Recent advances in the description of lepton-nucleus scattering

Noemi Rocco

QNP - The 10th International conference on Quark and Nuclear Physics
Universitat de Barcelona— July 8 - 12, 2024
Many fundamental questions are still open:

- Correct mass ordering
- Are there CP violations in the lepton sector
  \[ P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \]
- Measure \( \delta_{CP} \)
- What is the octant for \( \theta_{23} \)
Long baseline neutrino experiments

These questions are addressed with Long Baseline Neutrino Oscillation experiments

**T2K:** Tokai to Kamioka
295 km

**NOvA:** Fermilab to Ash River
810 km

Currently measuring:

\[ P(\nu_\mu \rightarrow \nu_\mu) \quad \nu_\mu \text{ disappearance} \]

\[ P(\nu_\mu \rightarrow \nu_e) \quad \nu_e \text{ appearance} \]
Why do we need more precision?

New generation of neutrino experiments will measure the unknown PMNS parameters with unprecedented accuracy.

\[ P(\nu_\mu \rightarrow \nu_e, E_\nu, L) = \frac{\Phi(E_\nu, L)}{\Phi_\mu(E_\nu, 0)} = \frac{N_e(E_\nu, L)/\sigma_e(E_\nu)}{N_\mu(E_\nu, L)/\sigma_\mu(E_\nu)} \]

Theory

Experiment

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Inputs for the nuclear model

More than 60% of the interactions at DUNE are non-quasielastic

**Unprecedented accuracy** in the determination of neutrino-argon cross section is required to achieve design sensitivity to CP violation at DUNE

Unprecedented accuracy in the determination of neutrino-argon cross section is required to achieve design sensitivity to CP violation at DUNE.

Theoretical tools for neutrino scattering, Contribution to: 2022 Snowmass Summer Study

Noemi Rocco, nrocco@fnal.gov
Why do we need more precision?

CLAS and e4ν collaboration,
Nature 599 (2021) 7886, 565-570

Used semi-exclusive electron scattering data to test models and event generators used in oscillation analyses

The results indicate the need for substantial improvement in the accuracy of the neutrino interactions' models and simulations

Current oscillation experiments report large systematic uncertainties associated with neutrino- nucleus interaction models.

Overview of neutrino cross section measurements,
M. Buizza Avanzini, Neutrino 2024
Short Baseline Neutrino program

The two sub-GeV neutrino beams (BNB and T2K) have very similar medium energy fluxes. The flux for T2K is narrower due to the off-axis effect.

<table>
<thead>
<tr>
<th>T2K beam</th>
<th>BNB beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutrino flux @SBND</td>
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</tr>
</tbody>
</table>

For BNB and T2K the dominant reaction mechanisms are quasi-elastic scattering. The contribution of π-production channels is ~25%.

For the sub-GeV experiments, the Delta is the only relevant resonance.

Three liquid argon TPCs in the Fermilab Booster Neutrino Beam: Definitive test of LSND oscillations using three baselines.

Noemi Rocco, nrocco@fnal.gov
Short Baseline Neutrino program

SBND will provide the world’s highest statistics cross section measurements in LAr: 2 million events for $\nu_\mu$ per year for the next 3 years

A. Papadopoulou Neutrino 2024

MicroBooNE provided the first simultaneous measurement of differential muon-neutrino CC cross sections on argon for final states with and without protons

D. Glbin, Neutrino 2024

ICARUS: new CC0$\pi$ analysis. Events with $1\mu + NP + 0\pi$
Theory of lepton-nucleus scattering

- The cross section of the process in which a lepton scatters off a nucleus is given by

\[ d\sigma \propto L^{\alpha \beta} R_{\alpha \beta} \]

Leptonic Tensor: determined by lepton kinematics

Hadronic Tensor: nuclear response function

\[ R_{\alpha \beta}(\omega, \mathbf{q}) = \sum_f \langle 0| J^+_\alpha(\mathbf{q}) | f \rangle \langle f| J_\beta(\mathbf{q}) |0 \rangle \delta(\omega - E_f + E_0) \]

The initial and final wave functions describe many-body states:

\[ |0\rangle = |\Psi_0^A\rangle, |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \ldots \]

One and two-body current operators

\[ \gamma, Z, W^\pm \]
Ab initio Methods

Ab-initio methods (CC, IMSRG, SCGF, QMC, etc) provide accurate predictions for ground state properties of nuclei + response functions in the low/moderate energy region.

\[ \omega_e \sim \frac{q^2}{2m} \]

QE

Meson Exchange
Hamiltonian and Currents

At low energy, the effective degrees of freedom are pions and nucleons:

\[ H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

1-body \hspace{1cm} 2-body \hspace{1cm} 3-body

\[ j^\mu(q) = \sum_i j_i^\mu + \sum_{i<j} j_{ij}^\mu + \ldots \]

The electromagnetic current is constrained by the Hamiltonian through the continuity equation

\[ \nabla \cdot j_{EM} + i[H, j^0_{EM}] = 0 \]

\[ [v_{ij}, j_i^0] \neq 0 \]

The above equation implies that the current operator includes one and two-body contributions.

• AV18+IL7
• chiral interactions
Many-Body method: GFMC

GFMC projects out the exact lowest-energy state:

\[ e^{-\left(H-E_0\right)\tau} |\Psi_T\rangle \rightarrow |\Psi_0\rangle \]

The computational cost of the calculation is \(2^A \times A!/Z!(A-Z)!\)

\[
|S\rangle = \begin{pmatrix}
S \uparrow\uparrow\uparrow \\
S \uparrow\uparrow\downarrow \\
S \uparrow\downarrow\uparrow \\
S \uparrow\downarrow\downarrow \\
S \downarrow\uparrow\uparrow \\
S \downarrow\uparrow\downarrow \\
S \downarrow\downarrow\uparrow \\
S \down\down\down
\end{pmatrix}
\]

Nuclear response function involves evaluating a number of transition amplitudes. Valuable information can be obtained from the integral transform of the response function

\[
E_{\alpha\beta}(\sigma, q) = \int d\omega K(\sigma, \omega) R_{\alpha\beta}(\omega, q) = \langle \psi_0 | J_{\alpha}^\dagger (q) K(\sigma, H - E_0) J_{\beta} (q) | \psi_0 \rangle
\]

Inverting the Laplace transform is a complicated problem

A. Lovato et al. PRL117 (2016), 082501, PRC97 (2018), 022502
Cross sections: Green’s Function Monte Carlo

Limitations:

Medium mass nuclei $A < 13$

Inclusive results which are virtually correct in the QE

Relies on non-relativistic treatment of the kinematics

Can not handle explicit pion degrees of freedom

Alessandro Lovato et al. PRL 117 082501 (2016)

A. Lovato, NR, et al, submitted to Universe

Noemi Rocco, nrocco@fnal.gov
Axial form factor determination

- The axial form-factor has been fit to the dipole form
  \[ F_A(q^2) = \frac{g_A}{(1 - q^2/m_A^2)^2} \]

- The intercept \( g_A = -1.2723 \) is known from neutron β decay

- Different values of \( m_A \) from experiments
  - \( m_A = 1.02 \) GeV q.e. scattering from deuterium
  - \( m_A = 1.35 \) GeV @ MiniBooNE

- Alternative derivation based on z-expansion
  —model independent parametrization
  \[ F_A(q^2) = \sum_{k=0}^{k_{\text{max}}} a_k z(q^2)^k, \]

Bhattacharya, Hill, and Paz  PRD 84 (2011) 073006

Noemi Rocco, nrocco@fnal.gov
Axial form factor determination

Comparison with recent MINERvA antineutrino-hydrogen charged-current measurements

Novel methods are needed to remove excited-state contributions and discretization errors

D2 Meyer et al: fits to neutrino-deuteron scattering data

LQCD result: general agreement between the different calculations

LQCD results are 2-3σ larger than D2 Meyer ones for $Q^2 > 0.3$ GeV$^2$

O. Tomalak, R. Gupta, T. Battcharaya, 2307.14920

Noemi Rocco, nrocco@fnal.gov
Study of model dependence in neutrino predictions

MiniBooNE results; study of the dependence on the axial form factor:


<table>
<thead>
<tr>
<th>MiniBooNE</th>
<th>0.2 &lt; cos(θ) &lt; 0.3</th>
<th>0.5 &lt; cos(θ) &lt; 0.6</th>
<th>0.8 &lt; cos(θ) &lt; 0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFMC Dipole (M_A = 1 GeV)</td>
<td></td>
<td></td>
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<tr>
<td>GFMC z expansion (D2)</td>
<td></td>
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<tr>
<td>GFMC z expansion (LQCD)</td>
<td></td>
<td></td>
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<tr>
<td>MB</td>
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</table>

GFMC Difference in dσ_{peak} (%)

<table>
<thead>
<tr>
<th>MiniBooNE</th>
<th>0.2 &lt; cos θ_μ &lt; 0.3</th>
<th>0.5 &lt; cos θ_μ &lt; 0.6</th>
<th>0.8 &lt; cos θ_μ &lt; 0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFMC Difference in dσ_{peak} (%)</td>
<td>18.6</td>
<td>17.1</td>
<td>12.2</td>
</tr>
</tbody>
</table>

D. Simons, N. Steinberg et al, 2210.02455
Study of model dependence in neutrino predictions

T2K results; study of the dependence on the axial form factor:


D. Simons, N. Steinberg et al, 2210.02455

<table>
<thead>
<tr>
<th>T2K</th>
<th>0.0 &lt; cos(θ) &lt; 0.6</th>
<th>0.8 &lt; cos(θ) &lt; 0.85</th>
<th>0.94 &lt; cos(θ) &lt; 0.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFMC difference in (d\sigma_{peak}) (%)</td>
<td>15.8</td>
<td>8.0</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Noemi Rocco, nrocco@fnal.gov
Coupled Cluster Method

Reference state Hartree Fock: \( |\Psi\rangle \)

Include correlations through \( e^T \) operator

Similarity transformed Hamiltonian

\[
e^{-T} H e^T |\Psi\rangle = \tilde{H} |\Psi\rangle = E |\Psi\rangle
\]

\( T \) is an expansion in particle- hole excitations with respect to the reference state \( |\Psi\rangle \)

\[
T = \sum t^i_a a^+_a a_i + \sum t^{ij}_{ab} a^+_a a^+_b a_i a_j + \ldots
\]

Polynomial scaling with the number of nucleons (predictions for \(^{132}\)Sn and \(^{208}\)Pb)

Electroweak response functions obtained using LIT

\[
K_\Gamma(\omega, \sigma) = \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \sigma)^2}
\]

JES, B. Acharya, S. Bacca, G. Hagen; PRL 127 (2021) 7, 072501, PRC 109 (2024) 2, 025502
Address new experimental capabilities

- Excellent spatial resolution
- Precise calorimetric information
- Powerful particle identification

\[ W = \sqrt{(p + q)^2}, \quad Q^2 = -q^2 = -(p_\nu - p_l)^2 \]
Factorization Based Approaches

Factorization of the hadronic final states: allows to tackle exclusive channels + higher energies relevant for DUNE
Short-Time Approximation

Response functions are given by the **scattering from pairs of fully interacting nucleons** that **propagate** into a **correlated pair** of nucleons.

The sum over all final states is replaced by a two nucleon propagator

\[ R_{\alpha}(q, \omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega + E_i)t} \langle \Psi_i | O_{\alpha}^{\dagger}(q) e^{-iHt} O_{\alpha}(q) | \Psi_i \rangle \]

Provides “more” exclusive information in terms of nucleon-pair kinematics via the Response Densities as functions of (E,e)

\[ R^{STA}(q, \omega) \sim \int \delta(\omega + E_0 - E_f) de dE_{cm} D(e, E_{cm}; q) \]

Pastore et al. PRC101(2020)044612

L. Andreoli, NR, et al. PRC 105, 014002 (2022)

**G. King** talk this afternoon
Spectral function approach

At large momentum transfer, the scattering reduces to the sum of individual terms

\[ J_\alpha = \sum_i j^i_\alpha \]

\[ |\Psi_f\rangle \rightarrow |p\rangle \otimes |\Psi_f\rangle_{A-1} \]

The incoherent contribution of the one-body response reads

\[ R_{\alpha\beta} \approx \int \frac{d^3k}{(2\pi)^3} dE P_h(k, E) \sum_i \langle k | j^i_\alpha \rangle \langle k + q | j^i_\beta \rangle |k\rangle \delta(\omega + E - e(k + q)) \]

The Spectral Function is the imaginary part of the two point Green’s Function

Different many-body methods can be adopted to determine it

\[ NR, \text{ Frontiers in Phys. 8 (2020) 116} \]
\[ J.E. \ Sobczyk \ et \ al, \ PRC \ 106 \ (2022) \ 3 \]
\[ J.E. \ Sobczyk \ et \ al, \ PRC \ 109 \ (2024) \]

O. Benhar et al, Rev.Mod.Phys. 80 (2008)
QMC Spectral function of light nuclei

- Single-nucleon spectral function:

\[
P_{p,n}(\mathbf{k}, E) = \sum_{n} \left| \langle \psi_{0}^{\mathbf{A}} | [\{ k \} \otimes \psi_{n}^{\mathbf{A}-1}] \rangle \right|^{2} \delta(E + E_{0}^{\mathbf{A}} - E_{n}^{\mathbf{A}-1}) = P_{MF}(\mathbf{k}, E) + P_{corr}(\mathbf{k}, E)
\]

\[
P_{MF}(\mathbf{k}, E) = \left| \langle \psi_{0}^{\mathbf{A}} | [\{ k \} \otimes \psi_{n}^{\mathbf{A}-1}] \rangle \right|^{2} \times \delta \left( E - B_{\mathbf{A}} + B_{\mathbf{A}-1} - \frac{k^{2}}{2m_{\mathbf{A}-1}} \right)
\]

- The single-nucleon overlap has been computed within VMC (center of mass motion fully accounted for).

\[
P_{corr}(\mathbf{k}, E) = \int d^{3}k' \left| \langle \psi_{0}^{\mathbf{A}} | [\{ k \} \otimes \psi_{n}^{\mathbf{A}-2}] \rangle \right|^{2} \times \delta \left( E - B_{\mathbf{A}} - e(k') + B_{\mathbf{A}-2} - \frac{(k + k')^{2}}{2m_{\mathbf{A}-2}} \right)
\]

- Written in terms of two-body momentum distribution

\[
n_{np}(q, Q)
\]
The hadronic tensor for two-body current factorizes as

$$ R_{2b}^{\mu \nu}(q, \omega) \propto \int dE d^3k d^3k' P_{2b}(k, k', E) \times d^3pd^3p' |\langle kk' | j_{2b}^{\mu} | pp' \rangle|^2 $$

Production of real $\pi$ in the final state

$$ R_{1b\pi}^{\mu \nu}(q, \omega) \propto \int dE d^3k P_{1b}(k, E) \times d^3pd^3k_\pi |\langle kk_\pi | j_{1b}^{\mu} | pp_\pi \rangle|^2 $$

*Pion production elementary amplitudes currently derived within the extremely sophisticated Dynamic Couple Chanel approach;

Axial Form Factors Uncertainty needs

D. Simons, N. Steinberg et al, 2210.02455

* Axial form factor dependence:

<table>
<thead>
<tr>
<th></th>
<th>MiniBooNE</th>
<th>0.2 &lt; cos ( \theta_\mu ) &lt; 0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF Difference in ( d\sigma_{\text{peak}} ) (%)</td>
<td>16.3</td>
<td></td>
</tr>
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</table>

* Many-body method dependence:

<table>
<thead>
<tr>
<th></th>
<th>MiniBooNE</th>
<th>0.2 &lt; cos ( \theta_\mu ) &lt; 0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFMC/SF difference in ( d\sigma_{\text{peak}} ) (%)</td>
<td>22.8</td>
<td></td>
</tr>
</tbody>
</table>
Resonance Uncertainty needs

The largest contributions to two-body currents arise from resonant $N \rightarrow \Delta$ transitions yielding pion production.

\[ p \Delta \rightarrow p' \]

\[ q \rightarrow k \]

\[ p \Delta \rightarrow p' \]

\[ q \rightarrow k' \]

The normalization of the dominant $N \rightarrow \Delta$ transition form factor needs be known to 3% precision to achieve 1% cross-section precision for MiniBooNE kinematics.

State-of-the-art determinations of this form factor from experimental data on pion electroproduction achieve 10-15% precision (under some assumptions).


Further constraints on $N \rightarrow \Delta$ transition relevant for two-body currents and $\pi$ production will be necessary to achieve few-percent cross-section precision.

D. Simons, N. Steinberg et al, 2210.02455
Including the one- and two-body interference

We recently included interference effects between one- and two-body currents yielding single nucleon knock-out

Observe a small quenching in the longitudinal channel and an enhancement in the q.e. peak in the transverse → agreement with the GFMC

N. Steinberg, NR, A. Lovato, arXiv: 2312.12545
Including the one- and two-body interference

FIG. 4. Inclusive electron cross sections on Carbon at several beam energies and scattering angles. Contributions are separated into pure one-body (red), pure two-body (orange), interference between one and two-body (purple), and total (blue).

FIG. 5. Flux-averaged $\nu_\mu$ differential cross sections on $^{12}$C for MiniBooNE. Three bins of $\cos\theta_\mu$ are shown with the one-body contributions in red, pure two-body contributions in orange, one- and two-body interference in purple, and total in blue. The width of the error band interpolates between the dipole axial form factor with $M_A = 1$ GeV, and the LQCD form factor of Ref. [53]. The open circles are the cross section to which the background reported in Ref. [79] is added.

While the choice of the LQCD form factor seems to significantly improve the agreement with data, the model dependent background subtraction method adopted by the MiniBooNE collaboration as well as the lack of a prediction including events with absorbed pions make quantitative comparisons difficult. We note that in our factorization scheme the enhancement from the LQCD form factor matches the enhancement seen in Green's Function Monte Carlo (GFMC) calculations of flux folded cross section using the same LQCD form factor [54]. As these are two completely different many body methods, only linked by the same underlying nuclear Hamiltonian, the sensitivity to the choice in axial form factor seems robust.

V. CONCLUSIONS

Providing accurate theoretical predictions, accompanied by reliable uncertainty quantification, for neutrino-nucleus scattering cross-sections in the energy regime relevant to the neutrino-oscillation problem is highly non-trivial. The primary challenges lie in combining a microscopic, quantum-mechanical description of real-time nuclear dynamics with relativistic kinematics and currents. In this regard, the extended factorization scheme, based on realistic spectral functions obtained from Quantum
Using Bayesian ANN for electron-nucleus scattering

The inclusive electron-nucleus cross section can be written in terms of the longitudinal and transverse response function

\[
\left( \frac{d^2 \sigma}{dE'd\Omega'} \right)_e = \left( \frac{d\sigma}{d\Omega'} \right)_M \left[ \frac{q^4}{q^4} R_L(q, \omega) + \left( \frac{\tan^2 \theta}{2} - \frac{1}{2} \frac{q^2}{q^2} \right) R_T(q, \omega) \right]
\]

Traditionally, the **Rosenbluth separation** is adopted to obtain \(R_L(q, \omega)\) and \(R_T(q, \omega)\)

\[
\Sigma(q, \omega, \epsilon) = \epsilon \frac{q^4}{Q^4} \left( \frac{d^2 \sigma}{dE'd\Omega'} \right)_e \left/ \left( \frac{d\sigma}{d\Omega'} \right)_M \right. = \epsilon R_L(q, \omega) + \frac{1}{2} \frac{q^2}{Q^2} R_T(q, \omega)
\]

**Photon polarization**

As \(\theta\) ranges between 180 to 0 degrees, \(\epsilon\) varies between 0 and 1. Within this approach, \(R_L\) is the **slope** while \((q^2/2Q^2)R_T\) is the **intercept** of the linear fit to data

This definition can only be applied if the Born approximation is valid and if the data have already been corrected to account for Coulomb distortions of the electron wave function.
Using Bayesian ANN for electron-nucleus scattering

We used ANN architecture to obtain the **longitudinal** and **transverse responses**

- We preprocess the input through a ‘scaling’ net whose output is $y(q, \omega, A, Z)$.

- We concatenate $y$ with the other inputs to compute the ‘Response’ net which gives a 32-dim output.

- This input is used to build **two completely independent nets**; each provides a single output corresponding to the longitudinal and transverse responses, respectively.

We train our ANN using the quasielastic electron nucleus scattering archive of [arXiv:nucl-ex/0603032](http://arxiv.org/abs/0603032) considering five different light and medium-mass nuclei, symmetric: $^4$He, $^6$Li, $^{12}$C, $^{16}$O and $^{40}$Ca.
Using Bayesian ANN for electron-nucleus scattering

We used **Bayesian statistics** to quantify the uncertainty of the ANN. We treat the weights \( \mathcal{W} \) as a probability distribution.

The posterior probability of the parameters \( \mathcal{W} \) given the measured cross sections \( Y \) can be written as

\[
P(\mathcal{W} \mid Y) = \frac{P(Y \mid \mathcal{W})P(\mathcal{W})}{P(Y)}
\]

We assign a normal Gaussian prior for each neural network parameter and assume a **Gaussian distribution for the likelihood** based on a loss function obtained from a **least-squares** fit to the empirical data

\[
P(Y \mid \mathcal{W}) = \exp \left( -\frac{\chi^2}{2} \right)
\]

\[
\chi^2 = \sum_{i=1}^{N} \frac{[y_i - \hat{y}_i(\mathcal{W})]^2}{\sigma_i^2}
\]

We increase the experimental errors \( \sigma \) listed in arXiv:nucl-ex/0603032 including an additional term proportional to the experimental cross section value: \( \sigma_i \rightarrow \sigma_i + 0.05y_i \).

The posterior distribution is sampled using the **NumPyro No-U-Turn Sampler** extension of HMC. We also implemented the standard HMC algorithm and validated results.
Using Bayesian ANN for electron-nucleus scattering

Results: Cross sections for different nuclei

- $^4\text{He}$, $E=0.60$ GeV, $\theta=75.00$
- $^{12}\text{C}$, $E=0.48$ GeV, $\theta=36.00$
- $^{16}\text{O}$, $E=0.88$ GeV, $\theta=32.00$
- $^{40}\text{Ca}$, $E=0.48$ GeV, $\theta=140.00$

Figure 2: Results on test data for four symmetric nuclei. The uncertainty band encompasses the total spread of the ANN predictions. Experimental data taken from Zghiche et al. (1994); Barreau et al. (1983); Anghinolfi et al. (1996); Meziani et al. (1984).

Following Kowal et al. (2023), we remove from our analysis the datasets on $^{12}\text{C}$ from Zeller (1973). Based on our preliminary analysis they stay in tension with all other experiments. For $^{16}\text{O}$, we add to our analysis the data from Anghinolfi et al. (1996), which are not included in quasielastic electron nucleus scattering archive of Benhar et al. (2006).

A critical aspect of this work consists in quantifying the uncertainty associated with the ANN predictions. To this aim, we leverage Bayesian statistics and treat $W$ as probability distributions (Neal, 2012). Using Bayes' theorem, the posterior probability of the parameters $W$ given the measured cross sections $Y$ can be written as

$$P(W|Y) = \frac{P(Y|W)P(W)}{P(Y)}$$

(9)

where $P(Y|W)$ is the likelihood and $P(W)$ is the prior density of the parameters (Utama et al., 2016). As in Neal (2012), we assign a normal Gaussian prior for each neural network parameter $P(W) = \frac{1}{(2\pi)^{D/2}}N/2 \exp \left(-\frac{1}{2} \sum^D_{i=1} w_i^2 \right)$.

(10)

Note that such prior corresponds to $l^2$ regularization with unit weight.

Following standard practice, we assume a Gaussian distribution for the likelihood based on a loss function obtained from a least-squares fit to the empirical data

$$P(Y|W) = \exp \left(-\frac{1}{2} \sum_i (y_i - \hat{y}_i(W))^2 \right)$$

(11)

$$\sum_i \hat{y}_i(W) = \sum_i y_i + \sum_i \epsilon_i$$

(12)

In the above equation, $y_i$ is the $i$-th experimental value of the cross section and the sum runs over the kinematics and nuclei included in the training dataset. We augment the experimental errors $\epsilon_i$ listed in Benhar et al. (2006) including an additional term proportional to the experimental cross section value: $\epsilon_i \propto y_i$. The primary reason behind this choice is that experimental errors are in general small and most experiments report an additional few-percent systematic uncertainty.

All of our numerical simulations are performed using the JAX Python library Bradbury et al. (2018). The posterior distribution is sampled leveraging the NumPyro No-U-Turn Sampler extension of Hamiltonian Monte Carlo (HMC) (Phan et al., 2019; Bingham et al., 2019). Additionally, we implemented the standard HMC algorithm as outlined in Ref. Ho et al. (2014).
Using Bayesian ANN for electron-nucleus scattering

Results: Electromagnetic responses

Low lying nuclear states

Contributions from elastic and low-lying inelastic transitions are explicitly removed from the GFMC responses and the Rosenbluth analysis, while they are present in the ANN curves.
Results: Electromagnetic responses

\[ ^{12}\text{C}, \, q = 380 \text{ MeV/c} \]

\[ R_L(\omega) \, \text{[GeV}^{-1}] \]

\[ 0 \leq \omega \leq 0.25 \text{ [GeV]} \]

\[ R_L(\omega) = \text{GFMC (1+2b) - ANN} \]

Low lying nuclear states

Contributions from elastic and low-lying inelastic transitions are explicitly removed from the GFMC responses and the Rosenbluth analysis, while they are present in the ANN curves.
Using Bayesian ANN for electron-nucleus scattering

**Results:** Electromagnetic responses

First separation of the longitudinal and transverse responses of $^{16}$O. Large uncertainty bands reflect the **scarcity of inclusive cross section data.**
Using Bayesian ANN for electron-nucleus scattering

**Results:** Electromagnetic responses

Note increasing error bars for large $\omega$ reflecting the scarcity of data for $^{40}$Ca in the high energy-momentum region. The net is learning from other nuclei in this region.

Dedicated discussion on the Rosenbluth separation carried out using two different experiments.
Conclusions

✱ Neutrino oscillation experiments are entering a new precision era

✱ To match these precision goals accurate predictions of neutrino cross sections are crucial

  Ab initio methods: almost exact results but limited in energy, fully inclusive

  Approaches based on factorization schemes are being further developed

✱ Uncertainty associated with the theory prediction of the hard interaction vertex needs to be assessed. Initial work has been carried out in this direction studying the dependence on:

  Form factors: one- and two-body currents, resonance/π production

  Error of factorizing the hard interaction vertex / using a non relativistic approach

✱ Study of ANN to extract and predict electromagnetic responses in scenarios where traditional methods fail due to the lack of data. Extend this framework to neutrino-nucleus scattering using near detector data
Thank you for your attention!
Two-body currents - Delta contribution

\[ j_\Delta^\mu = \frac{3}{2} \frac{f_{\pi NN} f^{*}}{m_\pi^2} \left\{ \Pi(k_2) \left[ \left( - \frac{2}{3} \tau^{(2)} + \frac{I_V}{3} \right) z F_{\pi NN}(k_2) F_{\pi N\Delta}(k_2) (J^\mu_a)_{(1)} \right. \\
- \left. \left( \frac{2}{3} \tau^{(2)} + \frac{I_V}{3} \right) z F_{\pi NN}(k_2) F_{\pi N\Delta}(k_2) (J^\mu_b)_{(1)} \right] + (1 \leftrightarrow 2) \right\} \]

where

Rarita Schwinger propagator

\[
(j_\alpha^\mu)^V = (k'_\pi)^\alpha G_{\alpha\beta}(p_\Delta) \left[ \frac{C_3^V}{m_N} (g_\beta \mu \, k q - q_\beta \gamma^\mu) \gamma_5 \right] \\
(j_\alpha^\mu)^A = (k'_\pi)^\alpha G_{\alpha\beta}(p_\Delta) C_5^A g_\beta \mu
\]
Resonance Uncertainty needs

The largest contributions to two-body currents arise from resonant $N \rightarrow \Delta$ transitions yielding pion production.

![Graph](image)

% Change in cross section vs. % Change in $C^2(0)$

The normalization of the dominant $N \rightarrow \Delta$ transition form factor needs be known to 3% precision to achieve 1% cross-section precision for MiniBooNE kinematics.

State-of-the-art determinations of this form factor from experimental data on pion electroproduction achieve 10-15% precision (under some assumptions).


Further constraints on $N \rightarrow \Delta$ transition relevant for two-body currents and π production will be necessary to achieve few-percent cross-section precision.

D. Simons, N. Steinberg et al, 2210.02455

Noemi Rocco, nrocco@fnal.gov
Using Bayesian ANN for electron-nucleus scattering

Results: Electromagnetic responses

**J. Sobczyk, NR, A. Lovato, arxiv:2406.06292**

We provide additional predictions of the ANN for the electromagnetic responses of $^4$He, $^{12}$C, and $^{40}$Ca for the kinematics where Rosenbluth separation data are available in the literature. The ANN responses of $^4$He, displayed in Figure 3, are in excellent agreement with the Rosenbluth separation analyses, and they share some distinctive features with the latter. The ANN extraction of the responses generally exhibits smaller uncertainties in the transverse channel than in the longitudinal, where the uncertainties tend to grow with the magnitude of the momentum transfer. At low energy transfers, the uncertainties in the predictions are large due to the elastic contribution and transitions to low-lying excited states, reducing the precision of the ANN responses. This behavior is primarily due to the scarcity of experimental data at low energies, which is insufficient to constrain the parameters of the ANN. Additionally, because the contributions from elastic and low-lying transitions strongly depend on the specific nuclear target, the ANN cannot leverage information from one nucleus to predict responses for a different one.

Figure 3: Rosenbluth separation for $^4$He at $q = 300$, 500, 600 MeV/c. Data taken from Carlson et al. (2002). Similar trends are observed in $^{12}$C and $^{40}$Ca, shown in 4. We note however that the ANN transverse response of $^{40}$Ca consistently overestimates the Rosenbluth-separation data from Jourdan (1996). In the following, we investigate closer the source of this discrepancy.

Experimental electron scattering on $^{40}$Ca was primarily measured at two facilities: Saclay Meziani et al. (1984) and MIT-Bates Williamson et al. (1997), collecting 1016 and 296 data-points respectively. Both experiments performed the Rosenbluth separation at several values of momentum transfer, leading to different results. Later Jourdan (1996) re-examined the data taking all available measurements into account.

We employed our ANN to investigate the source of discrepancy between the two experiments and to assess how our predictions depend on the datasets used in the training. In Fig. 5, we show...
Results: Electromagnetic responses

For $^{12}$C at $q = 300$ MeV/c, our predictions are shown in the left graph, compared to the Rosenbluth separation results (green line). The right graph compares our predictions at $q = 570$ MeV/c. The discrepancy between the two datasets is clear, with the combined dataset showing a smoother trend.

In the longitudinal response, the discrepancy is smaller at $q = 300$ MeV/c, as shown in the bottom left graph. For $q = 570$ MeV/c, the discrepancy is larger, as seen in the bottom right graph.

Overall, the Bayesian ANN model is able to reproduce the Rosenbluth separation results with high accuracy, and our predictions are consistent with the experiments.
Using Bayesian ANN for electron-nucleus scattering

Results: Electromagnetic responses

Figure 4: Rosenbluth separation for $^{12}$C and $^{40}$Ca at $q = 380, 570$ MeV/c. Data taken from Jourdan (1996).

Our predictions where we restrict the training datasets to either Saclay or Bates data. As we can see, our results coincide with the Rosenbluth separation performed by each experiment. When using the datasets combined, our ANN tends more towards Saclay predictions, since this experiment provides over three times more data points. In addition, we find a $q$-dependence in the discrepancy between both Rosenbluth-separation analyses. For instance, in Fig. 5, we show results for $q = 300$ MeV/c, where the discrepancy is considerably smaller for the longitudinal response.
Using Bayesian ANN for electron-nucleus scattering

Results: Electromagnetic responses

Figure 5: The ANN predictions when for 40Ca only data from a single experiment were used for training. Data taken from Meziani et al. (1984) and Williamson et al. (1997). In the left column we show our results compared to the Rosenbluth separation performed by each experiment for a similar momentum transfer $q = 400$ MeV/c (Bates) and $q = 410$ MeV/c (Saclay). In the right column we compare both predictions at $q = 300$ MeV/c.
Delta contribution to MEC

Diagrams including the Delta current depend on many parameters.

\[
\begin{align*}
(j_\alpha^\mu)_A &= (k_\pi')^\alpha G_{\alpha\beta}(p\Delta) \left[ \frac{C_3^A}{m_N} (g^{\beta\mu} q - q^\beta \gamma^\mu) ight. \\
&+ \frac{C_4^A}{m_N^2} (g^{\beta\mu} q \cdot p\Delta - q^\beta p_\Delta^\mu) \\
&\left. + C_5^A g^{\beta\mu} + \frac{C_6^A}{m_N^2} q^\mu q^\alpha \right],
\end{align*}
\]

Parametrization chosen for the vector ff:

\[
C_5^A = \frac{1.2}{(1 - q^2/M_\Delta)^2} \times \frac{1}{1 - q^2/(3M_\Delta^2)},
\]

Current extractions of \(C_A^5\) (0) rely on single pion production data from deuterium bubble chamber experiments; estimated uncertainty \(\sim 15\%\)

Delta decay width:

\[
\Gamma(p\Delta) = \left(\frac{4f_{\pi N\Delta}}{12\pi m_\pi^2}\right)^2 \frac{|d|^3}{\sqrt{s}} (m_N + E_d) R(r^2)
\]

\[
R(r^2) = \left(\frac{\Lambda_R^2}{\Lambda_R^2 - r^2}\right)
\]
QMC Spectral function of light nuclei

• Single-nucleon spectral function:

\[
P_{p,n}(k, E) = \sum_n \left| \langle \Psi_0^A | [\mid k \rangle | \Psi_{n}^{A-1} \rangle \right|^2 \\
\times \delta(E + E_0^A - E_n^{A-1}) \\
= P^{MF}(k, E) + P^{corr}(k, E)
\]

\[P_{p}^{MF}(k, E) = n_{p}^{MF}(k) \delta \left( E - B_{4He} + B_{3H} - \frac{k^2}{2m_{3H}} \right)\]

\[|\langle \Psi_0^{4He} | [\mid k \rangle \otimes | \Psi_0^{3H} \rangle |^2\]

• The single-nucleon overlap has been computed within VMC (center of mass motion fully accounted for)
Spectral function approach

\[
P^\text{corr}_p(k, E) = \sum_n \int \frac{d^3 k'}{(2\pi)^3} |\langle \Psi^A_0 | [k] | k' \rangle | \Psi^{A-2}_n \rangle|^2 \delta(E + E^A_0 - e(k') - E^{A-2}_n)
\]

Using QMC techniques

\[
\sum_{\tau_{k',=p,n}} n_{p,\tau_{k'}}(k, k') \delta \left( E - B_A - e(k') + B_{A-2} - \frac{(k + k')^2}{2m_{A-2}} \right)
\]

Only SRC pairs should be considered: \(|\Psi^{A-1}_0 \rangle \) and \(|k'\rangle|\psi^{A-2}_n \rangle \) be orthogonalized.

One can introduce cuts on the relative distance between the particles in the two-body momentum distribution.
QMC Spectral Function of $^{12}$C

• The p-shell contribution has been obtained by FT the radial overlaps:

\[
^{12}C(0^+) \rightarrow ^{11}B(3/2^-) + p \\
^{12}C(0^+) \rightarrow ^{11}B(1/2^-) + p \\
^{12}C(0^+) \rightarrow ^{11}B(3/2^-)^* + p.
\]


• The quenching of the spectroscopic factors automatically emerges from the VMC calculations

Computing the s-shell contribution is non trivial within VMC. We explored different alternatives:

• Quenched Harmonic Oscillator

• Quenched Wood Saxon

• VMC overlap associated for the $^4$He(0$^+$) → $^3$H(1/2$^+$) + p transition

Korover, et al, CLAS collaboration PRC 107 (2023) 6, L061301
Tuning

Discrepancies between generators and data often corrected by tuning an empirical model of the least well known mechanism: MEC (“meson exchange”/two-body currents)

Coyle, Li, and Machado, JHEP 12, 166 (2022)

Mis-modeling can distort signals of new physics, **biasing** measurement of **new physics parameters**

Studies on the impact of different neutrino interactions and nuclear models on determining neutrino oscillation parameters are critical. These enable us to assess the level of precision we aim at.

Study of model dependence in neutrino predictions

Percent change in the MiniBooNE cross section versus the percent change in the two-body current parameters for $0.5 < \cos \theta_\mu < 0.6$, $T_\mu = 325$ MeV

A 15% variation in either $C_5^A(0)$ or $\Lambda_R$ changes the flux-averaged cross section by roughly 5%
Two-body currents - Delta contribution

Rarita-Schwinger propagator

\[ G^{\alpha\beta}(p_\Delta) = \frac{P^{\alpha\beta}(p_\Delta)}{p_\Delta^2 - M_\Delta^2} \]

The spin 3/2 projection operator reads

\[ P^{\alpha\beta}(p_\Delta) = (p_\Delta + M_\Delta) \left[ g^{\alpha\beta} - \frac{1}{3} \gamma^\alpha \gamma^\beta - \frac{2}{3} \frac{p_\Delta^\alpha p_\Delta^\beta}{M_\Delta^2} + \frac{1}{3} \frac{p_\Delta^\alpha \gamma^\beta - p_\Delta^\beta \gamma^\alpha}{M_\Delta} \right] . \]

To account for the resonant behavior of the \( \Delta \):

\[ M_\Delta \to M_\Delta - i \Gamma(p_\Delta)/2 \]

\[ \Gamma(p_\Delta) = -2 \text{Im} \Sigma_{\pi N}(s) = \frac{(4f_{\pi N \Delta})^2 |d|^3}{12\pi m_\pi^2 \sqrt{s}} (m_N + E_d) R(r^2) \]

\( d \) is the decay three-momentum in the \( \pi N \) center of mass frame

In medium effects of the \( \Delta \)

\[ \Gamma_\Delta(p_\Delta) \to \Gamma_\Delta(p_\Delta) - 2 \text{Im} [U_\Delta(p_\Delta, \rho = \rho_0)] \]
Comparing different many-body methods

• \( e^{-3H} \): inclusive cross section

\[
\begin{align*}
E_e &= 0.3677 \text{ GeV}, \quad \Theta = 54.0 \\
E_e &= 0.5579 \text{ GeV}, \quad \Theta = 54.0
\end{align*}
\]

L. Andreoli, NR, et al, PRC 105 (2022) 1, 014002

• Comparisons among QMC, SF, and STA approaches: first step to precisely **quantify the uncertainties** inherent to the factorization of the final state.

• Gauge the role of **relativistic effects** in the energy region relevant for neutrino experiments.
Green’s Function Monte Carlo

Any trial wave function can be expanded in the complete set of eigenstates of the Hamiltonian according to

$$|\Psi_V\rangle = \sum_n c_n |\Psi_n\rangle \quad H|\Psi_n\rangle = E_n |\Psi_n\rangle$$

GFMC uses a projection technique to enhance the true ground-state component of a starting wave function.

$$\lim_{\tau \to \infty} e^{-(H-E_0)\tau} |\Psi_T\rangle = \lim_{\tau \to \infty} \sum_n c_n e^{-(E_n-E_0)\tau} |\Psi_n\rangle = c_0 |\Psi_0\rangle$$

The direct calculation of the imaginary-time propagator for strongly-interacting systems involves prohibitive difficulties.


The imaginary-time evolution is broken into N small imaginary-time steps, and complete sets of states are inserted

$$e^{-(H-E_0)\tau} |\Psi_V\rangle = \int dR_1 \ldots dR_N |R_N\rangle \langle R_N|e^{-(H-E_0)\Delta\tau}|R_{N-1}\rangle \ldots \langle R_2|e^{-(H-E_0)\Delta\tau}|R_1\rangle |\Psi_V(R_1)\rangle$$

Short Time Propagator
Phenomenological potential: av18 + IL7

Phenomenological potentials explicitly include the long-range one-pion exchange interaction and a set of intermediate- and short-range phenomenological terms.

- **Argonne** $v_{18}$ is a finite, local, configuration-space potential controlled by $\sim 4300$ np and pp scattering data below 350 MeV of the Nijmegen database.

  $$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij})O^p_{ij}$$

- Phenomenological three-nucleon interactions, like the **Illinois 7**, effectively include the lowest nucleon excitation, the $\Delta(1232)$ resonance, and other nuclear effects.

  $$V^{3N}_{ijk} = A^{PW}_{2\pi} O^{2\pi, PW}_{ijk} + A^{SW}_{2\pi} O^{2\pi, SW}_{ijk} + A^{3\pi, \Delta R}_{ijk} + A^{R}_{ijk} O^{R}_{ijk}$$

The parameters of the AV18 + IL7 are fit to properties of exactly solvable light nuclear systems.
Axial form factor determination

- The axial form-factor has been fit to the dipole form
  \[ F_A(q^2) = \frac{g_A}{(1 - q^2/m_A^2)^2} \]
- The intercept \( g_A = -1.2723 \) is known from neutron \( \beta \) decay
- Different values of \( m_A \) from experiments
  - \( m_A = 1.02 \) GeV q.e. scattering from deuterium
  - \( m_A = 1.35 \) GeV @ MiniBooNE
- Alternative derivation based on \textit{z-expansion}
  — model independent parametrization
  \[ F_A(q^2) = \sum_{k=0}^{k_{\text{max}}} a_k z(q^2)^k, \]
  known functions
  free parameters

Bhattacharya, Hill, and Paz  PRD 84 (2011) 073006

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Why relativity is important

\[ R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0 | J^\dagger_{\alpha}(\mathbf{q}) | f \rangle \langle f | J_{\beta}(\mathbf{q}) | 0 \rangle \delta(\omega - E_f + E_0) \]

Covariant expression of the e.m. current:

\[ j^\mu_{\gamma,S} = \bar{u}(\mathbf{p}') \left[ \frac{G^S_E + \tau G^S_M}{2(1 + \tau)} \gamma^\mu + i \frac{\sigma^{\mu
u} q_\nu}{4m_N} \frac{G^S_M - G^S_E}{1 + \tau} \right] u(\mathbf{p}) \]

Nonrelativistic expansion in powers of \( p/m_N \)

\[ j^0_{\gamma,S} = \frac{G^S_E}{2\sqrt{1 + Q^2/4m_N^2}} - i \frac{2G^S_M - G^S_E}{8m_N^2} \mathbf{q} \cdot (\mathbf{\sigma} \times \mathbf{p}) \]

Energy transfer at the quasi-elastic peak:

\[ w_{QE} = \sqrt{\mathbf{q}^2 + m_N^2} - m_N \]

\[ w_{nr}^{QE} = \mathbf{q}^2/(2m_N) \]
Frame dependence

LAB-frame

\[ q \]

\[ P_i = 0 \] (\( P_f = q \))

Active Nucleon Breit frame

\[ q^{fr} \]

\[ P^{fr}_i = -A/2q^{fr} \]

\[ P_f = -A/2 (q^{fr} + q^{fr}) \]

Lorentz Boost connects the two frames:

- ANB @ the single nucleon level:
  \[ p^{fr}_i = -q^{fr}/2 \]
  \[ p^{fr}_f = q^{fr}/2 \]

- Same position of the quasielastic peak
  \[ \omega_{QE} = \omega_{QE}^{nr} = 0 \]

**Equations**

\[ R^{\mu \nu}_{LAB}(\omega, q) = B^\mu_\alpha [\beta] B^\nu_\beta [\beta] R^{\alpha \beta}_{fr}(\omega_{fr}, q_{fr}) \]

- LAB (solid) and ANB (dashed) predictions

**Graphs**

- \( R_{T\perp}^{AA}/G_A \) at different energies (500, 700, 900 MeV)

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Cross sections: Green’s Function Monte Carlo

MiniBooNE results including relativistic corrections

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A.Nikolakopoulos, A.Lovato, NR, PRC 109 (2024) 1, 014623
Cross sections: Green’s Function Monte Carlo

Electron scattering results including relativistic corrections for some kinematics covered by the calculated responses

A.Lovato, A.Nikolakopoulos, NR, N. Steinberg, Universe 9 (2023) 8, 36
Neutrino-Nucleon scattering

- Sum rule can be enforced ensuring that the form factor falls smoothly to zero at large $Q^2$

$$\sum_{k=n}^{\infty} k(k-1)\cdots(k-n+1)a_k = 0, \quad n = 0, 1, 2, 3$$

Fit deuteron data replacing dipole axial form factor with z-expansion, enforce the sum rule constraints