QNP 2024 - 10th International Conference on Quarks and Nuclear Physics
Barcelona (Spain) – July 8-12, 2024

Hybrid baryons in a constituent model

LORENZO CIMINO*

UNIVERSITY OF MONS

* lorenzo.cimino@umons.ac.be

Table of contents

- Ordinary vs hybrid baryons
- Quark core model
  - Quark core Hamiltonian
  - Core – gluon interaction
- Helicity states
- Results and outlooks

Ordinary vs hybrid baryons

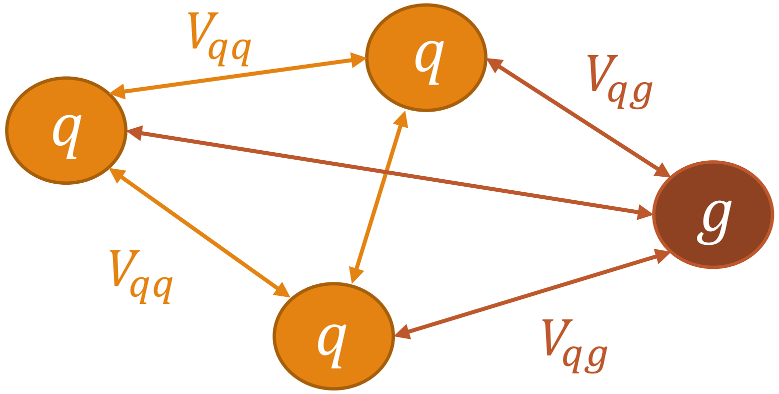
- Bound state of three quarks within a gluonic field: ordinary baryon
- Bound state of three quarks within an **excited** gluonic field: **hybrid baryon**

Ordinary vs hybrid baryons

- Bound state of three quarks within a gluonic field: ordinary baryon
- Bound state of three quarks within an **excited** gluonic field: **hybrid baryon**
  - Excited gluonic field ↔ Inclusion of a constituent gluon

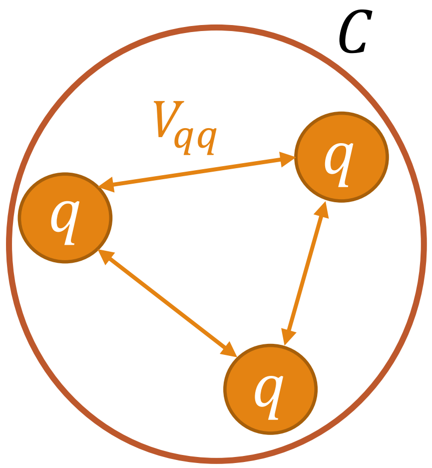
Ordinary vs hybrid baryons

- Bound state of three quarks within a gluonic field: ordinary baryon
- Bound state of three quarks within an **excited** gluonic field: **hybrid baryon**
  - ➢ Excited gluonic field $\leftrightarrow$ Inclusion of a constituent gluon
- Hybrid baryons in a constituent approach
  - ➢ QCD-inspired potentials
  - ➢ Semi-relativistic kinematics $\sqrt{p_i^2 + m_i^2}$



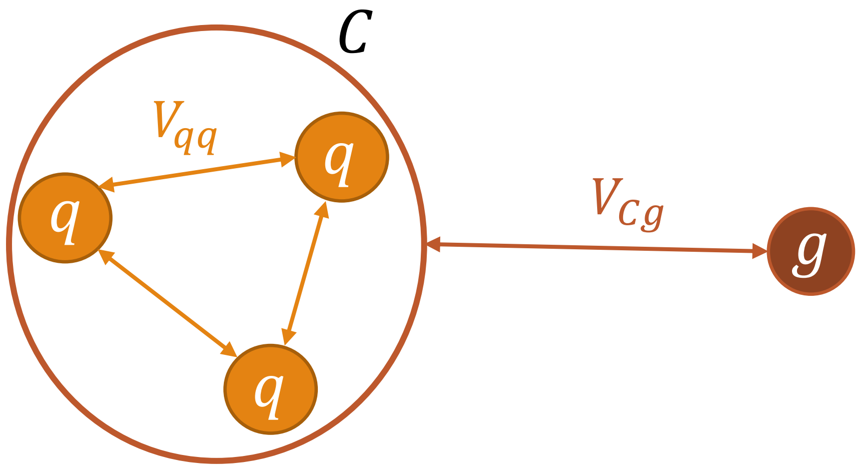
Quark core model

- Analogous to the quark-diquark model in baryons
- Interaction between quarks → quark core C
  - ➢ 3-body system



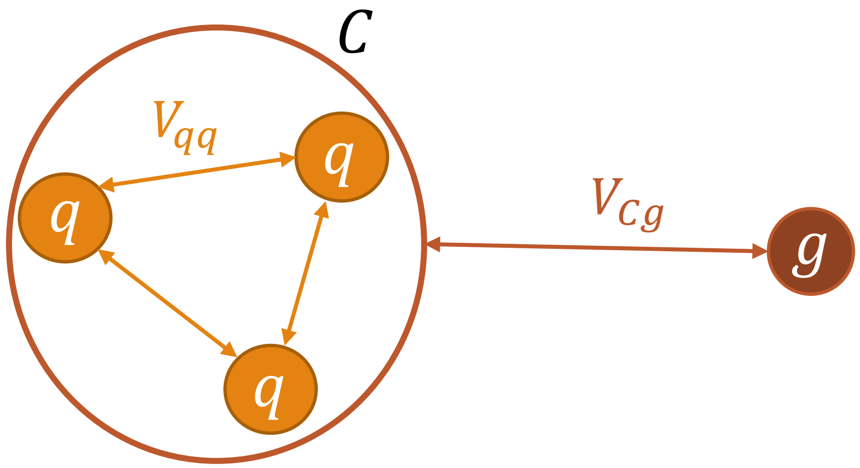
Quark core model

- Analogous to the quark-diquark model in baryons
- Interaction between quarks → quark core C
  - ➢ 3-body system
- Interaction between the core and the gluon
  - ➢ 2-body system



Quark core model

- Analogous to the quark-diquark model in baryons
- Interaction between quarks → quark core C
  - 3-body system
- Interaction between the core and the gluon
  - 2-body system
- Helicity of the gluon easier to consider



Quark core Hamiltonian

- Similar structure to the baryon (3 quarks)
  - Cornell-inspired potential

$$V_{qqq}(r) = \sum_{i<j} A\, F^2(i)\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right| + B\, F(i) \cdot F(j) \left(\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|\right)^{-\mathbf{1}}$$

Quark core Hamiltonian

- Similar structure to the baryon (3 quarks)
  - Cornell-inspired potential

$$V_{qqq}(r) = \sum_{i<j} \underbrace{A\, F^2(i) \left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|}_{\text{Confinement}} + B\, F(i) \cdot F(j) \left(\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|\right)^{\mathbf{-1}}$$

Quark core Hamiltonian

- Similar structure to the baryon (3 quarks)
  - Cornell-inspired potential

$$V_{qqq}(r) = \sum_{i<j} A\, F^2(i) \left| \boldsymbol{r}_i - \boldsymbol{r}_j \right| + B\, F(i) \cdot F(j) \left(\left| \boldsymbol{r}_i - \boldsymbol{r}_j \right| \right)^{-\mathbf{1}}$$

Confinement

Short-range interaction (OGE)

Quark core Hamiltonian

- Similar structure to the baryon (3 quarks)
  - Cornell-inspired potential

$$V_{qqq}(r) = \sum_{i<j} A\, F^2(i)\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right| + B\, F(i)\cdot F(j)\left(\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|\right)^{\mathbf{-1}}$$

  - A and B are constants [1]

[1] L.P. Fulcher, Phys. Rev. D **50**, 447 (1994)

Quark core Hamiltonian

- Similar structure to the baryon (3 quarks)
  - Cornell-inspired potential

$$V_{qqq}(r) = \sum_{i<j} A\, F^2(i) \left|\boldsymbol{r}_i - \boldsymbol{r}_j\right| + B\, F(i) \cdot F(j) \left(\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|\right)^{-\mathbf{1}}$$

  - A and B are constants [1]
  - $F^2(i)$ is the $SU(3)$ Casimir of the particle i (for a quark $F_q^2 = 4/3$)

[1] L.P. Fulcher, Phys. Rev. D **50**, 447 (1994)

Quark core Hamiltonian

- Similar structure to the baryon (3 quarks)
  - Cornell-inspired potential

$$V_{qqq}(r) = \sum_{i<j} A\, F^2(i)\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right| + B\, F(i) \cdot F(j) \left(\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|\right)^{-\mathbf{1}}$$

  - A and B are constants [1]
  - $F^2(i)$ is the $SU(3)$ Casimir of the particle i (for a quark $F_q^2 = 4/3$)
  - $F(i) \cdot F(j) = \frac{1}{2}\left[\left(F(i) + F(j)\right)^2 - F(i)^2 - F(j)^2\right]$

Casimir of the pair $q_i q_j$

[1] L.P. Fulcher, Phys. Rev. D **50**, 447 (1994)

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.
  - Quarks in hybrid baryon form a colour octet $\mathbf{8}$

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.
  - Quarks in hybrid baryon form a colour octet $\mathbf{8}$
  - Colour w.f. ϕ has a mixed symmetry

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.
  - Colour w.f. ϕ has a mixed symmetry
- Three identical quarks qqq and no excitations

$$|qqq\rangle = \psi^S \xi^S (\chi^{MS} \phi^{MA} - \chi^{MA} \phi^{MS})$$

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.
  - Colour w.f. ϕ has a mixed symmetry

- Three identical quarks qqq and no excitations

$$|qqq\rangle = \psi^S \xi^S (\chi^{MS}\phi^{MA} - \chi^{MA}\phi^{MS})$$

spatial w.f.

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.
  - Colour w.f. ϕ has a mixed symmetry

- Three identical quarks qqq and no excitations

$$|qqq\rangle = \psi^S \xi^S (\chi^{MS}\phi^{MA} - \chi^{MA}\phi^{MS})$$

spatial w.f. flavour w.f.

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.
  - Colour w.f. ϕ has a mixed symmetry
- Three identical quarks qqq and no excitations

$$|qqq\rangle = \psi^S \xi^S (\chi^{MS} \phi^{MA} - \chi^{MA} \phi^{MS})$$

spatial w.f.

flavour w.f.

Spin w.f.

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.
  - Colour w.f. ϕ has a mixed symmetry
- Three identical quarks qqq and no excitations

$$|qqq\rangle = \psi^S \xi^S (\chi^{MS}\phi^{MA} - \chi^{MA}\phi^{MS})$$

Spin w.f.

spatial w.f.

flavour w.f.

  - Spin-1/2 only

Quark core Hamiltonian

- Computation of $F(i) \cdot F(j)$ via the colour w.f.
  - Colour w.f. ϕ has a mixed symmetry

- Three identical quarks qqq and no excitations

$$|qqq\rangle = \psi^S \xi^S (\chi^{MS}\phi^{MA} - \chi^{MA}\phi^{MS})$$

Spin w.f.

spatial w.f.   flavour w.f.

  - Spin-1/2 only
  - $F(i) \cdot F(j) = -1/6$ for every pairs

Quark core Hamiltonian

- Quark core Hamiltonian

$$H_C = \sum_i \sqrt{\boldsymbol{p}_i^2 + m^2} + \sum_{i<j} \frac{4A}{3} |\boldsymbol{r}_i - \boldsymbol{r}_j| - \frac{B}{6} \left(|\boldsymbol{r}_i - \boldsymbol{r}_j|\right)^{-\mathbf{1}}$$

- Resolution of the Schrödinger equation by the expansion in oscillator bases [2]
  - Acces to the mass m_C of the core, and the « size » $1/\lambda$ of the system

[2] B. Silvestre-Brac, R. Bonnaz, C. Semay, and F. Brau, arXiv:2003.11028

Quark core Hamiltonian

- Quark core Hamiltonian

$$H_C = \sum_i \sqrt{\boldsymbol{p}_i^2 + m^2} + \sum_{i<j} \frac{4A}{3} |\boldsymbol{r}_i - \boldsymbol{r}_j| - \frac{B}{6} \left(|\boldsymbol{r}_i - \boldsymbol{r}_j| \right)^{-1}$$

- Resolution of the Schrödinger equation by the expansion in oscillator bases [2]
  - ➢ Acces to the mass m_C of the core, and the « size » $1/\lambda$ of the system

State	m_B	m_C	Δ	λ
ccc	4.822	5.119	0.297	0.825
bbb	14.401	14.894	0.493	1.261

Mass of ordinary baryon m_B, quark core m_C, difference Δ and size $1/\lambda$ [3]

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Core – gluon Hamiltonian

- Core – gluon interaction like gluon – gluon [4]

$$V_{gg}(r) = A'r - B'\frac{1}{r}$$

[4] V. Mathieu, F. Buisseret, C. Semay, Phys. Rev. D **77**, 114022 (2008)

Core – gluon Hamiltonian

- Core – gluon interaction like gluon – gluon [4]

$$V_{gg}(r) = A'r - B'\frac{1}{r}$$

- Spatial extension of the core considered

$$V_{Cg}(\boldsymbol{r}) = \int d\boldsymbol{r}\, \rho(\boldsymbol{r}) V_{gg}(|\boldsymbol{R} + \boldsymbol{r}|)$$

  - $\rho(\boldsymbol{r})$ is the quark density

$$\rho(\boldsymbol{r}) = \frac{\lambda^3}{\pi^{3/2}} e^{-\lambda^2 r^2}$$

[4] V. Mathieu, F. Buisseret, C. Semay, Phys. Rev. D **77**, 114022 (2008)

Core – gluon Hamiltonian

- Core – gluon Hamiltonian

$$H_{Cg} = \sqrt{p_C^2 + m_C^2} + \sqrt{p_g^2 + m_g^2} + A' \left[\frac{e^{-\lambda^2 r^2}}{\sqrt{\pi}\lambda} + \left(r + \frac{1}{2\lambda^2 r} \right) \operatorname{erf}(r) \right] - B' \frac{\operatorname{erf}(r)}{r}$$

- Resolution of the Schrödinger equation by the Lagrange-mesh method [5]

[5] D. Baye, Phys. Rep. **565**, 1 (2015)

Helicity formalism

- Coupling of the spin J_C of the quark core and the helicity $\lambda_g = \pm 1$ of the gluon
  - Helicity formalism of Jacob and Wick [6]
  - Well-known for 2-body systems but less for 3- or 4-body systems: **quark core model**

[6] M. Jacob, G.C. Wick, Ann. Phys. **7**, 404 (1959)

Helicity formalism

- Coupling of the spin J_C of the quark core and the helicity $\lambda_g = \pm 1$ of the gluon
  - Helicity formalism of Jacob and Wick [6]
  - Well-known for 2-body systems but less for 3- or 4-body systems: **quark core model**
- Basis of states $\left|H_{\pm}; J^P; \lambda_1 \lambda_2\right\rangle$ with well-defined J^P quantum numbers

See talk of Cyrille Chevalier
Thursday @ 15:20

[6] M. Jacob, G.C. Wick, Ann. Phys. **7**, 404 (1959)

Helicity formalism

- Coupling of the spin J_C of the quark core and the helicity $\lambda_g = \pm 1$ of the gluon
  - Helicity formalism of Jacob and Wick [6]
  - Well-known for 2-body systems but less for 3- or 4-body systems: **quark core model**
- Basis of states $\left|H_{\pm}; J^P; \lambda_1 \lambda_2\right\rangle$ with well-defined J^P quantum numbers
  - Expansion in canonical states $\left|{}^{2S+1}L_J\right\rangle$

$$|JM; \lambda_1 \lambda_2\rangle = \sum_{L,S} \left(\frac{2L+1}{2J+1}\right)^{1/2} (L\ 0\ S\ \lambda_1 - \lambda_2 \mid J \lambda_1 - \lambda_2)(s_1\ \lambda_1\ s_2\ - \lambda_2 \mid S \lambda_1 - \lambda_2) \left|{}^{2S+1}L_J\right\rangle$$

  - States of quantum mechanics

[6] M. Jacob, G.C. Wick, Ann. Phys. **7**, 404 (1959)

Helicity formalism

- Basis of states $\left|H_\pm; J^P; \lambda_1\lambda_2\right\rangle$ with well-defined J^P quantum numbers
  - Example for $J_C = 1/2$ [3]

$$\left|H_+; \left(k+\frac{1}{2}\right)^P; \frac{1}{2}1\right\rangle \text{ with } P=(-1)^k \Rightarrow \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \ldots$$

$$\left|H_-; \left(k+\frac{1}{2}\right)^P; \frac{1}{2}1\right\rangle \text{ with } P=-(-1)^k \Rightarrow \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \ldots$$

$$\left|H_+; \left(k+\frac{3}{2}\right)^P; -\frac{1}{2}1\right\rangle \text{ with } P=-(-1)^k \Rightarrow \frac{3}{2}^-, \frac{5}{2}^+, \ldots$$

$$\left|H_-; \left(k+\frac{3}{2}\right)^P; -\frac{1}{2}1\right\rangle \text{ with } P=(-1)^k \Rightarrow \frac{3}{2}^+, \frac{5}{2}^-, \ldots$$

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Helicity formalism

- Basis of states $\left|H_\pm; J^P; \lambda_1\lambda_2\right\rangle$ with well-defined J^P quantum numbers
  - Example for $J_C = 1/2$ [3]

$$\left|H_+; J^P; \frac{1}{2}1\right\rangle = \sqrt{\frac{2}{3}}\,|{}^2k+1_J\rangle + \sqrt{\frac{k}{2(2k+1)}}\,|{}^4k-1_J\rangle - \sqrt{\frac{k+2}{6(2k+1)}}\,|{}^4k+1_J\rangle\,,$$

$$\left|H_-; J^P; \frac{1}{2}1\right\rangle = \sqrt{\frac{2}{3}}\,|{}^2k_J\rangle + \sqrt{\frac{k}{6(2k+3)}}\,|{}^4k_J\rangle - \sqrt{\frac{k+2}{2(2k+3)}}\,|{}^4k+2_J\rangle\,,$$

$$\left|H_+; J^P; -\frac{1}{2}1\right\rangle = \sqrt{\frac{k+3}{2(2k+3)}}\,|{}^4k_J\rangle + \sqrt{\frac{3(k+1)}{2(2k+3)}}\,|{}^4k+2_J\rangle\,,$$

$$\left|H_-; J^P; -\frac{1}{2}1\right\rangle = \sqrt{\frac{3(k+3)}{2(2k+5)}}\,|{}^4k+1_J\rangle + \sqrt{\frac{k+1}{2(2k+5)}}\,|{}^4k+3_J\rangle\,.$$

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Helicity formalism

- Mixing of certain states through $\boldsymbol{L}^2$ operator
  - Present in the kinetic term

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Helicity formalism

- Mixing of certain states through $\boldsymbol{L}^2$ operator
  - Present in the kinetic term
  - Example for $J^P = \frac{3}{2}^+$ [3]

$$\langle \boldsymbol{L}^2 \rangle = \begin{pmatrix} 5 & -\sqrt{3} \\ -\sqrt{3} & 3 \end{pmatrix}$$

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Helicity formalism

- Mixing of certain states through $\boldsymbol{L}^2$ operator
  - Present in the kinetic term
  - Example for $J^P = \frac{3}{2}^+$ [3]

$$\langle \boldsymbol{L}^2 \rangle = \begin{pmatrix} 5 & -\sqrt{3} \\ -\sqrt{3} & 3 \end{pmatrix}$$

- Diagonalisation of $\boldsymbol{L}^2$ or adaptation of the Lagrange-mesh method

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Helicity formalism

- Mixing of certain states through $\boldsymbol{L}^2$ operator
  - Present in the kinetic term
  - Example for $J^P = \frac{3}{2}^+$ [3]

$$\langle \boldsymbol{L}^2 \rangle = \begin{pmatrix} 5 & -\sqrt{3} \\ -\sqrt{3} & 3 \end{pmatrix}$$

- Diagonalisation of $\boldsymbol{L}^2$ or adaptation of the Lagrange-mesh method

J^P	n_r	l_{eff}	$cccg$	$bbbg$
$1/2^\pm$	0	1	1.842	1.784
$3/2^\pm$	0	1	1.842	1.784
$3/2^\pm$	0	2	2.350	2.336
$1/2^\pm$	1	1	2.552	2.469
$3/2^\pm$	1	1	2.552	2.469
$3/2^\pm$	1	2	2.938	2.880

Mass gap between hybrid $qqqg$ and ordinary qqq baryon [3]

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Conclusion and outlooks

- Spectrum of **heavy** hybrid baryons computed
  - Quark core model
  - Helicity of gluon considered

J^P	n_r	l_{eff}	$cccg$	$bbbg$
$1/2^\pm$	0	1	1.842	1.784
$3/2^\pm$	0	1	1.842	1.784
$3/2^\pm$	0	2	2.350	2.336
$1/2^\pm$	1	1	2.552	2.469
$3/2^\pm$	1	1	2.552	2.469
$3/2^\pm$	1	2	2.938	2.880

Mass gap between hybrid $qqqg$ and ordinary qqq baryon [3]

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Conclusion and outlooks

- Spectrum of **heavy** hybrid baryons computed
  - Quark core model
  - Helicity of gluon considered
- Extension to **light** quarks
  - Current experiment at JLab
  - Universal potential model
  - Quark core favoured by heavy quarks

J^P	n_r	l_{eff}	$cccg$	$bbbg$
$1/2^\pm$	0	1	1.842	1.784
$3/2^\pm$	0	1	1.842	1.784
$3/2^\pm$	0	2	2.350	2.336
$1/2^\pm$	1	1	2.552	2.469
$3/2^\pm$	1	1	2.552	2.469
$3/2^\pm$	1	2	2.938	2.880

Mass gap between hybrid $qqqg$ and ordinary qqq baryon [3]

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Conclusion and outlooks

- Spectrum of **heavy** hybrid baryons computed
  - Quark core model
  - Helicity of gluon considered
- Extension to **light** quarks
  - Current experiment at JLab
  - Universal potential model
  - Quark core favoured by heavy quarks
- Extension to large-N_c QCD

J^P	n_r	l_{eff}	$cccg$	$bbbg$
$1/2^{\pm}$	0	1	1.842	1.784
$3/2^{\pm}$	0	1	1.842	1.784
$3/2^{\pm}$	0	2	2.350	2.336
$1/2^{\pm}$	1	1	2.552	2.469
$3/2^{\pm}$	1	1	2.552	2.469
$3/2^{\pm}$	1	2	2.938	2.880

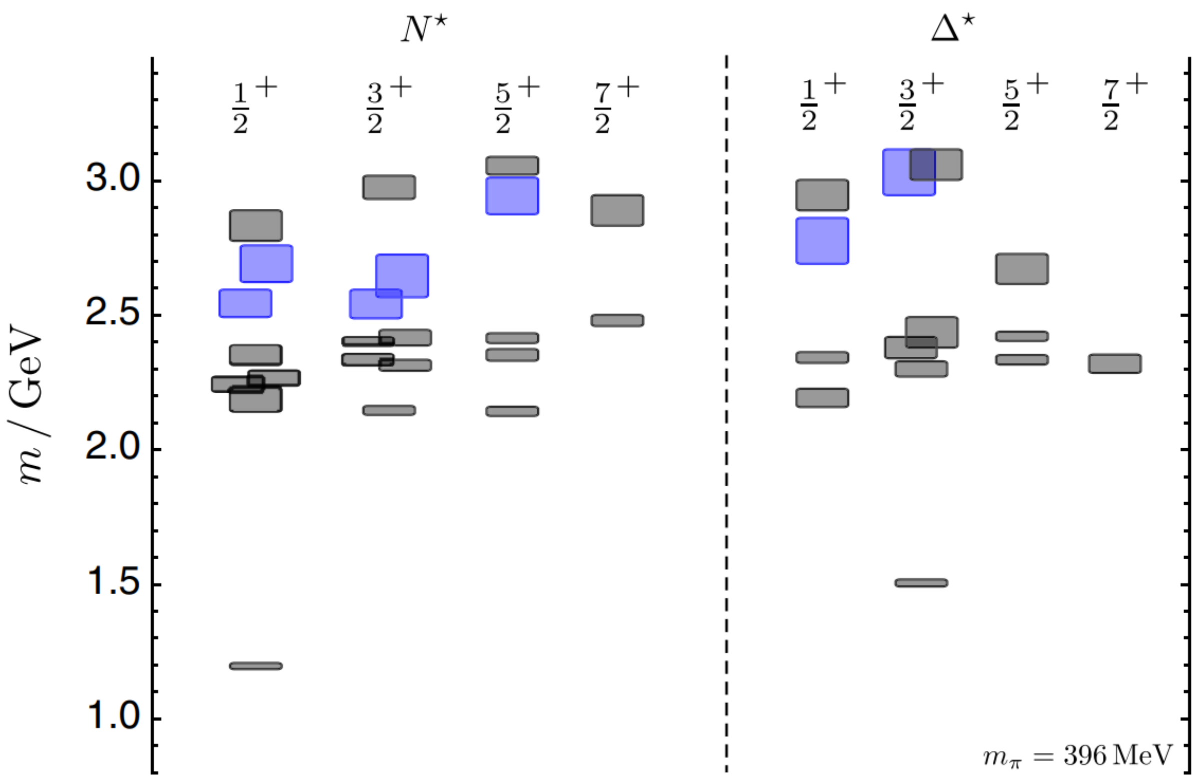
Mass gap between hybrid $qqqg$ and ordinary qqq baryon [3]

[3] L. Cimino, C.T. Willemyns, C. Semay, arXiv:2406.07912

Thank you for your attention

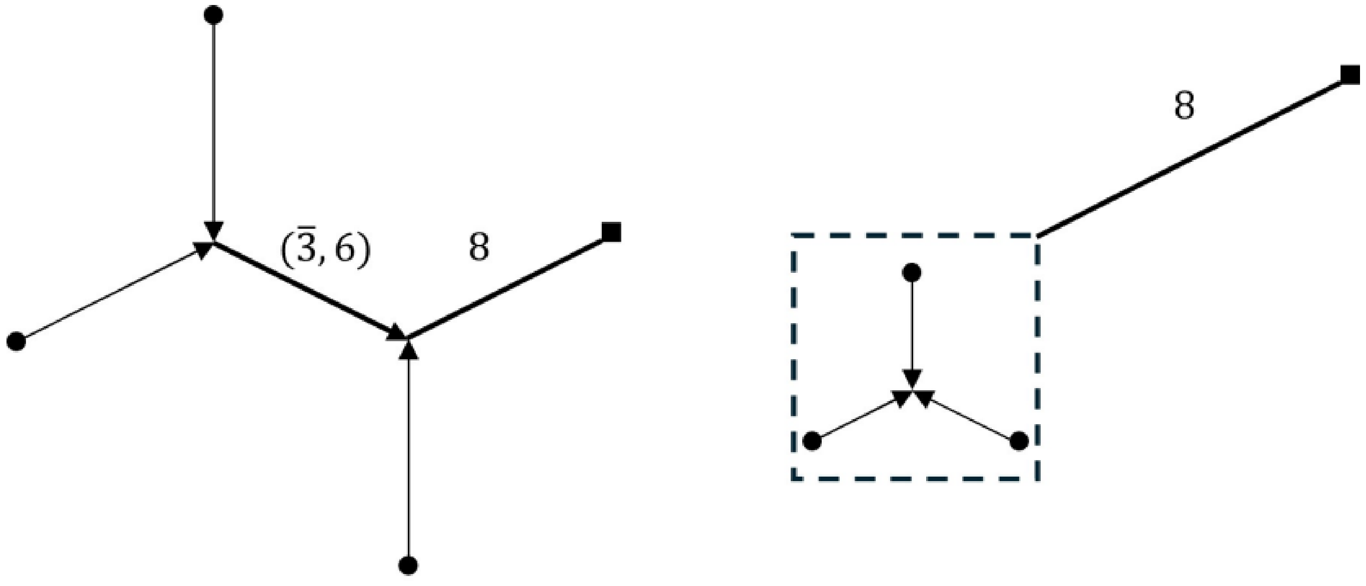
Lattice QCD results

- Lattice QCD results for light hybrid baryons [1]



[1] J.J. Dudek, R.G. Edwards, Phys. Rev. D **85**, 054016 (2012)

Flux tubes of hybrid baryons



(a) (b)