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# (VAH)-Vortical Waves in a plasma with massless fermions

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Sergio Morales Tejera<sup>1</sup>, Victor E. Ambruș<sup>1</sup>, Maxim Chernodub<sup>1,2</sup>

<sup>1</sup>Physics Faculty, West University of Timișoara, Romania

<sup>2</sup>Institut Denis Poisson, Université de Tours, France

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SMT, VEA & MC, arXiv:2403.19755; arXiv:2403.19756

The background features a series of concentric circles centered on the page. The innermost circle is a dark blue color, while the subsequent circles become progressively lighter, transitioning to a very light blue for the outermost visible circles. The word "Motivation" is centered within the innermost dark blue circle.

**Motivation**

# Motivation: QGP, Anomalous Transport and Helicity

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  - Chiral Separation Effect (CSE)  $J^5 \sim \mu B$

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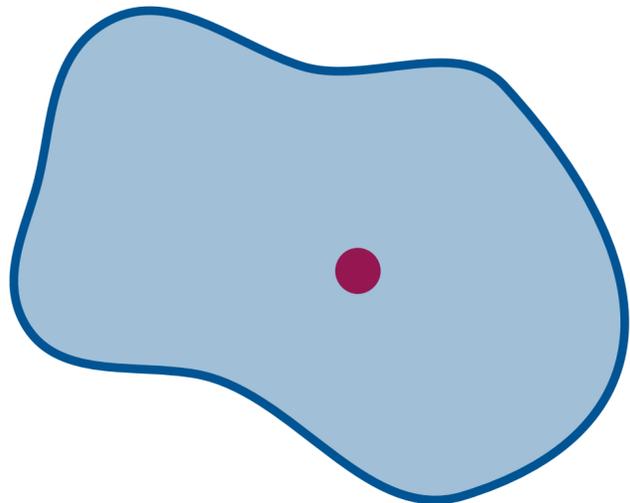
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# Motivation: QGP, Anomalous Transport and Helicity

- These studies on anomalous transport were focused on the vector (electric) and axial (chiral) degrees of freedom, related to the conserved charges of the  $U(1)_V \times U(1)_A$  symmetry of the Lagrangian.
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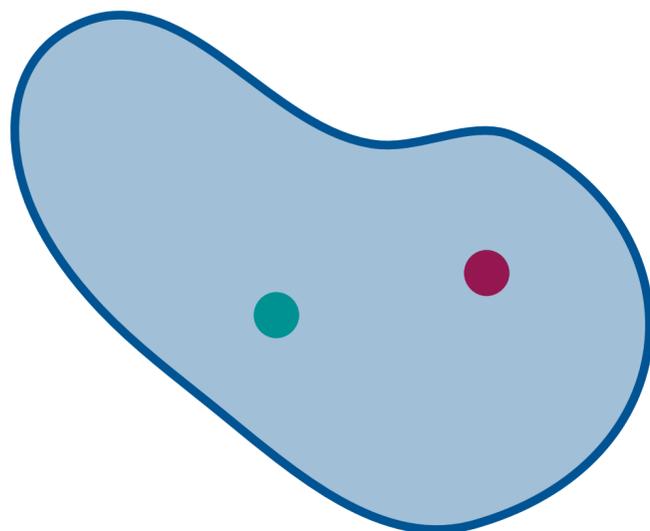
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|           | Particle |   | Anti-particle |   |
|-----------|----------|---|---------------|---|
| Chirality | +        | - | +             | - |
| Helicity  | +        | - | -             | + |

# Motivation: QGP, Anomalous Transport and Helicity

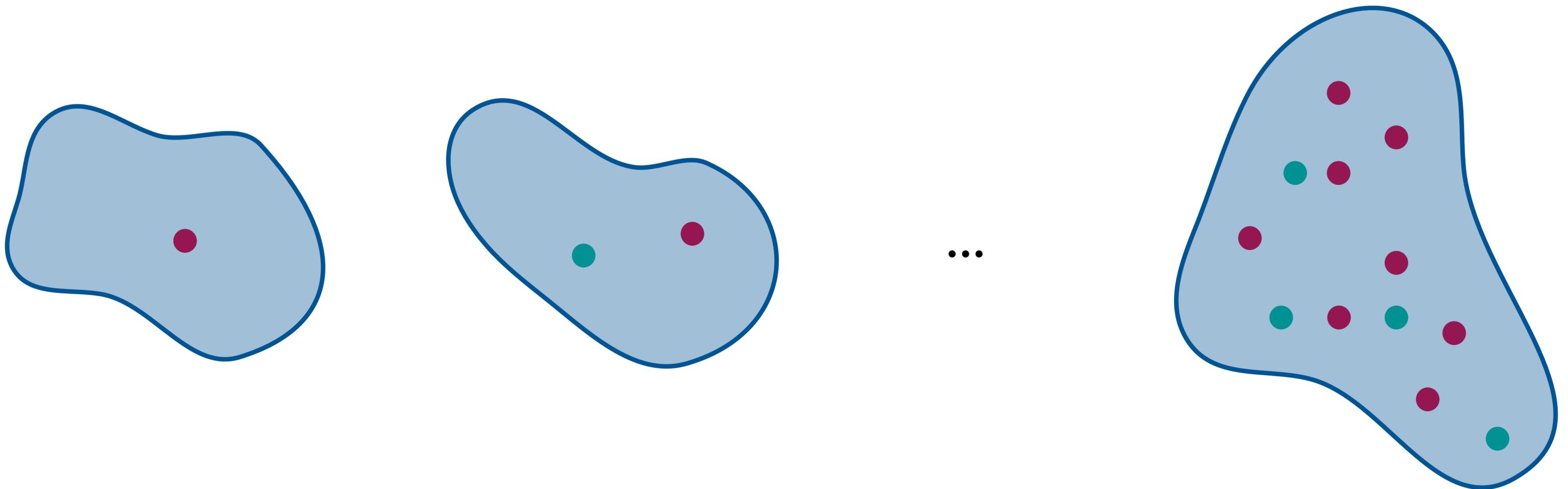
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|           | Particle/Anti-particle |          |           |           |
|-----------|------------------------|----------|-----------|-----------|
| Chirality | +/+<br>2               | +/-<br>0 | -/+<br>0  | -/-<br>-2 |
| Helicity  | +/-<br>0               | +/+<br>2 | -/-<br>-2 | -/+<br>0  |

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- These studies on anomalous transport were focused on the vector (electric) and axial (chiral) degrees of freedom, related to the conserved charges of the  $U(1)_V \times U(1)_A$  symmetry of the Lagrangian.
- There is another conserved charge: **Helicity**.
- Analogous to the CME, CVE... Transport phenomena has been uncovered involving the helicity current, as the **Helical Separation Effect** (HSE)  $J^H \sim TB$
- The inclusion of the helical degree of freedom is **necessary** for the study of collective excitations. It modifies the previously discussed CMW and CVW.

**Disclaimer:** Helicity is not conserved exactly. Its violation comes from pair creation-annihilation processes e.g.  $e_R^+ e_L^- \rightarrow e_L^+ e_R^-$ .

Axial charge is not conserved due to the axial anomaly.

# Motivation: QGP, Anomalous Transport and Helicity

## Goal

- Study the collective excitations of a fluid consisting of massless (anti)fermions in the presence of rotation.



**Setup**

# Hydrodynamic setup:

## Conservation Equations and Longitudinal Perturbations

- The waves are characterised by the dynamics of the (approximately)-conserved quantities.
- In the problem under consideration energy and momentum are conserved, as well as the vector, axial and helical charges.

- Therefore, we **impose** the **conservation equations**:

$$\partial_{\mu} T^{\mu\nu} = 0 \quad \partial_{\mu} J_V^{\mu} = 0 \quad \partial_{\mu} J_A^{\mu} = 0 \quad \partial_{\mu} J_H^{\mu} = 0$$

- These operators are found by means of the Noether's theorem and for a free massless fermion they are

$$T^{\mu\nu} = \frac{i}{2} \bar{\psi} \gamma^{(\mu} \partial^{\nu)} \psi, \quad J_V^{\mu} = \bar{\psi} \gamma^{\mu} \psi, \quad J_A^{\mu} = \bar{\psi} \gamma^{\mu} \gamma^5 \psi, \quad J_H^{\mu} = \bar{\psi} \gamma^{\mu} h \psi + \overline{h \psi} \gamma^{\mu} \psi.$$

# Hydrodynamic setup:

## Conservation Equations and Longitudinal Perturbations

- We **impose** the **conservation equations**:

$$\partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu J_V^\mu = 0 \quad \partial_\mu J_A^\mu = 0 \quad \partial_\mu J_H^\mu = 0$$

- We evaluate their expectation values  $\langle \cdot \rangle = Z^{-1} \text{Tr}\{\rho \cdot\}$  in a **thermal state of rotating massless fermions** described by the density operator:

$$\hat{\rho} = \exp \left[ -\beta(\hat{H} - \Omega \hat{J}^z - \boldsymbol{\mu} \cdot \hat{\mathbf{Q}}) \right] \quad \boldsymbol{\mu} = (\mu_V, \mu_A, \mu_H)$$

where we have introduced a chemical potential for each of the conserved U(1) charges.

- The above density operator induces a **preferred hydrodynamic frame**, the  $\beta$ -frame, defined by the following 4-velocity:

$$u_\Omega^\mu \partial_\mu = \Gamma(\partial_t + \Omega \partial_\varphi), \quad \Gamma_\Omega = \frac{1}{\sqrt{1 - \rho^2 \Omega^2}}$$

# Hydrodynamic setup:

## Conservation Equations and Longitudinal Perturbations

- As is customary in relativistic hydro, we decompose the expectation values in a particular frame.

$$J_\ell^\mu \simeq Q_{\ell;\beta} u_\Omega^\mu + \sigma_{\ell;\beta}^\omega \omega_\Omega^\mu, \quad T^{\mu\nu} \simeq (E_\beta + P_\beta) u_\Omega^\mu u_\Omega^\nu - P_\beta g^{\mu\nu} + \sigma_{\varepsilon;\beta}^\omega (\omega_\Omega^\mu u_\Omega^\nu + u_\Omega^\mu \omega_\Omega^\nu).$$

- The scalar quantities (Energy, Pressure, Charges, Conductivities) can be computed as a classical term (plus quadratic corrections which we neglect). In particular

$$P_{\beta;\text{cl}} = \frac{1}{3} \sum_{\sigma,\lambda} \int dP (p \cdot u) f_{\mathbf{p},\lambda}^{\text{eq};\sigma} = -\frac{T^4}{\pi^2} \sum_{\sigma,\lambda} \text{Li}_4(-e^{\mu_{\sigma,\lambda}/T}),$$

$$Q_{\ell;\beta;\text{cl}} = \frac{\partial P_{\beta;\text{cl}}}{\partial \mu_\ell} \quad \sigma_{\varepsilon;\text{cl}}^\omega = Q_{A;\text{cl}}$$

$$\sigma_{\ell;\beta;\text{cl}}^\omega = \frac{1}{2} \frac{\partial P}{\partial \mu_A \partial \mu_\ell}$$

$$f_{\mathbf{p},\lambda}^{\text{eq};\sigma} = \frac{1}{(2\pi)^3} \left[ \exp\left(\frac{p \cdot u - \mu_{\sigma,\lambda}}{T}\right) + 1 \right]^{-1}$$

$$\mu_{\sigma,\lambda} = \mathbf{q} \cdot \boldsymbol{\mu} = \sigma \mu_V - 2\sigma \lambda \mu_H - 2\lambda \mu_A,$$

$$\text{Li}_s(z) = \sum_{n=1}^{\infty} z^n / n^s$$

# Hydrodynamic setup: Conservation Equations and Longitudinal Perturbations

- In summary:

$$\partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu J_V^\mu = 0 \quad \partial_\mu J_A^\mu = 0 \quad \partial_\mu J_H^\mu = 0$$

$$T^{\mu\nu} = (E + P)u^\mu u^\nu - P g^{\mu\nu}, \quad J_\ell^\mu = Q_\ell u^\mu + \sigma_\ell^\omega \omega^\mu$$

$$P = \frac{E}{3} \simeq -\frac{T^4}{\pi^2} \sum_{\sigma,\lambda} \text{Li}_4(-e^{\mu_{\sigma,\lambda}/T}), \quad Q_\ell \simeq \frac{\partial P}{\partial \mu_\ell}, \quad \sigma_\ell^\omega = \frac{1}{2} \frac{\partial^2 P}{\partial \mu_\ell \partial \mu_A} - \frac{1}{4P} \frac{\partial P}{\partial \mu_\ell} \frac{\partial P}{\partial \mu_A}$$

Neglecting quantum corrections of order  $\Omega^2$

# Hydrodynamic setup:

## Conservation Equations and Longitudinal Perturbations

- Upon substitution of the constitutive relations, the previous equations turn into

$$\begin{aligned}
 DE + (E + P)\theta &= 0 & DQ_V + Q_V\theta + \omega^\mu \partial_\mu \sigma_V^\omega + \sigma_V^\omega \partial_\mu \omega^\mu &= 0 \\
 (E + P)Du^\mu - \nabla^\mu P &= 0 & DQ_A + Q_A\theta + \omega^\mu \partial_\mu \sigma_A^\omega + \sigma_A^\omega \partial_\mu \omega^\mu &= 0 \\
 & & DQ_H + Q_H\theta + \omega^\mu \partial_\mu \sigma_H^\omega + \sigma_H^\omega \partial_\mu \omega^\mu &= 0
 \end{aligned}$$

$$\begin{aligned}
 D &= u^\mu \partial_\mu \\
 \nabla^\mu &= \Delta^{\mu\nu} \partial_\nu \\
 \theta &= \partial_\mu u^\mu
 \end{aligned}$$

- We study perturbations around the rigidly rotating state along the rotation axis

$$\begin{aligned}
 u^\mu \partial_\mu &= u_\Omega^\mu \partial_\mu + \delta u e^{-ik(vt-z)} \partial_z, & \delta P &= \frac{\partial P}{\partial T} \delta T + \frac{\partial P}{\partial \mu_\ell} \delta \mu_\ell, & \delta Q_\ell &= \frac{\partial Q_\ell}{\partial T} \delta T + \frac{\partial Q_\ell}{\partial \mu_{\ell'}} \delta \mu_{\ell'}, \\
 Q_\ell &= Q_{\ell;0} + \delta Q_\ell e^{-ik(vt-z)}, \\
 P &= P_0 + \delta P e^{-ik(vt-z)}, \\
 \omega^\mu \partial_\mu &= \Omega \partial_z + \Omega \delta u e^{-ik(vt-z)} \left( \partial_t - \frac{i}{2} k v \rho \partial_\rho \right).
 \end{aligned}$$

# Hydrodynamic setup: Energy-Momentum sector

$$DE + (E + P)\theta = 0$$

$$(E + P)Du^\mu - \nabla^\mu P = 0$$

- The energy momentum sector gives a closed system for  $\delta P$  and  $\delta u$

$$\begin{pmatrix} -3v & 4P \\ 1 & -4Pv \end{pmatrix} \begin{pmatrix} \delta P \\ \delta u \end{pmatrix} = 0$$

whose non trivial solutions are the **sound modes**  $v = \pm 1/\sqrt{3}$

- It also admits the trivial solution  $\delta u = \delta P = 0$ . The non-trivial excitations in the charge sector satisfy this condition.

# Hydrodynamic setup: Charge sector

- The equations for the perturbations in the charge sector read:

$$(v\delta Q_\ell - \Omega\delta\sigma_\ell^\omega) \Big|_{\delta P=0} = 0$$

- The thermodynamic quantities naturally depend on the chemical potentials, thus we rewrite the equations in terms of  $\delta\mu_\ell$

$$\mathbb{M}_{\ell\ell'}\delta\mu_{\ell'} = 0$$

- The problem reduces to diagonalising  $\mathbb{M}$ .

$$\mathbb{M}_{\ell\ell'} = v \underbrace{\left( \frac{\partial Q_\ell}{\partial \mu_{\ell'}} - \frac{3Q_\ell Q_{\ell'}}{sT} + \frac{Q_{\ell'} \vec{\mu}}{sT} \cdot \frac{\partial Q_\ell}{\partial \vec{\mu}} \right)}_{\mathbb{M}_v} - \Omega \underbrace{\left( \frac{\partial \sigma_\ell^\omega}{\partial \mu_{\ell'}} - \frac{2\sigma_\ell^\omega Q_{\ell'}}{sT} + \frac{Q_{\ell'} \vec{\mu}}{sT} \cdot \frac{\partial \sigma_\ell^\omega}{\partial \vec{\mu}} \right)}_{\mathbb{M}_\Omega}$$

The image features a series of concentric circles centered on the page. The innermost circle is a dark blue color and contains the text "Results I". The subsequent circles are lighter shades of blue, with the outermost circle being a very light, almost white, blue. The circles are evenly spaced and create a sense of depth and focus on the central text.

**Results I**

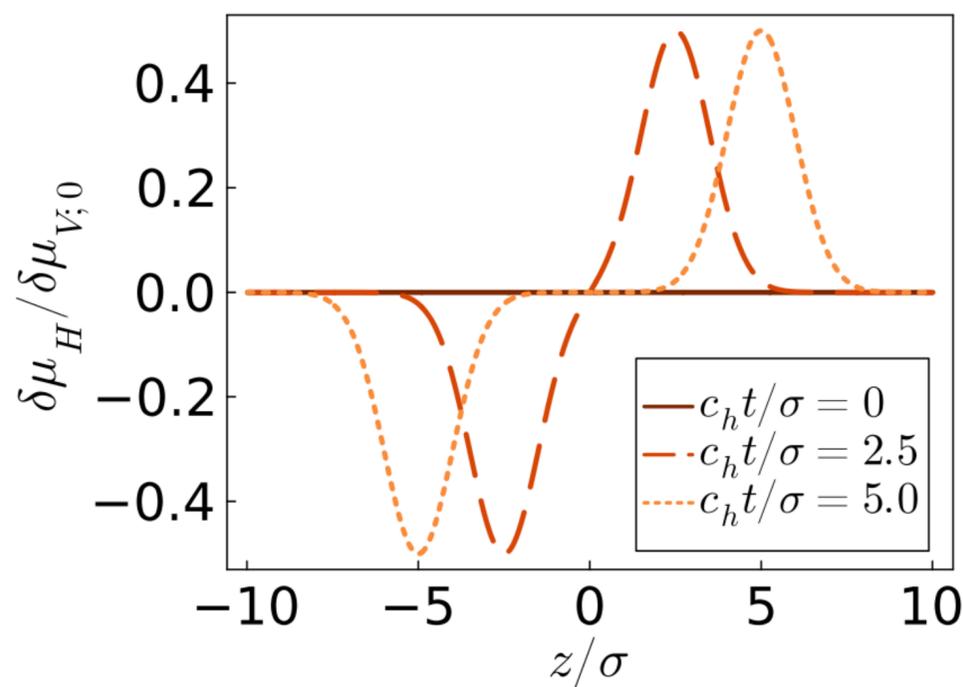
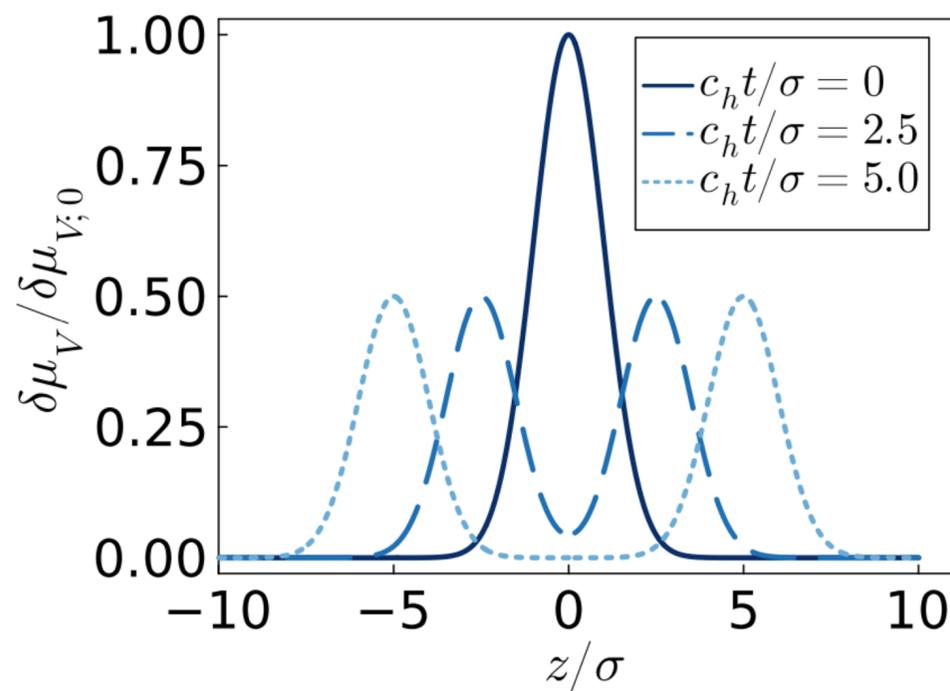
# CASE I: Conserved Charges

- Large Temperature Limit

- A. Helical Vortical Mode  
(Non-reciprocal!)

$$v_H^\pm = \pm \frac{c_H}{\pi^2} \frac{6 \ln 2 \Omega}{T} - \frac{6}{7\pi^2} \left[ \frac{84}{\pi^2} (\ln 2)^2 - 1 \right] \frac{\Omega \mu_A}{T^2} + O(T^{-3}).$$

$$\delta\mu_H^\pm = \pm \delta\mu_V^\pm \quad \delta\mu_A^\pm = - \left[ \frac{84(\ln 2)^2}{\pi^2} - 1 \right] \frac{\mu_H \pm \mu_V}{7T \ln 2} \delta\mu_V^\pm + O(T^{-2})$$



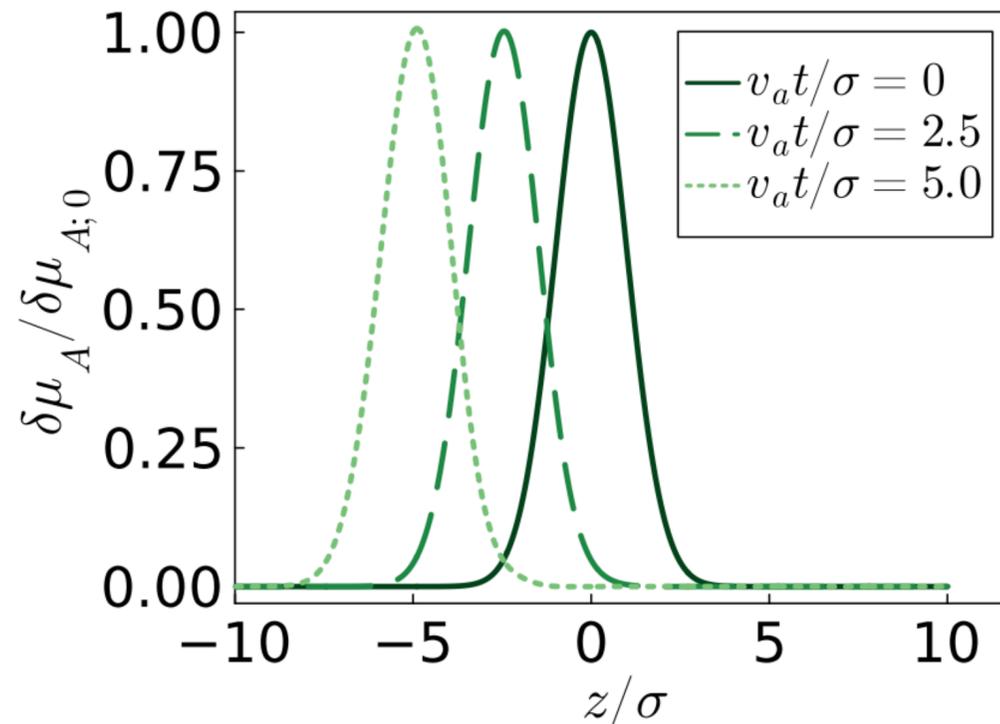
$\mu_V = \mu_A = \mu_H = 0$   
 $T = 300 \text{ MeV}$   
 $\Omega = 6.6 \text{ MeV}$

$T^{-2}$ )

# CASE I: Conserved Charges

- Large Temperature Limit

A. Helical Vortical Mode  
(Non-reciprocal!)



$$\begin{aligned} \mu_V &= \mu_H = 0 \\ \mu_A &= 30 \text{ MeV} \\ T &= 300 \text{ MeV} \\ \Omega &= 6.6 \text{ MeV} \end{aligned} \left[ \frac{\mu_A}{T^2} + O(T^{-3}) \right]$$

B. Axial Vortical Mode  
(Uni-directional!)

$$v_A = -\frac{24}{7\pi^2} \frac{\mu_A \Omega}{T^2} + O(T^{-3})$$

$$\delta\mu_{V/H,0}^A = -\frac{\mu_{H/V}}{7T \ln 2} \delta\mu_{A,0}^A + O(T^{-2})$$

$$\left[ \frac{(1/2)^2 - 1}{2} \right] \frac{\mu_H \pm \mu_V}{7T \ln 2} \delta\mu_V^\pm + O(T^{-2})$$

The image features a series of concentric circles centered on the page. The innermost circle is a dark blue color and contains the text 'Setup II'. Surrounding this are several more circles, each progressively lighter in shade, creating a ripple effect that fades into the white background.

# Setup II

# Non-Conservation of Charges

- Non-conservation can be implemented in the RTA approximation. Typically

$$\partial_\mu J^\mu = -\frac{\delta Q}{\tau} \quad \equiv \quad \partial_\mu J^\mu = -\frac{\chi \delta \mu}{\tau}$$

- If there are more than one charges in the system, both approaches are in general inequivalent due to non-vanishing crossed susceptibilities

$$\partial_\mu J^\mu = -\frac{\delta Q}{\tau} \quad \not\equiv \quad \partial_\mu J^\mu = -\frac{\chi \delta \mu}{\tau}$$

- We implement dissipation on the basis of the chemical potential:
  - A. On the one hand, a chemical potential is not well defined, in a thermodynamic sense, for non-conserved charges. It is reasonable to demand that it is absent in the final equilibrium state. (Note: The equilibrium equations are solved only for the **unpolarised plasma**  $\mu_A = \mu_H = 0$ )
  - B. Secondly, it can be shown that the other prescription can give rise to unphysical instabilities.

# Non-Conservation of Charges

- Helicity is not conserved in the presence of interactions. For example, in processes like  $e_R^+ e_L^- \rightarrow e_L^+ e_R^-$  chirality is conserved ( $\Delta Q_A = 0$ ) while helicity is violated ( $\Delta Q_H = 2$ ).
- Such processes will take place both in QED and in QCD, leading to 'relaxation of helicity'. QCD weak-coupling estimation given in [Ambrus, Chernodub (Mar 2023)]

$$\begin{aligned} \frac{dQ_H}{dt} &= -16\beta\mu_H g^2 \int dP dK dP' dK' s (2\pi)^6 \\ &\times \delta^4(p+k-p'-k') f_{0\mathbf{p}} f_{0\mathbf{k}} \tilde{f}_{0\mathbf{p}'} \tilde{f}_{0\mathbf{k}'} (\tilde{f}_{0\mathbf{p}} + f_{0\mathbf{p}'}) \\ &\times \frac{\alpha_{QCD}^2}{18E_{cm}^2} (1 - \cos\theta_{cm})^2. \end{aligned} \quad \longrightarrow \quad \begin{aligned} \frac{1}{2}\tau_H &\simeq \left(\frac{250 \text{ MeV}}{k_B T}\right) \left(\frac{1}{\alpha_{QCD}}\right)^2 \left(\frac{2}{N_f}\right) \\ &\times 2.54 \text{ fm}/c. \end{aligned}$$

- The chiral symmetry is actually anomalous (i.e. broken by quantum effects) and the axial current is not truly conserved:

$$\partial_\mu J_5^\mu = -\frac{1}{12\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr}\{G_{\mu\nu} G_{\rho\sigma}\}$$

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# Results II

## CASE II:

### Non-Conserved Helicity ( $0 \leq \tau_H < \infty$ )

- At small chemical potential we **recover** the **Helical Vortical Wave** [Ambrus, Chernodub (Mar 2023)]:

$$\omega_{h;0}^{\pm} = -\frac{i}{2\tau_H} \pm kc_h \sqrt{1 - \frac{k_{th}^2}{k^2}}, \quad k_{th} = \frac{1}{2\tau_H c_h} = \frac{\pi^2}{12 \ln 2} \frac{T}{\Omega\tau_H}.$$

- The wave propagates for wavenumbers above the threshold  $k_{th}$ . Thus, the non-conservation of the helical charge **inhibits propagation of the IR modes**.
- This mode **exists with vanishing chemical potentials!**  $\mu_V = \mu_A = \mu_H = 0$ . Contrary to the traditional CVW which requires  $\mu_V \neq 0$ .

## CASE II:

### Non-Conserved Helicity ( $0 \leq \tau_H < \infty$ )

- In the limit when helical charge dissipates very quickly ( $\tau_H \rightarrow 0$ ) the helical degree of freedom is effectively frozen and we **recover** the traditional **Chiral Vortical Wave** [Jiang, Huang, Liao (2015)]

$$\omega_h = -\frac{iT^2\sigma_A^\omega}{6\tau_H[(\sigma_A^\omega)^2 - (\sigma_H^\omega)^2]}, \quad \omega_a^\pm = \pm \frac{AT^2}{2\sigma_A^\omega\sqrt{H}} \left(1 - \frac{T^2\Delta H}{3\sigma_A^\omega}\right)^{-1/2} \frac{k\Omega}{T}.$$

$$\alpha_V = \mu_V/T$$

$$H = (e + P)/sT = 1 + \mu_V Q_V/sT$$

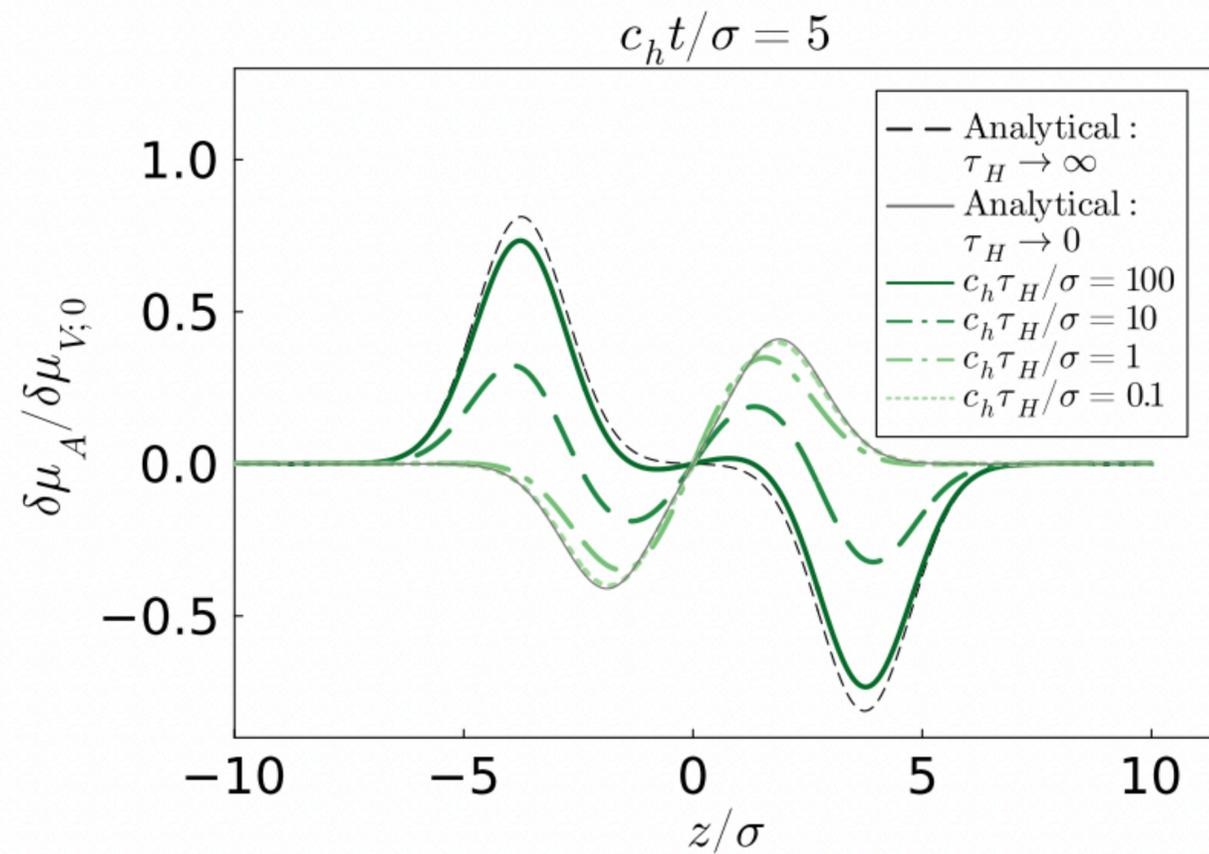
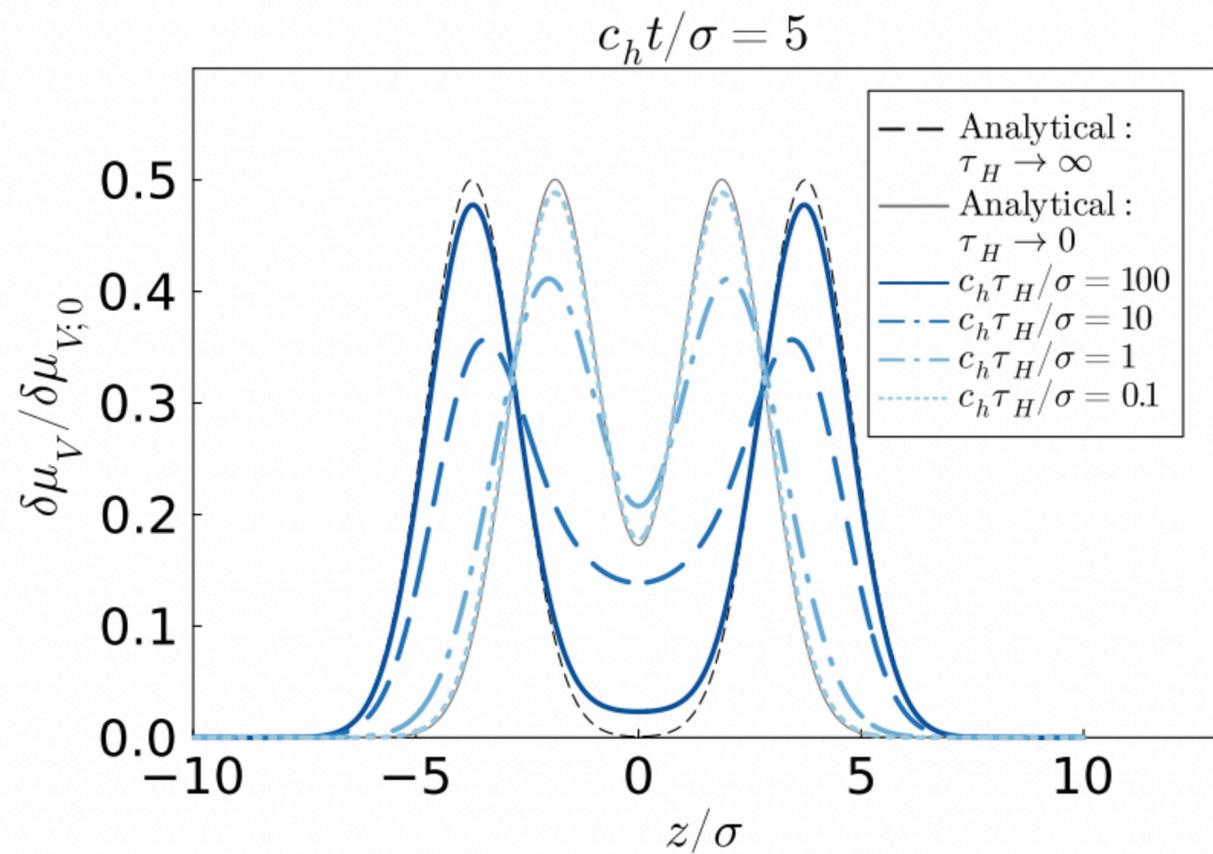
$$\Delta H = H - 1$$

$$A = \frac{\alpha_V}{\pi^2} - \frac{Q_V}{3s}$$

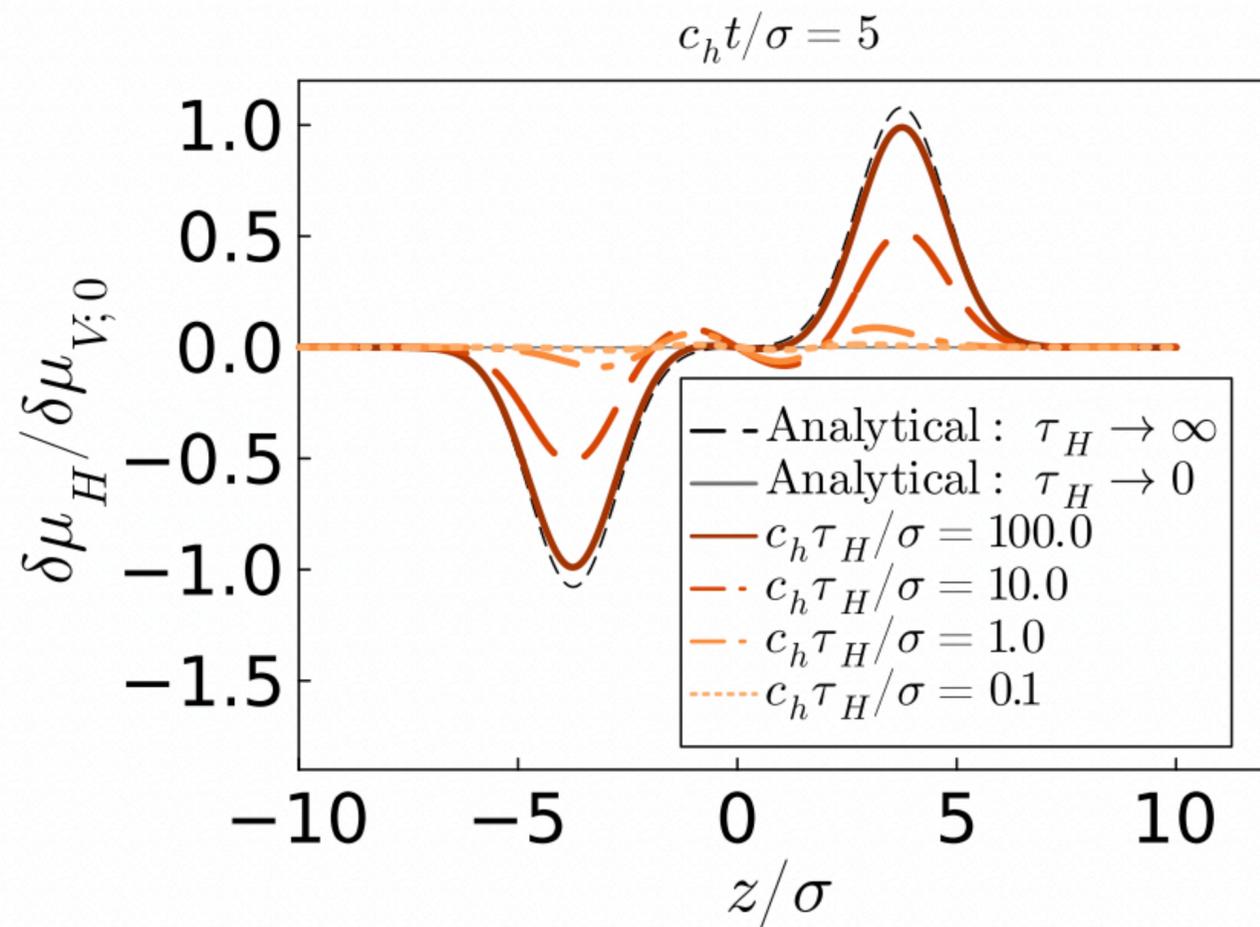
$$B = \frac{HL}{\pi^2} - \frac{2Q_V}{sT^2}\sigma_H^\omega$$

$$L = 2 \ln \left( 2 \cosh \frac{\mu_V}{2T} \right)$$

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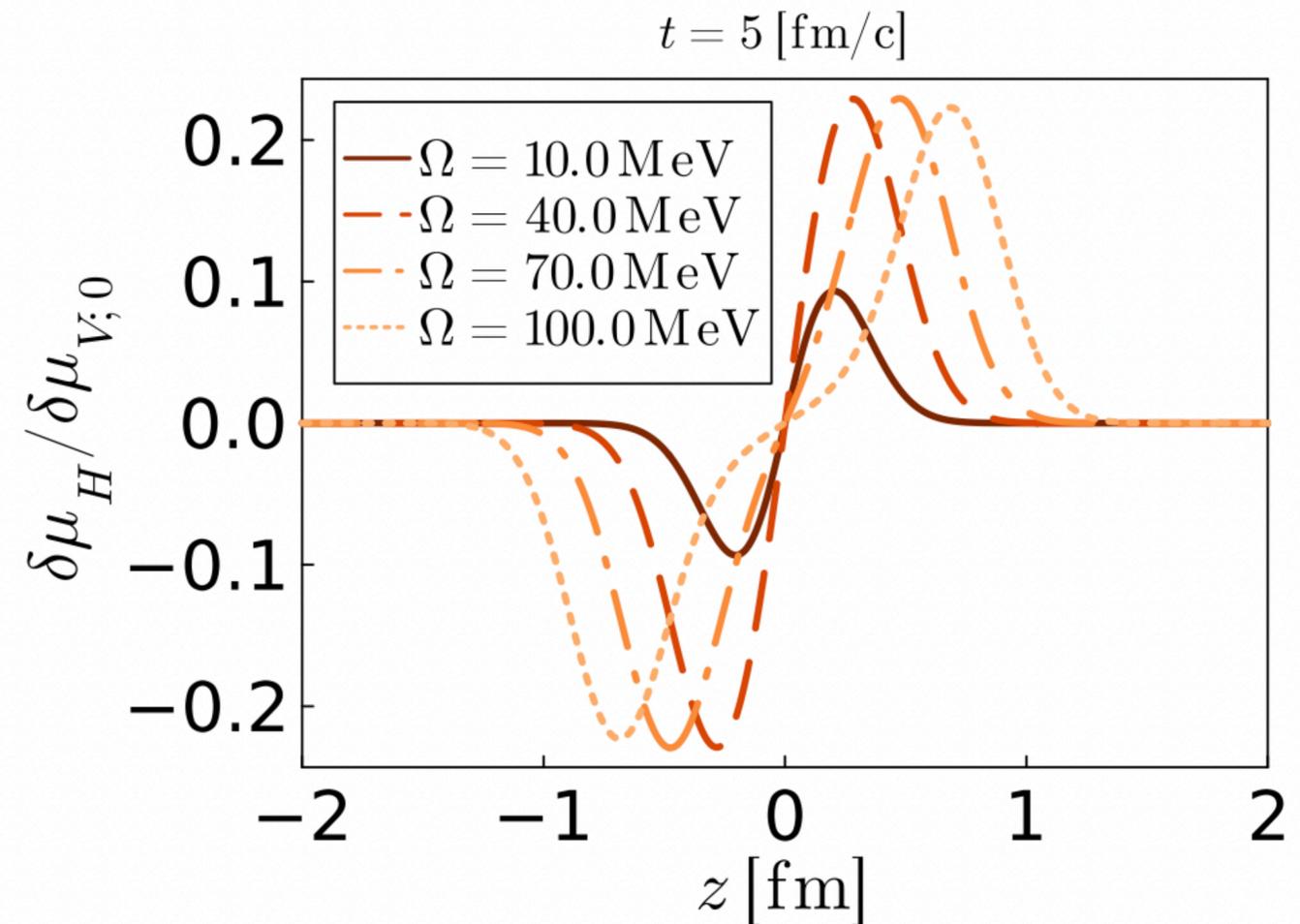
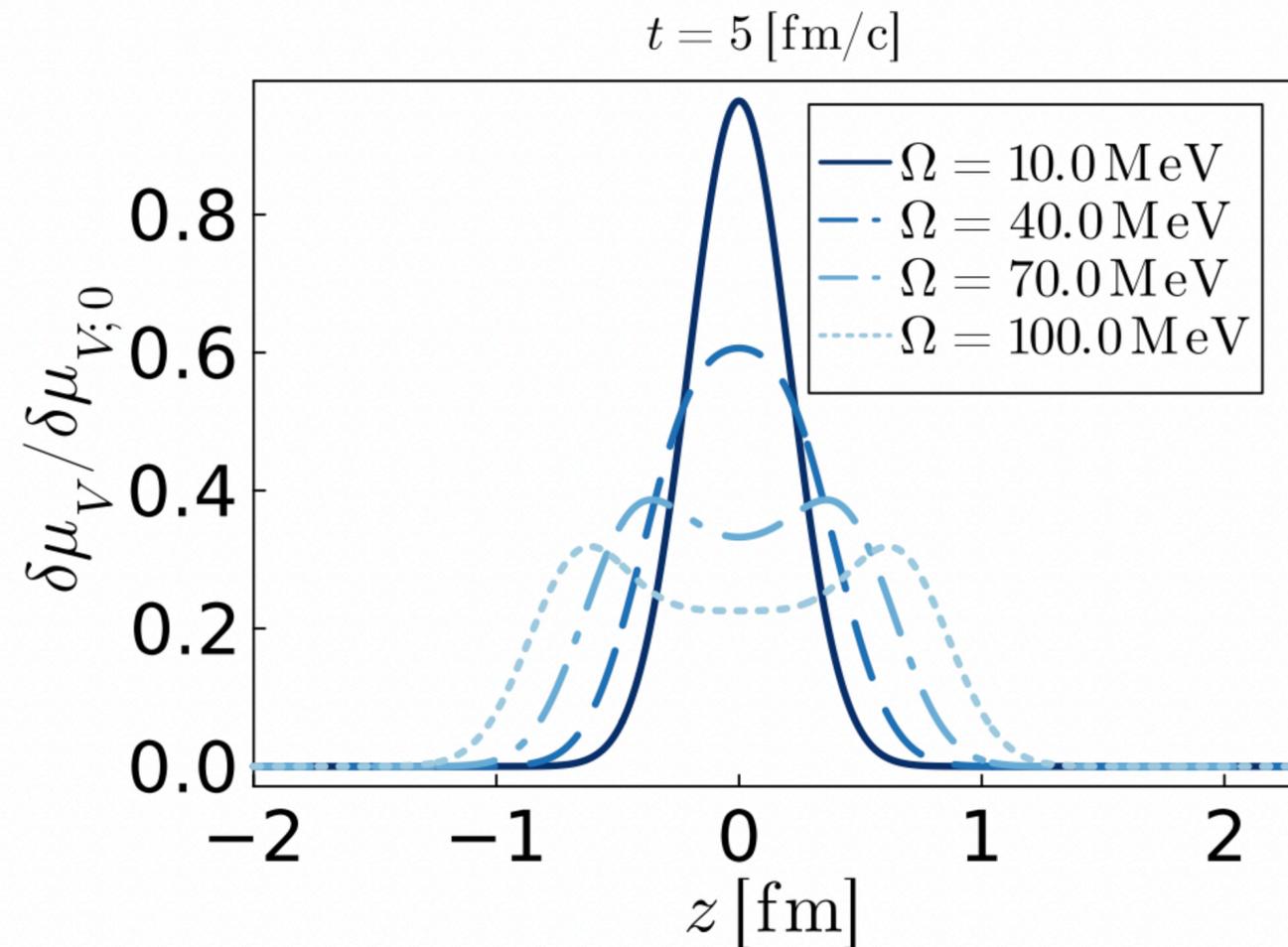


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## CASE III:

Non-Conserved Helicity and Chirality ( $0 \leq \tau_H, \tau_A < \infty$ )



$$\tau_H = 2.7 \text{ fm/c} \quad \tau_A = 0.25 \text{ fm/c}, \quad \tau_R = 0$$
$$\mu_V = \mu_A = \mu_H = 0 \quad T = 300 \text{ MeV}$$

# Dissipative Effects

- Finally, the presence of interactions gives rise to kinetic dissipative effects. Again these are implemented in the RTA and parametrically controlled by  $\tau_R$ .
- The dissipative contributions appear as deviations from the perfect fluid form.

$$T^{\mu\nu} \rightarrow T^{\mu\nu} = T_0^{\mu\nu} + \pi_d^{\mu\nu} - \Pi_d \Delta^{\mu\nu}, \quad J_\ell^\mu \rightarrow J_\ell^\mu = J_{\ell;0}^\mu + V_{\ell;d}^\mu$$

These are unconstrained and must be specified by constitutive relations. In 1st order

hydro:  $V_{\ell;d}^\mu = \sum_{\ell'} \kappa_{\ell,\ell'} \nabla^\mu \alpha_{\ell'}, \quad \pi_d^{\mu\nu} = 2\eta \sigma^{\mu\nu}$



A causal and stable theory requires  
2nd order hydro!

- The expectation values are now computed with the distribution function that solves the kinetic equation

$$p^\mu \partial_\mu f_{\mathbf{p},\lambda}^\sigma = -\frac{E_{\mathbf{p}} \cdot u}{\tau_R} \left( f_{\mathbf{p},\lambda}^\sigma - f_{\mathbf{p},\lambda}^{\text{eq};\sigma} \right)$$

$$f_{\mathbf{p},\lambda}^{\text{eq};\sigma} = \left[ \exp \left( \frac{p \cdot u - \mu_{\sigma,\lambda}}{T} \right) + 1 \right]^{-1}$$

# Dissipative Effects

- After some manipulations one finds

$$V_\ell^\mu = \tau_R \left( \frac{1}{3} \nabla^\mu Q_\ell - \frac{Q_\ell \nabla^\mu P}{E + P} \right) \quad \pi^{\mu\nu} = 2\eta \sigma^{\mu\nu}$$

$$\sigma^{\mu\nu} = \left( \frac{1}{2} \Delta^{\mu\lambda} \Delta^{\nu\kappa} + \frac{1}{2} \Delta^{\nu\lambda} \Delta^{\mu\kappa} - \frac{1}{3} \Delta^{\mu\nu} \Delta^{\lambda\kappa} \right) \partial_\lambda u_\kappa$$

- The diffusion Matrix becomes  $\kappa_{\ell\ell'} = \tau_R \left( \frac{1}{3} \frac{\partial Q_\ell}{\partial \alpha_{\ell'}} - \frac{Q_\ell Q_{\ell'} T}{E + P} \right)$  and the shear viscosity  $\eta = \frac{4}{5} \tau_R P$ .
- Fluctuations around rotating state + Fourier transform give for  $T^{\mu\nu}$  sector

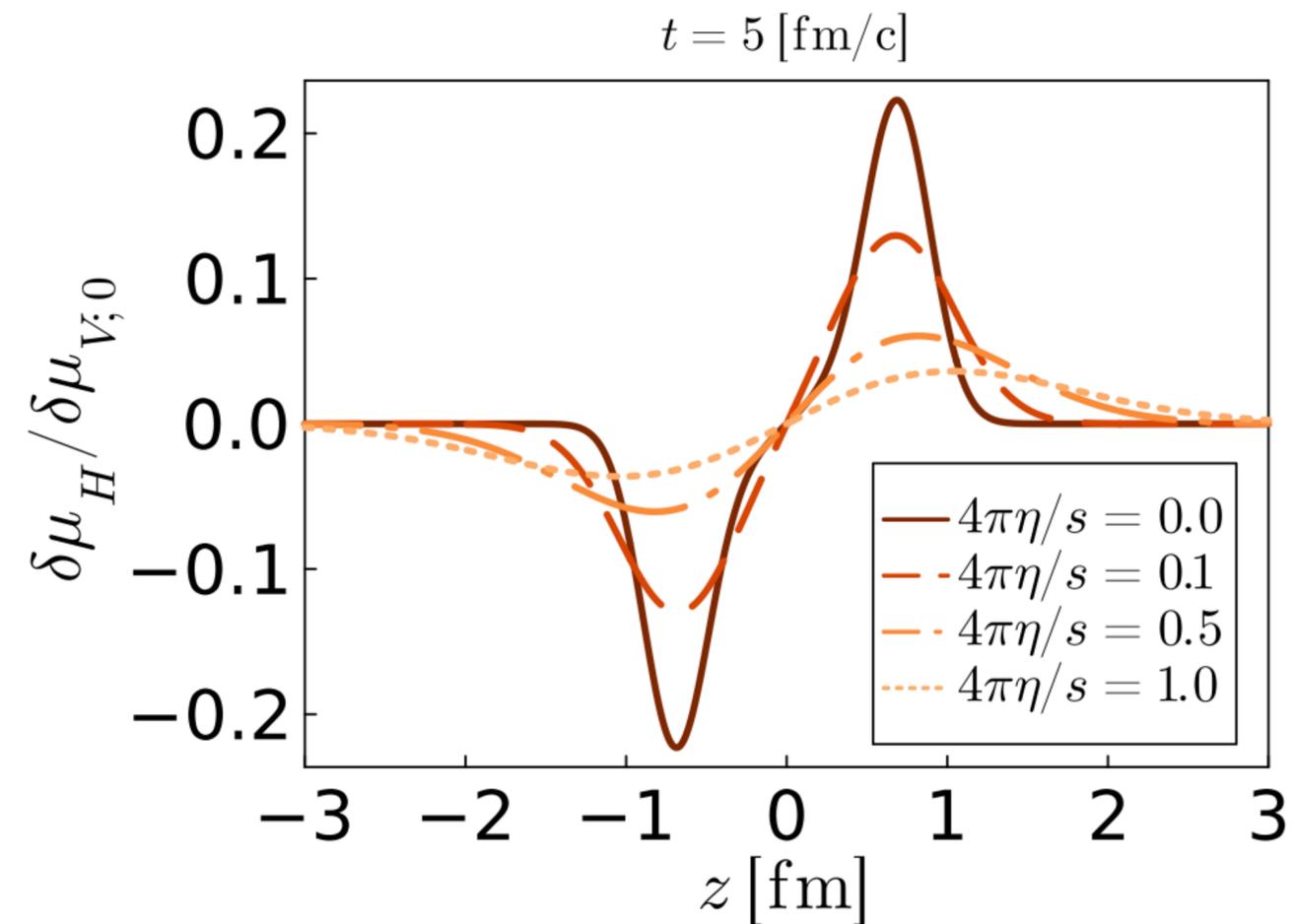
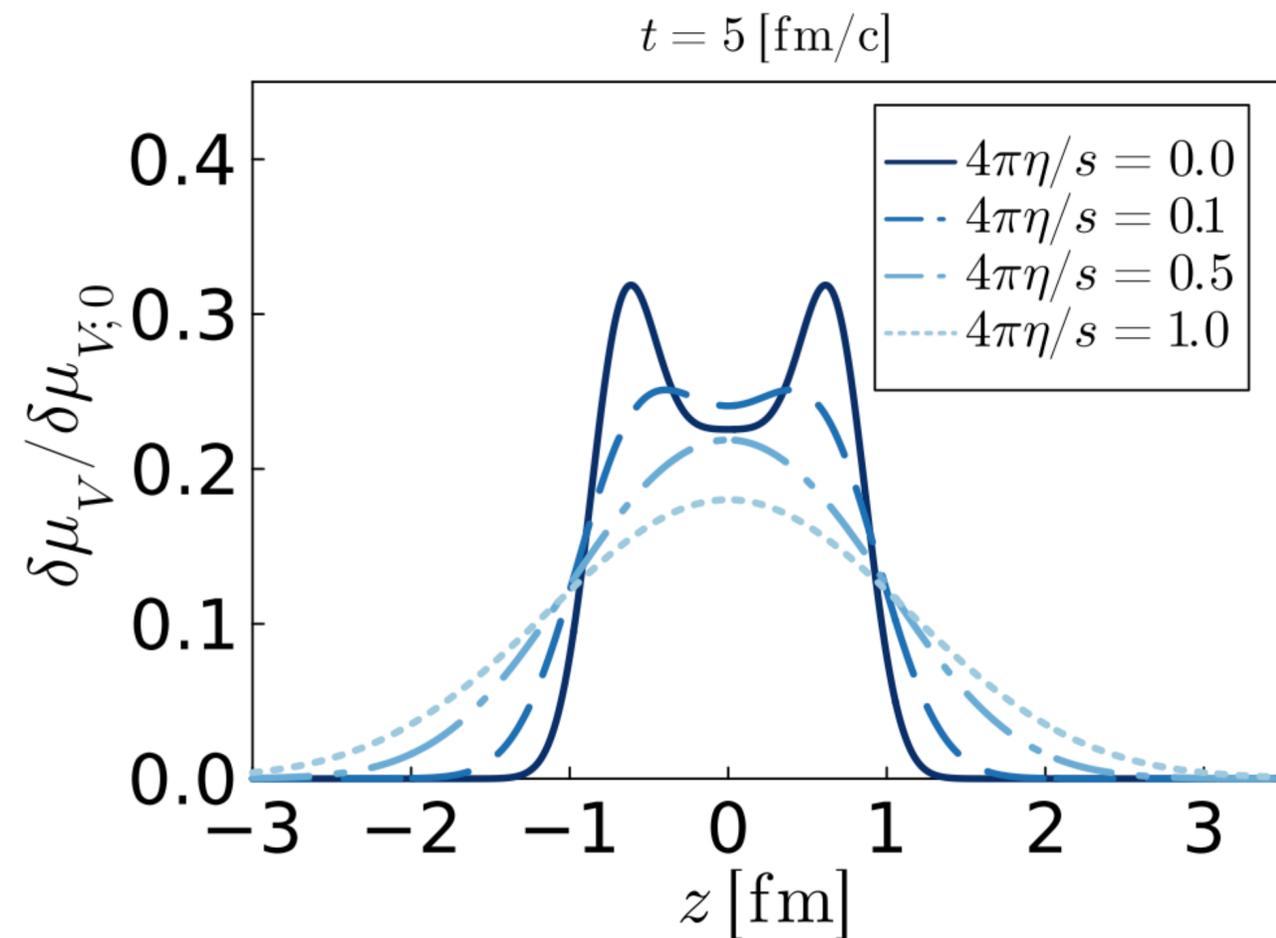
$$\begin{pmatrix} -3v & 4P \\ 1 & -4Pv - \frac{4}{3}\eta ik \end{pmatrix} \begin{pmatrix} \delta P \\ \delta u \end{pmatrix} = 0. \quad \implies \quad \omega_{R;ac.}^\pm = \pm k c_s(\eta) - \frac{ik^2 \eta}{6P}, \quad c_s(\eta) = \frac{1}{\sqrt{3}} \sqrt{1 - \frac{k^2 \eta^2}{12P^2}}$$

- The modes from the charge sector get a constant imaginary shift.

$$\left[ \left( \omega + \frac{ik^2 \tau_R}{3} \right) \delta Q_\ell - k \Omega \delta \sigma_\ell^\omega \right] \Big|_{\delta P=0} = -\frac{iT^2 \delta \mu_\ell}{3\tau_\ell} \quad \implies \quad \omega_R = \omega - \frac{ik^2 \tau_R}{3};$$

Kinetic dissipation  
**damps UV modes.**

# Final Case: Dissipation and charge non-conservation



$$\tau_H = 2.7 \text{ fm/c} \quad \tau_A = 0.25 \text{ fm/c}$$
$$\mu_V = \mu_A = \mu_H = 0 \quad T = 300 \text{ MeV} \quad \Omega = 100 \text{ MeV}$$

The background features a series of concentric circles in shades of blue, centered on the page. The circles are thin and spaced evenly, creating a ripple effect that draws the eye towards the center.

# Summary

# Summary & Outlook

- A fluid with vector axial and helical charges shows a rich variety of wave-like excitations.
- In a neutral unpolarised plasma the V, H d.o.f.s propagate as the **Helical Vortical Wave**.
- In a realistic plasma, axial and helical charges are not conserved  $\longrightarrow 0 < \tau_A, \tau_H < \infty$   
 $\longrightarrow$  IR (large wavelength) **propagation cutoff**.
- Conversely, kinetic dissipation **damps UV modes**.
- The traditional Chiral Vortical Wave arises **only** in the particular limit  $\tau_H \rightarrow 0$ .
- Both **non-reciprocity** and **uni-directionality** appear in a chirally imbalanced medium.
- The waves are unlikely to give a phenomenological imprint in the QGP at HIC.
  - I. Employ the framework for polarisation measurements in HIC.
  - II. Extension to more realistic EOS.
  - III. Investigate kinetic theory framework for fluids with these three charges.

# Thank you!

## **Acknowledgments:**

This work is supported by the European Union - NextGenerationEU through grant No. 760079/23.05.2023, funded by the Romanian ministry of research, innovation and digitalization through Romania's National Recovery and Resilience Plan, call no. PNRR-III-C9-2022-I8.



# Backup Slides

# Hydrodynamic setup:

## Conservation Equations and Longitudinal Perturbations

- The density operator induces a preferred hydrodynamic frame, the  $\beta$ -frame, defined by the following 4-velocity:

$$u_{\Omega}^{\mu} \partial_{\mu} = \Gamma(\partial_t + \Omega \partial_{\varphi}), \quad \Gamma_{\Omega} = \frac{1}{\sqrt{1 - \rho^2 \Omega^2}}$$

- The velocity allows to define three more (orthogonal) vectors, namely

$$\omega^{\mu} = \frac{1}{2} \varepsilon^{\mu\nu\lambda\sigma} u_{\nu} \nabla_{\lambda} u_{\sigma} = \Omega \Gamma_{\Omega}^2 \partial_z$$

$$a^{\mu} = u^{\nu} \nabla_{\nu} u^{\mu} = -\rho \Omega^2 \Gamma_{\Omega}^2 \partial_{\rho}$$

$$\tau^{\mu} = -\varepsilon^{\mu\nu\lambda\sigma} \omega_{\nu} a_{\lambda} u_{\sigma} = -\rho \Omega^3 \Gamma_{\Omega}^5 (\rho \Omega \partial_t + \rho^{-1} \partial_{\varphi})$$

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# Hydrodynamic setup:

## Conservation Equations and Longitudinal Perturbations

- As is customary in relativistic hydro, we decompose the expectation values in a particular frame.

$$J_\ell^\mu = Q_{\ell;\beta} u_\Omega^\mu + V_{\ell;\beta}^\mu, \quad T^{\mu\nu} = E_\beta u_\Omega^\mu u_\Omega^\nu - (P_\beta + \varpi_\beta) \Delta_\Omega^{\mu\nu} + \pi_\beta^{\mu\nu} + W_\beta^\mu u_\Omega^\nu + W_\beta^\nu u_\Omega^\mu.$$

- Conformal fluid ( $T_\mu^\mu = 0$ ) with equation of state  $E_\beta = 3P_\beta$ . Then  $\varpi_\beta = 0$

$$V_{\ell;\beta}^\mu = \sigma_{\ell;\beta}^\omega \omega_\Omega^\mu + \sigma_{\beta;\ell}^\tau \tau_\Omega^\mu, \quad W_\beta^\mu = \sigma_{\varepsilon;\beta}^\omega \omega_\Omega^\mu + \sigma_{\varepsilon;\beta}^\tau \tau_\Omega^\mu,$$

$$\pi_\beta^{\mu\nu} = \pi_{1;\beta} \left( \tau_\Omega^\mu \tau_\Omega^\nu - \frac{\omega_\Omega^2}{2} a_\Omega^\mu a_\Omega^\nu - \frac{\mathbf{a}_\Omega^2}{2} \omega_\Omega^\mu \omega_\Omega^\nu \right) + \pi_{2;\beta} (\omega_\Omega^\mu \tau_\Omega^\nu + \omega_\Omega^\nu \tau_\Omega^\mu),$$

 Quantum corrections to the perfect fluid form.

 Survives to linear order in  $\Omega$

# Hydrodynamic setup: Charge sector

- The equations for the perturbations in the charge sector read:

$$(v\delta Q_\ell - \Omega\delta\sigma_\ell^\omega) \Big|_{\delta P=0} = 0$$

- The thermodynamic quantities naturally depend on the chemical potentials, thus we rewrite the equations in terms of  $\delta\mu_\ell$

$$\delta Q_\ell = \frac{\partial Q_\ell}{\partial T}\delta T + \frac{\partial Q_\ell}{\partial\mu_{\ell'}}\delta\mu_{\ell'} = \left(\frac{\partial Q_\ell}{\partial\mu_{\ell'}} - \frac{Q_{\ell'}}{s}\frac{\partial Q_\ell}{\partial T}\right)\delta\mu_{\ell'}$$

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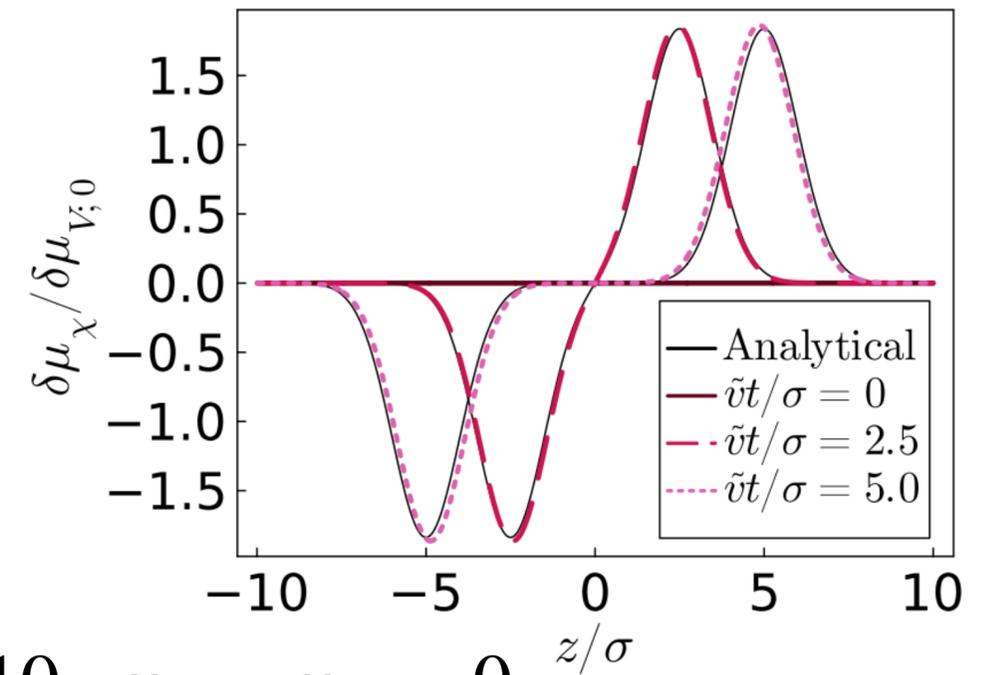
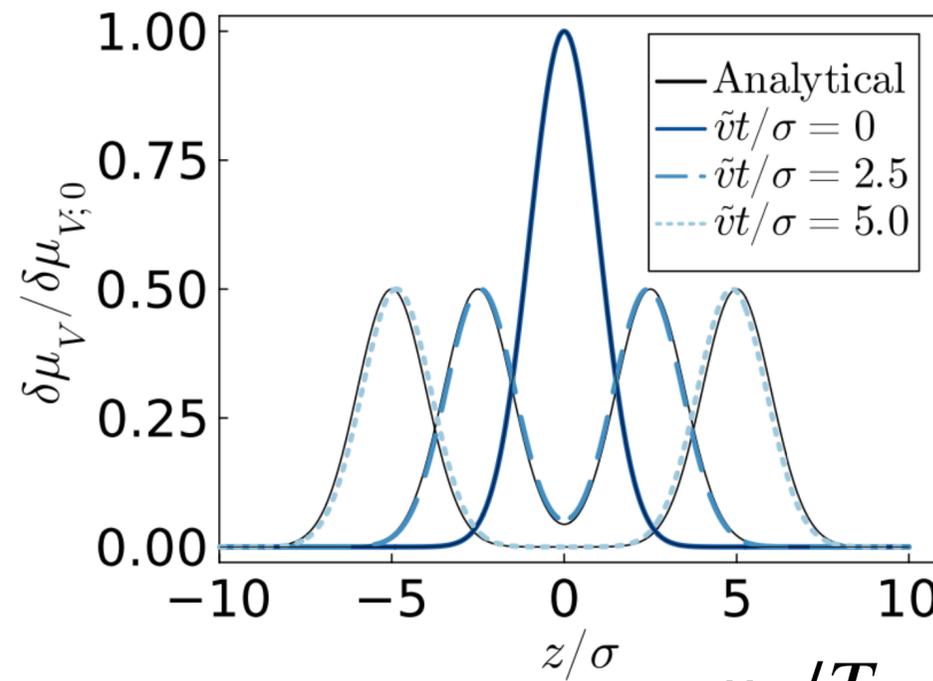
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# CASE I: Conserved Charges

- Degenerate Limit:  $|\mu_V| \gg T, |\mu_V| \gg |\mu_A \pm \mu_H|$ .
- The system composed of only particles (or antiparticles) up to exponentially small corrections. Hence, the **helical and axial degrees of freedom are not independent** and the solutions depend only on  $\mu_\chi = \mu_A + \mu_H$ .
- Neglecting exponentially suppressed corrections one finds two modes:

$$\tilde{v}_\pm = -\frac{5\Omega\mu_\chi}{\mu_V^2} \pm \frac{\Omega}{\mu_V^2} \sqrt{\frac{4\pi^2 T^2}{3} + 9\mu_\chi^2}$$

$$\tilde{\delta\mu}_V^\pm = \frac{3\tilde{\delta\mu}_\chi^\pm}{2\mu_V} \left( \mu_\chi \pm \sqrt{\frac{4\pi^2 T^2}{3} + 9\mu_\chi^2} \right)$$



$$\mu_V/T = 10, \mu_A = \mu_H = 0$$

# CASE I: Conserved Charges

- **Unpolarised Plasma:**  $\mu_A = \mu_H = 0$ .

$$\frac{1}{T^2} \mathbb{M} = v \mathbb{M}_v(\alpha_V) - \frac{\Omega}{T} \mathbb{M}_\Omega(\alpha_V)$$

$$\mathbb{M}_v = \frac{2}{T^2} \begin{pmatrix} \sigma_A^\omega - \frac{T^2}{3} \Delta H & 0 & 0 \\ 0 & \sigma_A^\omega & \sigma_H^\omega \\ 0 & \sigma_H^\omega & \sigma_A^\omega \end{pmatrix}$$

$$\mathbb{M}_\Omega = \begin{pmatrix} 0 & \frac{1}{H} A & \frac{1}{H} B \\ A & 0 & 0 \\ B & 0 & 0 \end{pmatrix}$$

$$\alpha_V = \mu_V / T$$

$$H = (e + P) / sT = 1 + \mu_V Q_V / sT$$

$$\Delta H = H - 1$$

$$A = \frac{\alpha_V}{\pi^2} - \frac{Q_V}{3s}$$

$$B = \frac{HL}{\pi^2} - \frac{2Q_V}{sT^2} \sigma_H^\omega$$

$$L = 2 \ln \left( 2 \cosh \frac{\mu_V}{2T} \right)$$

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- A. Axial-Helical Mode:  $v = 0$

$$\delta\mu_V = 0$$

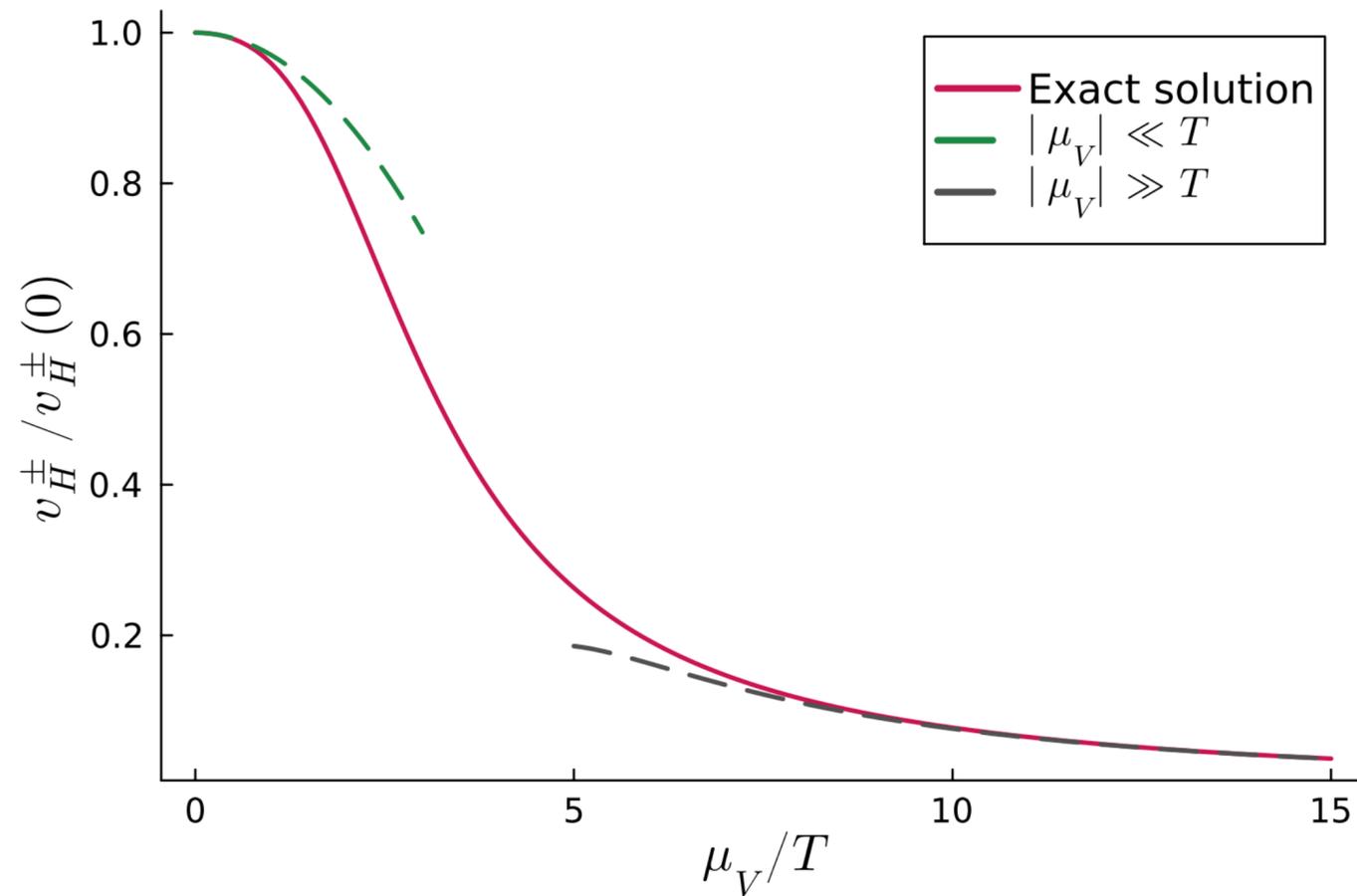
$$A\delta\mu_A + B\delta\mu_H = 0$$

- B. Helical Vortical Mode:

$$v_H^\pm = \pm \frac{\Omega T}{2} \sqrt{\frac{\sigma_A^\omega (A^2 + B^2) - 2AB\sigma_H^\omega}{H(\sigma_A^\omega - \frac{T^2}{3} \Delta H)[(\sigma_A^\omega)^2 - (\sigma_H^\omega)^2]}}$$

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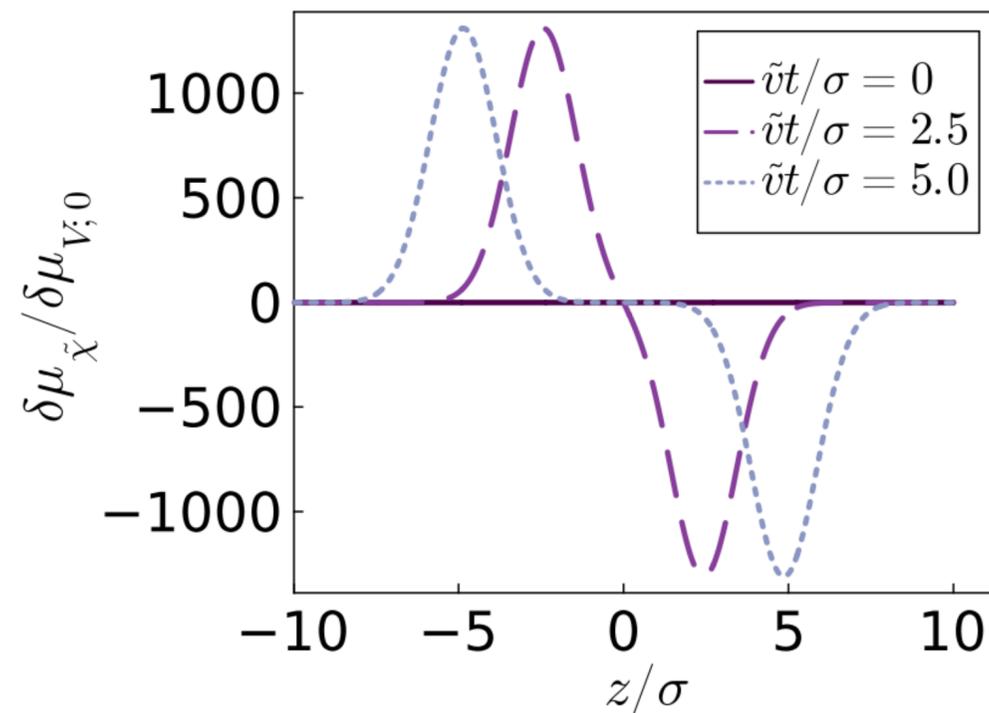
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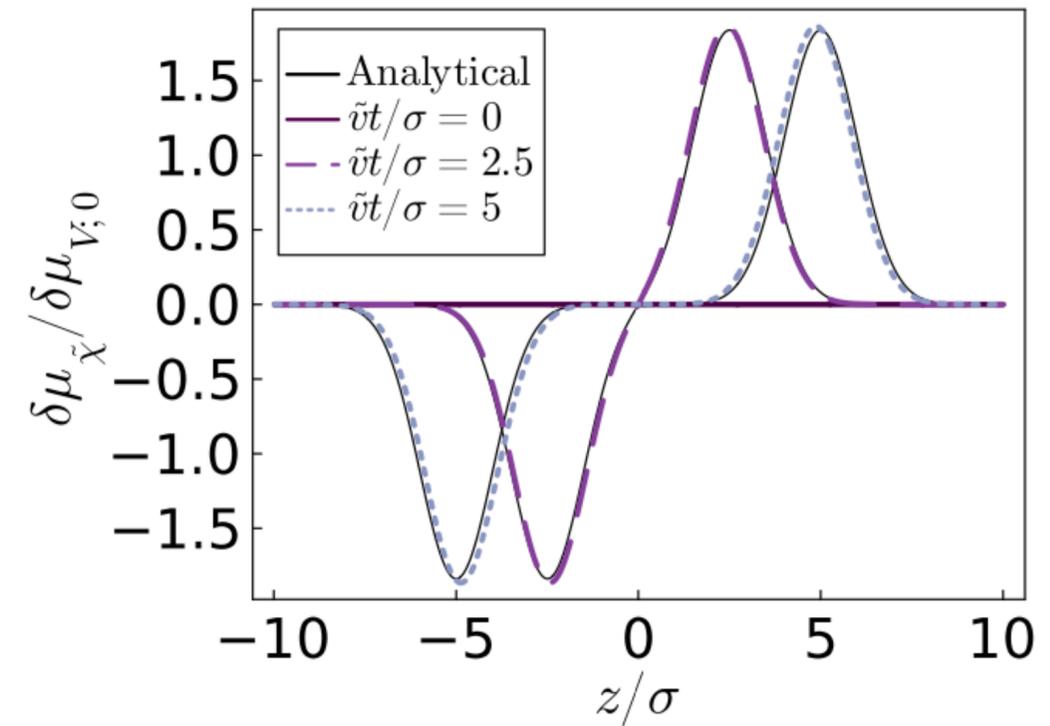
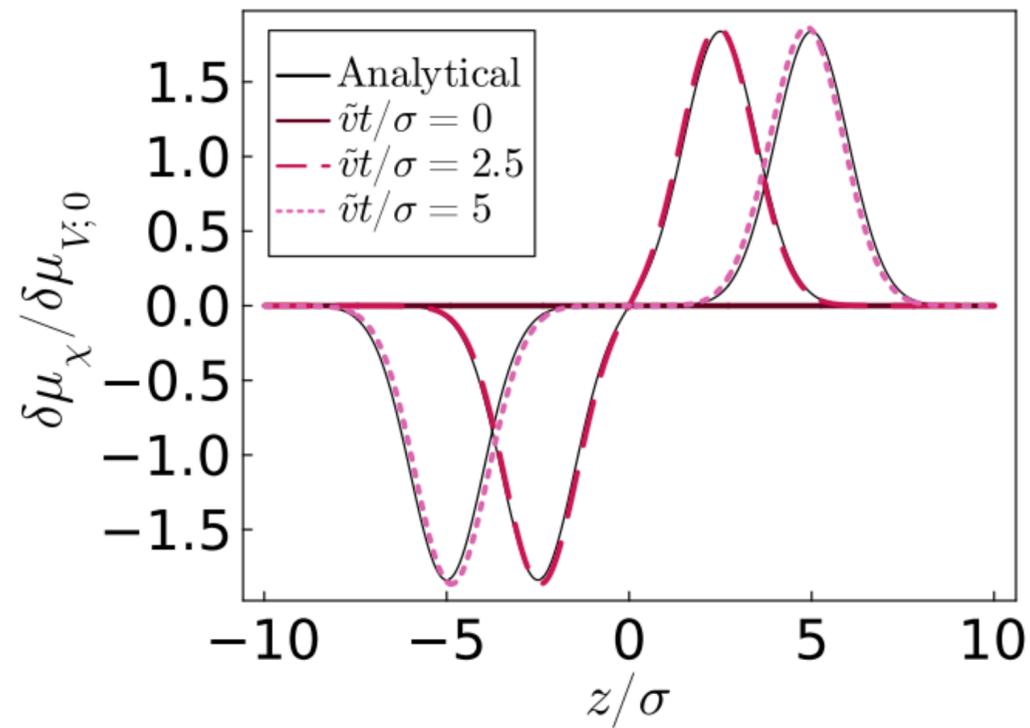
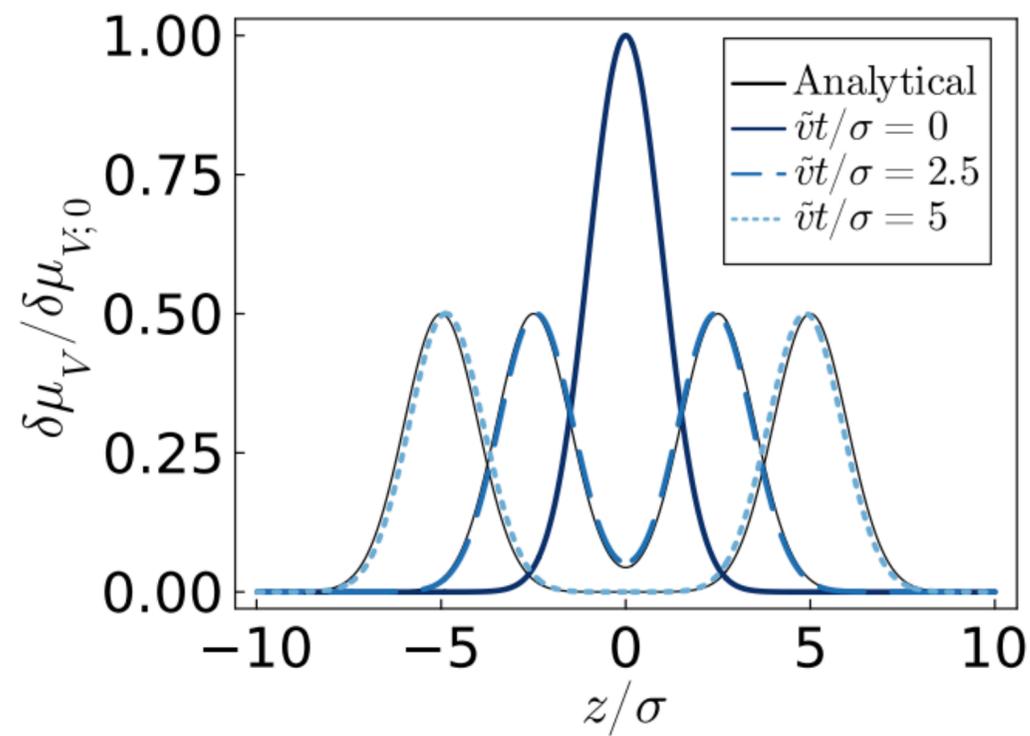


$\delta\mu_{\tilde{\chi}} \sim (\mu_V/T)^4$   
The linear perturbation regime is **unreliable** in this limit. Including dissipation will fix this problem.

## CASE II:

### Non-Conserved Helicity ( $0 \leq \tau_H < \infty$ )

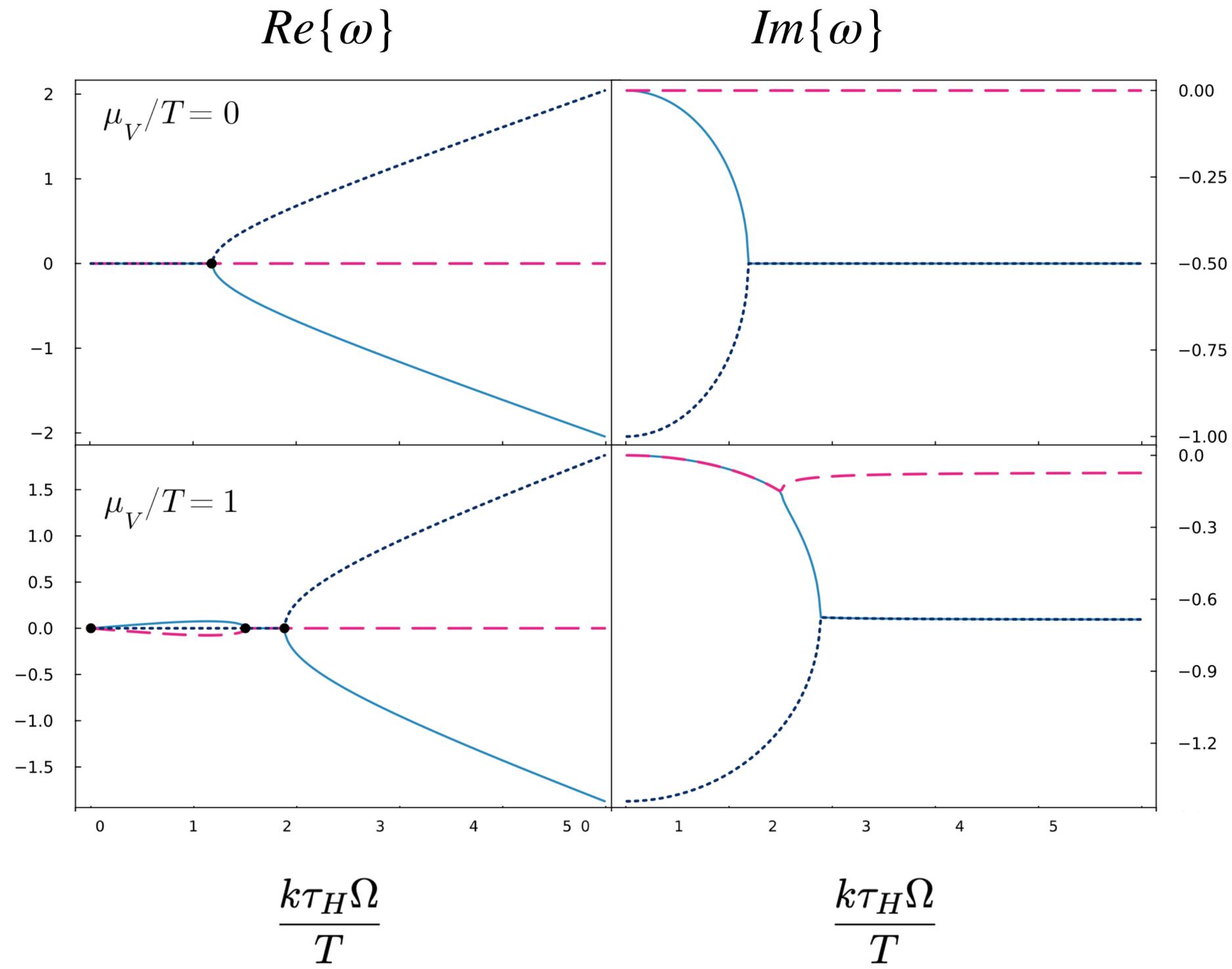
- In the degenerate limit ( $\mu_V/T \gg 1$ ), the finite lifetime  $\tau_H$  acts as a regulator and no perturbation grows to unreasonably large values.



# CASE II:

## Non-Conserved Helicity ( $0 \leq \tau_H < \infty$ )

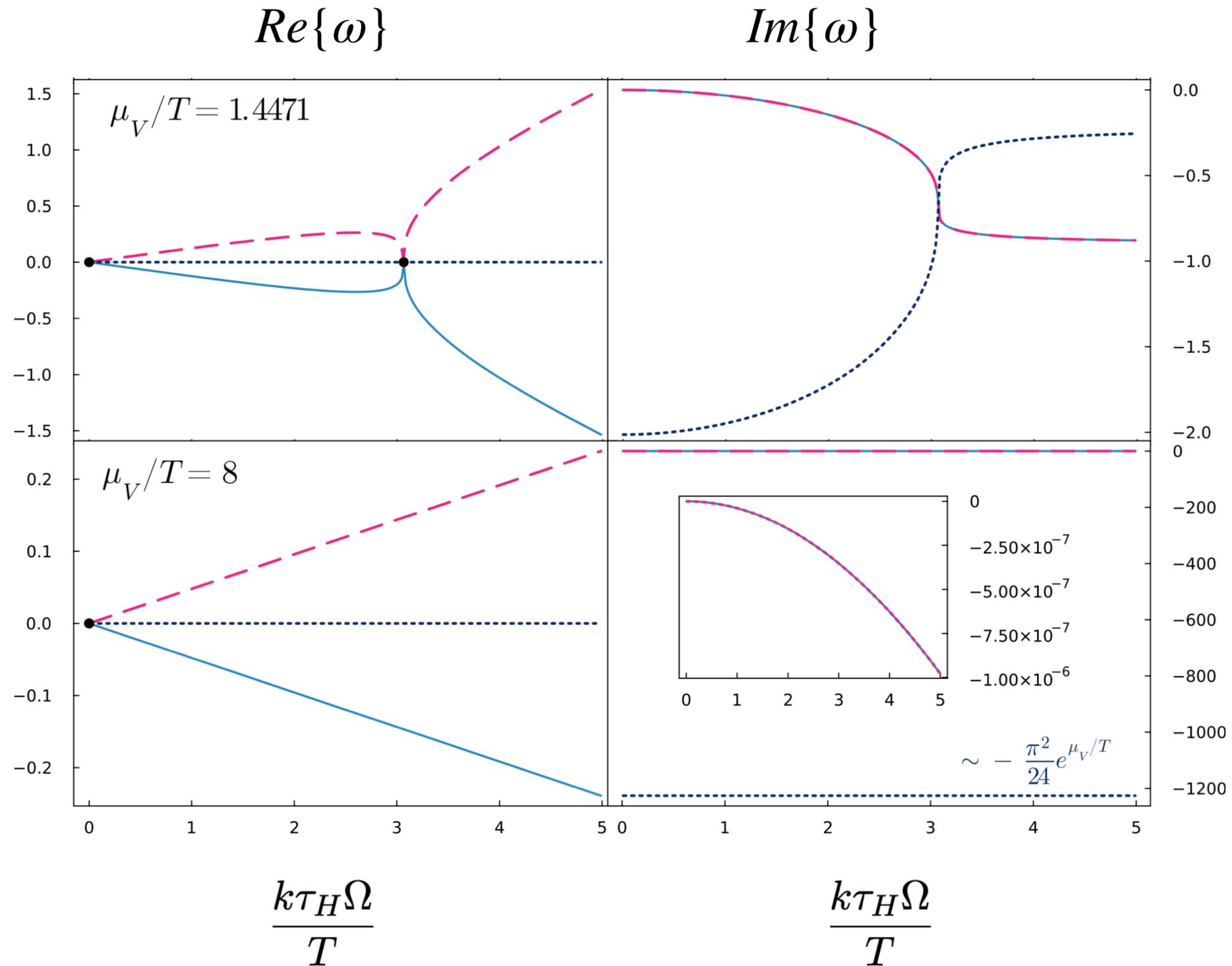
$$\omega = \tau_H k v$$



# CASE II:

## Non-Conserved Helicity ( $0 \leq \tau_H < \infty$ )

$$\omega = \tau_H k v$$



## CASE III:

### Non-Conserved Helicity and Chirality ( $0 \leq \tau_H, \tau_A < \infty$ )

- There are two remarkable novelties when axial charge is also dissipating:
  - A. The  $\tau_H \rightarrow 0$  limit gives the **generalisation** of the CVW (Vector-Axial) when axial charge dissipates. The presence of  $\tau_A$  also **eliminates the propagation of IR modes**.

$$\omega_a^\pm \simeq \frac{T^2}{12\sigma_A^\omega \tau_A} \left( -i \pm \sqrt{\frac{36A^2 \sigma_A^\omega}{H(\sigma_A^\omega - \frac{T^2}{3} \Delta H)} \tau_A^2 \kappa_\Omega^2 - 1} \right)$$

- B. The  $\tau_A \rightarrow 0$  limit, equivalent to neglecting/freezing the axial degree of freedom, gives a HVW (Vector-Helical). Note that  $\tau_A \sim 0.25 \text{ fm}/c \ll \tau_{QGP} \sim 10 \text{ fm}/c$

$$\omega_{h;0}^\pm \simeq \frac{T^2}{12\tau_H \sigma_A^\omega} \left( -i \pm \sqrt{\frac{36B^2 \sigma_A^\omega}{H(\sigma_A^\omega - \frac{T^2}{3} \Delta H)} \tau_H^2 \kappa_\Omega^2 - 1} \right)$$

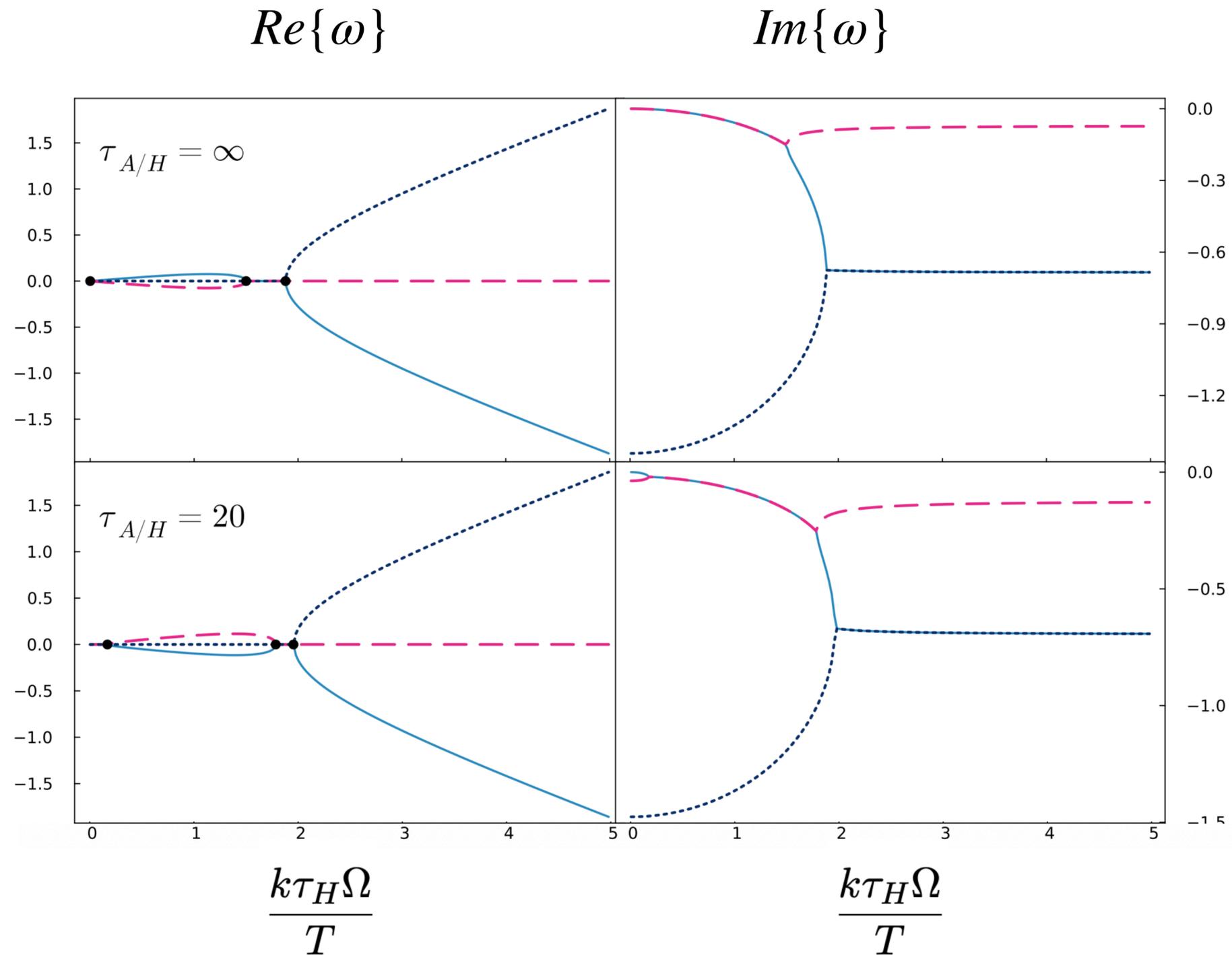
[Phys. Rev. D **102**,  
054516 (2020)]

$$\mu_V/T = 1$$

## CASE III:

$$\omega = \tau_H k v$$

Non-Conserved Helicity and Chirality ( $0 \leq \tau_H, \tau_A < \infty$ )

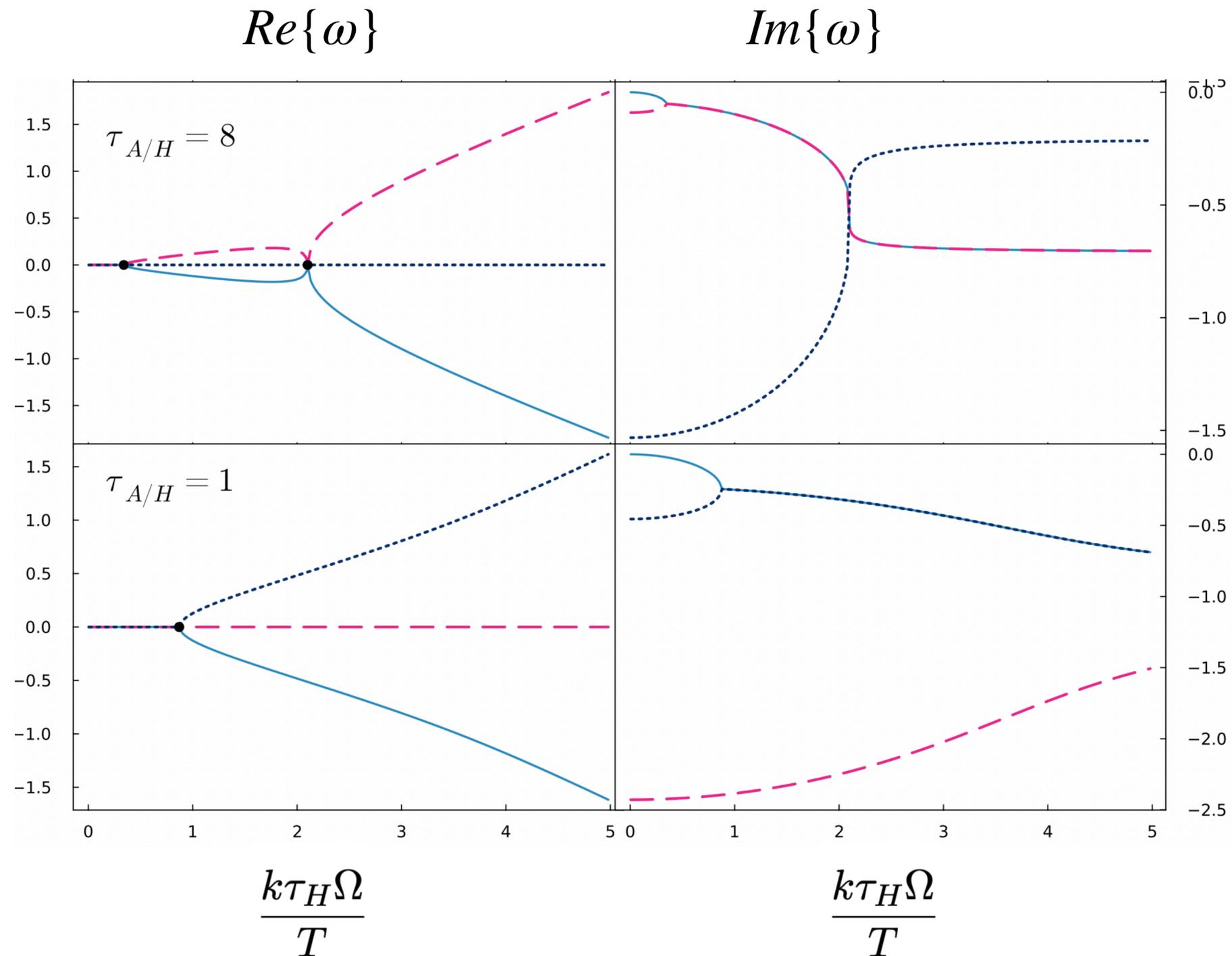


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### CASE III:

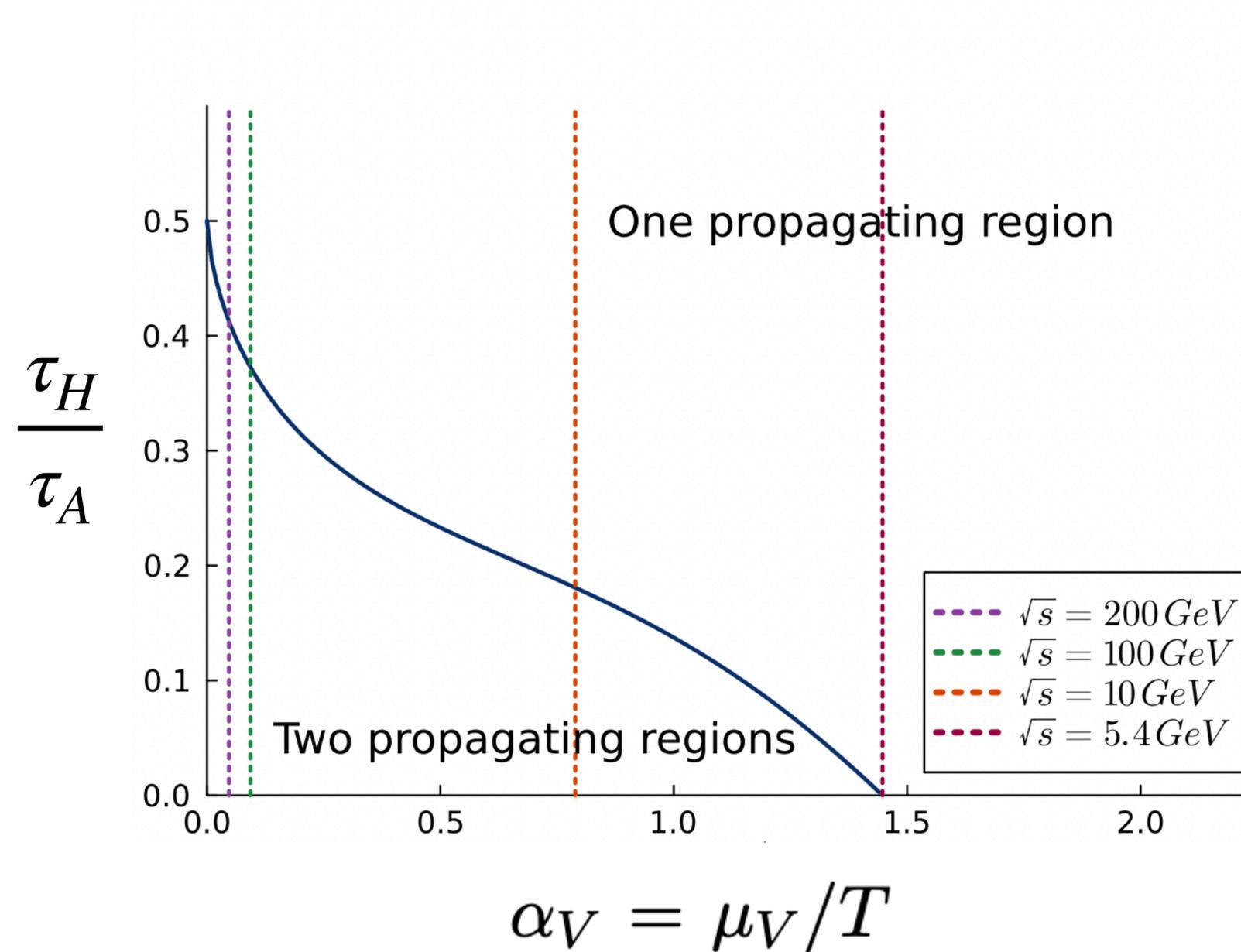
$$\omega = \tau_H k v$$

## Non-Conserved Helicity and Chirality ( $0 \leq \tau_H, \tau_A < \infty$ )



## CASE III:

Non-Conserved Helicity and Chirality ( $0 \leq \tau_H, \tau_A < \infty$ )



# Dissipative Effects

- The currents and the energy-momentum tensor get dissipative contributions, whose dynamics is not fixed by the conservation equations.
- A causal theory calls for second order hydro. For illustrative purposes we use 1st order.

$$\begin{aligned}
 \sum_q^{\{B,Q,S\}} \tau_{q'q} \dot{V}_q^{\langle\mu\rangle} + V_{q'}^\mu &= \sum_q^{\{B,Q,S\}} \kappa_{q'q} \nabla^\mu \alpha_q - \sum_q^{\{B,Q,S\}} \tau_{q'q} V_{q,\nu} \omega^{\nu\mu} - \sum_q^{\{B,Q,S\}} \delta_{VV}^{(q',q)} V_q^\mu \theta - \sum_q^{\{B,Q,S\}} \lambda_{VV}^{(q',q)} V_{q,\nu} \sigma^{\mu\nu} \\
 &\quad - \ell_{V\Pi}^{(q')} \nabla^\mu \Pi + \ell_{V\pi}^{(q')} \Delta^{\mu\nu} \nabla_\lambda \pi_\nu^\lambda + \tau_{V\Pi}^{(q')} \Pi \dot{u}^\mu - \tau_{V\pi}^{(q')} \pi^{\mu\nu} \dot{u}_\nu \\
 &\quad + \sum_q^{\{B,Q,S\}} \lambda_{V\Pi}^{(q',q)} \Pi \nabla^\mu \alpha_q - \sum_q^{\{B,Q,S\}} \lambda_{V\pi}^{(q',q)} \pi^{\mu\nu} \nabla_\nu \alpha_q,
 \end{aligned}$$

[Fotakis, Molnar, Niemi, Greiner, Rischke (2022)]

- We implement dissipation through relaxation time approximation  $\delta f_{\sigma,\lambda} = -\frac{\tau_R}{p \cdot u} p^\mu \partial_\mu f_{\sigma,\lambda}$ .

and the diss. terms

$$\begin{aligned}
 J_\ell^\mu &= \sum_{\sigma,\lambda} q_{\sigma,\lambda}^\ell Q_{\sigma,\lambda}, & Q_{\sigma,\lambda} &= \int \frac{d^3 p}{p^0} f_{\sigma,\lambda} p^\mu, & \text{under } f &\rightarrow \delta f \\
 T^{\mu\nu} &= \sum_{\sigma,\lambda} E_{\sigma,\lambda}, & E_{\sigma,\lambda} &= \int \frac{d^3 p}{p^0} f_{\sigma,\lambda} p^\mu p^\nu,
 \end{aligned}$$