Hadron interactions using three-quark potential in a constituent quark model

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- New three-quark potentials
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Introduction

Three-quark potential in a constituent quark model

$$\mathcal{C}_{123} = \begin{cases} \sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j \\ \quad = \mathrm{F}_1 \cdot \mathrm{F}_2 + \mathrm{F}_1 \cdot \mathrm{F}_3 + \mathrm{F}_2 \cdot \mathrm{F}_3, \\ d^{abc}\mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \\ i f^{abc}\mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \end{cases}$$

Intrinsic 3-body forces

$$\bar{\mathcal{C}}_{123} = \begin{cases} -d^{abc}\mathrm{F}_1^a \mathrm{F}_2^b \bar{\mathrm{F}}_3^c, \\ d^{abc}\mathrm{F}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c, \\ -d^{abc}\bar{\mathrm{F}}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c. \end{cases}$$

$$d^{abc}F_i^a F_j^b F_k^c = \frac{1}{6}\left[C^{(3)}_{i+j+k} - \frac{5}{2} C^{(2)}_{i+j+k} + \frac{20}{3} \right]$$

$$\langle C^{(2)} \rangle = \frac{1}{3}(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu)$$

$$\langle C^{(3)} \rangle = \frac{1}{18}(\lambda - \mu)(2\lambda + \mu + 3)(\lambda + 2\mu + 3)$$

V. Dmitrašinovic, PLB 499, 135(2001)
S. Pepin and Fl. Stancu, PRD 65, 054032(2002)

Three-quark potential in a constituent quark model

$$\mathcal{C}_{123} = \begin{cases} \sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j \\ \quad = \mathrm{F}_1 \cdot \mathrm{F}_2 + \mathrm{F}_1 \cdot \mathrm{F}_3 + \mathrm{F}_2 \cdot \mathrm{F}_3, \\ d^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \\ i f^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \end{cases}$$

Intrinsic 3-body forces

$$d^{abc} F_i^a F_j^b F_k^c = \frac{1}{4}[(ijk) + (ikj)] + \frac{1}{9} I - \frac{1}{6}[(ij) + (ik) + (jk)],$$

$$f^{abc} F_i^a F_j^b F_k^c = \frac{i}{4}[(ijk) - (ikj)].$$

$$\bar{\mathcal{C}}_{123} = \begin{cases} -d^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \bar{\mathrm{F}}_3^c, \\ d^{abc} \mathrm{F}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c, \\ -d^{abc} \bar{\mathrm{F}}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c. \end{cases}$$

V. Dmitrašinovic, PLB 499, 135(2001)
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Three-quark potential in a constituent quark model

$$\mathcal{C}_{123} = \begin{cases} \sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j \\ \quad = \mathrm{F}_1 \cdot \mathrm{F}_2 + \mathrm{F}_1 \cdot \mathrm{F}_3 + \mathrm{F}_2 \cdot \mathrm{F}_3, \\ d^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \\ i f^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \end{cases}$$

Intrinsic 3-body forces

$$\bar{\mathcal{C}}_{123} = \begin{cases} -d^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \bar{\mathrm{F}}_3^c, \\ d^{abc} \mathrm{F}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c, \\ -d^{abc} \bar{\mathrm{F}}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c. \end{cases}$$

$$d^{abc} F_i^a F_j^b F_k^c = \frac{1}{4}[(ijk) + (ikj)] + \frac{1}{9} I - \frac{1}{6}[(ij) + (ik) + (jk)],$$

$$f^{abc} F_i^a F_j^b F_k^c = \frac{i}{4}[(ijk) - (ikj)].$$

For a baryon,

$$d^{abc} F_i^a F_j^b F_k^c = \frac{10}{9},$$

$$f^{abc} F_i^a F_j^b F_k^c = 0.$$

V. Dmitrašinovic, PLB 499, 135(2001)

S. Pepin and Fl. Stancu, PRD 65, 054032(2002)

Three-quark potential in a constituent quark model

$$\mathcal{C}_{123} = \begin{cases} \sum_{i<j}^{3} \mathrm{F}_i \cdot \mathrm{F}_j \\ \quad = \mathrm{F}_1 \cdot \mathrm{F}_2 + \mathrm{F}_1 \cdot \mathrm{F}_3 + \mathrm{F}_2 \cdot \mathrm{F}_3, \\ d^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \\ i f^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \mathrm{F}_3^c, \end{cases}$$

Intrinsic 3-body forces

$$d^{abc} F_i^a F_j^b F_k^c = \frac{1}{4}[(ijk) + (ikj)] + \frac{1}{9} I - \frac{1}{6}[(ij) + (ik) + (jk)],$$

$$f^{abc} F_i^a F_j^b F_k^c = \frac{i}{4}[(ijk) - (ikj)].$$

$$\bar{\mathcal{C}}_{123} = \begin{cases} -d^{abc} \mathrm{F}_1^a \mathrm{F}_2^b \bar{\mathrm{F}}_3^c, \\ d^{abc} \mathrm{F}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c, \\ -d^{abc} \bar{\mathrm{F}}_1^a \bar{\mathrm{F}}_2^b \bar{\mathrm{F}}_3^c. \end{cases}$$

For a baryon,

$$d^{abc} F_i^a F_j^b F_k^c = \frac{10}{9},$$

$$f^{abc} F_i^a F_j^b F_k^c = 0.$$

For tetraquark,

$$V_s = c \frac{5}{18} \omega^2 (\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2),$$

$$V_a = -c \frac{5}{9} \omega^2 (\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2).$$

$[s \equiv 8, a \equiv 1]$

$$-\frac{3}{2} < c < \frac{78}{5}.$$

V. Dmitrašinovic, PLB 499, 135(2001)

S. Pepin and Fl. Stancu, PRD 65, 054032(2002)

Casimir Invariants

There are two Casimir operators in SU(3).

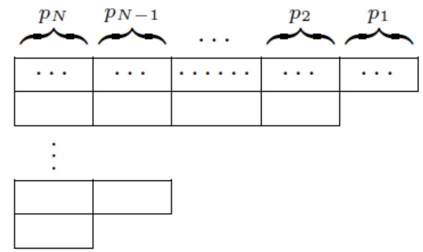
C_2: quadratic Casimir operator

C_3: cubic Casimir operator

$$C_2 = \sum_{i,j} F_i^c F_j^c = \sum_i F_i^c F_i^c + 2\sum_{i<j} F_i^c F_j^c$$

$$\begin{aligned} C_3 &= \sum_{i,j,k} d^{abc} F_i^a F_j^b F_k^c \\ &= \sum_i d^{abc} F_i^a F_i^b F_i^c + 6\sum_{i<j} d^{abc} F_i^a F_i^b F_j^c + 6\sum_{i<j<k} d^{abc} F_i^a F_j^b F_k^c \end{aligned}$$

Casimir Invariants



For SU(N),

total number of box : $n = p_1 + 2p_2 + \cdots + Np_N = \sum_{k=1}^{N} kp_k$

$$C_2^{SU(N)} = \frac{n}{2N}(N^2-1) + \frac{1}{N}(N-1)\left[\sum_{i=1}^{N}\binom{\sum_{j=i}^{N} p_j}{2}\right] - \sum_{k=1}^{N-1}\left(\sum_{j=k+1}^{N}(j-k)p_j\right)\left(1+\frac{1}{N}\sum_{i=k}^{N} p_i\right)$$

$$C_2^{\mathrm{SU}(3)} = p_1 + \frac{p_1^2}{3} + p_2 + \frac{1}{3}p_1p_2 + \frac{p_2^2}{3}$$

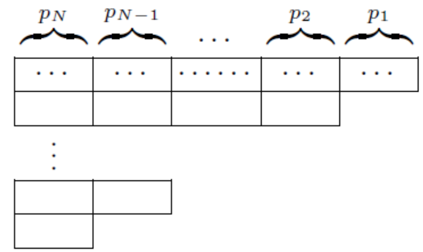
$$C_2^{\mathrm{SU}(4)} = \frac{3p_1}{2} + \frac{3p_1^2}{8} + 2p_2 + \frac{1}{2}p_1p_2 + \frac{p_2^2}{2} + \frac{3p_3}{2} + \frac{1}{4}p_1p_3 + \frac{1}{2}p_2p_3 + \frac{3p_3^2}{8}$$

$$C_2^{\mathrm{SU}(5)} = 2p_1 + \frac{2p_1^2}{5} + 3p_2 + \frac{3}{5}p_1p_2 + \frac{3p_2^2}{5} + 3p_3 + \frac{2}{5}p_1p_3 + \frac{4}{5}p_2p_3 + \frac{3p_3^2}{5} + 2p_4 + \frac{1}{5}p_1p_4 + \frac{2}{5}p_2p_4 + \frac{3}{5}p_3p_4 + \frac{2p_4^2}{5}$$

$$C_2^{\mathrm{SU}(6)} = \frac{5p_1}{2} + \frac{5p_1^2}{12} + 4p_2 + \frac{2}{3}p_1p_2 + \frac{2p_2^2}{3} + \frac{9p_3}{2} + \frac{1}{2}p_1p_3 + p_2p_3 + \frac{3p_3^2}{4} + 4p_4 + \frac{1}{3}p_1p_4 + \frac{2}{3}p_2p_4 + p_3p_4 + \frac{2p_4^2}{3} + \frac{5p_5}{2} + \frac{1}{6}p_1p_5 + \frac{1}{3}p_2p_5 + \frac{1}{2}p_3p_5 + \frac{2}{3}p_4p_5 + \frac{5p_5^2}{12}$$

Casimir invariants does not depend on p_N.

Casimir Invariants



For SU(N),

total number of box : $n = p_1 + 2p_2 + \cdots + Np_N = \sum_{k=1}^{N} kp_k$

$$C_3^{\mathrm{SU}(3)} = \frac{1}{18}(p_1 - p_2)(p_1 + 2p_2 + 3)(2p_1 + p_2 + 3)$$

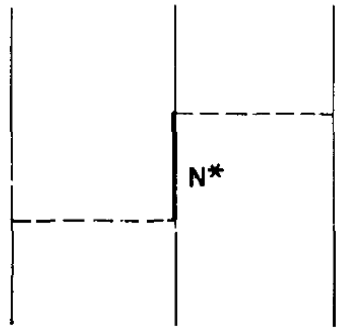
$$C_3^{\mathrm{SU}(4)} = \frac{3}{2}p_1 + \frac{9}{8}p_1^2 + \frac{3}{16}p_1^3 + \frac{3}{4}p_1p_2 + \frac{3}{8}p_1^2p_2 - \frac{3}{2}p_3 + \frac{3}{16}p_1^2p_3 - \frac{3}{4}p_2p_3 - \frac{9}{8}p_3^2 - \frac{3}{16}p_1p_3^2 - \frac{3}{8}p_2p_3^2 - \frac{3}{16}p_3^3$$

$$\begin{aligned} C_3^{\mathrm{SU}(5)} &= 3p_1 + \frac{9p_1^2}{5} + \frac{6p_1^3}{25} + \frac{3p_2}{2} + \frac{9}{5}p_1p_2 + \frac{27}{50}p_1^2p_2 + \frac{9p_2^2}{10} + \frac{9}{50}p_1p_2^2 + \frac{3p_2^3}{25} - \frac{3p_3}{2} + \frac{3}{5}p_1p_3 + \frac{9}{25}p_1^2p_3 + \frac{6}{25}p_1p_2p_3 \\ &+ \frac{6}{25}p_2^2p_3 - \frac{9p_3^2}{10} - \frac{3}{25}p_1p_3^2 - \frac{6}{25}p_2p_3^2 - \frac{3p_3^3}{25} - 3p_4 + \frac{9}{50}p_1^2p_4 - \frac{3}{5}p_2p_4 + \frac{3}{25}p_1p_2p_4 + \frac{3}{25}p_2^2p_4 - \frac{9}{5}p_3p_4 \\ &- \frac{3}{25}p_1p_3p_4 - \frac{6}{25}p_2p_3p_4 - \frac{9}{50}p_3^2p_4 - \frac{9p_4^2}{5} - \frac{9}{50}p_1p_4^2 - \frac{9}{25}p_2p_4^2 - \frac{27}{50}p_3p_4^2 - \frac{6p_4^3}{25} \end{aligned} \quad (3$$

$$\begin{aligned} C_3^{\mathrm{SU}(6)} &= 5p_1 + \frac{5p_1^2}{2} + \frac{5p_1^3}{18} + 4p_2 + 3p_1p_2 + \frac{2}{3}p_1^2p_2 + 2p_2^2 + \frac{1}{3}p_1p_2^2 + \frac{2p_2^3}{9} + \frac{3}{2}p_1p_3 + \frac{1}{2}p_1^2p_3 + \frac{3}{2}p_2p_3 + \frac{1}{2}p_1p_2p_3 \\ &+ \frac{1}{2}p_2^2p_3 - 4p_4 + \frac{1}{2}p_1p_4 + \frac{1}{3}p_1^2p_4 + \frac{1}{3}p_1p_2p_4 + \frac{1}{3}p_2^2p_4 - \frac{3}{2}p_3p_4 - 2p_4^2 - \frac{1}{6}p_1p_4^2 - \frac{1}{3}p_2p_4^2 - \frac{1}{2}p_3p_4^2 - \frac{2p_4^3}{9} \\ &- 5p_5 + \frac{1}{6}p_1^2p_5 - \frac{1}{2}p_2p_5 + \frac{1}{6}p_1p_2p_5 + \frac{1}{6}p_2^2p_5 - \frac{3}{2}p_3p_5 - 3p_4p_5 - \frac{1}{6}p_1p_4p_5 - \frac{1}{3}p_2p_4p_5 - \frac{1}{2}p_3p_4p_5 - \frac{1}{3}p_4^2p_5 \\ &- \frac{5p_5^2}{2} - \frac{1}{6}p_1p_5^2 - \frac{1}{3}p_2p_5^2 - \frac{1}{2}p_3p_5^2 - \frac{2}{3}p_4p_5^2 - \frac{5p_5^3}{18} \end{aligned} \quad ($$

Casimir invariants does not depend on p_N.
However, note that interaction factor can depend on total number of quarks.

Three-body forces in nuclear physics



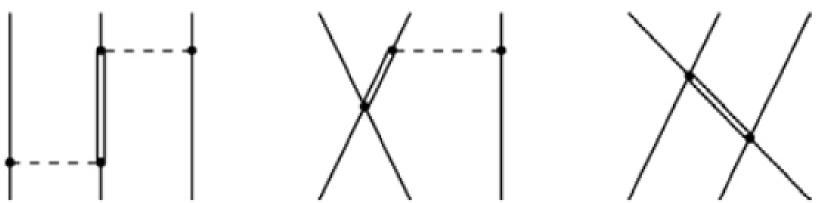
$$V^{(3)} = V(123) + V(231) + V(312),$$

$$V(123) = -[\{A(\tau^1\tau^2)(\tau^2\tau^3) + B(\tau^3\tau^2)(\tau^2\tau^1)\}(\sigma^1\nabla^1)(\sigma^2\nabla^1)(\sigma^2\nabla^3)(\sigma^3\nabla^3)$$
$$+ \{B(\tau^1\tau^2)(\tau^2\tau^3) + A(\tau^3\tau^2)(\tau^2\tau^1)\}(\sigma^3\nabla^3)(\sigma^2\nabla^3)(\sigma^2\nabla^1)(\sigma^1\nabla^1)$$
$$+ 2D(\tau^1\tau^3)(\sigma^1\nabla^1)(\sigma^3\nabla^3)]\, Y(12)\, Y(23),$$

G.E.Brown et al, Nucl. Phys. A 115, 435(1968)

J.Fujita and H.Miyazawa, Prog. Theor. Phys. 17, 360(1957)

The middle nucleon goes virtually to some excited state.



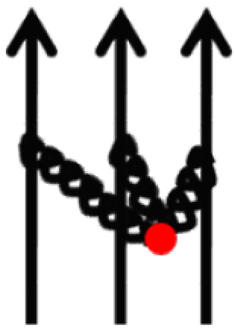
Leading contributions to the 3NF due to explicit Δ's.

E.Epelbaum, Prog. Part. Nucl. Phys. 57, 654(2006)

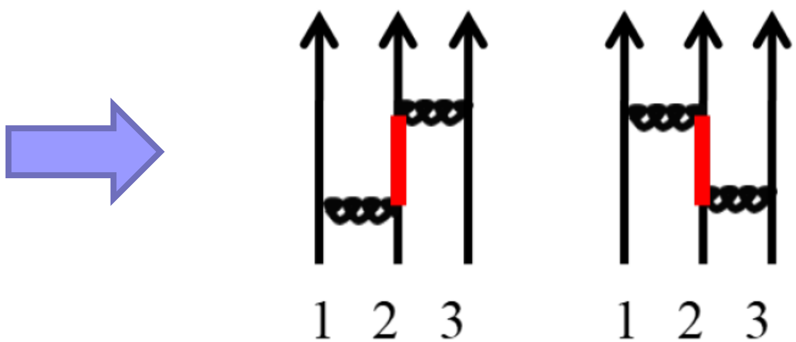
How about applying this concept to the three-quark potential?

New three-quark potentials

New three-quark potentials: color-color

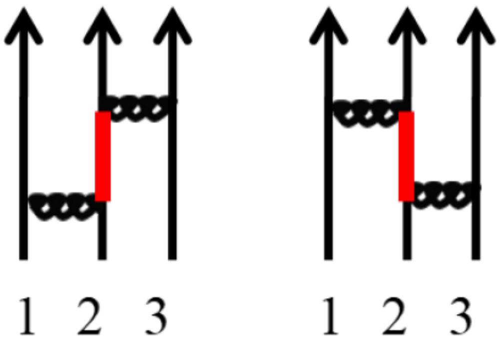


$$d^{abc}\mathrm{F}_1^a\mathrm{F}_2^b\mathrm{F}_3^c,$$
$$if^{abc}\mathrm{F}_1^a\mathrm{F}_2^b\mathrm{F}_3^c,$$



$$+ \quad (1 \leftrightarrow 2) \quad + \quad (2 \leftrightarrow 3)$$

New three-quark potentials: color-color



$$+ \quad (1 \leftrightarrow 2) \quad + \quad (2 \leftrightarrow 3)$$

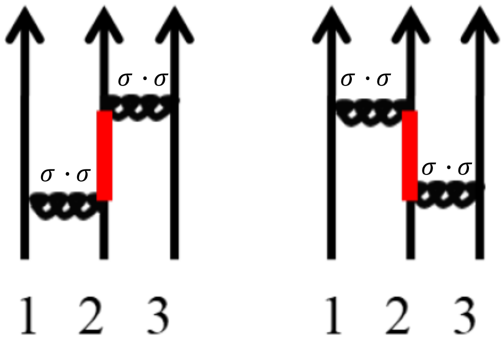
$$\{\lambda^a, \lambda^b\} = \frac{4}{3}\delta^{ab} + 2d^{abc}\lambda^c$$

$$[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c$$

$$\begin{aligned}
(\lambda_1^a\lambda_2^a)(\lambda_2^b\lambda_3^b) + (\lambda_2^a\lambda_3^a)(\lambda_1^b\lambda_2^b) &= \lambda_1^a\left(-\lambda_2^b\lambda_2^a + \frac{4}{3}\delta^{ab} + 2d^{abc}\lambda_2^c\right)\lambda_3^b + (\lambda_2^a\lambda_3^a)(\lambda_1^b\lambda_2^b) \\
&= -\lambda_1^a\lambda_2^b\lambda_2^a\lambda_3^b + \frac{4}{3}\lambda_1^a\lambda_3^a + 2d^{abc}\lambda_1^a\lambda_2^c\lambda_3^b + (\lambda_2^a\lambda_3^a)(\lambda_1^b\lambda_2^b) \\
&= -(\lambda_2^a\lambda_3^a)(\lambda_1^b\lambda_2^b) + \frac{4}{3}\lambda_1^a\lambda_3^a + 2d^{abc}\lambda_1^a\lambda_2^b\lambda_3^c + (\lambda_2^a\lambda_3^a)(\lambda_1^b\lambda_2^b) \\
&= \frac{4}{3}\lambda_1^a\lambda_3^a + 2d^{abc}\lambda_1^a\lambda_2^b\lambda_3^c
\end{aligned}$$

$$\Rightarrow \quad L_{123}^{C-C} = \frac{4}{3}\left(\frac{\lambda_2^c\lambda_3^c}{m_1} + \frac{\lambda_1^c\lambda_3^c}{m_2} + \frac{\lambda_1^c\lambda_2^c}{m_3}\right) + 2d^{abc}(\lambda_1^a\lambda_2^b\lambda_3^c)\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}\right)$$

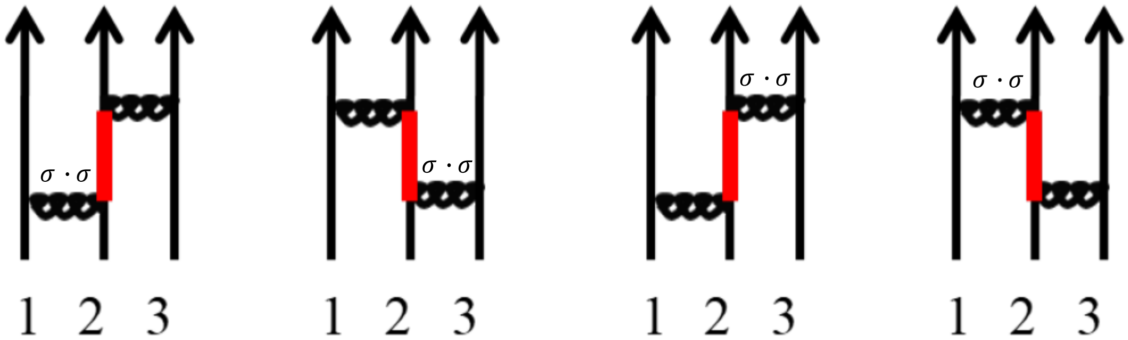
New three-quark potentials: color-spin



$$+ \ (1 \leftrightarrow 2) \ + \ (2 \leftrightarrow 3)$$

$$L_{123}^{S-S} = \frac{1}{m_1 m_2 m_3}\left[\frac{4}{3}\left(\frac{(\sigma_2\cdot\sigma_3)(\lambda_2^c\lambda_3^c)}{m_1^2} + \frac{(\sigma_1\cdot\sigma_3)(\lambda_1^c\lambda_3^c)}{m_2^2} + \frac{(\sigma_1\cdot\sigma_2)(\lambda_1^c\lambda_2^c)}{m_3^2}\right)\right.$$
$$\left. + 2d^{abc}(\lambda_1^a\lambda_2^b\lambda_3^c)\left(\frac{\sigma_2\cdot\sigma_3}{m_1^2} + \frac{\sigma_1\cdot\sigma_3}{m_2^2} + \frac{\sigma_1\cdot\sigma_2}{m_3^2}\right) - 2\epsilon_{ijk}\sigma_1^i\sigma_2^j\sigma_3^k f^{abc}\lambda_1^a\lambda_2^b\lambda_3^c\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{1}{m_3^2}\right)\right]$$

New three-quark potentials: color-spin hybrid



$+ \ (1 \leftrightarrow 2) \ + \ (2 \leftrightarrow 3)$

$$L_{123}^{C-S} = \frac{4}{3}\left[\frac{(\lambda_1^c\lambda_3^c)}{m_2}\left(\frac{\sigma_2\cdot\sigma_3}{m_2m_3}+\frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right)+\frac{(\lambda_1^c\lambda_2^c)}{m_3}\left(\frac{\sigma_3\cdot\sigma_2}{m_2m_3}+\frac{\sigma_1\cdot\sigma_3}{m_3m_1}\right)+\frac{(\lambda_2^c\lambda_3^c)}{m_1}\left(\frac{\sigma_1\cdot\sigma_3}{m_1m_3}+\frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right)\right]$$
$$+\,2d_{abc}(\lambda_1^a\lambda_2^b\lambda_3^b)\left[\frac{1}{m_2}\left(\frac{\sigma_2\cdot\sigma_3}{m_2m_3}+\frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right)+\frac{1}{m_3}\left(\frac{\sigma_3\cdot\sigma_2}{m_2m_3}+\frac{\sigma_1\cdot\sigma_3}{m_3m_1}\right)+\frac{1}{m_1}\left(\frac{\sigma_1\cdot\sigma_3}{m_1m_3}+\frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right)\right]$$

New three-quark potentials: antiquarks

For antiquarks,
$$\{\bar{\lambda}^a, \bar{\lambda}^b\} = \frac{4}{3}\delta^{ab} - 2d^{abc}\bar{\lambda}^c$$
$$[\bar{\lambda}^a, \bar{\lambda}^b] = 2if^{abc}\bar{\lambda}^c$$

$$L_{C-C} = \left[\sum_{i<j<k} \frac{4}{3}\left(\frac{\lambda_i^c\lambda_j^c}{m_k} + \frac{\lambda_i^c\lambda_k^c}{m_j} + \frac{\lambda_j^c\lambda_k^c}{m_i}\right) + 2d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c\left(\frac{\mathrm{sign}(i)}{m_i} + \frac{\mathrm{sign}(j)}{m_j} + \frac{\mathrm{sign}(k)}{m_k}\right)\right]$$

$$\begin{aligned} L_{S-S} = \sum_{i<j<k} \frac{1}{m_i m_j m_k}\Bigg[&\frac{4}{3}\left(\frac{\lambda_j^c\lambda_k^c\sigma_j\cdot\sigma_k}{m_i^2} + \frac{\lambda_i^c\lambda_k^c\sigma_i\cdot\sigma_k}{m_j^2} + \frac{\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j}{m_k^2}\right) \\ &+ 2d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c\left(\frac{\mathrm{sign}(i)\sigma_j\cdot\sigma_k}{m_i^2} + \frac{\mathrm{sign}(j)\sigma_i\cdot\sigma_k}{m_j^2} + \frac{\mathrm{sign}(k)\sigma_i\cdot\sigma_j}{m_k^2}\right) \\ &- 2\epsilon_{ijk}\sigma_1^i\sigma_2^j\sigma_3^k f^{abc}\lambda_1^a\lambda_2^b\lambda_3^c\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{1}{m_3^2}\right)\Bigg], \end{aligned}$$

$$\begin{aligned} L_{C-S} = \sum_{i<j<k} \frac{4}{3}\Bigg\{&\frac{\lambda_j^c\lambda_k^c}{m_i}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j} + \frac{\sigma_i\cdot\sigma_k}{m_i m_k}\right) + \frac{\lambda_i^c\lambda_k^c}{m_j}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j} + \frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right) + \frac{\lambda_i^c\lambda_j^c}{m_k}\left(\frac{\sigma_i\cdot\sigma_k}{m_i m_k} + \frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right)\Bigg] \\ &+ 2d^{abc}\lambda_i^c\lambda_j^c\lambda_k^c\left[\frac{\mathrm{sign}(i)}{m_i}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j} + \frac{\sigma_i\cdot\sigma_k}{m_i m_k}\right) + \frac{\mathrm{sign}(j)}{m_j}\left(\frac{\sigma_i\cdot\sigma_j}{m_i m_j} + \frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right) + \frac{\mathrm{sign}(k)}{m_k}\left(\frac{\sigma_i\cdot\sigma_k}{m_i m_k} + \frac{\sigma_j\cdot\sigma_k}{m_j m_k}\right)\right]\Bigg\} \end{aligned}$$

where $\mathrm{sign}(i) = -1$ for antiquarks.

Results
: Baryons, Tetraquarks, Dibaryons

Baryon fitting

$$H = \sum_{i=1}^{n}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - \frac{3}{4}\sum_{i<j}^{n}\frac{\lambda_i^c}{2}\frac{\lambda_j^c}{2}\left(V_{ij}^C + V_{ij}^{CS}\right),$$

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D,$$

$$V_{ij}^{CS} = \frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4}\frac{e^{-(r_{ij})^2/(r_{0ij})^2}}{(r_{0ij}) r_{ij}}\sigma_i \cdot \sigma_j .$$

$r_{0ij} = (\alpha + \beta m_{ij})^{-1}$, $\kappa' = \kappa_0 (1 + \gamma m_{ij})$ and $m_{ij} = \frac{m_i m_j}{m_i + m_j}$

$$V^{3-body} = AL_{C-C} + BL_{S-S} + CL_{C-S}$$

$A = -3.67522 \times 10^{-4}\,\mathrm{GeV}^2$, $B = -2.85156 \times 10^{-7}\,\mathrm{GeV}^6$, $C = -7.68351 \times 10^{-6}\,\mathrm{GeV}^4$

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameter (fm^{-2})
η_c	2983.6	2996.9	$a = 13.1$
$J\Psi$	3096.9	3089.6	$a = 11.1$
D	1864.8	1864.1	$a = 4.5$
D^*	2010.3	2010.7	$a = 3.7$
π	139.57	139.39	$a = 4.6$
ρ	775.11	775.49	$a = 2.2$
K	493.68	494.62	$a = 4.6$
K^*	891.66	888.82	$a = 2.8$

The standard deviation is $\sigma = 5.86$.

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameters (fm^{-2})
Λ_c	2286.5	2266.7 (2281.6)	$a_1 = 2.9$, $a_2 = 3.7$
Σ_c	2452.9	2441.6 (2480.9)	$a_1 = 2.1$, $a_2 = 3.8$
Λ	1115.7	1113.6 (1134.1)	$a_1 = 2.8$, $a_2 = 2.7$
Σ	1192.6	1196.5 (1231.6)	$a_1 = 2.1$, $a_2 = 3.1$
Σ_c^*	2518.5	2522.9 (2567.7)	$a_1 = 2.0$, $a_2 = 3.4$
Σ^*	1383.7	1398.9 (1455.2)	$a_1 = 1.9$, $a_2 = 2.4$
p	938.27	980.47 (1005.3)	$a_1 = 2.4$, $a_2 = 2.4$
Δ	1232	1272.1 (1346.8)	$a_1 = 1.8$, $a_2 = 1.8$

$\sigma = 24.44(63.10)$

With 3-quark potential

Three-body color matrices for tetraquarks

$|3_{12}\bar{3}_{34}\rangle, |\bar{6}_{12}6_{34}\rangle$ color basis $\quad T_{cc}: \bar{u}\bar{d}cc$

$$f^{abc}\overline{\lambda}_1^a\overline{\lambda}_2^b\lambda_3^c = \begin{pmatrix} 0 & -4i\sqrt{2} \\ 4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\overline{\lambda}_1^a\overline{\lambda}_2^b\lambda_3^c = \begin{pmatrix} -\frac{40}{9} & 0 \\ 0 & \frac{20}{9} \end{pmatrix}$$

$$f^{abc}\overline{\lambda}_1^a\overline{\lambda}_2^b\lambda_4^c = \begin{pmatrix} 0 & 4i\sqrt{2} \\ -4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\overline{\lambda}_1^a\overline{\lambda}_2^b\lambda_4^c = \begin{pmatrix} -\frac{40}{9} & 0 \\ 0 & \frac{20}{9} \end{pmatrix}$$

$$f^{abc}\overline{\lambda}_1^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & -4i\sqrt{2} \\ 4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\overline{\lambda}_1^a\lambda_3^b\lambda_4^c = \begin{pmatrix} \frac{40}{9} & 0 \\ 0 & -\frac{20}{9} \end{pmatrix}$$

$$f^{abc}\overline{\lambda}_2^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & 4i\sqrt{2} \\ -4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\overline{\lambda}_2^a\lambda_3^b\lambda_4^c = \begin{pmatrix} \frac{40}{9} & 0 \\ 0 & -\frac{20}{9} \end{pmatrix}$$

$|1_{13}1_{24}\rangle, |8_{13}8_{24}\rangle$ color basis $\quad \chi_{c1}: \bar{c}\bar{q}cq$

$$f^{abc}\overline{\lambda}_1^a\overline{\lambda}_2^b\lambda_3^c = \begin{pmatrix} 0 & -4i\sqrt{2} \\ 4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\overline{\lambda}_1^a\overline{\lambda}_2^b\lambda_3^c = \begin{pmatrix} 0 & \frac{20\sqrt{2}}{9} \\ \frac{20\sqrt{2}}{9} & -\frac{20}{9} \end{pmatrix}$$

$$f^{abc}\overline{\lambda}_1^a\overline{\lambda}_2^b\lambda_4^c = \begin{pmatrix} 0 & 4i\sqrt{2} \\ -4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\overline{\lambda}_1^a\overline{\lambda}_2^b\lambda_4^c = \begin{pmatrix} 0 & \frac{20\sqrt{2}}{9} \\ \frac{20\sqrt{2}}{9} & -\frac{20}{9} \end{pmatrix}$$

$$f^{abc}\overline{\lambda}_1^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & -4i\sqrt{2} \\ 4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\overline{\lambda}_1^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & -\frac{20\sqrt{2}}{9} \\ -\frac{20\sqrt{2}}{9} & \frac{20}{9} \end{pmatrix}$$

$$f^{abc}\overline{\lambda}_2^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & 4i\sqrt{2} \\ -4i\sqrt{2} & 0 \end{pmatrix} \qquad d^{abc}\overline{\lambda}_2^a\lambda_3^b\lambda_4^c = \begin{pmatrix} 0 & -\frac{20\sqrt{2}}{9} \\ -\frac{20\sqrt{2}}{9} & \frac{20}{9} \end{pmatrix}$$

Three-body potentials in tetraquarks

$$V^{3-body} = AL_{C-C} + BL_{S-S} + CL_{C-S}$$

$$A = -3.67522 \times 10^{-4}\,\mathrm{GeV}^2$$
$$B = -2.85156 \times 10^{-7}\,\mathrm{GeV}^6$$
$$C = -7.68351 \times 10^{-6}\,\mathrm{GeV}^4$$

T_{cc} $(|\mathbf{3}_{12}\overline{\mathbf{3}}_{34}\rangle, |\overline{\mathbf{6}}_{12}\mathbf{6}_{34}\rangle)$

Unit: MeV

Color	$\sum_{i<j<k} L_{ijk}^{C-C}$	$\sum_{i<j<k} L_{ijk}^{S-S}$	$\sum_{i<j<k} L_{ijk}^{C-S}$
$\mathbf{3}_{12}\overline{\mathbf{3}}_{34}$	-4.8424	0.0319	20.9444
$\overline{\mathbf{6}}_{12}\mathbf{6}_{34}$	31.4754	0.0907	8.6177

$$V^{3-body} = \begin{pmatrix} 16.134 & 0.7977 \\ 0.7977 & 40.1838 \end{pmatrix}$$

$\chi_{c1}(3872)$ $(|\mathbf{1}_{13}\mathbf{1}_{24}\rangle, |\mathbf{8}_{13}\mathbf{8}_{24}\rangle)$

Color	$\sum_{i<j<k} L_{ijk}^{C-C}$	$\sum_{i<j<k} L_{ijk}^{S-S}$	$\sum_{i<j<k} L_{ijk}^{C-S}$
$\mathbf{1}_{13}\mathbf{1}_{24}$	19.3695	0.0427	-1.3654
$\mathbf{8}_{13}\mathbf{8}_{24}$	7.2636	-0.2793	4.2077

$$V^{3-body} = \begin{pmatrix} 18.0468 & 15.9512 \\ 15.9512 & 11.1919 \end{pmatrix}$$

Three-body potentials in dibaryons

Unit: MeV

$H: uuddss(I = 0, S = 0), (F_1, F_{27})$

Flavor	$\sum_{i<j<k} L_{ijk}^{C-C}$	$\sum_{i<j<k} L_{ijk}^{S-S}$	$\sum_{i<j<k} L_{ijk}^{C-S}$
F_1	-21.0008	17.6644	34.0685
F_{27}	-21.0008	24.1615	-20.3534

$$V^{3-body} = \begin{pmatrix} 30.732 & -1.6934 \\ -1.6934 & -17.1928 \end{pmatrix}$$

$N\Omega: uudsss(I = 1/2, S = 2), (F_{27}, F_8)$

Flavor	$\sum_{i<j<k} L_{ijk}^{C-C}$	$\sum_{i<j<k} L_{ijk}^{S-S}$	$\sum_{i<j<k} L_{ijk}^{C-S}$
F_{27}	-19.056	5.3551	-6.2815
F_8	-19.056	1.3458	27.8001

$$V^{3-body} = \begin{pmatrix} -19.9824 & -3.89157 \\ -3.89157 & 10.0899 \end{pmatrix}$$

Summary

- Inspired by the three-body nuclear force, the three-quark potentials are newly constructed.
- When the three-quark potential is introduced, the baryon results fit better.
- When the three-quark potential is applied to exotic hadrons, it shows repulsive interaction for dominant states.

Thank you