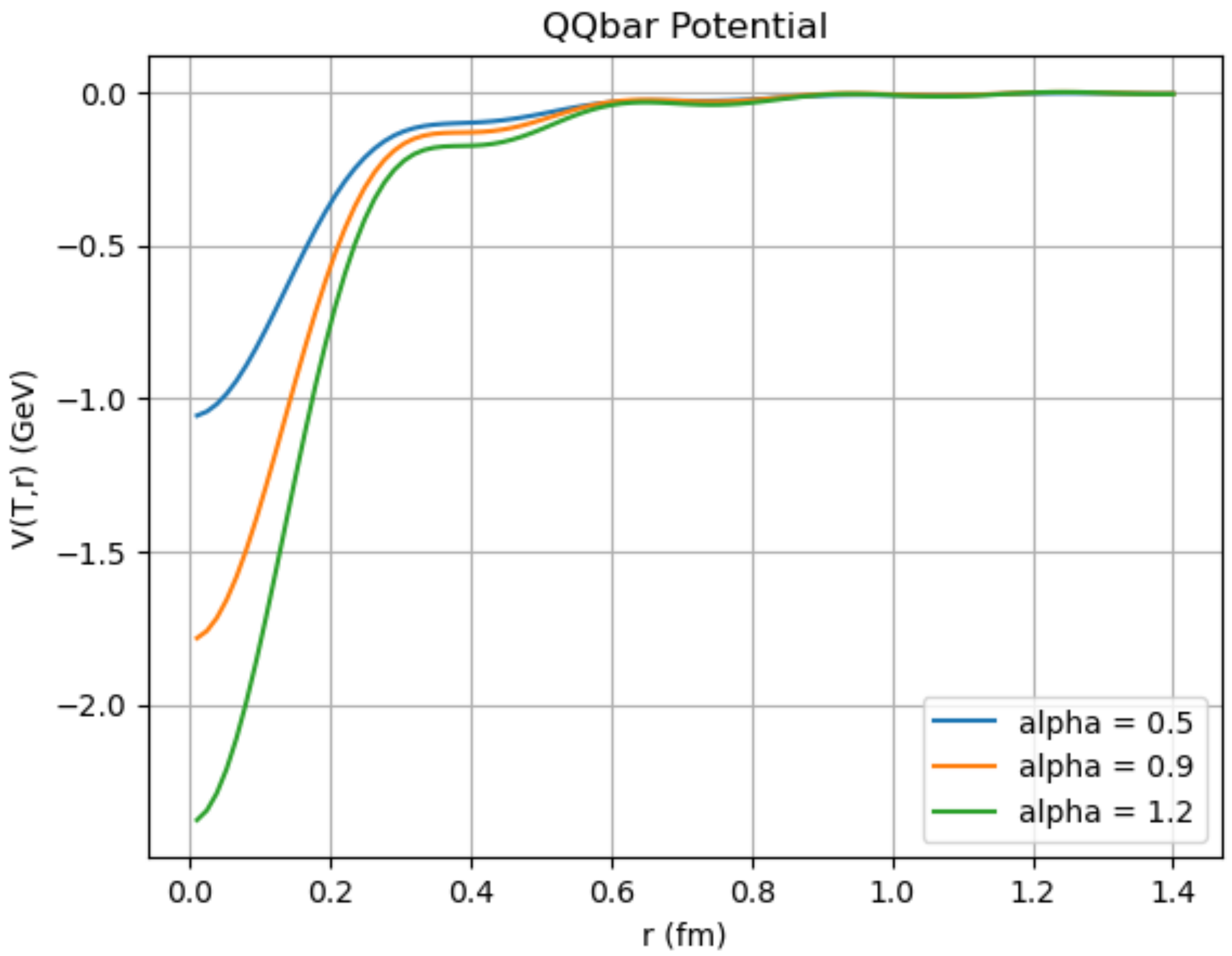


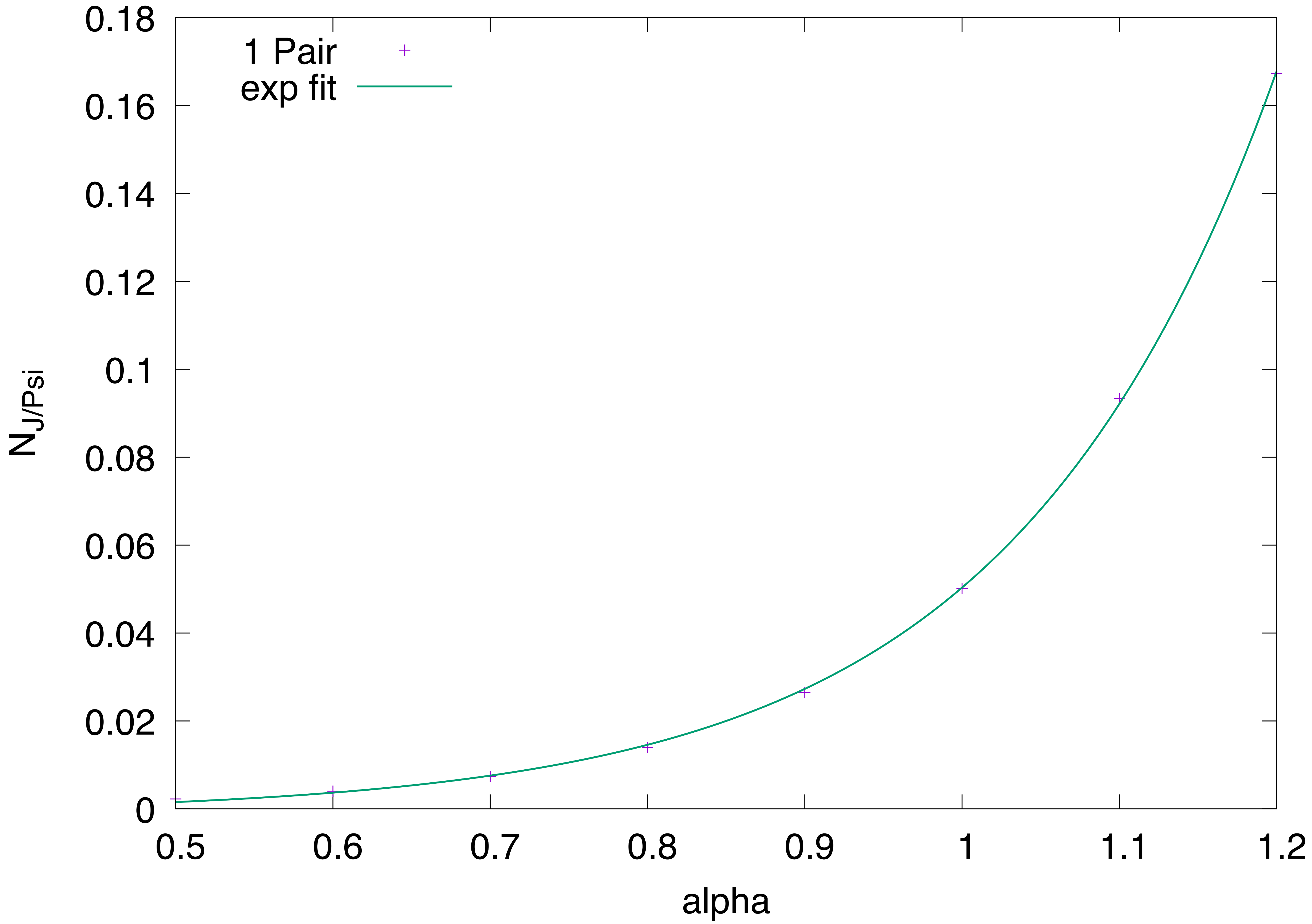
LHC, 20-40% Centrality



Strong Coupling of Potential

Influence on Number of Bound States





Strong Coupling of Potential

Comparison $\alpha_s = 0.5$ vs. $\alpha_s = 0.7$