Nucleons and vector mesons in holographic QCD

Alfonso Ballon-Bayona
(Rio de Janeiro Federal U.)

In collaboration with Adão S. da Silva Junior (Rio de Janeiro Federal U.)

Phys.Rev.D 109 (2024) 9, 094050
arXiv:2402.17950
Summary

1. Confinement in Einstein-dilaton holography
2. Vector mesons in confining holographic QCD
3. Nucleons in confining holographic QCD
4. Results
5. Conclusions
1. Confinement in Einstein-dilaton holography

Einstein-dilaton gravity in 5d:

\[ S = \sigma \int d^5 x \sqrt{-g} \left[ R - \frac{4}{3} (\partial_m \Phi)^2 + V(\Phi) \right] \]

5d ansatz in holographic QCD:

\[ ds^2 = \frac{1}{\zeta(z)^2} \left[ -dt^2 + d\vec{x}^2 + dz^2 \right] , \quad \Phi = \Phi(z) \]

Independent field equations:

\[ \zeta'' - \frac{4}{9} \zeta \Phi'^2 = 0 \]
\[ V - \zeta^5 (\zeta^{-3})'' = 0 \]

Warp factor in the Einstein frame

\[ A(z) = -\ln \zeta \]
Linear confinement is guaranteed by a quadratic dilaton asymptotic behaviour

\[ \Phi(z \to \infty) = k z^2 \]

*Karch-Katz-Son-Stephanov 2006, Gursoy-Kiritsis-Nitti 2007*

In this work we consider the following analytical solutions:

\[ \Phi_I = k z^2, \quad \zeta_I(z) = \Gamma \left( \frac{5}{4} \right) \left( \frac{3}{k} \right)^{1/4} \sqrt{z} I_{1/4} \left( 2 \frac{3}{2} k z^2 \right) \]  

\[ \Phi_{II} = \frac{1}{2} \sqrt{k} z \sqrt{9 + 4 k z^2} + \frac{9}{4} \sinh^{-1} \left( \frac{2}{3} \sqrt{k} z \right), \quad \zeta_{II}(z) = z \exp \left( \frac{2}{3} k z^2 \right) \]

Both models leads to a quadratic dilaton at large \( z \) and AdS asymptotics at small \( z \)

Warp factor in the string frame:

\[ A_s(z) = -\ln \zeta + \frac{2}{3} \Phi \]
Confinement criterion

The function \( f(z) = \sqrt{g_{tt}g_{xx}} \) defined in the string frame should have a minimum \( f(z^*) > 0 \)

\textit{Kinar, Schreiber and Sonnenschein 1998}

In terms of the warp factor we have \( f(z) = \exp(2A_s) \)

As shown in the figure, both models satisfy the confinement criterion
2. Vector mesons in confining holographic QCD

Pioneer works in the bottom-up approach:

*Erlich-Katz-Son-Stephanov 2005, Grigoryan-Radyushkin 2007*

Consider the vectorial currents associated with $SU(2)$ isospin symmetry

$$< J^{\mu,c} > = < \bar{q}(x) \gamma^\mu T^c q(x) > = < J^{\mu,c}_R > + < J^{\mu,c}_L >$$

Holographic QCD maps the 4d currents to 5d non-Abelian gauge fields described by the action

$$S = - \frac{1}{4g_5^2} \int d^4x \, dz \, \sqrt{-g} e^{-\Phi} \text{Tr} \left( F_{mn}^R + F_{mn}^L \right)^2$$

Expanding this action at second order in the perturbations, we find in the vectorial sector

$$S_V = - \frac{1}{4g_5^2} \int d^4x \, dz \, \sqrt{-g} e^{-\Phi} v^c_{mn} \, \text{Tr} \left( F_{mn}^R + F_{mn}^L \right)^2 = - \frac{1}{4g_5^2} \int d^4x \, dz \, e^{-A_5} \Phi v^c_{\hat{m}\hat{n}} \, \text{Tr} \left( F_{mn}^R + F_{mn}^L \right)^2$$

where $v^c_{\hat{m}\hat{n}} = \partial_{\hat{m}} V^c_{\hat{n}} - \partial_{\hat{n}} V^c_{\hat{m}}$ and $\hat{m}, \hat{n}$ are contracted with a 5d Minkowski metric.
The 5d gauge coupling can be fixed as

\[ g_5^2 = \frac{12\pi^2}{N_c} \]

with \( N_c = 3 \)

to reproduce the perturbative result for the current correlator at large energies

Varying the 5d action one finds the field equation

\[ \partial_m (e^{A_5 - \Phi} \nu_{\tilde{m}\tilde{n}}^c) = 0 \]

and the surface term

\[ \delta S_V = -\frac{1}{g_5^2} \int d^4x \, dz \partial_{\tilde{m}} (e^{A_5 - \Phi} \nu_{\tilde{m}\tilde{n}}^c \delta \nu_{\tilde{n}}^c) \]

The 5d vectorial field is decomposed as

\[ \nu_{\tilde{m}}^c = (\nu_{\tilde{2}}^c, \nu_{\tilde{\mu}}^c) , \quad \nu_{\tilde{\mu},c} = \nu_{\tilde{\mu},c}^\perp + \partial_{\tilde{\mu}} \xi^c \]

We use gauge symmetry to fix \( \nu_{\tilde{2}}^c = 0 \) and it turns out that \( \xi^c = 0 \)
The equation for the transverse sector takes the form (in momentum space)

\[ [(\partial_z + A'_s - \Phi')\partial_z - q^2]V^{\mu,c}_\perp = 0 \]

Taking the ansatz

\[ V^{\mu,c}_\perp(q, z) = e^{-B_V(z)}\eta^{\mu}\psi_V(q, z) \quad , \quad B_V = \frac{1}{2}(A_s - \Phi) \]

the equation takes the Schrödinger form

\[ [\partial_z^2 - q^2 - V_V]\psi_V = 0 \]

with

\[ V_V = B'_V + B''_V \]

The VEV of the current operator can be obtained from the surface term in the on-shell action

\[ < J^{\mu,c}(x) > = \frac{\delta S_V}{\delta V^{0,\perp}_{\mu,c}(x)} = \frac{1}{g_5^2} \left[ e^{A_s-\Phi}\partial_z V^{\mu,c}_\perp \right]_{z=\epsilon} \]

and we have introduced a UV regulator \( z = \epsilon \) for the AdS boundary
The bulk to boundary propagator and the current correlator

The field in 5 dimensions is mapped to the 4d source using the **bulk to boundary** propagator

\[
V_{\mu,c}^{\perp}(z, x) = \int d^4 y \, K_{\mu\nu}^{cd}(z, x; y)V_{\perp,0}^{\nu,d}(y)
\]

The on-shell action takes the form

\[
S_V^{o-s} = \frac{1}{2g_5^2} \int d^4 x \int d^4 y \, V_{\perp,0}^{\mu,c}(x)[e^{A_s-\Phi} \partial_z K_{\mu\nu}^{cd}(z, x; y)]V_{\perp,0}^{\nu,d}(y)
\]

Using the AdS/CFT dictionary we obtain the **current correlator**

\[
G_{\mu\nu}^{cd}(x - y) = < J_{\mu,c}(x)J_{\nu,d}(y) > = \frac{\delta S_V^{o-s}}{\delta V_{\perp,0}^{\mu,c}(x)\delta V_{\perp,0}^{\nu,d}(y)} = \frac{1}{g_5^2} [e^{A_s-\Phi} \partial_z K_{\mu\nu}^{cd}(z, x; y)]
\]
The Sturm-Liouville equation and the spectral decomposition

The bulk to boundary propagator can be written in momentum space as

\[ K_{\mu\nu}^{cd}(z, q) = \left( \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \delta^{cd} V(z, q) \]

The field \( V(z, q) \) satisfy the differential equation

\[ \left[ (\partial_z + A'_s - \Phi') \partial_z - q^2 \right] V(z, q) = 0 \]

which can be written in the Sturm-Liouville form

\[ [\partial_z (p(z) \partial_z) - s(z) + \lambda r(z)] V(z, q) = 0 \]

where

\[ p(z) = r(z) = e^{A_s - \Phi}, \quad s(z) = 0, \quad \lambda = -q^2 \]

We define the Green’s function by

\[ [\partial_z (p(z) \partial_z) - s(z) + \lambda r(z)] G(z; z') = \delta(z - z') \]
Following Sturm-Liouville theory we obtain

\[ G(z; z') = - \sum_n \frac{v^n(z)v^n(z')}{q^2 + m^2_{\nu n}} \]

where the Sturm-Liouville modes satisfy

\[ \left( \partial_z + A'_s - \Phi' \right) \partial_z + m^2_{\nu n} \right)v^n(z) = 0 \]

and are normalised as

\[ \int dz \, e^{A_s - \Phi} v^m(z)v^n(z) = \delta^{mn} \]

The bulk to boundary propagator takes the form

\[ V(z', q) = -\left[ e^{A_s - \Phi} \partial_z G(z; z') \right]_{z=\epsilon} = - \sum_n \frac{\left[ e^{A_s - \Phi} \partial_z v^n(z) \right]_{z=\epsilon} v^n(z')}{q^2 + m^2_{\nu n}} \]

and the current correlator becomes

\[ G^{cd}_{\mu \nu}(q) = \left( \eta_{\mu \nu} - \frac{q_\mu q_\nu}{q^2} \right) \delta^{cd} \sum_n \frac{F^2_{\nu n}}{q^2 + m^2_{\nu n}}, \quad F^{\nu n} = \frac{1}{g_5} \left[ e^{A_s - \Phi} \partial_z v^n(z) \right]_{z=\epsilon} \]

The coefficients \( F^{\nu n} \) are the vector meson decay constants consistent with large \( N_c \) QCD.
3. Nucleons in confining holographic QCD

Pioneer works in the bottom-up approach:

\[ \text{Brodsky-Teramond 2005, Hong-Inami-Yee 2006, Abidin-Carlson 2009} \]

Nucleons are usually described by interpolating fields. For a proton we use the Ioffe operator

\[ \langle O(x) \rangle = \langle \epsilon_{abc} \left( u_a^T(x) C \gamma_\mu u_b(x) \right) \gamma_5 \gamma^\mu d_c(x) \rangle \]

Ioffe 1981

In holographic QCD we start with a 5d Dirac spinor described by the action

\[ S_F = G_F \int d^4x \sqrt{-g_s} e^{-\Phi} \left( \frac{i}{2} \bar{\psi} \Gamma^n D_n \psi + c.c. -i \bar{\psi} \Gamma^5 \psi \right) \]

where

\[ \Gamma^n = e^a_{\hat{a}} \Gamma^{\hat{a}} \quad , \quad D_n = \partial_n + \frac{1}{8} \omega^{\hat{a} \hat{b}}_{\hat{n}} [\Gamma^{\hat{a}}, \Gamma^{\hat{b}}] \]

Redefining the Dirac field as

\[ \psi \rightarrow e^{\Phi/2} \psi \]

the action becomes

\[ S_F = G_F \int d^4x \sqrt{-g_s} \left( \frac{i}{2} \bar{\psi} \Gamma^n D_n \psi + c.c. -i \bar{\psi} \Gamma^5 \psi \right) \]
The 5d gauge coupling can be fixed as \( G_F = 2 \pi^{-4} \) to reproduce the perturbative QCD result for the nucleon correlator at large energies.

In holographic QCD the vielbein takes the form

\[
e^n_\alpha = e^{-A_s \delta^n_\alpha}
\]

and the non-vanishing components of the spin connection are

\[
\omega^\gamma_\mu = -\omega^\gamma_\mu = -A'_s \delta^\gamma_\mu
\]

The Dirac action becomes

\[
S_F = G_F \int d^4x \, dz \, e^{4A_s} \left( \frac{i}{2} \overline{\psi} \Gamma^\alpha \partial_\alpha \psi - \frac{i}{2} (\partial_\alpha \overline{\psi}) \Gamma^\alpha \psi - i e^{A_s} \overline{\tilde{m}} \overline{\psi} \psi \right)
\]

Varying this action we obtain the field equations

\[
(\Gamma^\alpha \partial_\alpha + 2A'_s \Gamma^\gamma - e^{A_s} \overline{\tilde{m}}) \psi = 0
\]

\[
\overline{\psi} (\overline{\partial_\alpha} \Gamma^\alpha + 2A'_s \Gamma^\gamma + e^{A_s} \overline{\tilde{m}}) = 0
\]

and the surface term

\[
\delta S_F = G_F \int d^4x \left( \frac{i}{2} e^{4A_s} \delta \overline{\psi} \Gamma^\gamma \psi \right)_{z=\epsilon} + c. c.
\]
Left and right decomposition:
\[ \psi = \psi_R + \psi_L \]

where
\[ \psi_{R/L} = P_{R/L} \psi \]
\[ \bar{\psi}_{R/L} = \bar{\psi} P_{L/R} \]
\[ P_{R/L} = \frac{1}{2} (1 \pm \Gamma^2) \]

The Dirac equation decomposes as
\[ \Gamma \bar{\psi}_R \psi_{R/L} = \pm \left( \partial_z + 2A' \pm e^A \bar{m} \right) \psi_{L/R} \]

and the surface term becomes
\[ \delta S_F = G_F \int d^4x \left( \frac{i}{2} e^{4A_s} \delta \bar{\psi}_L \psi_R - \frac{i}{2} e^{4A_s} \delta \bar{\psi}_R \psi_L \right)_{z=\epsilon} + c. c. \]

Since \( \psi_R \) and \( \psi_L \) are not independent we need to correct the Dirac action as
\[ S'_F = S_F + G_F \int d^4x \left( \sqrt{-\gamma} \frac{i}{2} \bar{\psi}_R \psi_L \right)_{z=\epsilon} = S_F + G_F \int d^4x \left( \frac{i}{2} e^{4A_s} \left( \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right) \right)_{z=\epsilon} \]

so that
\[ \delta S'_F = G_F \int d^4x \left( \frac{i}{2} e^{4A_s} \delta \bar{\psi}_L \psi_R \right)_{z=\epsilon} + c. c. \]
At small $z$ (near the boundary) we find the asymptotic behaviour

$$\psi_L(x, z) = \alpha_L(x)z^{2-m} + \ldots + \beta_L(x)z^{3+m} + \ldots$$
$$\psi_R(x, z) = \alpha_R(x)z^{3-m} + \ldots + \beta_R(x)z^{2+m} + \ldots$$

The source $\alpha_L(x)$ couples to the operator $O_R$ and we find

$$< O_R > = \frac{\delta S_F'}{\delta \alpha_L} = iG_F \left( z^{2-m}e^{4A_s} \psi_R \right)_{z=\epsilon} = iG_F \left( \frac{\Gamma \bar{\mu} \partial_{\bar{\mu}}}{2} \left( z^{2-m}e^{4A_s} \left( \partial_z + 2A'_s + e^{A_s}\tilde{m} \right) \psi_L \right) \right)_{z=\epsilon}$$

Combining the left and right coupled spinor equations we obtain

$$\left[ (\partial_z + 2A'_s \pm e^{A_s}\tilde{m}) \left( \partial_z + 2A'_s \mp e^{A_s}\tilde{m} \right) + \partial^2 \right] \psi_{R/L} = 0$$

We expand in the $x$ directions using a plane-wave basis

$$\psi_{R/L}(x, z) = \int d^4q \ e^{iq \cdot x} F_{R/L}(q, z) \alpha_{R/L}(q)$$

and we obtain

$$\left[ (\partial_z + 2A'_s \pm e^{A_s}\tilde{m}) \left( \partial_z + 2A'_s \mp e^{A_s}\tilde{m} \right) + Q^2 \right] F_{R/L} = 0$$

with $Q = \sqrt{-q^2}$
The Schrödinger equation

Using a Bogoliubov transformation

\[ F_{R/L}(q,z) = e^{-2A_s(z)}\xi_{R/L}(q,z) \]

we obtain the Schrödinger equations

\[ \left[ \partial_z^2 + Q^2 - V_{R/L} \right] \xi_{R/L} = 0 \]

with

\[ V_{R/L} = \pm \partial_z (e^{A_s} \tilde{m}) + (e^{A_s} \tilde{m})^2 \]

We propose the following ansatz for the mass term:

\[ \tilde{m} = e^{-A_s} \left( -m A'_s + \frac{1}{2} \Phi' \right) \]

The bulk to boundary propagator and the nucleon correlator

Introducing the bulk to boundary propagator by

the on-shell action takes the form

\[ S_F^{\rho-s} = G_F \int d^4x \int d^4y \frac{\hat{\rho}}{\partial^2} \left( i \alpha L(x) \left( z^2 - m e^{A_s} (\partial_z + 2 A'_s + e^{A_s} \tilde{m}) F_L(z; y) \right) \right)_{z=\epsilon} \alpha_L(y) + c. c. \]

where \( \hat{\rho} = \partial / \partial (x - y) \hat{\rho} \)
The 2-point nucleon correlator takes the form

\[
\Gamma_R(x - y) = \langle O_R(x) \bar{O}_R(y) \rangle = iG_F P_R \frac{\Gamma^{\mu \bar{\mu}}}{\delta^2} \left( z^2 - m^2 e^{4A_s} (\partial_z + 2 A'_s + e^{A_s} \tilde{m}) F_L(z, x, y) \right)_{z=\epsilon}
\]

The bulk to boundary propagator, in momentum space, satisfies the differential equation

\[
[(\partial_z + 4A'_s)\partial_z + \Theta_L + Q^2]F_L(z, q) = 0
\]

where

\[
\theta_L(z) = 2A''_s + 4A'^2_s + \partial_z (e^{A_s} \tilde{m}) - e^{2A_s} \tilde{m}^2
\]

The equation can be written in the Sturm-Liouville form

\[
[\partial_z (p(z) \partial_z) - s(z) + \lambda r(z)]F_L(z, q) = 0
\]

where

\[
p(z) = r(z) = e^{4A_s}, \quad s(z) = -e^{4A_s} \Theta_L, \quad \lambda = Q^2
\]

Using Sturm-Liouville theory we obtain the spectral decomposition

\[
G(z; z') = - \sum_n \frac{f_{L,n}(z) f_{L,n}(z')}{q^2 + m^2_n}
\]
where the Sturm-Liouville modes satisfy

$$[(\partial_z + 4A'_s)\partial_z + \Theta_L + m_n^2]f_{L,n}(z) = 0$$

and are normalised as

$$\int dz \, e^{4A_s} f_{L,m}(z) f_{L,n}(z) = \delta^{mn}$$

The bulk to boundary propagator takes the form

$$F_L(q, z') = -\left[ e^{4A_s} (F_L(z) \partial_z G_L(z; z') - G_L(z; z') \partial_z F_L(z)) \right]_{z=\epsilon} = \sum_n \frac{f_n m_n f_{L,n}(z')}{q^2 + m_n^2}$$

Using the AdS/CFT dictionary we obtain for the nucleon correlator

$$\Gamma_R(q) = -P_R \Gamma^\mu q_\mu \left( \frac{1}{Q^2} \sum_n \lambda_n^2 + \sum_n \frac{\lambda_n^2}{q^2 + m_n^2} \right), \quad \lambda_n = \sqrt{G_F f_n}, \quad f_n = [z^{-2-m} f_{R,n}(z)]_{z=\epsilon}$$

The coefficients $\lambda_n$ are the nucleon “decay constants” consistent with large $N_c$ QCD

The first term is a UV divergence that can be subtracted using holographic renormalisation
4. Results

**Spectrum of vector mesons**

Conformal dimension $\Delta = 3$ for the vectorial current

We solve the Schrödinger equation for the normalisable modes

In the figure we show the Schrödinger potentials for model I (blue), model II (red) and the soft wall model (black dashed)

The infrared parameter $k$ is the only parameter in the model

Mass ratios are independent of the choice of $k$

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Einstein-dilaton I</th>
<th>Einstein-dilaton II</th>
<th>Soft wall</th>
<th>Hard wall</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\rho_1}/m_{\rho_0}$</td>
<td>1.591</td>
<td>1.34</td>
<td>1.414</td>
<td>2.295</td>
<td>1.652 ± 0.048</td>
</tr>
<tr>
<td>$m_{\rho_2}/m_{\rho_0}$</td>
<td>2.015</td>
<td>1.611</td>
<td>1.732</td>
<td>3.598</td>
<td>1.888 ± 0.032</td>
</tr>
<tr>
<td>$m_{\rho_3}/m_{\rho_0}$</td>
<td>2.365</td>
<td>1.843</td>
<td>2</td>
<td>4.903</td>
<td>2.216 ± 0.026</td>
</tr>
<tr>
<td>$m_{\rho_4}/m_{\rho_0}$</td>
<td>2.67</td>
<td>2.049</td>
<td>2.236</td>
<td>6.209</td>
<td>2.46 ± 0.039</td>
</tr>
<tr>
<td>$m_{\rho_5}/m_{\rho_0}$</td>
<td>2.944</td>
<td>2.236</td>
<td>2.45</td>
<td>7.514</td>
<td>2.769 ± 0.022</td>
</tr>
</tbody>
</table>
Spectrum of nucleons

We consider two possible values for the conformal dimension: $\Delta = 7/2$ and $\Delta = 9/2$

We solve the Schrödinger equation for the normalisable modes

Mass ratios presented below are independent of the choice of $k$

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Einstein-dilaton I</th>
<th>Einstein-dilaton II</th>
<th>Soft wall</th>
<th>Hard wall</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{N_0}/m_{p_0}$</td>
<td>0.987</td>
<td>0.988</td>
<td>1.414</td>
<td>1.593</td>
<td>1.209 ± 0.002</td>
</tr>
<tr>
<td>$m_{N_1}/m_{p_0}$</td>
<td>1.623</td>
<td>1.339</td>
<td>1.732</td>
<td>2.917</td>
<td>1.856 ± 0.039</td>
</tr>
<tr>
<td>$m_{N_2}/m_{p_0}$</td>
<td>2.053</td>
<td>1.613</td>
<td>2</td>
<td>4.23</td>
<td>2.204 ± 0.039</td>
</tr>
<tr>
<td>$m_{N_3}/m_{p_0}$</td>
<td>2.403</td>
<td>1.847</td>
<td>2.236</td>
<td>5.54</td>
<td>2.423 ± 0.065</td>
</tr>
<tr>
<td>$m_{N_4}/m_{p_0}$</td>
<td>2.707</td>
<td>2.054</td>
<td>2.449</td>
<td>6.849</td>
<td>2.706 ± 0.065</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Einstein-dilaton I</th>
<th>Einstein-dilaton II</th>
<th>Soft wall</th>
<th>Hard wall</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{N_0}/m_{p_0}$</td>
<td>0.896</td>
<td>0.952</td>
<td>1.732</td>
<td>2.136</td>
<td>1.209 ± 0.002</td>
</tr>
<tr>
<td>$m_{N_1}/m_{p_0}$</td>
<td>1.593</td>
<td>1.314</td>
<td>2</td>
<td>3.5</td>
<td>1.856 ± 0.039</td>
</tr>
<tr>
<td>$m_{N_2}/m_{p_0}$</td>
<td>2.04</td>
<td>1.595</td>
<td>2.236</td>
<td>4.832</td>
<td>2.204 ± 0.039</td>
</tr>
<tr>
<td>$m_{N_3}/m_{p_0}$</td>
<td>2.399</td>
<td>1.833</td>
<td>2.449</td>
<td>6.153</td>
<td>2.423 ± 0.065</td>
</tr>
<tr>
<td>$m_{N_4}/m_{p_0}$</td>
<td>2.708</td>
<td>2.043</td>
<td>2.646</td>
<td>7.468</td>
<td>2.706 ± 0.065</td>
</tr>
</tbody>
</table>
Vector meson decay constants

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Einstein-dilaton I</th>
<th>Einstein-dilaton II</th>
<th>Soft wall</th>
<th>Hard wall</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{F_{\rho_0}/m_{\rho_0}}$</td>
<td>0.3719</td>
<td>0.283</td>
<td>0.3355</td>
<td>0.4246</td>
<td>0.446 ± 0.0019</td>
</tr>
<tr>
<td>$\sqrt{F_{\rho_1}/m_{\rho_0}}$</td>
<td>0.4704</td>
<td>0.3407</td>
<td>0.3989</td>
<td>0.7946</td>
<td>0.5588 ± 0.017</td>
</tr>
<tr>
<td>$\sqrt{F_{\rho_2}/m_{\rho_0}}$</td>
<td>0.5298</td>
<td>0.3798</td>
<td>0.4415</td>
<td>1.114</td>
<td>-</td>
</tr>
<tr>
<td>$\sqrt{F_{\rho_3}/m_{\rho_0}}$</td>
<td>0.5741</td>
<td>0.41</td>
<td>0.4744</td>
<td>1.405</td>
<td>-</td>
</tr>
<tr>
<td>$\sqrt{F_{\rho_4}/m_{\rho_0}}$</td>
<td>0.61</td>
<td>0.4351</td>
<td>0.5017</td>
<td>1.677</td>
<td>-</td>
</tr>
</tbody>
</table>

Nucleon “decay constants” $\left(\alpha = \Delta - \frac{3}{2}\right)$

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Einstein-dilaton I</th>
<th>Einstein-dilaton II</th>
<th>Soft wall</th>
<th>Hard wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_N/m_{\rho_0}^\alpha$</td>
<td>0.1108</td>
<td>0.09835</td>
<td>0.0507</td>
<td>0.1667</td>
</tr>
<tr>
<td>$\lambda_{N_1}/m_{\rho_0}^\alpha$</td>
<td>0.1302</td>
<td>0.1158</td>
<td>0.0716</td>
<td>0.4096</td>
</tr>
<tr>
<td>$\lambda_{N_2}/m_{\rho_0}^\alpha$</td>
<td>0.1519</td>
<td>0.1284</td>
<td>0.0877</td>
<td>0.7138</td>
</tr>
<tr>
<td>$\lambda_{N_3}/m_{\rho_0}^\alpha$</td>
<td>0.1708</td>
<td>0.1388</td>
<td>0.1013</td>
<td>1.069</td>
</tr>
<tr>
<td>$\lambda_{N_4}/m_{\rho_0}^\alpha$</td>
<td>0.1878</td>
<td>0.1478</td>
<td>0.1133</td>
<td>1.469</td>
</tr>
</tbody>
</table>

$\Delta = 7/2$

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Einstein-dilaton I</th>
<th>Einstein-dilaton II</th>
<th>Soft wall</th>
<th>Hard wall</th>
<th>Lattice QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_N/m_{\rho_0}^\alpha$</td>
<td>0.1055</td>
<td>0.158</td>
<td>0.01791</td>
<td>0.1414</td>
<td>0.05778 ± 0.0107</td>
</tr>
<tr>
<td>$\lambda_{N_1}/m_{\rho_0}^\alpha$</td>
<td>0.1201</td>
<td>0.1906</td>
<td>0.03102</td>
<td>0.4755</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{N_2}/m_{\rho_0}^\alpha$</td>
<td>0.1462</td>
<td>0.2172</td>
<td>0.04387</td>
<td>1.058</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{N_3}/m_{\rho_0}^\alpha$</td>
<td>0.172</td>
<td>0.2409</td>
<td>0.05664</td>
<td>1.931</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_{N_4}/m_{\rho_0}^\alpha$</td>
<td>0.1973</td>
<td>0.2627</td>
<td>0.06937</td>
<td>3.129</td>
<td>-</td>
</tr>
</tbody>
</table>

$\Delta = 9/2$
5. Conclusions

- We presented a minimal holographic QCD model that describes vector mesons and nucleons in a single fashion.

- The model contains only one free parameter associated with hadron mass generation and confinement.

- Comparison to experimental data is better for the higher excited states (light states require the addition of chiral symmetry breaking).

Next steps

- Calculate strong couplings between vector mesons and nucleons and find the electromagnetic and gravitational form factors.

- Turn on the temperature and investigate the transition to deconfinement, chiral symmetry restoration and the “melting” of hadrons in the quark-gluon plasma.