Effective Lagrangians and thermal resonances under extreme conditions

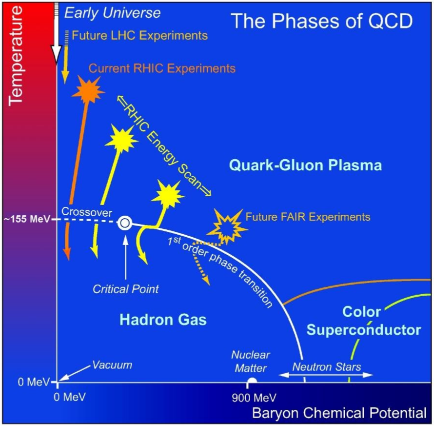
Andrea Vioque Rodríguez, Angel Gómez Nicola
and Jacobo Ruiz de Elvira

Universidad Complutense de Madrid

Outline

1. Aspects of the QCD phase diagram
2. Scattering and Resonances within finite-T Unitarized ChPT
3. Saturating scalar susceptibilities with light thermal resonances
4. Pion scattering and critical temperature at nonzero chiral imbalance

QCD transition



A. Bazavov, Quark Matter 2017

- Crossover-like transition in the physical case ($N_f = 2+1$, massive quarks)
- Phase transition in light chiral limit for $N_f = 2$, possibly of second order

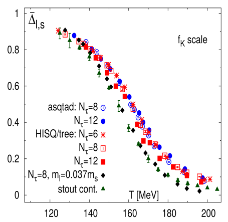
Signals of Chiral Symmetry Restoration

Inflection point for the light quark condensate $\langle\bar{q}q\rangle_l$

Subtracted quark condensate:

$$\Delta_{l,s} = m_s\langle\bar{q}q\rangle_l - 2m_l\langle\bar{s}s\rangle$$

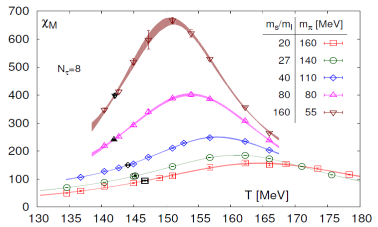
(avoids lattice divergences)



A. Bazavov et al PRD85, 054503 (2012)

Peak of scalar susceptibility

$$\chi_S = -\frac{\partial}{\partial m_l}\langle\bar{q}q\rangle_l$$



H. T. Ding et al PRL123, 062002 (2019)

Scattering and Resonances within finite-T Unitarized ChPT

Unitarized meson scattering from thermal unitarity including physical thermal-bath processes:

$$\text{IAM:} \quad t_{IAM}(s,T) = \frac{t_2(s)^2}{t_2(s) - t_4(s,T)}$$

$M_a \neq M_b$

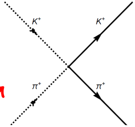
scattering πK

$$\text{Im}\, t_{IAM}(s,T) = \begin{cases} \sigma_{ab}^T(s)\,[t_{IAM}(s,T)]^2\,, & s \geq (M_a+M_b)^2 \text{ (unit.cut)} \\ \tilde{\sigma}_{ab}^T(s)\,[t_{IAM}(s,T)]^2\,, & 0 \leq s \leq (M_a-M_b)^2 \text{ (Landau thermal cut)} \end{cases}$$

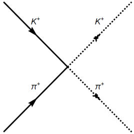
$$\Delta_{ab} = M_a^2 - M_b^2$$

Thermal phase space:

$$\sigma_{ab}^T(s) = \sigma_{ab}(s)\left[1 + n_B\left(\frac{s+\Delta_{ab}}{2\sqrt{s}}\right) + n_B\left(\frac{s-\Delta_{ab}}{2\sqrt{s}}\right)\right]$$



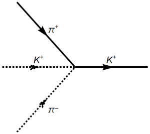
$(1+n_a)(1+n_b)$



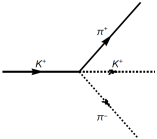
$-n_a n_b$

$$\tilde{\sigma}_{ab}^T(s) = \sigma_{ab}(s)\left[n_B\left(\frac{\Delta_{ab}-s}{2\sqrt{s}}\right) - n_B\left(\frac{s+\Delta_{ab}}{2\sqrt{s}}\right)\right]$$

two-body $T=0$ phase space

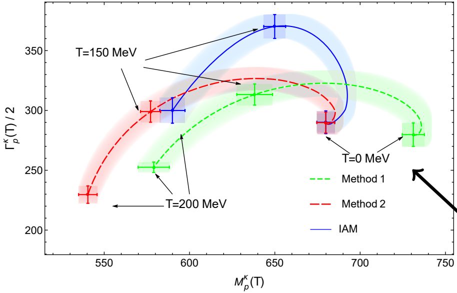


$(1+n_b)n_a$



$-(1+n_a)n_b$

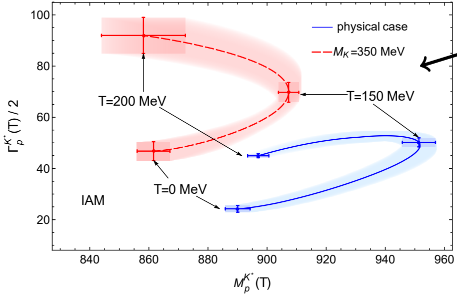
Scattering and Resonances within finite-T Unitarized ChPT



- M_p^κ stays constant up to temperatures around $T \sim 75$ MeV, from which it shows a decreasing behavior.
- Γ_p^κ increases at low temperatures and decreases for T closer to T_c.
- Similar behavior to that of the $f_0(500)$.

$K_0^*(700)/\kappa(I = 1/2, J = 0)$

$T = 0$ LECs: Molina, Ruiz de Elvira JHEP2020



$K^*(892)(I = 1/2, J = 1)$

- Softer temperature dependence.
- $SU(3)$ limit ($M_K = 350\,\text{MeV}$):

  $\rightarrow$ $\Gamma_p^{K^*}$ doubles its value from $T = 0$ to $T = 200$ MeV.

A. Gómez Nicola, J. R. de Elvira and AVR, JHEP **08** (2023), 148

Saturating scalar susceptibilities with light thermal resonances

S.Ferreres-Solé, A. Gómez Nicola, AVR, PRD99, 036018 (2019)

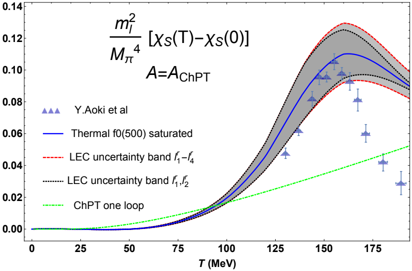
χ_S saturated by lightest $IJ = 00$ state, i.e. $f_0(500)$
generated in unitarized finite-T $\pi\pi$ scattering

$$\chi_S(T) \simeq \chi_S(0)\frac{M_S^2(0)}{M_S^2(T)}$$

$$M_S^2(T) \;=\; \mathrm{Re}\, s_p(T) \;\sim\; \mathrm{Re}\,\Sigma_{f_0}$$

behaves as $p = 0$ thermal mass in this channel
(scaling near T_c checked with LSM analysis)

- Reproduces expected peak $T_c \sim 158$ MeV
- Agrees with lattice below the peak within uncertainties
- Consistent T_c reduction and χ_S growth near chiral limit



LECs FLAG coll. Hanhart, Peláez, Ríos PRL100 (2008)

Thermal interactions crucial!

$I = 1/2$ sector (K/κ)

$$K^b = i\bar{q}\gamma_5\lambda^b q \xleftrightarrow[U(1)_A]{SU(2)_A} \kappa^b = \bar{q}\lambda^b q$$

degenerate under both $O(4)$ and $U(1)_A$
(lowest states K and $K_0^*(700)/\kappa$)

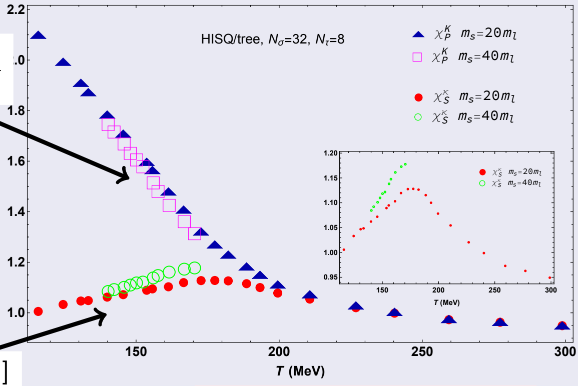
Reconstructed susceptibilities from WIs and lattice condensate data

$$\chi_P^K(T) = \frac{[\langle\bar{q}q\rangle_l + 2\langle\bar{s}s\rangle]}{m_s - m_l}$$

Lattice points from

A.Bazavov et al (Hot QCD)

2012-14 (Nf = 2+1)



$$\chi_S^\kappa(T) = -\frac{[\langle\bar{q}q\rangle_l - 2\langle\bar{s}s\rangle]}{m_s + m_l}$$

$\chi_S^\kappa \to \chi_P^K$ above the χ_S^κ peak

$I = 1/2$ sector (K/κ)

From WIs in this sector:

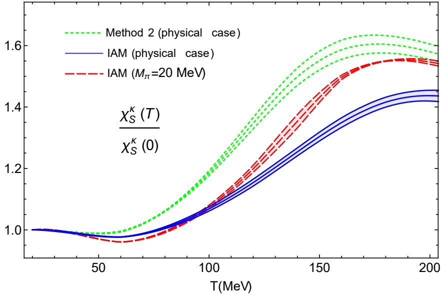
$$\chi_S^\kappa(T) - \chi_P^K(T) = \tfrac{2}{m_s^2 - m_l^2}\, \Delta_{l,s}(T) \longleftarrow \text{dictated by subtracted condensate}$$

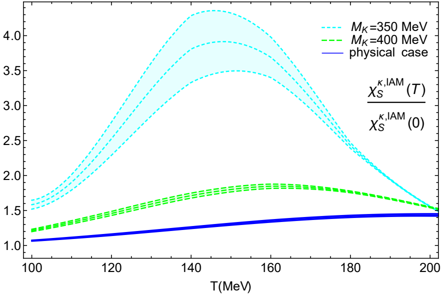
- In physical case strength of $U(1)_A$ above T_c well determined and driven by $\langle \bar{s}s \rangle$.
- In $N_f = 2$ limit, exact $O(4)$ $\times$U$(1)_A$ degeneration for $m_l, \langle \bar{q}q \rangle_l \to 0$.

$$m_s \gg m_l: \boxed{\chi_S^\kappa(T) - \chi_P^K(T)\Big|_{m_s \gg m_l} = \frac{2}{m_s} \langle \bar{q}q \rangle_l \Big|_{SU(2)} + \mathcal{O}(1/m_s^2)}$$

- May help to clarify the role of strangeness.

χ_S^κ saturated by $I = 1/2$ $K_0^*(700)$ scalar pole





$$\chi_S^\kappa = \chi_S^{\kappa, ChPT}(0)\,\frac{M_\kappa^2(0)}{M_\kappa^2(T)}$$

- The peak is reproduced.
- Chiral limit ($M_\pi = 20\,\mathrm{MeV}$):

  → Larger growth below peak enhanced by chiral symmetry.

↓

$K - \kappa$ degeneration takes place at a lower temperature.

- $SU(3)$ limit:

  → Peak grows.

  → Displacement of the peak towards T_c.

↓

Consistently with its degeneracy with χ_S.

Chiral imbalance in ChPT

μ_5 chemical potential for approximate conservation of the chiral charge.

QCD Lagrangian for $\mu_5 \neq 0$: $\mathcal{L}_{QCD} \to \mathcal{L}_{QCD} + \mu_5 \bar{q}\gamma_5\gamma^0 q$

We have constructed the most general meson effective Lagrangian for $\mu_5 \neq 0$ and two light flavours.

The construction is carried out using the framework of the external source method.

New terms coming from:

- Covariant derivatives.
- Explicit axial source terms.

$\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ effective Lagrangian

$$\mathcal{L}_2 \to \mathcal{L}_2 + 2\mu_5^2 F^2 (1 + \kappa_0)$$

$$\mathcal{L}_4 \to \mathcal{L}_4 + \kappa_1 \mu_5^2 \mathrm{tr}\left(\partial^\mu U^\dagger \partial_\mu U\right) + \kappa_2 \mu_5^2 \left(\partial_0 U^\dagger \partial^0 U\right) + \kappa_3 \mu_5^2 \mathrm{tr}\left(\chi^\dagger U + U^\dagger \chi\right) + \kappa_4 \mu_5^4$$

$$\uparrow$$

constants to be determined from different observables

D. Espriu, A. Gómez Nicola, AVR, JHEP. 2020, 62

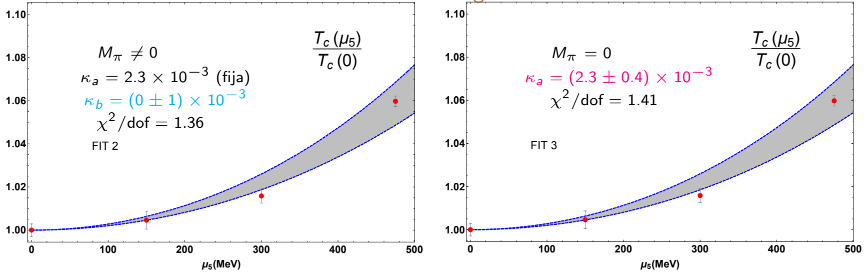
Quark condensate and critical temperature at NNLO

$$\langle \bar{q}q \rangle_l^{NNLO}(T,\mu_5)/\langle \bar{q}q \rangle_l^{NNLO}(T=0,\mu_5) \longrightarrow \boxed{\kappa_a = 2\kappa_1 - \kappa_2} \quad \text{y} \quad \boxed{\kappa_b = \kappa_1 + \kappa_2 - \kappa_3}$$

$$\frac{\langle \bar{q}q \rangle_l(T_c,\mu_5)}{\langle \bar{q}q \rangle_l(T=0,\mu_5)} = 0 \xrightarrow{M_{0\pi}=0} [T_c(\mu_5)]^2 = 24F^2\left[\sqrt{\frac{2}{3}+\left[1-2\kappa_a\frac{\mu_5^2}{F^2}\right]^2} - 1 + 2\kappa_a\frac{\mu_5^2}{F^2}\right]$$

$M_{0\pi} \neq 0$

Lattice data Braguta et al 2015



An alternative method for calculating T_c:

$$\text{Peak of } \chi_S(T,\mu_5) = \chi_S(T=0,\mu_5)\frac{M_S^2(0,\mu_5)}{M_S^2(T,\mu_5)}$$

Scattering $\pi\pi$ at nonzero μ_5

Pion scattering, $T_c(\mu_5)$ and fits of κ_i to lattice

μ_5 corrections to the pion scattering amplitude:

- Tree level coming from $\mathcal{L}_4$.
- Dispersion relation.
- Residue of the LSZ formula.

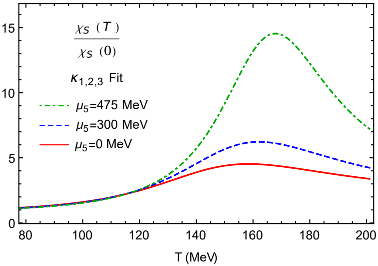
A. Gómez Nicola, Patricia Roa-Bravo, AVR, PRD **109** (2024) no.3, 034011

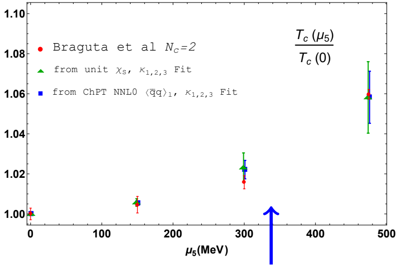
Combined fit of χ_{top} and T_c:

$\kappa_1 \times 10^4$	$\kappa_2 \times 10^4$	$\kappa_3 \times 10^4$	χ^2/dof
$9.4^{+1.1}_{-1.3}$	$-4.5^{+1.5}_{-1.4}$	$3.6^{+9.1}_{-8.7}$	1.37

$$\downarrow$$

$$\Delta t^{00} \rightrightarrows \begin{array}{l} \kappa_1' = 6\kappa_1 + 5\kappa_2 \\ \kappa_2' = -8\kappa_1 - 4\kappa_2 + 5\kappa_3 \end{array}$$





The growing behaviour of $T_c(\mu_5)$ is compatible with lattice results

Conclusions

- Scalar thermal resonances crucial for chiral and $U(1)_A$ restorations.
- Saturating χ_S with thermal $f_0(500)$, we reproduce the crossover peak of χ_S and most of the lattice data fall into the uncertainty band.
- K/κ alternative channel for $O(4) \times U(1)_A$ restoration.
- Saturated χ_S^κ with thermal $K_0^*(700)$ develops a peak and is consistent with $O(4) \times U(1)_A$ pattern.
- We have analyzed the effective chiral Lagrangian for nonzero chiral imbalance for two light flavours.
- The critical temperature increases with μ_5, in agreement with the lattice results.