Two-neutrino double-beta decay to excited states of heavy nuclei

Beatriz Benavente de Lucas

Collaboration: Javier Ménendez, Dorian Frycz
Motivation

- Neutrinoless double-beta decay violates the Standard model of particles physics (D. Castillo’s previous talk)
- Two-neutrino double-beta decay is permitted in the Standard model of particles physics
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**Beta decay**

- **Antineutrino**

**Double beta decay**

- **Antineutrinos**

**Neutrinoless double beta decay**

- **e^-**

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- Initial and final states are common in both cases
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Study of $2\nu\beta\beta$ decay $\rightarrow$ information for $0\nu\beta\beta$ decay
- Initial and final states are common in both cases
- Many-body methods applicable in both cases
Motivation

- Experimental interest in the decay to the first excited $0^+$ state

![Diagram](https://www.nndc.bnl.gov)

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- Decay to ground state measured for 13 nuclei

![Diagram showing decay processes and energies](image.png)

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- $^{76}$Ge $\rightarrow$ $^{76}$Se, $E(76\text{Se}, 0^+_2) = 1.122$ MeV.
- $^{82}$Se $\rightarrow$ $^{82}$Kr, $E(82\text{Kr}, 0^+_2) = 1.488$ MeV.
- $^{130}$Te $\rightarrow$ $^{130}$Xe, $E(130\text{Xe}, 0^+_2) = 1.793$ MeV.


Nuclear shell model

Schrödinger equation:

\[ H_{\text{eff}} |\psi_{\text{eff}}\rangle = E |\psi_{\text{eff}}\rangle \]
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Restriction to the valence space:

\(^{76}\text{Ge}, \ ^{82}\text{Se}: (1p_{3/2}, 0f_{5/2}, 1p_{1/2}, 0g_{9/2})\)

\(^{130}\text{Te}: (0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2})\)
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**ANTOINE** shell-model code:

Lanczos method


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Linear combinations of the Slater determinants:

\[ |\psi\rangle = \sum_\alpha C_\alpha |\phi_\alpha\rangle \]

\[ |\phi_\alpha\rangle = \sum_{i=nljm} a_i^\dagger |0\rangle \]

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Two-neutrino $\beta \beta$ decay

- $\beta$ decay governed by the Gamow-Teller operator: $\sigma \tau^-$

Matrix element:

$$M_{\nu}^2 = X_n \langle 0^+ f | P_a \sigma^a \tau^{-a} | 1^+ n \rangle \langle 1^+ n | P_b \sigma^b \tau^{-b} | 0^+ i \rangle E_n - (E_i - E_f) / 2 E_n$$

- Energy of intermediate state $E_i$
- Energy of initial state $E_f$
- Energy of final state $E_n$

Correction by quenching factor, $q$, on the matrix elements

Half-life:

$$T_{\nu} = \frac{G_{\nu}}{G_{\nu}} \frac{4 A}{m_e c^2} M_{\nu}^2$$

- $G_{\nu}$: phase-space factor
- $A$: axial coupling
- $m_e$: electron mass
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**Two-neutrino $\beta\beta$ decay**

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- Correction by quenching factor, $q$, on the matrix elements
- Half-life:

$$\left( T_{1/2}^{2\nu} \right)^{-1} = G^{2\nu} g_A^4 \left( M^{2\nu} m_e c^2 \right)^2$$

- $G^{2\nu}$: phase-space factor
- $g_A$: axial coupling
- $m_e$: electron mass
4 different interactions
No interaction significantly better than the others
RGPROLATE: least descriptive of the nuclear structure
Spectrum

- 4 different interactions
- No interaction significantly better than the others
- RGPROLATE: least descriptive of the nuclear structure

Occupation study: \[ |n_{g_9/2} n_{p_{1/2}} n_{f_{5/2}} n_{p_{3/2}} p_{g_{9/2}} p_{p_{1/2}} p_{f_{5/2}} p_{p_{3/2}} \rangle \]

JJ4BB: \[ |0^{+}_{gs} \rangle = 0.60 |8 2 6 4 0 0 4 2 \rangle + \ldots \]

JUN45: \[ |0^{+}_{gs} \rangle = 0.62 |8 2 6 4 0 0 4 2 \rangle + \ldots \]

RG545: \[ |0^{+}_{gs} \rangle = 0.61 |8 2 6 4 0 0 4 2 \rangle + \ldots \]

RGPROLATE: \[ |0^{+}_{gs} \rangle = 0.44 |1 0 0 6 4 0 0 6 0 \rangle + 0.32 |1 0 0 6 4 0 0 4 2 \rangle + \ldots \]
- JJ4BB, RGPROLATE: best description of $0_2^+$ energy
- JUN45, RG545: higher $0_2^+$ energy prediction
- $W_f$ describing $0_2^+$ state fragmented across all interactions
- JJ4BB, RGPROLATE: best description of $0^+_2$ energy
- JUN45, RG545: higher $0^+_2$ energy prediction
- Wf describing $0^+_2$ state fragmented across all interactions
Matrix elements

\[ M^{2\nu} = \sum_n \frac{\langle 0_f^+ | \sum_a \sigma_a \tau_a^- | 1_n^+ \rangle \langle 1_n^+ | \sum_b \sigma_b \tau_b^- | 0_i^+ \rangle}{E_n - (E_i - E_f)/2} \]
Matrix elements

\[ M_{n_{\text{max}}}^{2\nu} = \sum_{n}^{n_{\text{max}}} \frac{\langle 0_f^+ \| \sum_a \sigma_a \tau_a^- \| 1_n^+ \rangle \langle 1_n^+ \| \sum_b \sigma_b \tau_b^- \| 0_i^+ \rangle}{E_n - (E_i - E_f)/2} \]

• Convergence of matrix elements
• Running matrix element: truncation of matrix elements for \( n \) intermediate states
• Calculated using Lanczos strength function

\[^{130}\text{Te}(0_{gs}^+) \rightarrow ^{130}\text{Xe}(0^+)\]
Matrix elements

- **JJ4BB and RGPROLATE:**
  - no cancellation of terms,
  - larger running matrix element

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Projected generator coordinate method (PGCM)

Variational approach:

- Configuration mixing of Hartree-Fock-Bogoliubov (HFB) states:
  \[ \Psi_{\text{GCM}} = \sum_q f_q \phi_{\text{HFB}}(q) \]

- Similar deformations for all interactions


Figure: Contribution of each HFB wavefunction to fully mixed state for \(^{82}\text{Kr} (0^+_2)\) with all interactions

D. Frycz: 11/07, 17:30h, M1
Quenching

Quenching factor to correct the overestimation of matrix elements

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  Predicted half-life g.s. to g.s. $\xrightarrow{q_{2\nu}}$ Experimental half-life g.s. to g.s.

  A. Barabash. Universe. 6(10): 159 (2020)
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<tr>
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Half-life predictions

Experimental limit: \( T_{1/2}^{2\nu}(^{82}\text{Se}, 0_{gs}^+ \rightarrow ^{82}\text{Kr}, 0_2^+) > 1.3 \cdot 10^{21}\text{yr} \)

Range of predictions: \( T_{1/2}^{2\nu}(^{82}\text{Se}, 0_{gs}^+ \rightarrow ^{82}\text{Kr}, 0_2^+) = (3.5 - 170) \cdot 10^{21}\text{yr} \)
Half-life predictions

- Prediction: longer half-lives than most obtained with other methods

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Half-life predictions

- Prediction: longer half-lives than most obtained with other methods
- Prediction consistent with experimental limit

Experimental limit: \( T_{1/2}^{2\nu}(^{82}\text{Se}, 0_{gs}^+ \rightarrow ^{82}\text{Kr}, 0_{2}^+) > 1.3 \cdot 10^{21} \text{yr} \)

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Half-life predictions

Range of predictions:

\[ T_{1/2}^{2\nu}(\text{Ge}, 0_{gs}^+ \rightarrow \text{Se}, 0^+_2) = (2 - 260) \cdot 10^{24} \text{yr} \]

Range of predictions:

\[ T_{1/2}^{2\nu}(\text{Te}, 0_{gs}^+ \rightarrow \text{Xe}, 0^+_2) = (7.7 - 130) \cdot 10^{25} \text{yr} \]

References:


Summary

- Study of 3 different $2\nu\beta\beta$ decays with different interactions in the context of the NSM
- Many-body methods applicable in the study of the neutrinoless double-beta decay
- Different running matrix elements, but are yet to find a plausible explanation for it
- Predicted half-lives consistent with experimental limits, close to it in some cases

The discrepancies between the prediction of half-lives makes the testing of these values a good way of validating the many-body methods used in the study. We really look forward to these results being tested!
Thank you!