The QCD static potential

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Introduction

static potential

$\rightarrow$ dominant interaction between heavy q-$\bar{q}$ at low energy

$T = 0$

- attractive

- coulomb-like at small r (linearly rising at large r)

$T \neq 0$

- the short-distance potential screened

- yukawa-like with screening mass $\propto T$

proposed signal for QGP formation:

suppression of heavy q-$\bar{q}$ bound state production at high T

T. Matsui and H. Satz, Phys. Lett. B **178***, 416-422 (1986)* at LO and in thermal equilibrium:

$V(r)$ from f-transform of temporal component of thermal gluon propagator in zero frequency limit

real part: (using $\alpha = g^2 C_F/(4\pi)$)

$$G_{\rm lo}(0,p) = -\frac{1}{m_D^2 + p^2}$$

$$V_{1\rm lo}(\vec{r}) = g^2 C_F \int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\cdot\vec{r}}\, G_{\rm lo}(0,p) = -\frac{\alpha}{r} e^{-m_D r}$$

will consider:

1. effects of an aniostropic momentum distribution on the real part

MEC, G. Kustatter and A. Mukherjee, arXiv:2405.05622

2. corrections in thermal equilibrium beyond LO

MEC, C. Manuel and J. Soto, arXiv:2407.00310

anisotropy

anisotropic distribution function:

$$n(\vec{k}) = C_\xi \, n\big(kH_\xi(\vec{v})\big) \quad \text{with} \quad n(k) = \frac{1}{e^{\beta(k)} \mp 1} \quad \text{and} \quad \vec{v} = \hat{k}$$

spheroidal anisotropy $(\hat{n}_3 = \hat{z})$

$$H_\xi^2(\vec{v}) = 1 + \xi(\vec{n}_3 \cdot \vec{v})^2$$

P. Romatschke and M. Strickland, Phys. Rev. D **68**, *036004 (2003).*

generalization $(\hat{n}_1 \cdot \hat{n}_3 = 0)$

$$\begin{aligned} H_\xi^2(\vec{v}) \quad = \quad & (1+\xi_0) + \xi_2(\vec{n}_1\cdot\vec{v})^2 + \xi_9(\vec{n}_3\cdot\vec{v})^2 + \xi_6(\vec{n}_1\cdot\vec{v})(\vec{n}_3\cdot\vec{v}) + \xi_4(\vec{n}_1\cdot\vec{v})^4 \\ & + \xi_8(\vec{n}_1\cdot\vec{v})^3(\vec{n}_3\cdot\vec{v}) + \xi_{11}(\vec{n}_1\cdot\vec{v})^2(\vec{n}_3\cdot\vec{v})^2 + \xi_{13}(\vec{n}_1\cdot\vec{v})(\vec{n}_3\cdot\vec{v})^3 + \xi_{14}(\vec{n}_3\cdot\vec{v})^4 \,. \end{aligned}$$

set ξ_i: iso distro expanded/contracted in dirn $\hat{v}$ if $H_\xi(\vec{v}) < / > 1$

MEC, B.M. Forster and S. Makar, Phys. Rev. C **104**, *064908 (2021).*

normalization:

$$[m_D^2]_\xi = 8g^2 \frac{C_\xi}{(2\pi)^3} \int d\Omega \int dk\, k\, n_f(kH) \equiv m_D^2$$

motivation: makes threshold of potential independent of ξ_i

- will calculate binding energy from schrödinger equation

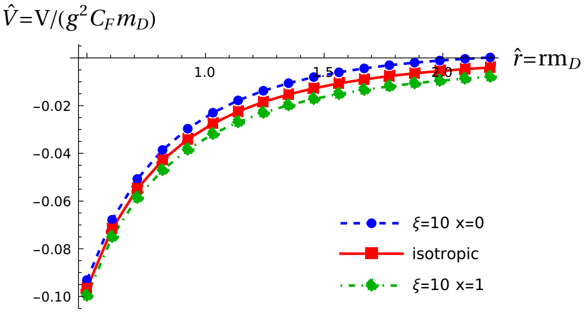
$\Rightarrow$ want to change ξ_i without moving the whole potential up/down

the calculation:

1. $n(\vec{k}) \to$ calculate $\Pi_{\mu\nu}$ in the hard-loop approximation
2. find the propagator by inverting $D^{-1}_{\mu\nu} = D^{-1}_{0\mu\nu} - \Pi_{\mu\nu}$
3. temporal component $D_{00}(\vec{p}) \to V(\vec{r})$

results:

spheroidal distribution:



if q-$\bar{q}$ pair aligned/anti-aligned with dirn of anisotropy ($x = \cos(\theta)$)
$\rightarrow$ potential is deeper/shallower than isotropic

** for more general distributions the structure is richer

results: weak anisotropy

$\bar{m}_D = m_D(1 - \sum_i \xi_i h_i)$ where h_i are numerical coefficients

$$V(r,\theta,\phi) = -\alpha \bar{m}_D - \frac{\alpha e^{-rm_D}}{r}(1 - \Sigma_i \xi_i f_i(rm_D,\theta,\phi)) - \alpha(1 - e^{-rm_D})(m_D - \bar{m}_D)$$

satisfies:

$\lim_{\xi_i \to 0} V(r,\theta,\phi) = -\alpha m_D - \frac{\alpha}{r} e^{-rm_D}$

$\lim_{r \to 0} V(r,\theta,\phi) = -\frac{\alpha}{r}$

$\lim_{r \to \infty} V(r,\theta,\phi) = -\alpha m_D$

effect of anisotropy on binding energy

method for weak aniosotropy:

- ansatz: $V(\vec{r}) \rightarrow$ effective screening mass $\tilde{m}_D(rm_D, \theta, \phi)$
- reformulate result with an angle averaged screening masses

- constructed from integrals weighted with spherical harmonics

$\Rightarrow$ physical effects of the anisotropy are packaged into screening masses that depend on the anisotropy parameters

A. Dumitru, Y. Guo, A. Mocsy and M. Strickland, Phys. Rev. D **79**, 054019 (2009).
L. Dong, Y. Guo, A. Islam and M. Strickland, Phys. Rev. D **104**, 096017 (2021).
L. Dong, Y. Guo, A. Islam, A. Rothkopf and M. Strickland, JHEP **09**, 200 (2022).
A. Islam, L. Dong, Y. Guo, A. Rothkopf and M. Strickland, EPJ Web Conf. **274**, 04015 (2022).

$\Rightarrow$ calculate binding energies by solving 1d schrödinger equation results: ground state E_b minus binding energy of isotropic state

-	ξ_2	ξ_4	ξ_{14}	ξ_{11}
-0.617	-4.30	-1.52	-1.52	-0.866

$\xi_9 = 0.95$ and $g = 1.85$

parameter in top row is 0.8 and all others set to zero

$|E_b|$ increases - anisotropy promotes binding in the ground state

- *not always true for excited states*

$$H_\xi^2(\vec{v}) = 1 + \xi_2(\vec{n}_1 \cdot \vec{v})^2 + \xi_9(\vec{n}_3 \cdot \vec{v})^2 + \xi_4(\vec{n}_1 \cdot \vec{v})^4 + \xi_{11}(\vec{n}_1 \cdot \vec{v})^2(\vec{n}_3 \cdot \vec{v})^2 + \xi_{14}(\vec{n}_3 \cdot \vec{v})^4 + \dots$$

real time static potential beyond LO in equilibrium

MEC, C. Manuel and J. Soto, arXiv:2407.00310

motivation:

static potential has an imaginary part

$\mathrm{Im}[V] > \mathrm{Re}[V]$ when screening effects become important

- bound states disappear because decay (become wide resonances)
- not because V is screened *too shallow to support them*

M. Laine, O. Philipsen, P. Romatschke and M. Tassler, JHEP **03**, *054 (2007).*

- work beyond leading order
- in the temperature range where bound states start melting

• a check of the idea of quarkonium dissociation

• provides a wider set of physically motivated forms of the potential

- to use as input for methods to extract V from lattice correlators

calculational method

potential obtained from real time QCD (rectangular) wilson loop

$$W(t,r) = \frac{1}{N_c}\left\langle \mathcal{P}\mathrm{exp}\left(ig\int A_\mu(z)z^\mu\right)\right\rangle$$

$$V(r) = \lim_{t\to\infty}\frac{i}{t}\ln[W(t,r)]$$

- thermalized plasma
- $M_q \gg$ all other physical scales

  static quark and antiquark are (unthermalised) probe particles
- couple to A_0 on t-ordered branch of CTP contour
- we use Coulomb gauge and dimensional regularization

$\tilde{V}_{\rm lo}(p)$: $g \ll 1$ & typical $\vec{p}$ exchange btwn q-$\bar{q}$ is $p \ll T$

$$\tilde{V}_{\rm lo}(p) = g^2 C_F \, G_{\rm lo}(0,p)$$
$$G_{\rm lo}(0,p) = -\frac{1}{m_D^2 + p^2} + \frac{i\pi T m_D^2}{p\left(m_D^2+p^2\right)^2}$$

we calculate beyond LO corrections to $\tilde{V}_{\rm lo}(p)$

we consider $p \sim g^a T$ with $1/3 < a < 2/3$

$\Rightarrow$ which gives $m_D \ll p \ll T$

- upper bound on p: from condition ${\rm Re}\tilde{V}_{\rm lo}(p) \sim {\rm Im}\tilde{V}_{\rm lo}(p)$

\- width is so big the bound state decays

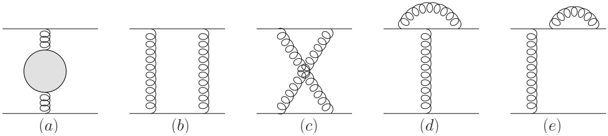
- lower bound on p: require p semi-hard

\- calculation of beyond leading order potential is simplified

consequences: $V(r)$ valid for $r\, m_D \ll 1 \ll r\, T$

how to calculate static potential beyond leading order

- expand W to higher order in g
- dress the propagator in the LO contribution



- iterate the LO potential (not shown)

determine how to dress lines/vertices for $p \sim g^a T$ with $\frac{1}{3} < a < \frac{2}{3}$

comment about power counting

calculation of nlo htl n-point functions

$\rightarrow$ follow prescription ...

for the static potential there are two important differences

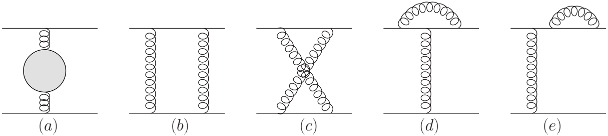
1. fermion lines have the form $\frac{1}{p_0 \pm i\eta}$ ($M_q \gg$ all other scales)
2. external frequencies are taken to zero

$\Rightarrow$ external momenta don't flow through the diagram as expected

** power counting is different from standard thermal field theory

- we take into account corrections to LO:
  real part larger than g^2 & imag part larger than (g^{3a}, g^{2-a})
- denominators $\sim p^2 + m_D^2$ kept unexpanded *(damped approximation)*

$\rightarrow$ extends region that coordinate space potential is valid



in the self-energy diagram we need:

- power correction to htl gluon bubble

- one loop gluon bubble with loop momenta semi-hard

Re[*V*] *in A. K. Rebhan, Phys. Rev. D* **48**, *R3967 (1993)*
Im[*V*] *in J. Q. Zhu, Z. L. Ma, C. Y. Shi and Y. D. Li, Nucl. Phys. A* **942**, *54-64 (2015)*

for all other diagrams (ladder diagrams):

- htl propagators and bare vertices

also: static quark self-energies

$\rightarrow$ *constant contributions that we have not calculated* coordinate space potential beyond leading order

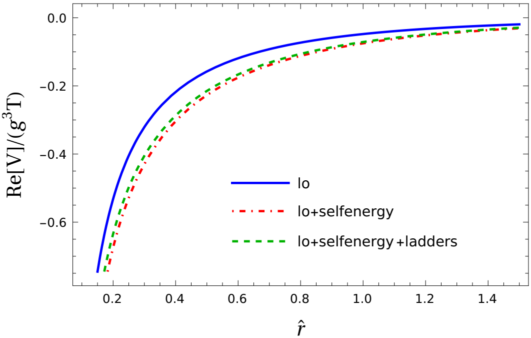
$$\hat{r} = r m_D$$

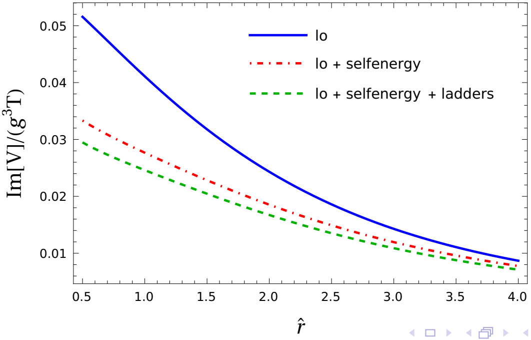
$$I_j(\hat{r}) = \int_0^\infty d\hat{p}\, \sin(\hat{p}\hat{r})\,(\hat{p}^2+1)^{-j}$$

$$V_{1\mathrm{lo}} = -\frac{g^2 C_F}{4\pi\hat{r}}\left(m_D e^{-\hat{r}} - 2iT\, I_2(\hat{r})\right)$$

$$\mathrm{Re}[V_2] = \frac{g^4 N_c C_F T}{64\pi^2\hat{r}}\left\{8\left(I_2(\hat{r}) - I_1(\hat{r})\right) + \frac{e^{-\hat{r}}}{16}\left(3\pi^2 - 16 + \frac{\hat{r}}{6}\left(16-\pi^2\right)\right)\right\}$$

$$i\mathrm{Im}[V_2] = -i\frac{g^3 C_F\, T}{16\pi^2\hat{m}_D}\left\{\frac{3\pi^2-16}{32\,\hat{r}} I_2(\hat{r}) + \frac{7}{3}\, N_c e^{-\hat{r}} - \frac{2g\,\hat{m}_D}{\pi\hat{r}}\left(N_c - \frac{N_f}{2}\right)\left(I_1(\hat{r}) - I_2(\hat{r})\right)\right\}$$


0.0
−0.2
−0.4
−0.6
Re[V]/(g^3T)
lo
lo+selfenergy
lo+selfenergy +ladders
0.2
0.4
0.6
0.8
1.0
1.2
1.4
r̂


0.05
0.04
0.03
0.02
0.01
Im[V]/(g^3T)
lo
lo + selfenergy
lo + selfenergy + ladders
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
r̂

results: compare to lattice calculations

- include contributions to $V(r)$ from $p \sim m_D$

- have a universal form at any order in g

since $m_D r \ll 1$ can expand exponential $e^{i\vec{p}\cdot\vec{r}}$ in this region

$\rightarrow$ a polynomial in r (up to possible logarithms)

- add contributions:

$$\text{Re}[V] = C + g^3 r_0 T$$
$$\text{Im}[V] = g^3 i_0 T + g^5 i_2 r^2 T^3$$

coefficients obtained by fitting to lattice results

C is a global constant that adjusts the origins of the energies

note also:

- heavy q self-energy contributions don't need to be calculated

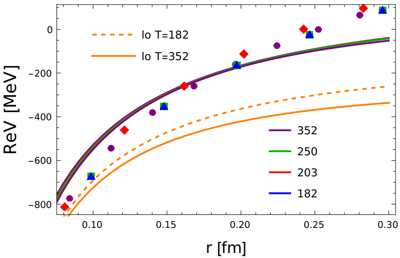
- they are absorbed into the fitted constants *lattice calculation: A. Bazavov, D. Hoying, O. Kaczmarek, R. N. Larsen, S. Mukherjee, P. Petreczky, A. Rothkopf and J. H. Weber, [arXiv:2308.16587 [hep-lat]].*

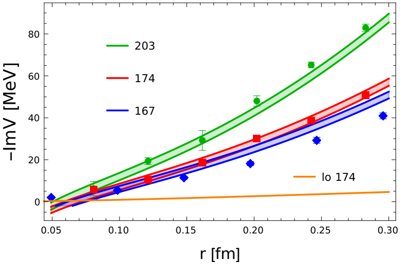
use $g = 1.8$ from fit to $T = 0$ lattice data with $r \in (0.0, 0.3)$ fm

find (C, r_0, i_0, i_2) with fit to all available T and $r \in (0.02, 0.3)$ fm

- real part of potential varies little with T (like data)
- imaginary part gets big contro from soft region

– solid bands are uncertainties in fitted coefficients inherited from lattice data





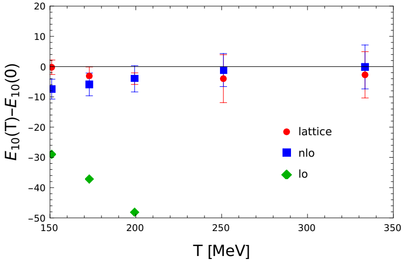
lattice calculation: R. Larsen, S. Meinel, S. Mukherjee and P. Petreczky, Phys. Lett. B **800***, 135119 (2020).*

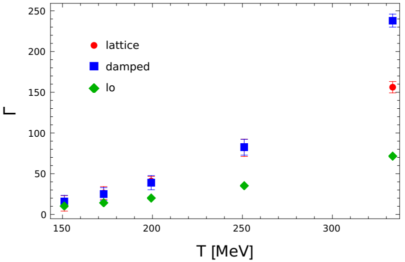
solve the schrödinger equation using our result for Re$[V]$

$\to$ binding energies and $\Gamma = -\langle \mathrm{Im}[V] \rangle$

find coefficients by fitting to all available temperatures

- error bars from fitting to upper/lower values





$\Rightarrow$ reasonable description of data for both E-bind and Γ

fitted soft contribution

contribution to $V(r)$ from $p \sim m_D$

$\mathrm{Re}[V] = C + g^3 r_0 T$

$\mathrm{Im}[V] = g^3 i_0 T + g^5 i_2 r^2 T^3$

find values of coefficients by fitting to 2 sets of lattice data

in our calculation all scales are explicit

$\rightarrow$ expect same size for numerical coefficients from the two fits

$(r_0, i_0, i_2) = (0.049, \quad -0.021 \pm 0.002, 0.205 \pm 0.001)$

$(r_0, i_0, i_2) = (0.078 \pm -0.004, -0.026 \pm 0.009, 0.053 \pm 0.002)$

i_2 from the first fit is significantly larger

C=219 MeV from first calculation

in second the coulomb binding energy is subtracted (C plays no role) dissociation:

- bound states disappear because decay (become wide resonances)
- not because V is screened *too shallow to support them*

$T_{\rm diss} \approx$ temperature where ground state $E_{\rm bind} = \Gamma = -2\langle {\rm Im}\, V\rangle$

- define $E_{\rm bind}$ as eigenvalue of V with threshold set to 0

lo result: $T_{\rm diss} = 193.2$ MeV

beyond-lo result: $T_{\rm diss} = 151.8 \pm 1.2$ MeV $\leftarrow$ using first fit

*** outlying result for i_2*

beyond-lo result: $T_{\rm diss} = 225 \pm 10$ MeV $\leftarrow$ using second fit

consistent with other lattice studies predict $>$ crossover T

G. Aarts, C. Allton, T. Harris, S. Kim, M. P. Lombardo, S. M. Ryan and J. I. Skullerud, JHEP 07, 097 (2014).

conclusions

- anisotropy changes real part of the LO potential

- general effect is to promote binding

- calculated beyond-lo corrections to momentum space potential

- when the typical momentum transfer p fulfils $m_D \ll p \ll T$

- relevant region to obtain dissociation T for heavy quarkonium

$\Rightarrow V(r)$ determined up to a polynomial in r^2 (and possible $\log(r)$)

- encodes the contribution for $p \lesssim m_D$

- describes reasonably well 2 different sets of lattice data (LO fails)