Theoretical study of the ΣN cusp in the $K^- d \to \pi \Lambda N$ reaction

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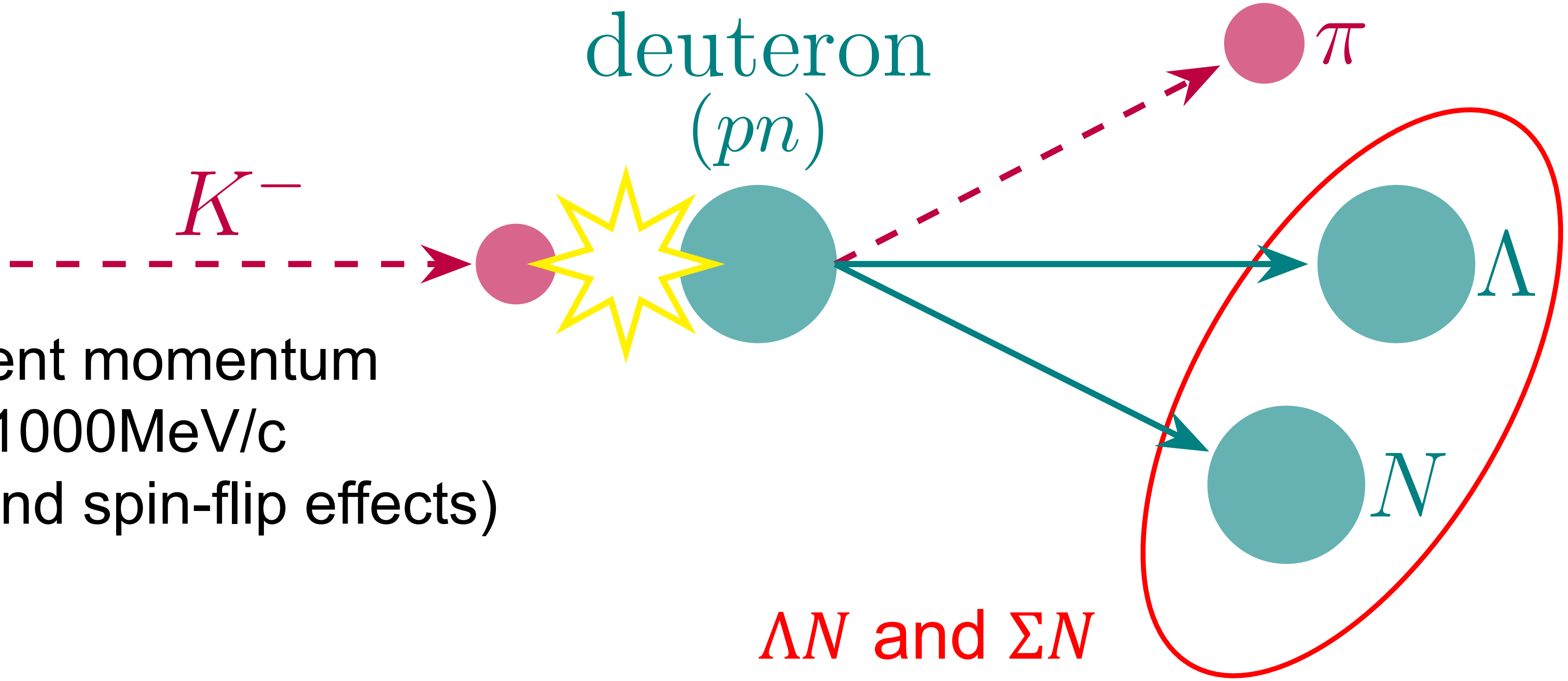
$K^- d \to \pi \Lambda N$ reaction



Incident momentum
~1000MeV/c
(p-wave and spin-flip effects)

ΛN and ΣN
(final state interaction)

Final state interaction of $YN \to YN$

- Hyperon-nucleon interaction is an attractive subject. There are applications to hyper-nucleus structure and neutron stars.
- It is difficult to conduct experiments on low energy two body scatterings of YN.
- An alternative approach is analyzing two body interaction of hyperon-nucleon through final state interaction.

ΣN cusp structure

- Threshold cusp is general feature in spectra representing the sharp peak at thresholds.

- Two factors on the ΣN threshold:

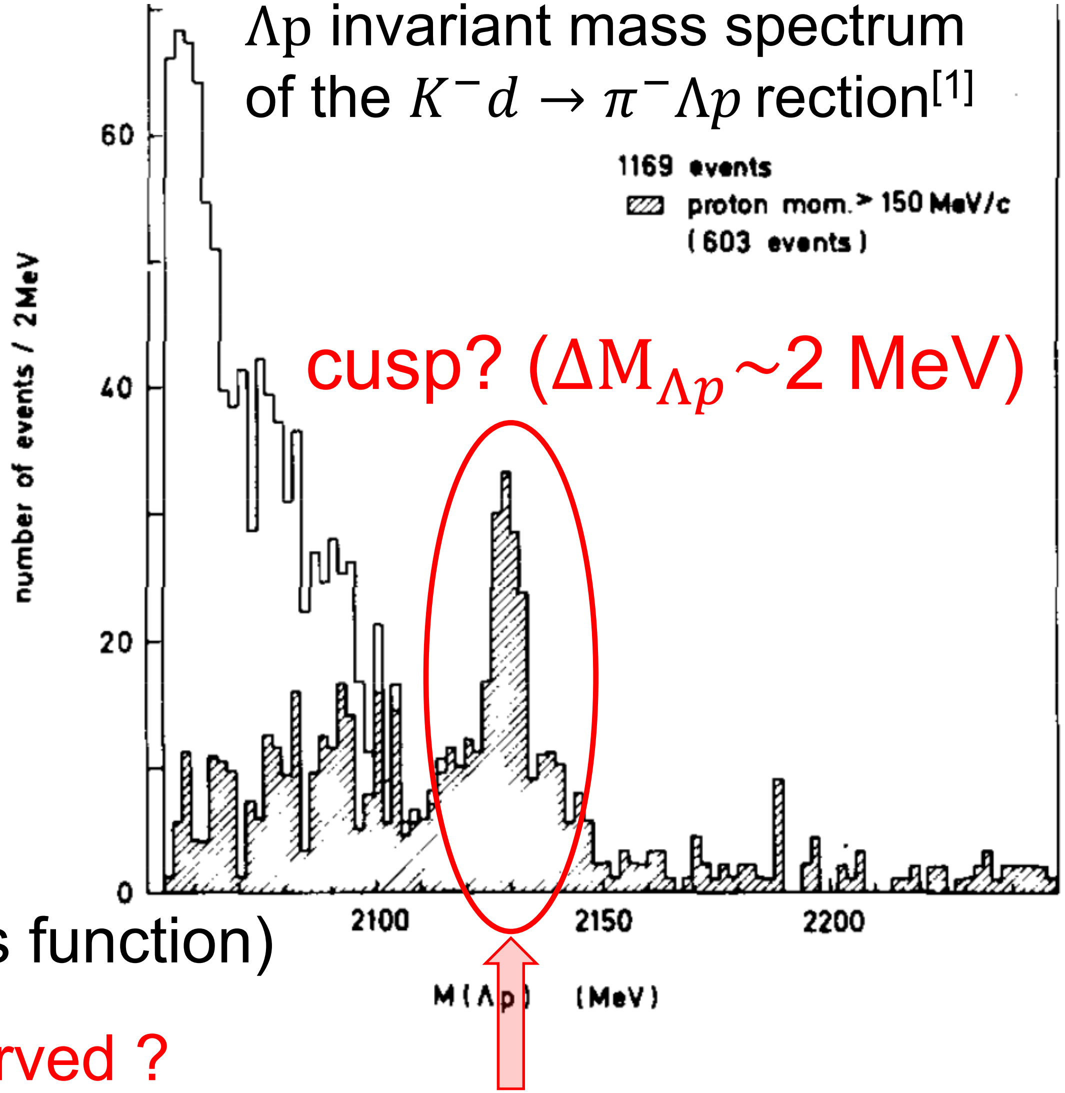
  1. ΣN is coupled to lower channel ΛN

  2. Intermediate Σ (imaginary part of Green's function)

  Is cusp structure (sharpness) really observed ?

  Can we obtain the amplitude of $\Sigma N \to \Lambda N$?

Λp invariant mass spectrum of the $K^- d \to \pi^- \Lambda p$ rection[1]



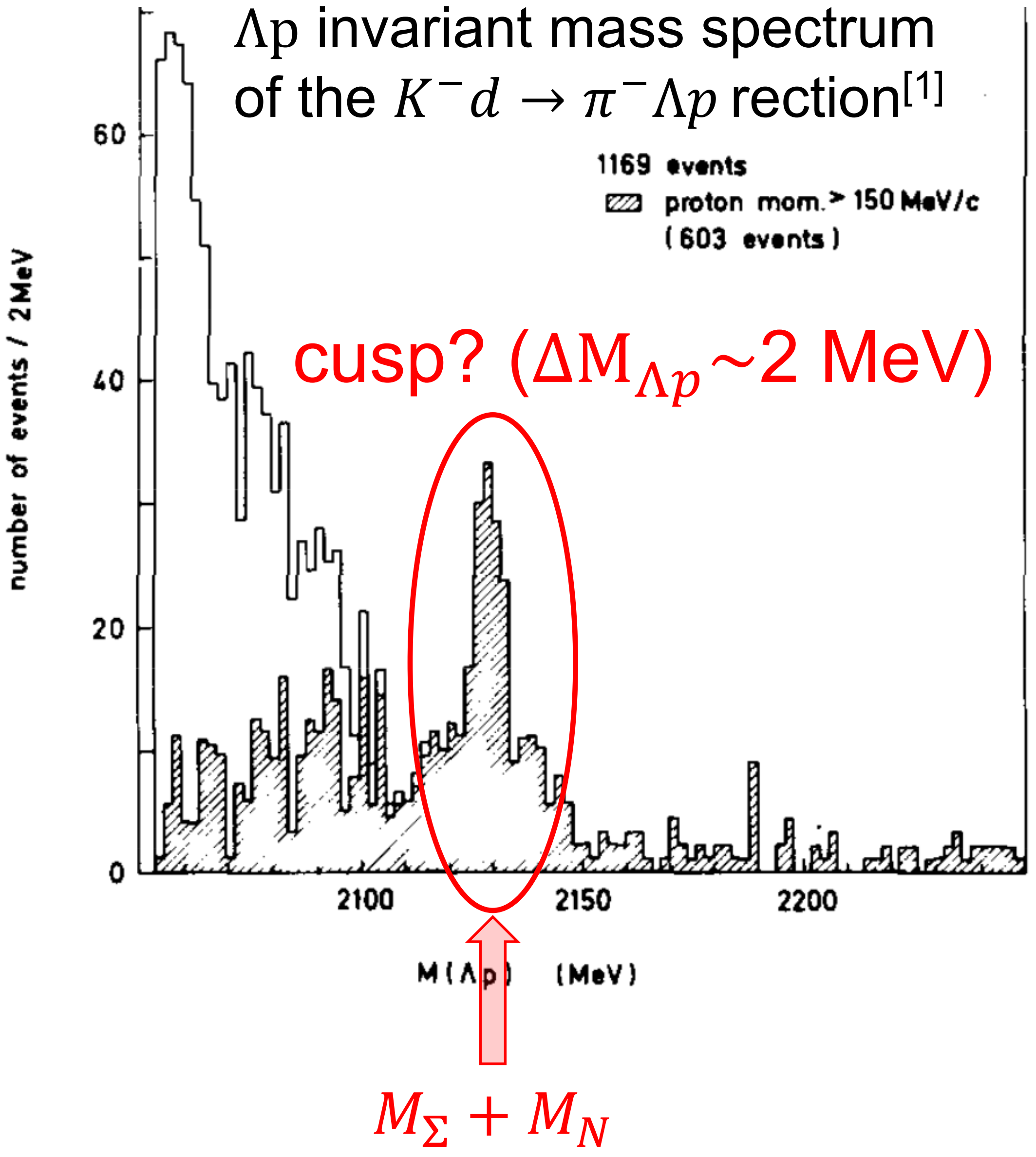
cusp? ($\Delta M_{\Lambda p}$~2 MeV)

$M_\Sigma + M_N$

[1] O. Braun, et al, Nuclear Physics B 124, 45 (1977).

ΣN cusp structure

- Ongoing experiment J-PARC E90[2]:
  incident kaon momentum 1.4 GeV/c
  expected resolution $\Delta M_{\Lambda p} = 0.4$ MeV
- This study is focused on the theoretical analysis of the ΛN invariant mass spectrum near the ΣN threshold.



[1] O. Braun, et al, Nuclear Physics B 124, 45 (1977).
[2]Y. Ichikawa et al. (J-PARC E90 Collaboration), EPJ Web Conf. 271, 02012 (2022).

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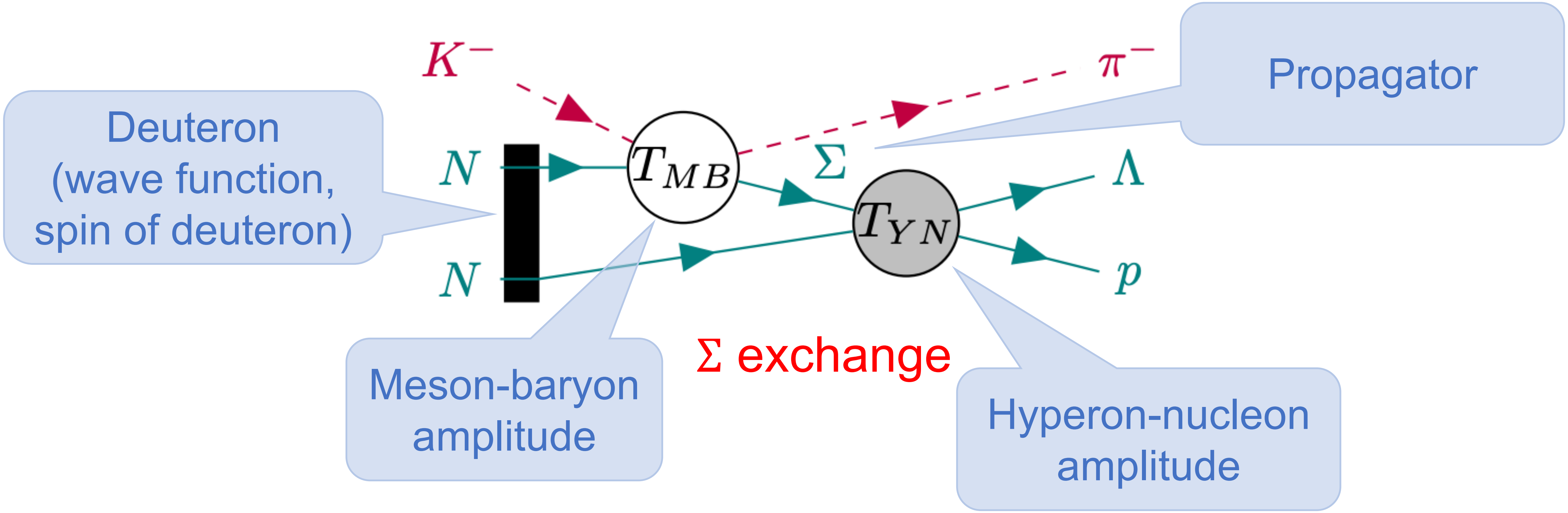
- Methods
- Numerical results
- Summary

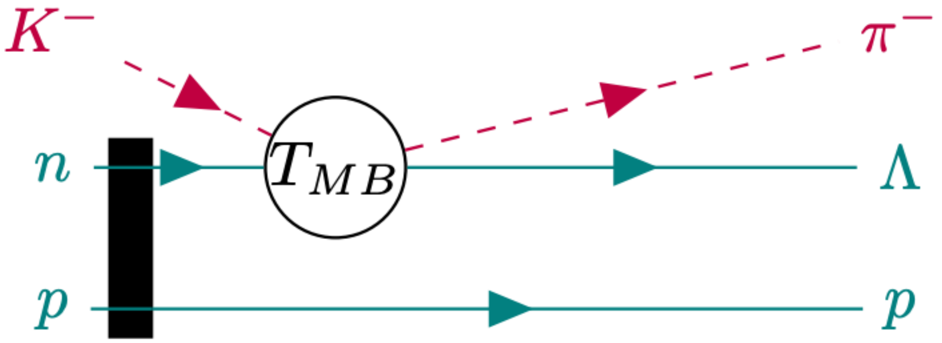
Scattering process (Foreground)

Σ exchange diagram for the $K^- d \to \pi^- \Lambda p$ reaction

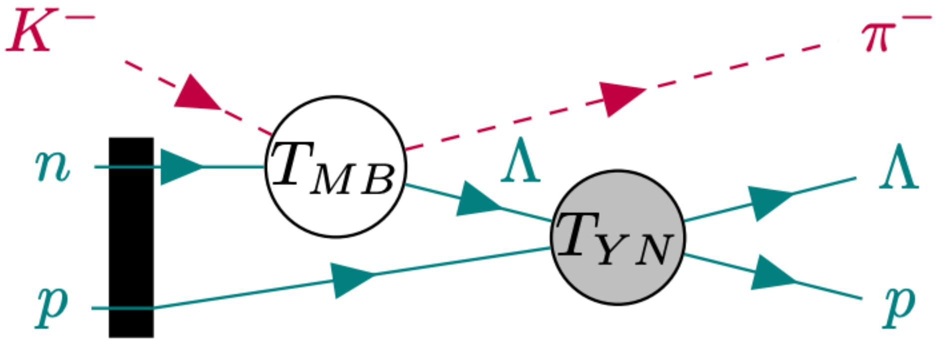


Scattering processes (Backgrounds)

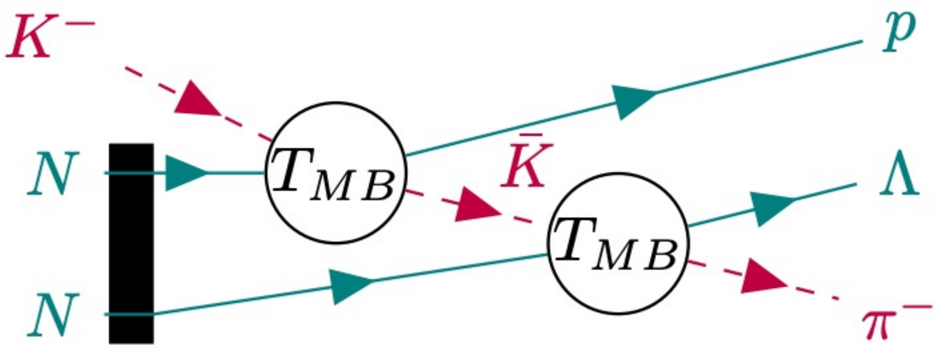
Background diagrams for the $K^- d \to \pi^- \Lambda p$ reaction



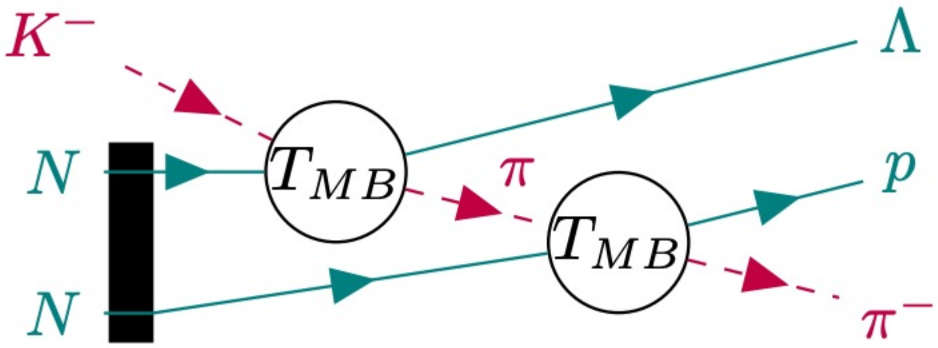
impulse



Λ exchange



$\bar{K}$ exchange



π exchange

Background reduction

- In our previous study[3], we examined kinematic conditions to reduce the backgrounds.

- Two Kinematic conditions to reduce the backgrounds:

1. Pions going forward: $\cos\theta_\pi \sim 1$

   $\Longrightarrow$ suppression of meson exchange

   (& dominance of spin triplet interaction between baryons)

2. Nucleons having higher momenta: $p_N \sim 100$ MeV

   $\Longrightarrow$ suppression of impulse process

Same conditions as today's talk

[3] S.Y. and D. Jido, T. Ishikawa, (2024), arxiv:2405.15534 [nucl-th]

Our previous study on ΛN interaction

- There are some experimental data of the scattering length $a_{\Lambda p}$.
  However, $a_{\Lambda n}$ is not measured due to the experimental difficulties.
  Thus, its isospin symmetry breaking ($a_{\Lambda n} / a_{\Lambda p}$) is unknown.

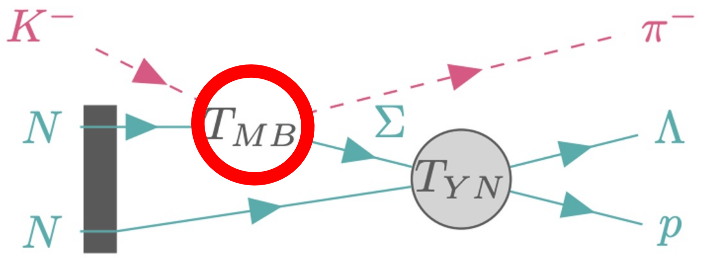
- We calculated ΛN invariant mass spectra for the $K^- d \to \pi^0 \Lambda n$, $\pi^- \Lambda p$ and concluded that ratios of them can be extracted from that of the spectra[3].

$$a_{\Lambda p}^{\text{spin=1}} / a_{\Lambda n}^{\text{spin=1}} \approx 2 \frac{d\sigma(\pi^0 \Lambda n)}{dM_{\Lambda n}} \Big/ \frac{d\sigma(\pi^- \Lambda p)}{dM_{\Lambda p}}$$

numerically

[3] S.Y. and D. Jido, T. Ishikawa, (2024), arxiv:2405.15534 [nucl-th]

Meson-Baryon amplitude: T_{MB}



- Partial wave expansion (s-wave + p-wave)

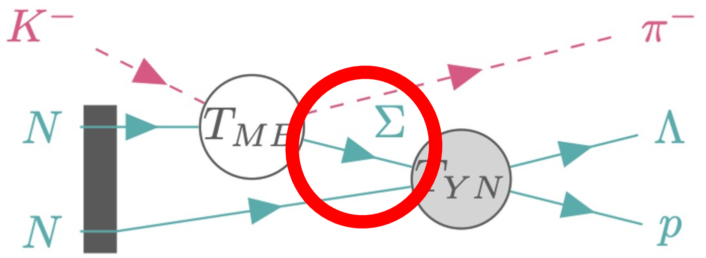
- $\bar{K}N$ amplitudes $(\bar{K}N \to \bar{K}N, \pi Y)$:

  Chiral unitary approach[4] (amplitude for each channel)

- πN amplitudes: experimental data[5]

[4] D. Jido, E. Oset, A. Ramos, Phys. Rev. C 66, 055203 (2002)
[5] R. L. Workman, R. A. Arndt, W. J. Briscoe, M. W. Paris, and I. I. Strakovsky, Phys. Rev. C 86, 035202 (2012)

Propagator of Σ



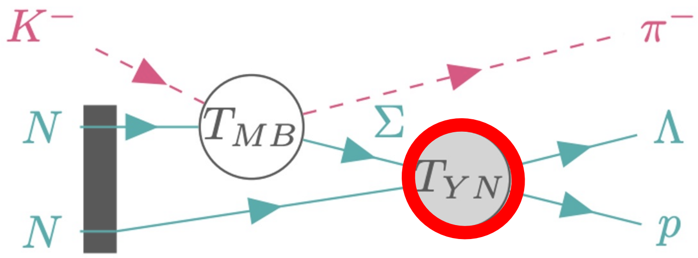
- There is a propagator from intermediate Σ, and its off-shellness is linked to that of deuteron. Using the deuteron wave function[6] φ, their convolutive amplitude T_G is denoted as

$$T_G = \int \frac{d^3\vec{q}}{(2\pi)^3} \varphi(|\vec{q} + \vec{p}_{\pi^-} - \vec{p}_{K^-}|) \frac{1}{q_0^2 - |\vec{q}|^2 - M_\Sigma^2 + i\epsilon} .$$

- Imaginary part of the propagator is on-shell distribution denoted by delta function. Therefore, $\mathrm{Im}(T_G)$ shows the threshold.

[6] R. Machleidt, Phys. Rev. C 63, 024001 (2001).

$\Sigma N \to \Lambda N$ conversion amplitude



- By Lippmann-Schwinger equation near the mass threshold of ΣN $(M_{\Lambda N} \sim M_\Sigma + M_N)$, conversion amplitude $T_{\Sigma N \to \Lambda N}$ is obtained as off-diagonal part of 2-channel T-matrix

$$T = \begin{pmatrix} T_{\Lambda N \to \Lambda N} & T_{\Lambda N \to \Sigma N} \\ T_{\Sigma N \to \Lambda N} & T_{\Sigma N \to \Sigma N} \end{pmatrix}$$

- We use scattering lengths $a_{\Lambda N}$(empirical) and $a_{\Sigma N}$(theoretical models) as parameters of T.

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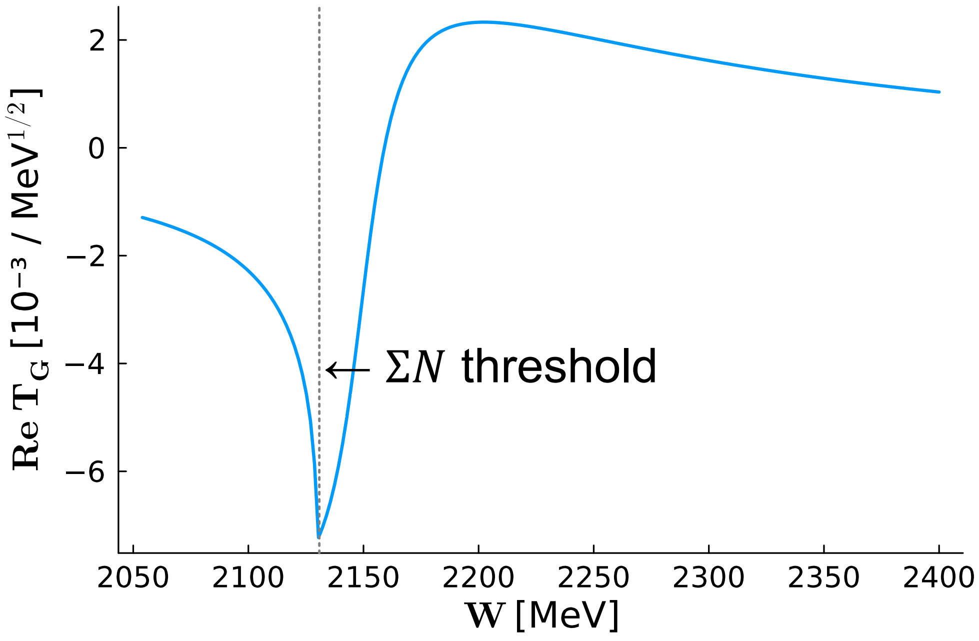
[7] S.Y. and D. Jido, in preparation

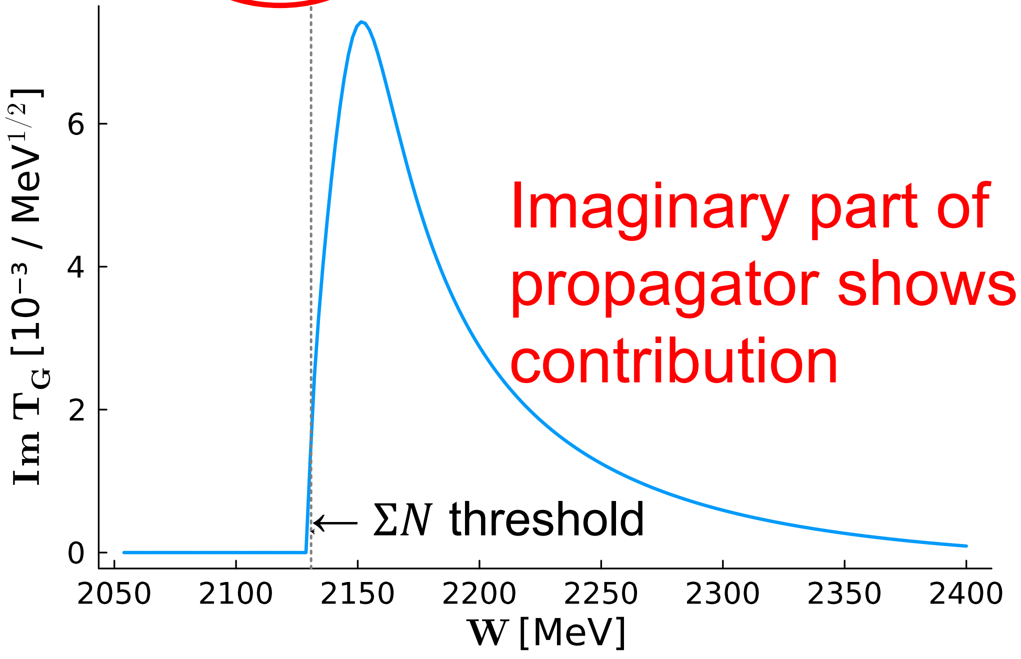
Amplitude from propagator

- Amplitude from propagator and deuteron wave function φ

Important to the location of the threshold

$$T_G(W) = \int \frac{d^3\vec{q}}{(2\pi)^3} \varphi(|\vec{q} + \vec{p}_{\pi^-} - \vec{p}_{K^-}|) \frac{1}{q_0^2(W) - |\vec{q}|^2 - M_\Sigma^2 + i\epsilon}$$



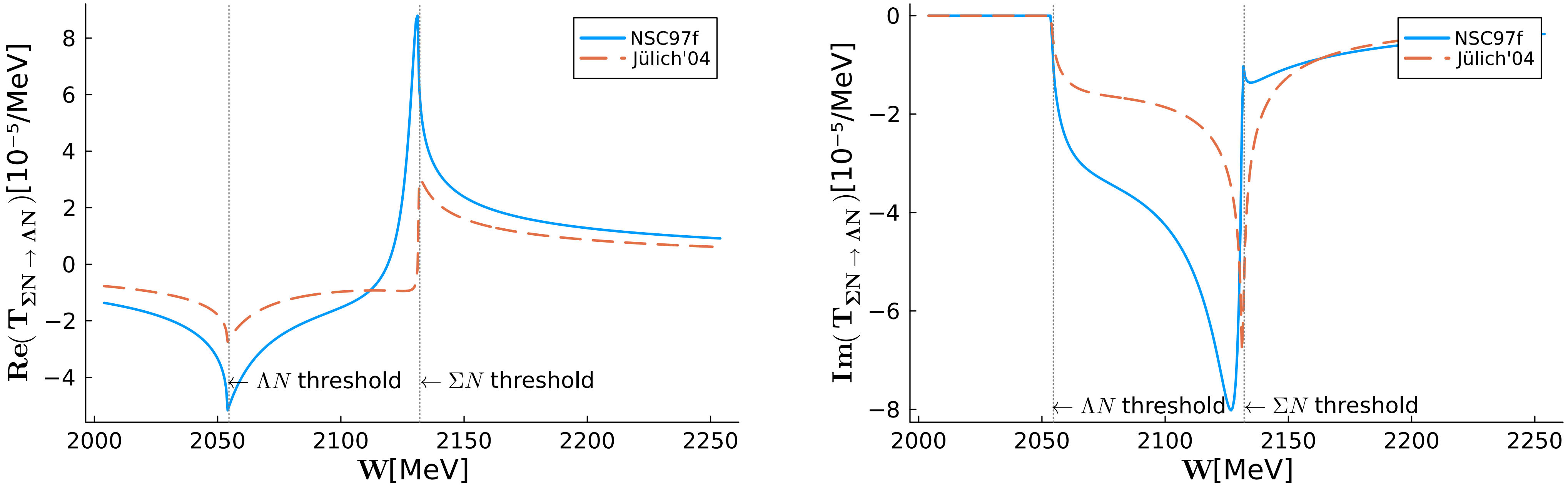


Imaginary part of propagator shows on-shell contribution

Amplitude of $\Sigma N \rightarrow \Lambda N$

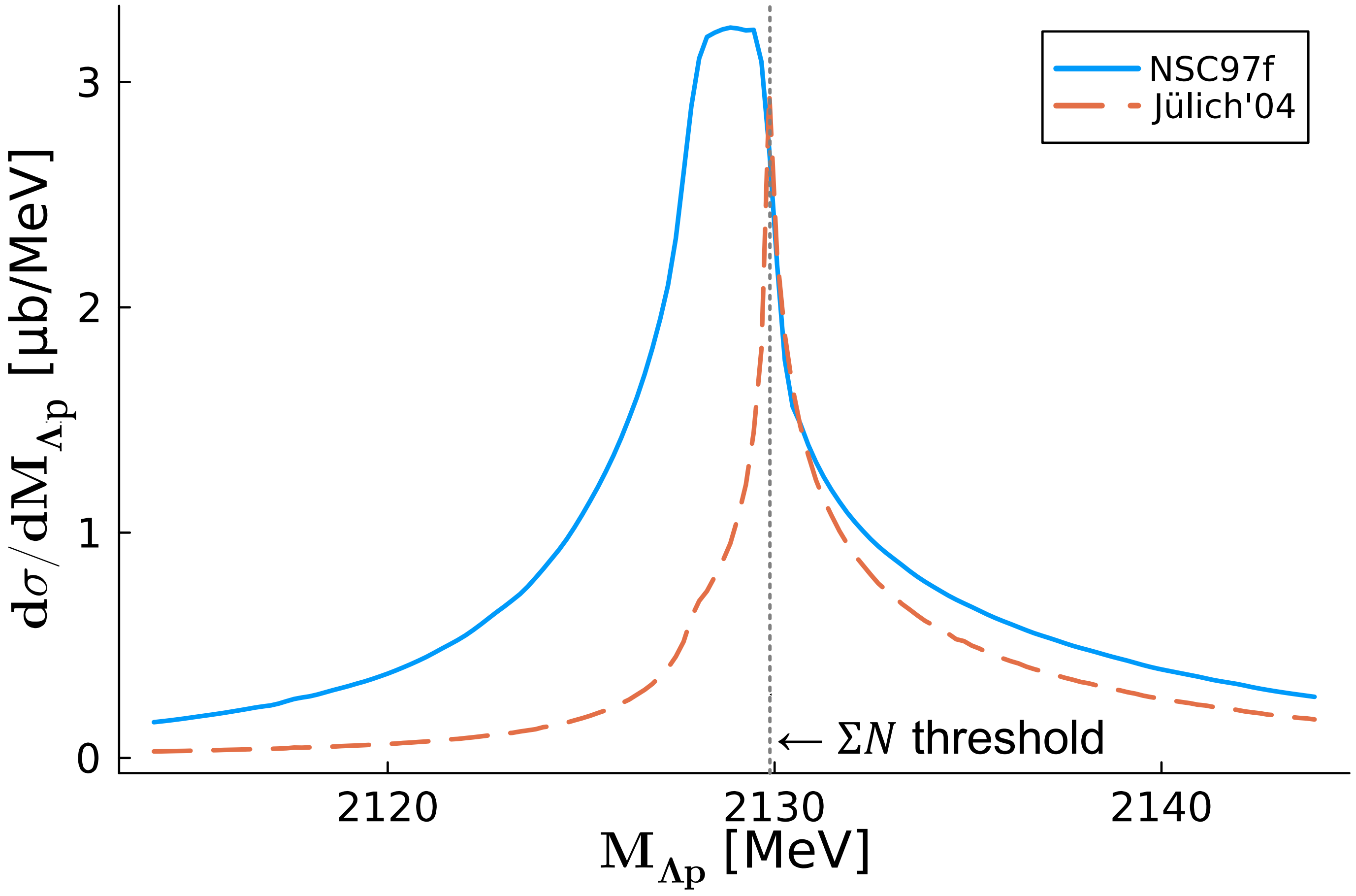
- Spin-triplet scattering length $a_{\Lambda N}^{\mathrm{spin}=1} = -1.56^{+0.19}_{-0.22}$ [fm][8], and two parameters

  NSC97f: $a_{\Sigma N}^{\mathrm{spin}=1} = 1.68 - 2.35i$[9] and Jülich'04: $a_{\Sigma N}^{\mathrm{spin}=1} = -3.83 - 3.01i$[10]



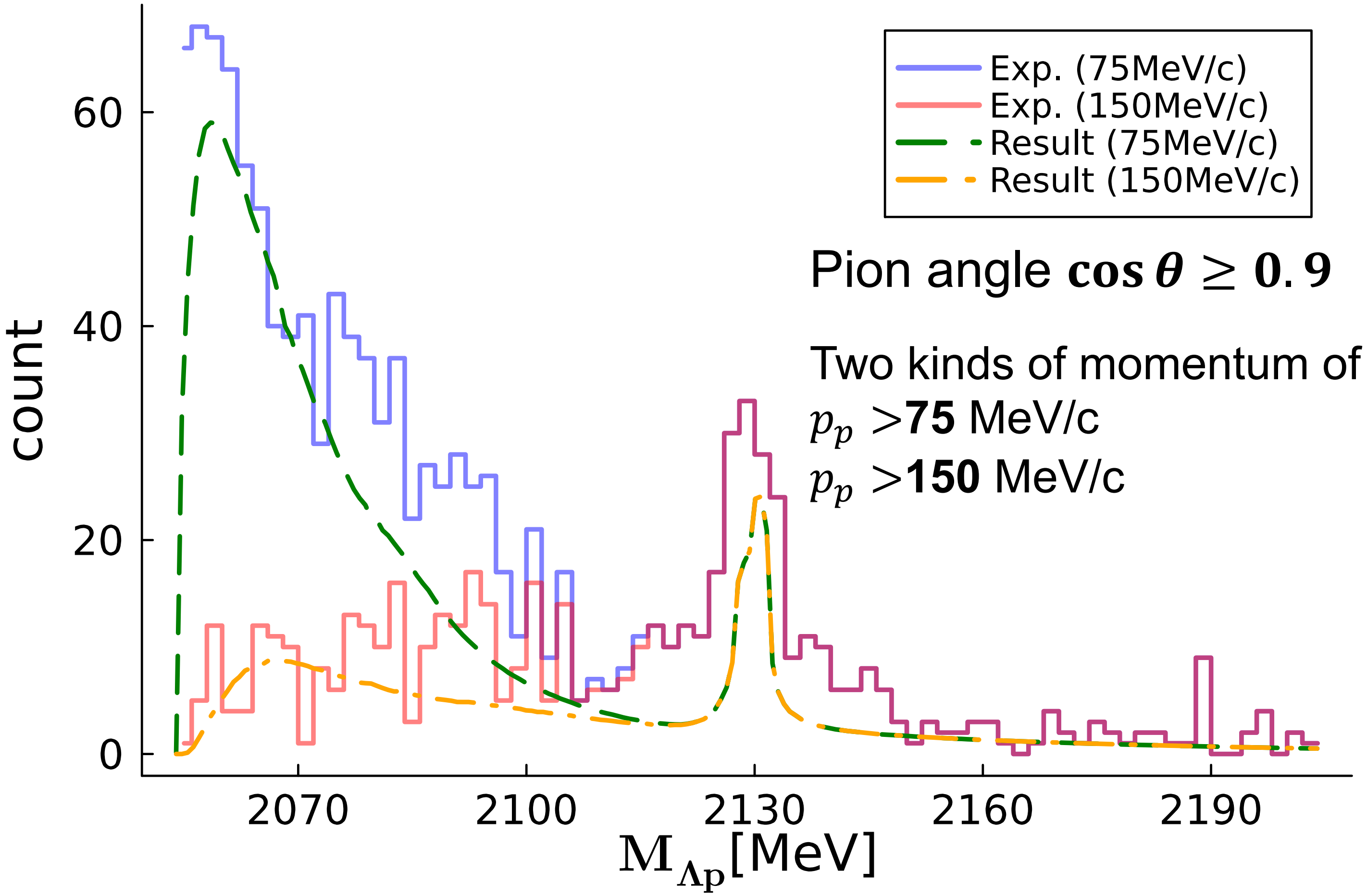
[8] A. Budzanowski *et al.* (HIRES Collaboration), Phys. Lett. B 687, 31 (2010)
[9] T. A. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C 59, 21 (1999).
[10]J. Haidenbauer and U.-G. Meißner, Phys. Rev. C 72, 044005 (2005)

ΣN cusp with two parameters



- ΛN invariant mass spectra for the $K^- d \to \pi^- \Lambda p$ reaction (only with foreground process)
- Incident kaon: **1000 MeV/c**
- The spectrum shape at ΣN threshold (narrow or wide peak) varies depending on the parameter $a_{\Sigma N}$

Comparison with experiment



Pion angle $\cos\theta \geq 0.9$

Two kinds of momentum of p:
p_p >**75** MeV/c
p_p >**150** MeV/c

- Incident kaon
  Experiment[1]: 680~840MeV/c
  Our result (including backgrounds): 760 MeV/c
- Scaling from numerical cross section [μb/MeV] to data [count] is done by considering the incident flux of experiment, 7.1[events/μb].
- The parameter of Σ exchange term
  NSC97f : $a_{\Sigma N}^{\text{spin}=1} = 1.68 - 2.35i$

[1] O. Braun, et al, Nuclear Physics B 124, 45 (1977).

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Summary

- The cusp (sharp peak) structure at ΣN threshold in the ΛN invariant mass spectrum for $K^{-}d \rightarrow \pi\Lambda N$ is numerically observed.

- There is mainly two factors that causes threshold peak:
  ① propagator with deuteron wave function
  ② two body amplitude of the final state interaction

- The shape of the spectrum varies depending on the value of $a_{\Sigma N}^{\mathrm{spin}=1}$.
  ⇒ possibility of detecting the amplitude of $\Sigma N \rightarrow \Lambda N$

Additional slides

Detail of $\Sigma N \to \Lambda N$ conversion amplitude ①

- Treat ΛN and ΣN as doublet states

$$T = \begin{pmatrix} T_{\Lambda N \to \Lambda N} & T_{\Lambda N \to \Sigma N} \\ T_{\Sigma N \to \Lambda N} & T_{\Sigma N \to \Sigma N} \end{pmatrix}$$

- Amplitude F is constructed by the unitarity with kernel V

$$F = (-V^{-1} - iP)^{-1}$$

$$P = \begin{pmatrix} p_\Lambda^* & 0 \\ 0 & p_\Sigma^* \end{pmatrix}$$

p_Λ^* and p_Σ^* : momenta of Λ and Σ in the YN rest frame

Detail of $\Sigma N \to \Lambda N$ conversion amplitude ②

- We assume kernel V is constant and symmetric,
  and satisfy the condition at thresholds of ΛN and ΣN

$$T_{11}(W = M_\Lambda + M_N) = a_{\Lambda N}$$

$$T_{22}(W = M_\Sigma + M_N) = a_{\Sigma N}$$

- V is determined by 3 parameters, $a_{\Lambda N}$, $\mathrm{Re}(a_{\Sigma N})$, and $\mathrm{Im}(a_{\Sigma N})$
  (3 conditions for 3 components of V, V_{11}, $V_{12} = V_{21}$, and V_{22})