Dipion transitions between heavy hybrids and heavy quarkonium excitations

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QNP2024, Barcelona, 11/7/24

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JS, Sandra Tomàs Valls, in preparation

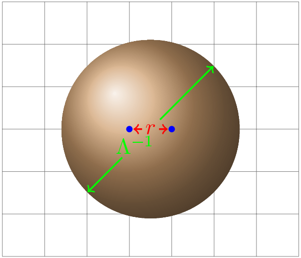
Introduction

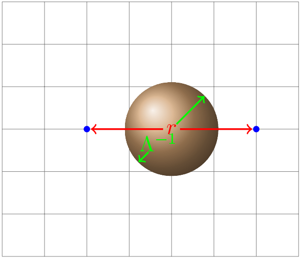
- Heavy Quarks: $Q = c, b,\ m_Q \gg \Lambda_{QCD}$ ($\sim 400\,\mathrm{MeV}$)
- Heavy Quarkonium: $S = \bar{Q}Q$
- Heavy Hybrids: $H = \bar{Q}Qg$
- Heavy quarks move slowly $\Longrightarrow$ use non-relativistic approximations: p ($\bar{Q}Q$ relative three momentum), r ($\bar{Q}Q$ typical distance), E (binding energy scale)

$$m_Q \gg p\,,1/r \gg E$$

- A universal way to encode them is using Effective Field Theories
- NRQCD exploits $m_Q \gg p\,,1/r\,,\Lambda_{QCD}\,,E$
  - Implies heavy quark spin symmetry at leading order
  - pNRQCD (weak coupling) exploits in addition $p\,,1/r \gg \Lambda_{QCD}\,,E$
  - BOEFT (strong coupling pNRQCD) exploits in addition $p\,,1/r\,,\Lambda_{QCD} \gg E$

How does the hadron look like ?





$1/r \gg E \gtrsim \Lambda_{QCD}$

weak coupling pNRQCD

$1/r \gtrsim \Lambda_{QCD} \gg E$

strong coupling pNRQCD

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Born-Oppenheimer EFT

Figures: Najjar, Bali, 2009

$QQ/Q\bar{Q}$ + light quarks and gluons in BOEFT

$$\mathcal{L}_{\rm BOEFT} = \sum_{\kappa^p} \Psi^\dagger_{\kappa^p} \left[i\partial_t - h_{\kappa^p} \right] \Psi_{\kappa^p}$$

(JS, Tarrús Castellà, 20)

$$h_{\kappa^p} = \frac{\boldsymbol{p}^2}{m_Q} + \frac{\boldsymbol{P}^2}{4m_Q} + V^{(0)}_{\kappa^p}(\boldsymbol{r}) + \frac{1}{m_Q} V^{(1)}_{\kappa^p}(\boldsymbol{r}, \boldsymbol{p}) + \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

- LDF$\equiv$ light quarks + gluons, characterized by their quantum numbers (κ, p ...)
  - $\kappa \equiv$ total angular momentum, $p \equiv$ parity (P/PC)
  - Quantum numbers not explicitely displayed: baryon number (B), isospin (I), strangeness (S), principal quantum number (n)
- $V^{(0)}_{\kappa^p}$, $V^{(1)}_{\kappa^p}$, ... must be calculated non-perturbatively
- Here, we will only be concerned with the $Q\bar{Q}$ case with $\kappa^p = 0^{++}$ (Quarkonia) and the $Q\bar{Q}g$ case with $\kappa^p = 1^{+-}$ (Lowest Lying Hybrids)

- $V^{(0)}_{\kappa^p}$:
  - Lattice data exists (abundant for 0^{++}, scarce for 1^{+-}: Kuti, Juge, Morningstar, 00; Capitani, Philipsen , Reisinger, Riehl, Wagner, 18; Müller, Philipsen, Reisinger, Wagner, 19; Schlosser, Wagner, 21; Höllwieser , Knechtli , Korzec , Peardon , Urrea-Nino, 24)
  - Well described by an interpolation between the known short distance behavior (Coulomb) and the long distance behaviour (linear) calculated in the QCD effective string theory (EST).
- $V^{(1)}_{\kappa^p}$:
  - Lattice data exist for 0^{++} but not for 1^{+-}
  - The heavy quark spin dependent part has been estimated using an interpolation between the short distance behavior calculated in pNRQCD and the long distance behaviour calculated in the EST (JS, Tomàs Valls, 23; see Tomàs Valls poster).
- Spectrum known at LO (since long for 0^{++}, for some time 1^{+-}: Braaten, Langmack, Hudson Smith, 14; Berwein, Brambilla, Tarrús Castellà, Vairo 15; Oncala, JS, 17)
- Hyperfine splittings known (since long for 0^{++}, recently for 1^{+-}: Brambilla, Kim Lai, Segovia, Tarrús Castellà, 18, 19; JS, Tomàs Valls, 23)

$Q\bar{Q} \to Q\bar{Q} + \pi\pi$ in BOEFT

- At LO in $1/m_Q$ and the chiral expansion

$$\mathrm{L}_{\rm int} = \int d^3\mathbf{R} \int d^3\mathbf{r}\, \mathrm{tr}\left[S^\dagger(\mathbf{R},\mathbf{r},t)\left(g_0(\mathbf{r})\partial_0 U^\dagger \partial^0 U + g_1(\mathbf{r})\partial_i U^\dagger \partial^i U + \right.\right.$$

$$\left.\left. + g_2(\mathbf{r}) r^i r^j \partial_i U^\dagger \partial_j U + g_3(\mathbf{r})\left(U^\dagger \mathcal{M} + \mathcal{M}^\dagger U\right)\right) S(\mathbf{R},\mathbf{r},t)\right]$$

$$U = e^{\frac{i\vec{\pi}\vec{\tau}}{f_\pi}} \quad , \quad \vec{\pi}\vec{\tau} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

  - $\vec{\pi} = \vec{\pi}(\mathbf{R}, t)$, $\mathcal{M} = m_q \mathbb{I}$ (light quark mass), $m_\pi^2 = 2 m_q B_0$, $B_0 \sim \Lambda_{QCD}$
  - It implicitly assumes that $1/r, \Lambda_{QCD} \gg \Delta E$
  - For $r \ll 1/\Lambda_{QCD}$, $g_k(\mathbf{r})$, $k = 0, 1, 2, 3$, are analytic in $\mathbf{r}$ (multipole expansion: Gottfried, 78; Voloshin 79; Pineda, Tarrús Castellà, 19; Passemar, Tarrús Castellà, 21)
  - For $r \gg 1/\Lambda_{QCD}$, one should be able to estimate them in the EST
    - ⋆ We need an interaction Lagrangian of pions with the QCD string

The interaction of pions with the QCD string

- Pions: Lorentz invariance, chiral symmetry
- String: Lorentz invariance, reparameterization invariance
- Pions+String: Locality $\Longrightarrow$ Embed the string in the Chiral Lagrangian

$$S_{\rm int} = \int d^2\xi \sqrt{-\det(\partial_a x^\mu \partial_b x_\mu)}\mathcal{L}_{\rm Ch}(x(\xi))$$

$$\mathcal{L}_{\rm Ch}^{LO} = \lambda {\rm Tr}(\partial_\mu U^\dagger \partial^\mu U) + \lambda' {\rm Tr}(U^\dagger \mathcal{M} + \mathcal{M}^\dagger U)\,,\ \mathcal{L}_{\rm Ch}^{NLO} = \lambda'' {\rm Tr}(\mathcal{M}^\dagger \mathcal{M}) + \cdots$$

- We fix the frame such that $\xi = (x^0, x^3) = (t, z)$, then $x^i = x^i(t, z)$, $i = 1, 2$
- We assume $1/r \sim m_\pi$ and count $x^i \sim 1/\Lambda_{QCD}$ and $\partial_\mu \sim 1/r$ (on x^i) $\sim m_\pi$ (on U)
- It implies the following light quark mass (m_q) dependence ($m_\pi^2 = 2m_q B_0$) of the string tension

$$\sigma \to \sigma - \left(\frac{2\lambda'}{B_0} m_\pi^2 + \left[\frac{3}{8\pi^2}\frac{1}{f_\pi^2}(\lambda - \frac{\lambda'}{2B_0})\left(\ln\frac{m_\pi^2}{\mu^2} - 1\right) + \frac{\lambda''}{2B_0^2}\right] m_\pi^4\right)$$

Scattering and decay

- Pion scattering off the string
  - String states are characterized by the representations of $D_{\infty\, h} \subset O(3)$, $|L_z|^h_{CP}$. $|L_z| = 0(\Sigma), 1(\Pi), 2(\Delta), \ldots$, $CP = +(g), -(u)$, $h = +, -$

$$0^{++} \supset \Sigma_g^+ \quad , \quad 1^{+-} \supset \Sigma_u^- \,, \Pi_u \quad , \quad 2^{--} \supset \Sigma_g^- \,, \Pi_g \,, \Delta_g \quad , \quad \ldots$$

  - $\pi(q)\Sigma_g^+ \to \pi(q')\Sigma_g^+$

$$\mathcal{A} = -i\frac{4}{f_\pi^2}(\lambda q_\mu {q'}^\mu - \frac{\lambda' m_\pi^2}{2B_0})\frac{\sin\left[(q_z - q_z')\frac{r}{2}\right]}{(q_z - q_z')}$$

- Decay of string excitations by pion emission
  - $\Sigma_u^- \to \Sigma_g^+ \pi(q)\pi(q')$

$$\mathcal{A} = 0$$

  - $\Pi_u^{L/R} \to \Sigma_g^+ \pi(q)\pi(q')$

$$\mathcal{A} = \pm\frac{4\sqrt{\pi}}{f_\pi^2\sqrt{2\sigma r}}(\lambda q_\mu {q'}^\mu + \frac{\lambda' m_\pi^2}{2B_0})\frac{\cos\left[(q_z + {q'}_z)\frac{r}{2}\right]}{\frac{\pi^2}{r^2} - (q_z + {q'}_z)^2}\left[(q_1 + {q'}_1) \pm i(q_2 + {q'}_2)\right]$$

Excited Quarkonium and Hybrid dipion transitions

- For large principal quantum number, the long distance form of $g_k(\mathbf{r})$, $k = 0, 1, 2, 3$, is expected to dominate
- The following interactions match the result of EST in the static limit
  - $\Sigma_g^+ \to \Sigma_g^+ \, \pi(q)\pi(q')$

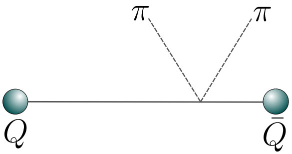
$$\mathrm{L}_{\mathrm{int}} = \int d^3\mathbf{R} \int d^3\mathbf{r} \, \mathrm{tr}\left[S^\dagger(\mathbf{R}, \mathbf{r}, t) \int_{-r/2}^{r/2} dz \, g(r, z) \mathcal{L}_{\mathrm{Ch}}(t, \mathbf{R} + z\hat{\mathbf{r}}) S(\mathbf{R}, \mathbf{r}, t) \right]$$

  - $\Sigma_u^-, \Pi_u^{L/R} \to \Sigma_g^+ \, \pi(q)\pi(q')$

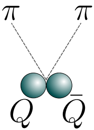
$$\mathrm{L}_{\mathrm{int}} = \int d^3\mathbf{R} \int d^3\mathbf{r} \, \mathrm{tr}\left[S^\dagger(\mathbf{R}, \mathbf{r}, t) \int_{-r/2}^{r/2} dz \, g(r, z) \partial_i \, \mathcal{L}_{\mathrm{Ch}}(t, \mathbf{R} + z\hat{\mathbf{r}}) H^i(\mathbf{R}, \mathbf{r}, t) \right]$$

  - For $r \ll 1/\Delta E \sim 1/m_\pi$ the expected form in the BOEFT is recovered with $g_0(\mathbf{r}) = g_1(\mathbf{r}) = \int_{-r/2}^{r/2} dz \, g(r, z)\lambda$, $g_2(\mathbf{r}) = 0$ and $g_3(\mathbf{r}) = \int_{-r/2}^{r/2} dz \, g(r, z)\lambda'/2B_0$

String picture



Multipole expansion



Dipion invariant mass distribution

- The dipion invariant mass distribution reads $m_{\pi\pi} \in (2m_\pi, \Delta E)$

$$\frac{d\Gamma}{dm^2_{\pi\pi}} = \frac{2}{f_\pi^4 (2\pi)^3} \left[\frac{\lambda}{2} m^2_{\pi\pi} + \left(\frac{\lambda'}{2B_0} - \lambda \right) m_\pi^2 \right]^2 \sqrt{1 - \frac{4m_\pi^2}{m^2_{\pi\pi}}} s I_\theta^2(s)$$

$s = \sqrt{\Delta E^2 - m^2_{\pi\pi}}$. $I_\theta(s)$ depends on the wave functions of the initial and final state.

- $\bar{Q}Q \to \bar{Q}Q + \pi\pi$

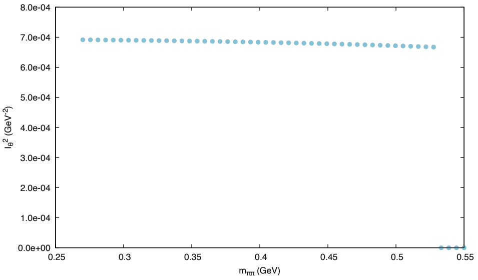
$$I_\theta(s) \sim \int dr \int d\Omega R_{n'l'}(r) Y^*_{l'm'}(\theta,\phi) \frac{\sin\left(sr\frac{\cos\theta}{2}\right)}{s\cos\theta} Y_{lm}(\theta,\phi) R_{nl}(r)$$

- $\bar{Q}Qg \to \bar{Q}Q + \pi\pi$ $(Y_{lm} = Y_{lm}(\theta,\phi),\ Y^*_{l'm'} = Y^*_{l'm'}(\theta,\phi))$

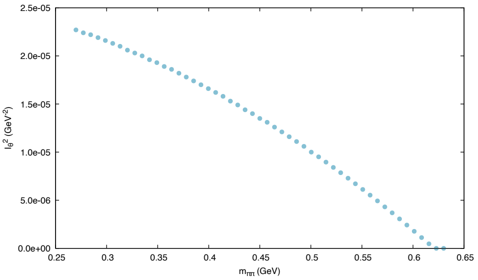
$$I_\theta(s) \sim \int dr \int d\Omega R_{n'l'}(r) Y^*_{l'm'} \frac{\cos\left(sr\frac{\cos\theta}{2}\right)}{\frac{\pi^2}{r^2} - (s\cos\theta)^2} \left(\pm\frac{i}{\sqrt{2}}\right) s \sin\theta e^{\pm i\phi} Y^{\pm}_{lm} R_{nJl}(r)$$

Examples

$\Upsilon(5s) \to \Upsilon(3s)\pi\pi$



$\Upsilon(2(s/d)_1) \to h_b(2p)\pi\pi$



Lessons

- The dipion spectrum is qualitative different if the initial state is a hybrid or a quarkonium
- $\Upsilon(10860)$ could be a mixure of a quarkonium $5s$ state and a hybrid $2(s/d)_1$ (H_1'), (Oncala, JS, 17)
  - Then we have a prediction for the following ratio of dipion spectra:

$$\frac{d\Gamma}{dm^2_{\pi\pi}}(\Upsilon(10860) \rightarrow h_b(2p)\pi\pi) : \frac{d\Gamma}{dm^2_{\pi\pi}}(\Upsilon(10860) \rightarrow \Upsilon(3s)\pi\pi)$$

  - This is expected to provide information on the mixing pattern
- We also have preliminary results for $\psi(4415) \rightarrow \psi_2(3823)\pi\pi$, $\psi(4415) \rightarrow \psi(3770)\pi\pi$, $\psi(4230) \rightarrow \psi(2s)\pi\pi$, $\psi(4660) \rightarrow \psi(2s)\pi\pi$, $\psi(4660) \rightarrow \psi_2(3823)\pi\pi$, $\psi(4040) \rightarrow h_c(1p)\pi\pi$, $\psi(4360) \rightarrow h_c(1p)\pi\pi$, $\Upsilon(4s) \rightarrow \Upsilon(3s)\pi\pi$, $\Upsilon(10753) \rightarrow \Upsilon(3s)\pi\pi$, $\Upsilon(11020) \rightarrow \Upsilon(3s)\pi\pi$, $\Upsilon(10860) \rightarrow h_b(1s)\pi\pi$

Conclusions

- We propose an interaction Lagrangian for pions and the QCD string
  - Compatible with chiral symmetry, reparameterization invariance, Lorentz invariance and locality
  - Can be systematically improved by adding higher dimensional operators
  - Provides the quark mass dependence of the string tension
  - It allows to calculate the long distance behavior of the transition form factors $(g_k(r))$
  - It allows to calculate the dipion transition from exited heavy quarkonium/hybrid states (dominated by long distances)
- It may help to elucidate the nature of some charmonium and bottomonium states (quarkonium vs hybrid)