$\pi\pi$-Scattering with IAM in the Finite Volume

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Overview

Introduction

Tasks to do:

- Current methods:
  - Volume-dependence
  - *LHC* is ignored at loops
  - $\tilde{T} = T$
- Generalization:
  - Volume- and Exponential-dependencies
  - Correct treatment of the *LHC*
  - $\tilde{T} = T + \Delta T$

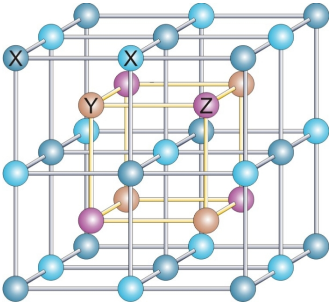


Figure: Lattice QCD

M. Luscher (DESY). Volume Dependence of the Energy Spectrum. Commun. Math. Phys. 105 (1986) 153-188.

Consequence of the Discretization

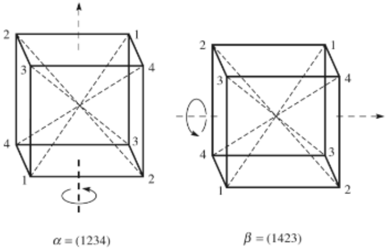


Figure: Cube rotations

- The $SO(3)$-symmetry is broken (3 generators) $\rightarrow O_h$ (24 generators)
- Lorentz-symmetry is broken $\rightarrow$ PV is not applicable.
- Volume- and Exponential-dependence on amplitudes.
- Discretization of the momentum $\rightarrow$ Shell-labeling
- $T(p,p') \rightarrow T(p.p',p',p)$

Inverse Amplitude Method (IAM)

For elastic scattering, every t partially projected, an amplitude:

$$t = t_2 + t_4 + \dots$$

must fulfill: $Im\, t^{-1} = \sigma |t|^2$ ($Im\, t_2 = 0$, $Im\, t_4 = \sigma t_2^2$). Then for a given $F(s) = \frac{t_2^2}{t}$, we have:

$$F(s) = F(0) + F'(s)s + \frac{1}{2}F''(0)s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{Im\, F(s')}{s'^3(s'-s)} + LC(F) + PC$$

The perturbative conditions imply:

$$\frac{t_2^2}{t} = F = t_2 - t_4$$

then,

$$t_{IAM}(s) = \frac{t_2^2}{t_2 - t_4 + A^{mIAM}}$$

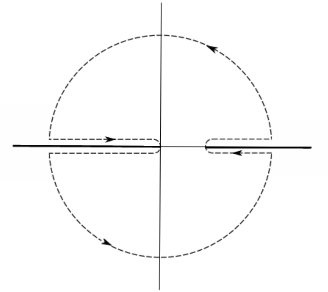


Figure: Contour of integration in the complex s-plane

ArXiv: 1205.35082v1 [hep-lat] (2012)

Inverse Amplitude Method (IAM)

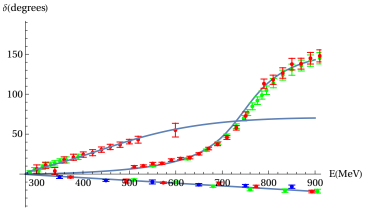


Figure: δ-shift for $I = \{0, 1, 2\}$ at different energies.

From

$$A^{mIAM} = t_4(s_2) - \frac{(s_2 - s_A)(s - s_2)}{s - s_A}(t_2'(s_2) - t_4'(s_2))$$

Phys. Rev D 59, 074001 (2007). J.A. Oller. E. Oset, and J. R. Peláez.

arXiv: 0418.2769v1 [hep-ph]. A. Gómez. J. R. Peláez, and G. Ríos.

Finite Volume $\pi\pi$-Scattering

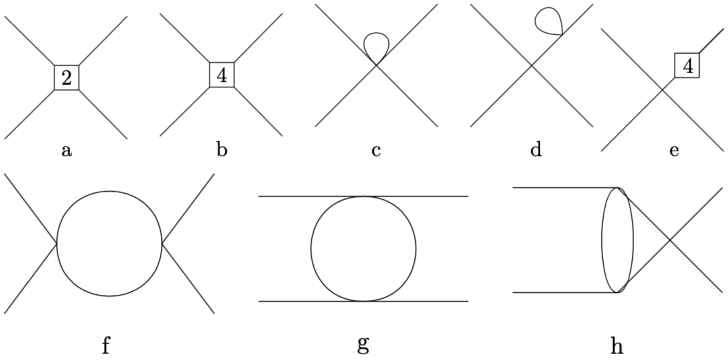


Figure: Feynman diagrams for $\pi\pi$ scattering amplitude up to fourth order in ChPT.

Finite Volume $\pi\pi$-Scattering

Amplitudes

$$\mathcal{T} = T_2 + T_4 + \underbrace{\sum_{i=\{s,t,u\}} c_i \bar{J}(i)}_{T_4^{(loop)}} = T_\infty + f_s \bar{J}(s)$$

$$\tilde{\mathcal{T}} = \underbrace{T_\infty + f_s \bar{J}(s)}_{\mathcal{T}} + \underbrace{\sum_{i=\{H,t,u,t^2,u^2\}} f_i \Delta J_i}_{\Delta T} = T_\infty + \Delta T + f_s \tilde{J}$$

$$\mathcal{T}^{IAM} = (1 - \mathcal{B}\mathcal{J})\mathcal{V}$$

Phys. Rev. D, vol 65, 054009 (2002). A. Gómez Nicola and J. R. Peláez

Phys. Rev. D 73, 074501 (2006). P.F. Bedaque, I. Sato and A. Walker-Loud

Dynamical equation

IAM

$$\mathcal{T}^{IAM} = (1 - \mathcal{B}\mathcal{J})^{1}\mathcal{V}$$

where $\mathcal{V} = T_2(T_2 - T_4 - \Delta T + A_z)^{-1}T_2$, $\mathcal{B} = (T_2 - T_4 - \Delta T + A_z)^{-1}f_s$, and
$\mathcal{J} = \sum_r^{N\max} J_r = \sum_r^{N\max} \frac{\vartheta_r}{L^3} \frac{1}{\omega_{\vec{q}}(4\omega_{\vec{q}}^2 - E^2)}$

Dynamical equation

$$\tilde{\mathcal{T}} = \mathcal{V} + \mathcal{B}\mathcal{J}\tilde{\mathcal{T}}$$

Remark

The Adler Zeros is relevant only in S-wave projections, and surprisingly, that is also the case in the finite volume, due to the expansion in the A_1^+ Irreps..

Phys. Rev. D 73, 074501 (2006). P.F. Bedaque, I. Sato and A. Walker-Loud

Integral - Differences

The loop sum-integrals can be defined as follows:

$$\tilde{H} = \sum\!\!\!\!\!\!\int \frac{1}{q^2 - m^2}$$

$$\tilde{J}_q = \sum\!\!\!\!\!\!\int \frac{1}{(q-Q)^2 - m^2}$$

$$\tilde{J}_{2q} = -\sum\!\!\!\!\!\!\int \frac{q_4^2}{q^2 - m^2} \frac{1}{(q-Q)^2 - m^2}$$

We should note that our discretization is only spatial, meaning that we divide the space into cubes, while the temporal component remains continuous.
We define the difference between the finite and infinite volume as follows:

$$\Delta f = \int \frac{dq_0}{2\pi i} \left[\frac{1}{L^3} \sum_{\vec{q} = \frac{2\pi \vec{n}}{L}} - \int \frac{d^3 q}{(2\pi)^3} \right] f(\vec{q})$$

ΔJ_H Computation

The tadpole integral contributes to modifications in m_π and f_π, resulting in the following volume corrections:

$$\Delta J_H = \int \frac{dq_0}{2\pi i}\left[\frac{1}{L^3}\sum_{\vec{q}=\frac{2\pi\vec{n}}{L}} - \int\frac{d^3q}{(2\pi)^3}\right]\frac{i}{q^2-m^2}$$

$$= \left[\frac{1}{L^3}\sum_{\vec{q}=\frac{2\pi\vec{n}}{L}} - \int\frac{d^3q}{(2\pi)^3}\right]\frac{1}{2\omega_q}$$

$$= \frac{m}{4\pi^2 L}\sum_n \frac{\vartheta_n}{\sqrt{n}} K_1(\sqrt{n}mL)$$

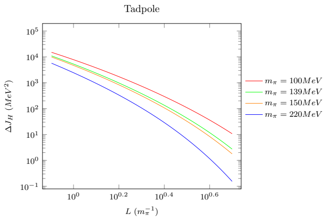
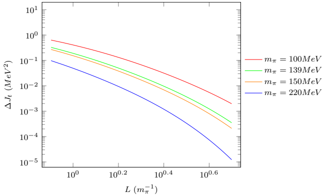
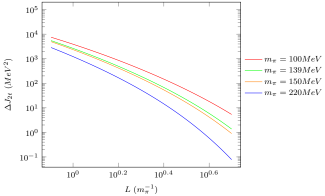


Figure: Comparison between analytic and numeric ΔJ_H

t- and u-Differences



(a) Comparison between analytic and numeric $\Delta J_{t,u}$ at BSW's approach.



(b) Comparison between analytic and numeric $\Delta J_{2t,2u}$ at BSW's approach.

The integral/sum $J_{2t,2u}$ is given by:

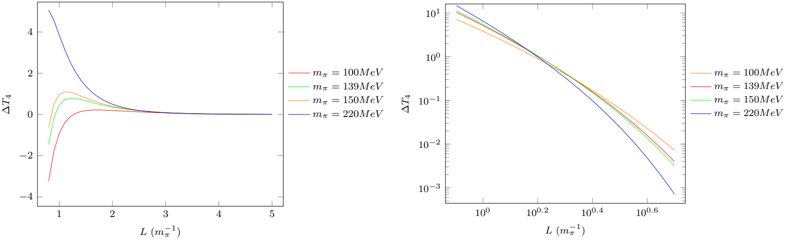
$$\Delta J_{2t,2u} = \left[\frac{1}{L^3}\sum_{\vec{q}=\frac{2\pi\vec{n}}{L}} - \int\frac{d^3q}{(2\pi)^3}\right]\frac{-1}{2\left(\omega_q + \omega_{q-Q}\right)}$$

where $\vec{Q} = \vec{T}, \vec{U}$. In the BSW limit, $\Delta J_{2u,2t} = -\frac{1}{2}\Delta J_H$. With respect to $\Delta J_{u,t}$, we have:

$$\Delta J_{t,u} = \left[\frac{1}{L^3}\sum_{\vec{q}=\frac{2\pi\vec{n}}{L}} - \int\frac{d^3q}{(2\pi)^3}\right]\frac{1}{2\omega_q\omega_{q-Q}\left(\omega_q + \omega_{q-Q}\right)}$$

Indeed, it is straightforward to observe that $\Delta J_{u,t} = -\frac{1}{2}\frac{\partial}{\partial m}\Delta J_H$.

Exponential Volume Correction



(a) $I = 0$

(b) $I = 2$

Figure: Finite exponential correction of T_4

Spectrum in a Finite Volume

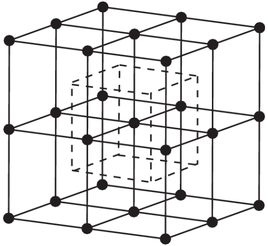


Figure: Cubic Lattice.

arXiv:1206.4141v2 [hep-lat] (2012)

arXiv: 0806.4495v2 [hep-lat] (2012)

Phys. Rev. D 97, 114508 (2018).

The symmetry group G of the cubic lattice. The irreps of the octahedral group of 24 elements (pure rotations) are:

- A_1: trivial one-dimensional rotation.
- A_2: one-dimensional representation, which assigns -1 to the conjugacy classes: $6C_4$ and $6C_2'$.
- E : two-dimensional rotations.
- T_1: three-dimensional rotations. $T_{\sigma\rho} = \cos(\omega_a)\delta_{\sigma\rho} + (1 - \cos\omega_a)n_\sigma^{(a)}n_\rho^{(a)} - \sin\omega_a\epsilon_{\sigma\rho\lambda}n_\lambda^{(a)}$
- T_2: three-dimensional rotations, which assigns -1 to the conjugacy classes: $6C_4$ and $6C_2'$.

The shells are the given surfaces where: $n_x^2 + n_y^2 + n_z^2 = \left|\frac{\vec{p}L}{2\pi}\right|^2$ and $\vec{p} = g\vec{p}_0$.

Alternative of Expansion

$$\tilde{T}(p,p') = \tilde{V}(p,p') + \sum_k B(p,k)\mathcal{J}(k)\tilde{T}(k,p') \tag{1}$$

An arbitrary function $f(\vec{p})$ can be characterized by the shell of the momentum $\vec{p}$ belongs to and the orientation. So, the expansion over the cubic lattice is given by,

$$f(\vec{p}) = f(g\vec{p}_0) = \sum_{\Gamma}\sum_{\rho\sigma} T^{\Gamma}_{\rho\sigma}(g) f^{\Gamma}_{\sigma\rho}(\vec{p}_0)$$

The quantization condition is given by,

$$\det\left(\delta_{ss'}\delta_{\delta\sigma} - J_r b^{\Gamma}_{\delta\sigma}(s,s')\right) = 0$$

Phys. Rev. D 97, 114508 (2018).

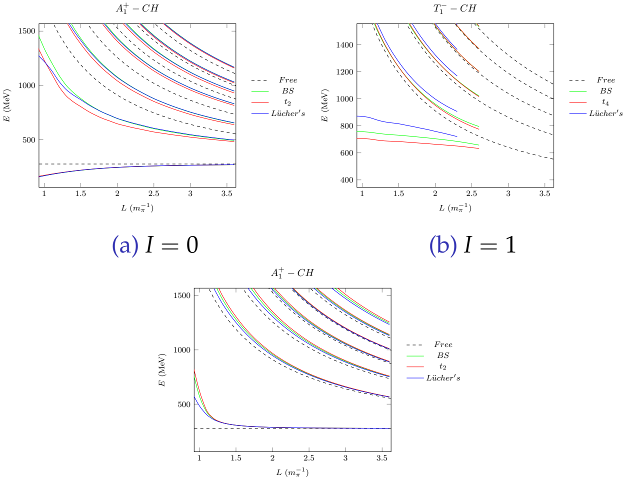
The expansion in cubic harmonic (CH) basis is given by,

$$f^s(\hat{p}_j) = \sqrt{4\pi}\sum_{\Gamma\alpha}\sum_u f^{\Gamma\alpha s}_u \chi^{\Gamma us}_u(\hat{p}_j)$$

The quantization condition is given by,

$$\det\left(\delta_{uu'}\delta_{ss'} - J_r b^{\Gamma}_{su;s'u'}\right) = 0$$

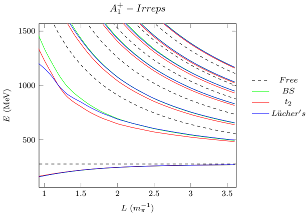
CH projection



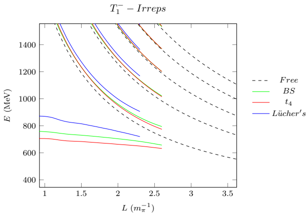
(a) $I = 0$ (b) $I = 1$ (c) $I = 2$

Figure: CH energy levels

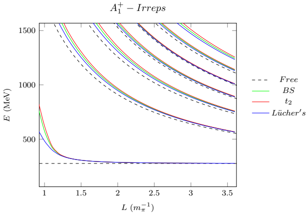
Irreps projection



(a) $I = 0$



(b) $I = 1$



(c) $I = 2$

Figure: Irreps energy levels

Conclusions and perspectives

Conclusions:

- The inclusion of the $t-$ and $u-$channel loop is useful for the correct computation of the energy levels.
- The finite volume effects are stronger in Isospin $I = 0$ and $I = 1$, over $I = 2$. Due to the presence resonances.

Perpectives:

- To expand the formalism of CH and Irreps to the moving frame case.
- To study the dependence of the spectrum with the masses


That's all Folks!