

From hadron resonance gas to quark-gluon plasma via Mott - dissociation of multiquark clusters

David Blaschke (IFT UWr, HZDR/CASUS)



CASUS
CENTER FOR ADVANCED
SYSTEMS UNDERSTANDING

www.casus.science



A NEW RESEARCH TRIANGLE



A NEW RESEARCH TRIANGLE



A NEW RESEARCH TRIANGLE



A NEW RESEARCH TRIANGLE



HZDR
HELMHOLTZ ZENTRUM
DRESDEN ROSSENDORF

**CASUS**
CENTER FOR ADVANCED
SYSTEMS UNDERSTANDING

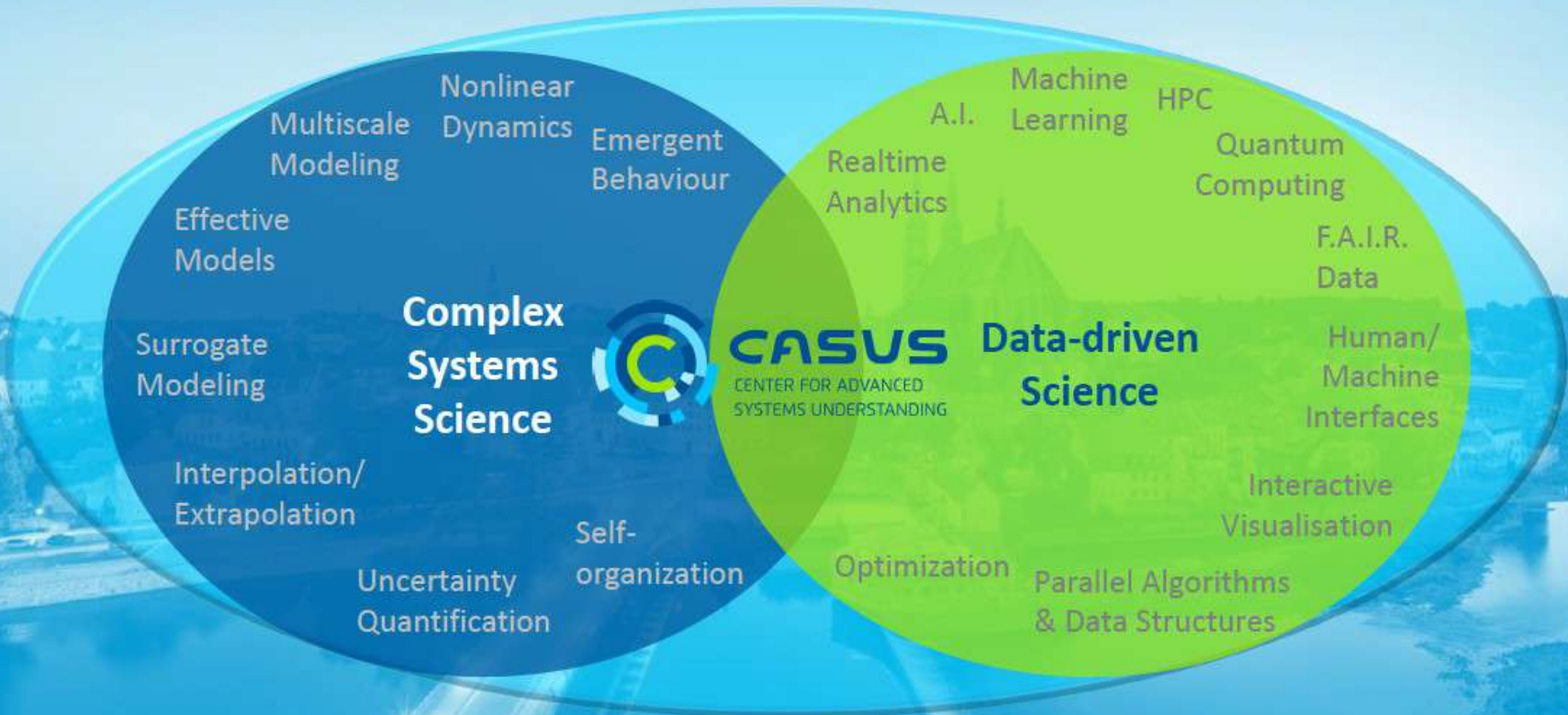
Uniwersytet
Wrocławski

A NEW RESEARCH TRIANGLE



Understanding complex systems with data

CASUS pushes the frontier of data-driven complex systems science



70 people, 20 countries, the brightest minds



Prof. Dr. Justin Calabrese
Ecological Data Science



Dr. Ricardo Martinez-Garcia
Dynamics of Complex Living Systems



Dr. Attila Cangi
Matter under Extreme Conditions



Dr. Michael Hecht
Mathematical Foundations of Complex System Science



Dr. Artur Yakimovich
Machine Learning for Infection and Disease



Dr. Weronika Schlechte-Welnicz
SCULTETUS Center





CASUS

CENTER FOR ADVANCED
SYSTEMS UNDERSTANDING

www.casus.science

HZDR
HELMHOLTZ ZENTRUM
DRESDEN ROSSENDORF

**TECHNISCHE
UNIVERSITÄT
DRESDEN**

CBG
Max Planck Institute
of Molecular Cell Biology
and Genetics

**HELMHOLTZ
CENTRE FOR
ENVIRONMENTAL
RESEARCH – UFZ**

**Uniwersytet
Wrocławski**

SPONSORED BY THE

**Federal Ministry
of Education
and Research**

**STAATSMINISTERIUM
FÜR WISSENSCHAFT
UND KUNST**

**Freistaat
SACHSEN**

Research Technology Digitization

„Science Creating Prospects
for the Region!“



Scientific Commission: 13. July 2022

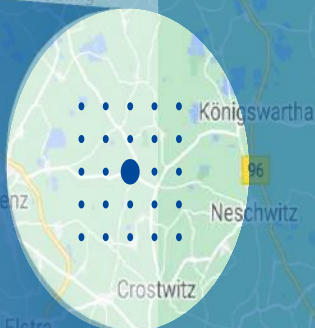
Structural and Transfer-Commission: 30. August 2022

Final decision (Approval): 29. September 2022

Why in Saxony? Lusatia is a unique region for Astrophysics, Technology and Digitization



Location for the Low Seismic Lab



TECHNISCHE UNIVERSITÄT DRESDEN

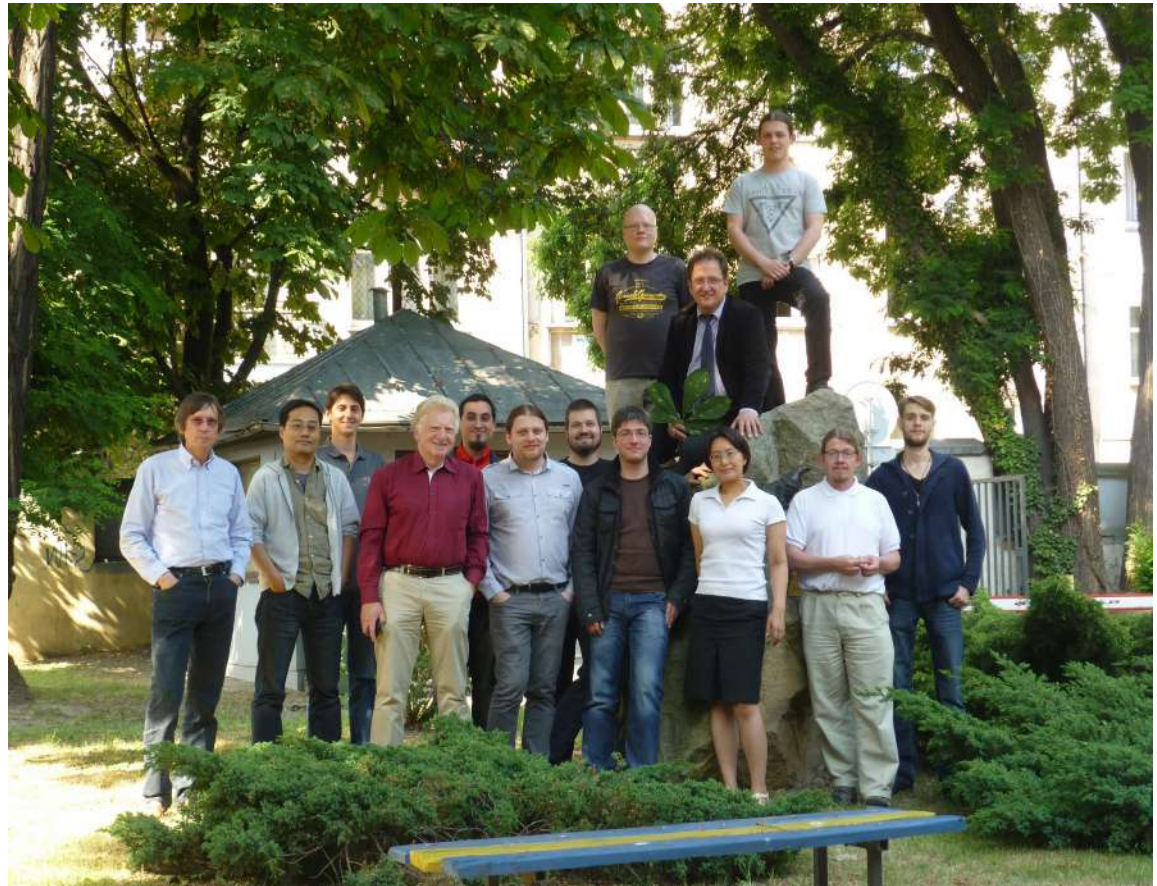


A center for astrophysics with advanced data intensive computing and technology development.



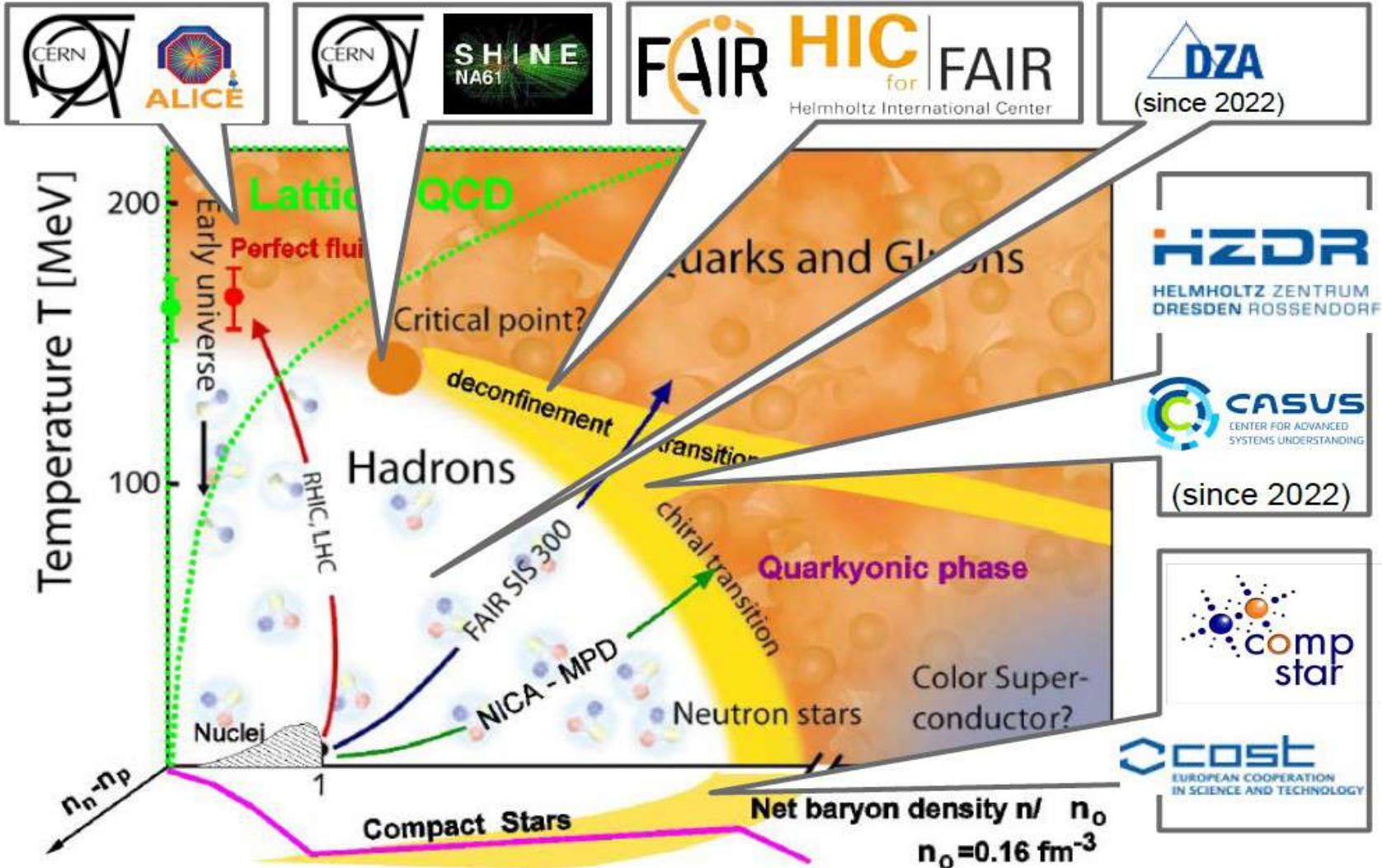
Thanks to my collaborators:

T. Fischer, G. Röpke, A. Bauswein, O. Ivanytskyi,
N. Bastian, M. Cierniak, U. Shukla, S. Liebing, K. Maslov,
A. Ayriyan,
H. Grigorian,
D.N. Voskresensky,
M. Kaltenborn,
G. Grunfeld,
D. Alvarez-Castillo,
B. Dönigus, ...



Wroclaw Group ...

Division: Theory of Elementary Particles - Collaborations



QCD Phase Diagram

Landscape of our investigations

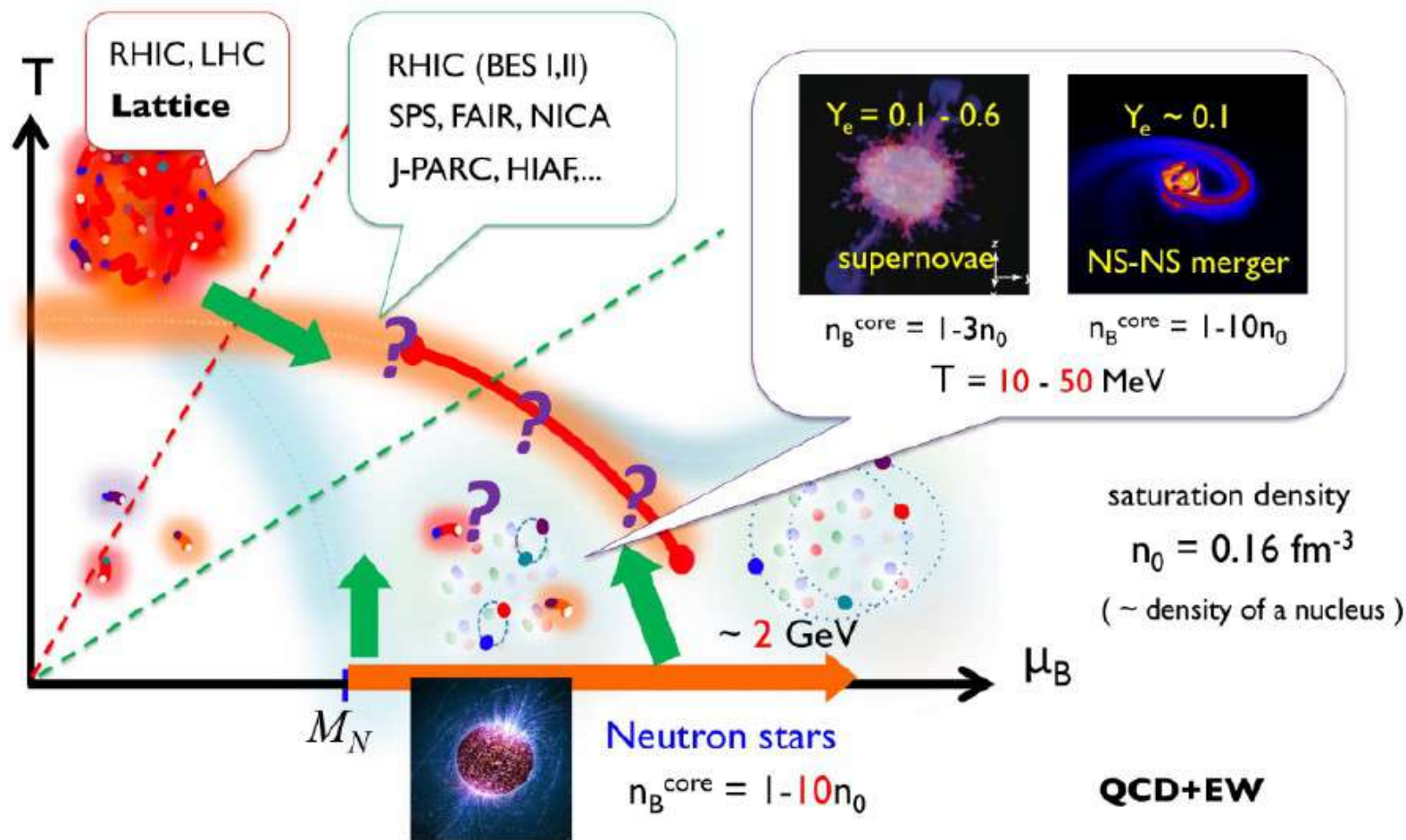
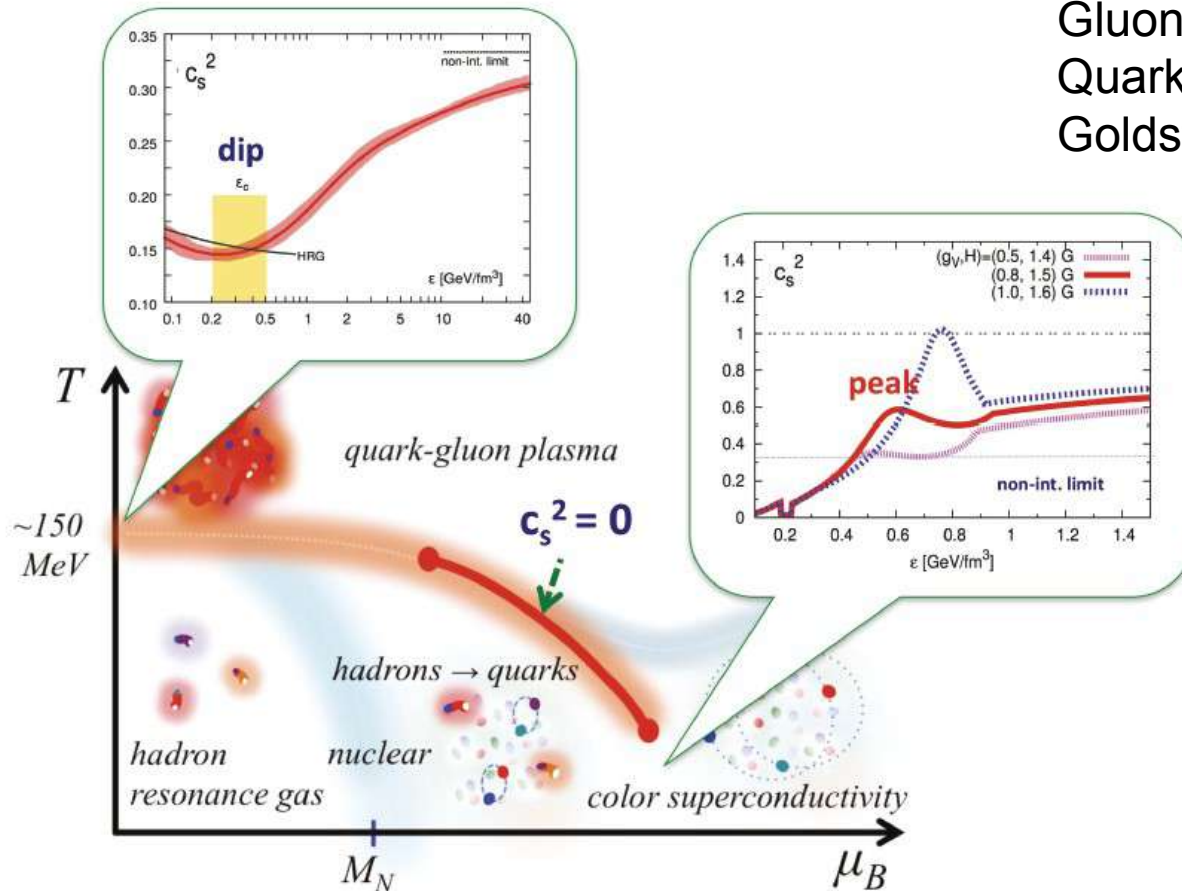


Figure from T. Kojo arXiv:1912.05326 [nucl-th]

QCD Phase Diagram

Landscape of our investigations



Gluons \leftrightarrow Vector mesons

Quarks \leftrightarrow Baryons

Goldstones \leftrightarrow Pseudoscalar mesons

**Quark-Hadron
Duality?**

**Mutual influence of
Order parameters for
 χ SB and CSC**

From: T. Kojo,
"QCD equations of state in
quark-hadron continuity",
Universe 4 (2018) 42

T. Schaefer & F. Wilczek, Phys. Rev. Lett. 82 (1999) 3956

C. Wetterich, Phys. Lett. B 462 (1999) 164

T. Hatsuda, M. Tachibana, T. Yamamoto & G. Baym, Phys. Rev. Lett. 97 (2006) 122001

Introduction

- New research triangle Wrocław – Görlitz – Dresden/Rossendorf: UWr – CASUS & DZA - HZDR
- Landscape of investigations: QCD Phase Diagram

Towards a unified approach to quark-nuclear matter

- Generalized Φ -derivable approach with clusters; cluster virial expansion
- Hadrons (mesons, baryons, multi-quark states) as clusters in quark matter – Mott dissociation of clusters
- Beth-Uhlenbeck approach to thermodynamics of quark-hadron matter

Relativistic density functionals for quark matter with confinement

- Density functional for warm, dense quark matter; chiral symmetry breaking and color superconductivity
- Quark confinement as density functional \rightarrow effective Nambu model with density-dependent couplings
- Phase transition construction and hybrid neutron star properties

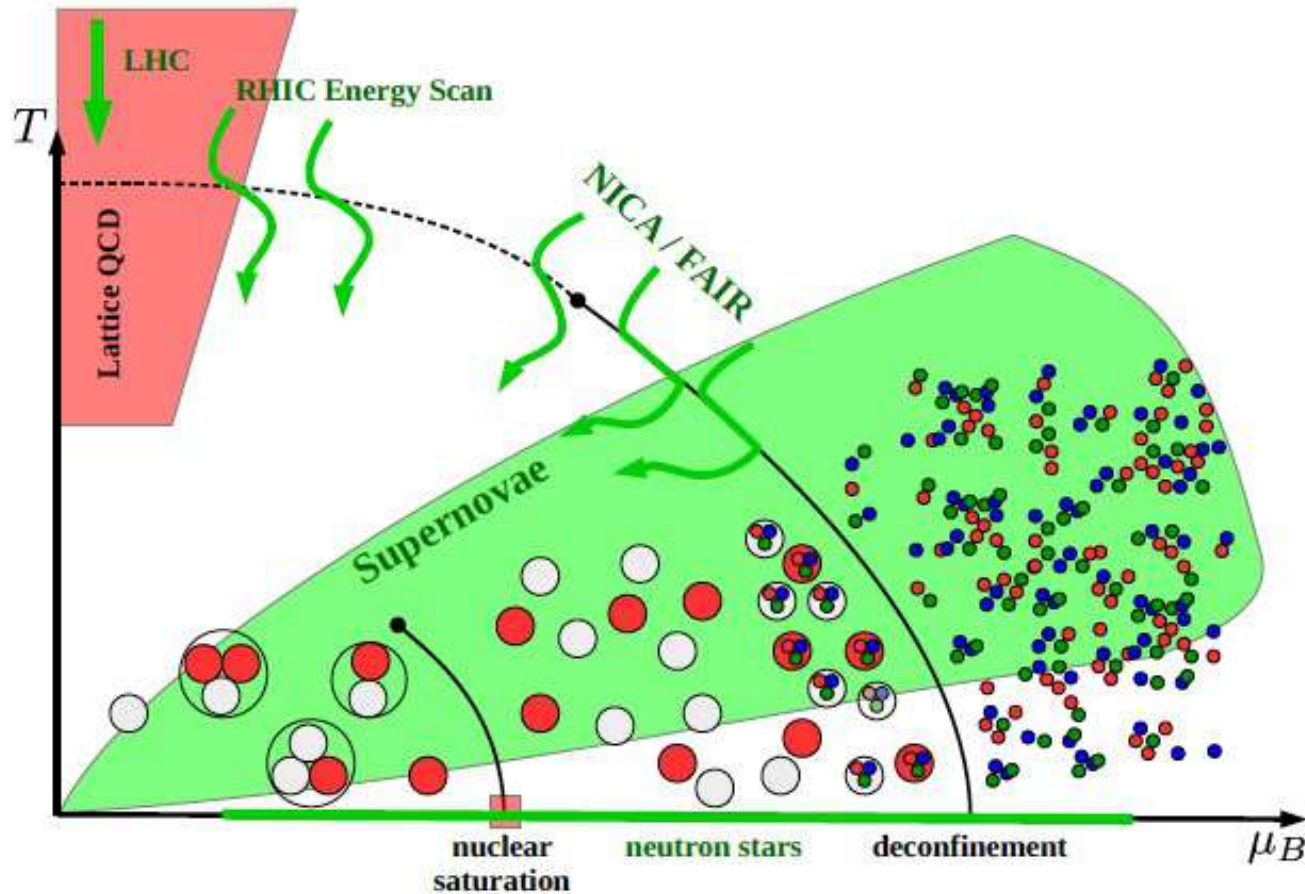
Unified EOS for quark-hadron matter

Cluster virial expansion & Beth-Uhlenbeck EoS



Unified approach to quark-nuclear matter

Clustering aspects in the QCD phase diagram



From: N.-U. Bastian, D.B., et al., Universe 4 (2018) 67; arxiv:1804.10178

Unified approach to quark-nuclear matter

Φ -derivable approach to cluster virial expansion

$$\Omega = \sum_{l=1}^A \Omega_l = \sum_{l=1}^A \left\{ c_l [\text{Tr} \ln (-G_l^{-1}) + \text{Tr} (\Sigma_l G_l)] + \sum_{\substack{i,j \\ i+j=l}} \Phi[G_i, G_j, G_{i+j}] \right\},$$

$$G_A^{-1} = G_A^{(0)-1} - \Sigma_A, \quad \Sigma_A(1 \dots A, 1' \dots A', z_A) = \frac{\delta \Phi}{\delta G_A(1 \dots A, 1' \dots A', z_A)}$$

Stationarity of the thermodynamical potential is implied

$$\frac{\delta \Omega}{\delta G_A(1 \dots A, 1' \dots A', z_A)} = 0.$$

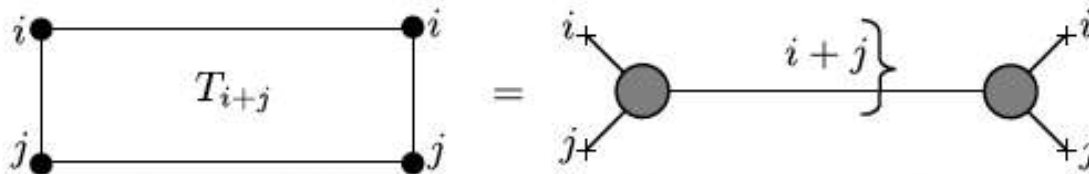
Cluster virial expansion follows for this Φ - functional



Figure: The Φ functional for A -particle correlations with bipartitions $A = i + j$.

Unified approach to quark-nuclear matter

Green's function and T-matrix, separable approx.



The T_A matrix fulfills the Bethe-Salpeter equation in ladder approximation

$$T_{i+j}(1, 2, \dots, A; 1', 2', \dots, A'; z) = V_{i+j} + V_{i+j} G_{i+j}^{(0)} T_{i+j},$$

which in the separable approximation for the interaction potential,

$$V_{i+j} = \Gamma_{i+j}(1, 2, \dots, i; i+1, i+2, \dots, i+j) \Gamma_{i+j}(1', 2', \dots, i'; (i+1)', (i+2)', \dots, (i+j)'),$$

leads to the closed expression for the T_A matrix

$$T_{i+j}(1, 2, \dots, i+j; 1', 2', \dots, (i+j)'; z) = V_{i+j} \{1 - \Pi_{i+j}\}^{-1},$$

with the generalized polarization function

$$\Pi_{i+j} = \text{Tr} \left\{ \Gamma_{i+j} G_i^{(0)} \Gamma_{i+j} G_j^{(0)} \right\}$$

The one-frequency free i -particle Green's function is defined by the $(i-1)$ -fold Matsubara sum

$$\begin{aligned} G_i^{(0)}(1, 2, \dots, i; \Omega_i) &= \sum_{\omega_1 \dots \omega_{i-1}} \frac{1}{\omega_1 - E(1)} \frac{1}{\omega_2 - E(2)} \cdots \frac{1}{\Omega_i - (\omega_1 + \dots + \omega_{i-1}) - E(i)} \\ &= \frac{(1-f_1)(1-f_2)\dots(1-f_i) - (-)^i f_1 f_2 \dots f_i}{\Omega_i - E(1) - E(2) - \dots - E(i)}. \end{aligned}$$

Unified approach to quark-nuclear matter

Useful relationships for many-particle functions

$$G_{i+j}^{(0)} = G_{i+j}^{(0)}(1, 2, \dots, i+j; \Omega_{i+j}) = \sum_{\Omega_i} G_i^{(0)}(1, 2, \dots, i; \Omega_i) G_j^{(0)}(i+1, i+2, \dots, i+j; \Omega_j) .$$

Another set of useful relationships follows from the fact that in the ladder approximation both, the full two-cluster ($i+j$ particle) T matrix and the corresponding Greens' function

$$G_{i+j} = G_{i+j}^{(0)} \{1 - \Pi_{i+j}\}^{-1} \quad (1)$$

have similar analytic properties determined by the $i+j$ cluster polarization loop integral and are related by the identity

$$T_{i+j} G_{i+j}^{(0)} = V_{i+j} G_{i+j} . \quad (2)$$

which is straightforwardly proven by multiplying Equation for the T_{i+j} - matrix with $G_{i+j}^{(0)}$ and using Equation (1). Since these two equivalent expressions in Equation (2) are at the same time equivalent to the two-cluster irreducible Φ functional these functional relations follow

$$T_{i+j} = \delta\Phi / \delta G_{i+j}^{(0)} ,$$
$$V_{i+j} = \delta\Phi / \delta G_{i+j} .$$

Unified approach to quark-nuclear matter

Generalized Beth-Uhlenbeck EOS from Φ -deriv.

Consider the partial density of the A -particle state defined as

$$n_A(T, \mu) = -\frac{\partial \Omega_A}{\partial \mu} = -\frac{\partial}{\partial \mu} d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left[\ln(-G_A^{-1}) + \text{Tr}(\Sigma_A G_A) \right] + \sum_{\substack{i,j \\ i+j=A}} \Phi[G_i, G_j, G_{i+j}] .$$

Using spectral representation for $F(\omega)$ and Matsubara summation

$$F(iz_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\text{Im}F(\omega)}{\omega - iz_n}, \quad \sum_{z_n} \frac{c_A}{\omega - iz_n} = f_A(\omega) = \frac{1}{\exp[(\omega - \mu)/T] - (-1)^A}$$

with the relation $\partial f_A(\omega)/\partial \mu = -\partial f_A(\omega)/\partial \omega$ we get for Equation (3) now

$$n_A(T, \mu) = -d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_A(\omega) \frac{\partial}{\partial \omega} \left[\text{Im} \ln(-G_A^{-1}) + \text{Im}(\Sigma_A G_A) \right] + \sum_{\substack{i,j \\ i+j=A}} \frac{\partial \Phi[G_i, G_j, G_A]}{\partial \mu},$$

where a partial integration over ω has been performed For two-loop diagrams of the sunset type holds a cancellation³ which generalize here for cluster states

$$d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_A(\omega) \frac{\partial}{\partial \omega} (\text{Re}\Sigma_A \text{Im}G_A) - \sum_{\substack{i,j \\ i+j=A}} \frac{\partial \Phi[G_i, G_j, G_A]}{\partial \mu} = 0 .$$

Using generalized optical theorems we can show that ($G_A = |G_A| \exp(i\delta_A)$)

$$\frac{\partial}{\partial \omega} \left[\text{Im} \ln(-G_A^{-1}) + \text{Im}\Sigma_A \text{Re}G_A \right] = 2\text{Im} \left[G_A \text{Im}\Sigma_A \frac{\partial}{\partial \omega} G_A^* \text{Im}\Sigma_A \right] = -2 \sin^2 \delta_A \frac{\partial \delta_A}{\partial \omega} .$$

The density in the form of a generalized Beth-Uhlenbeck EoS follows

$$n(T, \mu) = \sum_{i=1}^A n_i(T, \mu) = \sum_{i=1}^A d_i \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_i(\omega) 2 \sin^2 \delta_i \frac{\partial \delta_i}{\partial \omega} .$$

³B. Vanderheyden & G. Baym, J. Stat. Phys. (1998), J.-P. Blaizot et al., PRD (2001)

Unified approach to quark-nuclear matter

Example: deuterons in nuclear matter

The Φ -derivable thermodynamical potential for the nucleon-deuteron system reads

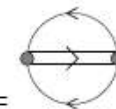
$$\Omega = -\text{Tr} \{ \ln(-G_1) \} - \text{Tr} \{ \Sigma_1 G_1 \} + \text{Tr} \{ \ln(-G_2) \} + \text{Tr} \{ \Sigma_2 G_2 \} + \Phi[G_1, G_2] ,$$

where the full propagators obey the Dyson-Schwinger equations

$$G_1^{-1}(1, z) = z - E_1(p_1) - \Sigma_1(1, z); \quad G_2^{-1}(12, 1'2', z) = z - E_1(p_1) - E_2(p_2) - \Sigma_2(12, 1'2', z),$$

with selfenergies and Φ functional

$$\Sigma_1(1, 1') = \frac{\delta\Phi}{\delta G_1(1, 1')} ; \quad \Sigma_2(12, 1'2', z) = \frac{\delta\Phi}{\delta G_2(12, 1'2', z)} , \quad \Phi = \text{diagram} ,$$



fulfilling stationarity of the thermodynamic potential $\partial\Omega/\partial G_1 = \partial\Omega/\partial G_2 = 0$.

For the density we obtain the cluster virial expansion

$$n = -\frac{1}{V} \frac{\partial\Omega}{\partial\mu} = n_{\text{qu}}(\mu, T) + 2n_{\text{corr}}(\mu, T) ,$$

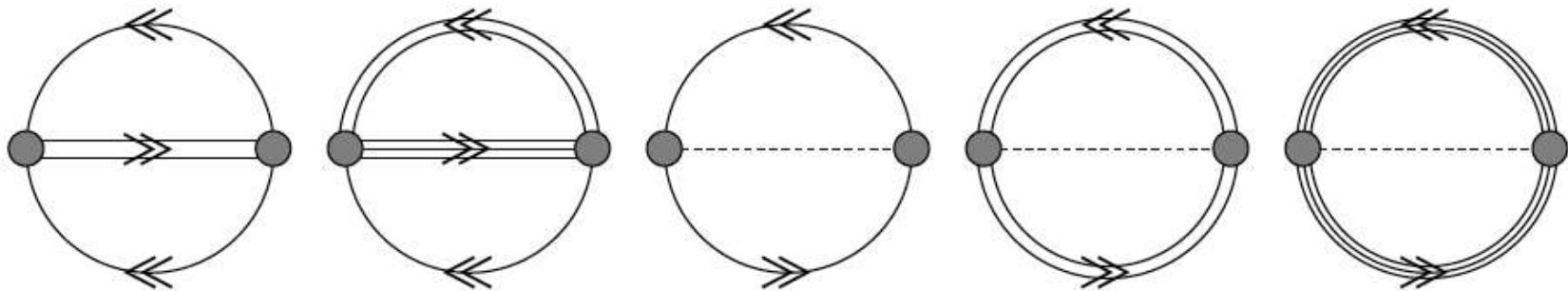
with the correlation density in the generalized Beth-Uhlenbeck form

$$n_{\text{corr}} = \int \frac{dE}{2\pi} g(E) 2 \sin^2 \delta(E) \frac{d\delta(E)}{dE} .$$

Unified approach to quark-nuclear matter

Cluster virial expansion for quark-hadron matter

$$\Omega = \sum_{i=Q,M,D,B} c_i [\text{Tr} \ln (-G_i^{-1}) + \text{Tr} (\Sigma_i G_i)] + \Phi [G_Q, G_M, G_D, G_B] ,$$



When Φ functional for the system is given by 2-loop diagrams holds

$$\begin{aligned} n &= -\frac{\partial \Omega}{\partial \mu} = \sum_a a n_a(T, \mu) \\ &= \sum_a a d_a \int \frac{d\omega}{\pi} \int \frac{d^3 q}{(2\pi)^3} \left\{ f_{\phi}^{(a),+} - \left[f_{\phi}^{(a),-} \right]^* \right\} 2 \sin^2 \delta_a(\omega, q) \frac{\partial \delta_a(\omega, q)}{\partial \omega} , \end{aligned}$$

Analogous for the entropy density $s = -\partial \Omega / \partial T$.

Unified approach to quark-nuclear matter

Cluster virial expansion for quark-hadron matter

The cluster decomposition of the thermodynamic potential is given as

$$\Omega_{\text{total}}(T, \mu, \phi, \bar{\phi}) = \Omega_{PNJL}(T, \mu, \phi, \bar{\phi}) + \Omega_{\text{pert}}(T, \mu, \phi, \bar{\phi}) + \Omega_{MHRG}(T, \mu, \phi, \bar{\phi}),$$

where the first two terms describe the quark and gluon degrees of freedom via the mean-field thermodynamic potential for quark matter in a gluon background field \mathcal{U}

$$\Omega_{PNJL}(T, \mu, \phi, \bar{\phi}) = \Omega_Q(T, \mu, \phi, \bar{\phi}) + \mathcal{U}(T, \phi, \bar{\phi})$$

with a perturbative correction $\Omega_{\text{pert}}(T, \mu, \phi, \bar{\phi})$.

The Mott-Hadron-Resonance-Gas (MHRG) part for the multi-quark clusters is

$$\Omega_{MHRG}(T, \mu, \phi, \bar{\phi}) = \sum_{i=M,B,\dots} \Omega_i(T, \mu, \phi, \bar{\phi}),$$

where the multi-quark states are described by the GBU formula:

$$\begin{aligned} n &= -\frac{\partial \Omega}{\partial \mu} = \sum_a a n_a(T, \mu) \\ &= \sum_a a d_a \int \frac{d\omega}{\pi} \int \frac{d^3 q}{(2\pi)^3} \left\{ f_{\phi}^{(a),+} - \left[f_{\phi}^{(a),-} \right]^* \right\} 2 \sin^2 \delta_a(\omega, q) \frac{\partial \delta_a(\omega, q)}{\partial \omega}, \end{aligned}$$

where d_i is the degeneracy factor, a is the number of valence quarks in the cluster and $f_{\phi}^{(a),+}$, $\left[f_{\phi}^{(a),-} \right]^*$ are the Polyakov-loop modified distribution functions.

Analogous for the entropy density $s = -\partial \Omega / \partial T$.

Unified approach to quark-nuclear matter

Polyakov-loop modified distribution functions

For multiquark clusters with net number a of valence quarks holds

$$f_{\phi}^{(a),\pm} \stackrel{(a \text{ even})}{=} \frac{(\phi - 2\bar{\phi}y_a^{\pm})y_a^{\pm} + y_a^{\pm 3}}{1 - 3(\phi - \bar{\phi}y_a^{\pm})y_a^{\pm} - y_a^{\pm 3}},$$
$$f_{\phi}^{(a),\pm} \stackrel{(a \text{ odd})}{=} \frac{(\bar{\phi} + 2\phi y_a^{\pm})y_a^{\pm} + y_a^{\pm 3}}{1 + 3(\bar{\phi} + \phi y_a^{\pm})y_a^{\pm} + y_a^{\pm 3}},$$

where $y_a^{\pm} = e^{-(E_p \mp a\mu)/T}$ and $E_p = \sqrt{p^2 + M^2}$.

It is instructive to consider the two limits $\phi = \bar{\phi} = 1$ (deconfinement)

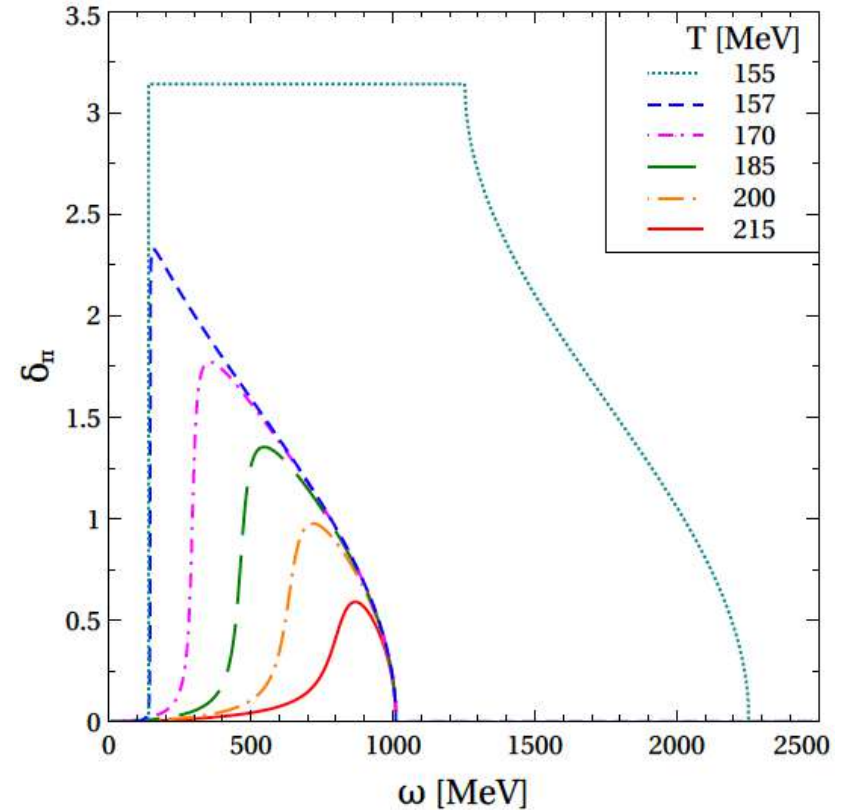
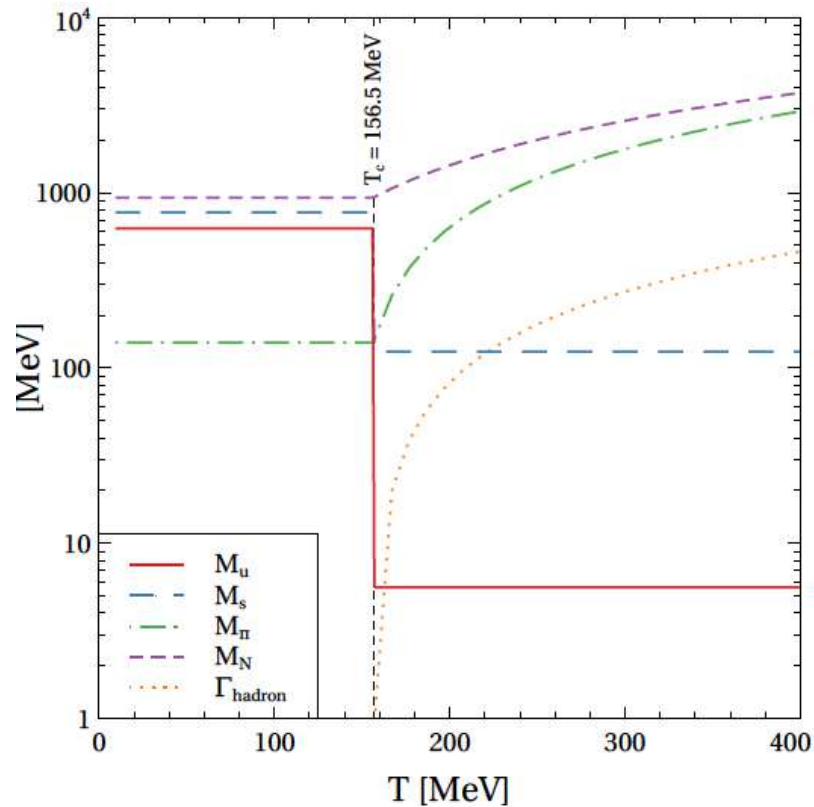
$$f_{\phi=1}^{(a=0,2,4,\dots),\pm} = \frac{y_a^{\pm}}{1 - y_a^{\pm}}, \quad f_{\phi=1}^{(a=1,3,5,\dots),\pm} = \frac{y_a^{\pm}}{1 + y_a^{\pm}},$$

and $\phi = \bar{\phi} = 0$ (confinement),

$$f_{\phi=0}^{(a=0,2,4,\dots),\pm} = \frac{y_a^{\pm 3}}{1 - y_a^{\pm 3}}, \quad f_{\phi=0}^{(a=1,3,5,\dots),\pm} = \frac{y_a^{\pm 3}}{1 + y_a^{\pm 3}}.$$

Unified approach to quark-hadron matter

Inputs: mass spectrum & phase shifts (models)



Unified approach to quark-hadron matter

Inputs: mass spectrum (Particle Data Tables)

Mesons

PDG mesons	d_i	M_{PDG} [MeV]	M_i [MeV]	$M_{\text{th},i}^<$ [MeV]	$M_{\text{th},i}^>$ [MeV]
π^+/π^0	3	140	140	1254	11.2
K^+/K^0	4	494	494	1397	129.6
η	1	548	878	1349	90.1
ρ^+/ρ^0	9	775	783	1254	11.2
ω	9	783	783	1254	11.2
K^{*+}/K^{*0}	12	895	806 [*])	2651	140.8
η'	1	960	878	1349	90.1
a_0	3	980	1095 [*])	2508	22.4
f_0	1	980	1095 [*])	2508	22.4
ϕ	3	1020	1069	1540	248
..					
$\pi_2(1880)$	15	1895	1095 [*])	2508	22.4
$f_2(1950)$	5	1944	1095 [*])	2508	22.4
$a_4(2040)$	27	1996	1095 [*])	2508	22.4
$f_2(2010)$	5	2011	1095 [*])	2508	22.4
$f_4(2050)$	9	2018	1095 [*])	2508	22.4
$K_4^*(2045)$	36	2045	1238 [*])	2651	140.8
$\phi(2170)$	3	2175	1381 [*])	2794	259.2
$f_2(2300)$	5	2297	1095 [*])	2508	22.4
$f_2(2340)$	5	2339	1095 [*])	2508	22.4

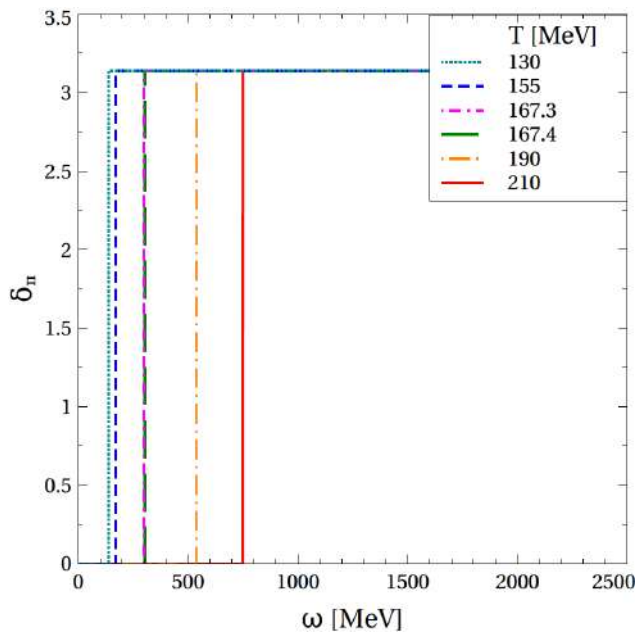
Baryons

PDG baryons	d_i	M_{PDG} [MeV]	M_i [MeV]	$M_{\text{th},i}^<$ [MeV]	$M_{\text{th},i}^>$ [MeV]
n/p	4	939	939	1881	16.8
Λ	2	1116	1082	2024	135.2
Σ	6	1193	1082	2024	135.2
Δ	16	1232	1251 ^{**)}	3135	28
Ξ^0	2	1315	1225	2167	253.6
Ξ^-	2	1322	1225	2167	253.6
$\Sigma(1385)$	6	1385	1394 ^{**)}	3278	146.4
$\Lambda(1405)$	2	1405	1394 ^{**)}	3278	146.4
$N(1440)$	4	1440	1251 ^{**)}	3135	28
..					
$N(2195)$	36	2220	1251 ^{**)}	3135	28
$\Sigma(2250)$	6	2250	1394 ^{**)}	3278	146.4
$\Omega^-(2250)$	2	2252	1680 ^{**)}	3564	383.2
$N(2250)$	20	2275	1251 ^{**)}	3135	28
$\Lambda(2350)$	10	2350	1394 ^{**)}	3278	146.4
$\Delta(2420)$	48	2420	1251 ^{**)}	3135	28
$N(2600)$	24	2600	1251 ^{**)}	3135	28

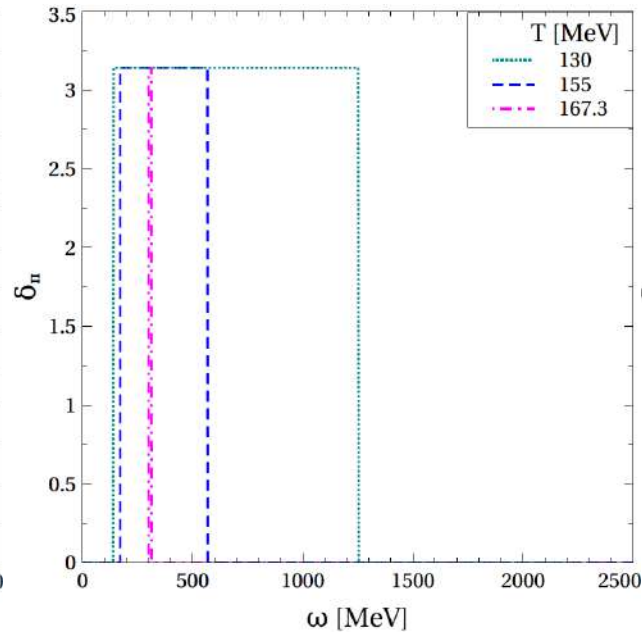
... and colored clusters (model) !

Unified approach to quark-hadron matter

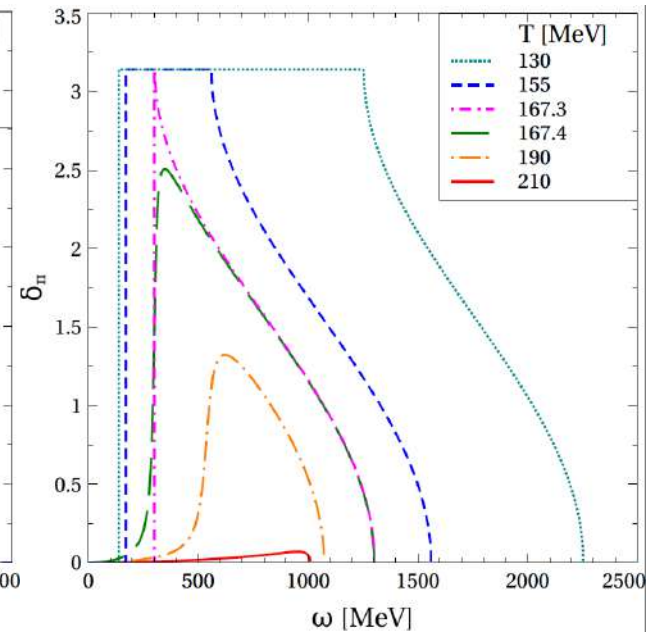
Inputs for the phase shifts (models)



Step-up (SU) model →
Hadron Resonance Gas



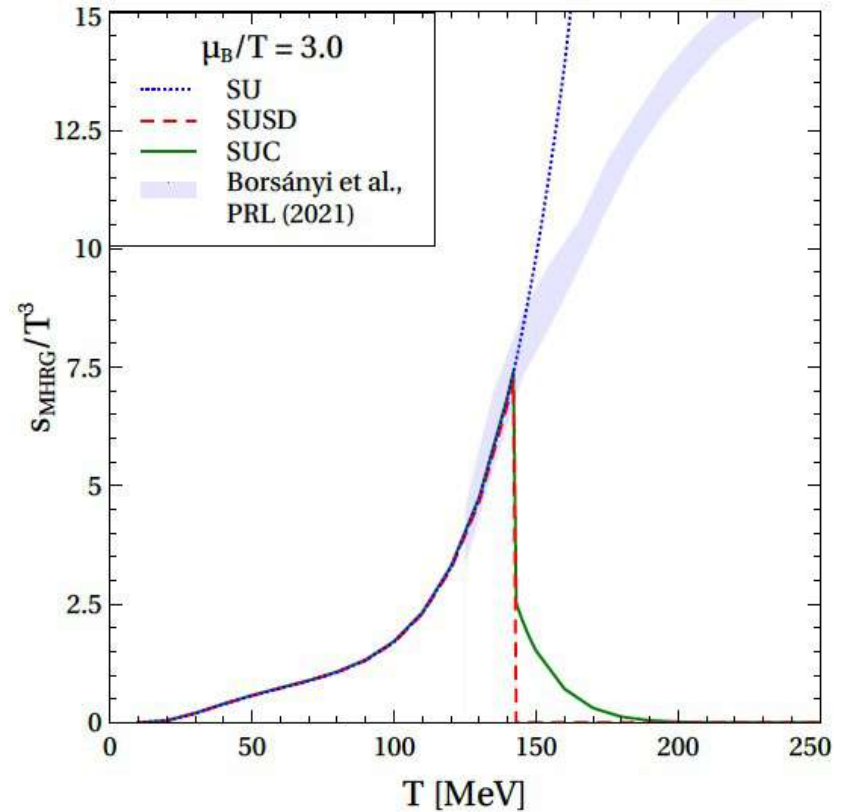
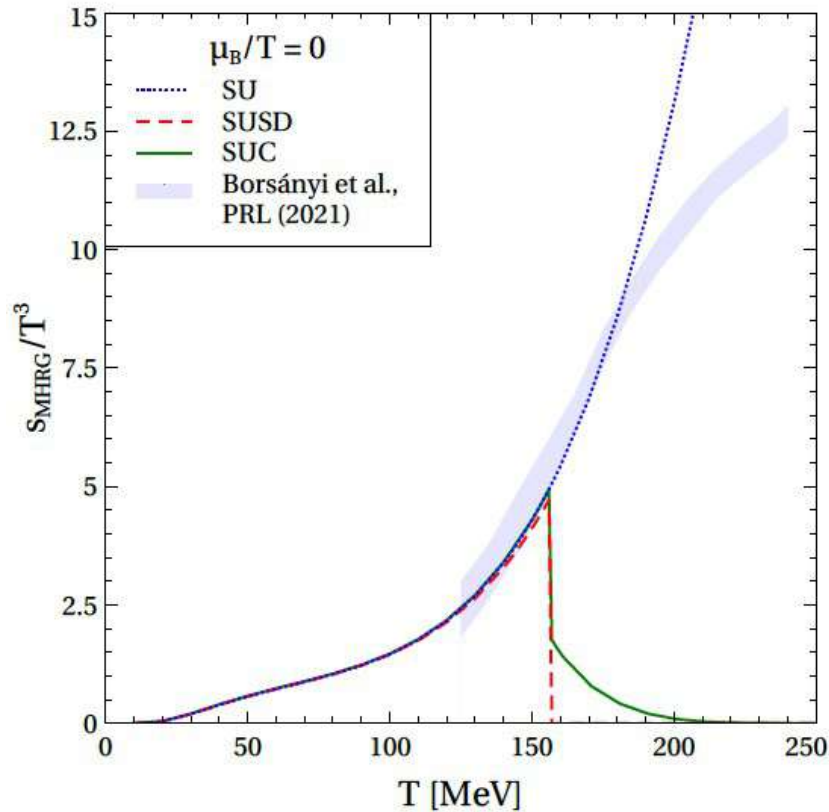
Step-up-step-down model
→ Mott Hadron Resonance Gas (MHRG)



Step-up-continuum model

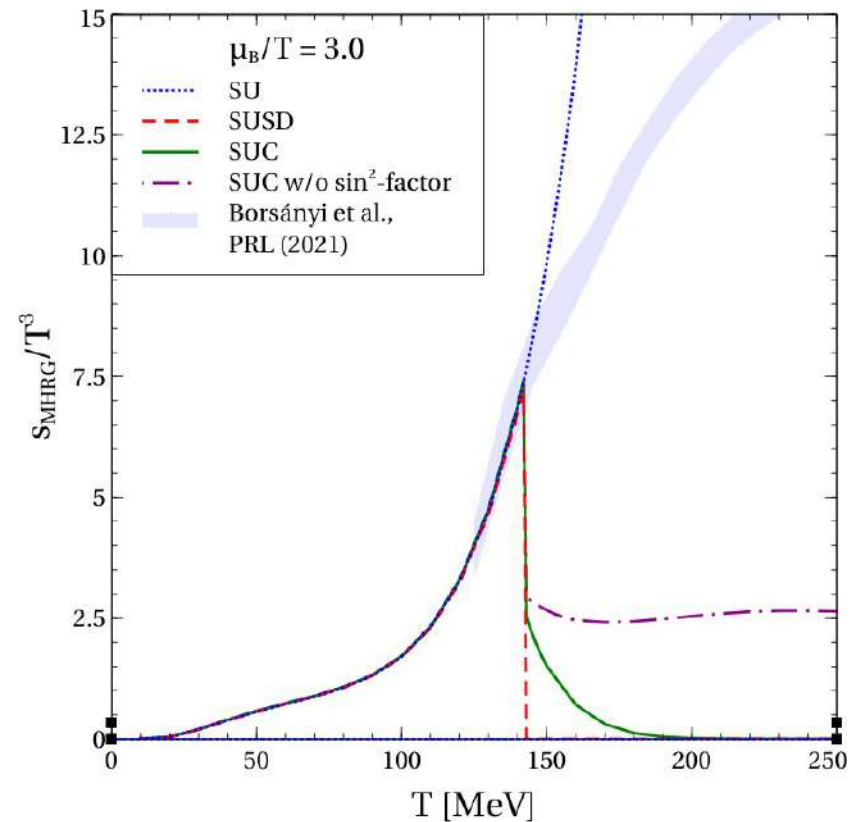
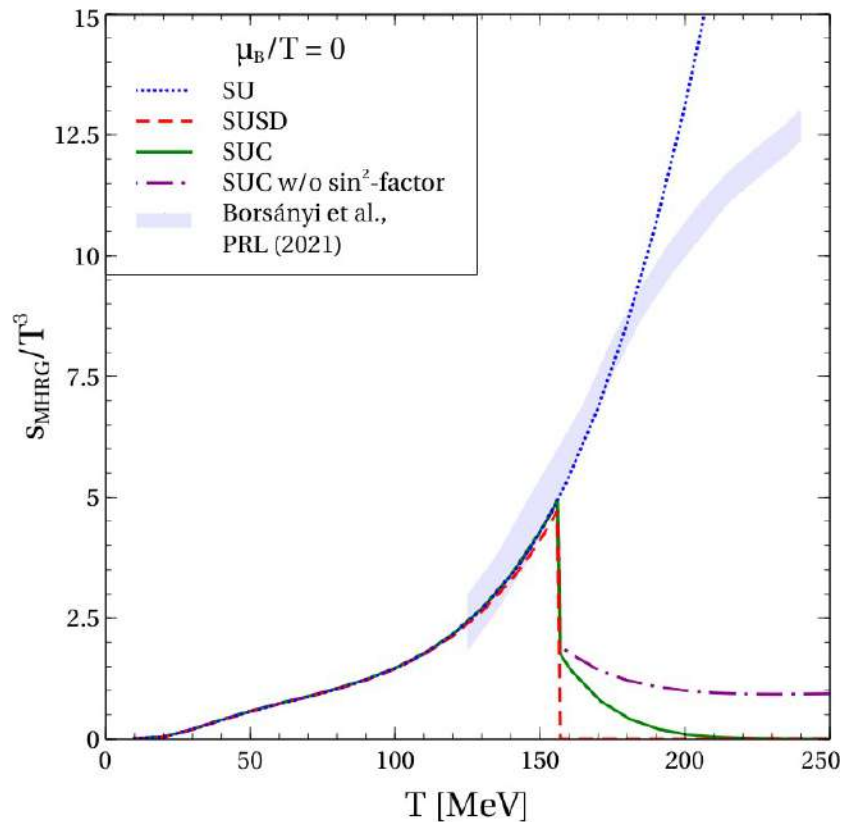
Unified approach to quark-hadron matter

Results for Mott-Hadron Resonance Gas (MHRG)



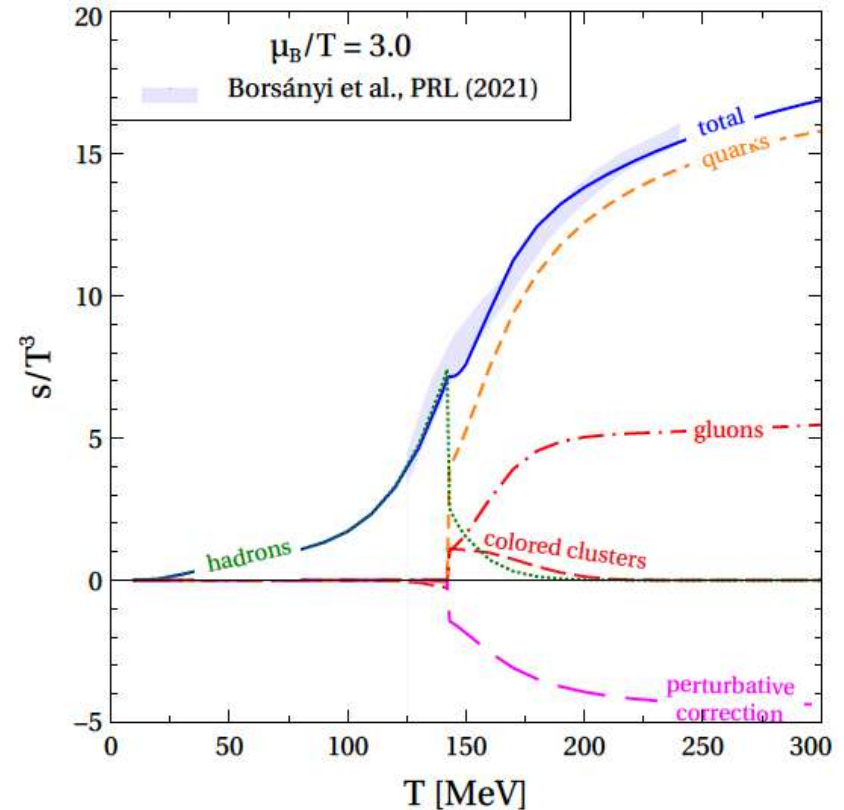
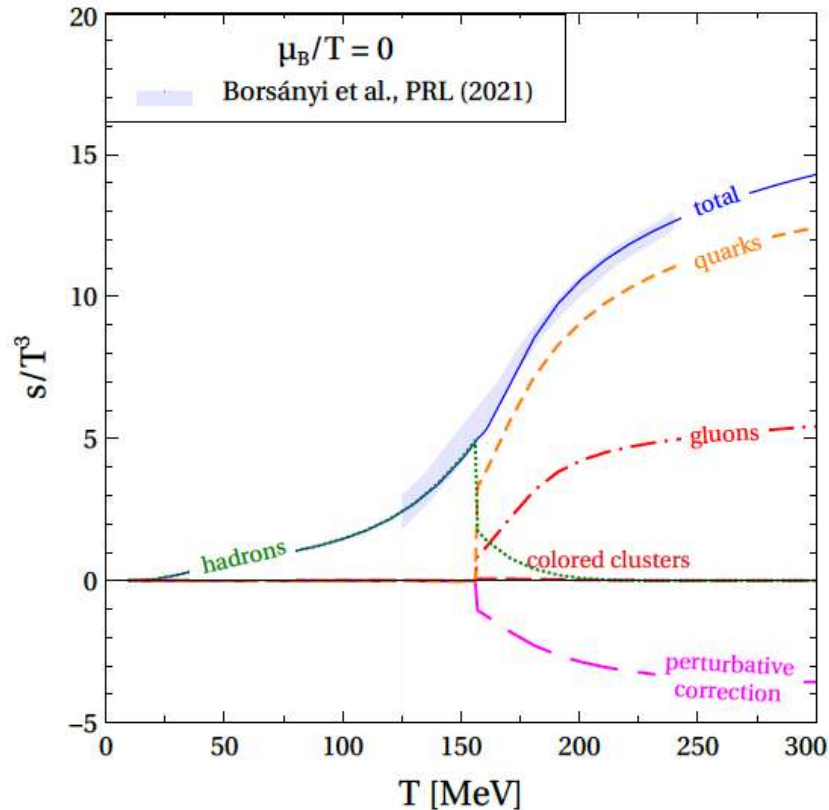
Unified approach to quark-hadron matter

Entropy for MHRG: role of the \sin^2 -term



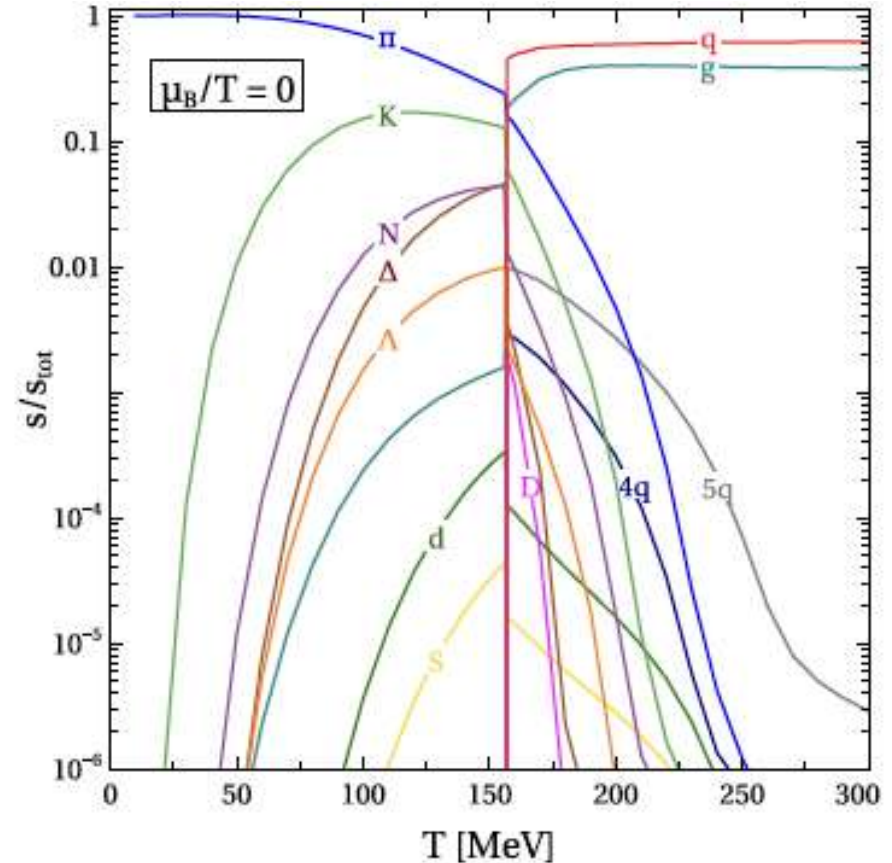
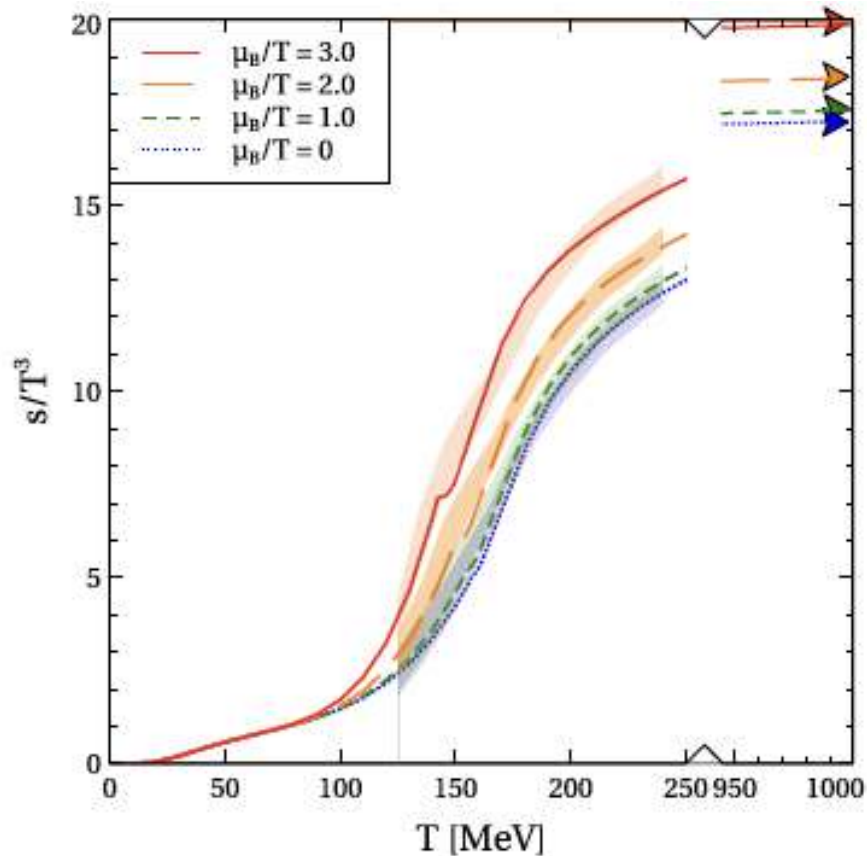
Unified approach to quark-hadron matter

Results for the entropy density



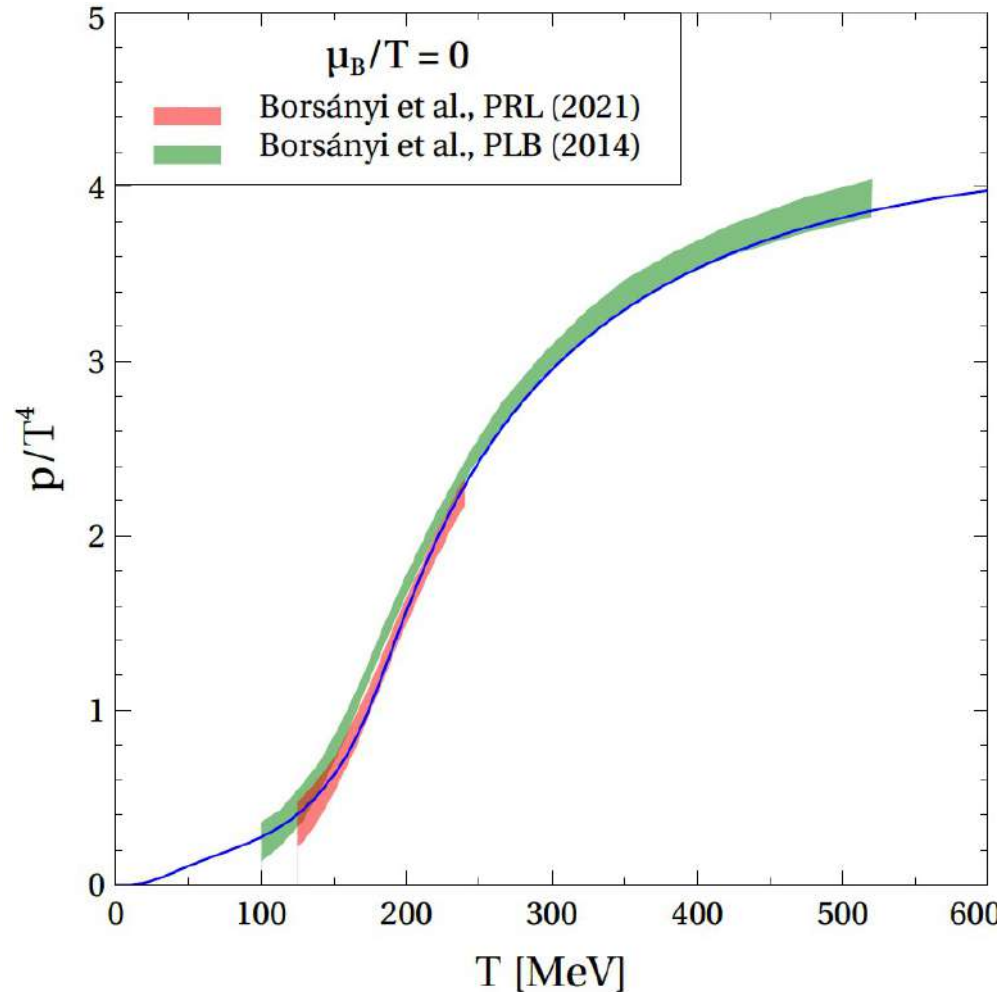
Unified approach to quark-hadron matter

Results for the entropy density & composition



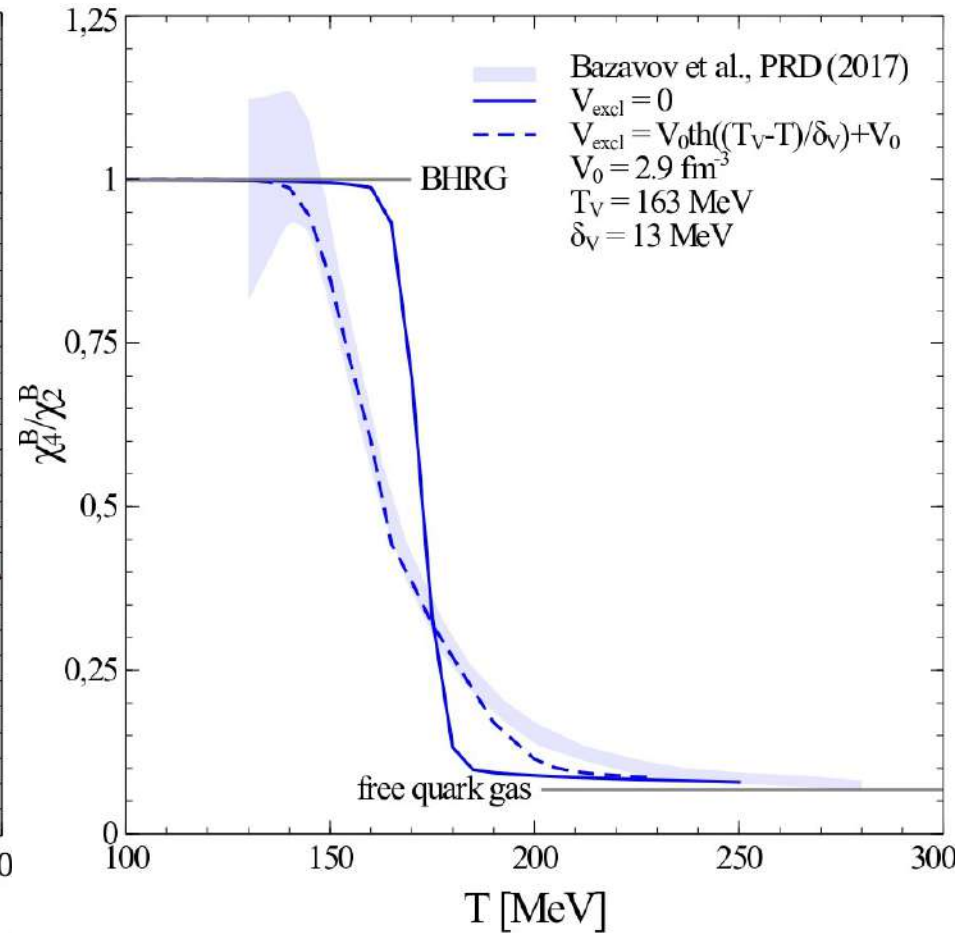
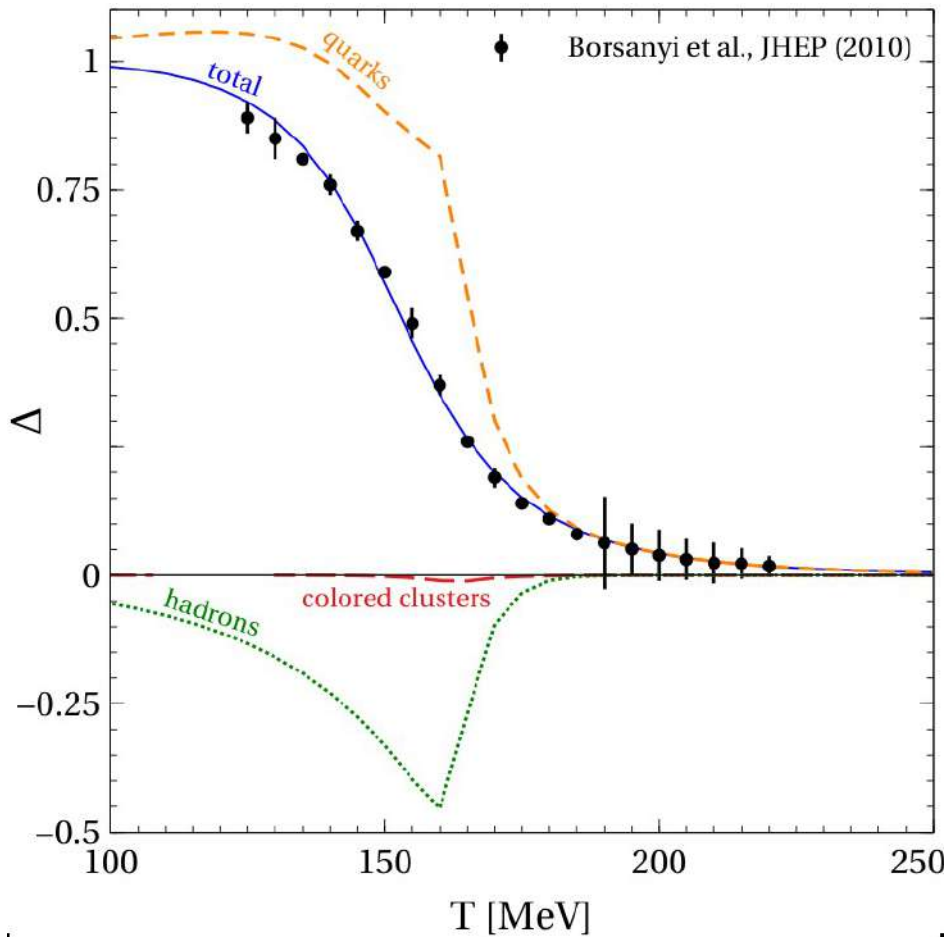
Unified approach to quark-hadron matter

Results for pressure=thermodynamic potential



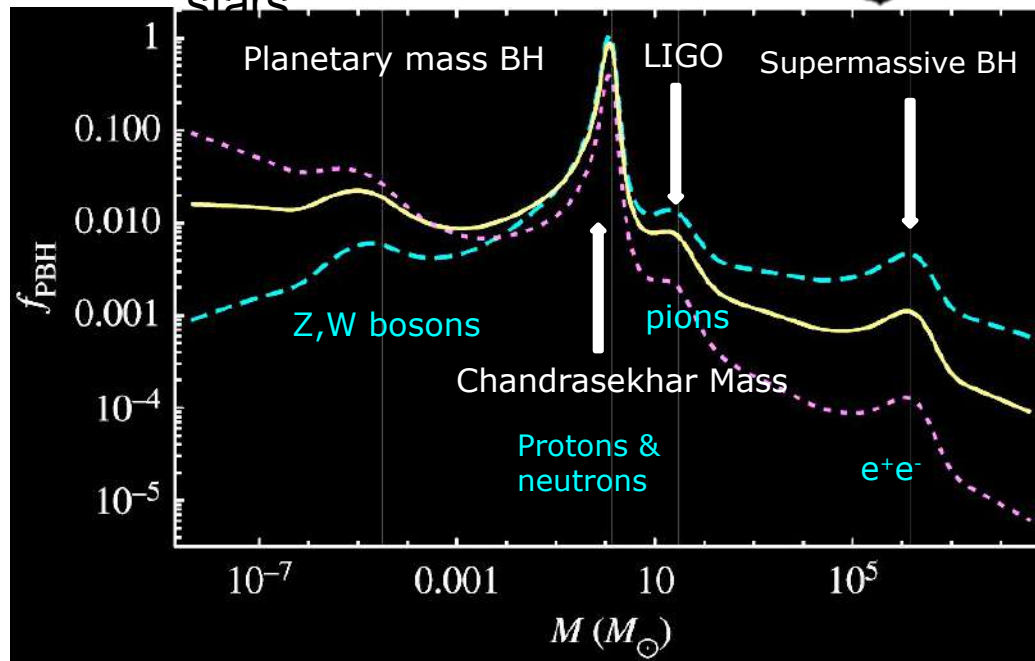
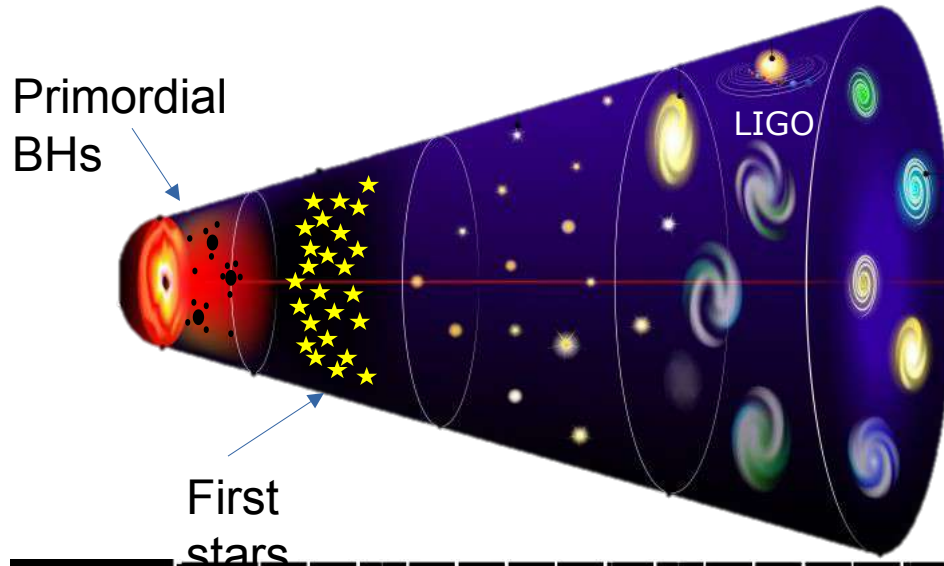
Unified approach to quark-hadron matter

Chiral condensate & baryon susceptibilities



D.B., O. Ivanytskyi and G. Röpke, in preparation (2024)

JWST results – primordial black holes !



Talk at University of Wroclaw
by Günther Hasinger,
Founding director of the
German Centre for Astrophysics
In Görlitz:



**Key role plays the QCD
hadronization transition !**

Different peaks correspond to
different particles created at the
early universe phase transitions and
the corresponding reduction in the
sound velocity.

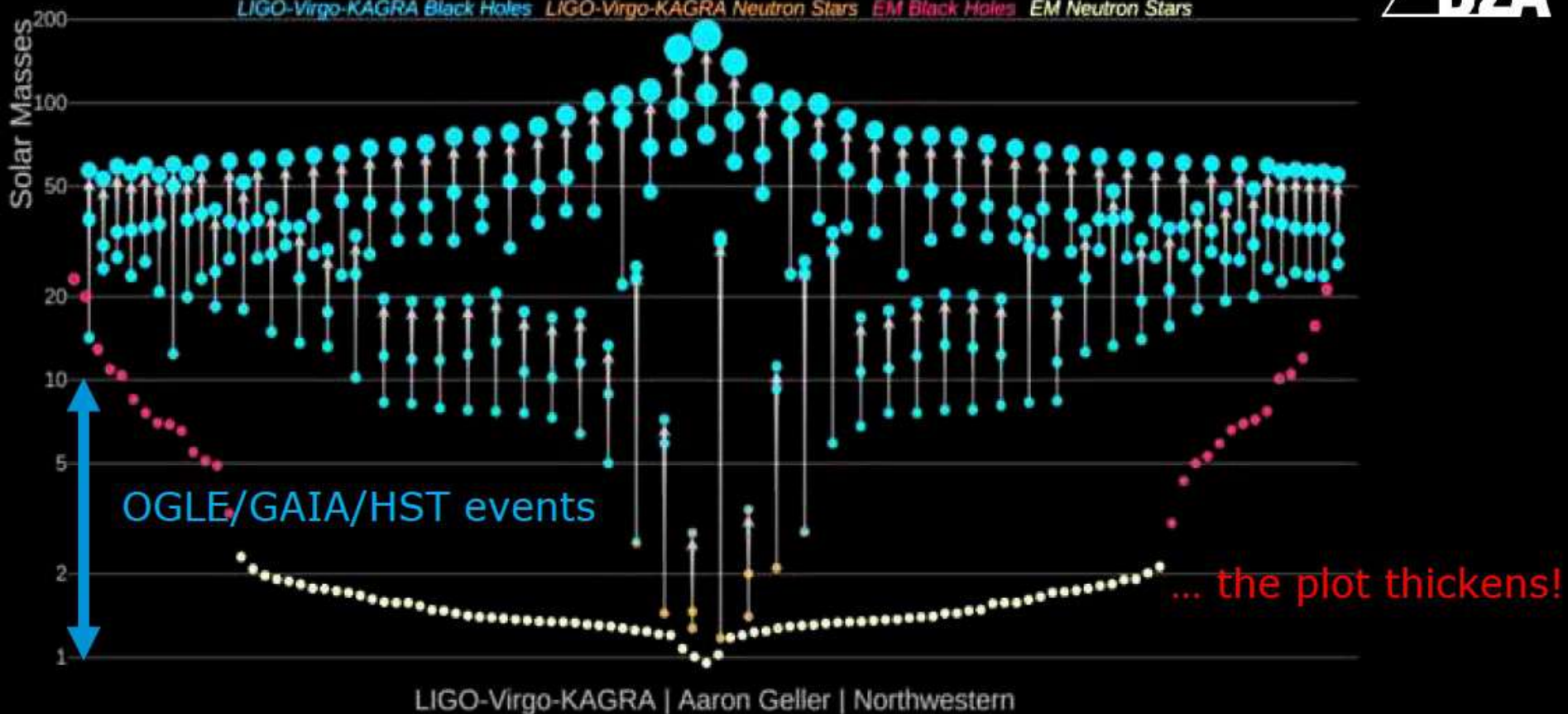
BH mass corresponds to the horizon
size at each time.

Only requirement is enough
fluctuation power in a volume fraction
of 10^{-9} of the early Universe.

Carr, Clesse, García-Bellido 2019

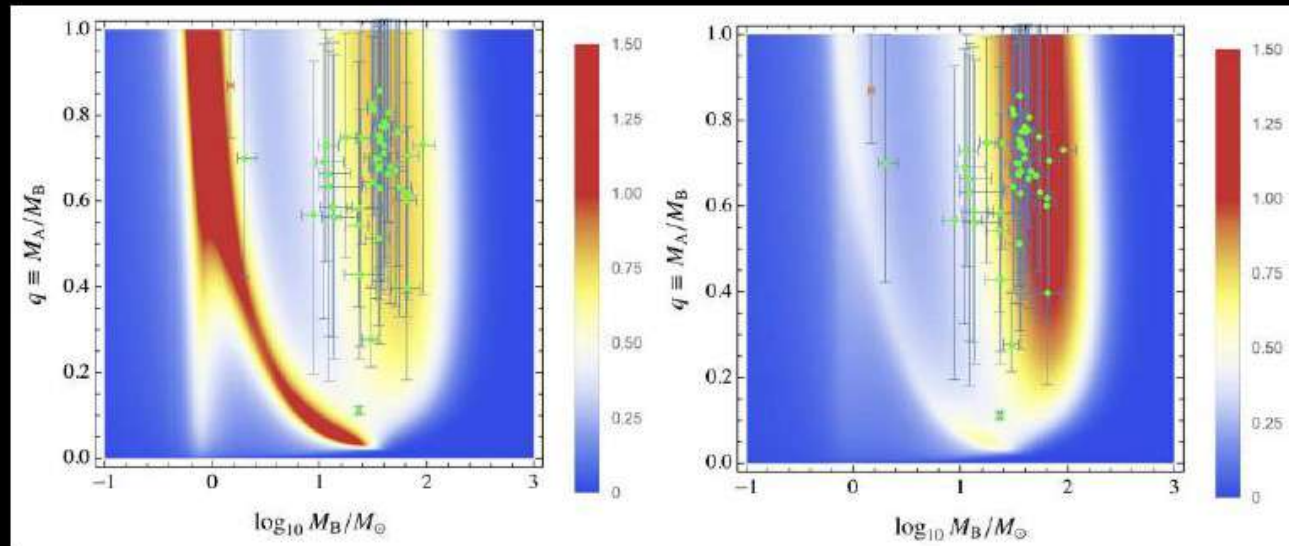
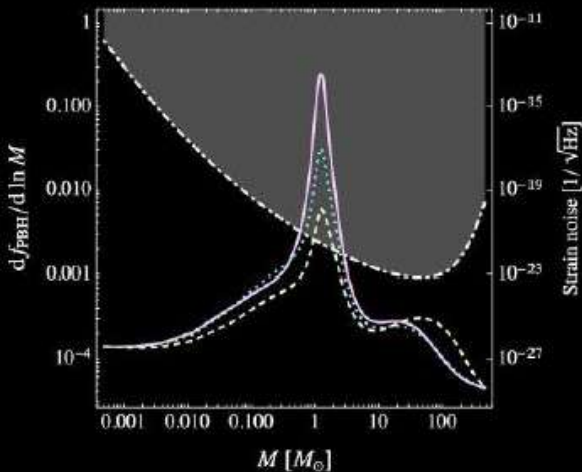
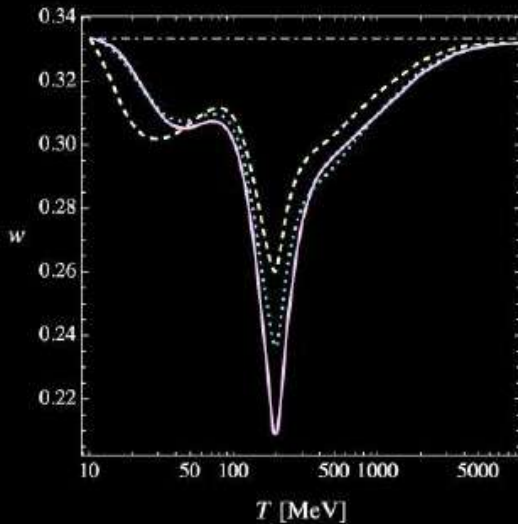
JWST results – primordial black holes !

Masses in the Stellar Graveyard



Lepton Flavor Asymmetries

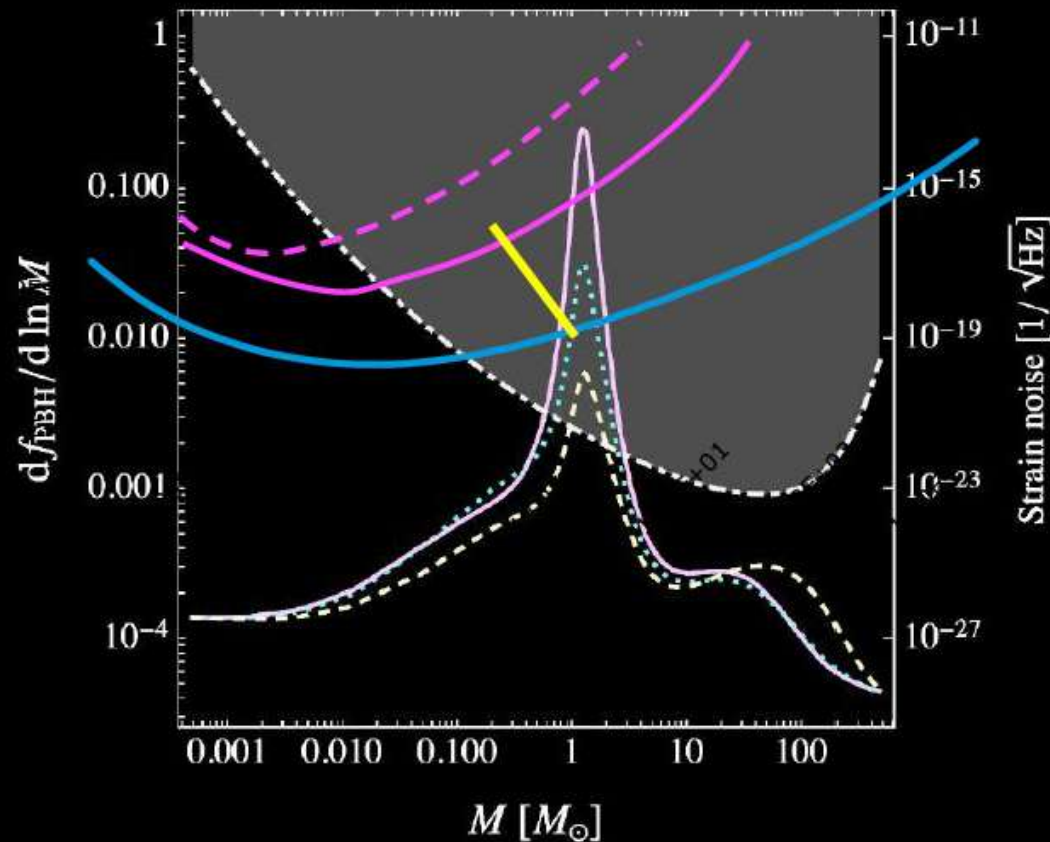
Baryon asymmetry is roughly 10^{-11} .
 Lepton flavor asymmetry could be as large as 10^{-2} .
 This has significant consequences for the QCD phase transition!



Bödecker, D., et al. 2021, Phys. Rev. D

Deutsches Zentrum für Astrophysik

New constraints on PBH mass function



Original MACHO & OGLE microlensing constraints (Wyrzykowski, L., et al. 2011, solid). Reanalysis of the MACHO constraints on PBH in the light of the new Gaia MW rotation curve (Garcia-Bellido, J. & Hawkins, M., 2024, dashed)

New 20-yr OGLE microlensing constraints (Mroz, P. et al., arXiv 2403.02386).

Search for Subsolar-Mass Binaries in the First Half of Advanced LIGO's and Advanced Virgo's Third Observing Run.

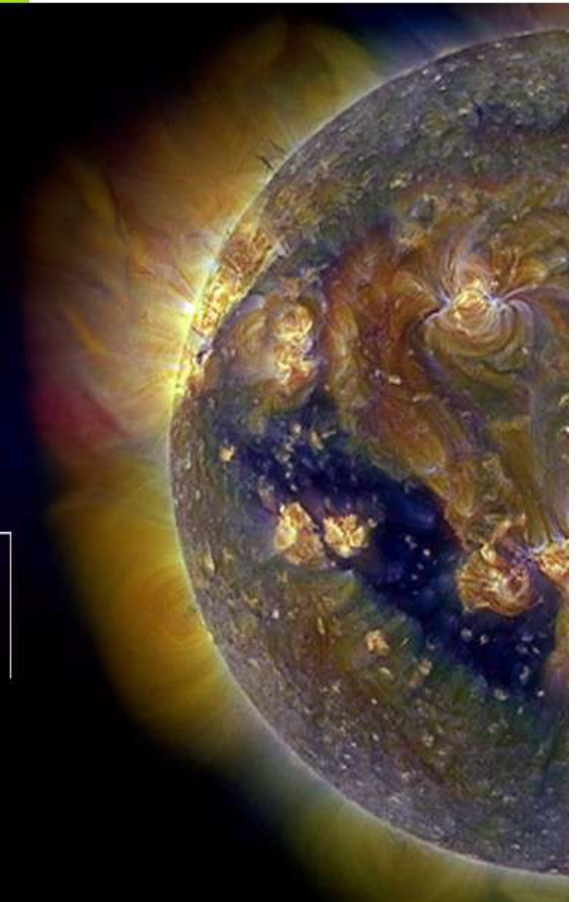
→ Just about fits!

Polish-German WE-Heraeus Seminar & Max Born Symposium:



03.12.
06.12.
2023

Many-particle systems
under extreme conditions



<https://events.hifis.net/event/1076>

