Effective Lagrangians and thermal resonances under extreme conditions

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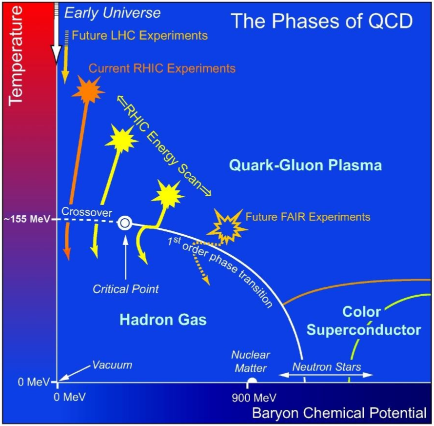
Outline

1. Aspects of the QCD phase diagram
2. Scattering and Resonances within finite-T Unitarized ChPT
3. Saturating scalar susceptibilities with light thermal resonances
4. Pion scattering and critical temperature at nonzero chiral imbalance

QCD transition



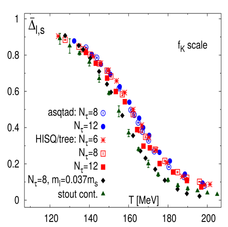
A. Bazavov, Quark Matter 2017

- Crossover-like transition in the physical case ($N_f = 2 + 1$, massive quarks)
- Phase transition in light chiral limit for $N_f = 2$, possibly of second order

Signals of Chiral Symmetry Restoration

Inflection point for the light quark condensate $\langle \bar{q}q \rangle_l$

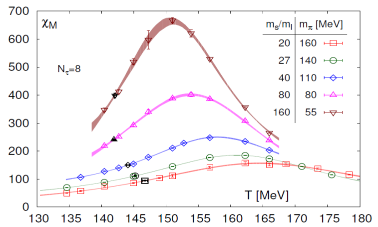
Subtracted quark condensate:
$$\Delta_{l,s} = m_s \langle \bar{q}q \rangle_l - 2m_l \langle \bar{s}s \rangle$$
(avoids lattice divergences)



A. Bazavov et al PRD85, 054503 (2012)

Peak of scalar susceptibility

$$\chi_S = -\frac{\partial}{\partial m_l} \langle \bar{q}q \rangle_l$$



H. T. Ding et al PRL123, 062002 (2019)

Scattering and Resonances within finite-T Unitarized ChPT

Unitarized meson scattering from thermal unitarity including physical thermal-bath processes:

$$\text{IAM:} \quad t_{IAM}(s,T) = \frac{t_2(s)^2}{t_2(s) - t_4(s,T)}$$

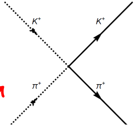
$M_a \neq M_b$
scattering πK

$$\text{Im}\, t_{IAM}(s,T) = \begin{cases} \sigma^T_{ab}(s)\,[t_{IAM}(s,T)]^2\,, & s \geq (M_a+M_b)^2 \text{ (unit.cut)} \\ \tilde{\sigma}^T_{ab}(s)\,[t_{IAM}(s,T)]^2\,, & 0 \leq s \leq (M_a-M_b)^2 \text{ (Landau thermal cut)} \end{cases}$$

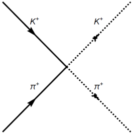
$$\Delta_{ab} = M_a^2 - M_b^2$$

Thermal phase space:

$$\sigma^T_{ab}(s) = \sigma_{ab}(s)\left[1 + n_B\left(\frac{s+\Delta_{ab}}{2\sqrt{s}}\right) + n_B\left(\frac{s-\Delta_{ab}}{2\sqrt{s}}\right)\right]$$



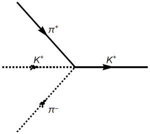
$(1+n_a)(1+n_b)$



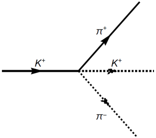
$-n_a n_b$

$$\tilde{\sigma}^T_{ab}(s) = \sigma_{ab}(s)\left[n_B\left(\frac{\Delta_{ab}-s}{2\sqrt{s}}\right) - n_B\left(\frac{s+\Delta_{ab}}{2\sqrt{s}}\right)\right]$$

two-body $T=0$ phase space

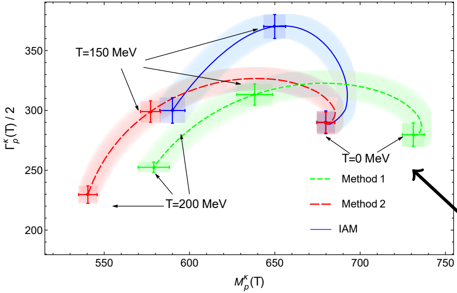


$(1+n_b)n_a$



$-(1+n_a)n_b$

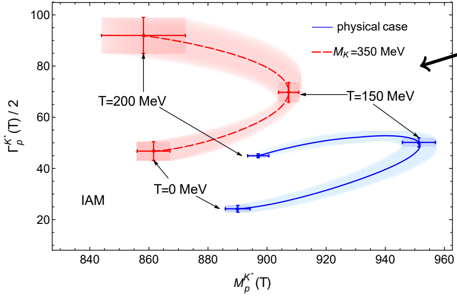
Scattering and Resonances within finite-T Unitarized ChPT



- M_p^κ stays constant up to temperatures around $T \sim 75$ MeV, from which it shows a decreasing behavior.
- Γ_p^κ increases at low temperatures and decreases for T closer to T_c.
- Similar behavior to that of the $f_0(500)$.

$K_0^*(700)/\kappa(I = 1/2, J = 0)$

$T = 0$ LECs: Molina, Ruiz de Elvira JHEP2020



$K^*(892)(I = 1/2, J = 1)$

- Softer temperature dependence.
- $SU(3)$ limit ($M_K = 350$ MeV):

  $\rightarrow$ $\Gamma_p^{K^*}$ doubles its value from $T = 0$ to $T = 200$ MeV.

A. Gómez Nicola, J. R. de Elvira and AVR, JHEP **08** (2023), 148

S.Ferreres-Solé, A. Gómez Nicola, AVR, PRD99, 036018 (2019)

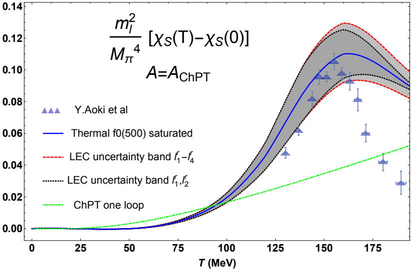
χ_S saturated by lightest $IJ = 00$ state, i.e. $f_0(500)$
generated in unitarized finite-T $\pi\pi$ scattering

$$\chi_S(T) \simeq \chi_S(0)\frac{M_S^2(0)}{M_S^2(T)}$$

$$M_S^2(T) \ = \ \mathrm{Re}\, s_p(T) \ \sim \ \mathrm{Re}\,\Sigma_{f_0}$$

behaves as $p = 0$ thermal mass in this channel
(scaling near T_c checked with LSM analysis)

- Reproduces expected peak $T_c \sim 158$ MeV
- Agrees with lattice below the peak within uncertainties
- Consistent T_c reduction and χ_S growth near chiral limit



LECs FLAG coll. Hanhart, Peláez, Ríos PRL100 (2008)

Thermal interactions crucial!

$I = 1/2$ sector (K/κ)

$$K^b = i\bar{q}\gamma_5\lambda^b q \xleftrightarrow[U(1)_A]{SU(2)_A} \kappa^b = \bar{q}\lambda^b q$$

degenerate under both $O(4)$ and $U(1)_A$
(lowest states K and $K_0^*(700)/\kappa$)

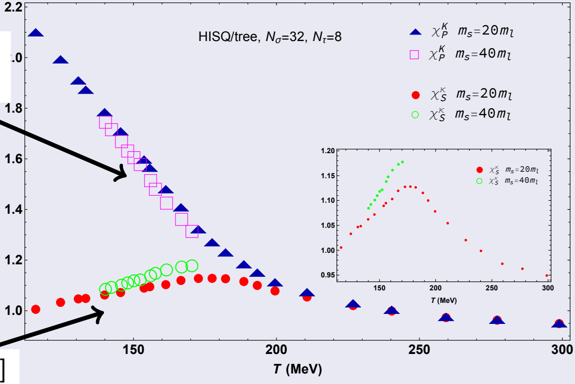
Reconstructed susceptibilities from WIs and lattice condensate data

$$\chi_P^K(T) = \frac{[\langle\bar{q}q\rangle_l + 2\langle\bar{s}s\rangle]}{m_s - m_l}$$

Lattice points from

A.Bazavov et al (Hot QCD)

2012-14 (Nf = 2+1)



$$\chi_S^\kappa(T) = -\frac{[\langle\bar{q}q\rangle_l - 2\langle\bar{s}s\rangle]}{m_s + m_l}$$

$\chi_S^\kappa \to \chi_P^K$ above the χ_S^κ peak

$I = 1/2$ sector (K/κ)

From WIs in this sector:

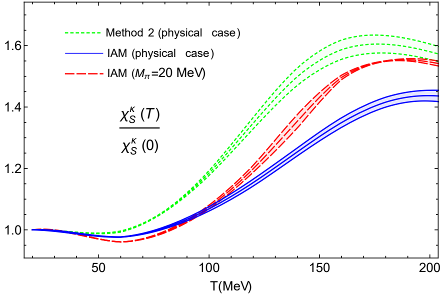
$$\chi_S^\kappa(T) - \chi_P^K(T) = \frac{2}{m_s^2 - m_l^2} \Delta_{l,s}(T) \longleftarrow \text{dictated by subtracted condensate}$$

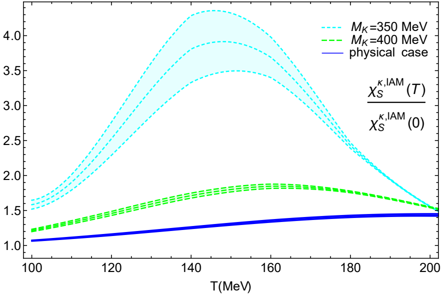
- In physical case strength of $U(1)_A$ above T_c well determined and driven by $\langle \bar{s}s \rangle$.
- In $N_f = 2$ limit, exact $O(4) \times \mathrm{U}(1)_A$ degeneration for $m_l, \langle \bar{q}q \rangle_l \to 0$.

$$m_s \gg m_l: \boxed{\chi_S^\kappa(T) - \chi_P^K(T)\Big|_{m_s \gg m_l} = \frac{2}{m_s} \langle \bar{q}q \rangle_l \Big|_{SU(2)} + \mathcal{O}(1/m_s^2)}$$

- May help to clarify the role of strangeness.

χ_S^κ saturated by $I = 1/2$ $K_0^*(700)$ scalar pole





$$\chi_S^\kappa = \chi_S^{\kappa,ChPT}(0)\frac{M_\kappa^2(0)}{M_\kappa^2(T)}$$

- The peak is reproduced.
- Chiral limit ($M_\pi = 20\,\mathrm{MeV}$):
  - $\rightarrow$ Larger growth below peak enhanced by chiral symmetry.

$\downarrow$

$K - \kappa$ degeneration takes place at a lower temperature.

- $SU(3)$ limit:
  - $\rightarrow$ Peak grows.
  - $\rightarrow$ Displacement of the peak towards T_c.

$\downarrow$

Consistently with its degeneracy with χ_S.

Chiral imbalance in ChPT

μ_5 chemical potential for approximate conservation of the chiral charge.

$$\text{QCD Lagrangian for } \mu_5 \neq 0\text{: } \mathcal{L}_{QCD} \to \mathcal{L}_{QCD} + \mu_5 \bar{q}\gamma_5\gamma^0 q$$

We have constructed the most general meson effective Lagrangian for $\mu_5 \neq 0$ and two light flavours.

The construction is carried out using the framework of the external source method.

New terms coming from:

- Covariant derivatives.
- Explicit axial source terms.

$\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ effective Lagrangian

$$\mathcal{L}_2 \to \mathcal{L}_2 + 2\mu_5^2 F^2 (1 + \kappa_0)$$

$$\mathcal{L}_4 \to \mathcal{L}_4 + \kappa_1 \mu_5^2 \mathrm{tr}\left(\partial^\mu U^\dagger \partial_\mu U\right) + \kappa_2 \mu_5^2 \left(\partial_0 U^\dagger \partial^0 U\right) + \kappa_3 \mu_5^2 \mathrm{tr}\left(\chi^\dagger U + U^\dagger \chi\right) + \kappa_4 \mu_5^4$$

$\uparrow$

constants to be determined from different observables

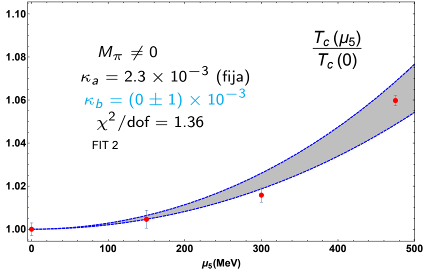
D. Espriu, A. Gómez Nicola, AVR, JHEP. 2020, 62

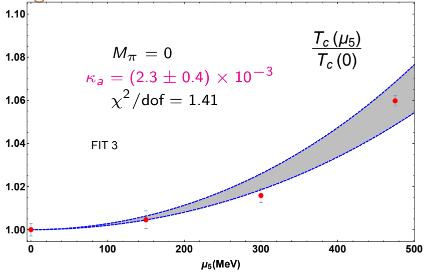
$$\langle \bar{q}q \rangle_l^{NNLO}(T,\mu_5) / \langle \bar{q}q \rangle_l^{NNLO}(T=0,\mu_5) \longrightarrow \boxed{\kappa_a = 2\kappa_1 - \kappa_2} \text{ y } \boxed{\kappa_b = \kappa_1 + \kappa_2 - \kappa_3}$$

$$\frac{\langle \bar{q}q \rangle_l (T_c,\mu_5)}{\langle \bar{q}q \rangle_l (T=0,\mu_5)} = 0 \xrightarrow{M_{0\pi}=0} [T_c(\mu_5)]^2 = 24F^2 \left[\sqrt{\frac{2}{3} + \left[1 - 2\kappa_a \frac{\mu_5^2}{F^2}\right]^2} - 1 + 2\kappa_a \frac{\mu_5^2}{F^2} \right]$$

$M_{0\pi} \neq 0$

Lattice data Braguta et al 2015





An alternative method for calculating T_c:

Peak of $\chi_S(T,\mu_5) = \chi_S(T=0,\mu_5) \dfrac{M_S^2(0,\mu_5)}{M_S^2(T,\mu_5)}$

Scattering $\pi\pi$ at nonzero μ_5

Pion scattering, $T_c(\mu_5)$ and fits of κ_i to lattice

μ_5 corrections to the pion scattering amplitude:

- Tree level coming from $\mathcal{L}_4$.
- Dispersion relation.
- Residue of the LSZ formula.

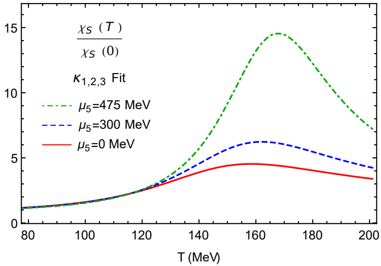
$\downarrow$

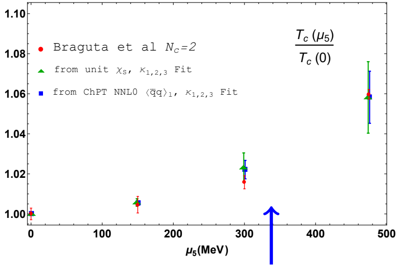
$$\Delta t^{00} \rightrightarrows \begin{array}{l} \kappa_1' = 6\kappa_1 + 5\kappa_2 \\ \kappa_2' = -8\kappa_1 - 4\kappa_2 + 5\kappa_3 \end{array}$$

A. Gómez Nicola, Patricia Roa-Bravo, AVR, PRD **109** (2024) no.3, 034011

Combined fit of χ_{top} and T_c:

$\kappa_1 \times 10^4$	$\kappa_2 \times 10^4$	$\kappa_3 \times 10^4$	χ^2/dof
$9.4^{+1.1}_{-1.3}$	$-4.5^{+1.5}_{-1.4}$	$3.6^{+9.1}_{-8.7}$	1.37





The growing behaviour of $T_c(\mu_5)$ is compatible with lattice results

Conclusions

- Scalar thermal resonances crucial for chiral and $U(1)_A$ restorations.
- Saturating χ_S with thermal $f_0(500)$, we reproduce the crossover peak of χ_S and most of the lattice data fall into the uncertainty band.
- K/κ alternative channel for $O(4) \times U(1)_A$ restoration.
- Saturated χ_S^κ with thermal $K_0^*(700)$ develops a peak and is consistent with $O(4) \times U(1)_A$ pattern.
- We have analyzed the effective chiral Lagrangian for nonzero chiral imbalance for two light flavours.
- The critical temperature increases with μ_5, in agreement with the lattice results.