Flow harmonics of charmonium states in heavy ion collisions

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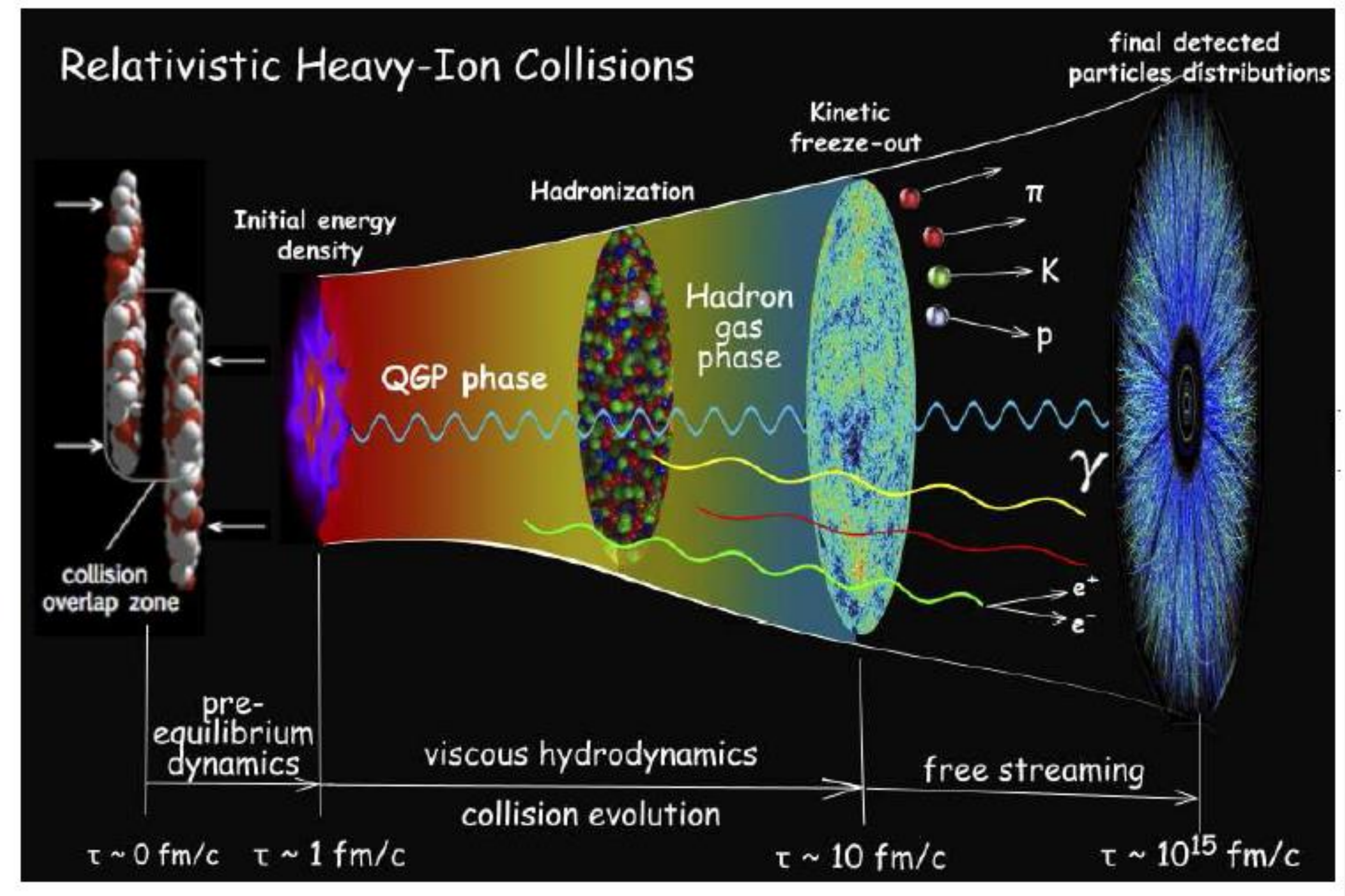
Introduction

– Relativistic heavy ion collisions

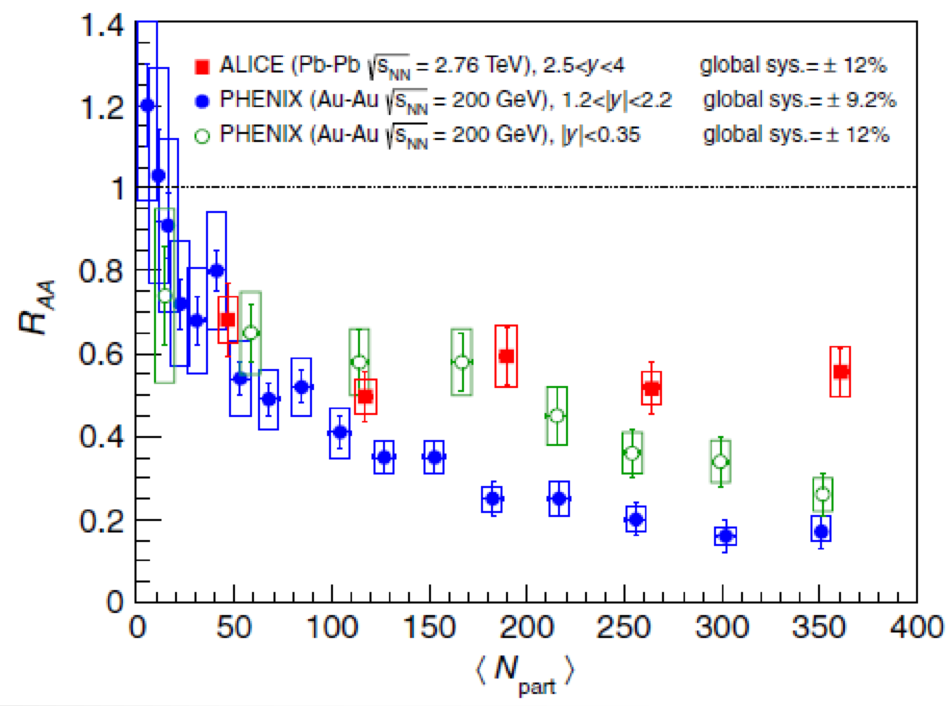


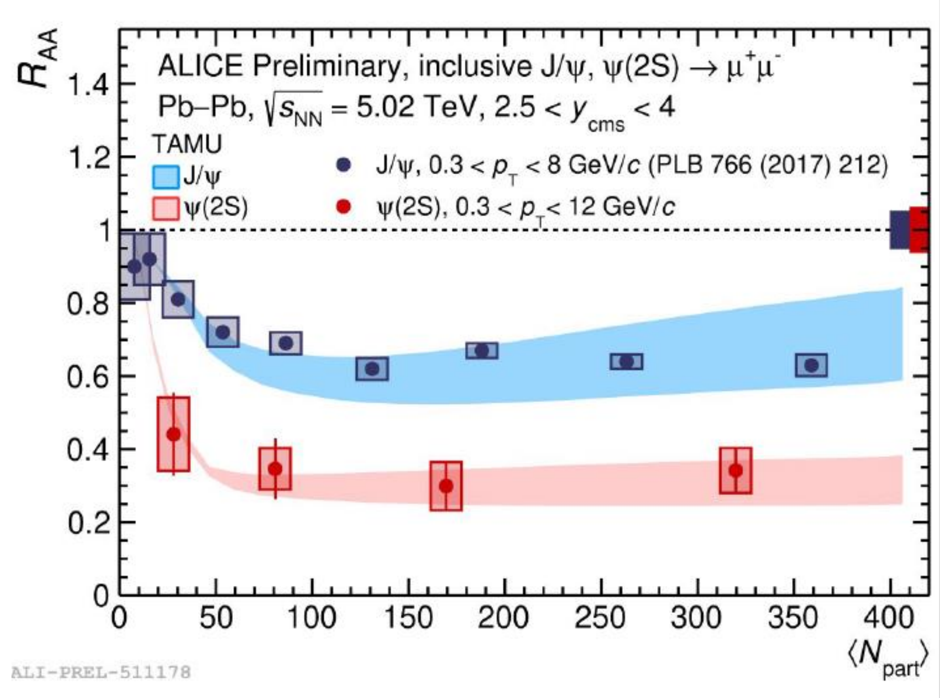
U. W. Heinz, J. Phys. Conf. Ser. **455**, 012044 (2013)

– Regeneration of charmonium states

1) The nuclear modification factor of charmonium states

B. Abelev et al, (ALICE Collaboration), Phys. Rev. Lett. **109**, 072301
Jon-Are Saetre (Univ. of Bergen), Quark Matter 2022, Krakow, April 4-10

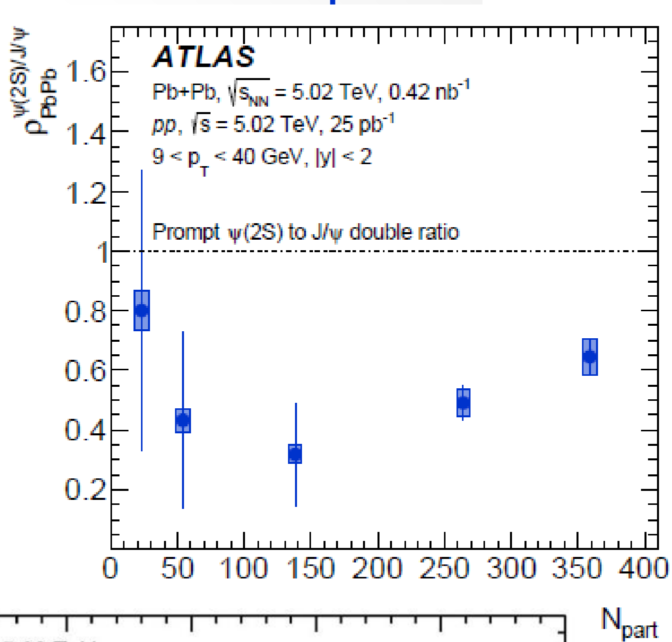


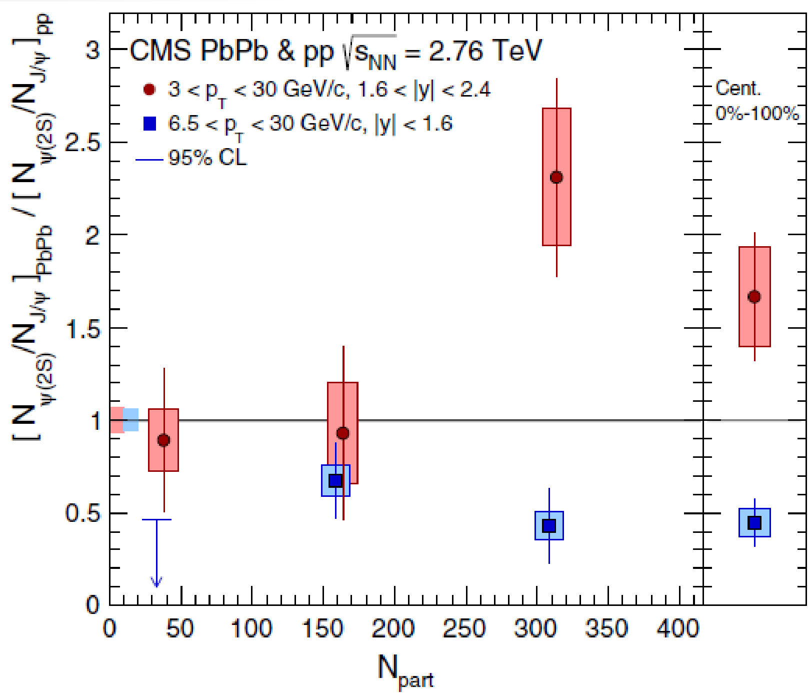


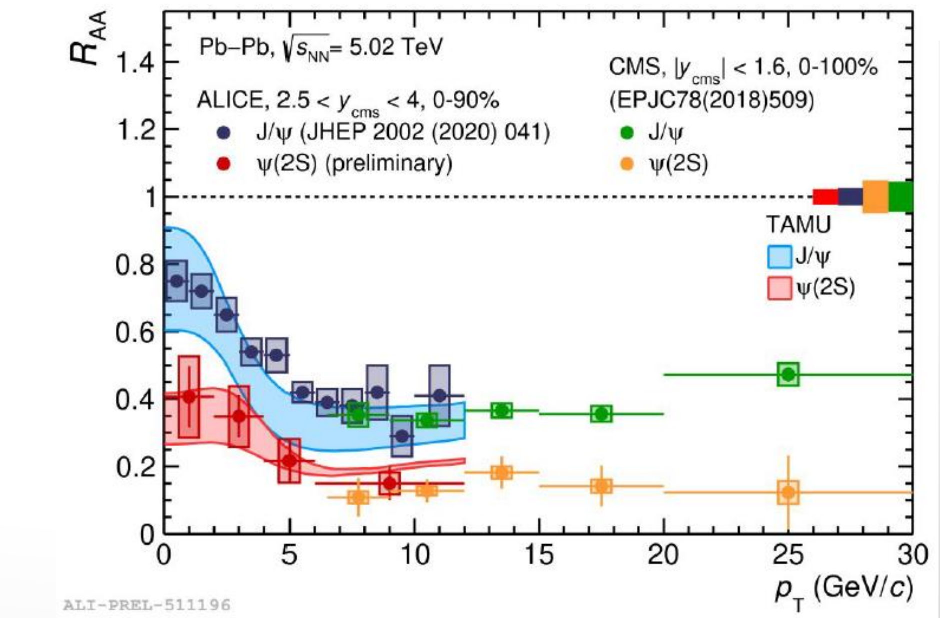
– Nuclear modification factors of the J/ψ and ψ(2S)

1) The nuclear modification factor ratio between the J/ψ and ψ(2S)

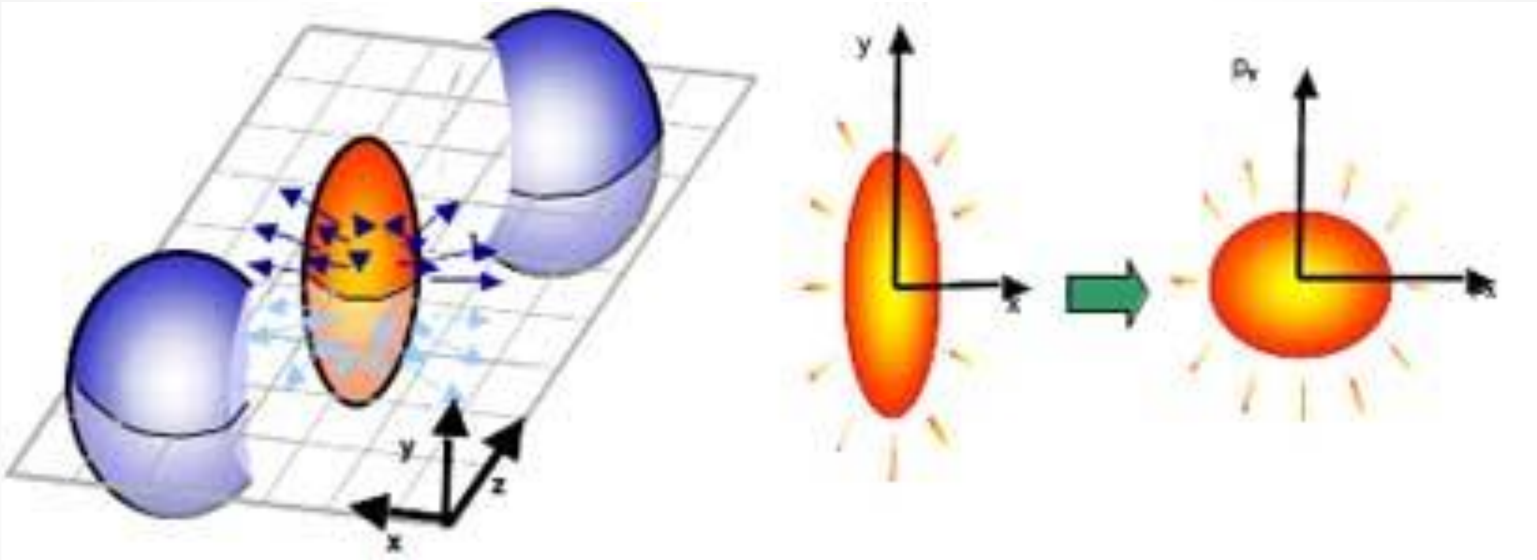
V. Khachatryan et al, Phys. Rev. Lett. **113,** 262301 (2014)
M. Aaboud et al, Eur. Phys. J. C **78,** 762 (2018)







– Non-central collisions, anisotropic flows



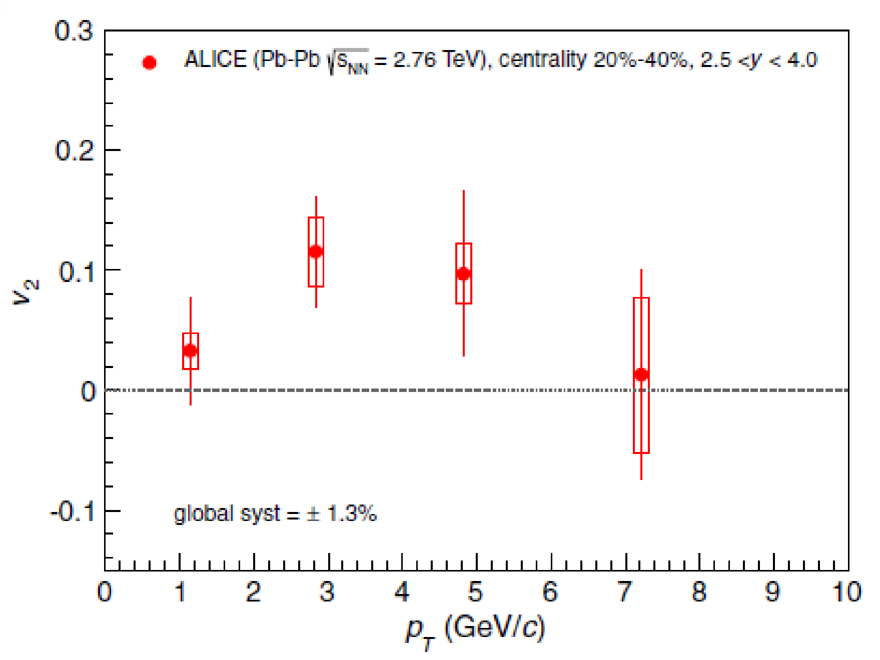
$$E\frac{d^3N_q}{d^3p} = \frac{dN_q}{p_T dp_T d\varphi dy} = \frac{1}{2\pi}\frac{dN_q}{p_T dp_T dy}\left[1+\sum_{n=1} 2v_{n,q}(p_T)\cos(n\varphi)\right]$$

$$= \frac{1}{2\pi}\frac{dN_q}{p_T dp_T dy}\left[1+2v_{1,q}(p_T)\cos(\varphi)+2v_{2,q}(p_T)\cos(2\varphi)+\cdots\right]$$

A. M. Poskanzer and S. A Voloshin, Phys. Rev. C **58**, 1671 (1998)

1) Elliptic flow of the J/ψ

E. Abbas et al, Phys. Rev. Lett. **111**, 162301 (2013)



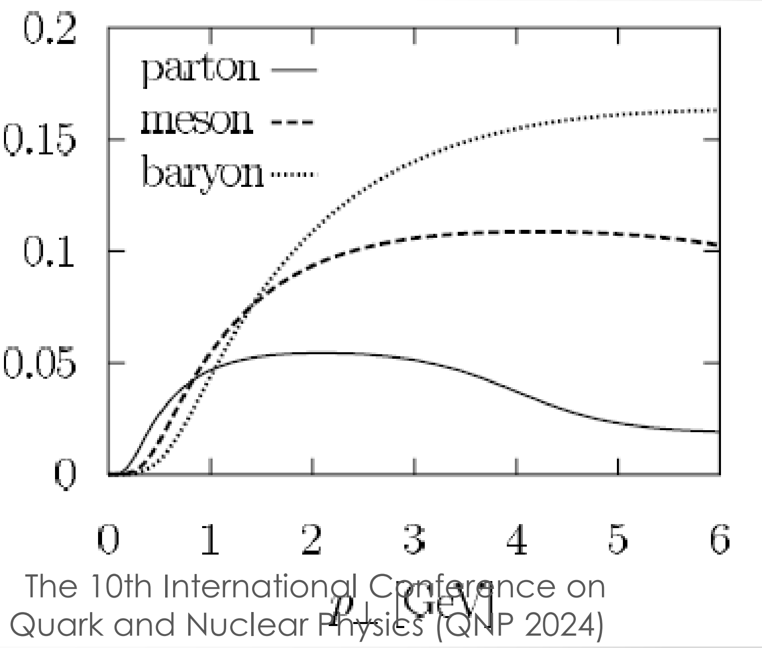
2) Quark number scaling of the elliptic flow

D. Molnar and S. A. Voloshin, Phys. Rev. Lett **91**, 092301 (2003)

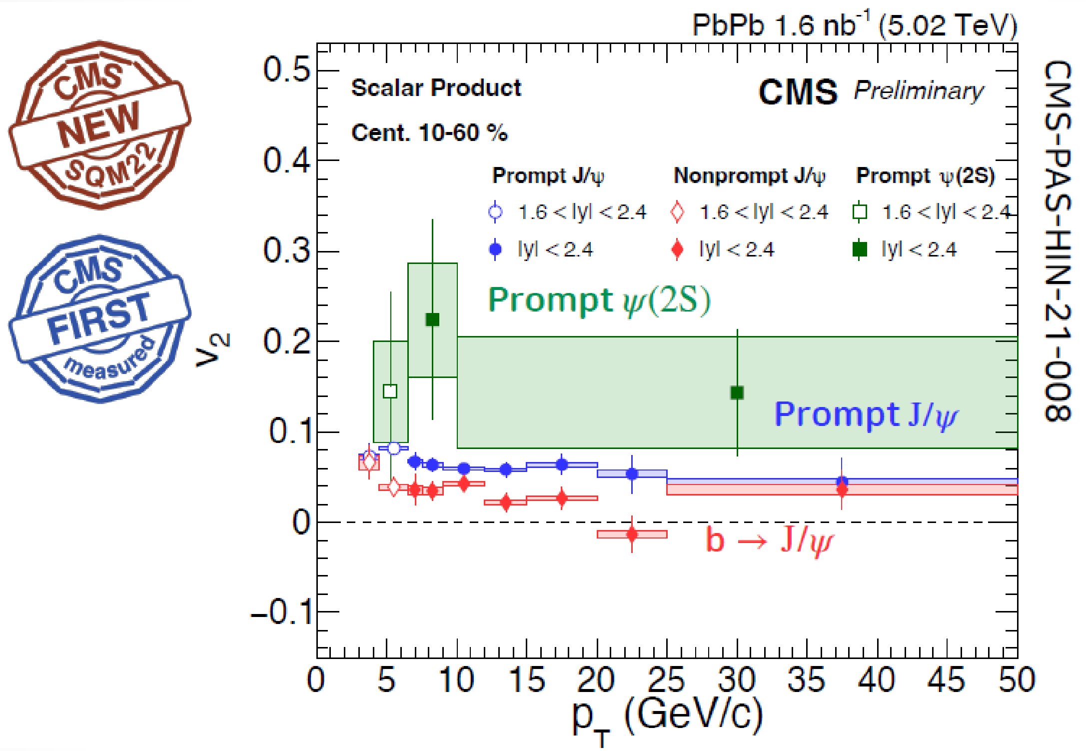
$$v_{2,M}(p_T) = \frac{2v_{2,q}(p_T/2)}{1+2v_{2,q}^2(p_T/2)}$$

$$v_{2,B}(p_T) = \frac{3v_{2,q}(p_T/3)+3v_{2,q}^3(p_T/3)}{1+6v_{2,q}^2(p_T/3)}$$

$$v_{2,h}(p_T) \approx n v_{2,q}\left(\frac{1}{n}p_T\right)$$



3) Recent measurements of elliptic flow of charmonium states, the J/ψ and ψ(2S) meson at LHC by CMS Collaboration



G. Bak [CMS Collaboration], Strangeness in Quark Matter 2022, Busan, June, 13-17
CMS-PAS-HIN-21-008

$$v_n(p_T) = \langle \cos(n(\psi - \Psi_n)) \rangle = \frac{\int d\psi \cos(n(\psi - \Psi_n)) \frac{d^2N}{dp_T^2}}{\int d\psi \frac{d^2N}{dp_T^2}},$$

$$\Psi_n = \frac{1}{n} \tan^{-1}\left(\frac{\langle p_T \sin(n\psi)\rangle}{\langle p_T \cos(n\psi)\rangle}\right),$$

Transverse momentum distributions and yields of charmonium states

– Yields of hadrons in the coalescence model

V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C **68**, 034904 (2003)
R. J. Freis. B. Muller, C. Nonaka, and S. Bass, Phys. Rev. C **68**, 044902 (2003)

$$N^{Coal} = g\int\left[\prod_{i=1}^{n}\frac{1}{g_i}\frac{p_i\cdot d\sigma_i}{(2\pi)^3}\frac{d^3p_i}{E_i}f(x_i,p_i)\right]f^W(x_1,\cdots,x_n:p_1,\cdots,p_n)$$

1) The Wigner function, the coalescence probability function

$$f^W(x_1,\cdots,x_n:p_1,\cdots,p_n) = \int\prod_{i=1}^{n}dy_i e^{p_i y_i}\psi^*\left(x_1+\frac{y_1}{2},\cdots,x_n+\frac{y_n}{2}\right)\psi\left(x_1-\frac{y_1}{2},\cdots,x_n-\frac{y_n}{2}\right)$$

2) A Lorentz-invariant phase space integration of a space-like hyper-surface constraints the number of particles in the system

$$\int p_i\cdot d\sigma_i\frac{d^3p_i}{(2\pi)^3E_i}f(x_i,p_i)=N_i$$

– Production of charmonium states by recombination

S. Cho, Phys. Rev. C **91**, 054914 (2015)

1) Coalescence production of charmonium states

$$N_\psi = g_\psi \int p_c \cdot d\sigma_c p_{\bar{c}} \cdot d\sigma_{\bar{c}} \frac{d^3\vec{p}_c}{(2\pi)^3 E_c} \frac{d^3\vec{p}_{\bar{c}}}{(2\pi)^3 E_{\bar{c}}} f_c(r_c, p_c) f_{\bar{c}}(r_{\bar{c}}, p_{\bar{c}}) W_\psi(r_c, r_{\bar{c}}; p_c, p_{\bar{c}}),$$

The transverse momentum distribution of the charmonium yield

$$\frac{dN_\psi}{d^2\vec{p}_T} = \frac{g_\psi}{V} \int d^3\vec{r} d^2\vec{p}_{cT} d^2\vec{p}_{\bar{c}T} \delta^{(2)}(\vec{p}_T - \vec{p}_{cT} - \vec{p}_{\bar{c}T}) \frac{dN_c}{d^2\vec{p}_{cT}} \frac{dN_{\bar{c}}}{d^2\vec{p}_{\bar{c}T}} W_\psi(\vec{r}, \vec{k})$$

2) Gaussian Wigner functions for different charmonium states

$$W_s(\vec{r}, \vec{k}) = 8 e^{-\frac{r^2}{\sigma^2} - k^2\sigma^2}$$

$$W_p(\vec{r}, \vec{k}) = \left(\frac{16}{3}\frac{r^2}{\sigma^2} - 8 + \frac{16}{3}\sigma^2 k^2\right) e^{-\frac{r^2}{\sigma^2} - k^2 \sigma^2}$$

$$W_{\psi_{10}}(\vec{r}, \vec{k}) = \frac{16}{3}\Big(\frac{r^4}{\sigma^4} - 2\frac{r^2}{\sigma^2} + \frac{3}{2} - 2\sigma^2 k^2 + \sigma^4 k^4 - 2r^2k^2 + 4(\vec{r}\cdot\vec{k})^2\Big) e^{-\frac{r^2}{\sigma^2} - k^2\sigma^2}.$$

3) Integration of the Wigner function over the spatial coordinates

$$\int d^3\vec{r} W_\psi(\vec{r},\vec{k}) = \begin{cases} (2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} & \psi_s^G; J/\psi \\ \frac{2}{3}(2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2}\sigma^2 k^2 & \psi_p^G; \chi_c \\ \frac{2}{3}(2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2}\left(\sigma^2 k^2 - \frac{3}{2}\right)^2 & \psi_{10}^G; \psi(2S) \\ 64\pi \frac{a_0^3}{(a_0^2k^2+1)^4} & \psi_{1S}^C; J/\psi \\ 8\pi a_0^3 \frac{(a_0^2k^2-1/4)^2}{(a_0^2k^2+1/4)^6} & \psi_{2S}^C; \psi(2S) \end{cases}$$

$$\int d^3\vec{r} W(\vec{r},\vec{k}) = \left|\tilde{\psi}(\vec{k})\right|^2$$

M. Hillery, R. F. O'Connel, M. O. Scully and E. P. Wigner, Phys. Rept. **106**, 121 (1984)

$$\frac{dN_\psi}{d\vec{p}_T} = \frac{g_\psi}{V}\int d\vec{p}_{cT} d\vec{p}_{\bar{c}T} \delta(\vec{p}_T - \vec{p}_{cT} - \vec{p}_{cT}) \frac{dN_c}{d\vec{p}_{cT}} \frac{dN_{\bar{c}}}{d\vec{p}_{\bar{c}T}} \left|\tilde{\psi}(\vec{k})\right|^2$$

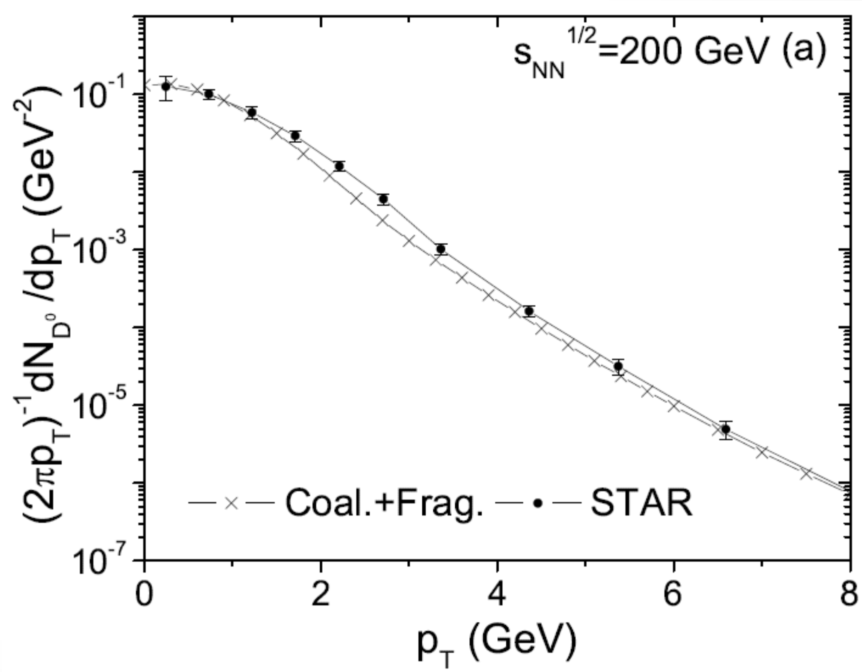
4) Transverse momentum distributions of charm quarks

$$\frac{dN_c}{d^2p_T} = \begin{cases} a_0 \exp[-a_1 p_T^{a_2}] & p_T \le p_0 \\ a_0 \exp[-a_1 p_T^{a_2}] + a_3(1+p_T^{a_4})^{-a_5} & p_T \ge p_0 \end{cases}$$

RHIC	a_0	a_1	a_2	a_3	a_4	a_5
$p_T \le p_0$	0.69	1.22	1.57			
$p_T \ge p_0$	1.08	3.04	0.71	3.79	2.02	3.48
LHC	a_0	a_1	a_2	a_3	a_4	a_5
$p_T \le p_0$	1.97	0.35	2.47			
$p_T \ge p_0$	7.95	3.49	3.59	87335	0.5	14.31

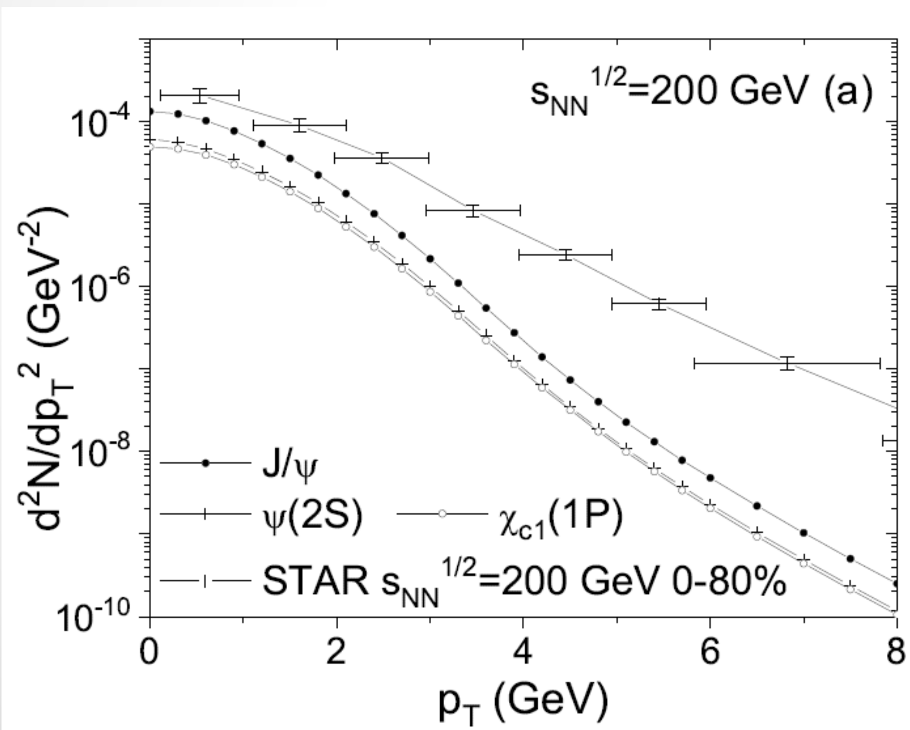
S. Plumari, V. Minissale, S. K. Das, G. Coci and V. Greco, Eur. Phys. J. C **78**:348 (2017)
Y. Oh, C. M. Ko, S.-H. Lee, and S. Yasui, Phys. Rev. C **79** 044905 (2009)
S. Cho *et al*. (EXHIC Collaboration), Prog. Part. Nucl. Phys. **95**, 279 (2017)
S. Cho and S. H. Lee, Phys. Rev. C **101**, 024902 (2020)

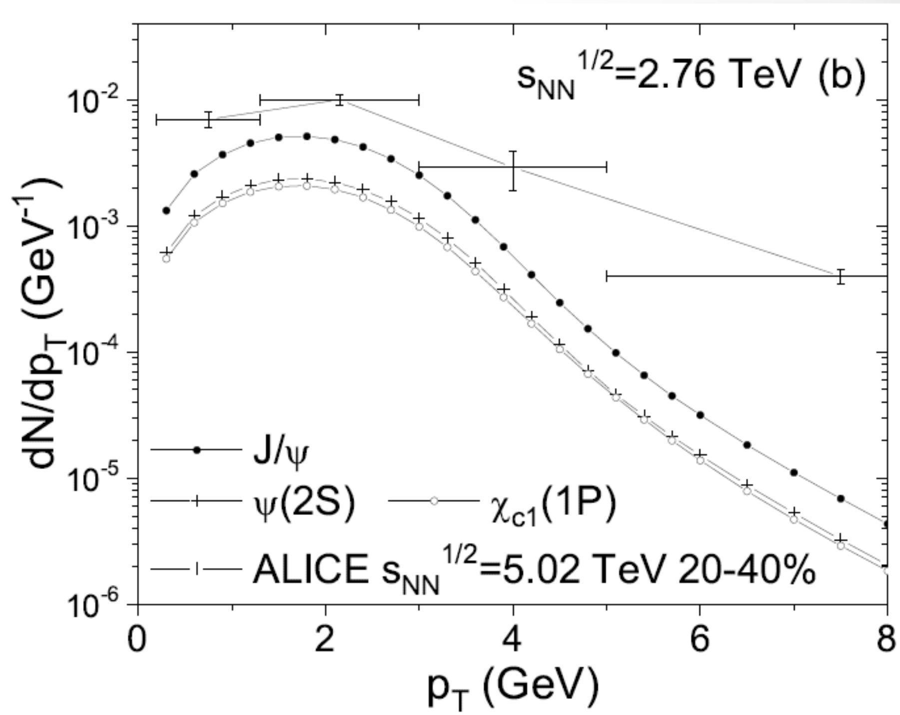
	RHIC		LHC (2.76 TeV)	
	Sc. 1	Sc. 2	Sc. 1	Sc. 2
T_H (MeV)	162		156	
V_H (fm^3)	2100		5380	
μ_B (MeV)	24		0	
μ_s (MeV)	10		0	
γ_c	22		39	
γ_b	4.0×10^7		8.6×10^8	
T_C (MeV)	162	166	156	166
V_C (fm^3)	2100	1791	5380	3533
$N_u = N_d$	320	302	700	593
$N_s = N_{\bar{s}}$	183	176	386	347
$N_c = N_{\bar{c}}$	4.1		11	
$N_b = N_{\bar{b}}$	0.03		0.44	



5) Transverse momentum distributions and yields of charmonium states in midrapidities at RHIC and LHC

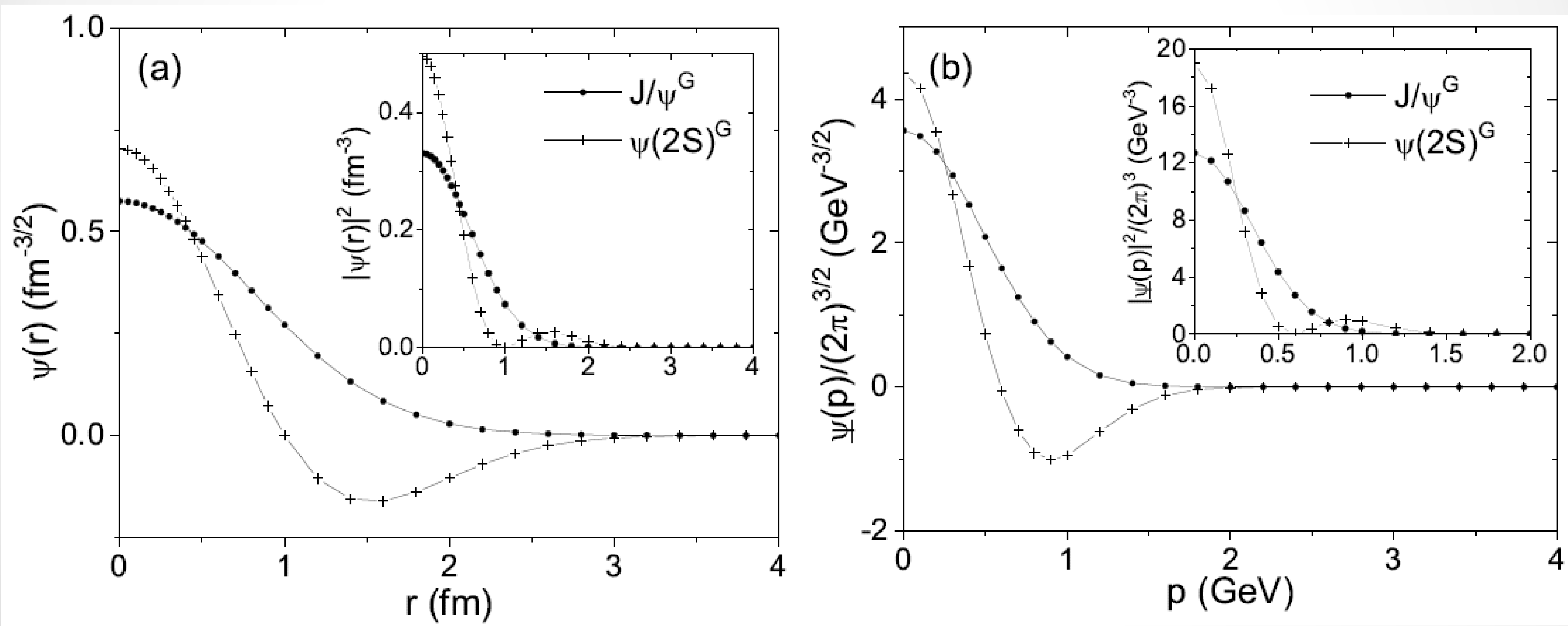
J. Adam et al. [STAR Collaboration], Phys. Lett. B **797**, 134917 (2019).
S. Acharya et al. [ALICE Collaboration], Phys. Lett. B **805**, 135434 (2020).





	RHIC	LHC
J/ψ	7.6×10^{-4}	1.3×10^{-2}
$\psi(2S)$	3.5×10^{-4}	5.8×10^{-3}
$\chi_{c1}(1P)$	3.0×10^{-4}	5.1×10^{-3}

6) The dependence of transverse momentum distributions and yields of charmonium states on their internal structures, or wave function distributions



Elliptic and triangular flow of charmonium states

– Flow harmonics of charmonium states

$$v_n(p_T) = \langle \cos(n(\psi - \Psi_n)) \rangle = \frac{\int d\psi \cos(n(\psi - \Psi_n)) \frac{d^2N}{dp_T^2}}{\int d\psi \frac{d^2N}{dp_T^2}}, \qquad \Psi_n = \frac{1}{n} \tan^{-1}\left(\frac{\langle p_T \sin(n\psi) \rangle}{\langle p_T \cos(n\psi) \rangle} \right),$$

1) Transverse momentum distribution of charm quarks with flow harmonics

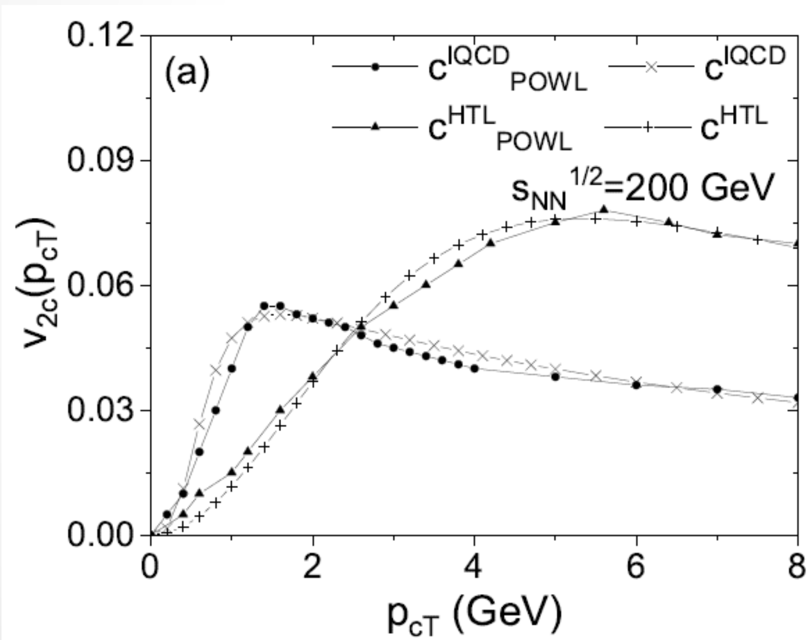
$$\frac{d^2N_c}{dp_{cT}^2} = \frac{1}{2\pi p_{cT}} \frac{dN_c}{dp_{cT}} \left(1 + \sum_{n=1}^{\infty} 2v_{nc}(p_{cT}) \cos(n(\phi_c - \Psi_n)) \right),$$

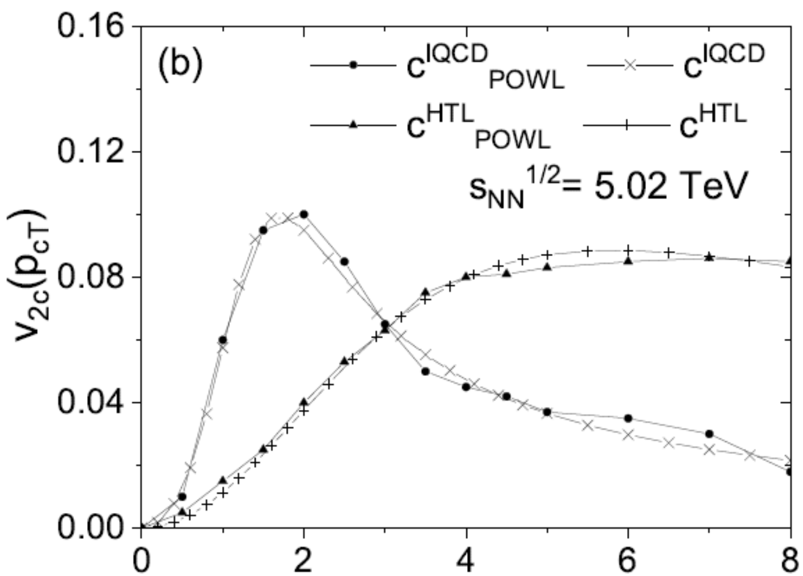
2) Event plane averaged flow harmonics

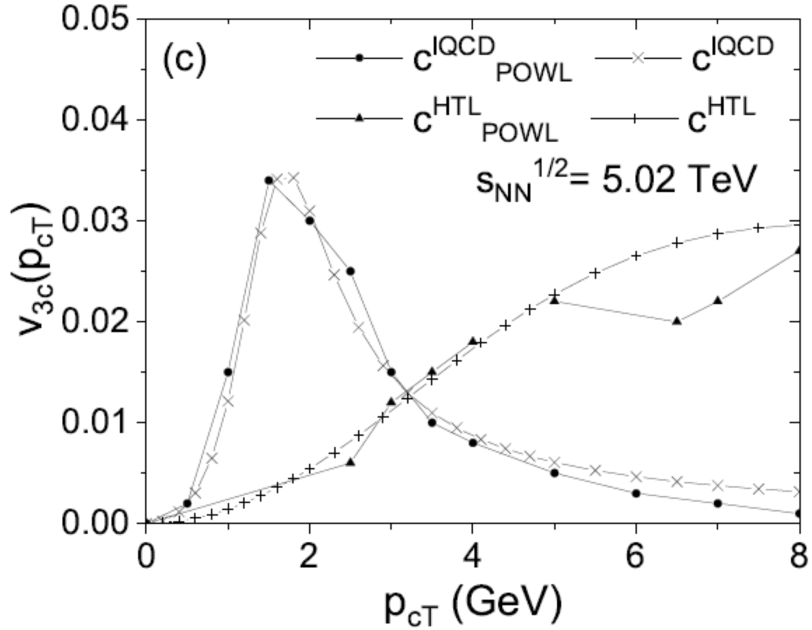
$$v_n(p_T) = \frac{\frac{n}{2\pi} \int_{-\frac{\pi}{n}}^{\frac{\pi}{n}} \int d\psi \cos(n(\psi - \Psi_n)) \frac{d^2N}{dp_T^2} d\Psi_n}{\frac{n}{2\pi} \int_{-\frac{\pi}{n}}^{\frac{\pi}{n}} \int d\psi \frac{d^2N}{dp_T^2} d\Psi_n}.$$

3) Charm quark flow harmonics from POWLANG
: Pade approximation on charm quark flow harmonics

$$v_{nc}(p_{cT}) = \frac{a_3 p_{cT}^3 + a_2 p_{cT}^2 + a_1 p_{cT}}{b_4 p_{cT}^4 + b_3 p_{cT}^3 + b_2 p_{cT}^2 + b_1 p_{cT} + 1},$$

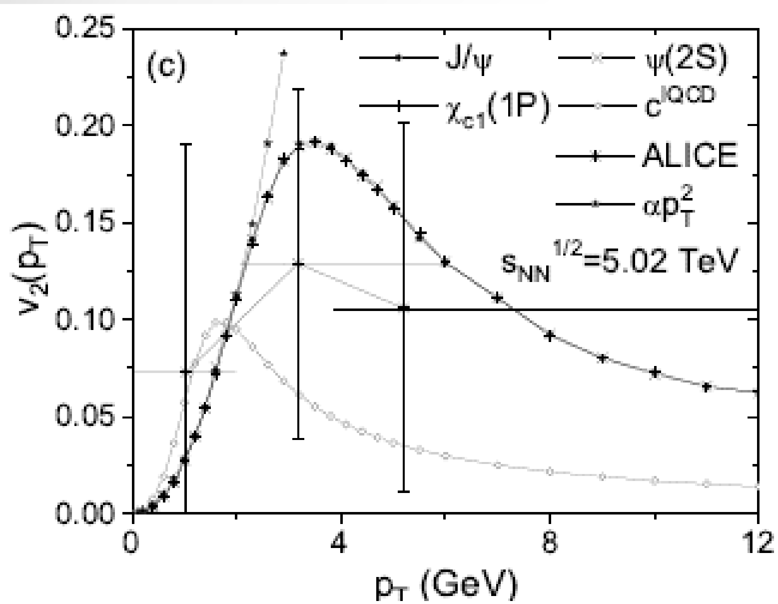


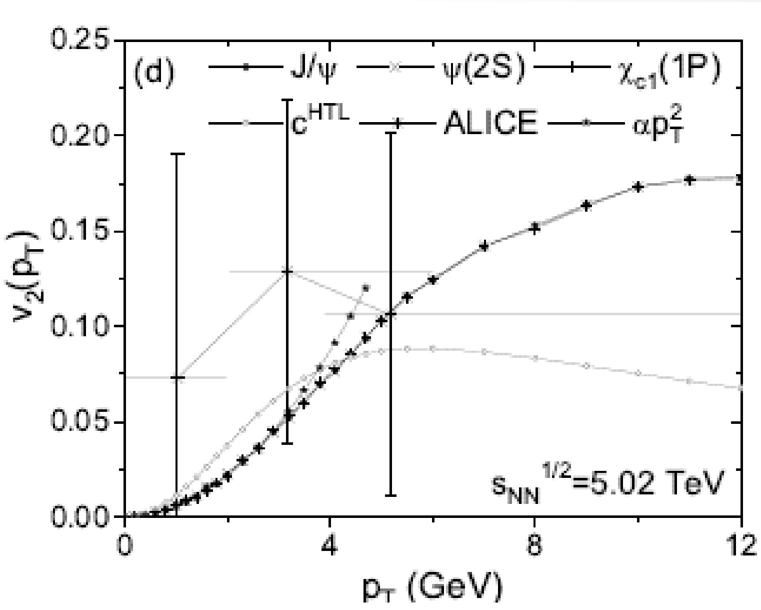


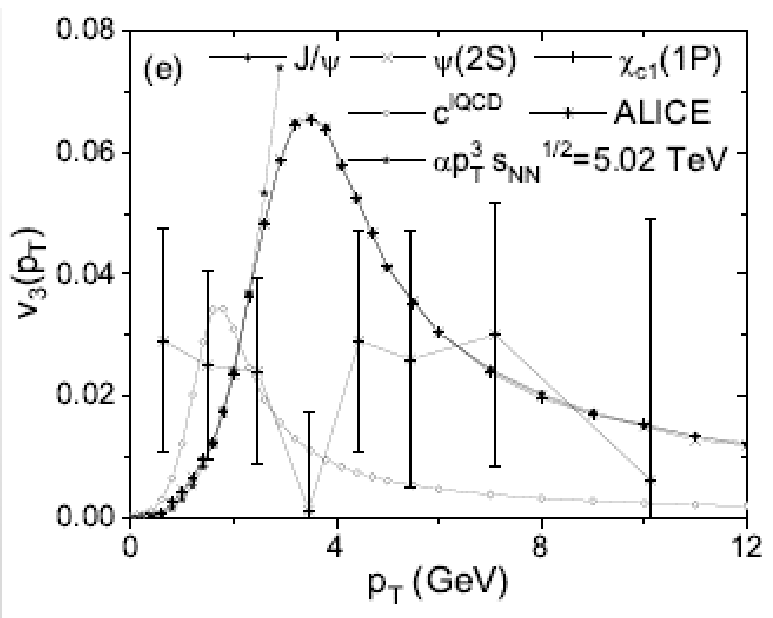


A. Beraudo, A. De Pace, M. Monteno,
M. Nardi andF. Prino, JHEP **02**, 043 (2018).

4) Elliptic and triangular flow of charmonium states at LHC





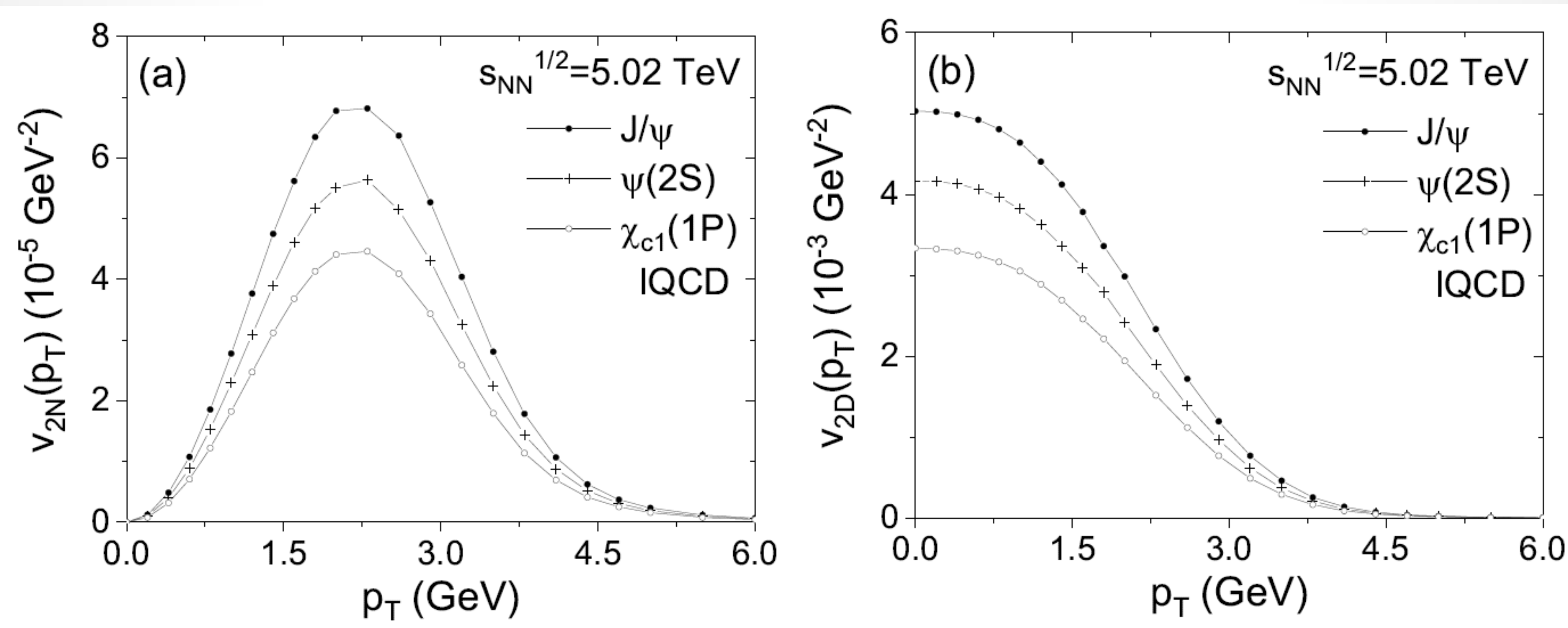


S. Cho, Phys. Rev. C **109**, no. 5, 054904 (2024)
S. Acharya et al. [ALICE Collaboration],
Phys. Rev. Lett. **119**, no. 24, 242301 (2017).
S. Acharya et al. [ALICE Collaboration],
JHEP **2010**, 141 (2020).

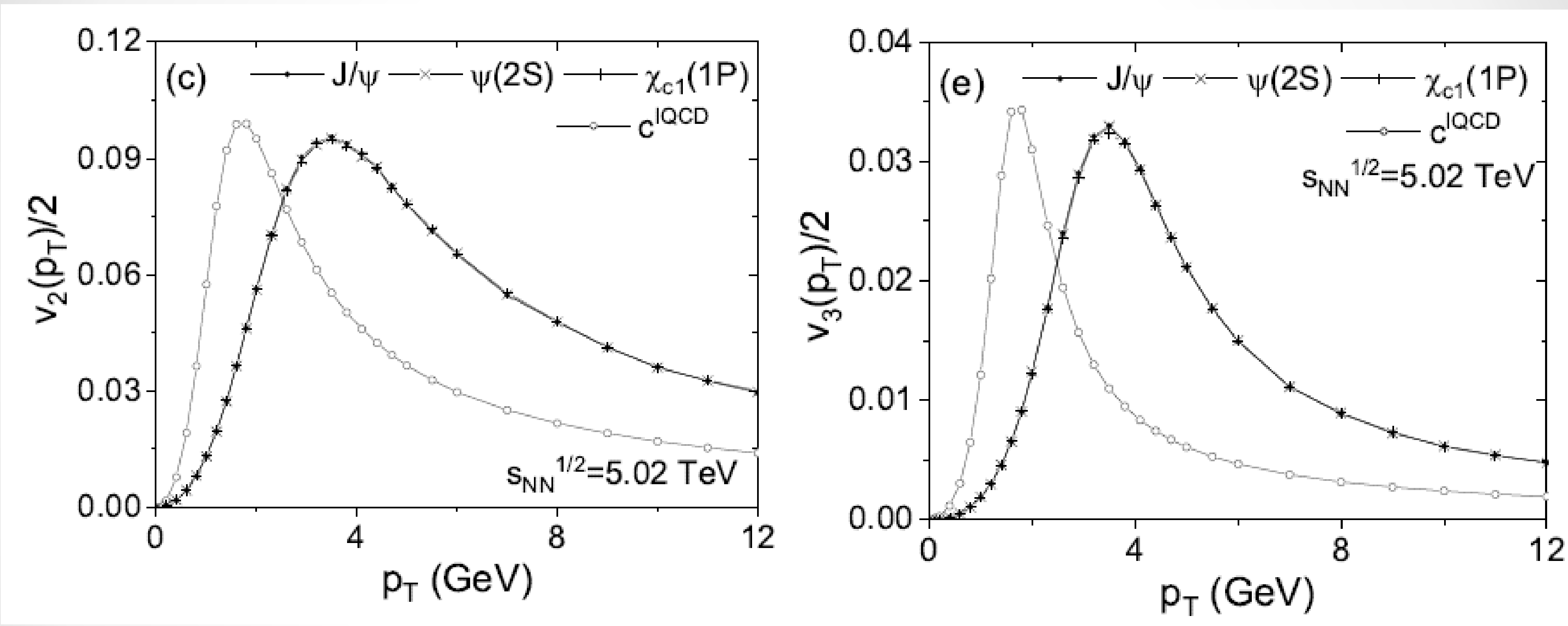
5) The dependence of the elliptic flow of charmonium states on their internal structures, or their wave function distributions

$$v_{2N}(p_T) = \frac{2}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int d\psi \cos(2(\psi - \Psi_2))\frac{d^2N}{dp_T^2}d\Psi_2$$

$$v_{2D}(p_T) = \frac{2}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int d\psi \frac{d^2N}{dp_T^2}d\Psi_2. \quad (18)$$



6) The quark number scaling of elliptic and triangular flow



$$v_{n,c\bar{c}}(p_T) \approx 2v_{n,c}(p_T/2)$$

Conclusion

– Flow harmonics of charmonium states in heavy ion collisions

1) The production of charmonium states in heavy ion collisions can be understood in the coalescence model
2) The transverse momentum distribution and yield of charmonium states are dependent on their internal structures
3) The enhanced transverse momentum distribution of $\psi(2S)$ mesons, compared to that of J/ψ mesons, is originated from intrinsic wave function distributions between $\psi(2S)$ and J/ψ mesons
4) The elliptic and triangular flow of charmonium states are also affected by wave function distributions of charmonium states

Thank you for your attention!