Dense Baryonic Matter Equation of State with Quark Pauli Blocking

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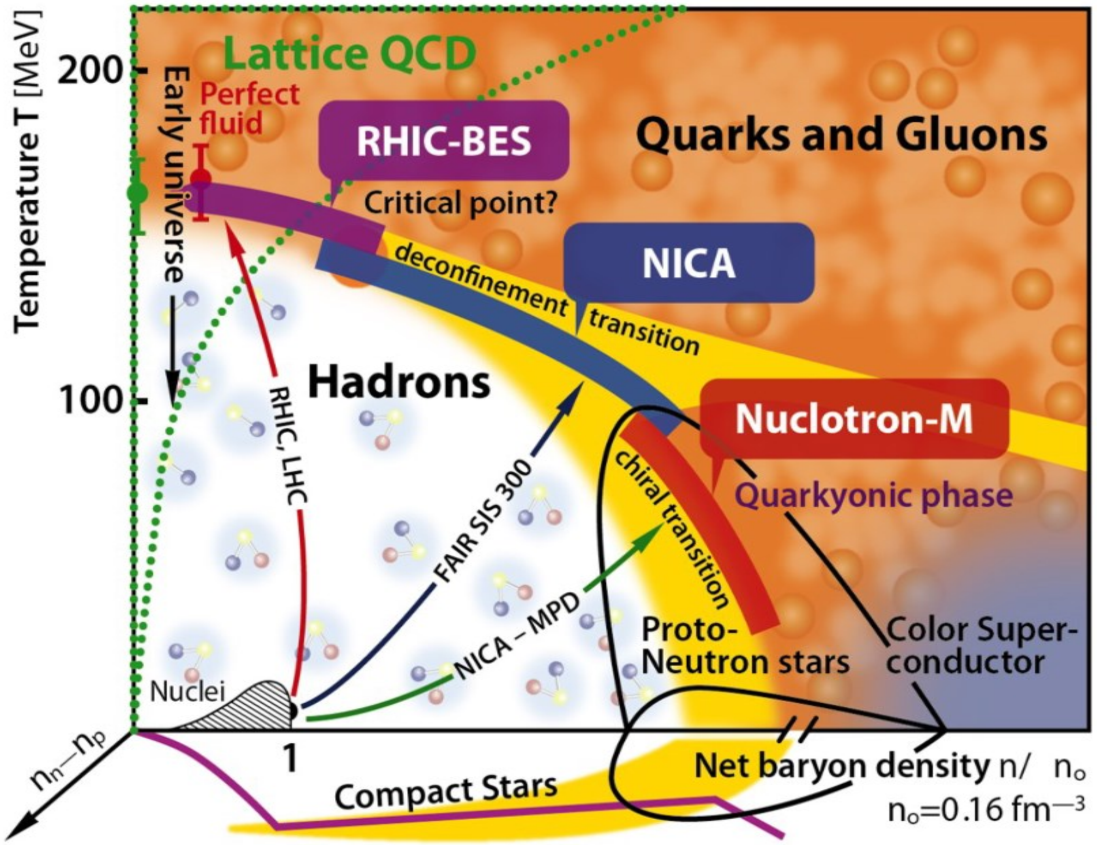
Introduction



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[Figures: J. Phys.: Conf. Ser. 912 012016 (2017) & MNRAS 497, 3118–3130 (2020)]

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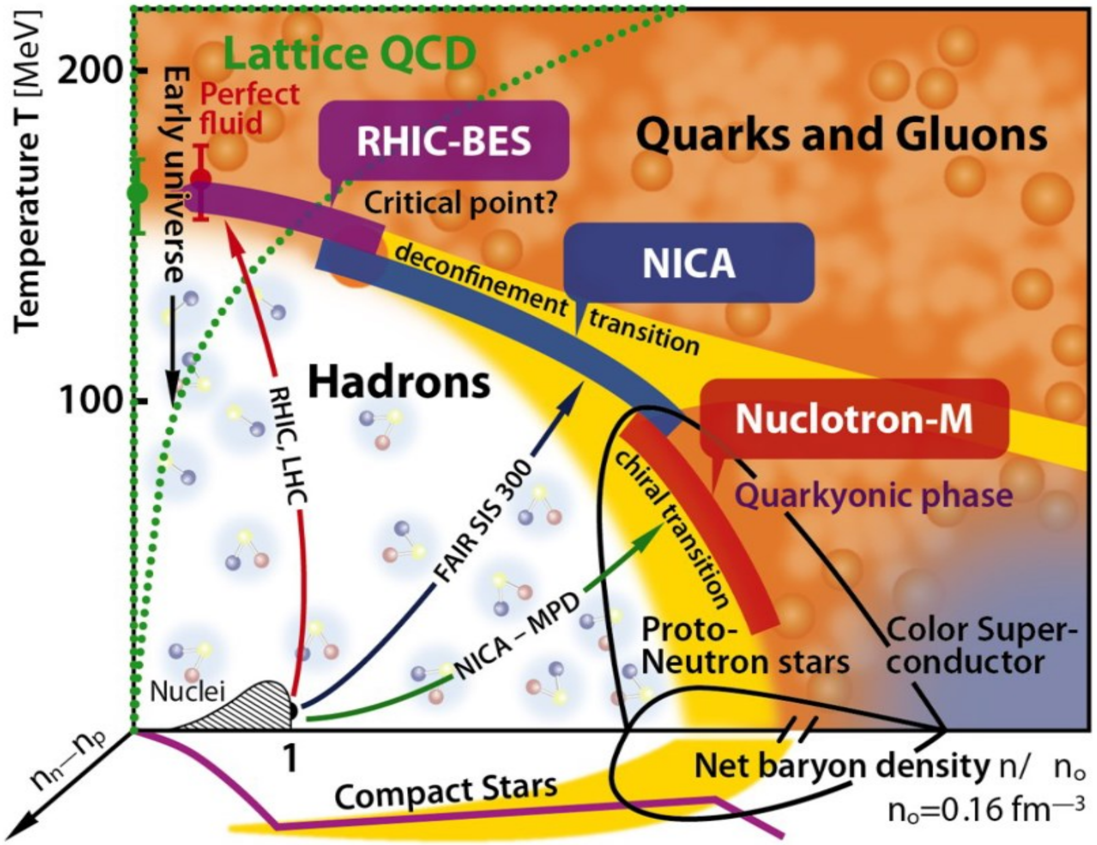

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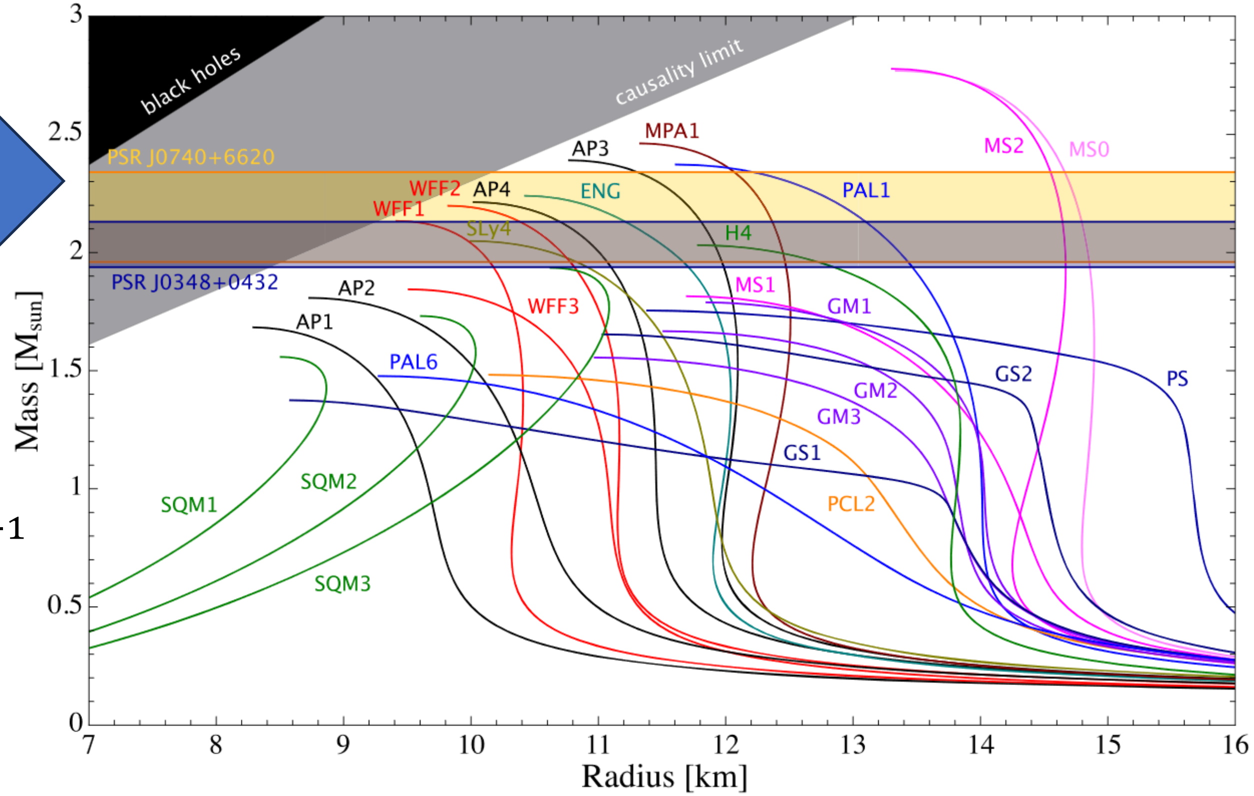

$$\frac{dP}{dr} = -\frac{M(r)\varepsilon(r)}{r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)}\right)\left(1-\frac{2M(r)}{r}\right)^{-1}$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon(r)$$

$T = 0$ region can be probed by neutron stars!

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$\sigma - \omega$ Model

Let us consider a minimal model of relativistic nuclear dynamics where the degrees of freedom are nucleons which interact with each other by an exchange of scalar meson σ and vector meson ω_μ:

$$\mathcal{L}_{\sigma-\omega} = \bar{\psi}(i\partial_\mu\gamma^\mu - m)\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + g_\sigma\sigma\bar{\psi}\psi - g_\omega\omega_\mu\bar{\psi}\gamma^\mu\psi$$

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or regrouping the terms:

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu\underbrace{(\partial_\mu + ig_\omega\omega_\mu)}_{D_\mu} - \underbrace{(m - g_\sigma\sigma)}_{m^*}]\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$

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The Euler-Lagrange equations for this model are given by:

$$(\partial^2 + m_\sigma^2)\sigma = g_\sigma\bar{\psi}\psi$$

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MFA →

$$m_\sigma^2\langle\sigma\rangle=g_\sigma\langle\bar{\psi}\psi\rangle$$

$$m_\omega^2\langle\omega_\mu\rangle=g_\omega\langle\bar{\psi}\gamma_\mu\psi\rangle\to m_\omega^2\langle\omega_0\rangle=g_\omega\langle\bar{\psi}\gamma_0\psi\rangle$$

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$\sigma - \omega$ Model

To extract thermodynamics one can compute the energy momentum tensor $T^{\mu\nu}$ and relate its components to the pressure and energy density:

$$\varepsilon = \langle T^{00} \rangle = -\langle \mathcal{L} \rangle + \langle \bar{\psi} \gamma_0 k_0 \psi \rangle, \qquad P = \frac{1}{3} \langle T^{ii} \rangle = \langle \mathcal{L} \rangle + \frac{1}{3} \langle \bar{\psi} \vec{\gamma} \vec{k} \psi \rangle$$

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In our case this yields to:

$$P_{\sigma-\omega} = \sum_{\tau=p,n} P_{FG}(P_{F,\tau}, m_\tau^*) + \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{2} m_\sigma^2 \sigma^2, \qquad \varepsilon_{\sigma-\omega} = \sum_{\tau=p,n} \varepsilon_{FG}(P_{F,\tau}, m_\tau^*) + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\sigma^2 \sigma^2$$

With the equations of motion of mesonic fields:

$$g_\sigma \sigma = \left(\frac{g_\sigma}{m_\sigma}\right)^2 n_s = \left(\frac{g_\sigma}{m_\sigma}\right)^2 \sum_{\tau=p,n} n_{s,\tau}(P_{F,\tau}, m_\tau^*), \qquad g_\omega \omega_0 = \left(\frac{g_\omega}{m_\omega}\right)^2 n = \left(\frac{g_\omega}{m_\omega}\right)^2 \sum_{\tau=p,n} n_\tau(P_{F,\tau})$$

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Where:

$$E_\tau^* = \sqrt{P_{F,\tau}^2 + m_\tau^{*2}}, \qquad n_\tau(P_{F,\tau}) = \frac{g_\tau P_{F,\tau}^3}{6\pi^2},$$

$$P_{FG}(P_{F,\tau}, m_\tau^*) = g_\tau \int_0^{P_{F,\tau}} \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_\tau^*}, \qquad \varepsilon_{FG}(P_{F,\tau}, m_\tau^*) = g_\tau \int_0^{P_{F,\tau}} \frac{d^3p}{(2\pi)^3} E_\tau^*, \qquad n_{s,\tau}(P_{F,\tau}, m_\tau^*) = g_\tau \int_0^{P_{F,\tau}} \frac{d^3p}{(2\pi)^3} \frac{m_\tau^*}{E_\tau^*}$$

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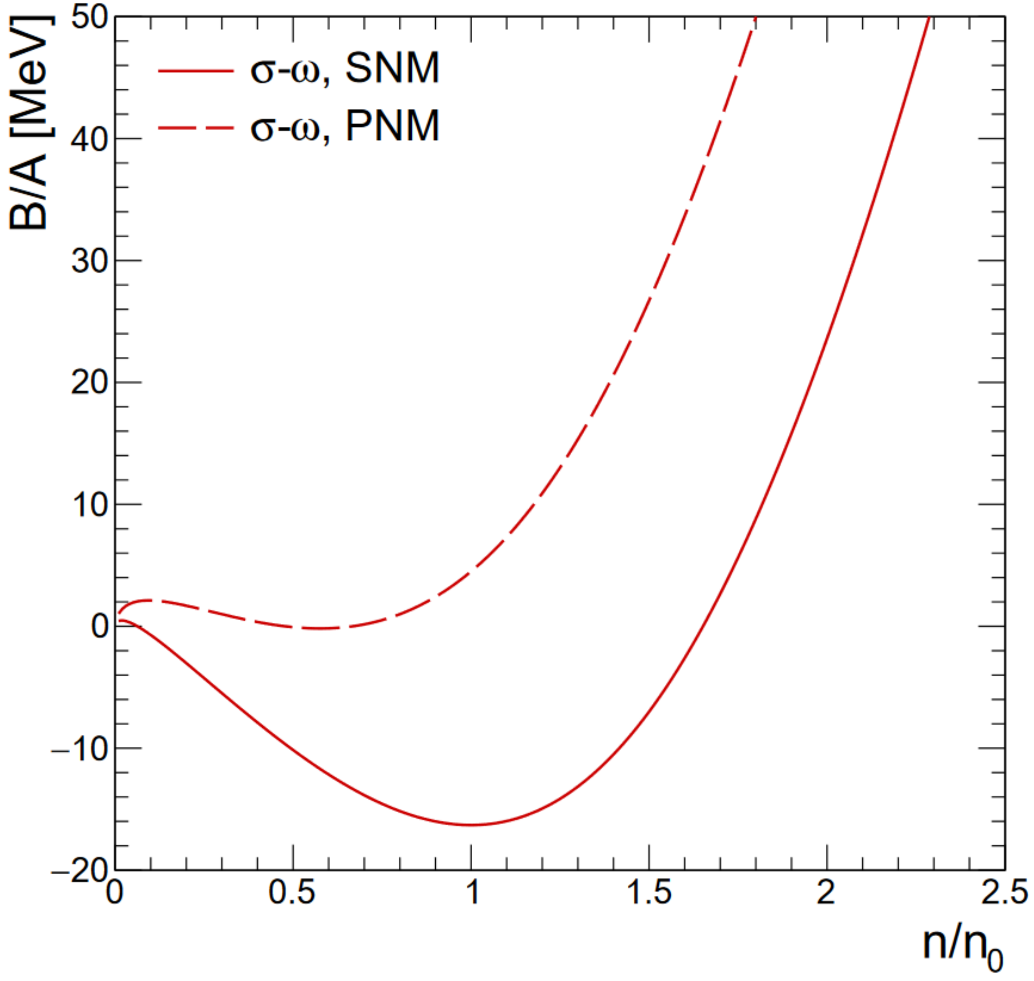

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4) $K = 9\left[n^2 \frac{d^2}{dn^2}\frac{\varepsilon}{n}\right]_{n_0} \approx 240\ \mathrm{MeV}$
5) $a_{sym} = \frac{1}{2}\left[\frac{d^2}{dt^2}\frac{\varepsilon}{n}\right]_{t=0} = 32.5\ \mathrm{MeV}$

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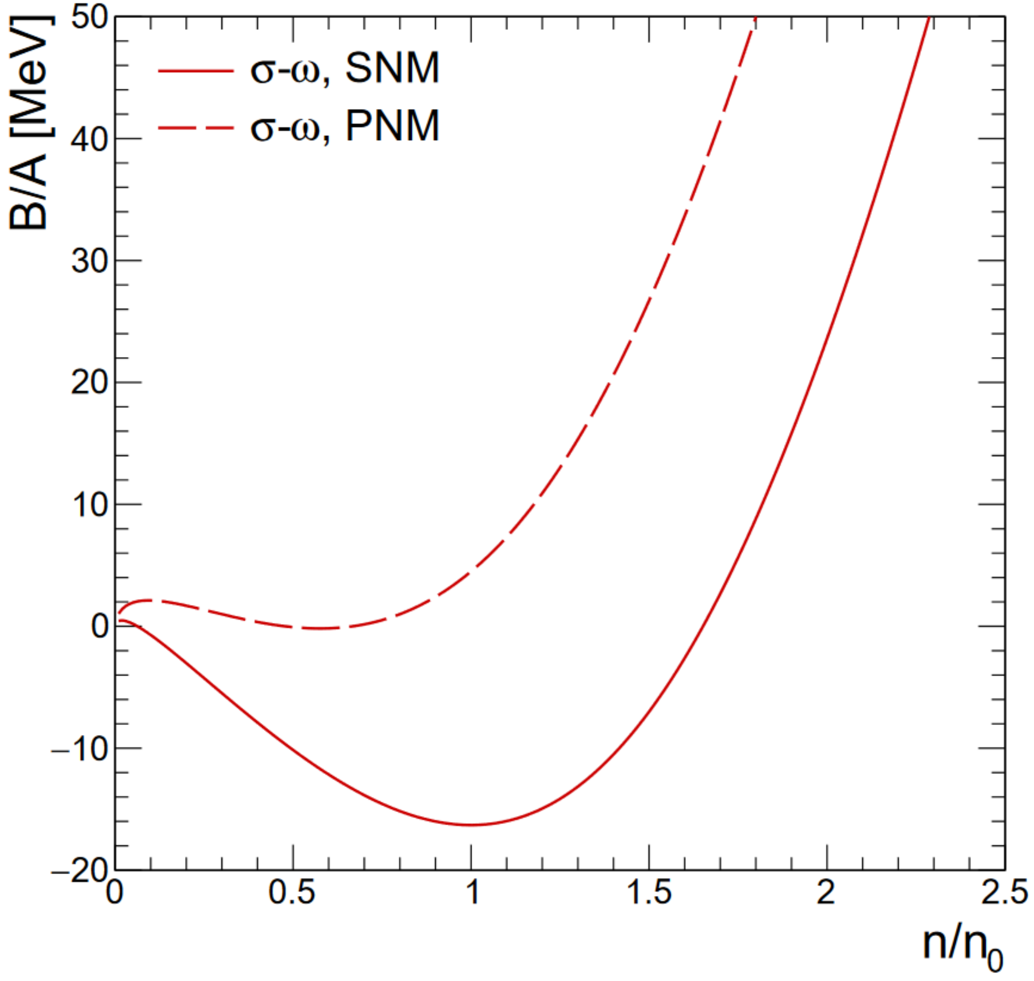

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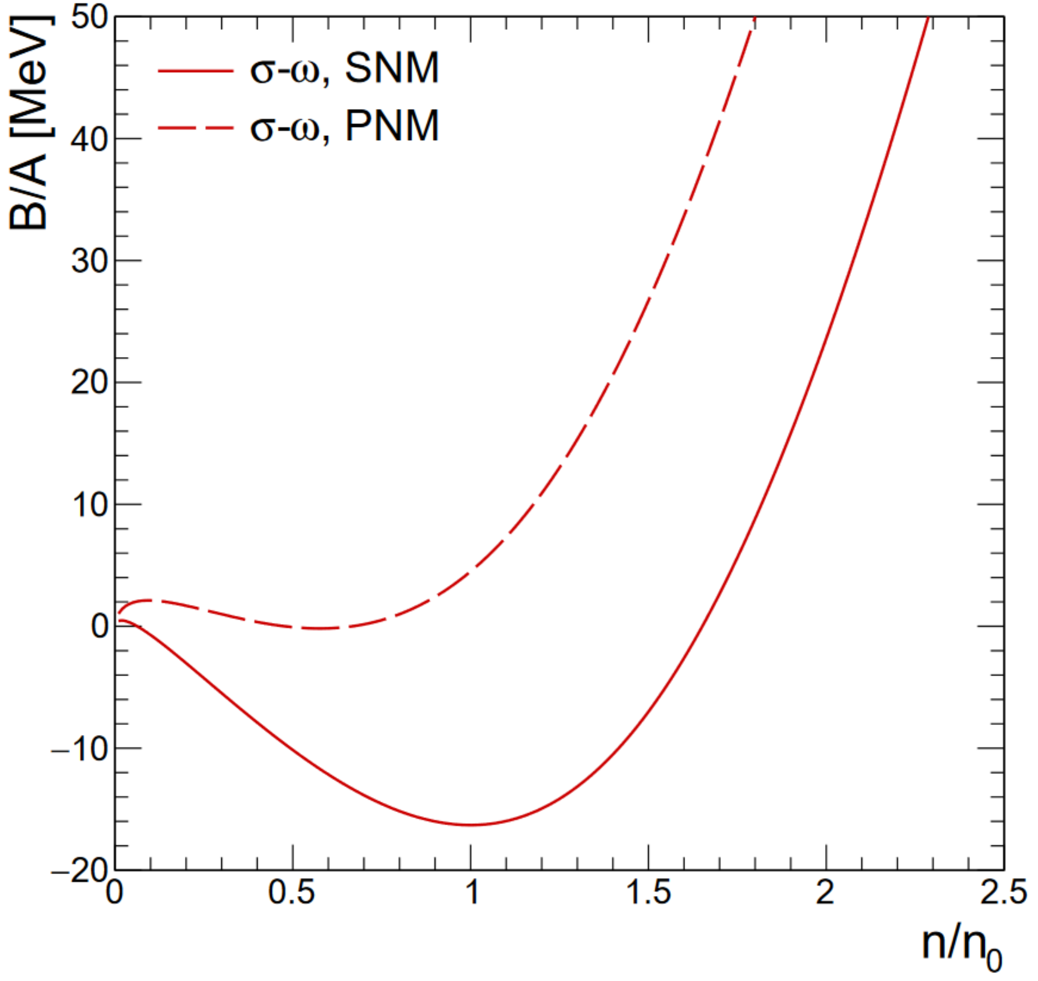

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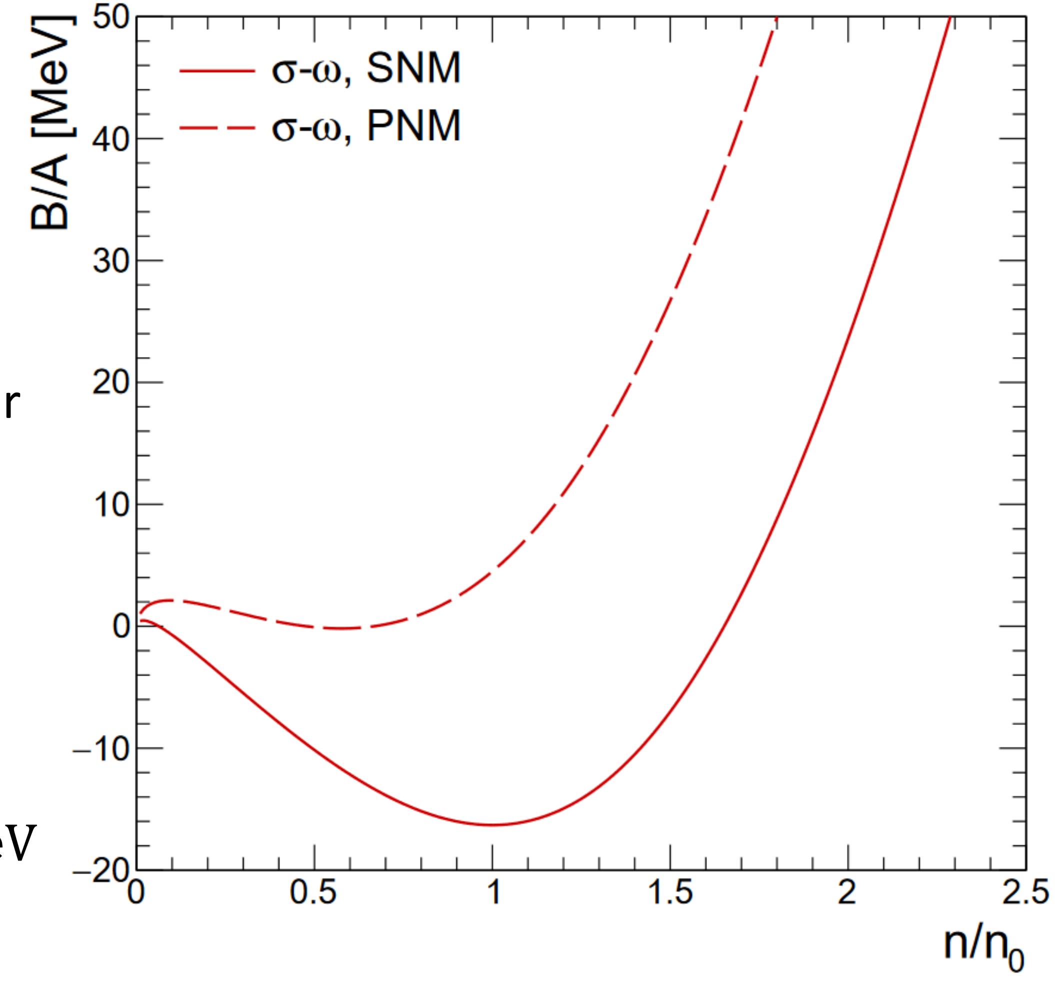
$\frac{m^*}{m} = 0.538$

$K = 562.9\ \mathrm{MeV}$

$a_{sym} = 19.7\ \mathrm{MeV}$



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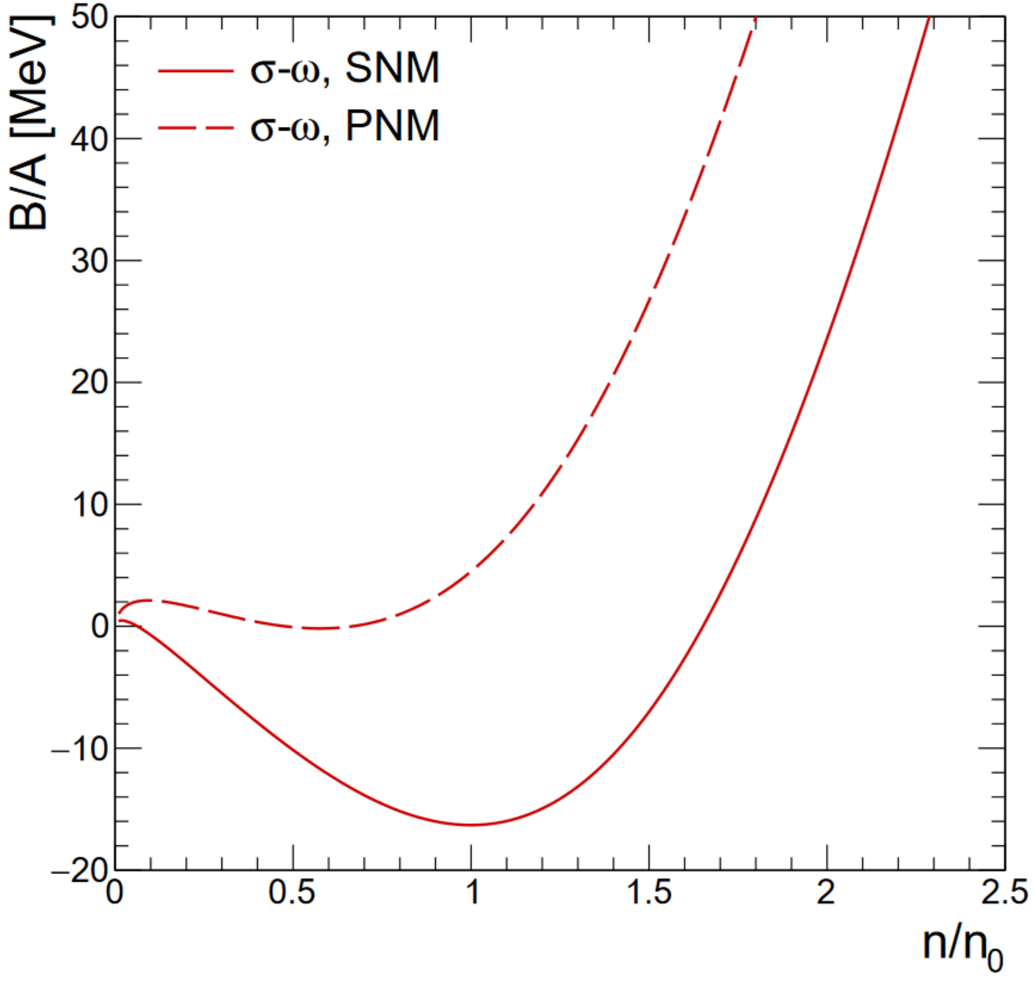

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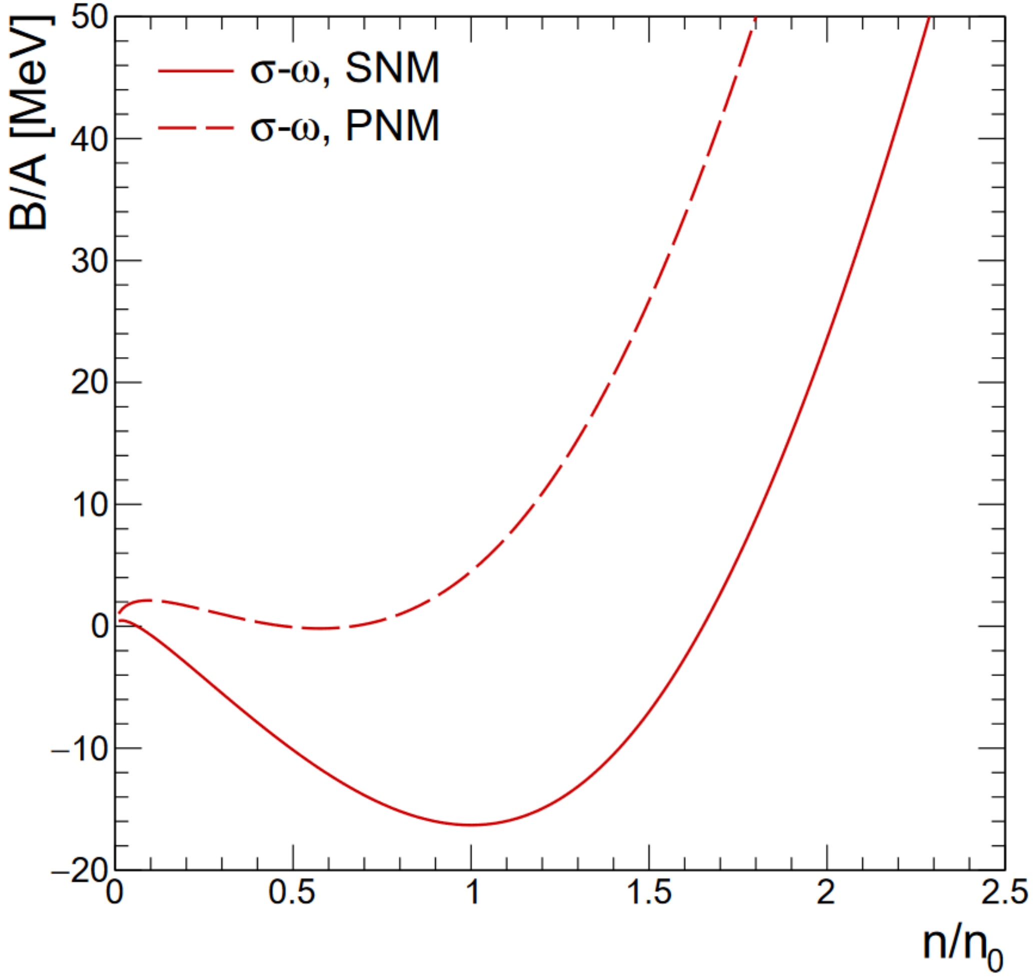

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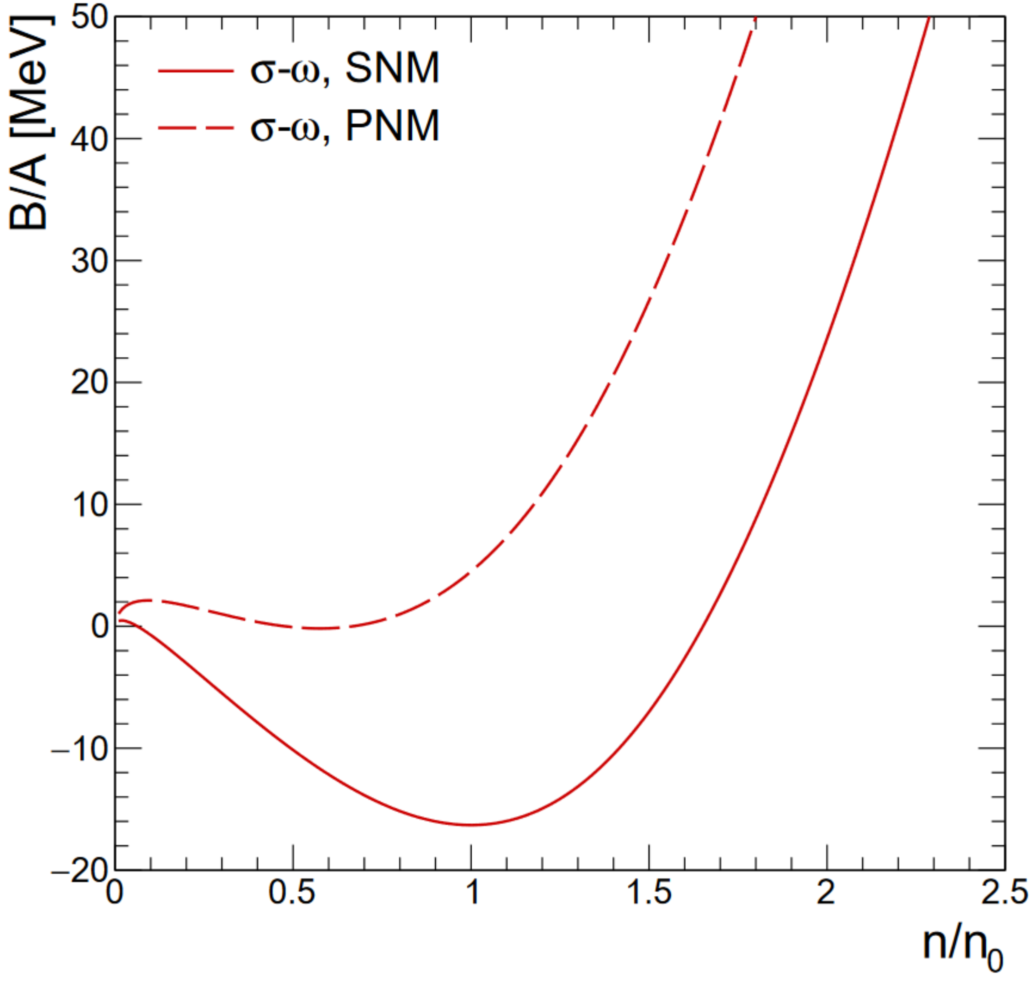
- Add self-interactions
  $$\mathcal{L} = \mathcal{L}_{\sigma-\omega} - \frac{1}{3} bm(g_\sigma \sigma)^3 - \frac{1}{4} c(g_\sigma \sigma)^4$$
  This allows to fulfil two more constraints with two more parameters
- Add ρ meson
  $$\mathcal{L} = \mathcal{L}_{\sigma-\omega} - g_\rho \vec{\rho}_\mu \vec{I}^\mu$$
- Use density dependent couplings
- Add form factors
- Account for the quark substructure of the nucleons
- …



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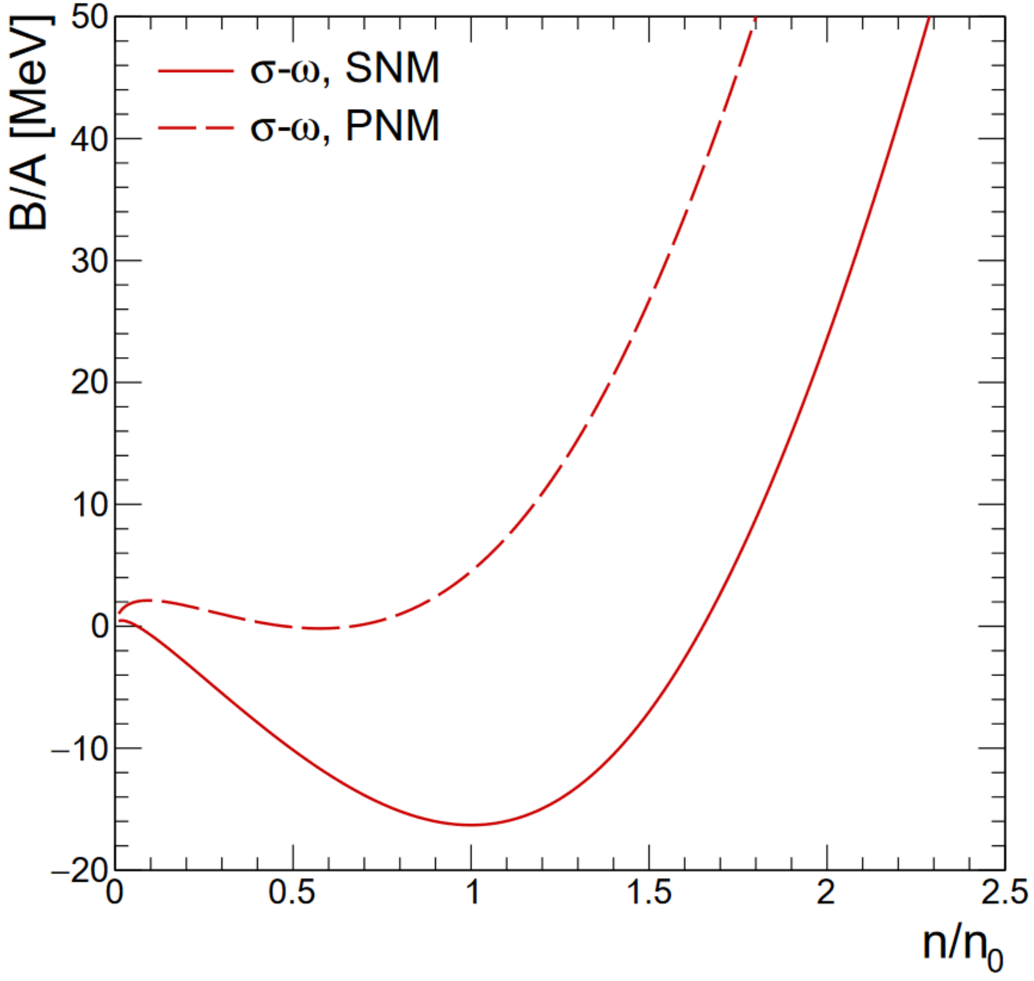
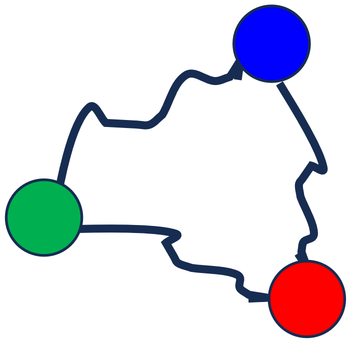


Quark Substructure Effect

Let us consider nonrelativistic massive quarks with a confining two-body interactions in the form of a harmonic oscillator potential [Particles 2020, 3, 477–499]:

$$H(123) = \sum_{i=1}^{3}\left(m + \frac{p_i^2}{2m}\right) + \sum_{i<j=2}^{3} V_{ij}\,, \text{where } V_{ij} = \frac{m\omega^2}{2}\left(\vec{r}_i - \vec{r}_j\right)^2$$



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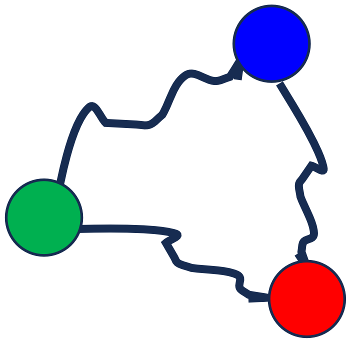
$$H(123) = \sum_{i=1}^{3}\left(m + \frac{p_i^2}{2m}\right) + \sum_{i<j=2}^{3} V_{ij}\,, \text{where } V_{ij} = \frac{m\omega^2}{2}\left(\vec{r}_i - \vec{r}_j\right)^2$$



Now, we can solve Schrödinger equation:

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Orbital part → $\phi_P(123)$

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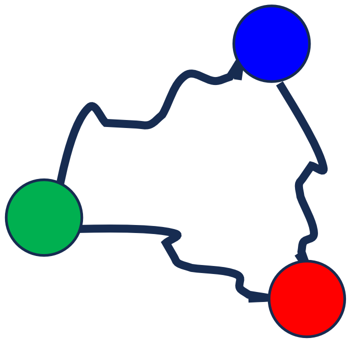

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$$\phi_P(123) = \frac{8\pi^3}{V}\left(\frac{\sqrt{3}b^2}{\pi}\right)^{3/2} \delta_{\vec{P},\vec{P}_R} e^{-\frac{b}{2}(p_\rho^2+p_\lambda^2)}, \qquad E_n = \frac{P^2}{6m} + 3m + 3\sqrt{3}\omega$$

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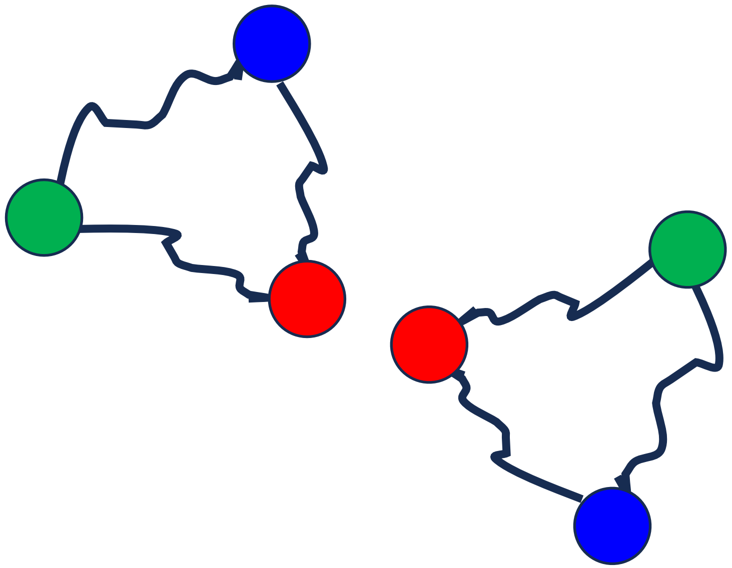

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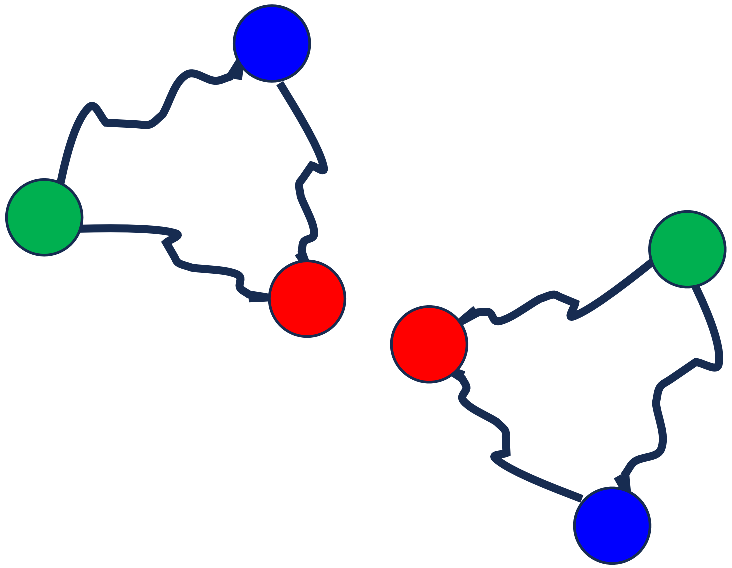

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We want to represent the six-quark wave function as a product of two nucleonic wave functions that behaves antisymmetrically with respect to each exchange of quantum numbers belonging to quarks or nucleons:

$$\Phi_{nn'}(123456) = \left(1 - \textstyle\sum_{i=1}^{3} \hat{P}_{i,i+3}\right)\left(1 - \hat{P}_{nn'}\right) \psi_n(123)\psi_{n'}(456)$$

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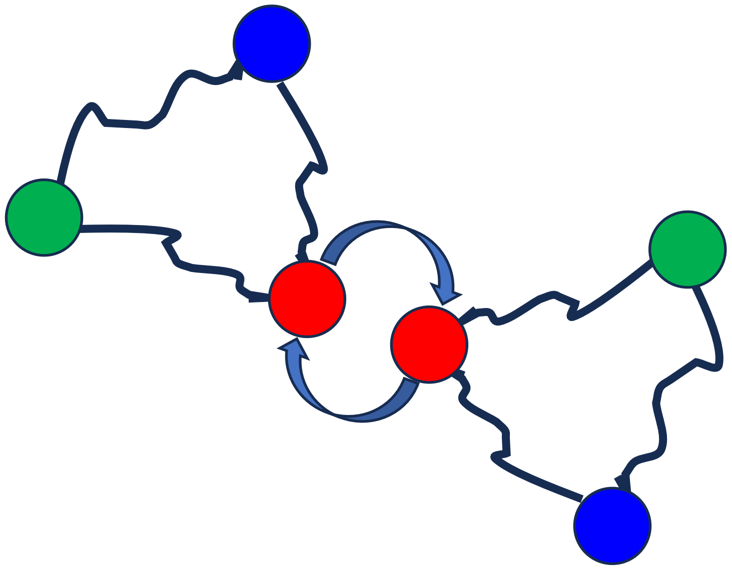

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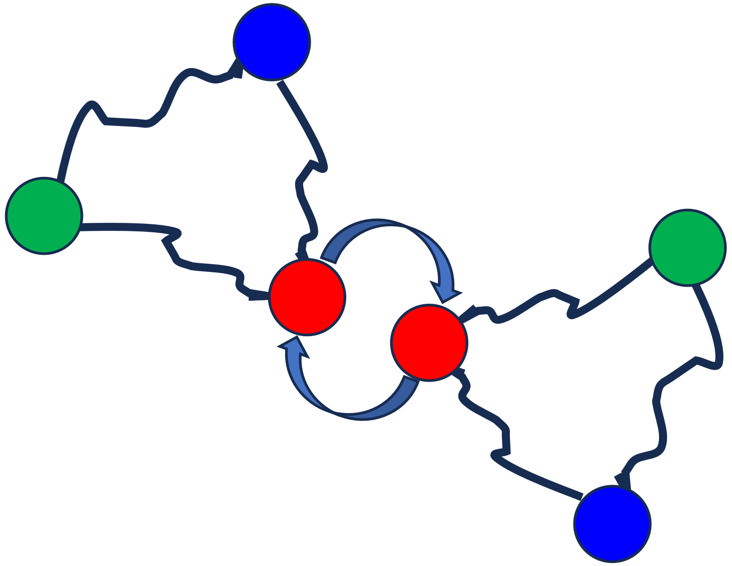
$$\Delta E_{nn'}^{\text{Pauli}} = \frac{\langle \Phi_{nn'} | H | \Phi_{nn'} \rangle}{\langle \Phi_{nn'} | \Phi_{nn'} \rangle} - E_n - E_{n'}$$



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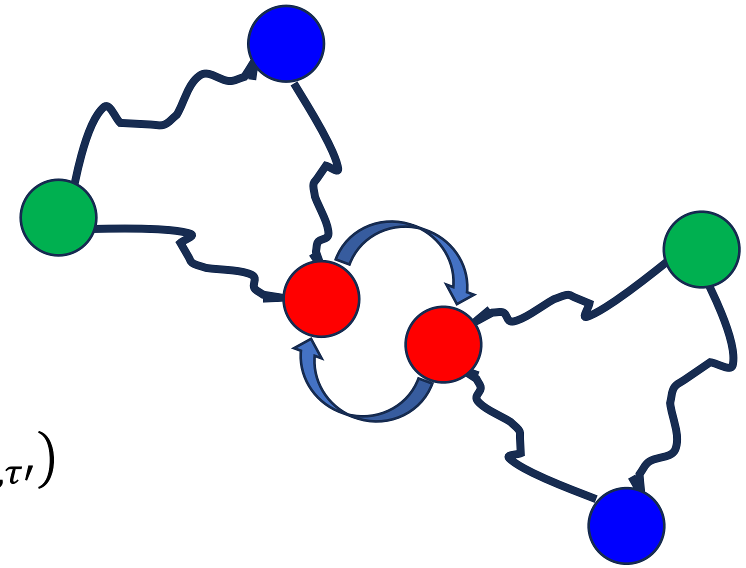

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$$\Delta E_{\tau P}^{\text{Pauli}}(P_{F,n}, P_{F,p}) = \sum_{n'} V \int_{P' < P_F} \frac{d^3 \vec{P}'}{(2\pi)^3} \Delta E_{nn'}^{\text{Pauli}} = \sum_{\tau' = p,n} \sum_{\alpha = 1,2} c_{\tau\tau'}^{(\alpha)} W_\alpha(P, P_{F,\tau'})$$

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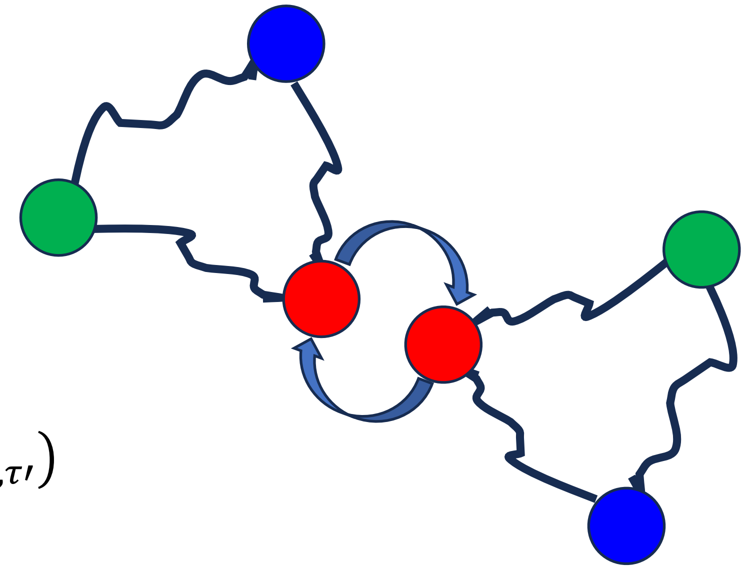

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Where:

$$W_\alpha(P, P_{F,\tau'}) = \frac{9\sqrt{3}}{64\sqrt{\pi}} \frac{b}{m} \frac{1}{\lambda_\alpha^3} \Bigg\{ 12\sqrt{\pi}\left[\operatorname{erf}\left(\lambda_\alpha (P_{F\tau'} - P)\right) + \operatorname{erf}\left(\lambda_\alpha (P_{F\tau'} + P)\right)\right] + \frac{1}{\lambda_\alpha P}\left\{[11 - 2\lambda_\alpha^2 P_{F\tau'}(P_{F\tau'} + P)] e^{-\lambda_\alpha^2 (P_{F\tau'}+P)^2}\right\} - \frac{1}{\lambda_\alpha P}\left\{[11 - 2\lambda_\alpha^2 P_{F\tau'}(P_{F\tau'} - P)] e^{-\lambda_\alpha^2 (P_{F\tau'}-P)^2}\right\} \Bigg\}$$

$\boldsymbol{\tau}$	$\boldsymbol{c_{n\tau}^{(1)}}$	$\boldsymbol{c_{n\tau}^{(2)}}$
$\boldsymbol{n}$	$\frac{15}{81}$	$-\frac{16}{81}$
$\boldsymbol{p}$	$\frac{12}{81}$	$-\frac{14}{81}$

Coefficients $c_{nn'}^{(\alpha)}$ entail the symmetry condition: $\Delta E_{pP}^{\text{Pauli}}(P_{F,n}, P_{F,p}) = \Delta E_{nP}^{\text{Pauli}}(P_{F,p}, P_{F,n})$

Now, we introduce the quark exchange self-energy contribution in the relativistic mean-field EoS:

$$\mu_{ex,\tau} = \Delta E^{\mathrm{Pauli}}_{\tau P_{F,\tau}}\left(P_{F,n}, P_{F,p}\right) = \Delta_\tau(n, x)$$

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- Constant quark mass: $m_q = m_{q0} = 350\ \text{MeV}$
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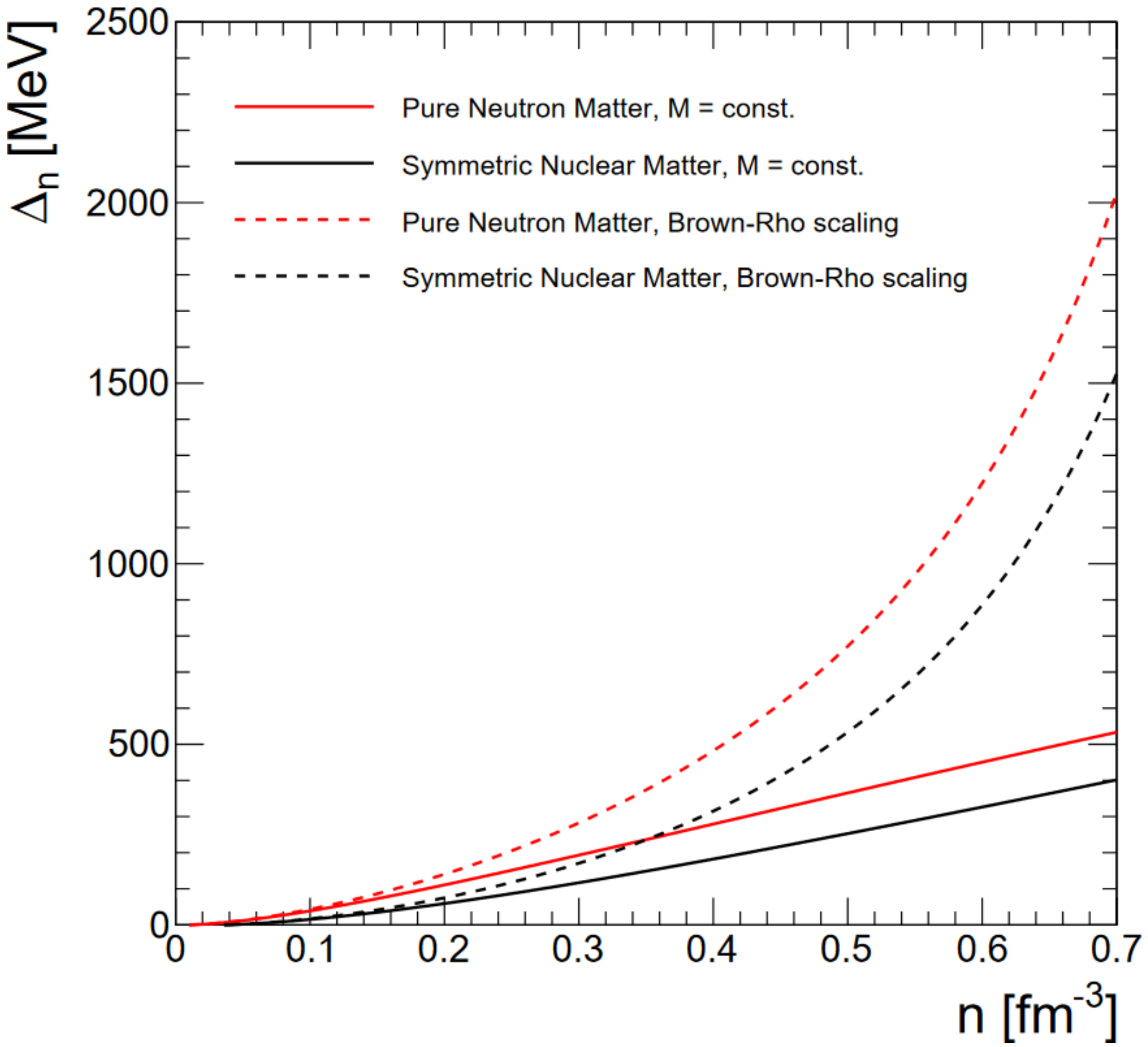

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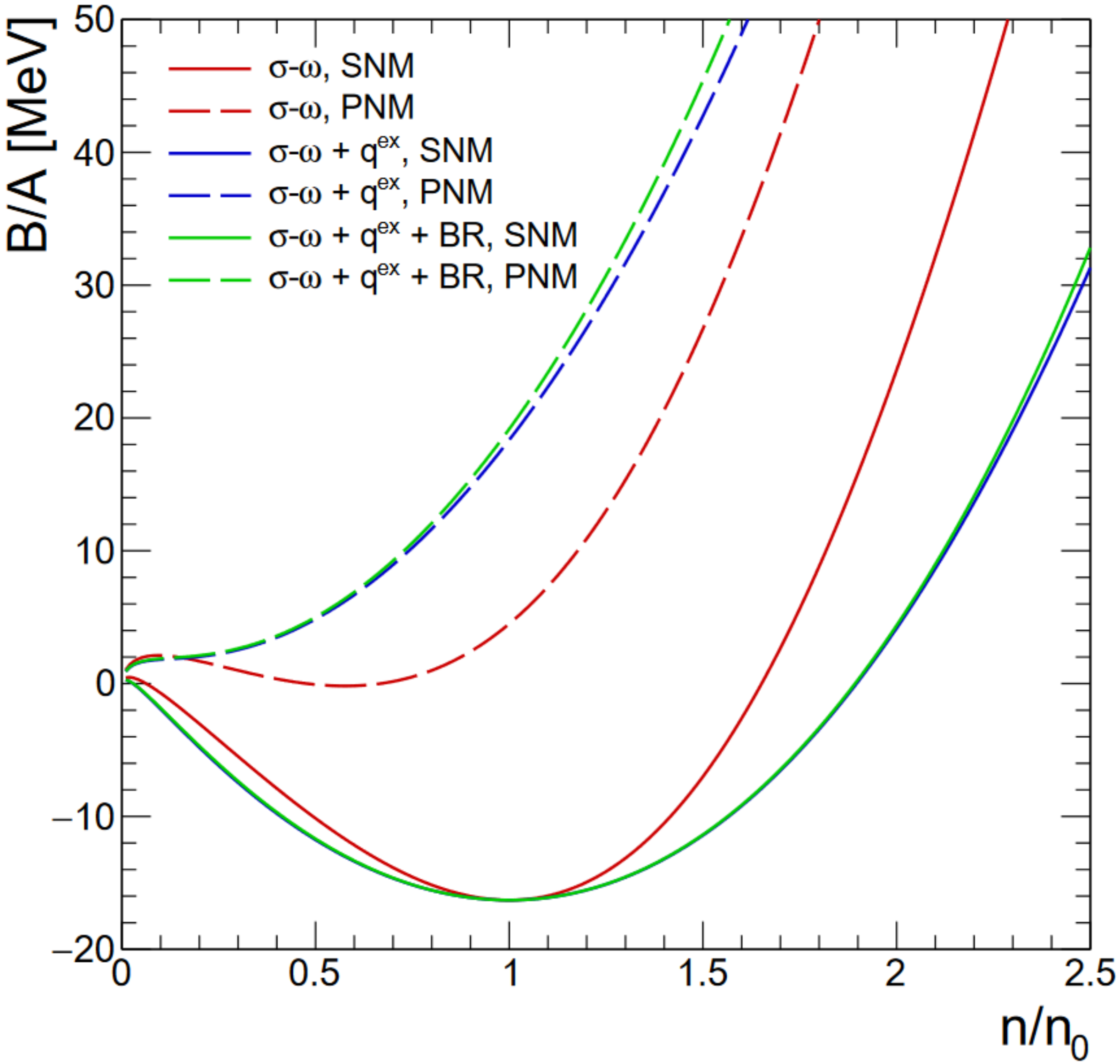
G_ω, fm^2	11.4741	6.3356	4.0066
m^*/m	0.538	0.687	0.759
K, MeV	562.9	335.9	341.5
a_{sym}, MeV	19.7	33.8	34.7



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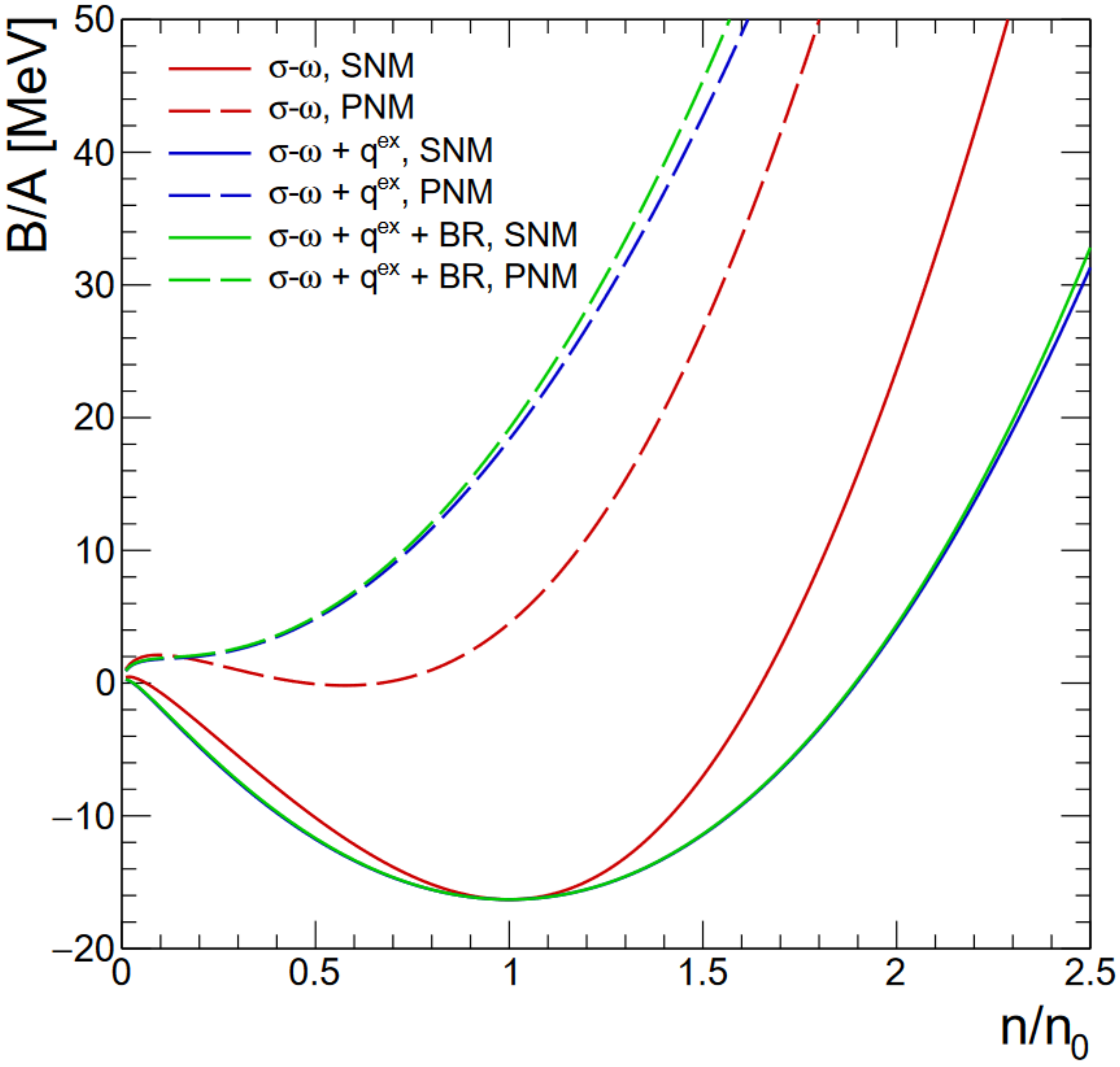

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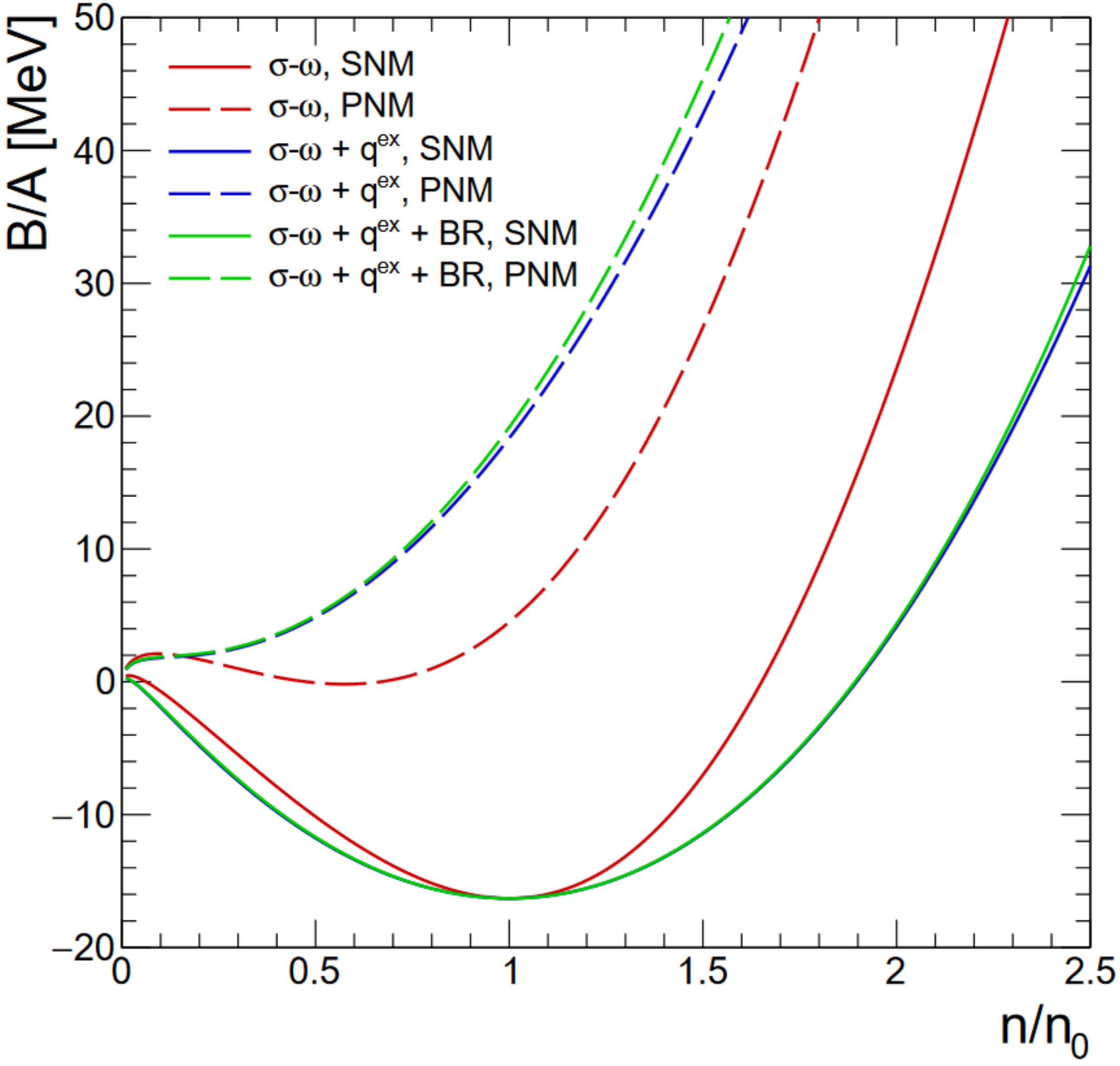

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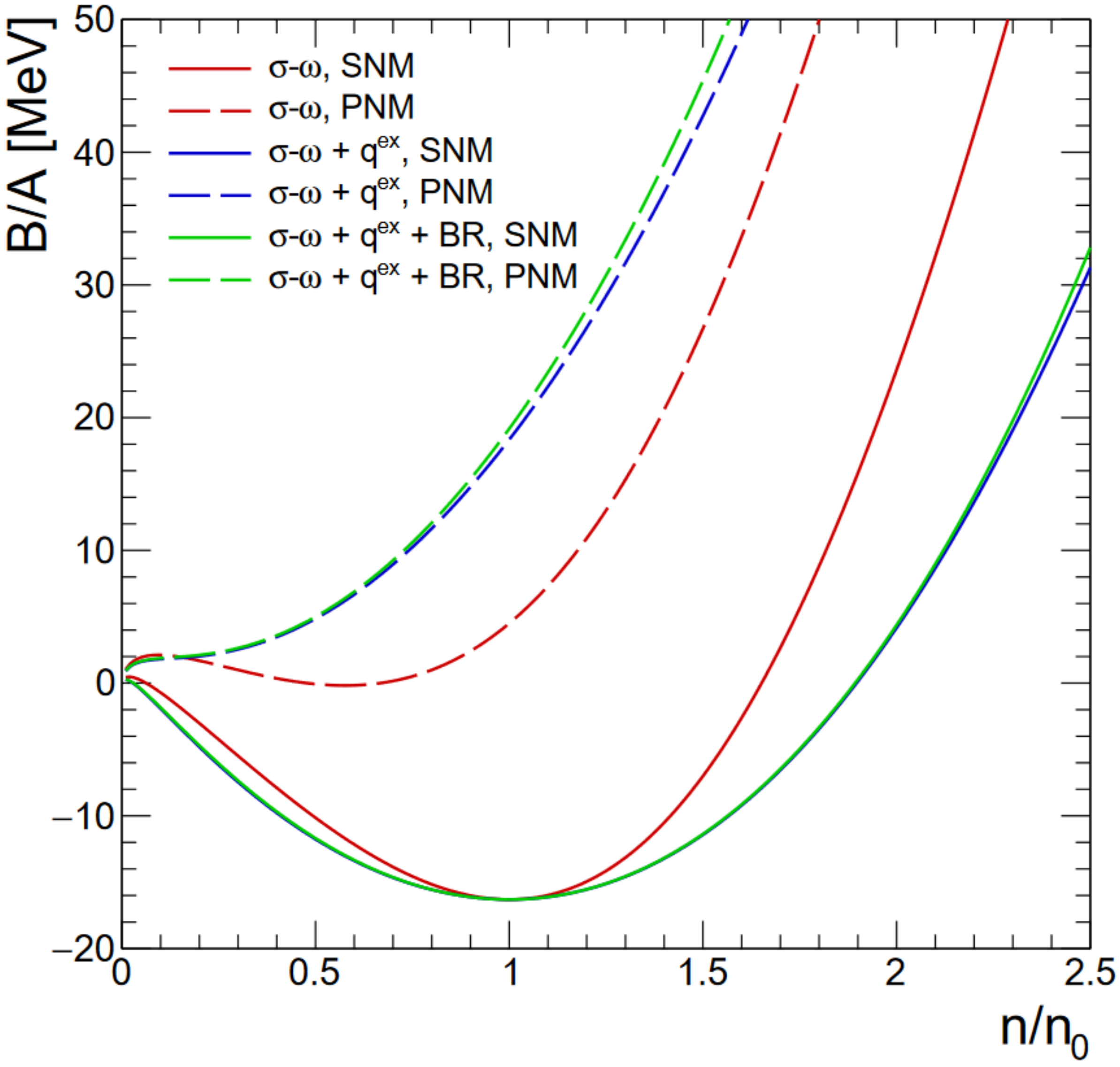

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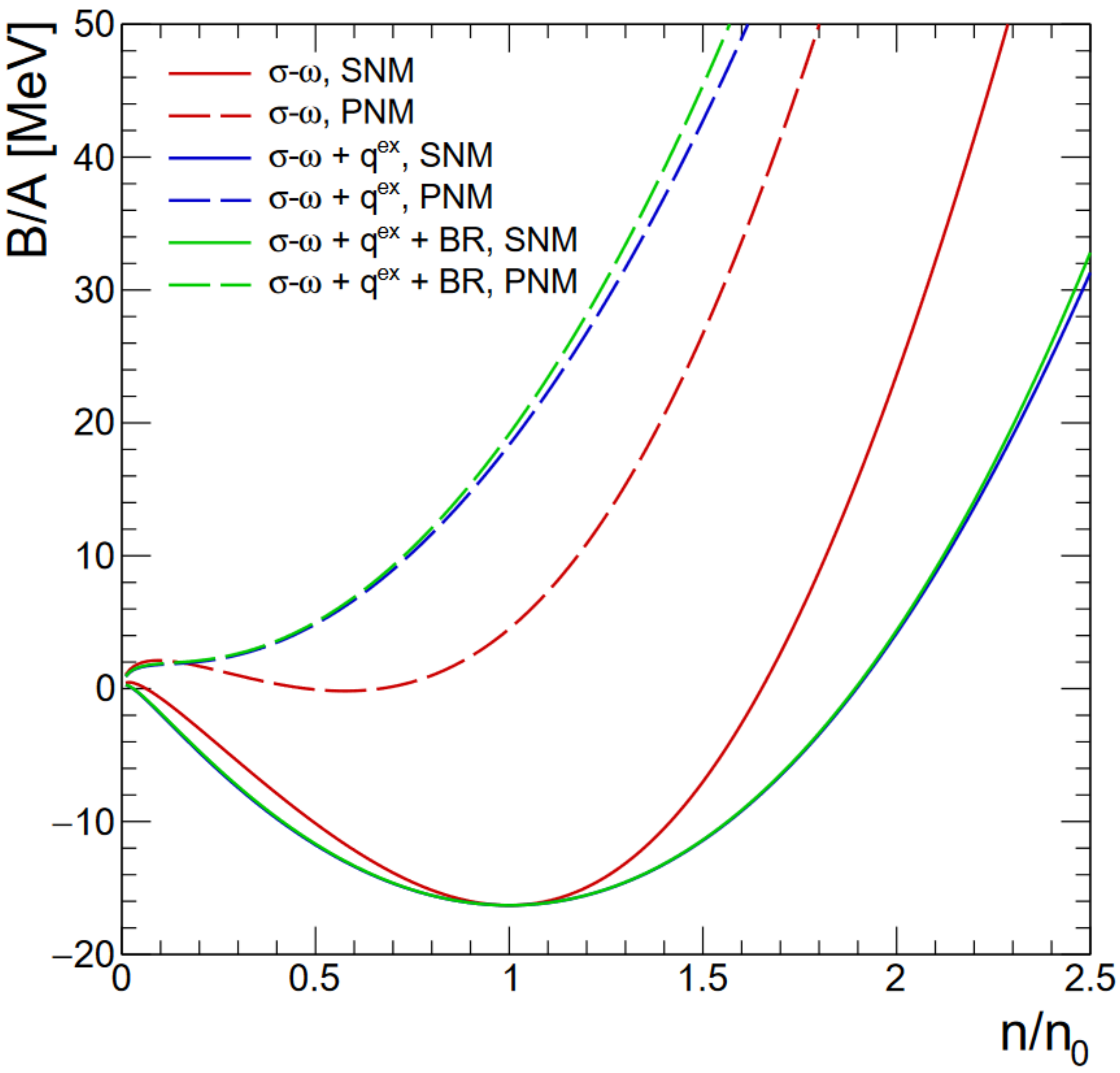
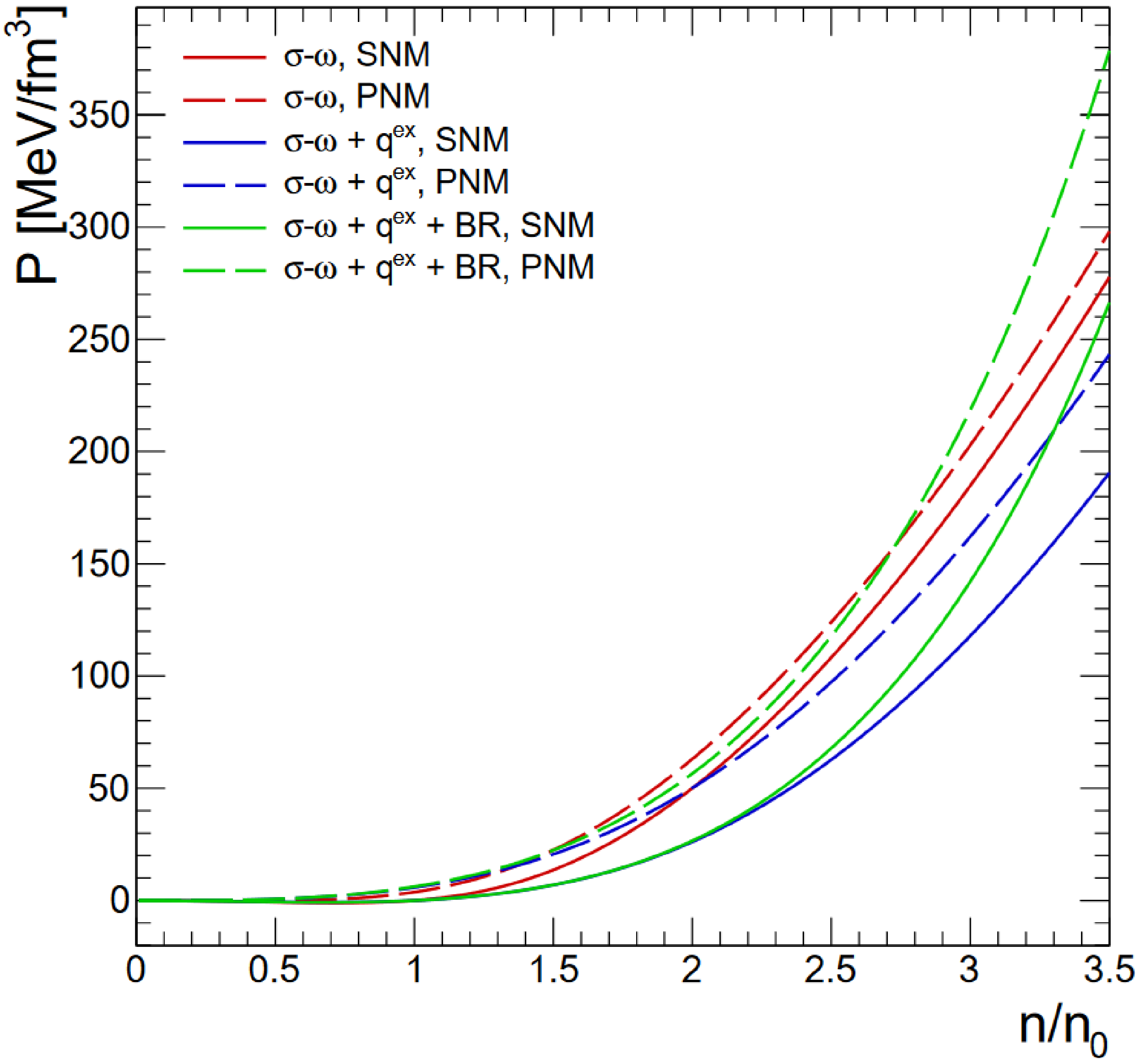
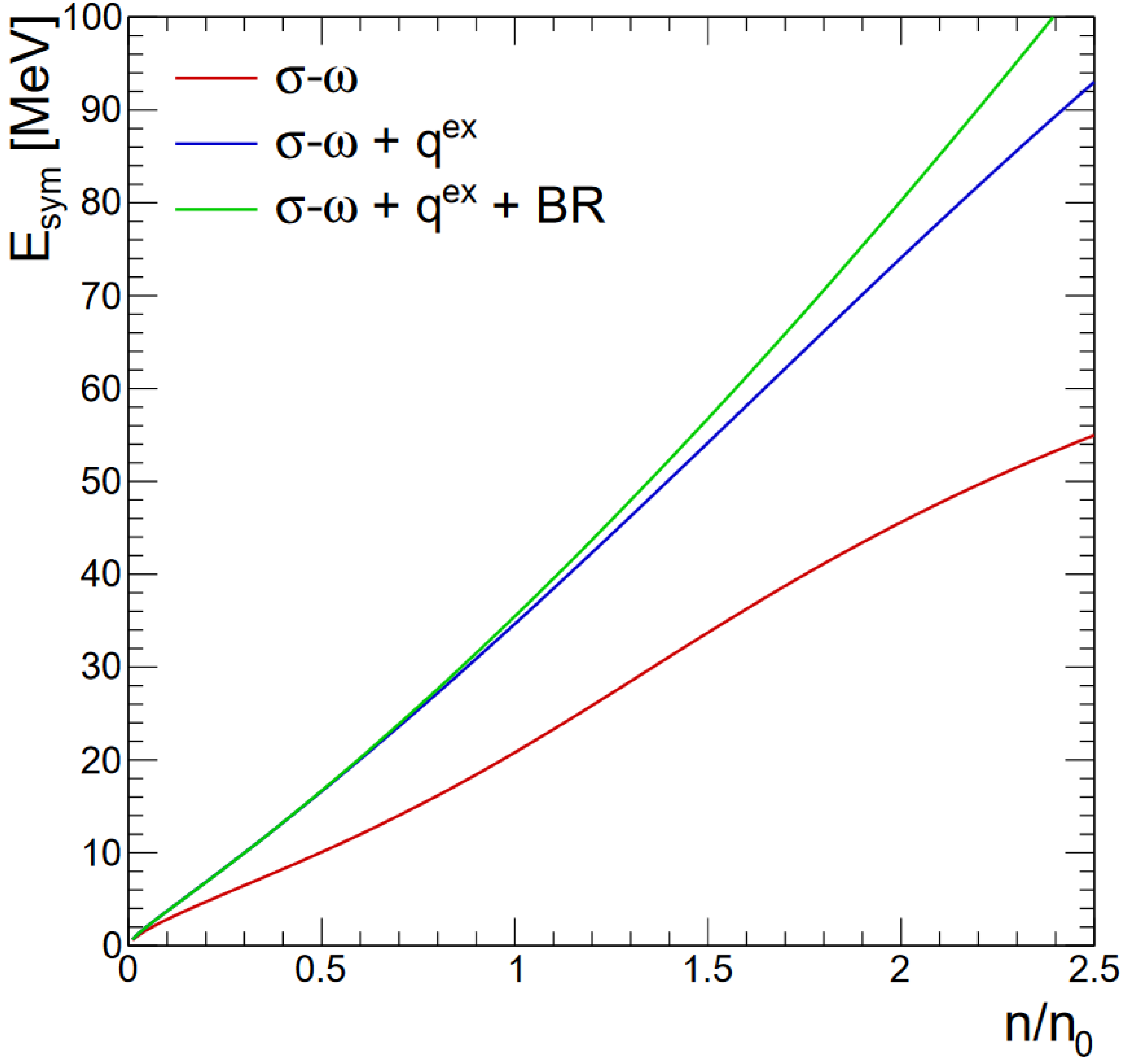
Correct value of the symmetry energy coefficient without iso-vector meson



Results: SNM & PNM





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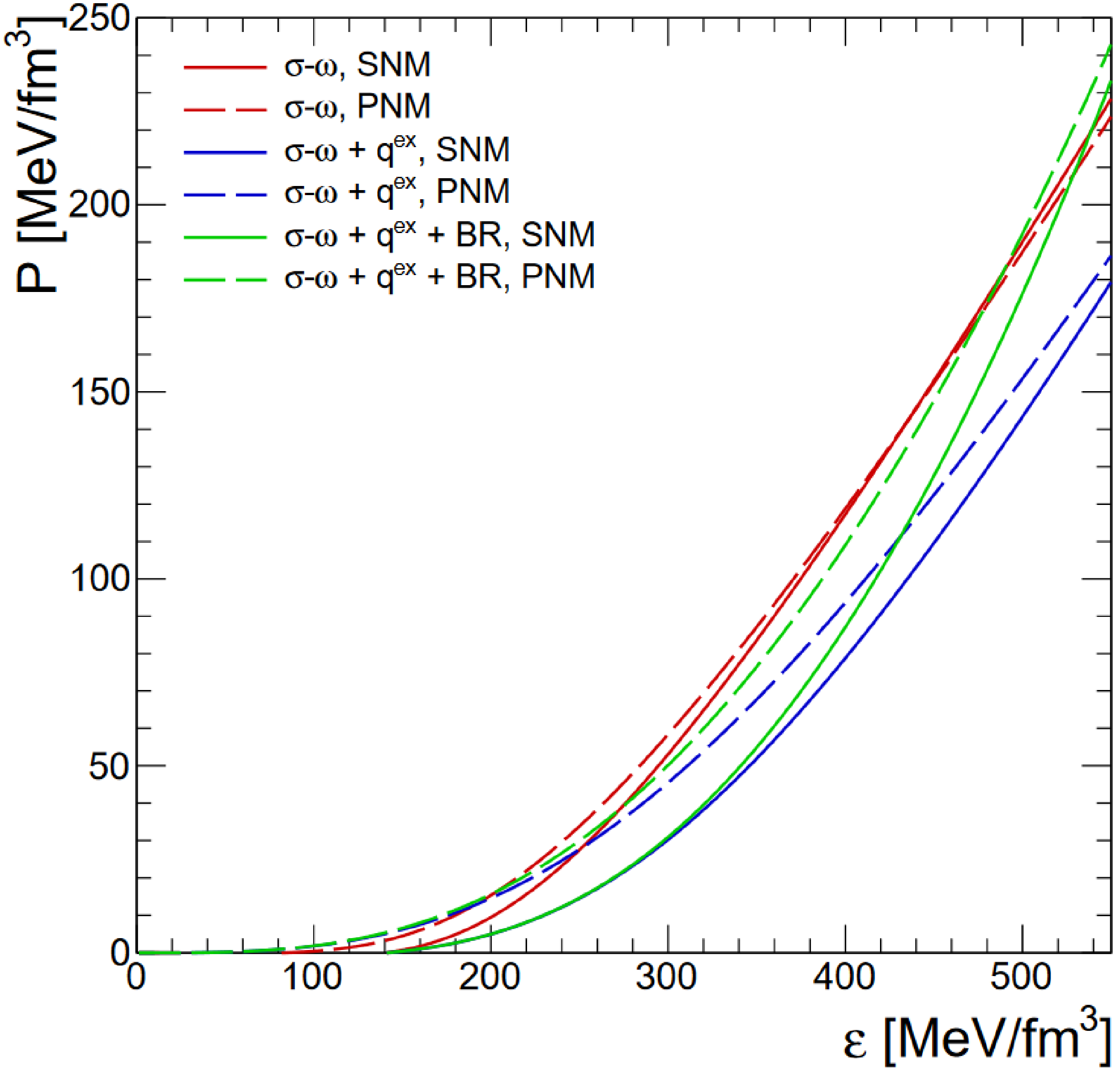

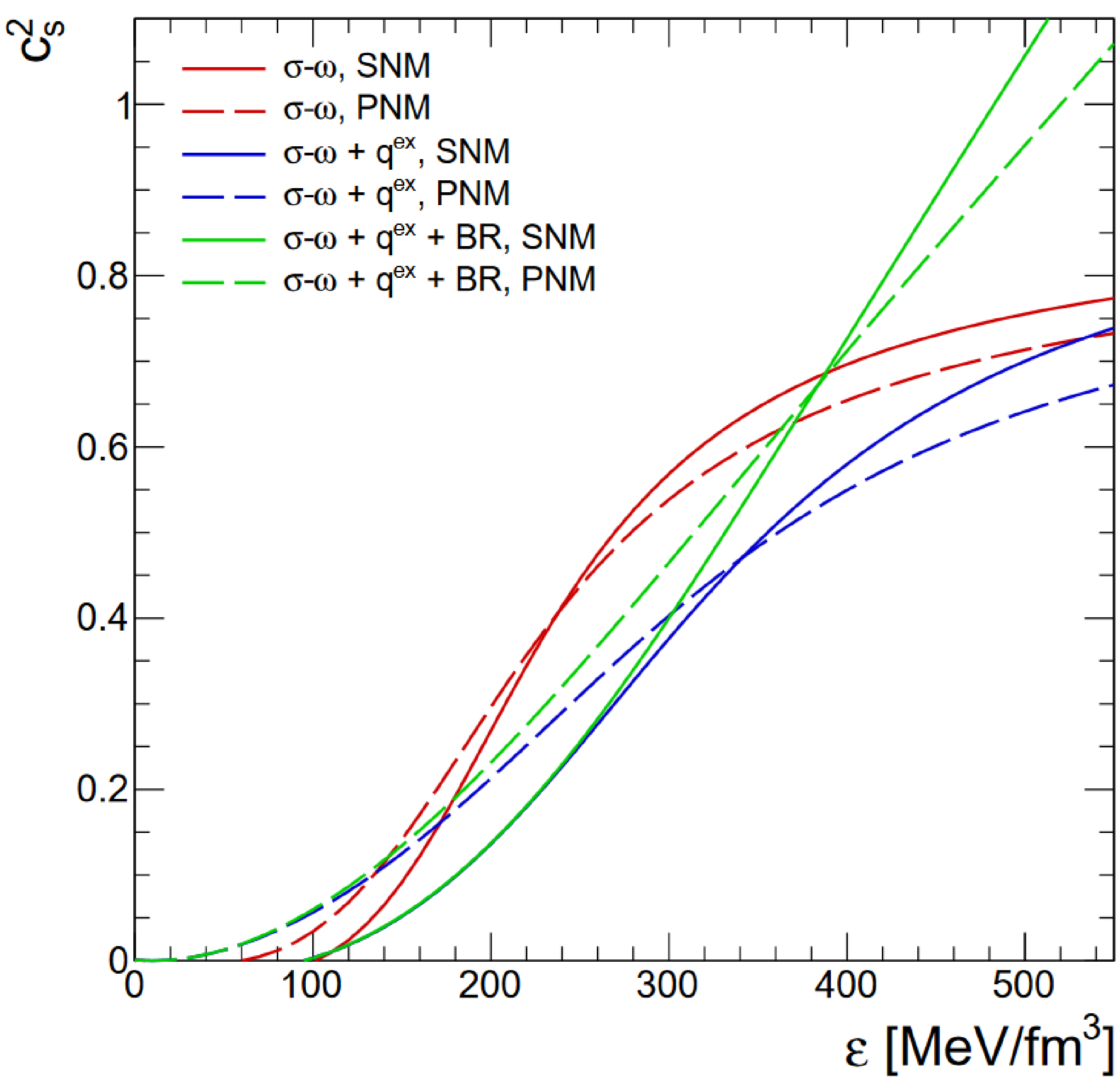

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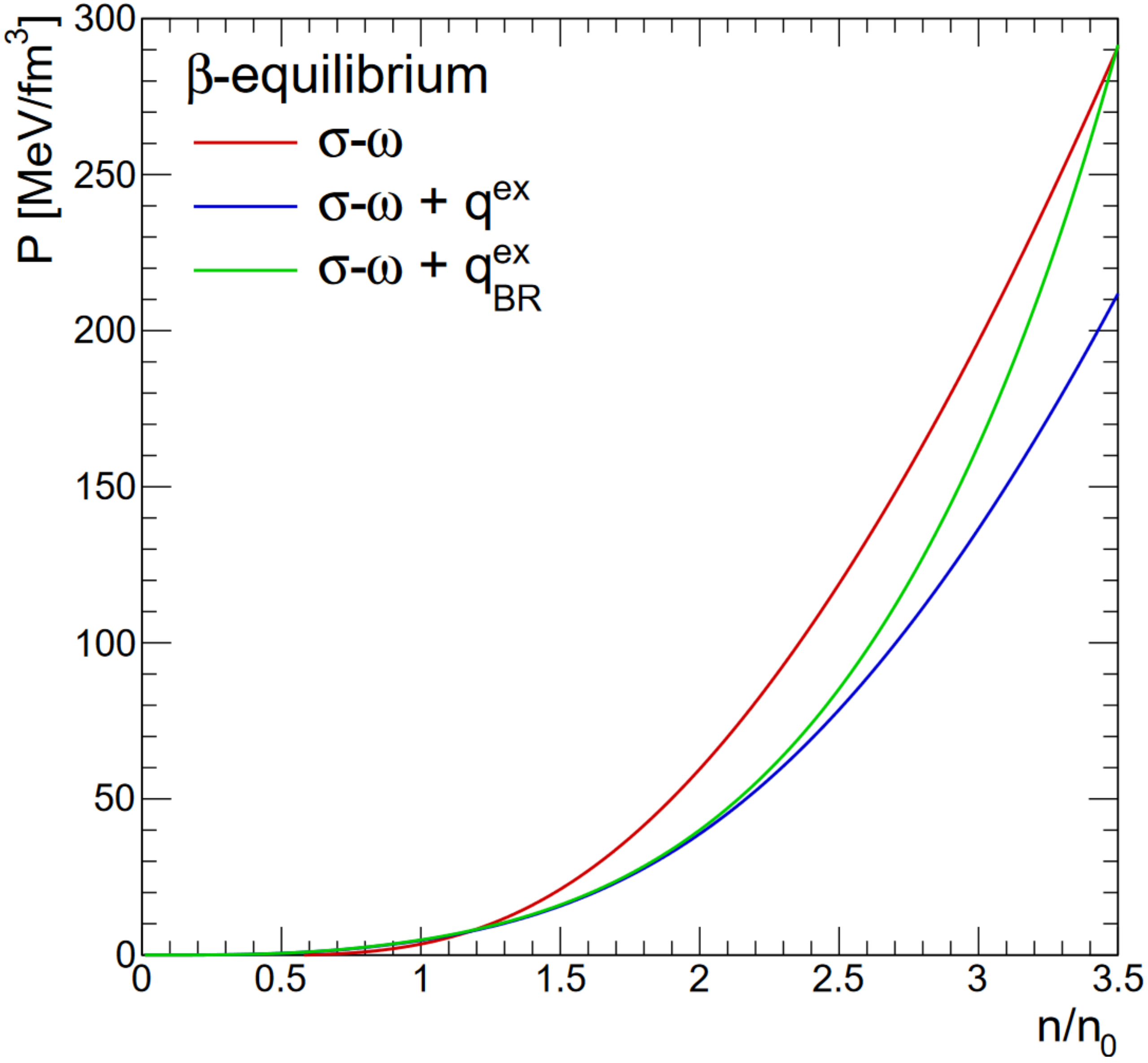

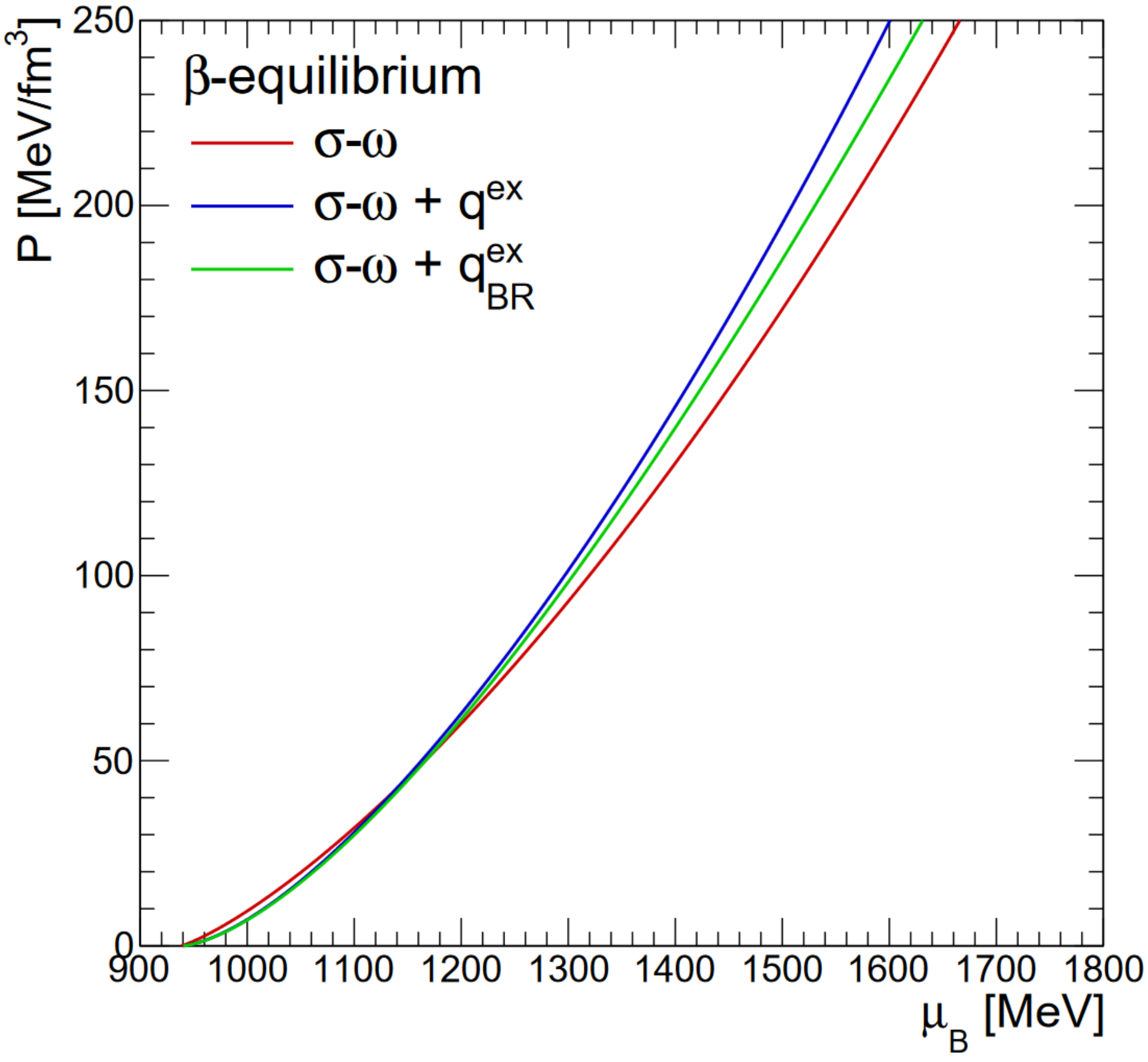

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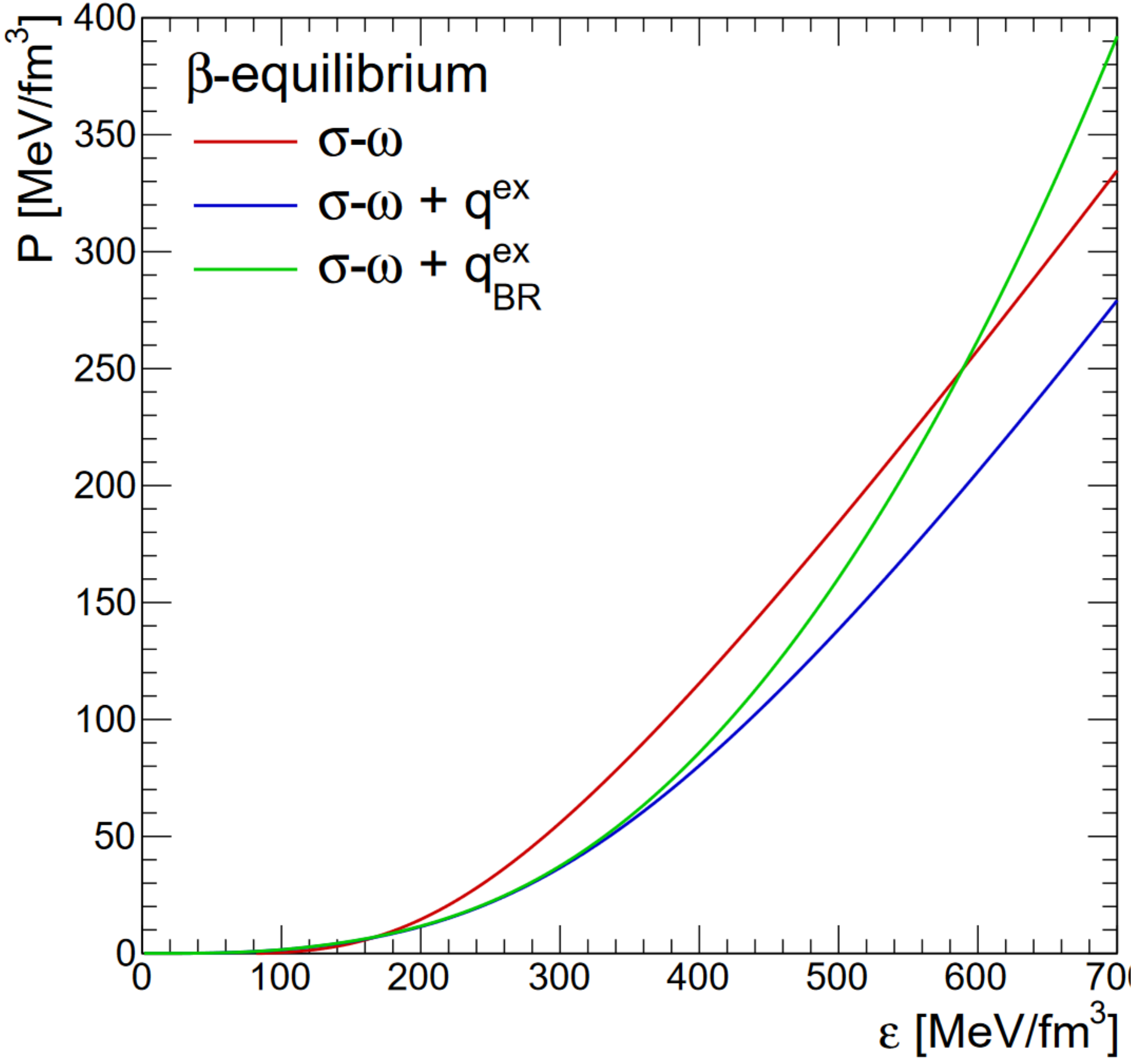

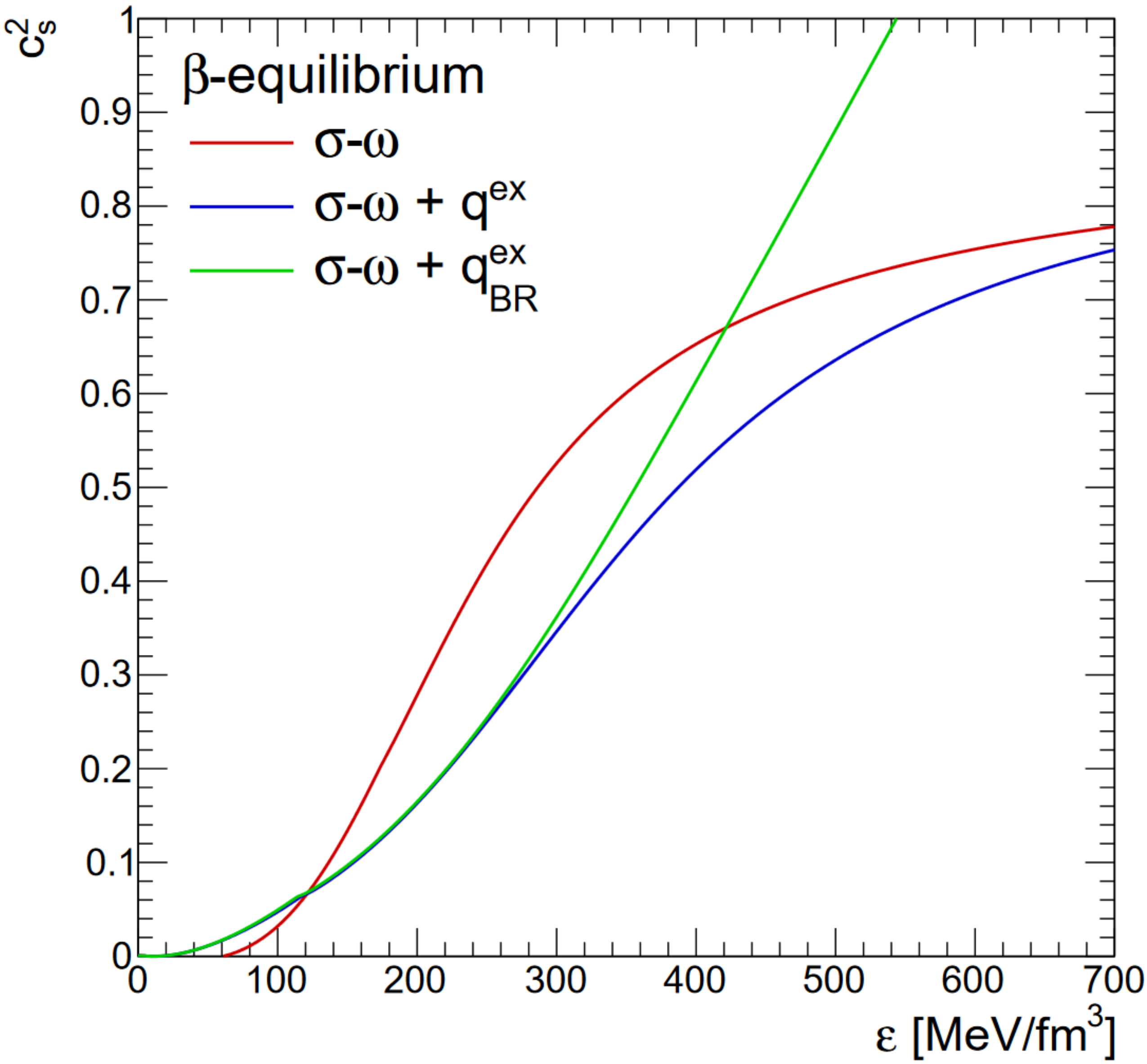

Summary & Outlook

Summary:

- The relativistic mean field model of the nuclear matter equation of state was modified by including the effect of Pauli-blocking owing to quark exchange between the baryons
- A chiral enhancement of the quark Pauli blocking due to a density-dependent reduction of the value of the dynamical quark mass was considered
- Onset of chiral symmetry restoration (mimicked by Brown-Rho scaling) enhances nucleon swelling and Pauli blocking at high baryon densities
- Partial replacement of other short-range repulsion mechanisms (ω_μ exchange)
- The nuclear symmetry energy was obtained in fair accordance with phenomenological constraints, i.e. full replacement of the iso-vector vector $\vec{\rho}_\mu$ meson

Outlook:

- Introduction of other hadronic states (hyperons)
- Relativistic generalization
- Finite temperature case