# Possible scenario of dynamical chiral symmetry breaking in the instanton liquid

Based on arXiv:2402.05425 to be published in PRD

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Tokyo Institute of Technology

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# introduction

- we revisit chiral symmetry breaking ($\chi$SB) in interacting instanton liquid model (IILM)
- even though ordinary $\chi$SB condition is NOT satisfied, chiral symmetry can be broken in anomaly driven way (explain later)
- interestingly, anomaly driven scenario has direct connection to nature of hadron, e.g., sigma meson

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# outline

- introduction to ordinary $\chi$SB
- introduction to anomaly driven $\chi$SB
- application to instanton liquid model
- summary

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NJL model:

$$\vec{\pi} = \bar{q}\vec{\tau}\gamma_5 q$$

$$\sigma = \bar{q}q$$

$$\mathcal{L}_{\mathrm{int}} = \sum_{a=0}^{8} \frac{g_S}{2}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\lambda_a\gamma_5 q)^2\right]$$

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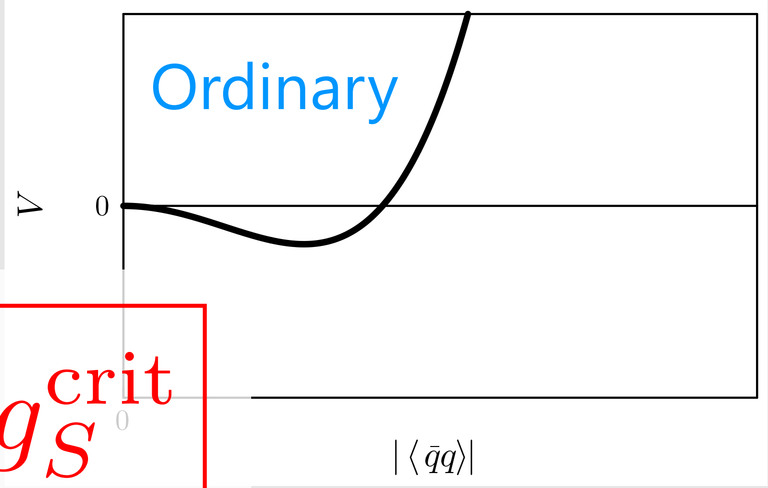
**ordinary $\chi$SB** in three-flavor NJL model with chiral limit:
large enough $g_S$ breaks $\chi$S dynamically, otherwise, it doesn't break $\chi$S



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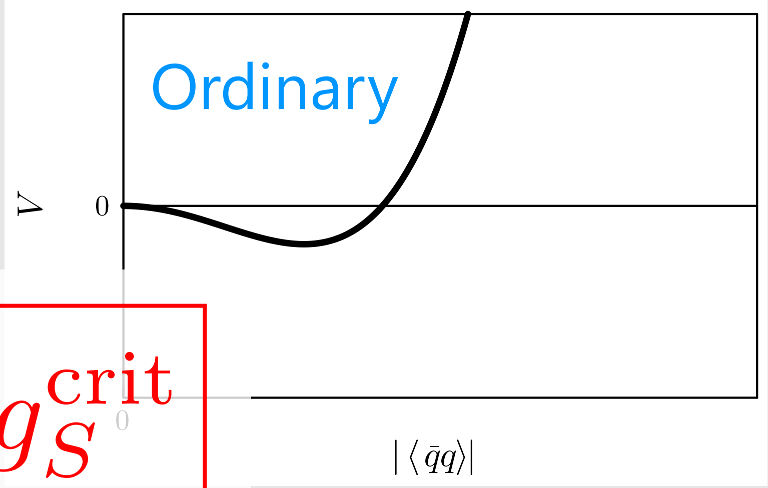

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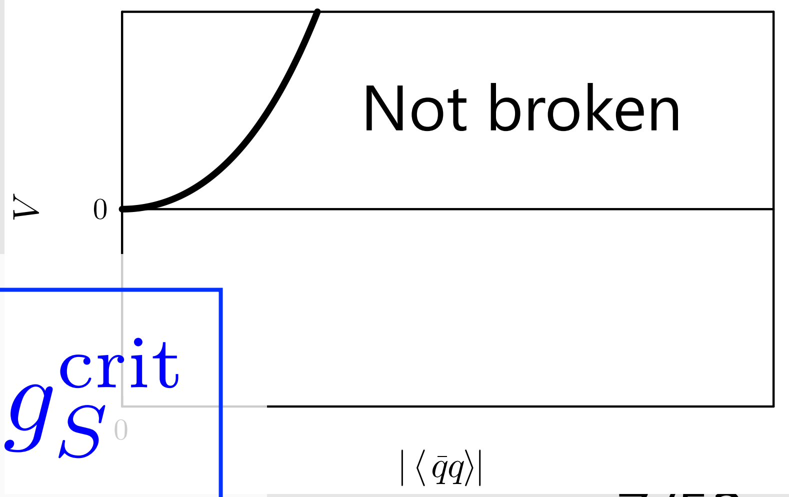

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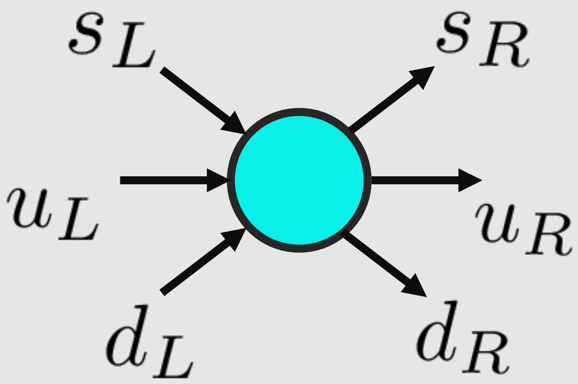

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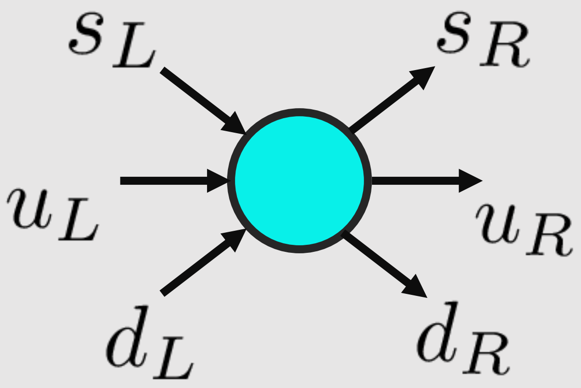

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E. Shuryak developed
instanton liquid model
instead of $g_s$ and $\Lambda$ of NJL
another two parameters
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$$n_{\mathrm{inst}} \approx 1\ \mathrm{fm}^{-4}$$

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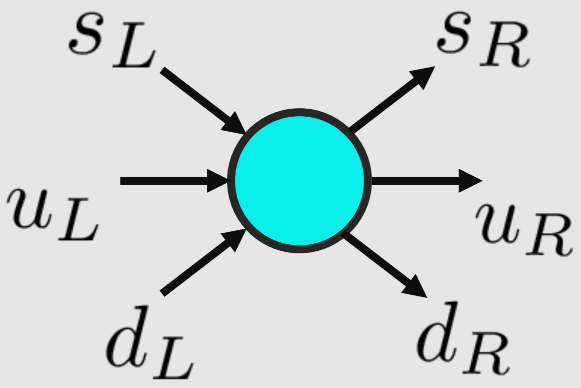

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Interacting instanton liquid model 1990s ← we use (explain later)
summed all orders of ‘t Hooft vertex

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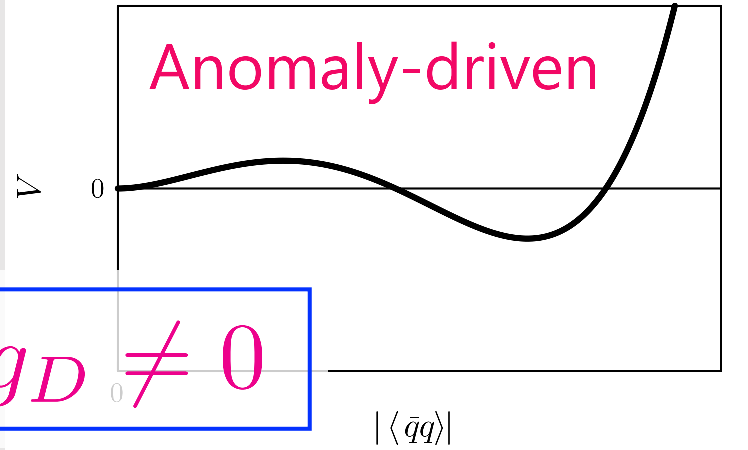

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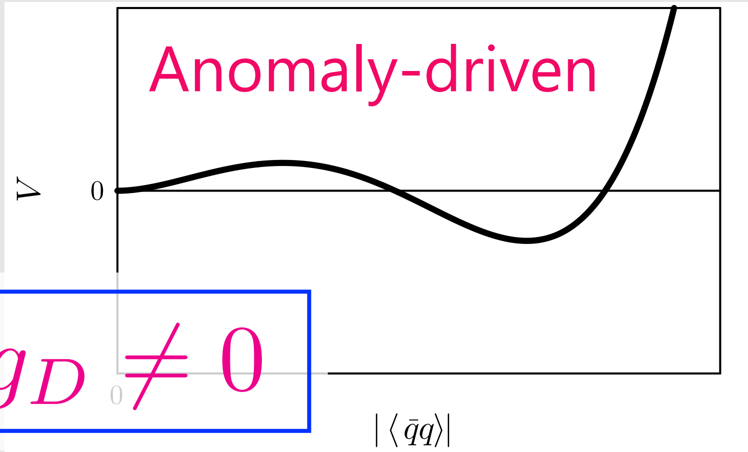

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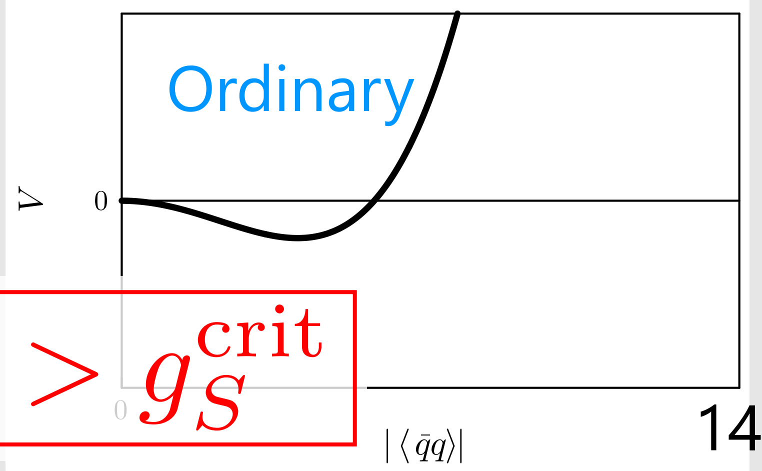

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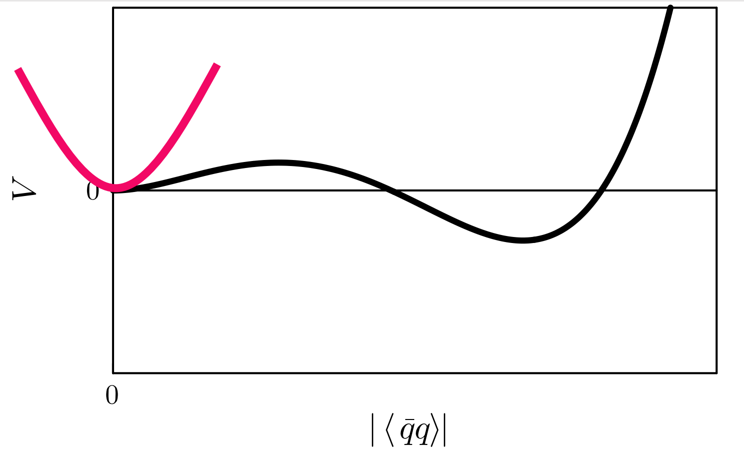

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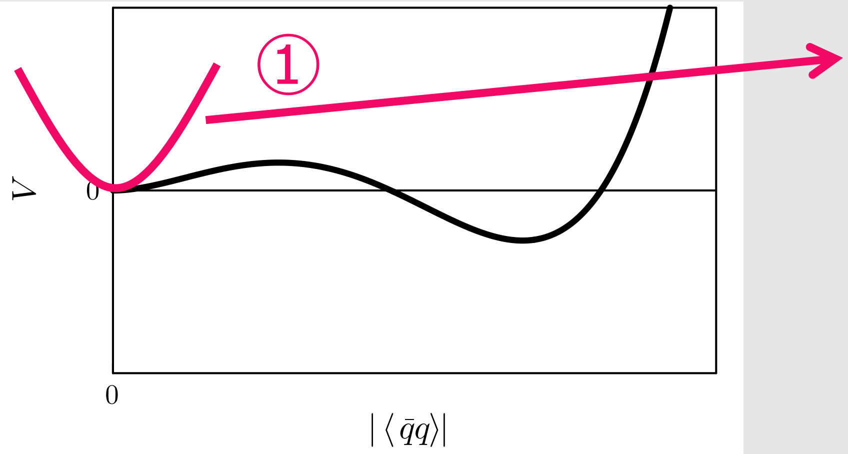

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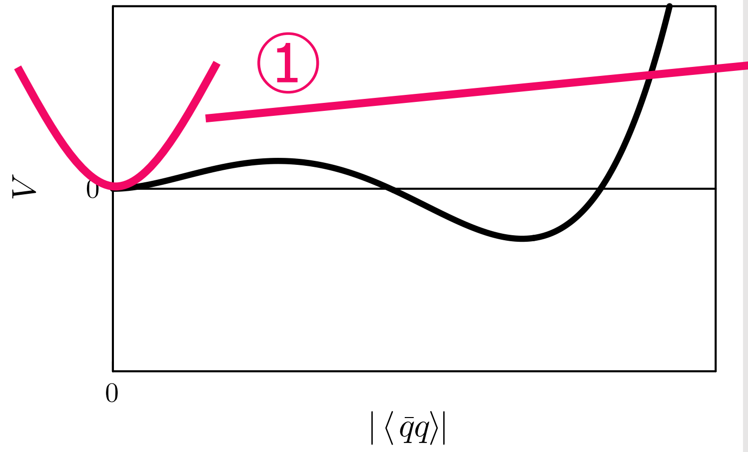

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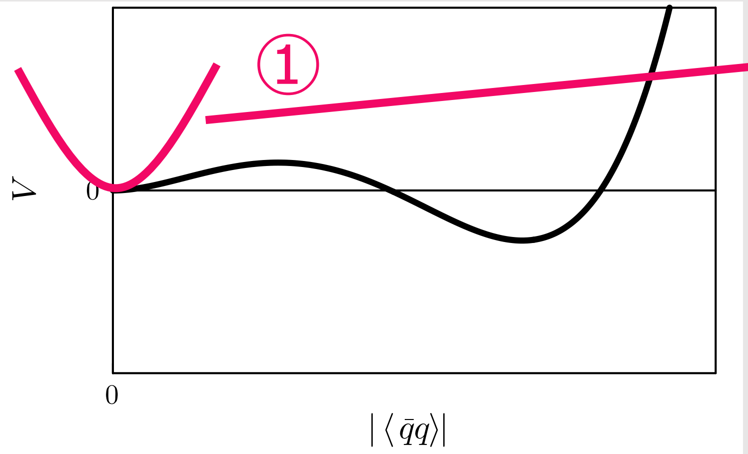

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Our definition is
sign of curvature: $\left.\frac{\partial^2 V}{\partial\langle\bar{q}q\rangle^2}\right|_{\langle\bar{q}q\rangle=0} > 0$

Positive → Anomaly driven breaking
Negative → Normal breaking

by the way

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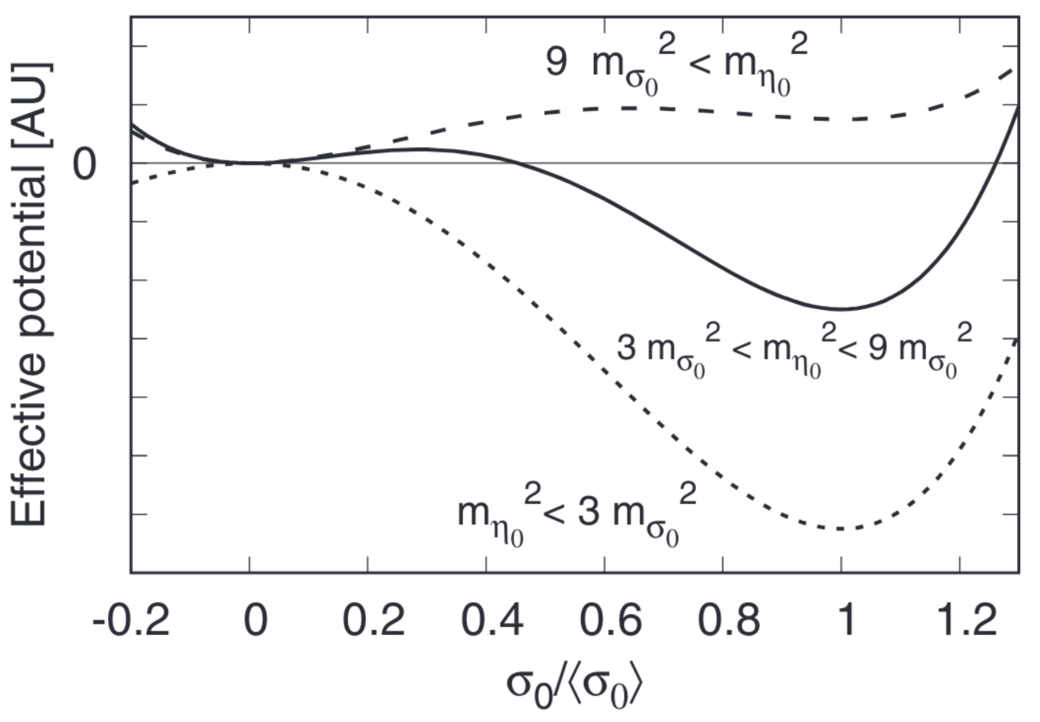

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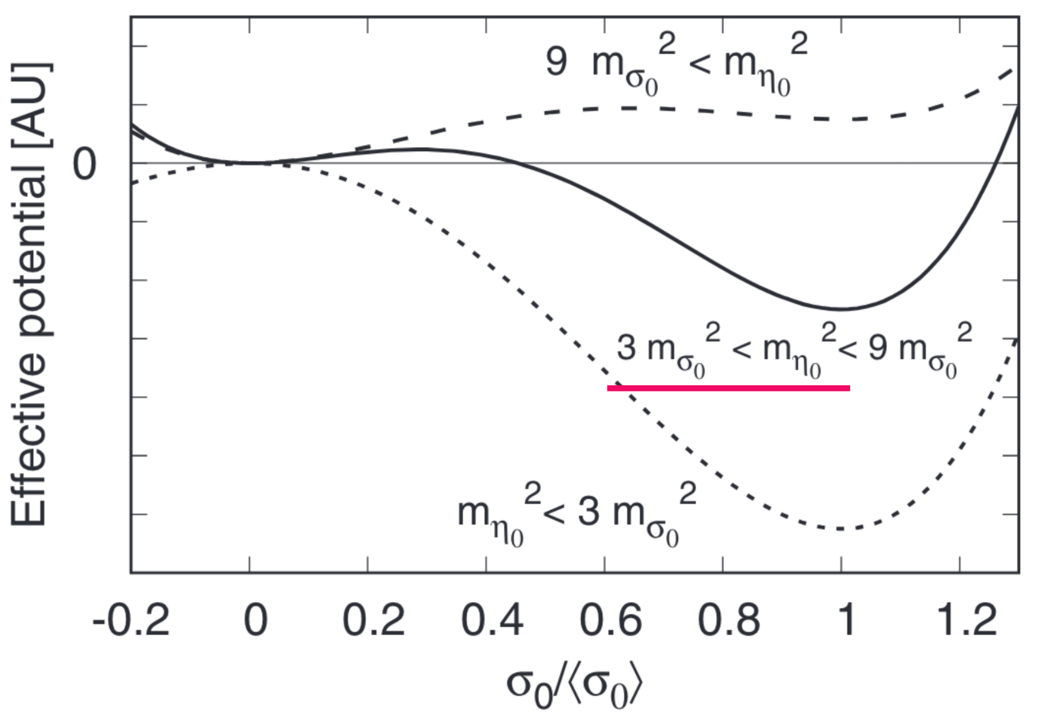

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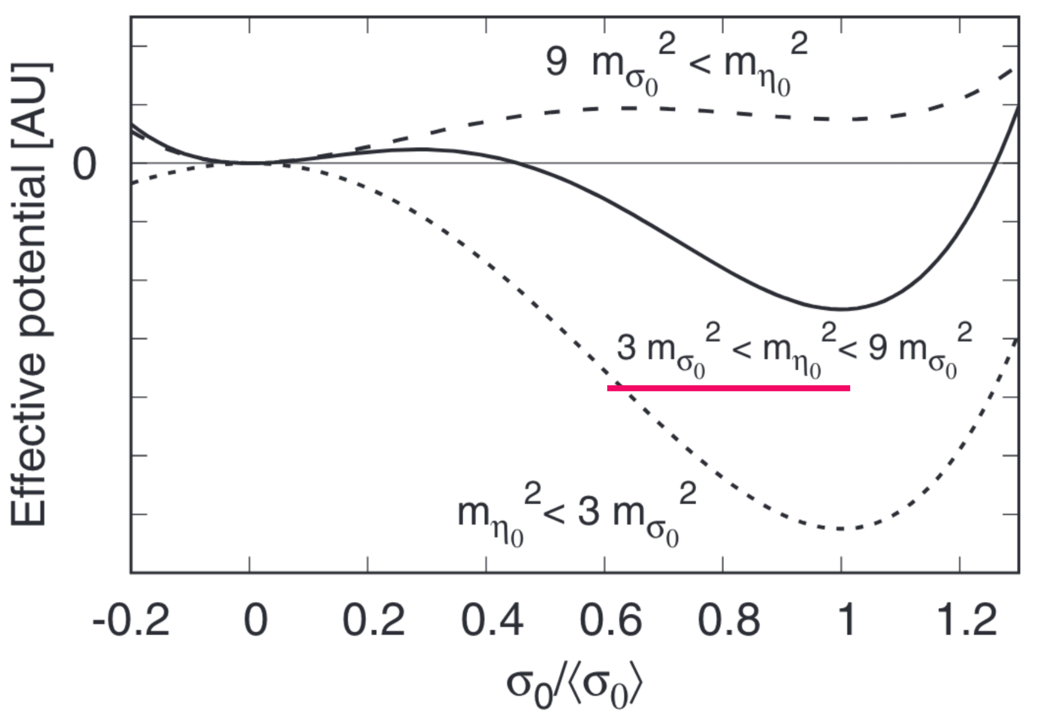

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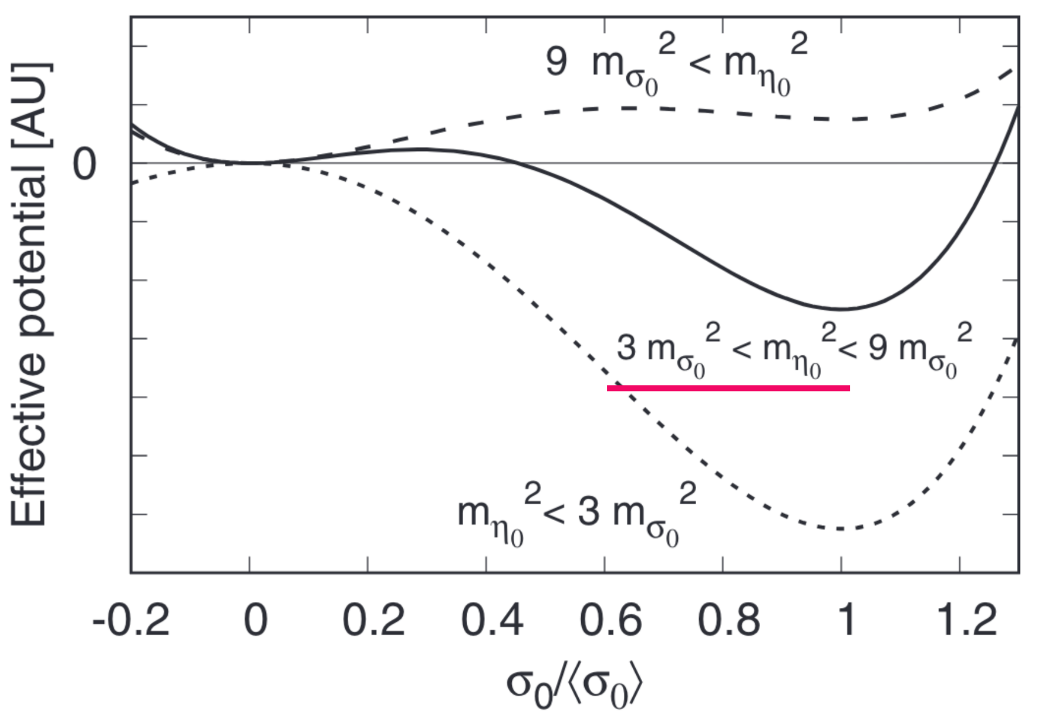

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alternatively, if we could rule out anomaly driven solution, we would have the lower limit of the mass of chiral sigma

# Application to instanton liquid model

# Motivation of study

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to verify that anomaly driven breaking in other systems
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-> positive curvature is our criterion to determine anomaly driven $\chi$SB

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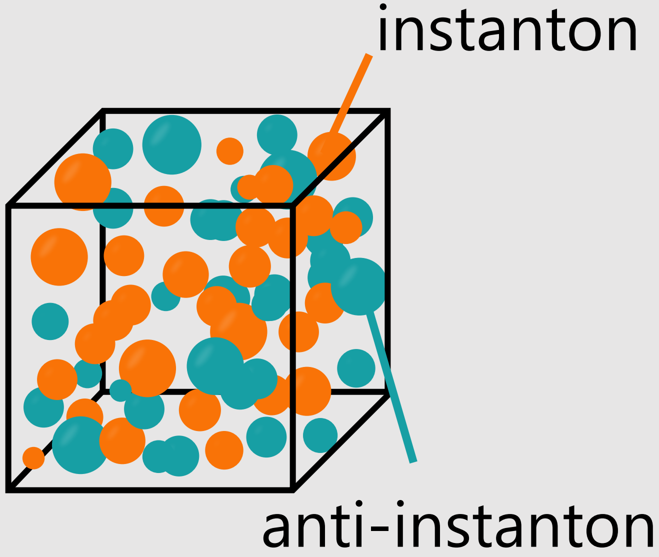

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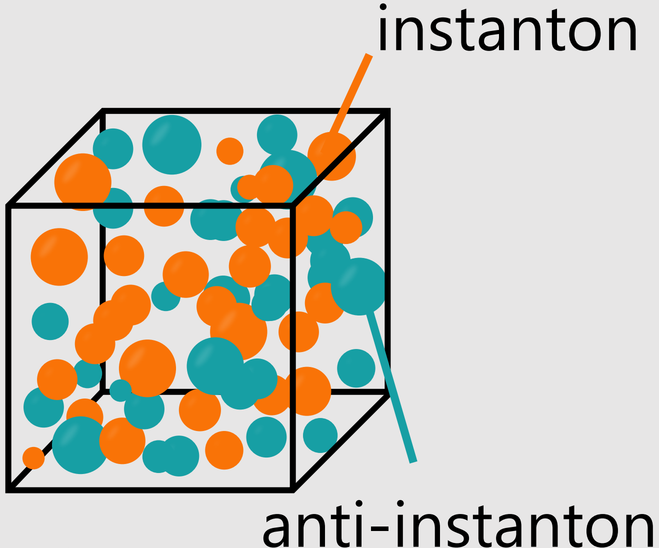

described by Euclidean partition function saturated by instantons

$$Z_{\mathrm{IILM}} = \frac{1}{N_+!N_-!}\int\left(\prod_{i=1}^{N_++N_-} \underbrace{d\Omega_i}_{\text{Collective coordinates of instantons}}\,\underbrace{f(\rho_i)}_{\text{Semiclassical Instanton amplitude}}\right)\exp(-\overbrace{S_{\mathrm{int}}}^{\text{Instanton-instanton interaction}})\prod_{f=1}^{N_f}\overbrace{\mathrm{Det}\left(\gamma_\mu D_\mu + m_f\right)}^{\text{Instanton-quark interaction}}$$

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Model action (weight func. for Monte Carlo calc.)

- Full

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Setup

- color & flavor : Nc=3 & SU(3)_f limit / Quench
- $m_q$ (MeV) : $37 < m_q < 70$ for SU(3)_f, $2.8 < m_q < 28$ for Quench
- # of $I$&$\bar{I}$ (fixed) : 16+16
- # of conf. : 5000

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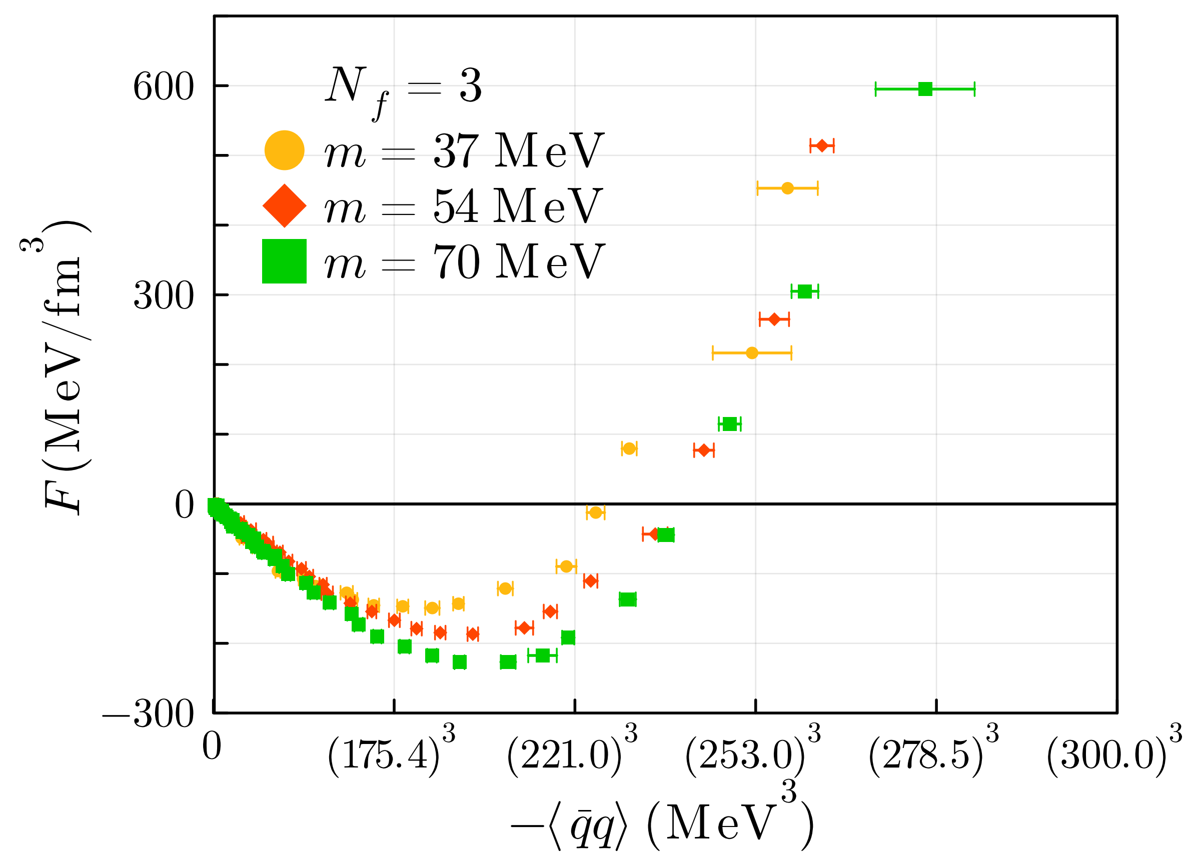
Observables

- Vacuum energy $F = -\ln Z / V$ (for zero temperature)
- Quark condensate $\langle \bar{q}q \rangle$ (one flavor amount w/o free contribution)

# Result: F vs. qq [full]

Vacuum energy density (effective pot.)
vs. quark condensate
shows chiral symmetry breaking



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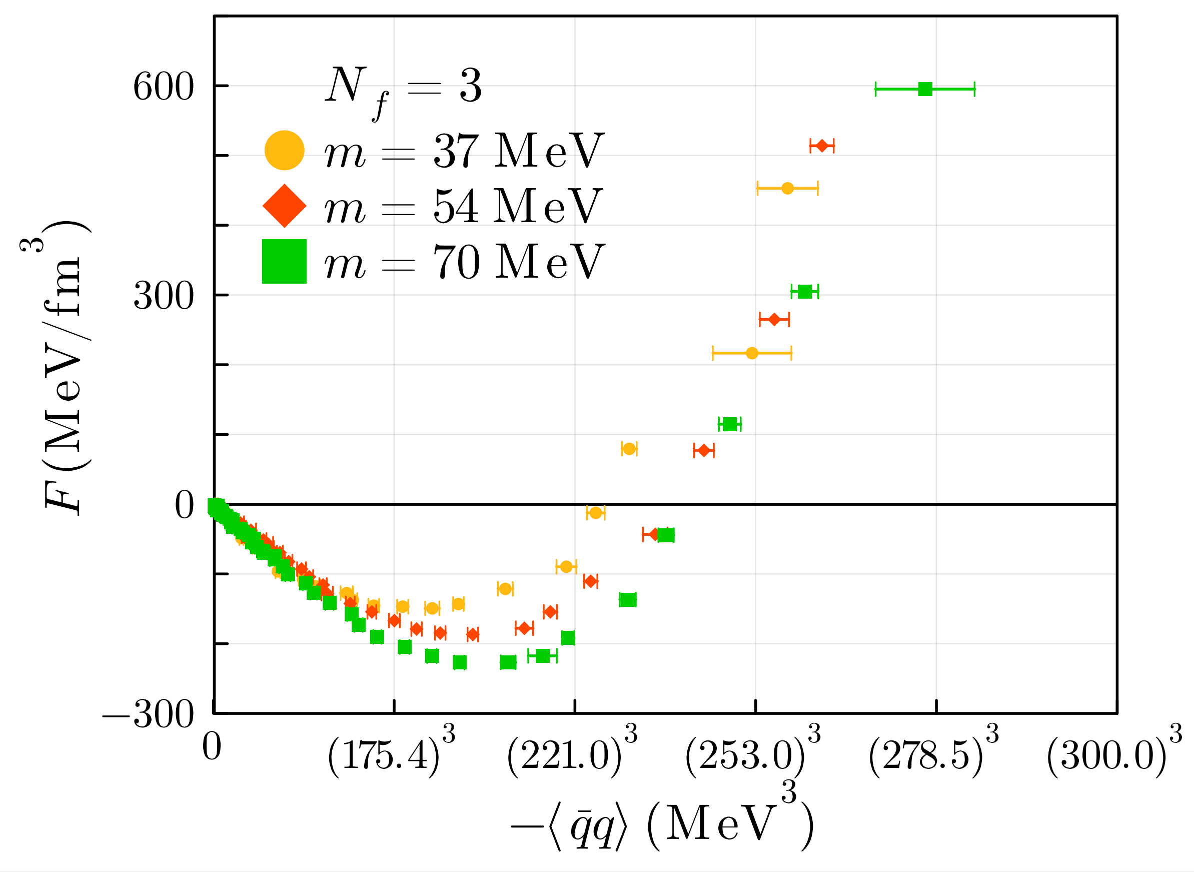

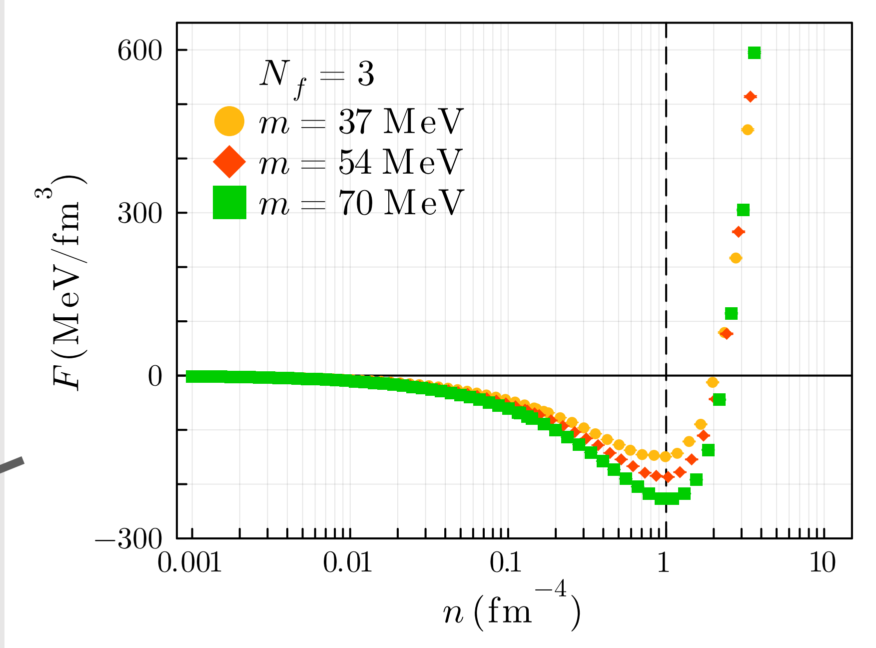

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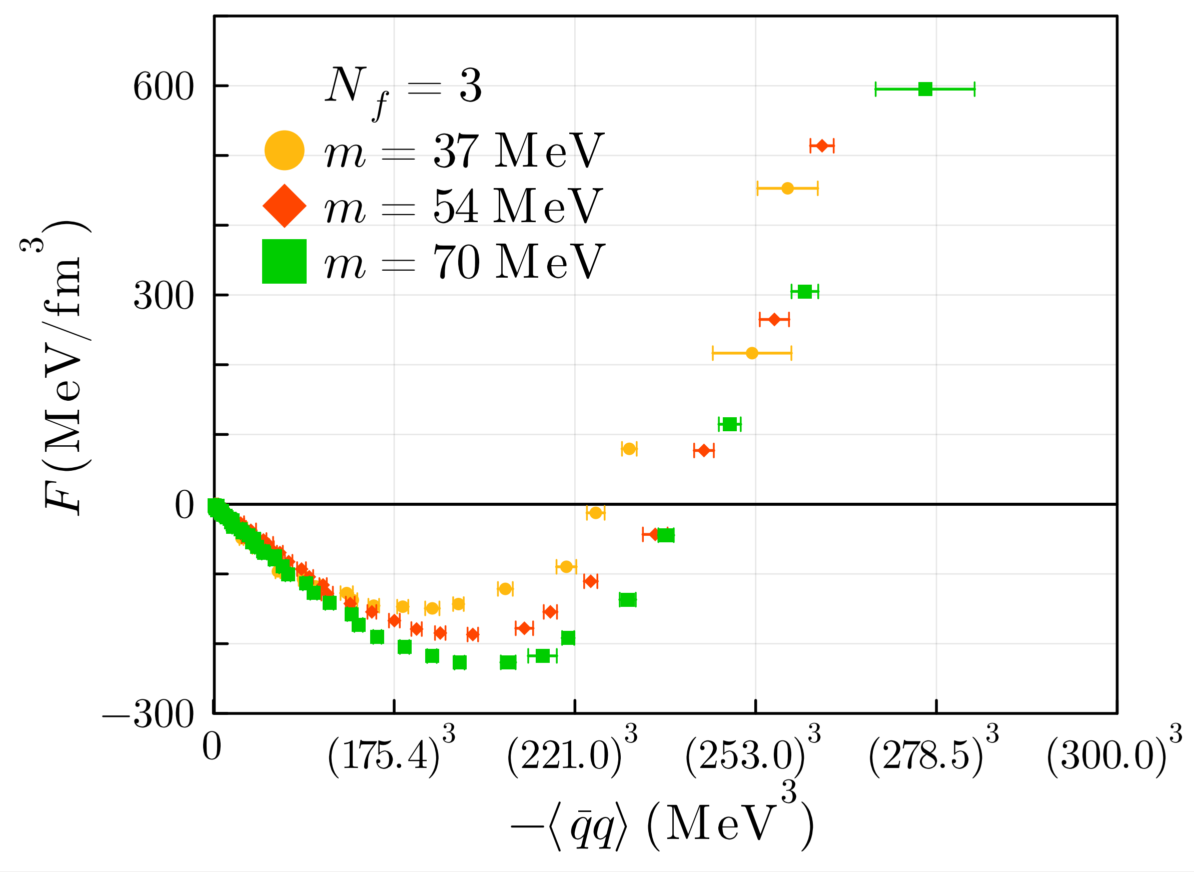

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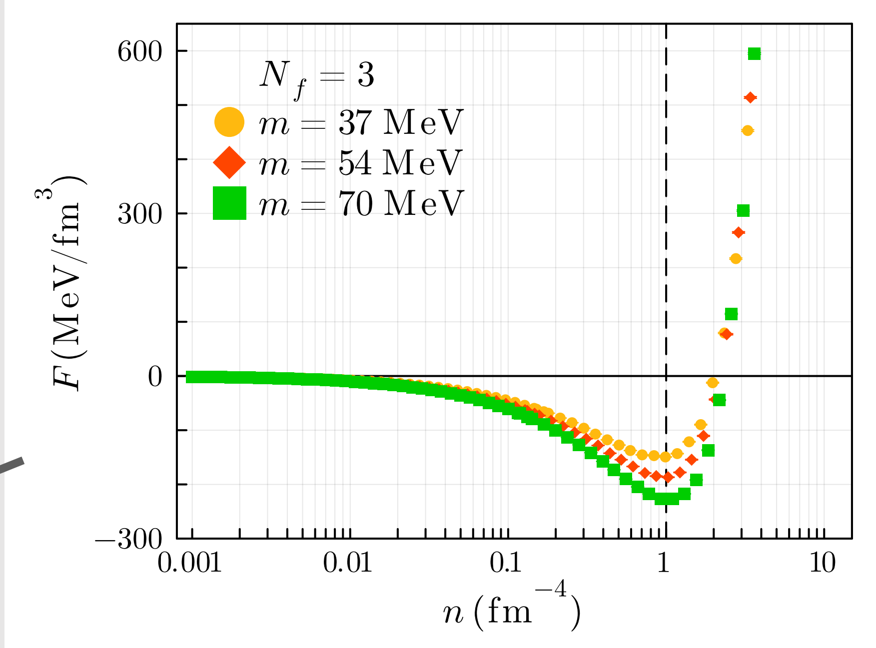
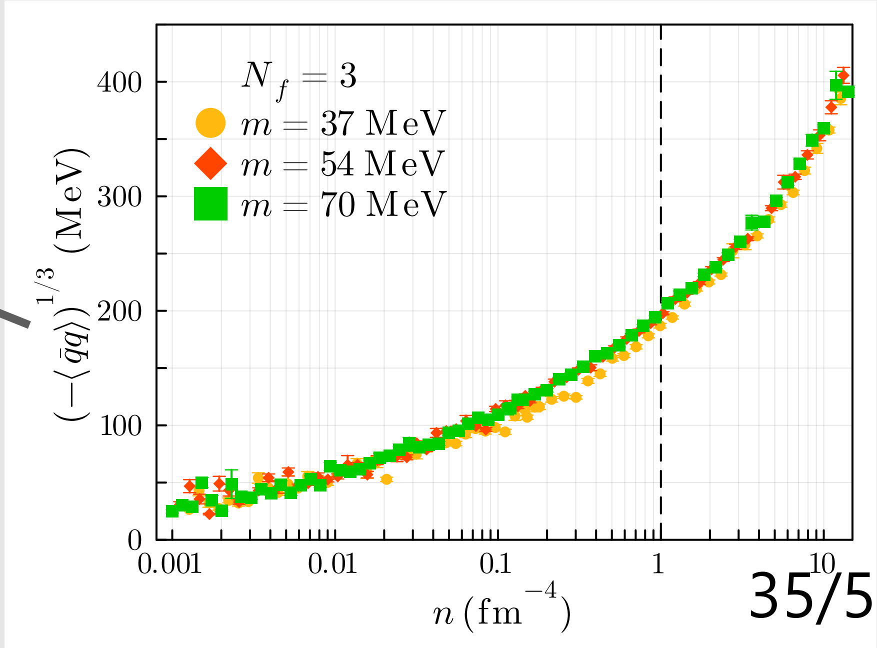
combining





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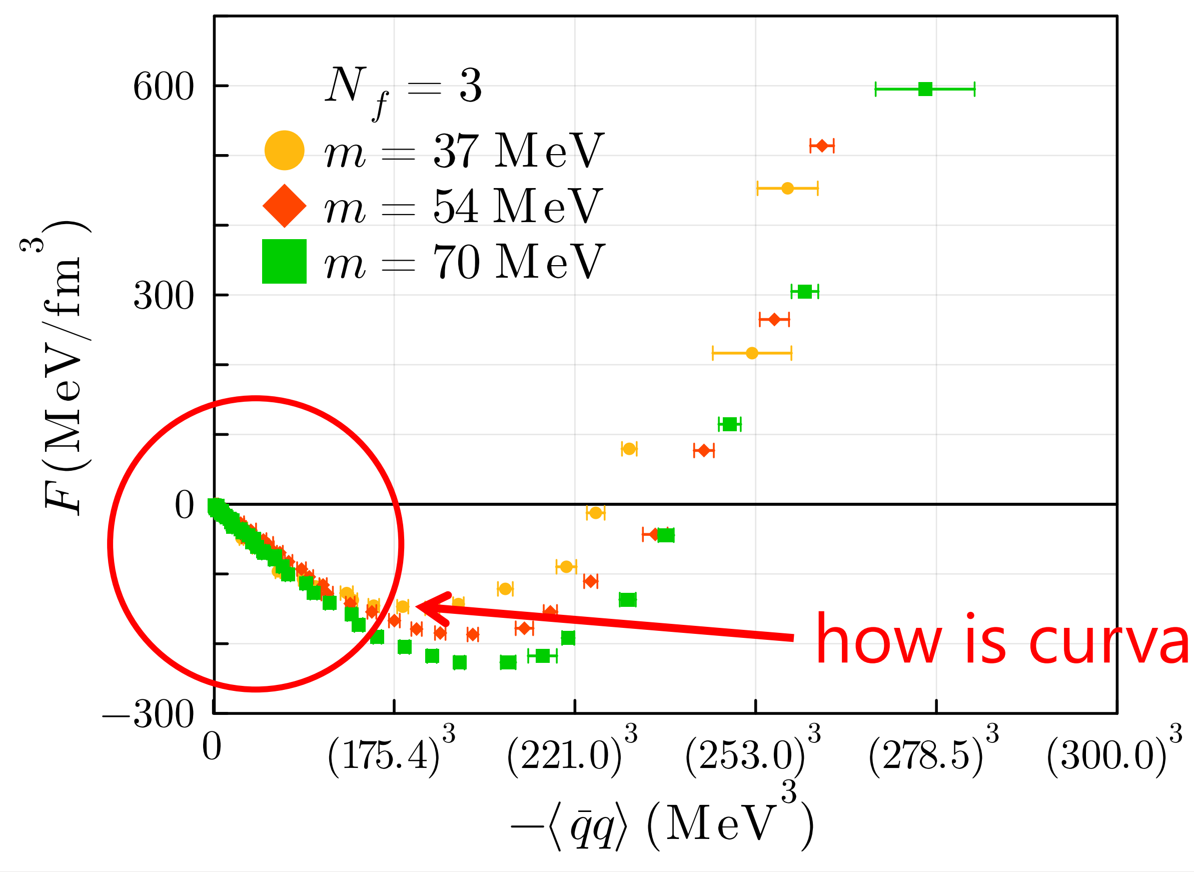
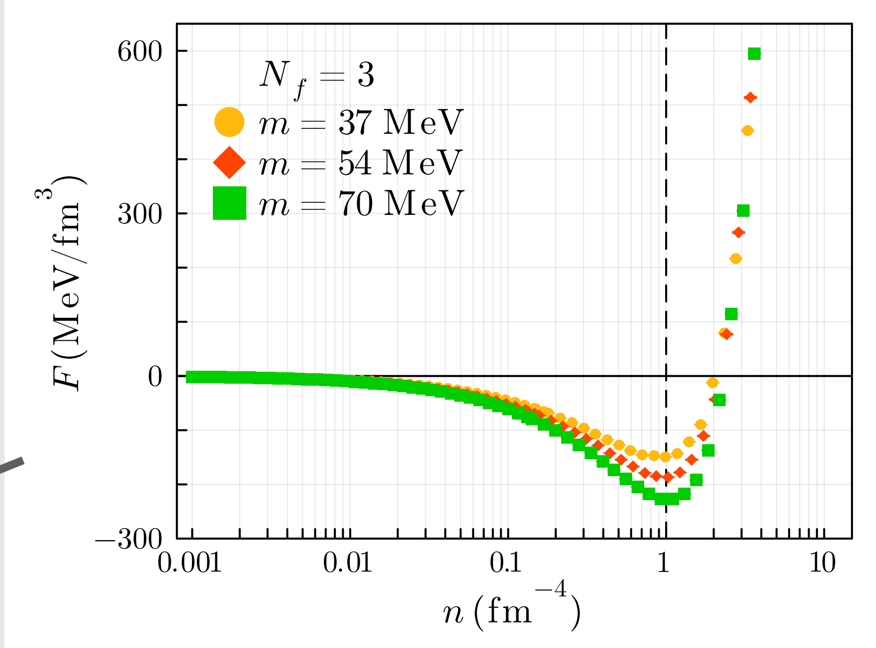
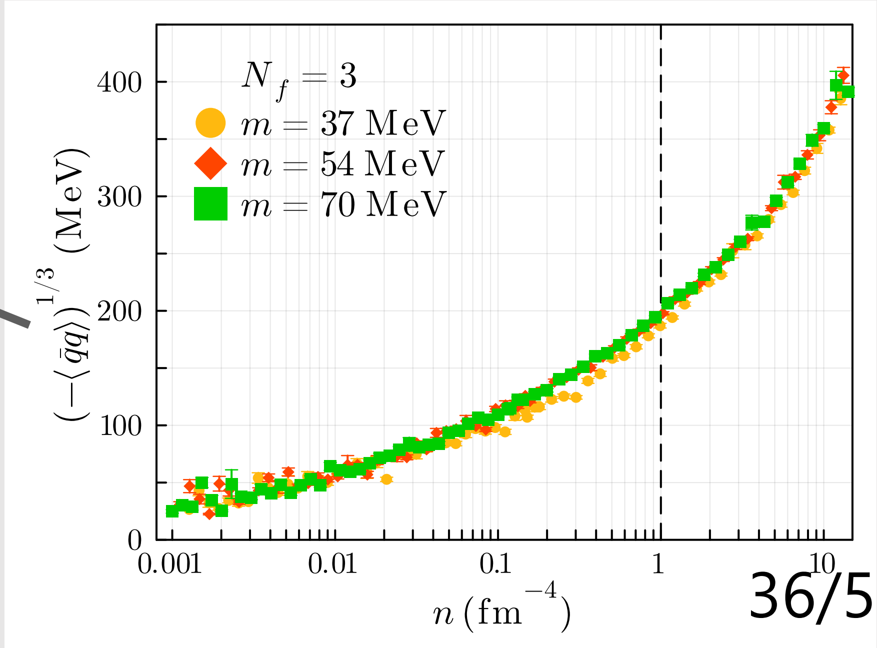


how is curvature?

[7] YS and D. Jido, arXiv:2402.05425 to be published in PRD

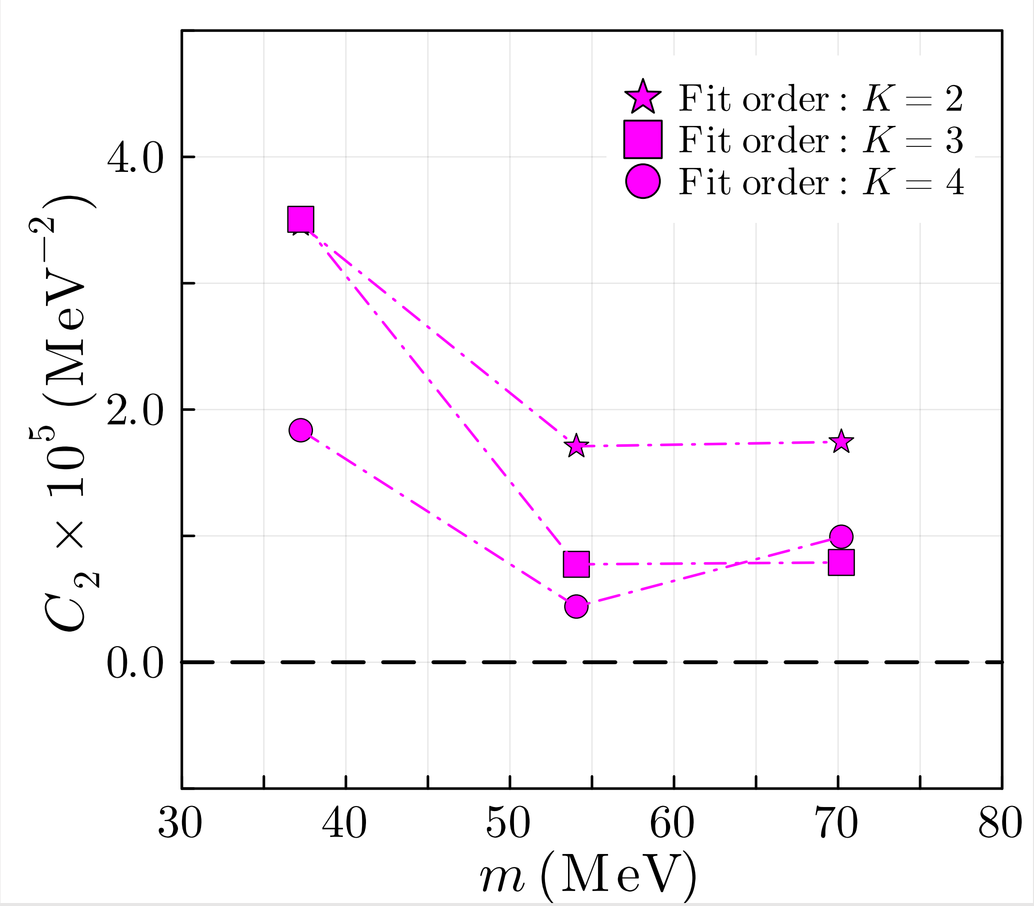
combining





# Result: Curvature [full]

positive curvature (here $C_2$) is obtained
by polynomial fitting of data
in wide quark mass ranges



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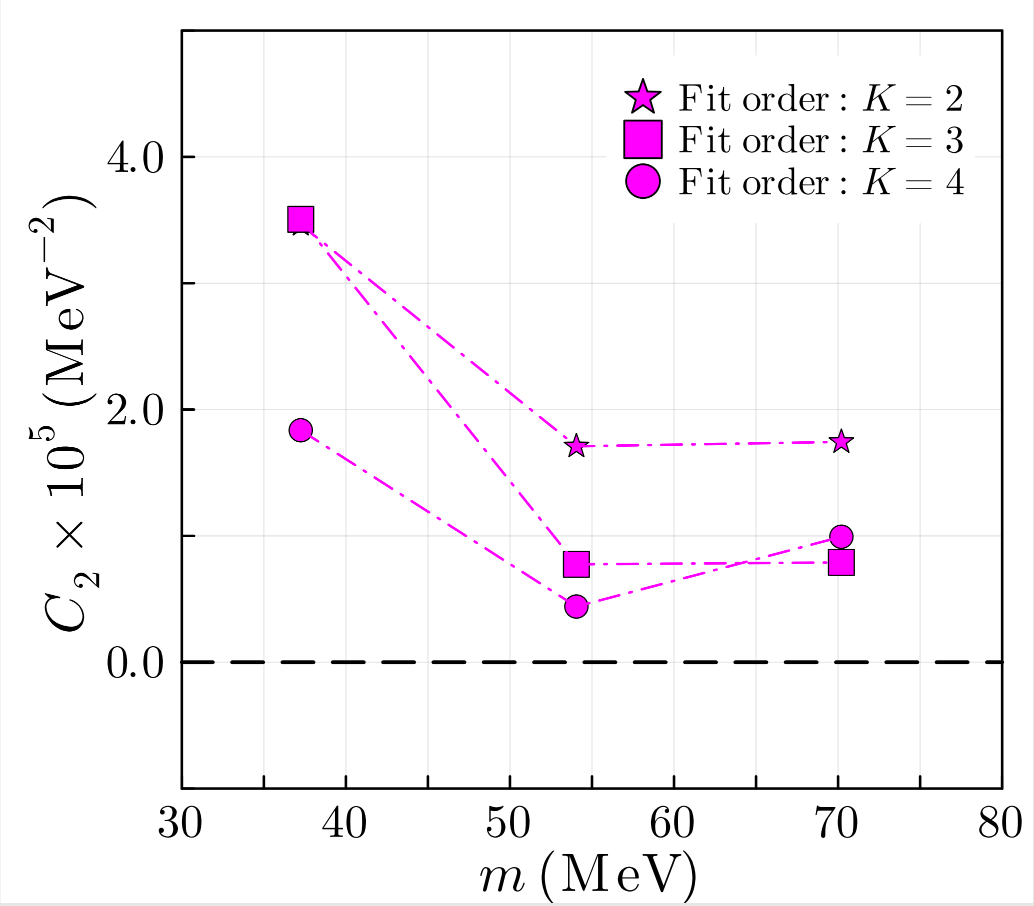

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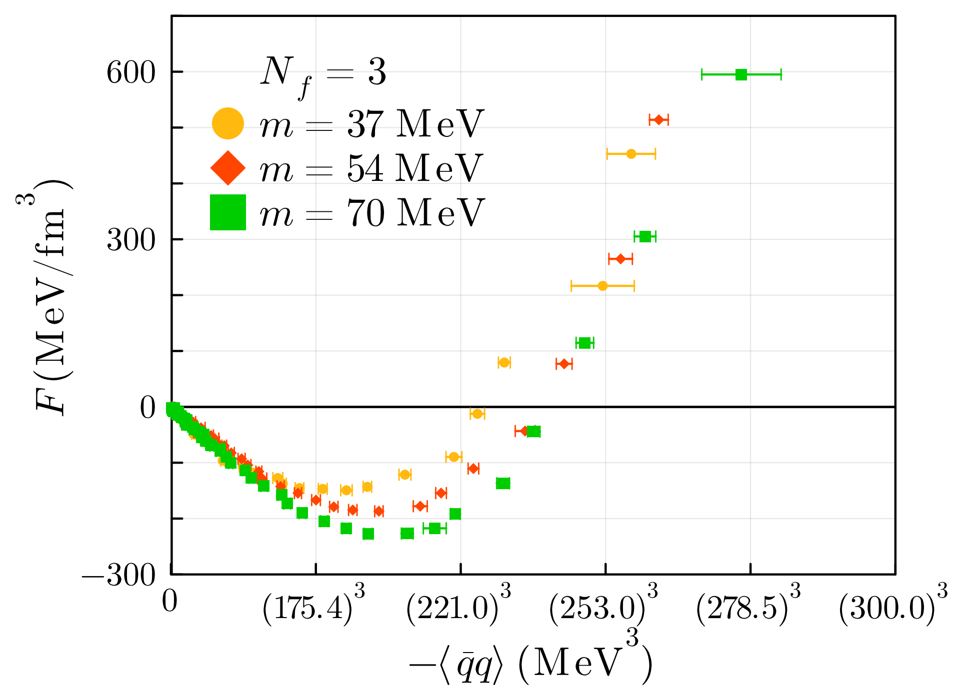

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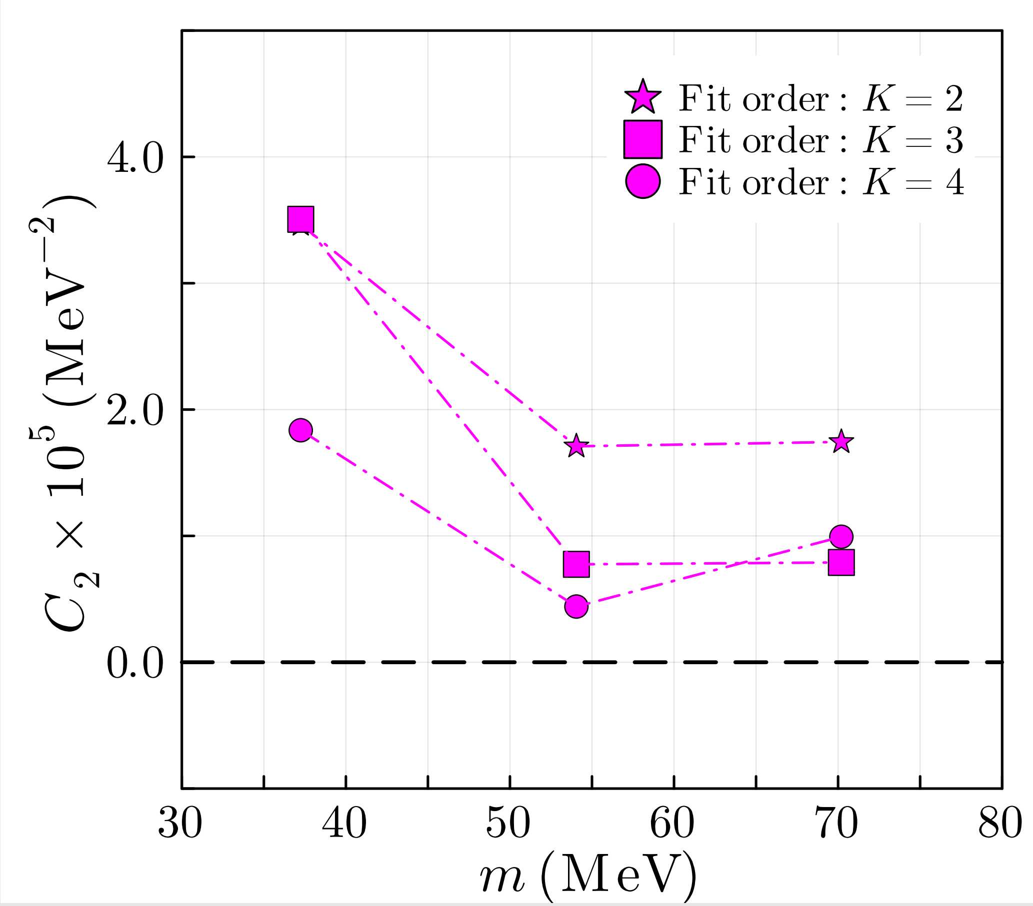

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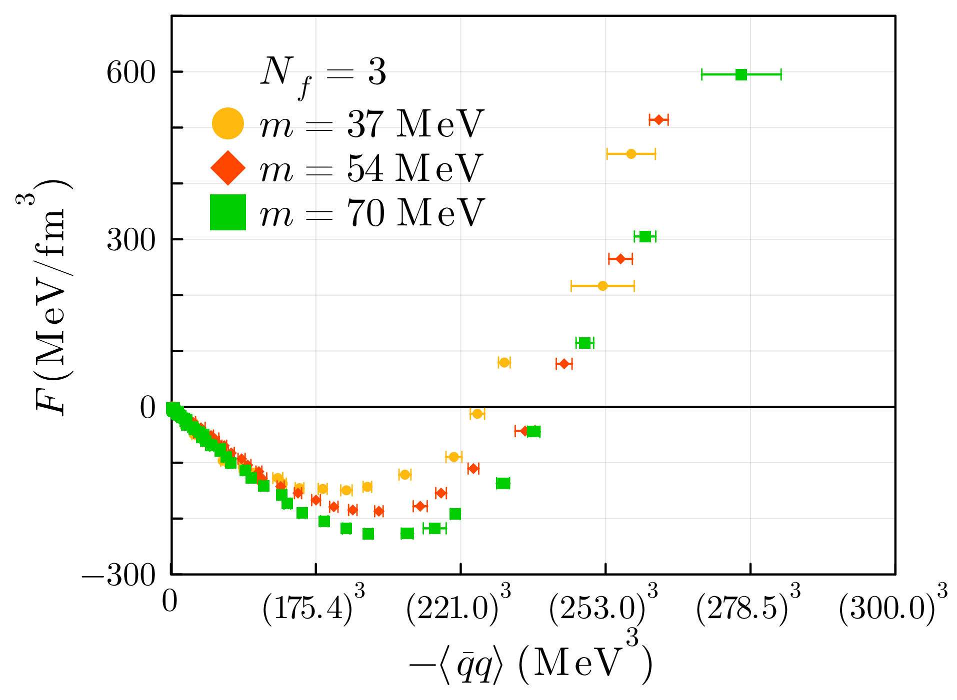

fitting

$$F(\langle\bar{q}q\rangle) = \sum_{j=0}^{K} C_j \,\langle\bar{q}q\rangle^j$$

chi-square/d.o.f.
with x&y errors:

$$\chi^2_{\mathrm{d.o.f.}} = \frac{1}{N_{\mathrm{d.o.f.}}} \sum_{i=1}^{M} \frac{(y_i - f(x_i))^2}{\sigma^2_{y_i} + \sigma^2_{x_i}[f'(x_i)]^2}$$

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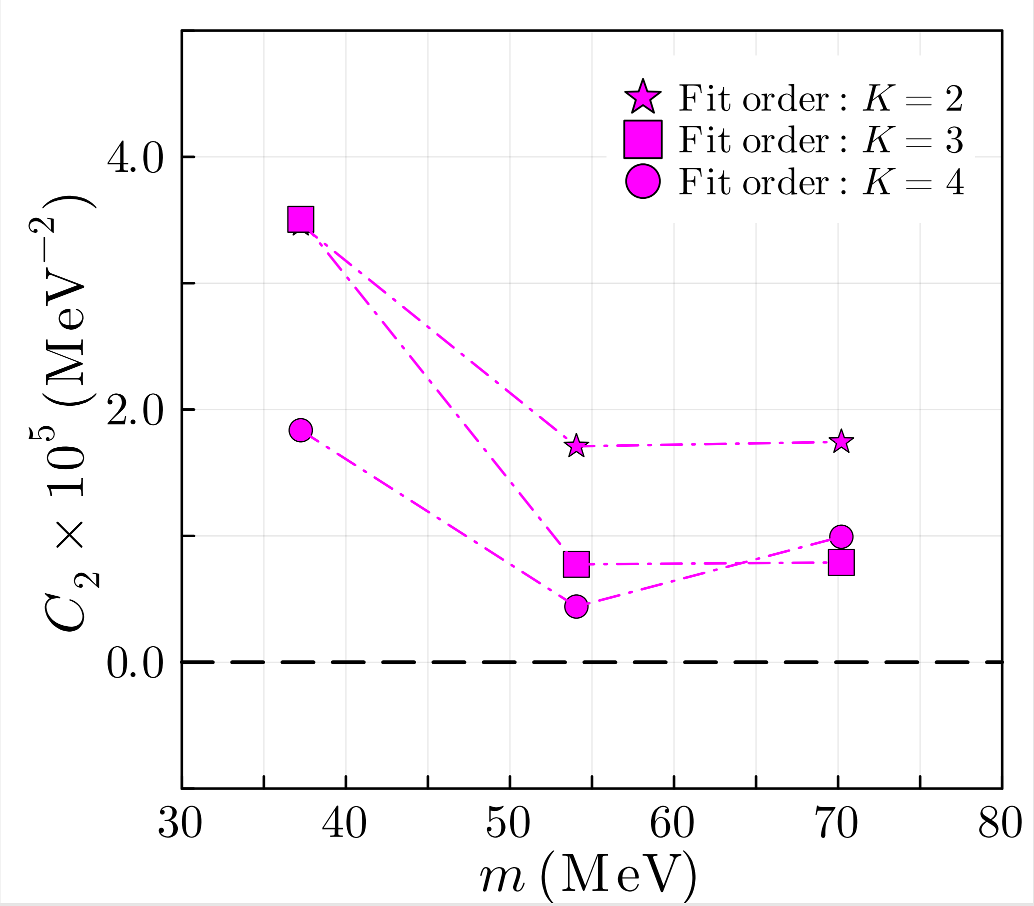

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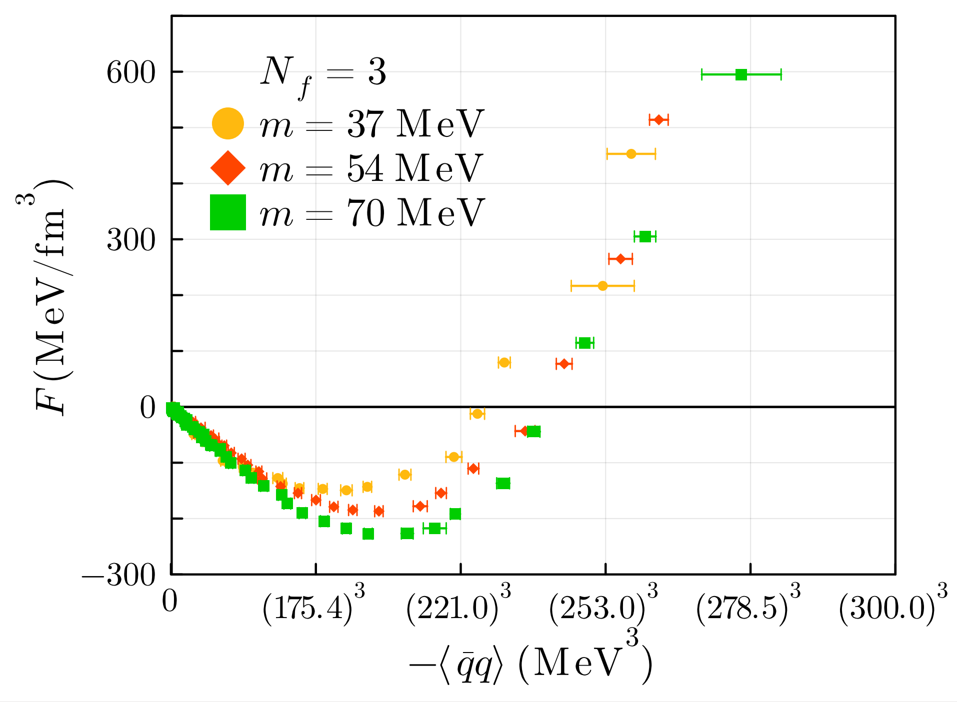

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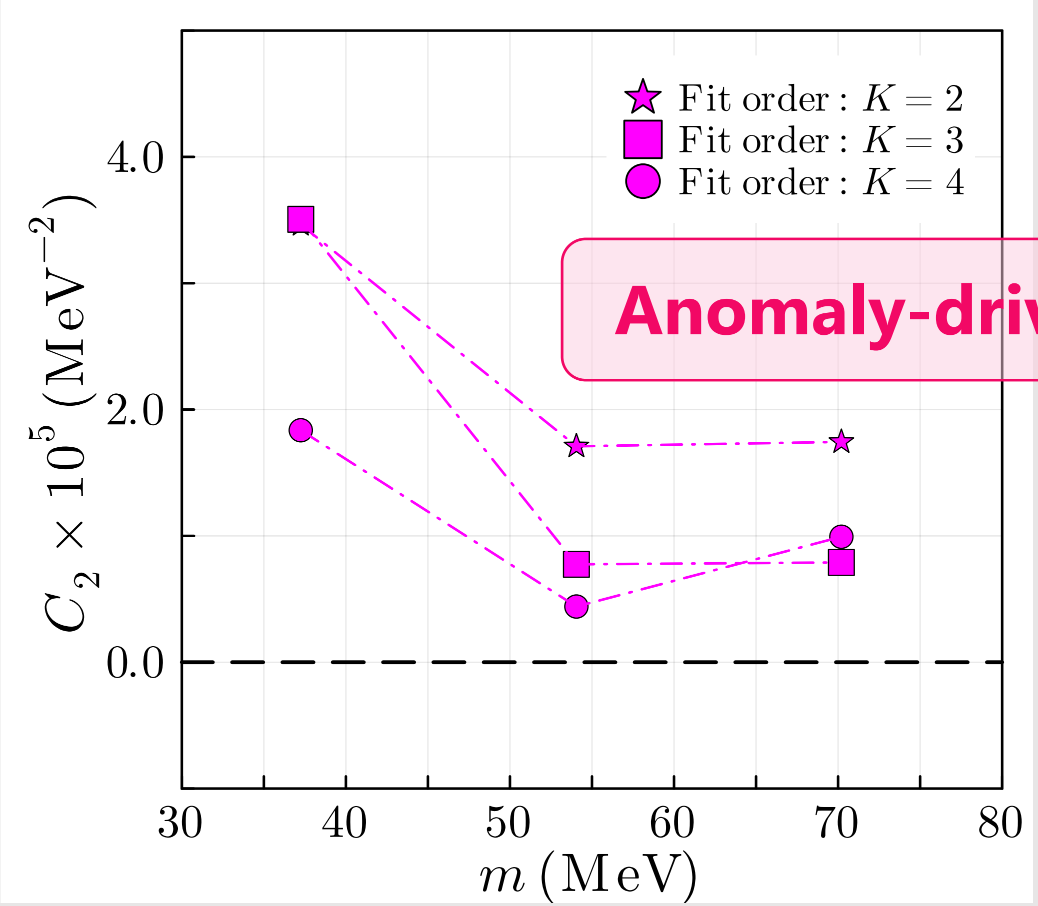
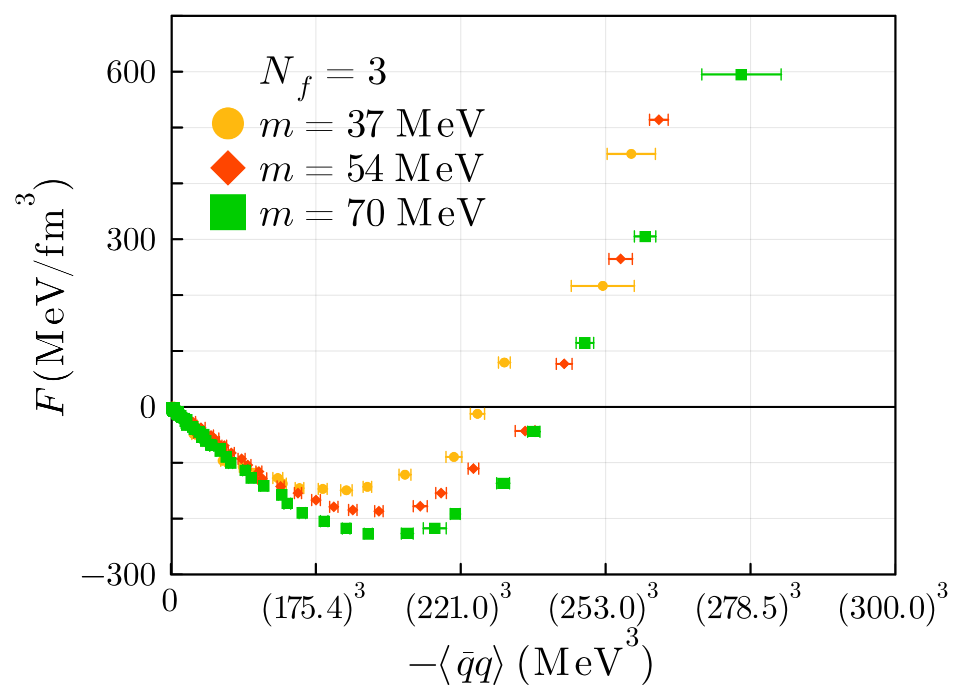


**Anomaly-driven $\chi$SB!!**

[7] YS and D. Jido, arXiv:2402.05425 to be published in PRD



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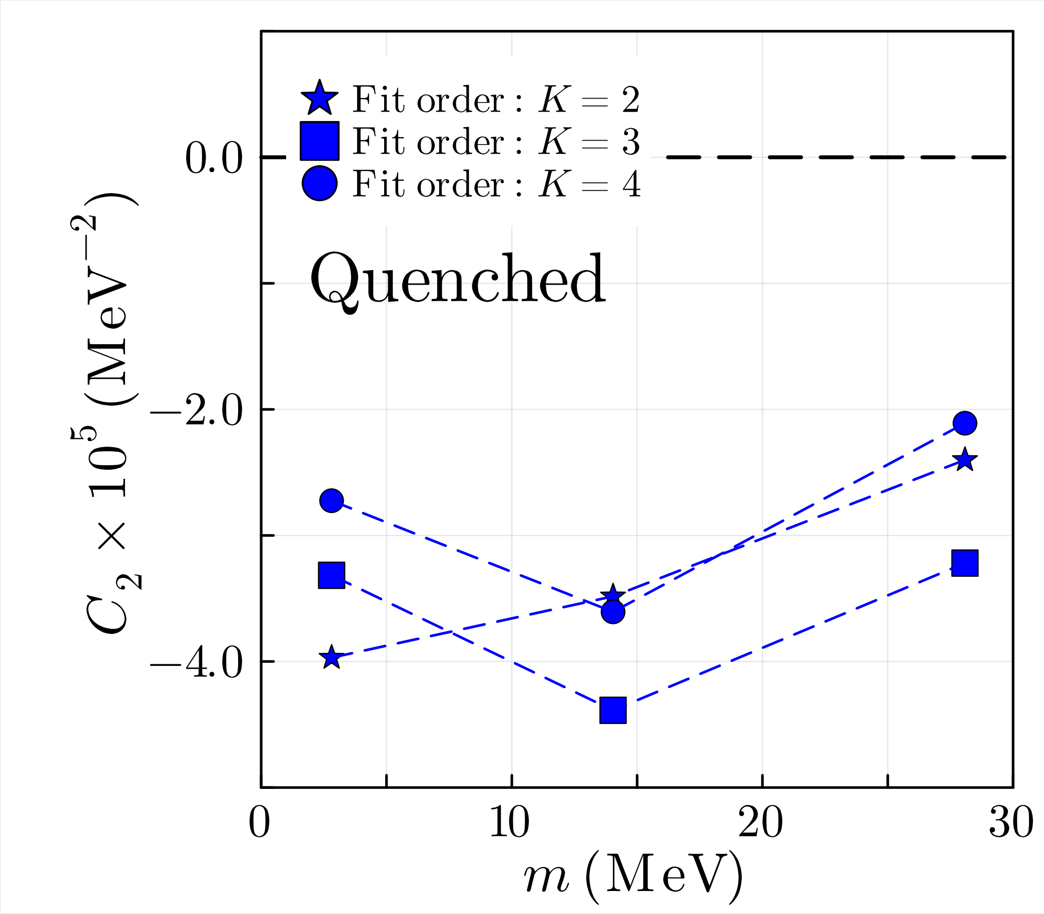
# Result: Curvature [quench]

negative curvature (here $C_2$) is obtained
by polynomial fitting of data
in wide quark mass ranges



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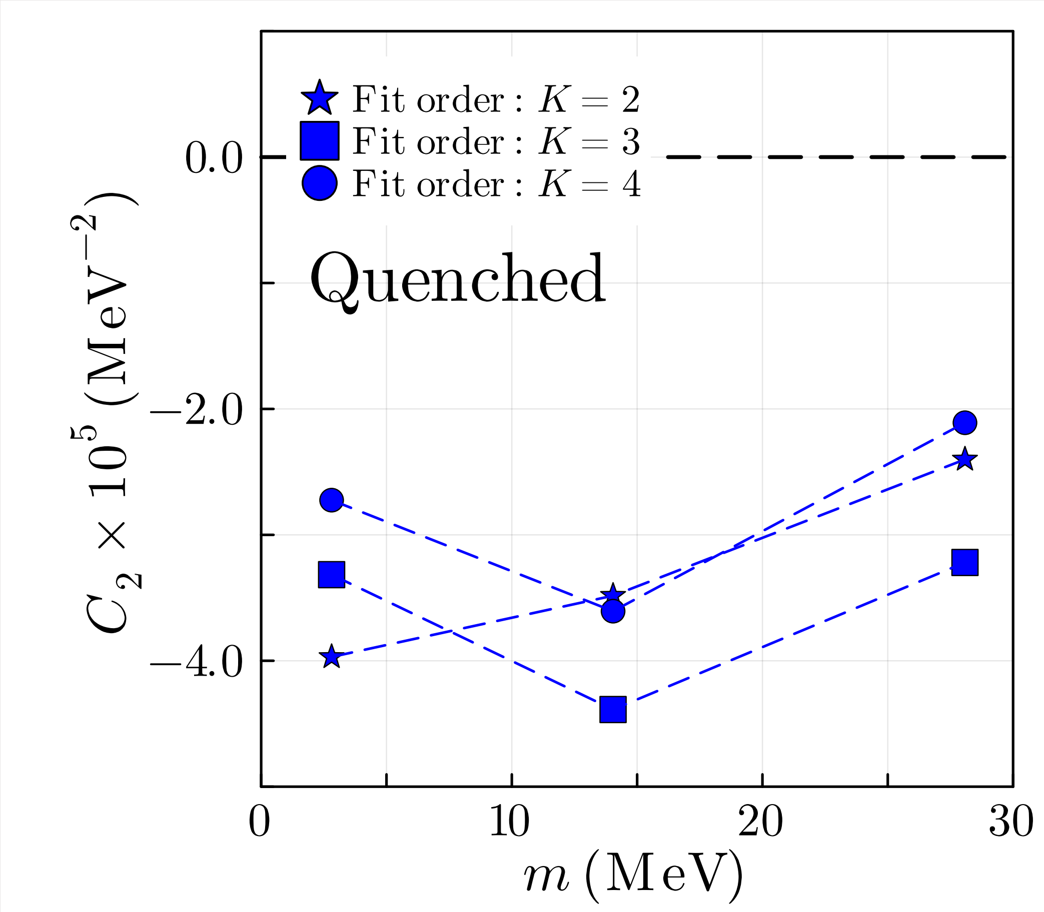

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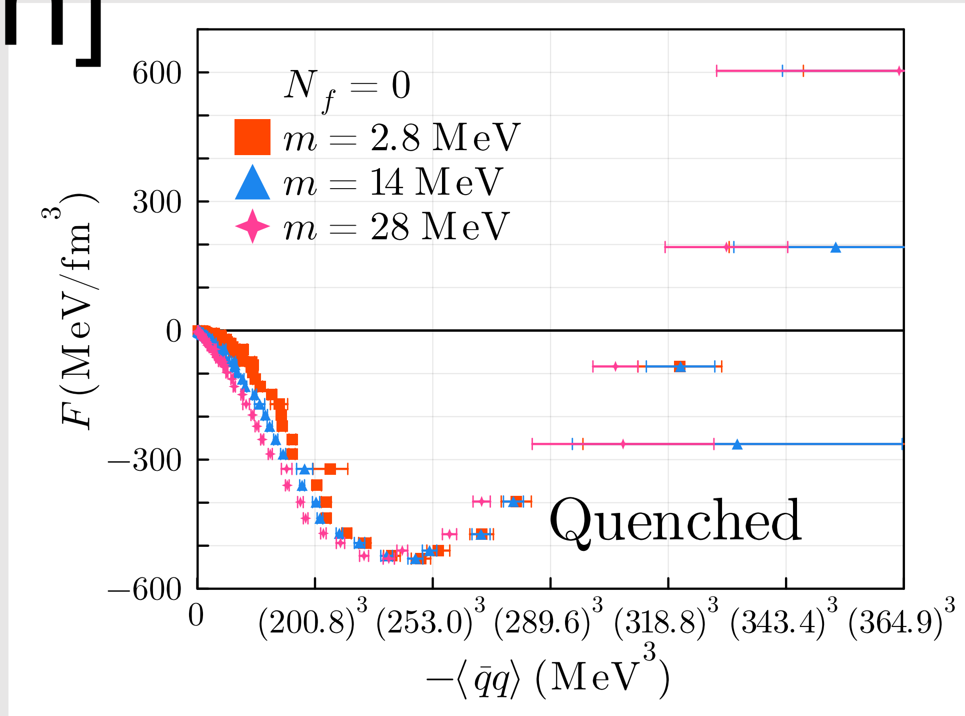

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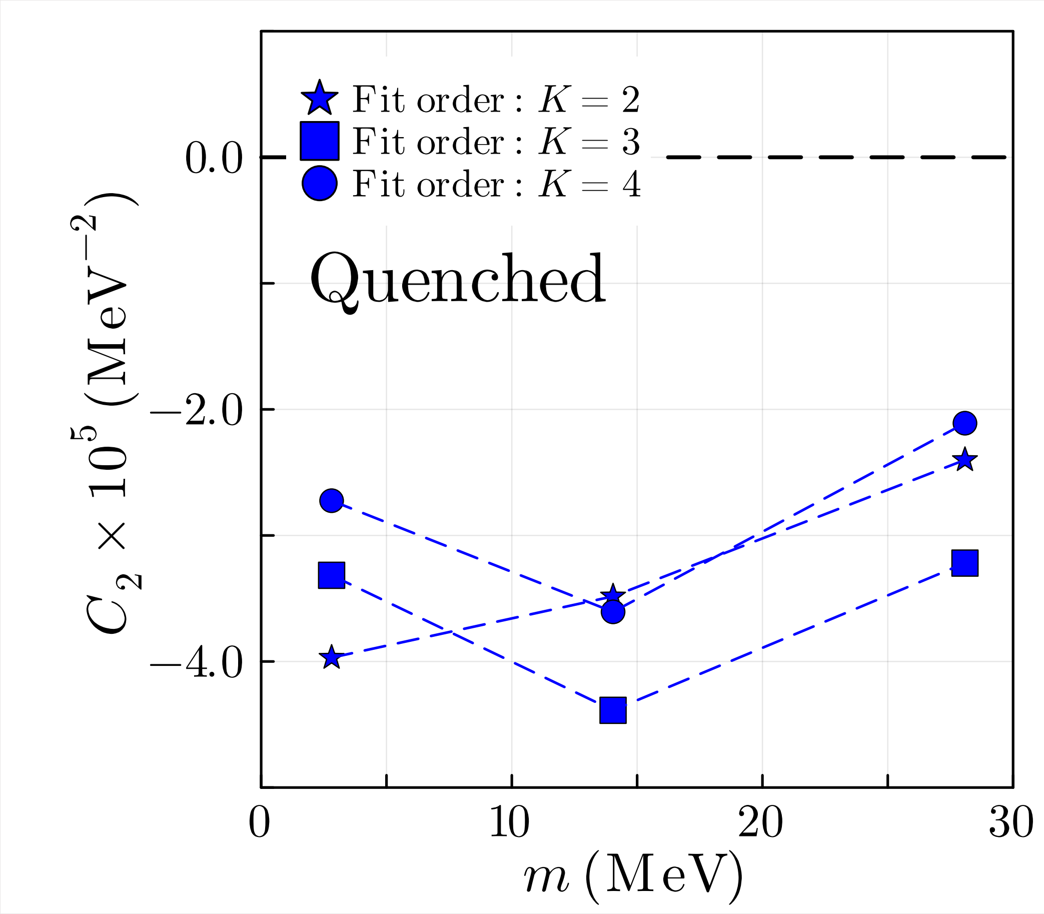

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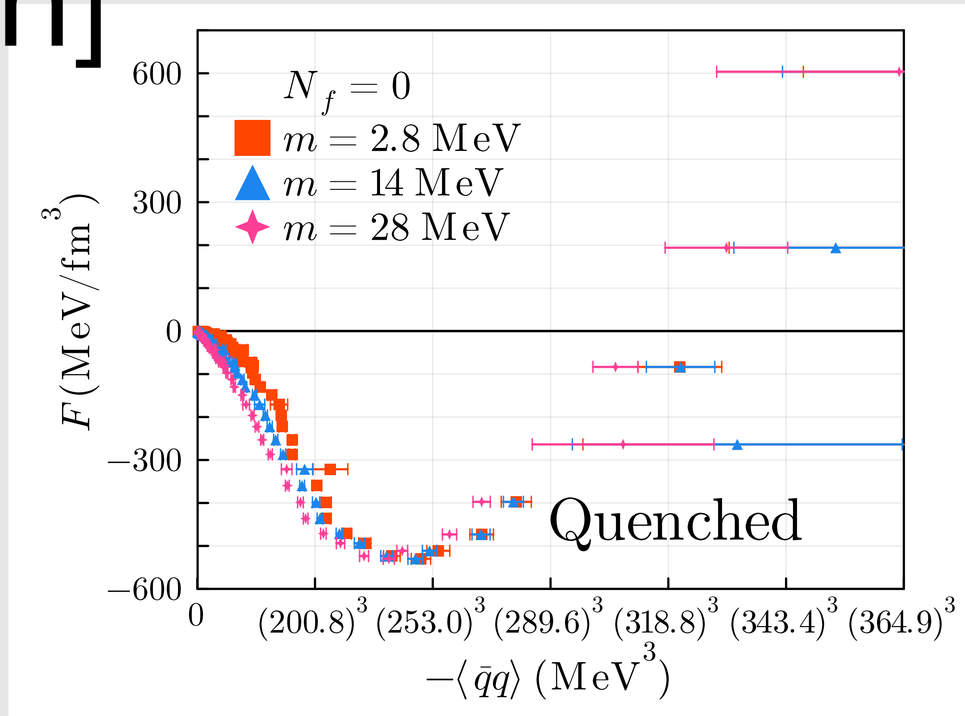

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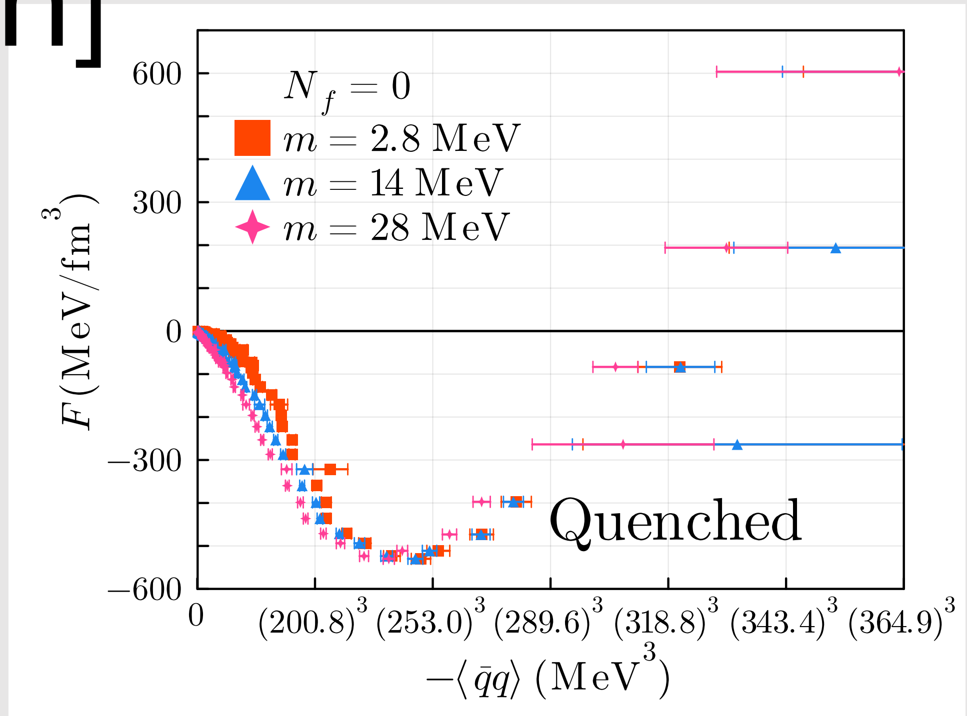

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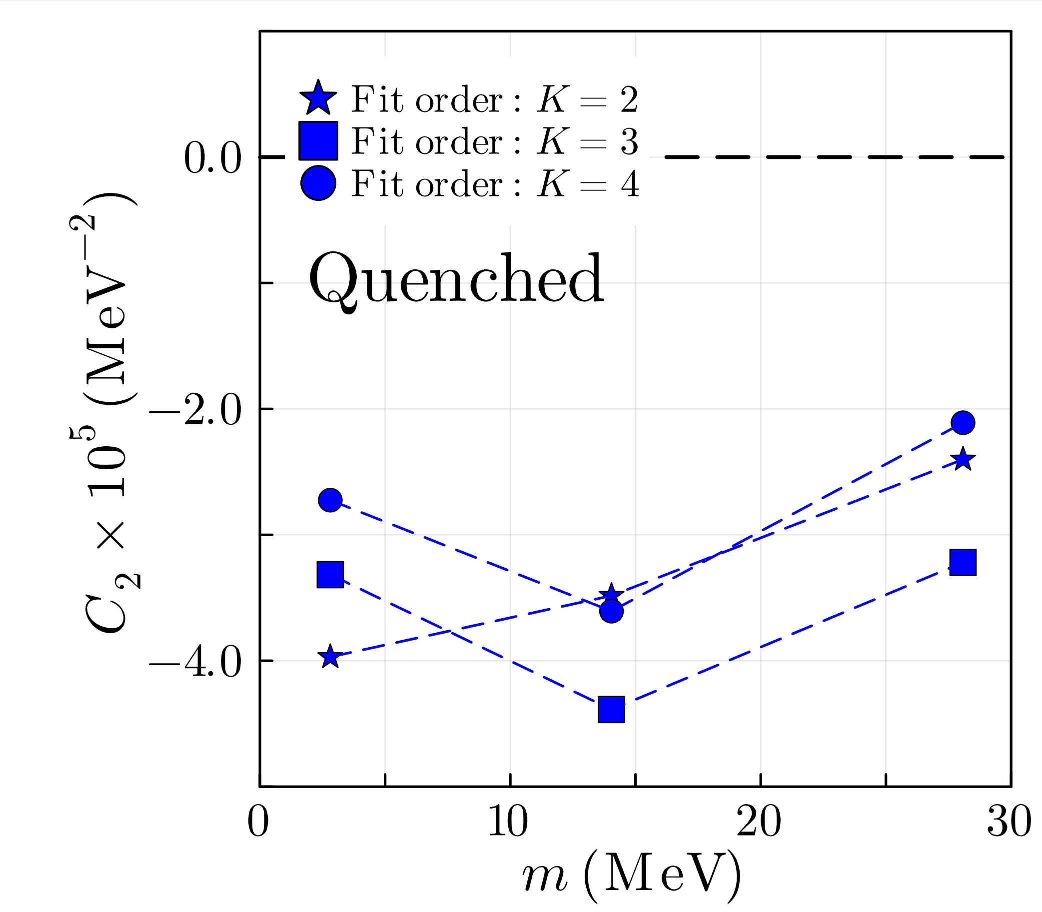

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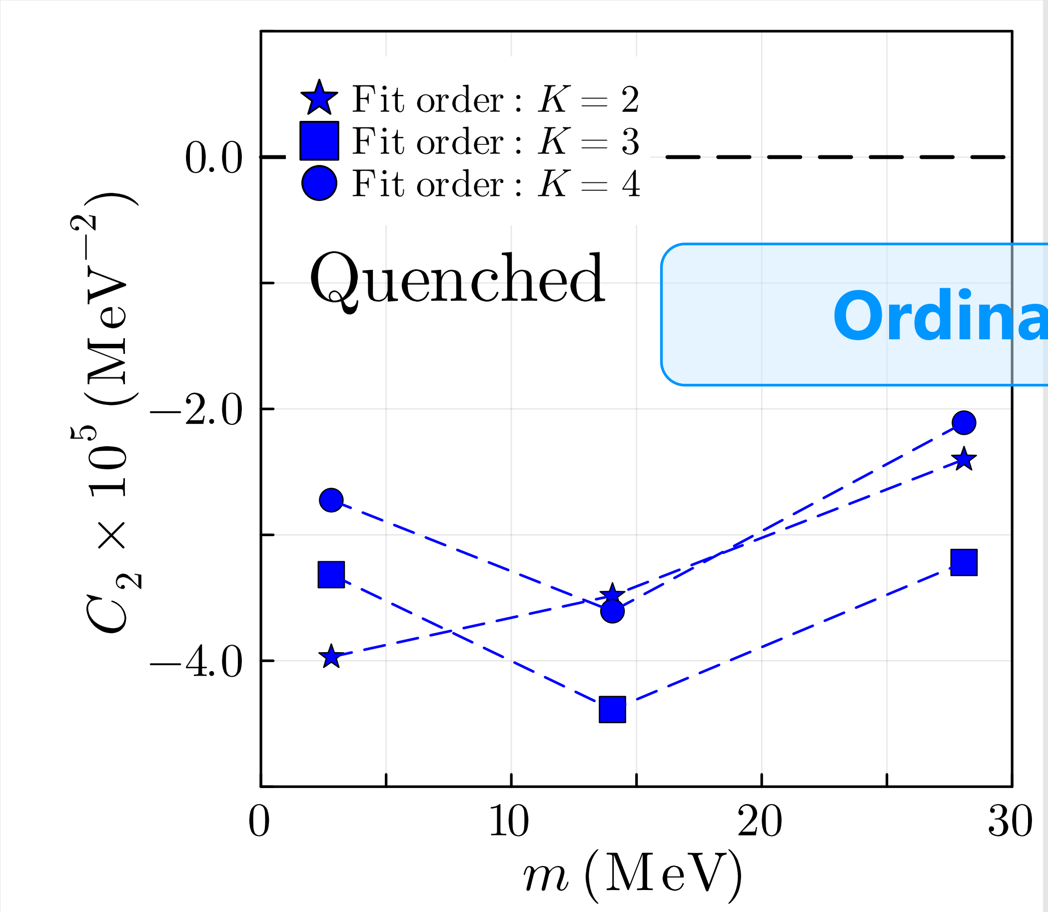
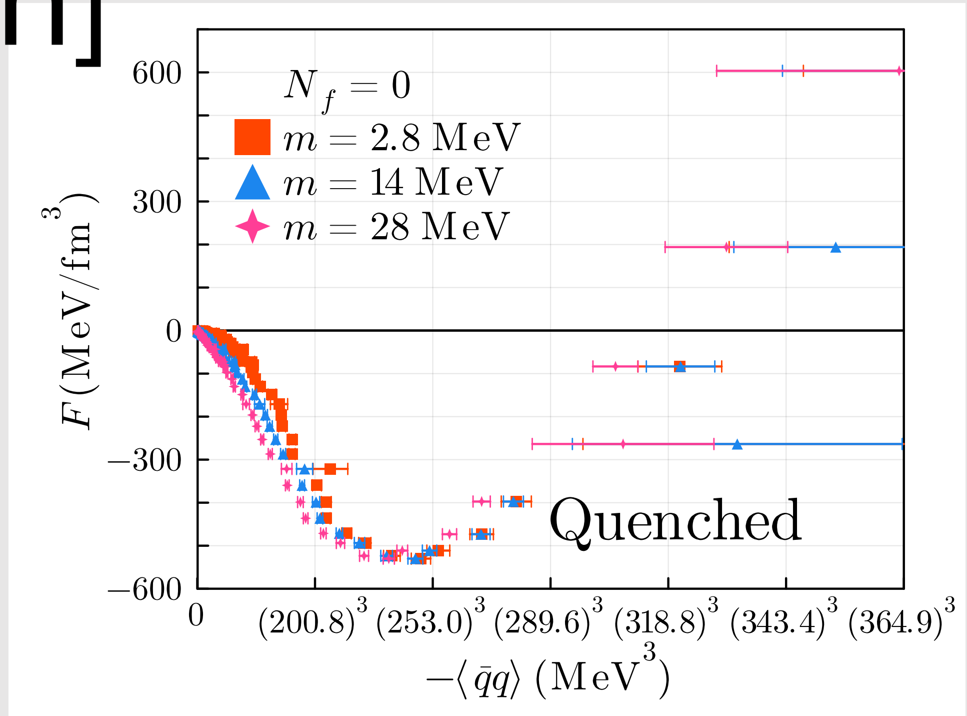


**Ordinary $\chi$SB!!**



fitting

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chi-square/d.o.f.
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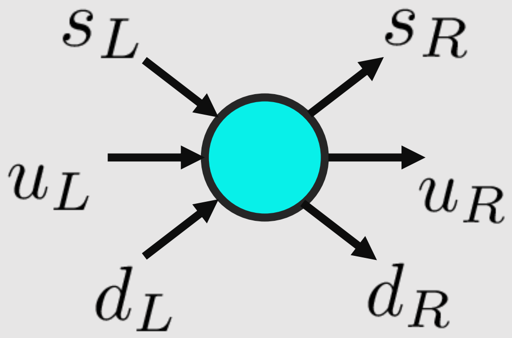
# Discussion

KMT term for three-flavor NJL model



three-flavor NJL model with U(1)_A anomaly term
includes only 6-quark interaction

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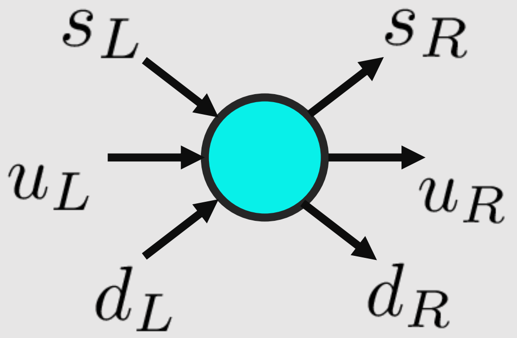

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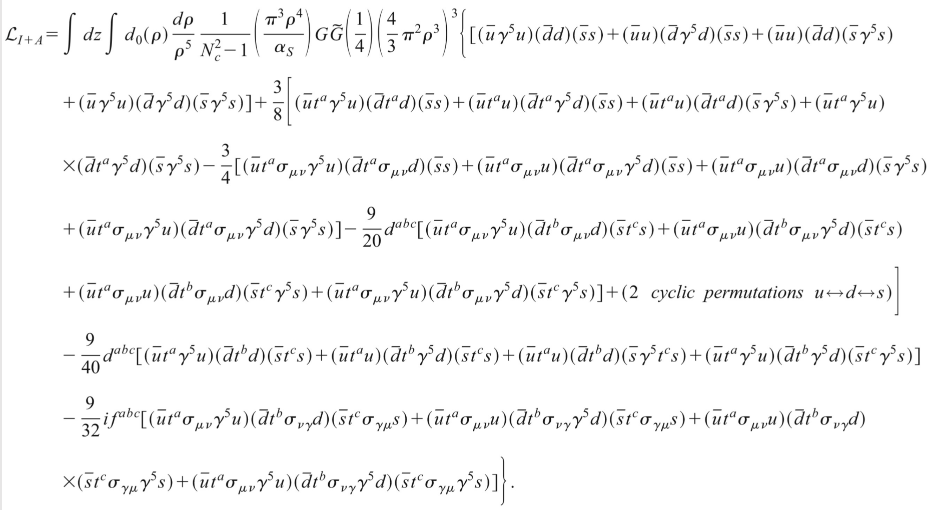
the form of the $N_f = 3$ effective 't Hooft Lagrangian

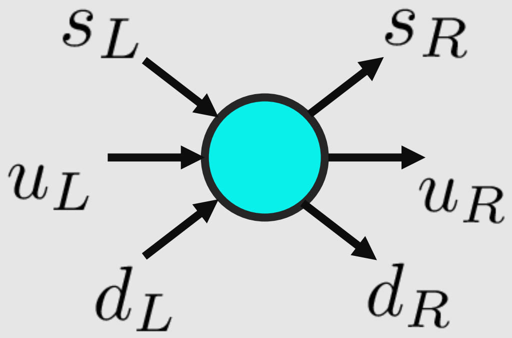


$$\begin{aligned}\mathcal{L}_{I+A} = &\int dz \int d_0(\rho)\frac{d\rho}{\rho^5}\frac{1}{N_c^2-1}\left(\frac{\pi^3\rho^4}{\alpha_S}\right)G\tilde{G}\left(\frac{1}{4}\right)\left(\frac{4}{3}\pi^2\rho^3\right)^3\Big\{[(\bar{u}\gamma^5 u)(\bar{d}d)(\bar{s}s)+(\bar{u}u)(\bar{d}\gamma^5 d)(\bar{s}s)+(\bar{u}u)(\bar{d}d)(\bar{s}\gamma^5 s)\\
&+(\bar{u}\gamma^5 u)(\bar{d}\gamma^5 d)(\bar{s}\gamma^5 s)]+\frac{3}{8}\Big[(\bar{u}t^a\gamma^5 u)(\bar{d}t^a d)(\bar{s}s)+(\bar{u}t^a u)(\bar{d}t^a\gamma^5 d)(\bar{s}s)+(\bar{u}t^a u)(\bar{d}t^a d)(\bar{s}\gamma^5 s)+(\bar{u}t^a\gamma^5 u)\\
&\times(\bar{d}t^a\gamma^5 d)(\bar{s}\gamma^5 s)-\frac{3}{4}[(\bar{u}t^a\sigma_{\mu\nu}\gamma^5 u)(\bar{d}t^a\sigma_{\mu\nu}d)(\bar{s}s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^a\sigma_{\mu\nu}\gamma^5 d)(\bar{s}s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^a\sigma_{\mu\nu}d)(\bar{s}\gamma^5 s)\\
&+(\bar{u}t^a\sigma_{\mu\nu}\gamma^5 u)(\bar{d}t^a\sigma_{\mu\nu}\gamma^5 d)(\bar{s}\gamma^5 s)]-\frac{9}{20}d^{abc}[(\bar{u}t^a\sigma_{\mu\nu}\gamma^5 u)(\bar{d}t^b\sigma_{\mu\nu}d)(\bar{s}t^c s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\mu\nu}\gamma^5 d)(\bar{s}t^c s)\\
&+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\mu\nu}d)(\bar{s}t^c\gamma^5 s)+(\bar{u}t^a\sigma_{\mu\nu}\gamma^5 u)(\bar{d}t^b\sigma_{\mu\nu}\gamma^5 d)(\bar{s}t^c\gamma^5 s)]+(2\ \ cyclic\ \ permutations\ \ u\leftrightarrow d\leftrightarrow s)\Big]\\
&-\frac{9}{40}d^{abc}[(\bar{u}t^a\gamma^5 u)(\bar{d}t^b d)(\bar{s}t^c s)+(\bar{u}t^a u)(\bar{d}t^b\gamma^5 d)(\bar{s}t^c s)+(\bar{u}t^a u)(\bar{d}t^b d)(\bar{s}\gamma^5 t^c s)+(\bar{u}t^a\gamma^5 u)(\bar{d}t^b\gamma^5 d)(\bar{s}t^c\gamma^5 s)]\\
&-\frac{9}{32}if^{abc}[(\bar{u}t^a\sigma_{\mu\nu}\gamma^5 u)(\bar{d}t^b\sigma_{\nu\gamma}d)(\bar{s}t^c\sigma_{\gamma\mu}s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\nu\gamma}\gamma^5 d)(\bar{s}t^c\sigma_{\gamma\mu}s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\nu\gamma}d)\\
&\times(\bar{s}t^c\sigma_{\gamma\mu}\gamma^5 s)+(\bar{u}t^a\sigma_{\mu\nu}\gamma^5 u)(\bar{d}t^b\sigma_{\nu\gamma}\gamma^5 d)(\bar{s}t^c\sigma_{\gamma\mu}\gamma^5 s)]\Big\}.\end{aligned}$$

on the other hand, in principle,
IILM sums all orders of 't Hooft vertex
including 6-quark interaction
(c.f. textbook by E. Shuryak in 2021)

# Discussion

KMT term for three-flavor NJL model



three-flavor NJL model with U(1)_A anomaly term
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the form of the $N_f = 3$ effective 't Hooft Lagrangian

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\mathcal{L}_{I+A} = & \int dz \int d_0(\rho)\frac{d\rho}{\rho^5}\frac{1}{N_c^2-1}\left(\frac{\pi^3\rho^4}{\alpha_S}\right)G\tilde{G}\left(\frac{1}{4}\right)\left(\frac{4}{3}\pi^2\rho^3\right)^3\Big\{[(\bar{u}\gamma^5u)(\bar{d}d)(\bar{s}s)+(\bar{u}u)(\bar{d}\gamma^5d)(\bar{s}s)+(\bar{u}u)(\bar{d}d)(\bar{s}\gamma^5s) \\
& +(\bar{u}\gamma^5u)(\bar{d}\gamma^5d)(\bar{s}\gamma^5s)]+\frac{3}{8}\Big[(\bar{u}t^a\gamma^5u)(\bar{d}t^ad)(\bar{s}s)+(\bar{u}t^au)(\bar{d}t^a\gamma^5d)(\bar{s}s)+(\bar{u}t^au)(\bar{d}t^ad)(\bar{s}\gamma^5s)+(\bar{u}t^a\gamma^5u) \\
& \times(\bar{d}t^a\gamma^5d)(\bar{s}\gamma^5s)-\frac{3}{4}[(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^a\sigma_{\mu\nu}d)(\bar{s}s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^a\sigma_{\mu\nu}\gamma^5d)(\bar{s}s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^a\sigma_{\mu\nu}d)(\bar{s}\gamma^5s) \\
& +(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^a\sigma_{\mu\nu}\gamma^5d)(\bar{s}\gamma^5s)]-\frac{9}{20}d^{abc}[(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^b\sigma_{\mu\nu}d)(\bar{s}t^cs)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\mu\nu}\gamma^5d)(\bar{s}t^cs) \\
& +(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\mu\nu}d)(\bar{s}t^c\gamma^5s)+(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^b\sigma_{\mu\nu}\gamma^5d)(\bar{s}t^c\gamma^5s)]+(2\ \ \textit{cyclic permutations}\ \ u\leftrightarrow d\leftrightarrow s)\Big] \\
& -\frac{9}{40}d^{abc}[(\bar{u}t^a\gamma^5u)(\bar{d}t^bd)(\bar{s}t^cs)+(\bar{u}t^au)(\bar{d}t^b\gamma^5d)(\bar{s}t^cs)+(\bar{u}t^au)(\bar{d}t^bd)(\bar{s}\gamma^5t^cs)+(\bar{u}t^a\gamma^5u)(\bar{d}t^b\gamma^5d)(\bar{s}t^c\gamma^5s)] \\
& -\frac{9}{32}if^{abc}[(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^b\sigma_{\nu\gamma}d)(\bar{s}t^c\sigma_{\gamma\mu}s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\nu\gamma}\gamma^5d)(\bar{s}t^c\sigma_{\gamma\mu}s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\nu\gamma}d) \\
& \times(\bar{s}t^c\sigma_{\gamma\mu}\gamma^5s)+(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^b\sigma_{\nu\gamma}\gamma^5d)(\bar{s}t^c\sigma_{\gamma\mu}\gamma^5s)]\Big\}.
\end{aligned}$$

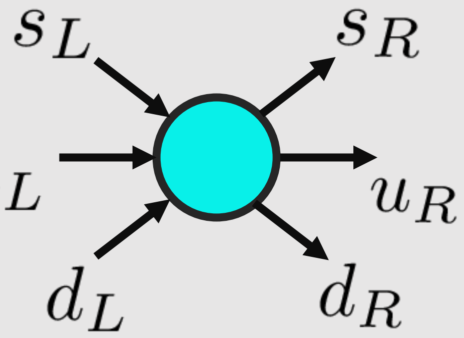
thus, it is natural to reproduce
anomaly driven $\chi$SB in IILM as in the NJL model
-> have shown by our work

$$\begin{aligned}
\mathcal{L}_{N_f=2} = & \int d\rho\, n_0(\rho)\left[\prod_f\left(m\rho-\frac{4}{3}\pi^2\rho^3\bar{q}_{f,R}q_{f,L}\right)+\frac{3}{32}\left(\frac{4}{3}\pi^2\rho^3\right)^2\right. \\
& \left.\times\left(\bar{u}_R\lambda^a u_L\bar{d}_R\lambda^a d_L-\frac{3}{4}\bar{u}_R\sigma_{\mu\nu}\lambda^a u_L\bar{d}_R\sigma_{\mu\nu}\lambda^a d_L\right)\right]+(L\leftrightarrow R)
\end{aligned}$$

the form of $N_f = 2$ case

# Discussion

KMT term for three-flavor NJL model



three-flavor NJL model with U(1)_A anomaly term
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the form of the $N_f = 3$ effective ‘t Hooft Lagrangian

$$\begin{aligned}\mathcal{L}_{I+A}=&\int dz\int d_0(\rho)\frac{d\rho}{\rho^5}\frac{1}{N_c^2-1}\left(\frac{\pi^3\rho^4}{\alpha_S}\right)G\tilde{G}\left(\frac{1}{4}\right)\left(\frac{4}{3}\pi^2\rho^3\right)^3\Big\{[(\bar{u}\gamma^5u)(\bar{d}d)(\bar{s}s)+(\bar{u}u)(\bar{d}\gamma^5d)(\bar{s}s)+(\bar{u}u)(\bar{d}d)(\bar{s}\gamma^5s)\\
&+(\bar{u}\gamma^5u)(\bar{d}\gamma^5d)(\bar{s}\gamma^5s)]+\frac{3}{8}\Big[(\bar{u}t^a\gamma^5u)(\bar{d}t^ad)(\bar{s}s)+(\bar{u}t^au)(\bar{d}t^a\gamma^5d)(\bar{s}s)+(\bar{u}t^au)(\bar{d}t^ad)(\bar{s}\gamma^5s)+(\bar{u}t^a\gamma^5u)\\
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&+(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^a\sigma_{\mu\nu}\gamma^5d)(\bar{s}\gamma^5s)]-\frac{9}{20}d^{abc}[(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^b\sigma_{\mu\nu}d)(\bar{s}t^cs)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\mu\nu}\gamma^5d)(\bar{s}t^cs)\\
&+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\mu\nu}d)(\bar{s}t^c\gamma^5s)+(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^b\sigma_{\mu\nu}\gamma^5d)(\bar{s}t^c\gamma^5s)]+(2\ \ \textit{cyclic permutations}\ \ u\leftrightarrow d\leftrightarrow s)\Big]\\
&-\frac{9}{40}d^{abc}[(\bar{u}t^a\gamma^5u)(\bar{d}t^bd)(\bar{s}t^cs)+(\bar{u}t^au)(\bar{d}t^b\gamma^5d)(\bar{s}t^cs)+(\bar{u}t^au)(\bar{d}t^bd)(\bar{s}\gamma^5t^cs)+(\bar{u}t^a\gamma^5u)(\bar{d}t^b\gamma^5d)(\bar{s}t^c\gamma^5s)]\\
&-\frac{9}{32}if^{abc}[(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^b\sigma_{\nu\gamma}d)(\bar{s}t^c\sigma_{\gamma\mu}s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\nu\gamma}\gamma^5d)(\bar{s}t^c\sigma_{\gamma\mu}s)+(\bar{u}t^a\sigma_{\mu\nu}u)(\bar{d}t^b\sigma_{\nu\gamma}d)\\
&\times(\bar{s}t^c\sigma_{\gamma\mu}\gamma^5s)+(\bar{u}t^a\sigma_{\mu\nu}\gamma^5u)(\bar{d}t^b\sigma_{\nu\gamma}\gamma^5d)(\bar{s}t^c\sigma_{\gamma\mu}\gamma^5s)]\Big\}.\end{aligned}$$

on the other hand, in principle,
IILM sums all orders of ‘t Hooft vertex
including 6-quark interaction
(c.f. textbook by E. Shuryak in 2021)

thus, it is natural to reproduce
anomaly driven $\chi$SB in IILM as in the NJL model
-> have shown by our work

what would happen in $N_f = 2$ world?
-> no KMT term, but ‘t Hooft vertex exists

$$\begin{aligned}\mathcal{L}_{N_f=2}=&\int d\rho\, n_0(\rho)\left[\prod_f\left(m\rho-\frac{4}{3}\pi^2\rho^3\bar{q}_{f,R}q_{f,L}\right)+\frac{3}{32}\left(\frac{4}{3}\pi^2\rho^3\right)^2\right.\\
&\left.\times\left(\bar{u}_R\lambda^a u_L\bar{d}_R\lambda^a d_L-\frac{3}{4}\bar{u}_R\sigma_{\mu\nu}\lambda^a u_L\bar{d}_R\sigma_{\mu\nu}\lambda^a d_L\right)\right]+(L\leftrightarrow R)\end{aligned}$$

the form of $N_f = 2$ case

# Summary & More

- U(1)_A anomaly contrib. to dynamical chiral sym. breaking is studied
- focus on sign of curvature of energy density w.r.t. the quark condensate
- in the IILM, the curvature is positive in full (unquench) simulation
- that implies anomaly driven breaking can be taken place in the IILM

# Summary & More

- U(1)_A anomaly contrib. to dynamical chiral sym. breaking is studied
- focus on sign of curvature of energy density w.r.t. the quark condensate
- in the IILM, the curvature is positive in full (unquench) simulation
- that implies anomaly driven breaking can be taken place in the IILM

- what does happen in Nf=2 world?
- how does the meson correlation function behave?

  correlation functions are calculated in many literature,

  but no studies are found in such context