QNP 2024

PRECISION NEUTRINOLESS $\beta\beta$ DECAY NUCLEAR MATRIX ELEMENTS

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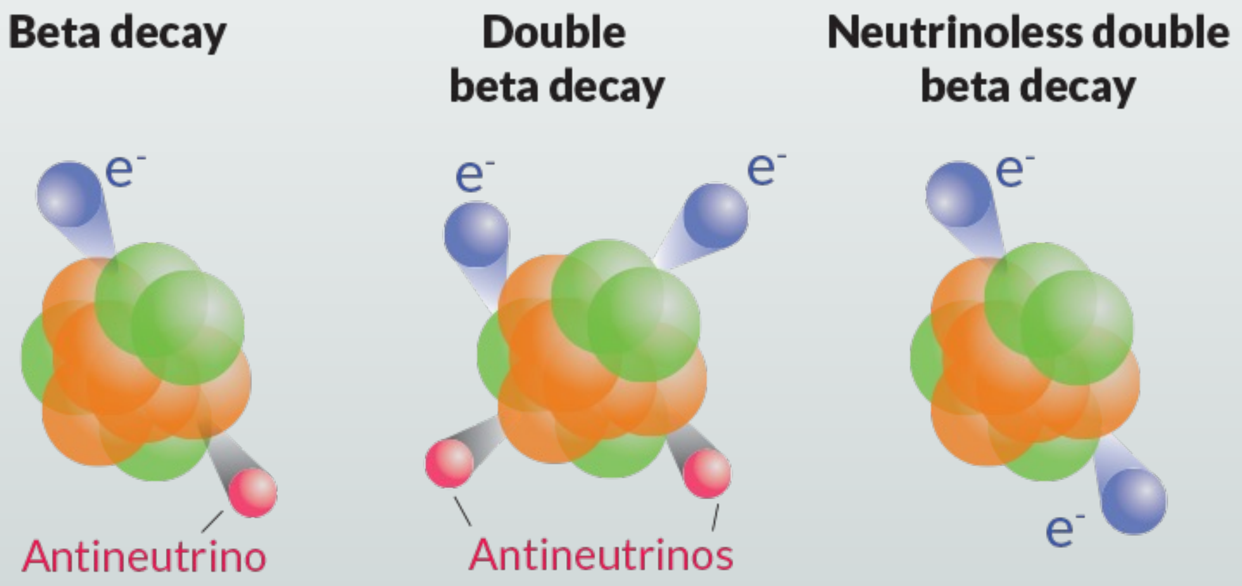
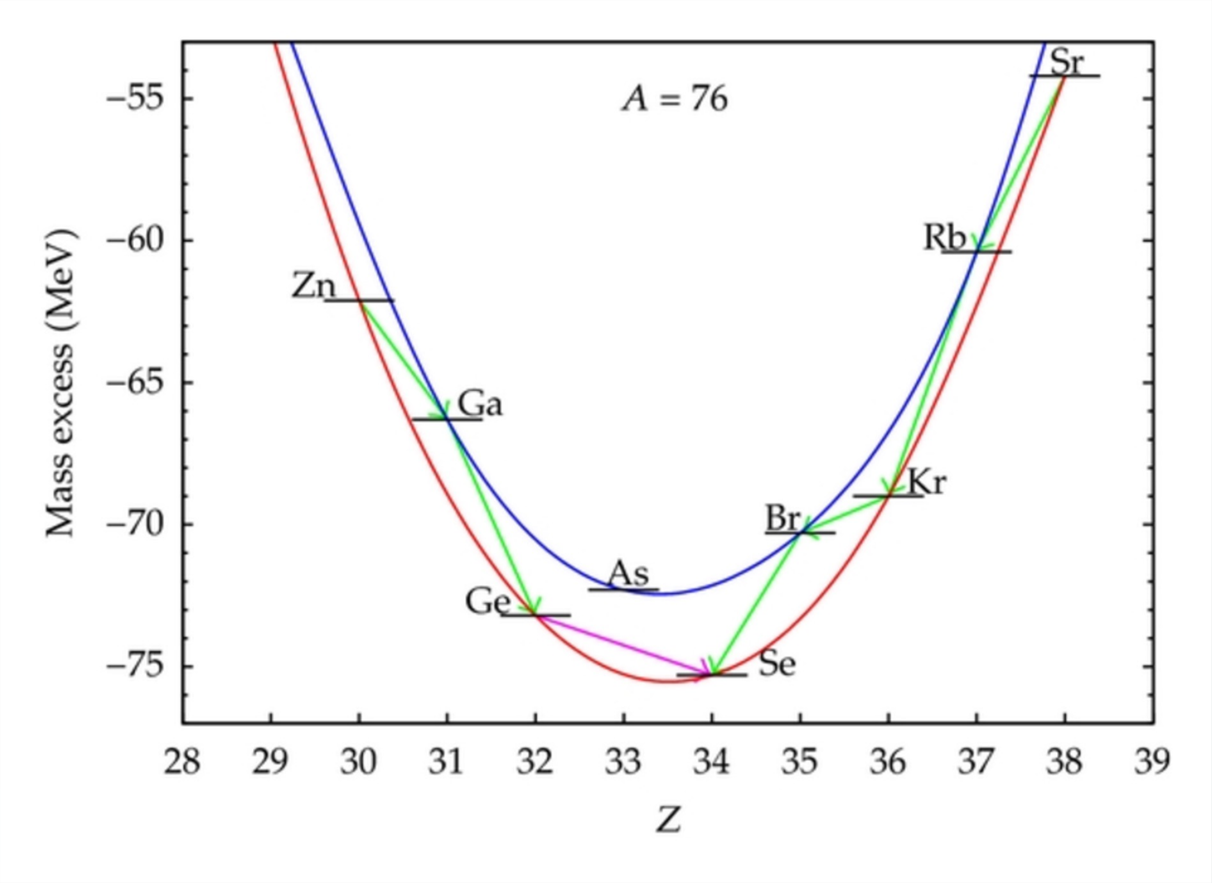
EXCELENCIA MARÍA DE MAEZTU 2020-2023

NEUTRINOLESS $\beta\beta$ DECAY

$\mathbf{0\nu\beta\beta}$ decay : $\mathbf{2n \rightarrow 2p + 2e^-}$



https://www.sciencenews.org/article/quest-identify-nature-neutrinos-alter-ego-heating



Giuliani, et al, Adv. High Energy Phys, 2012, Article, 2012.

New information:
Absolute mass of neutrino
Neutrino as Majorana fermion
Dominance of matter in the universe

$0\nu\beta\beta$ NUCLEAR MATRIX ELEMENTS

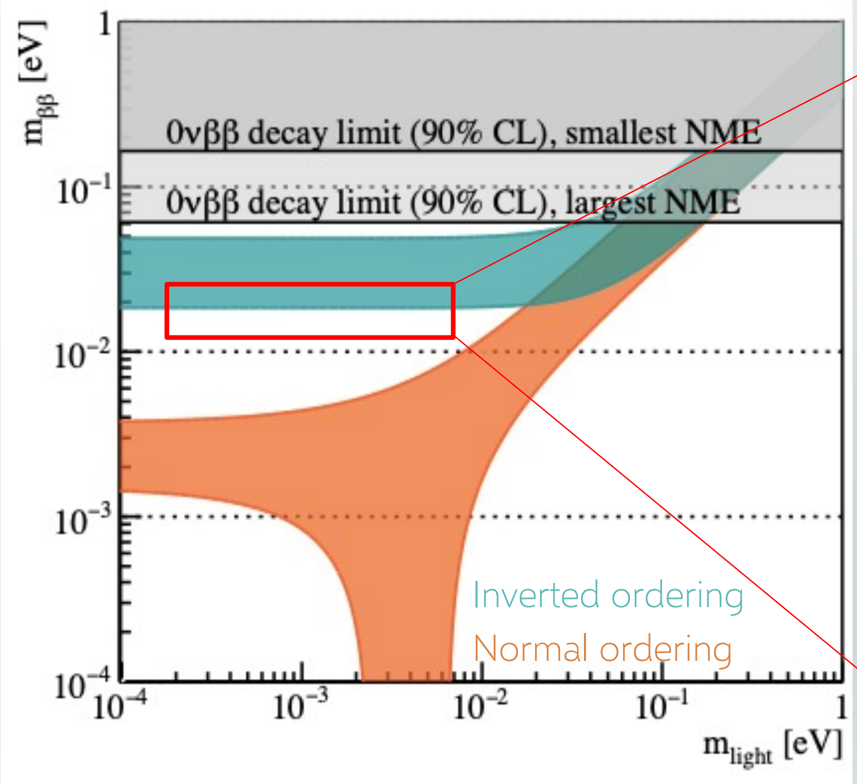
$$\left(t_{1/2}^{0\nu}\right)^{-1} = g_A^4 G_{0\nu} |\mathcal{M}^{0\nu}|^2 m_{\beta\beta}^2$$

$\boldsymbol{\mathcal{M}^{0\nu}} = \langle \mathbf{0}_f^+ | \hat{O} | \mathbf{0}_i^+ \rangle$ ≡ Nuclear matrix elements

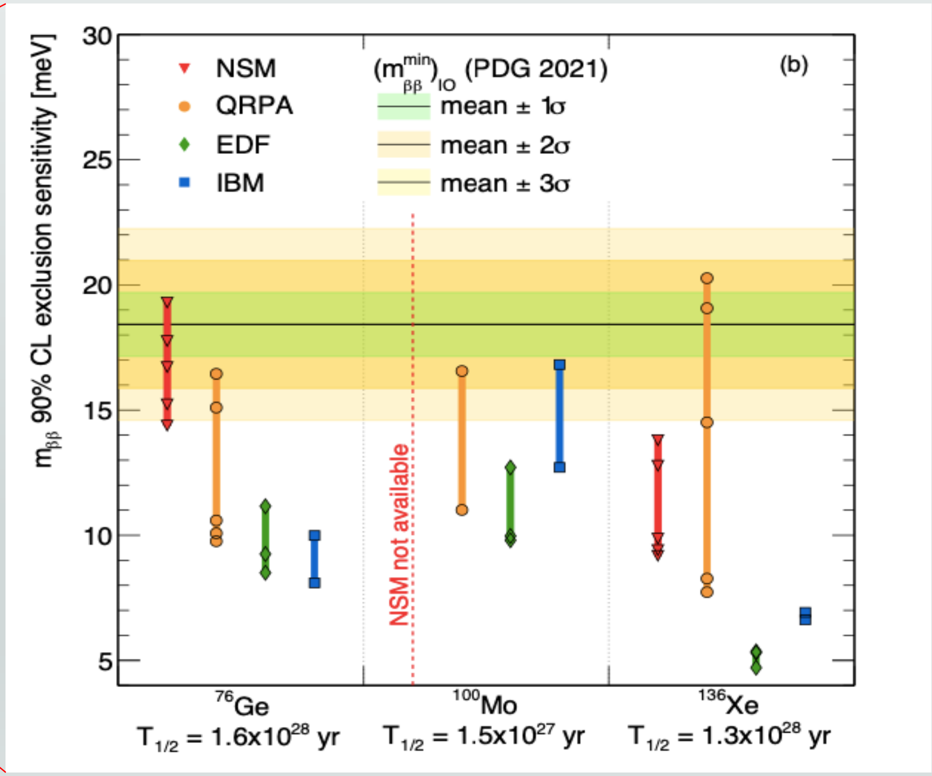
$G_{0\nu}$ ≡ Phase-space factor

$m_{\beta\beta} = \sum_{j=light} U_{ej} m_j$ ≡ Effective neutrino mass

g_A ≡ Axial coupling



Agostini et al. Rev. Mod. Phys. 95, 025002, 2023



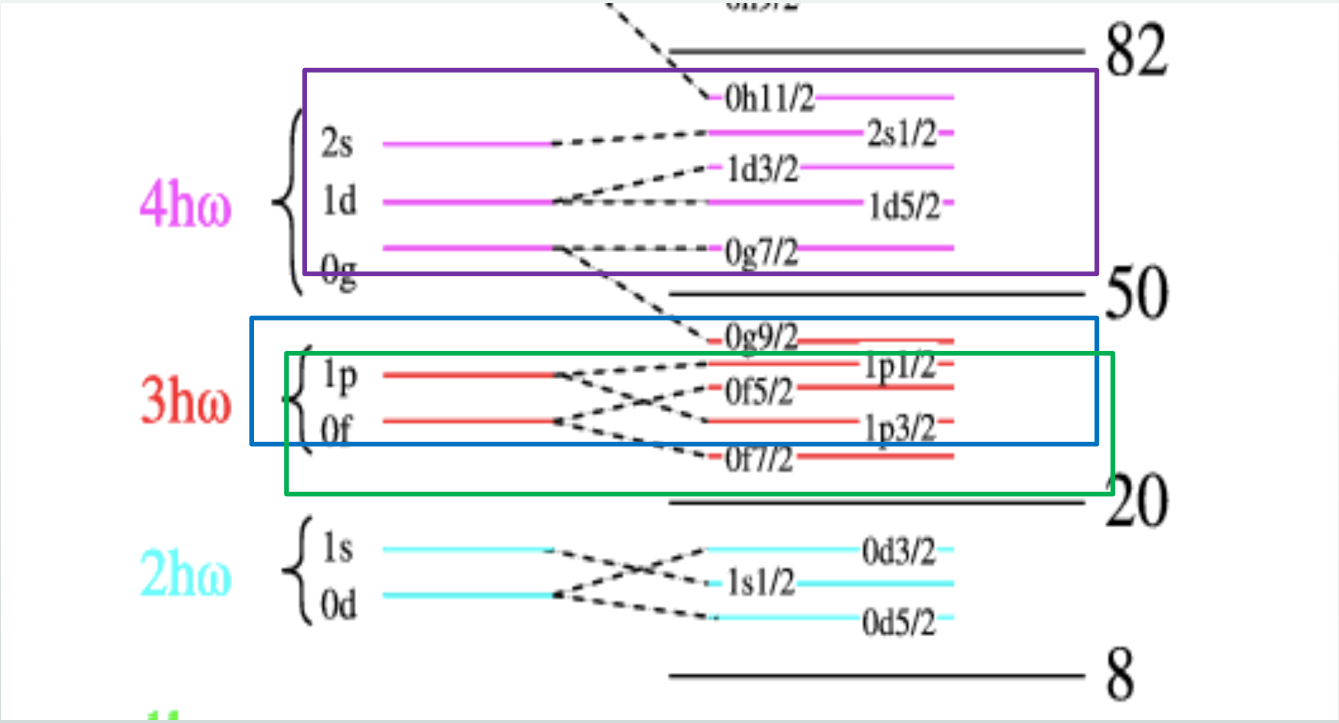
Agostini et al. Phys. Rev. C 104, L042501, 2021

NUCLEAR SHELL MODEL: WAVE FUNCTIONS

Interacting Shell model: $H_{eff} = \sum_i t_i + u_i + \frac{1}{2}\sum_{ij} v_{ij}$

Wavefunctions → Linear combination of Slater Determinants

$$|\Phi\rangle = \sum_{\alpha} C_\alpha |\phi_\alpha\rangle$$



https://oer.physics.manchester.ac.uk/NP/Notes/Notesse23.xht

- Shell-model codes:
  - ANTOINE: m-scheme
  - NATHAN: J-coupled scheme

Caurier et al. Rev. Mod. Phys 77, 427-488, 2005

Valence space:

- ^{124}Sn, ^{130}Te, ^{136}Xe: 0g7/2, 1d5/2, 1d3/2, 2s1/2, 0h11/2
- ^{76}Ge, ^{82}Se: 1p3/2, 0f5/2, 1p1/2, 0g9/2
- ^{48}Ca: 0f7/2, 1p3/2, 0f5/2, 1p1/2

SPHERICAL PROTON-NEUTRON QUASIPARTICLE RANDOM-PHASE APPROXIMATION: WAVE FUNCTIONS

- Single-particle bases ⟶ Woods-Saxon potential
- Quasiparticle bases ⟶ BCS equations with Bonn-A two-body G matrix
- Intermediate states ≡ Two-quasiparticles excitations

$$|J_k^\pi\rangle = \sum_{pn} \left(X_{pn}^{J_\pi^k} \left[a_p^\dagger a_n^\dagger\right]_J - Y_{pn}^{J_\pi^k} \left[a_p^\dagger a_n^\dagger\right]_J^\dagger \right) |QRPA\rangle$$

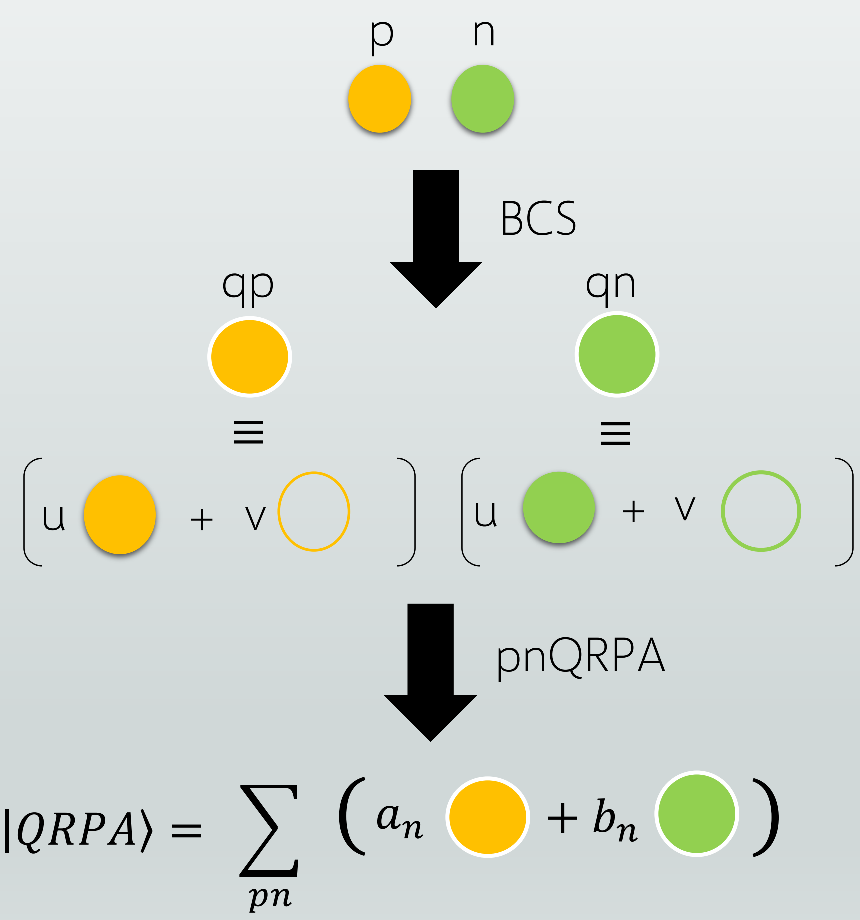
- Adjustable parameters: $\boldsymbol{g_{ph}}$ and $\boldsymbol{g_{pp}}$

  J. Suhonen, Springer-Verlag, Berlin Heidelberg, 2007

- ^{76}Ge, ^{82}Se: 18 orbitals
- ^{96}Zr, ^{100}Mo: 25 orbitals
- ^{116}Cd, ^{124}Sn, ^{130}Te, ^{136}Xe: 26 orbitals

} $n = 0 \rightarrow 2$ harmonic oscillators shells above the fermi level

- RPA details: D. Gambacurta (Previous Talk)



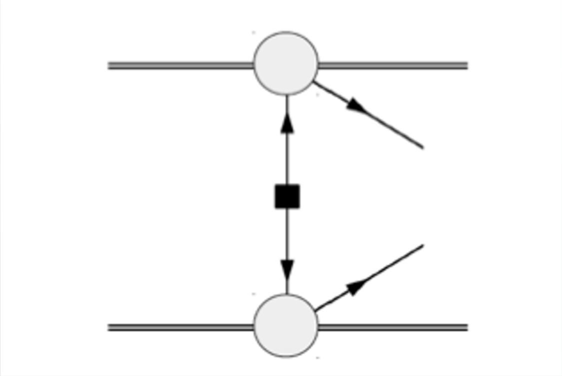
Jokiniemi, seminar UB

$0\nu\beta\beta$ DIAGRAMS

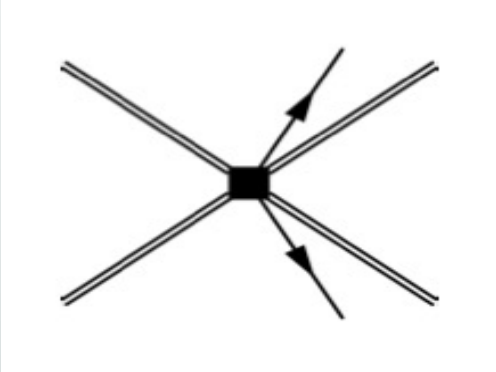
$$\mathcal{M}^{0\nu} = \mathcal{M}^{0\nu}_{LO,L} + \mathcal{M}^{0\nu}_{LO,S} + \mathcal{M}^{0\nu}_{N2LO,\,US} + \mathcal{M}^{0\nu}_{N2LO,loop}$$

LO

Long-range (L) +FSC (N^2LO)



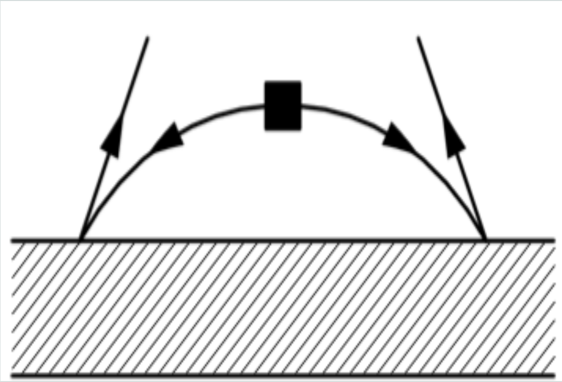
Short-range (S)



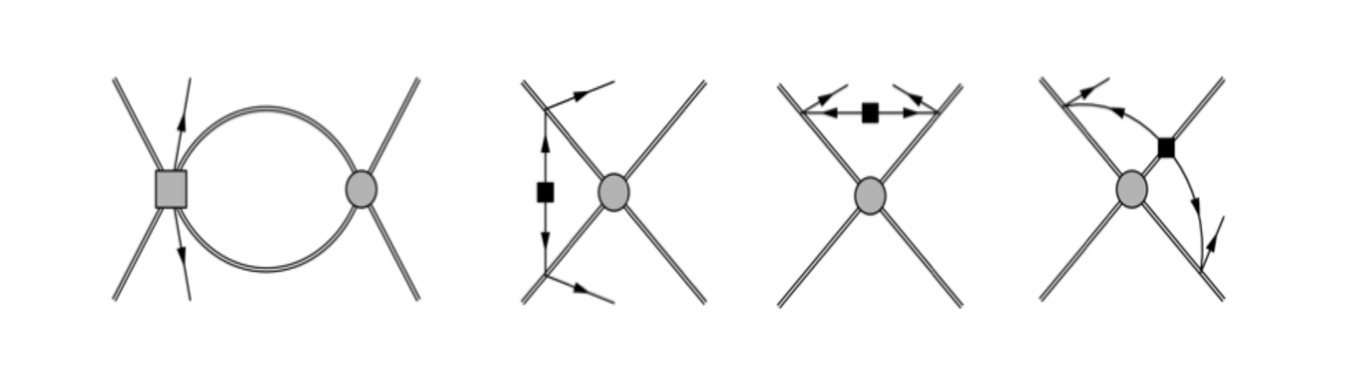
→ already computed

N^2LO

Ultrasoft ($k_F \ll 100$ MeV)



One-loop



→ New terms

Cirigliano, et al. Phys. Rev. C 97, 065501, 2018

ULTRASOFT NME: NSM vs pnQRPA

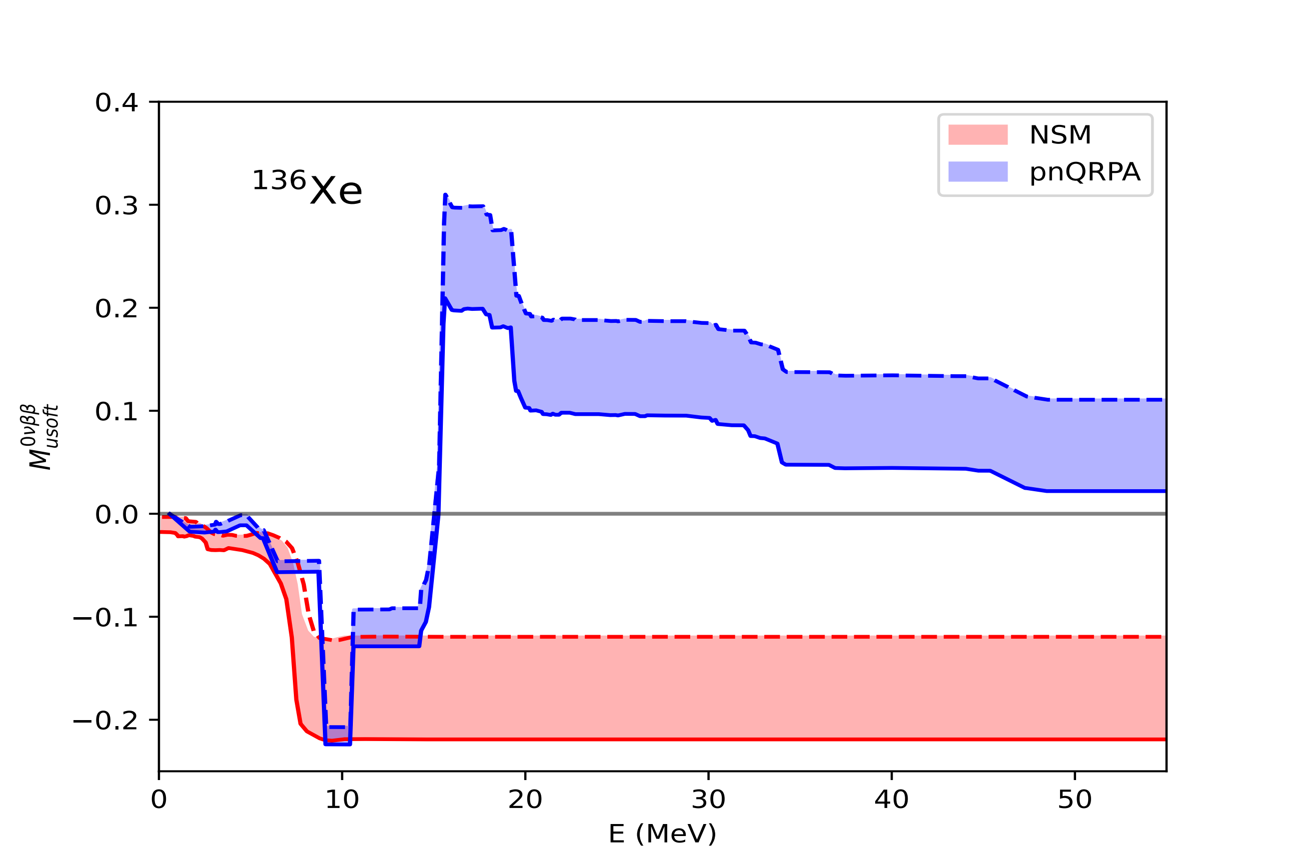
- $\mathcal{M}^{0\nu}_{N2LO,usoft} = -\frac{R}{\pi}\sum_n \left\langle 0^+_f \left\| \sum_k \tau^-_k \sigma_k \right\| 1^+_n \right\rangle \left\langle 1^+_n \left\| \sum_k \tau^-_k \sigma_k \right\| 0^+_i \right\rangle \times 2\left(\frac{Q_{\beta\beta}}{2} + m_e + E_n - E_i\right) \cdot \ln\left(\frac{\mu_{us}}{2\left(\frac{Q_{\beta\beta}}{2}+m_e+E_n-E_i\right)} + 1\right)$

- NSM: $\mathcal{M}^{0\nu}_{usoft} / \mathcal{M}^{0\nu}_{LO} \sim -(1 - 11)\%$ reduction
- pnQRPA: $\mathcal{M}^{0\nu}_{usoft} / \mathcal{M}^{0\nu}_{LO} \sim (-2 - 11)\%$ enhancement

- $m_e \equiv$ electron mass
- $\mu_{us} \equiv$ renorm. scale
- $\sigma_k \equiv$ Pauli matrices
- $Q_{\beta\beta} \equiv Q$-value for $\beta\beta$

^{136}Xe

- NSM: Different hamiltonians
  - GCN5082 → $\mathcal{M}^{0\nu}_{usoft} = -0.220$
  - QX5082 → $\mathcal{M}^{0\nu}_{usoft} = -0.120$
- pnQRPA: Different proton-neutron pairing
  - $g^{T=0}_{pp} = 0.67 \rightarrow \mathcal{M}^{0\nu}_{usoft} = 0.022$
  - $g^{T=0}_{pp} = 0.69 \rightarrow \mathcal{M}^{0\nu}_{usoft} = 0.110$



Jokiniemi, D.C. et al., in progress

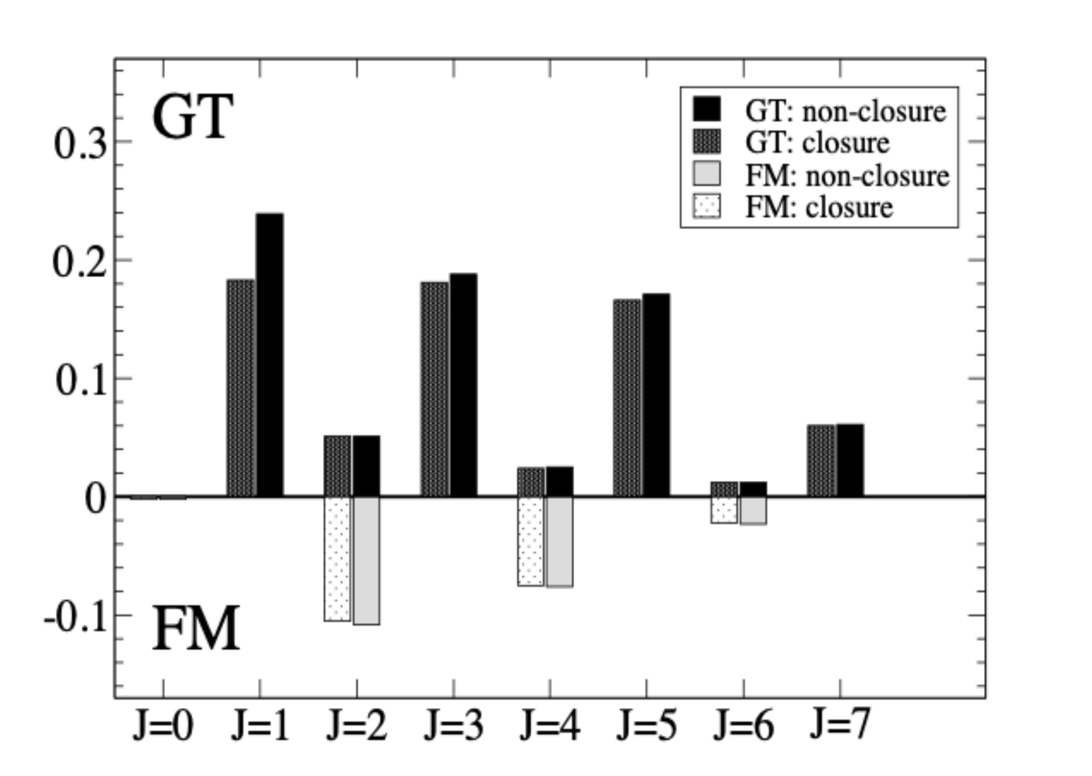
CLOSURE VS NON-CLOSURE

- $\mathcal{M}^{0\nu}_{non-cl} = \frac{2R}{\pi g_A^2}\int_0^{\infty} q^2 dq \sum_n \sum_{a,b} \frac{j_\lambda(qr)\langle 0_f^+|J_\mu(x)|n\rangle\langle n|J^\mu\,(y)|0_i^+\rangle}{q(q+E_n-\frac{1}{2}(E_i+E_f))}$
- $\mathcal{M}^{0\nu}_{LO,k} = \frac{2R}{\pi g_A^2}\int_0^{\infty} q^2 dq \sum_n \sum_{a,b} \frac{j_\lambda(qr)\langle 0_f^+|J_\mu J^\mu)|0_i^+\rangle}{q^2}$

- $q \approx k_F \approx 100$ MeV
- $\boldsymbol{E_n - \frac{1}{2}(E_i + E_f) \rightarrow 0}$ (CLOSURE)
- $J_\mu J^\mu = h_{GT}(q^2) + h_F(q^2) + h_T(q^2)$

- $E_n \equiv$ intermediate state energy
- $E_i(E_f) \equiv$ initial (final) states energy
- $|n\rangle \equiv$ Intermediate states

- Large effect of the intermediate states:
  - Ultrasoft NME
  - $2\nu\beta\beta$: B. Benavente (Next Talk)



Sen'kov and Horoi Phys. Rev. C 88, 064312, 2013

ULTRASOFT NME: χEFT PREDICTION

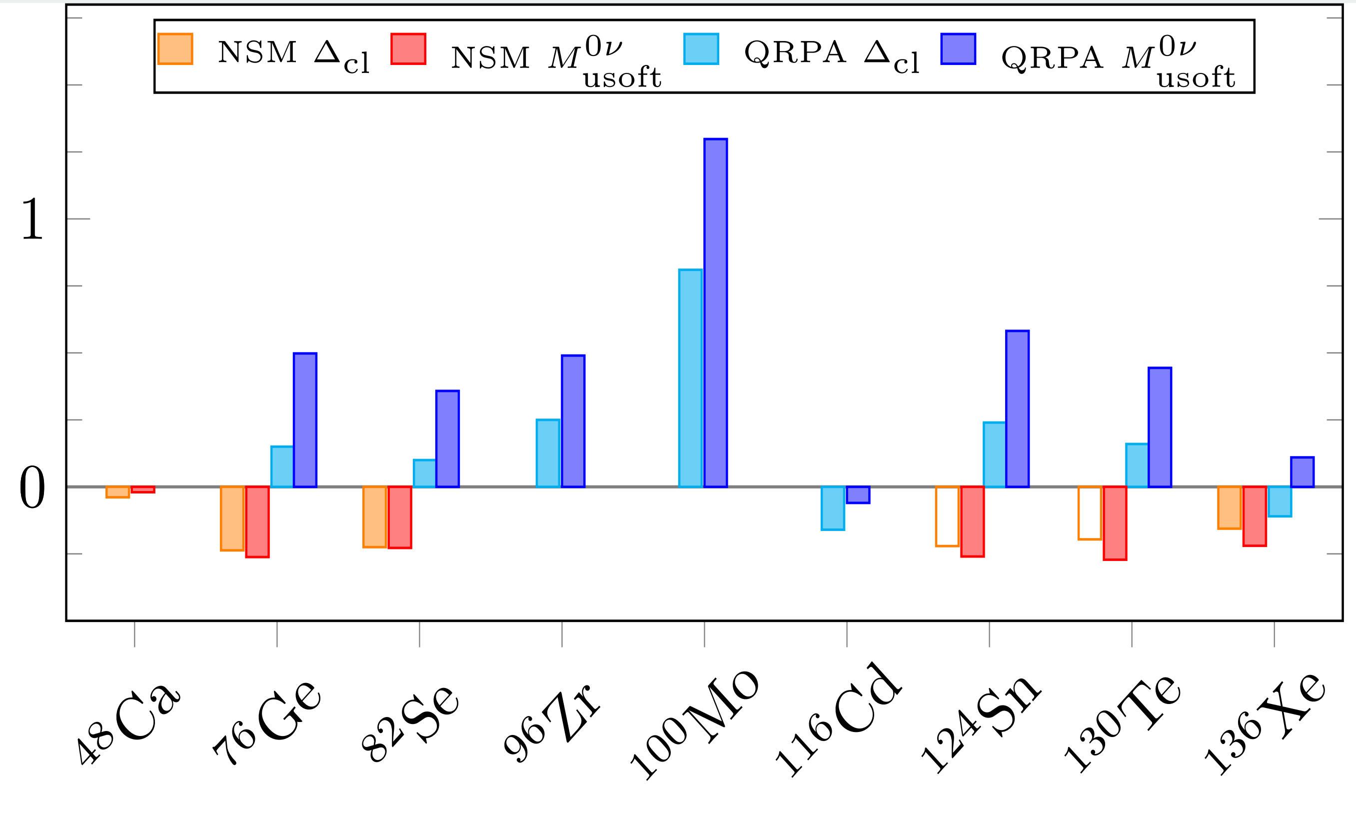
- According to chiral efective field theory (χEFT) the ultrasoft term must be the main contribution beyond the closure approximation.
$\mathcal{M}^{0\nu}_{non-cl} - \mathcal{M}^{0\nu}_{cl} = \Delta_{cl} \approx \mathcal{M}^{0\nu}_{N2LO,usoft}$

Cirigliano, et al. Phys. Rev. C 97, 065501, 2018

- NSM: $|\Delta_{cl}/\mathcal{M}^{0\nu}_{N2LO,usoft}| \sim 80\%$
- QRPA: $|\Delta_{cl}/\mathcal{M}^{0\nu}_{N2LO,usoft}| \sim 60\%$

- NSM: Negative results for $\mathcal{M}^{0\nu}_{N2LO,usoft}$ and Δ_{cl}
- QRPA: Positive results for $\mathcal{M}^{0\nu}_{N2LO,usoft}$ and Δ_{cl}

- The results obtained are in agreement with χEFT prediction.



Jokiniemi, D.C. et al., in progress

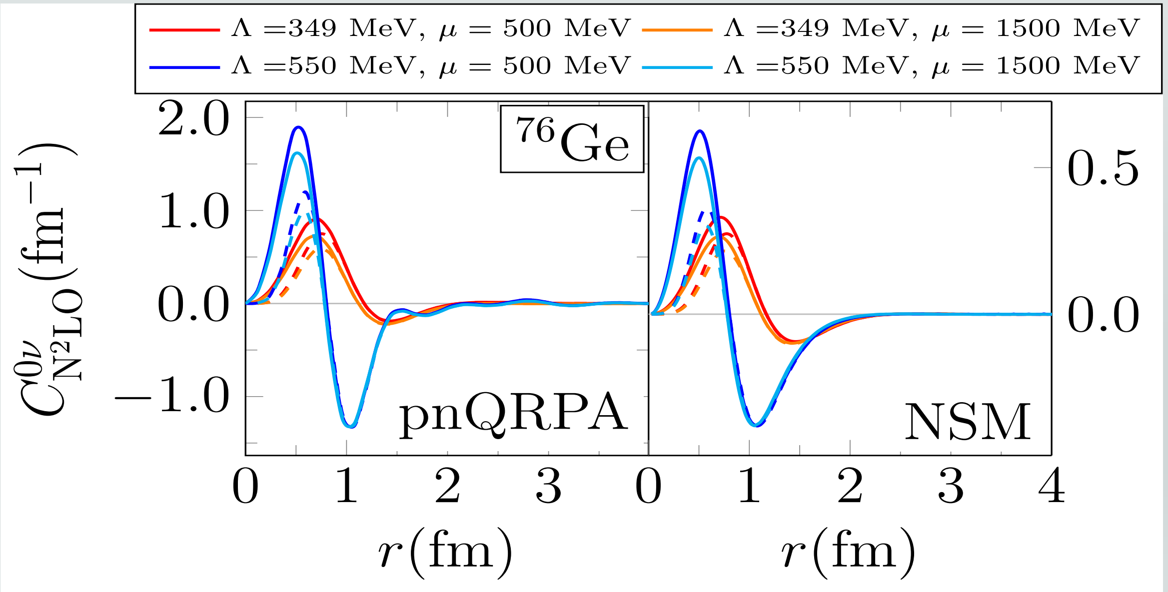
LOOP CORRECTIONS (N^2LO)

- $\mathcal{M}^{0\nu}_{N2LO,k} = \frac{4R}{\pi g_A^2} \langle 0_f^+ || \hat{O} \int_0^\infty q^2 j_\lambda(qr) h_{N2LO,k}(q) dq \, || 0_i^+ \rangle$
- Dependencies:
  - Effective Interactions
  - Short-Range Correlations
  - Regulator cutoff
  - Energy scale

- NSM: $\mathcal{M}^{0\nu}_{N2LO,\,loop} / \, \mathcal{M}^{0\nu}_{LO} \sim (-4 \text{ - } 11)\%$
- QRPA: $\mathcal{M}^{0\nu}_{N2LO,\,loop} / \, \mathcal{M}^{0\nu}_{LO} \sim (-3 \text{ - } 14)\%$

- Large Uncertainty in the loop NME

- $\mathcal{M}^{0\nu}_{N2LO,loop} = \int C^{0\nu}_{N2LO,loop}(r) dr$
- $\mathcal{M}^{0\nu}_{N2LO,loop} = \int \tilde{C}^{0\nu}_{N2LO,loop}(q) dq$



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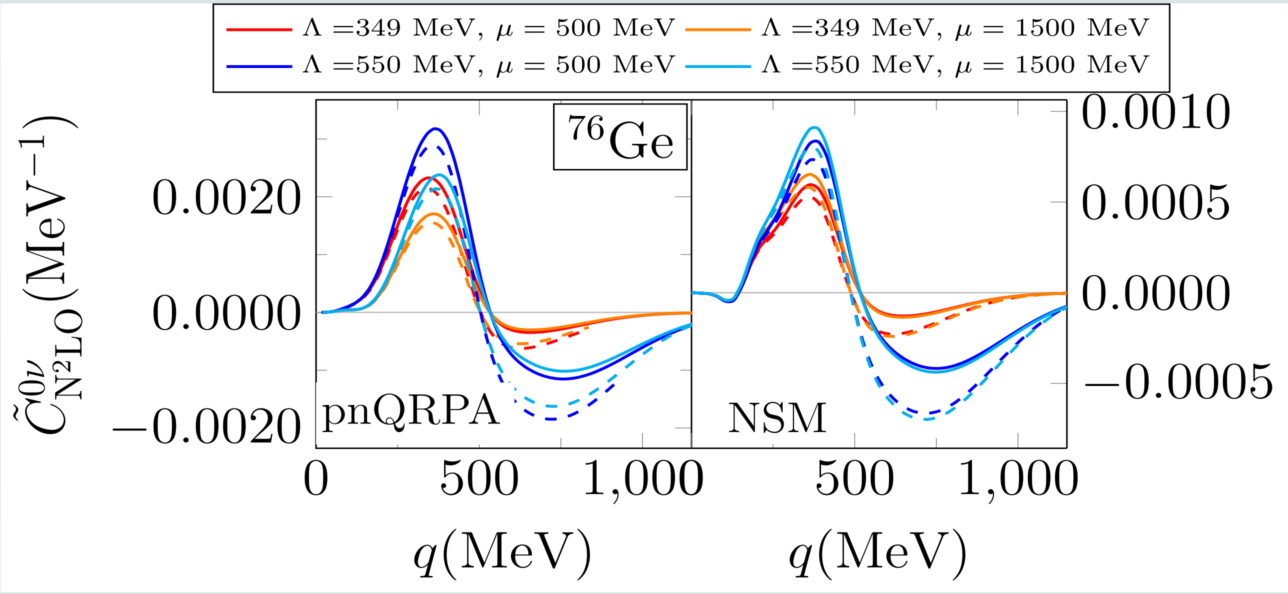
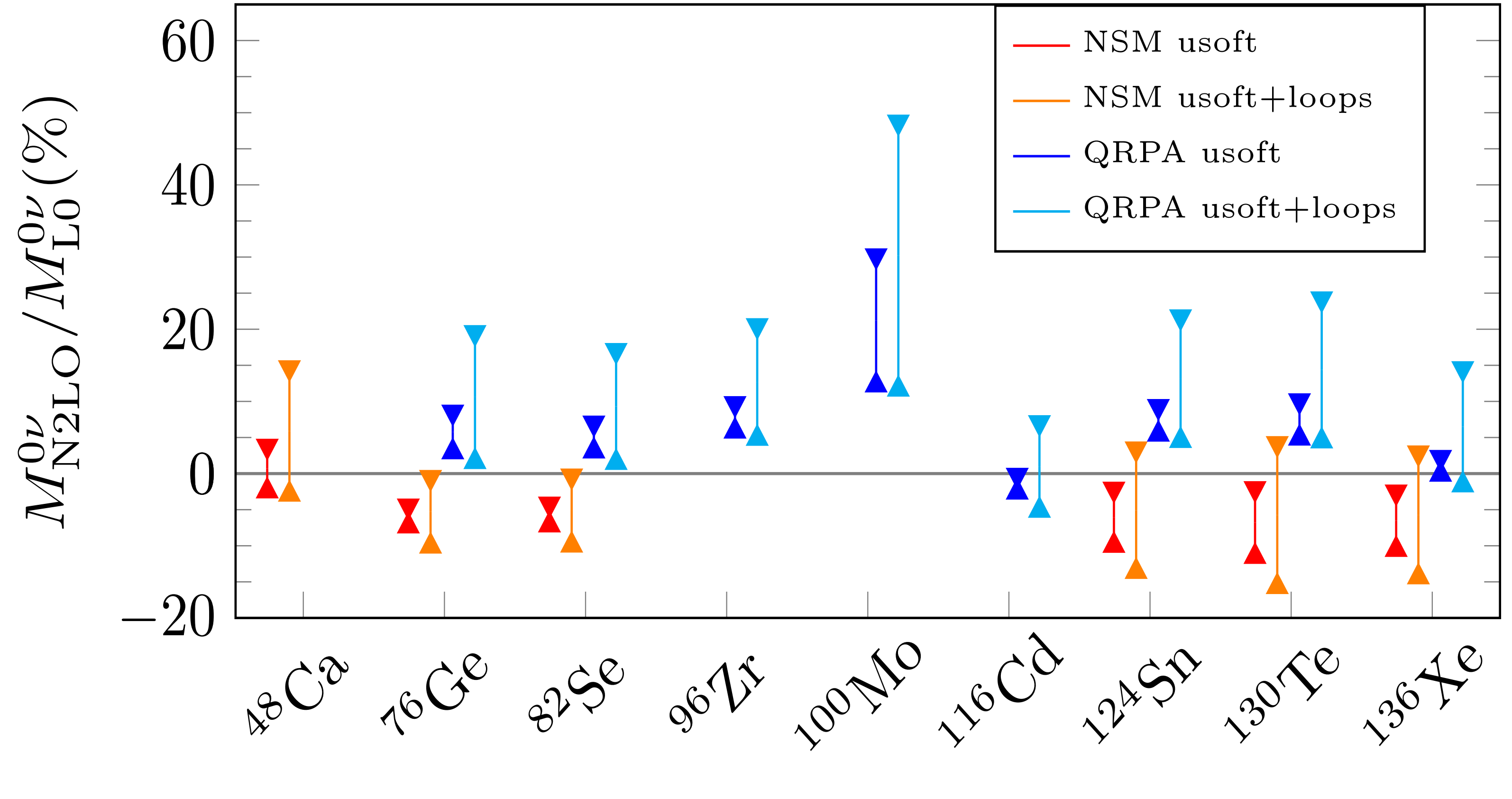


Jokiniemi, D.C. et al., in progress

TOTAL N^2LO CORRECTIONS



Jokiniemi, D.C. et al., in progress

- NSM:
  $|\mathcal{M}^{0\nu}_{N2LO}/\ \mathcal{M}^{0\nu}_{LO}| \sim (0\ \text{-}\ 15)\%$
- QRPA:
  $|\mathcal{M}^{0\nu}_{N2LO}/\ \mathcal{M}^{0\nu}_{LO}| \sim (0\ \text{-}\ 25)\%$
  For ^{100}Mo up to **50%**

- χEFT expectations ~ **10%**

- QRPA: $M^{0\nu}$ Enhancement
- NSM: $M^{0\nu}$ Reduction

- Loop: Uncertainty increase

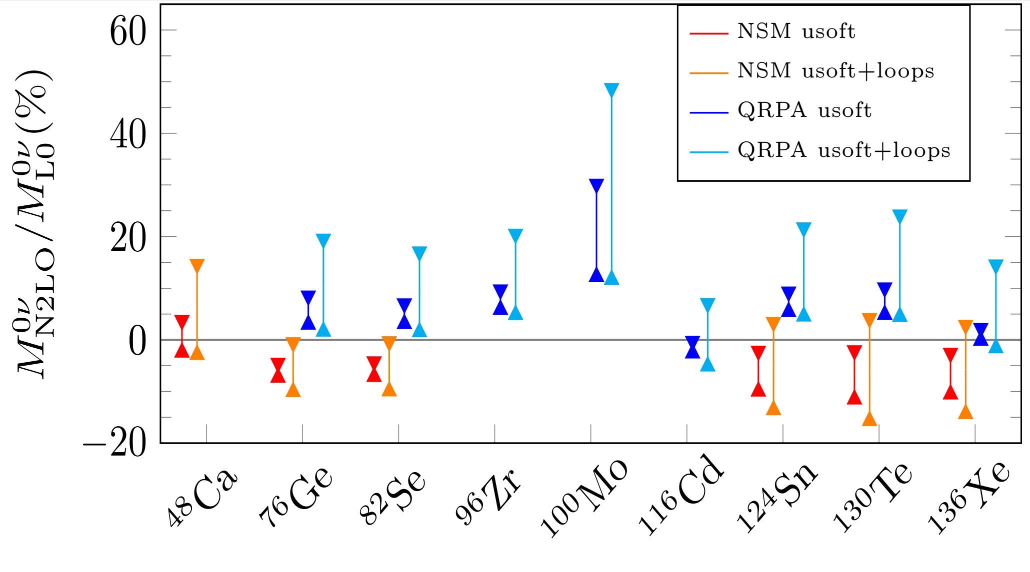
SUMMARY AND OUTLOOK

- $0\nu\beta\beta$ decay: ^{48}Ca, ^{76}Ge, ^{82}Se, ^{124}Sn, ^{130}Te and ^{136}Xe within NSM
  ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{124}Sn, ^{130}Te and ^{136}Xe within QRPA

- The usoft NME: $-\mathbf{11\%}$ correction within NSM
  $+\mathbf{11\%}$ within the QRPA model

- χEFT prediction: agreement with our results

- The N^{2}LO terms: NSM $< \mathbf{20\%}$ correction
  QRPA $< \mathbf{30\%}$ correction



- For future studies:
  - More accurate study of the uncertainties involved in the N^{2}LO corrections.
  - Improvement of interactions, SRCs, couplings' values…
  - Extend the χEFT study to N^{3}LO corrections: two-body currents

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THANK YOU FOR YOUR ATTENTION!

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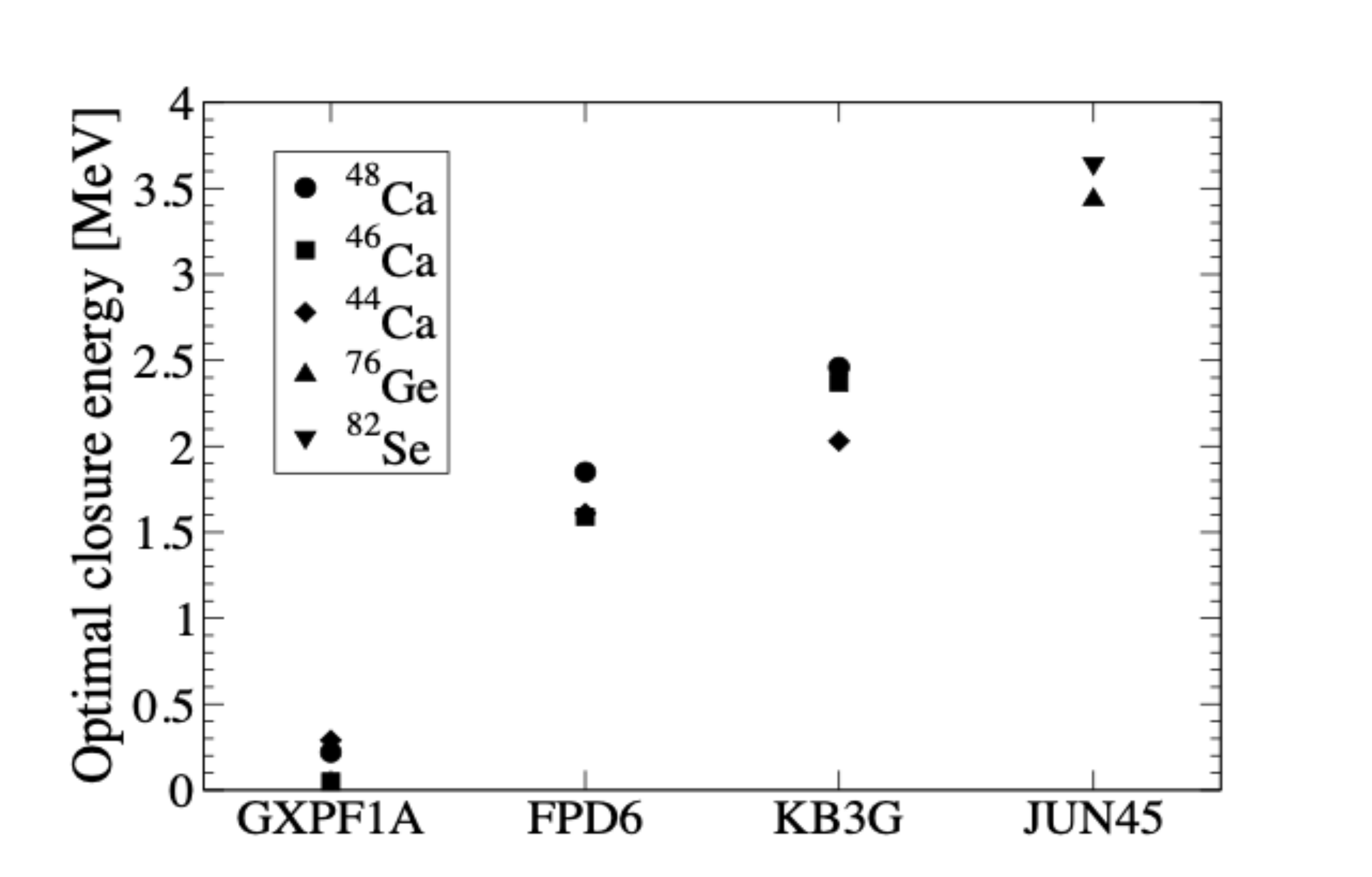
EXTRA SLIDES

NON-CLOSURE ENERGY



A. Neacsu and M. Horoi Phys. Rev. C **91**, 024309, 2015

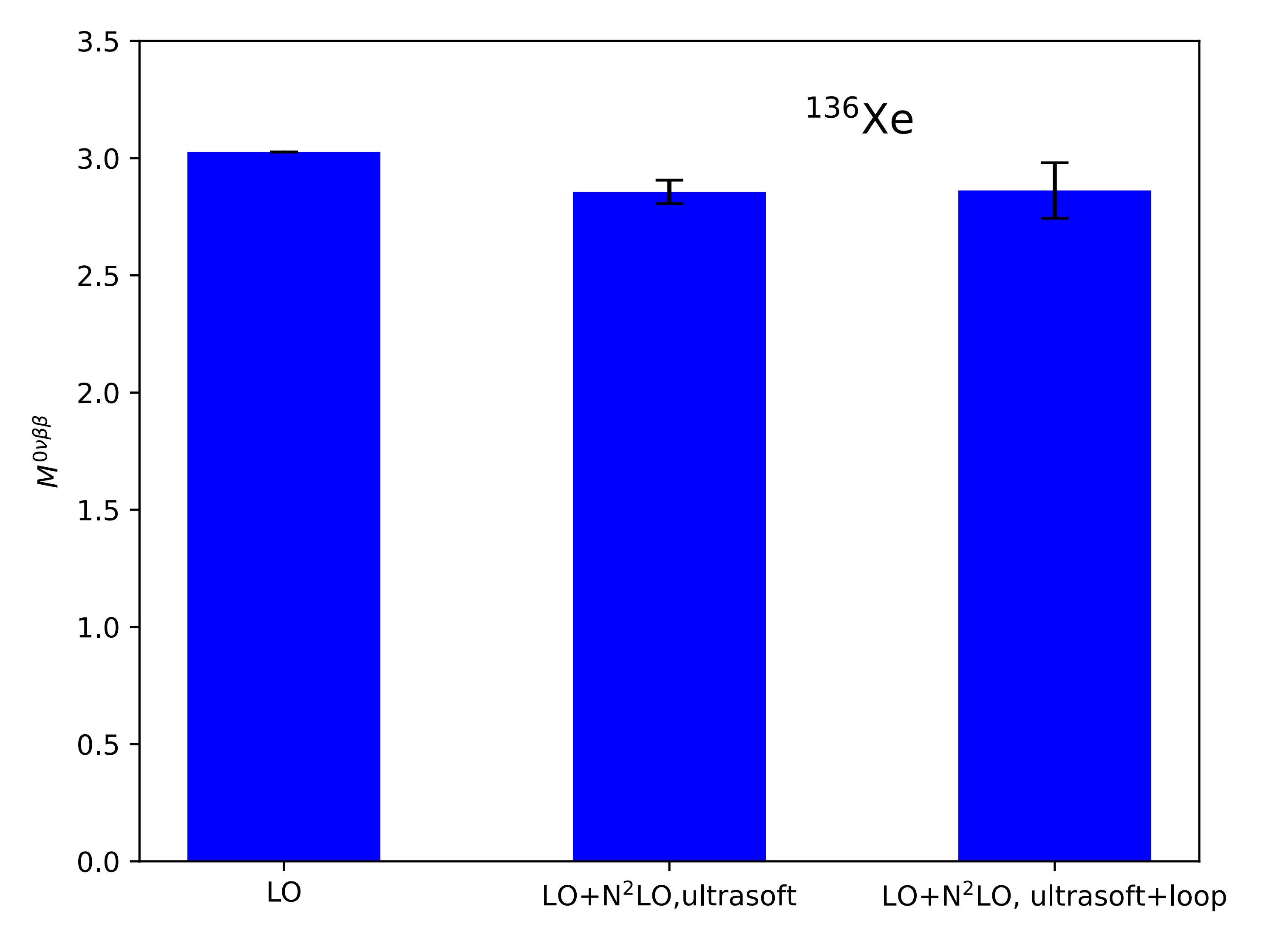
^{76}Ge:

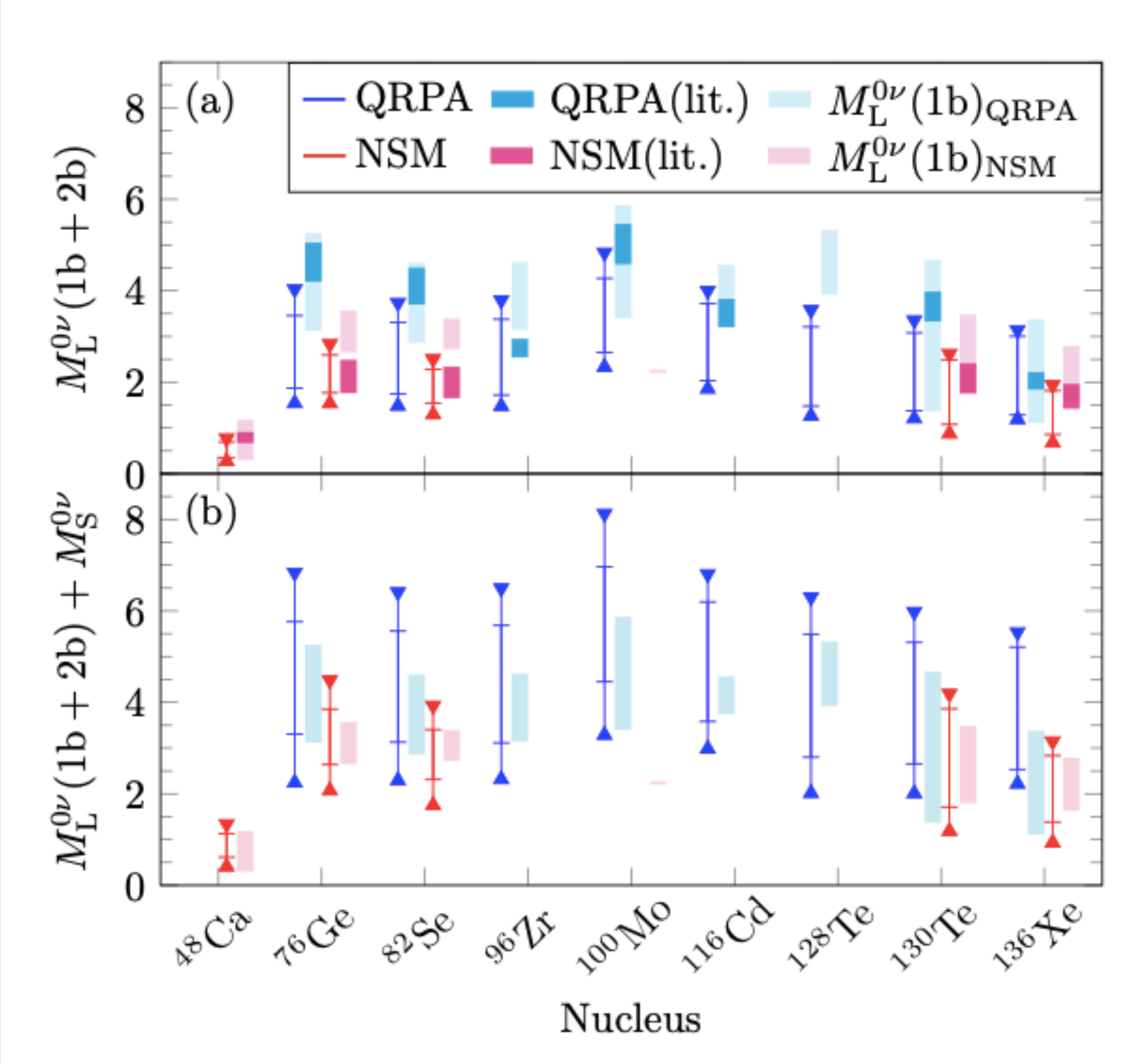
Argonne: $\mathcal{M}_{ncl}^{0\nu} - \mathcal{M}_{cl}^{0\nu} = -0.237$
CD-Bonn: $\mathcal{M}_{ncl}^{0\nu} - \mathcal{M}_{cl}^{0\nu} = -0.244$
$\mathcal{M}_{usoft}^{0\nu} = -0.262$

^{82}Se:

Argonne: $\mathcal{M}_{ncl}^{0\nu} - \mathcal{M}_{cl}^{0\nu} = -0.229$
CD-Bonn: $\mathcal{M}_{ncl}^{0\nu} - \mathcal{M}_{cl}^{0\nu} = -0.236$
$\mathcal{M}_{usoft}^{0\nu} = -0.228$

FULL $0\nu\beta\beta$ NUCLEAR MATRIX ELEMENT





Jokiniemi, et al. Phys. Rev. C 107, 044305, 2023

OBJECTIVE

- Obtain nuclear matrix elements (NMEs) for $M1M1$ transitions of [20]Ne, [48]Ti, [40]Ca and [72]Ge

$$M1 = \mu_n \sqrt{\frac{3}{4\pi}} (g_i^l \vec{l_i} + g_i^s \vec{s_i});$$

$g_i^l, g_i^s \equiv$ g-factors, $\vec{l_i} \equiv$ orbital angular momentum

$\vec{s_i} \equiv$ spin

$$\mathcal{M}^{\gamma\gamma} = \sum_n \frac{\langle 0_f^+ || M1 || 1_n^+ \rangle \langle 1_n^+ || M1 || 0_i^+ \rangle}{\epsilon_n \left(1 - \frac{\Delta\epsilon^2}{2\epsilon_n^2}\right)}$$

$$\Delta\epsilon = k_0 - k_0'$$

$k_0, k_0' \equiv$ photon energies

$k_0 = k_0{}'$

→ Approximation

$$\mathcal{M}^{\gamma\gamma} = \sum_n \frac{\langle 0_f^+ || M1 || 1_n^+ \rangle \langle 1_n^+ || M1 || 0_i^+ \rangle}{\epsilon_n}$$

$$\epsilon_n = E_n - \frac{1}{2}(E_i + E_f)$$

METHOD

1. Determine the final ($|0^+_{GS}\rangle$) and initial state ($|0^+_i\rangle$) by solving the Schrödinger equation

$$H_{eff}|0^+_{GS}\rangle = E_{GS}|0^+_{GS}\rangle, \quad H_{eff}|0^+_i\rangle = E_i|0^+_i\rangle$$

2. Apply the M1 operator for both states, obtaining

$$M1|0^+_{GS}\rangle, M1|0^+_i\rangle.$$

3. Apply the Lanczos' strength function method to expand

$$M1|0^+_i\rangle = \sum_{n=1}^{max} a_n|1^+_n\rangle .$$

4. Calculate the necessary overlaps to get the NMEs

$$\langle 0^+_{GS}|M1|1^+_n\rangle.$$

$$\mathcal{M}^{\gamma\gamma} = \sum_{n}^{max} \frac{\langle 0^+_{GS}||M1||1^+_n\rangle\langle 1^+_n||M1||0^+_i\rangle}{\epsilon_n}$$

↓

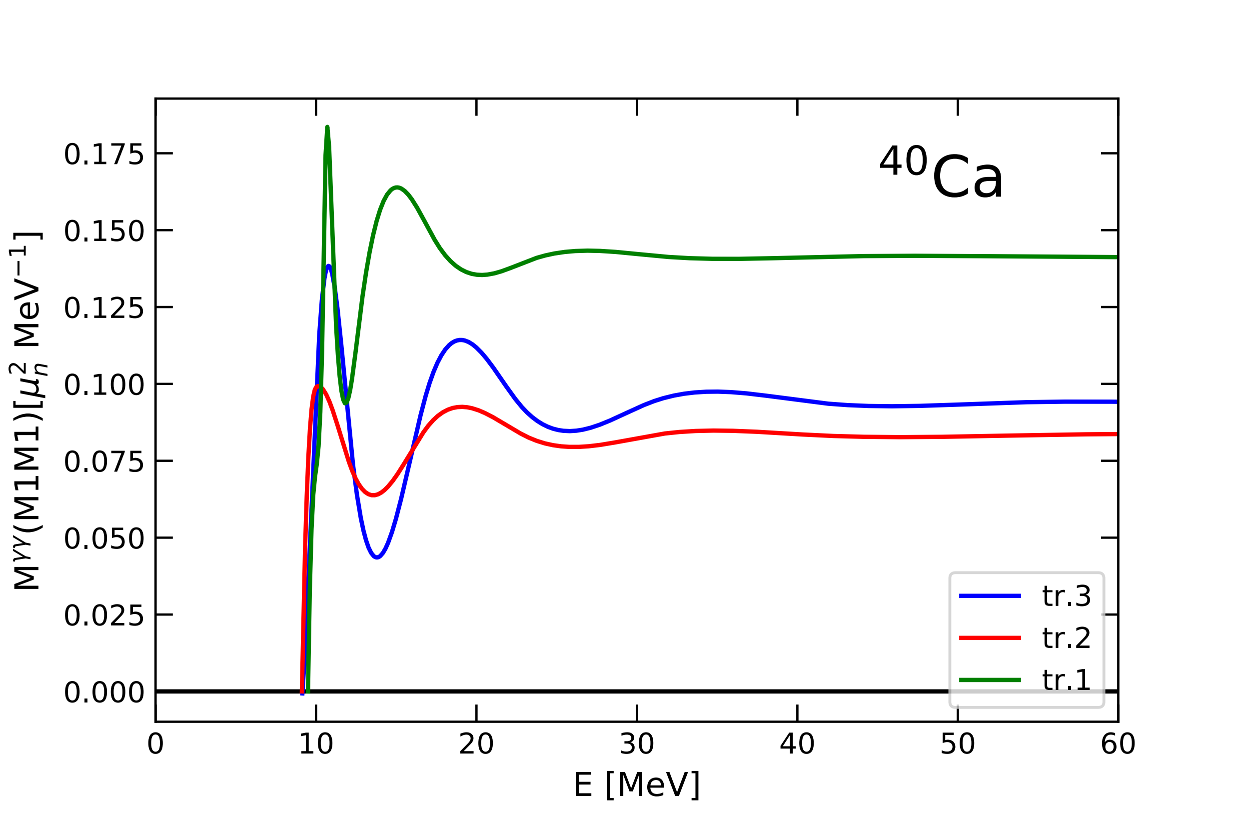
Convergence test of Lanczos' method:

$$\sum_n \langle 0^+_{GS}||M1||1^+_n\rangle\langle 1^+_n||M1||0^+_i\rangle = \langle 0^+_{GS}||M1M1||0^+_i\rangle$$

^{40}Ca

- ^{40}Ca $(0_2^+ \rightarrow 0_{GS}^+)$
  Valence space: sd- and pf-shell
  $0d_{5/2}$ full occupied
  Interaction: sd.pf.ca40.pcr [4]

Dimension is too large:
10^{12} Slater determinants.
Truncate the valence space.



- Tr.1 (no $1p_{1/2}$ orbital) → $\mathcal{M}^{\gamma\gamma} = 0.14\ \mu_n^2$ MeV-1
- Tr.2 (pf $shell$) → $\mathcal{M}^{\gamma\gamma} = 0.083\ \mu_n^2$ MeV-1
- Tr.3 $(0f_{5/2}, 1p_{3/2}, 1p_{1/2})$ → $\mathcal{M}^{\gamma\gamma} = 0.094\ \mu_n^2$ MeV-1

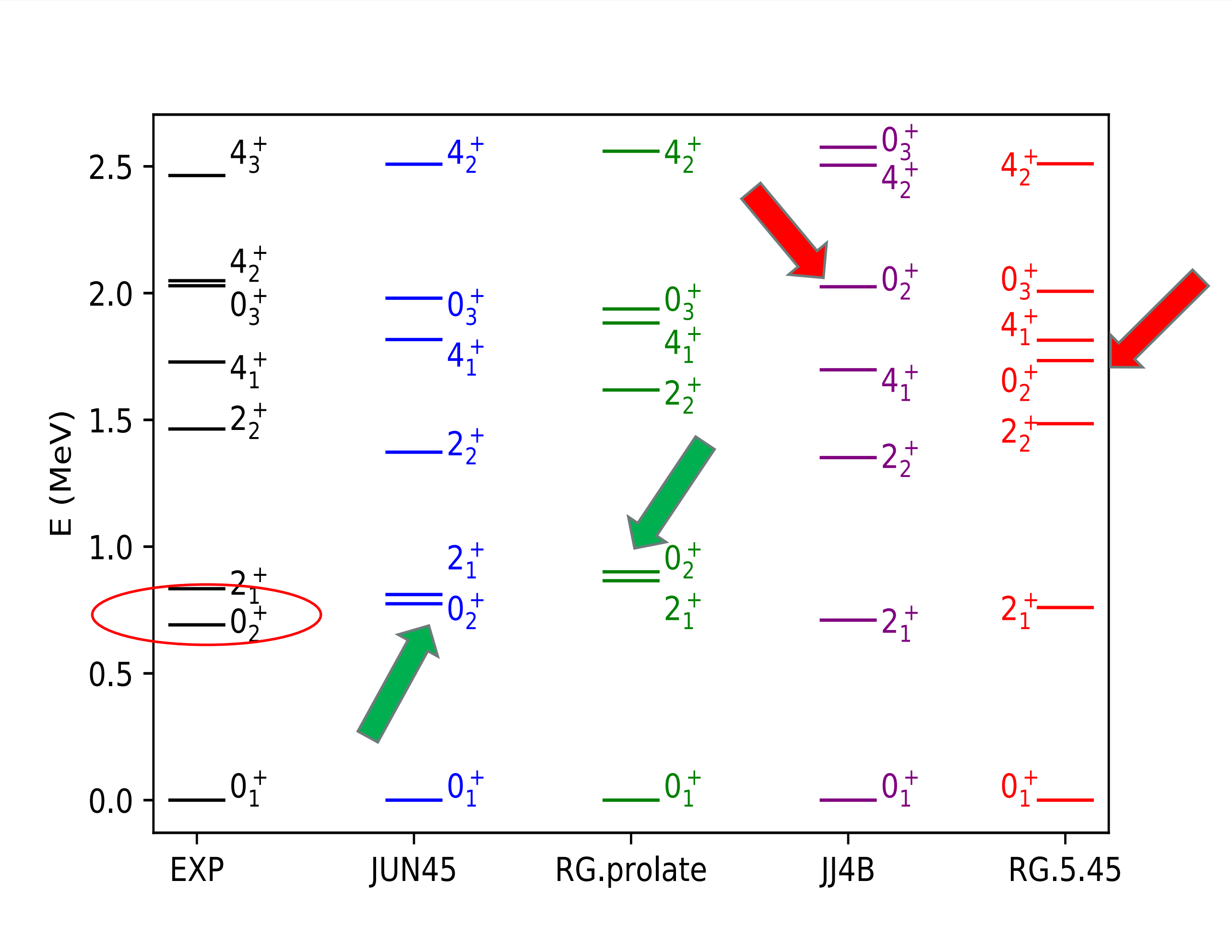
- Tr.1 → $Q_{EM} = 5.25$ MeV
- Tr.2 → $Q_{EM} = 3.49$ MeV
- Tr.3 → $Q_{EM} = 3.91$ MeV
- EXP. → $Q_{EM} = 3.35$ MeV

[5] A. Poves, F. Nowacki. "The nuclear shell model". Springer:070101 (2001)

^{72}Ge INTERACTIONS

- ^{72}Ge $(0_2^+ \rightarrow 0_{GS}^+)$
  Valence space: r3g
  $(1p_{3/2}, 0f_{5/2}, 1p_{1/2}, 0g_{9/2})$
  Interactions [5]:
  - JUN45
  - RG.prolate
  - JJ4B
  - RG.5.45

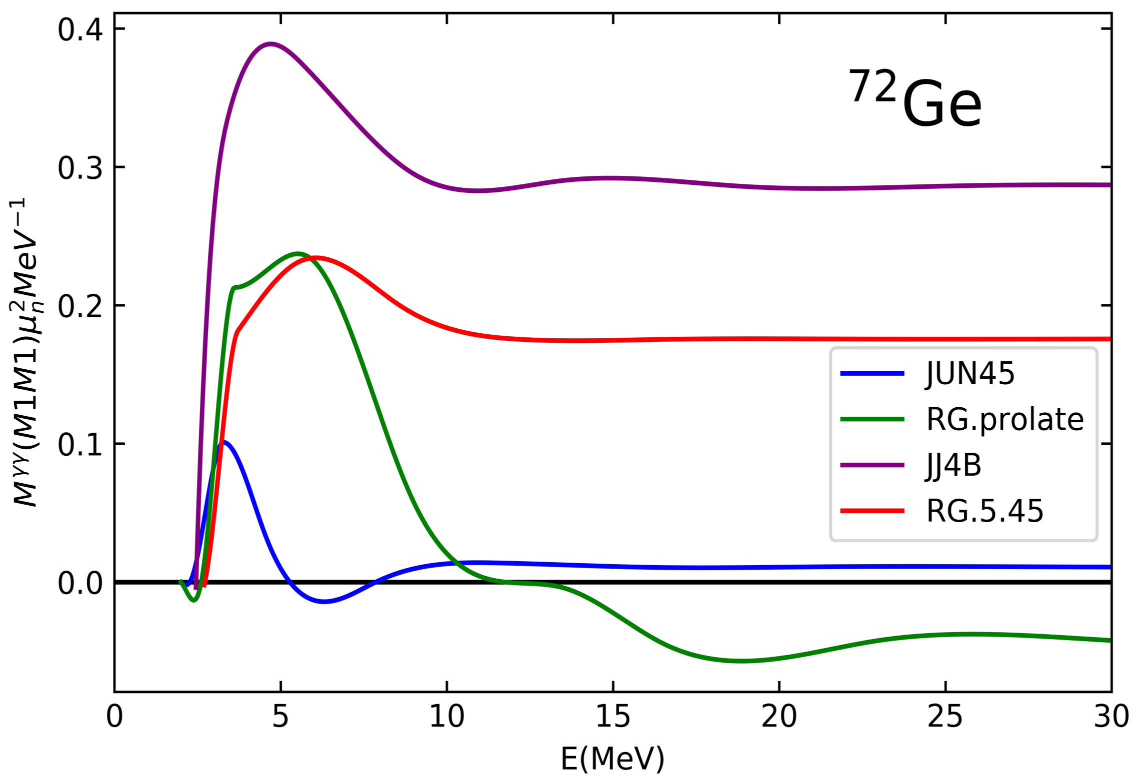
- Energy difference of experimental 0_2^+ with theoretical 0_2^+:
  - JUN45: $\Delta E = 0.08$ MeV
  - RG.prolate: $\Delta E = 0.21$ MeV
  - JJ4B: $\Delta E = 1.34$ MeV
  - RG.5.45: $\Delta E = 1.04$ MeV



[5] B. Romeo, J. Menéndez, C. Peña Garay. Phys. Lett. B 827: 136965 (2022).

^{72}Ge NUCLEAR MATRIX ELEMENTS

- JUN45:
  $\mathcal{M}^{\gamma\gamma} = 0.011\ \mu_n^2$ MeV^{-1}
- RG.prolate:
  $\mathcal{M}^{\gamma\gamma} = -0.043\ \mu_n^2$ MeV^{-1}
- JJ4B:
  $\mathcal{M}^{\gamma\gamma} = 0.29\ \mu_n^2$ MeV^{-1}
- RG.5.45:
  $\mathcal{M}^{\gamma\gamma} = 0.19\ \mu_n^2$ MeV^{-1}



JUN45
RG.prolate
} Cancellation

JJ4B
RG.5.45
} Dominance of the first contribution

SUMMARY AND OUTLOOK

- We have studied the second-order $M1M1$ transitions for the nuclei in the shell-model.
- For ^{40}Ca, tr.2 have the most similar transition energy compared to experimental data.
  $\mathcal{M}^{\gamma\gamma} = 0.08\, \mu_n^2$ MeV^{-1} is the most reliable NME.
- For ^{72}Ge, the NME could be between, $\mathcal{M}^{\gamma\gamma} = 0.01 - 0.04\, \mu_n^2$ MeV^{-1},
  we can not disregard larger values, $\mathcal{M}^{\gamma\gamma} = 0.18 - 0.29\, \mu_n^2$ MeV^{-1}.
- Largest value obtained is the ^{48}Ti NME, $\mathcal{M}^{\gamma\gamma} = 0.97\, \mu_n^2$ MeV^{-1}.
- $M1M1$ NME are sensitive on the nuclear interaction and final and initial states of the transition.

- **For future studies:**
  - Compared our results to NME calculated without assuming that the two photons energies equal.
  - Calculate half-lives and energy widths to compare the probability of observing these decays with the first-order EM transitions..