From hadron resonance gas to quark-gluon plasma via Mott-dissociation of multiquark clusters

David Blaschke (IFT UWr, HZDR/CASUS)
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Understanding complex systems with data

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„Science Creating Prospects for the Region!“

Structural and Transfer-Commission: 30. August 2022
Final decision (Approval): 29. September 2022
Why in Saxony? Lusatia is a unique region for Astrophysics, Technology and Digitization

A center for astrophysics with advanced data intensive computing and technology development.

Location for the Low Seismic Lab
Thanks to my collaborators:


Wroclaw Group ...
QCD Phase Diagram

Landscape of our investigations

RHIC, LHC Lattice
RHIC (BES I,II)
SPS, FAIR, NICA
J-PARC, HIAF...

Y_e = 0.1 - 0.6
supernovae
n_B^{\text{core}} = 1 - 3n_0
T = 10 - 50 MeV

Y_e \sim 0.1
NS-NS merger
n_B^{\text{core}} = 1 - 10n_0

saturation density
n_0 = 0.16 \text{ fm}^{-3}
(\sim \text{density of a nucleus})

Neutron stars
n_B^{\text{core}} = 1 - 10n_0

\sim 2 \text{ GeV}

M_N

QCD+EW

Figure from T. Kojo arXiv:1912.05326 [nucl-th]
QCD Phase Diagram
Landscape of our investigations

Gluons ↔ Vector mesons
Quarks ↔ Baryons
Goldstones ↔ Pseudoscalar mesons

Quark-Hadron Duality?
Mutual influence of Order parameters for χSB and CSC


Introduction

- New research triangle Wroclaw – Görlitz – Dresden/Rossendorf: UWr – CASUS & DZA - HZDR
- Landscape of investigations: QCD Phase Diagram

Towards a unified approach to quark-nuclear matter

- Generalized $\Phi$-derivable approach with clusters; cluster virial expansion
- Hadrons (mesons, baryons, multiquark states) as clusters in quark matter – Mott dissociation of clusters
- Beth-Uhlenbeck approach to thermodynamics of quark-hadron matter

Relativistic density functionals for quark matter with confinement

- Density functional for warm, dense quark matter; chiral symmetry breaking and color superconductivity
- Quark confinement as density functional $\rightarrow$ effective Nambu model with density-dependent couplings
- Phase transition construction and hybrid neutron star properties
Unified EOS for quark-hadron matter

Cluster virial expansion & Beth-Uhlenbeck EoS
Unified approach to quark-nuclear matter
Clustering aspects in the QCD phase diagram

Unified approach to quark-nuclear matter

$\Phi$-derivable approach to cluster virial expansion

$$\Omega = \sum_{l=1}^{A} \Omega_l = \sum_{l=1}^{A} \left\{ c_l \left[ \mathrm{Tr} \ln (-G_l^{-1}) + \mathrm{Tr} (\Sigma_l G_l) \right] + \sum_{i+j=l}^{\Phi[G_i, G_j, G_{i+j}]} \right\},$$

$$G_A^{-1} = G_A^{(0)-1} - \Sigma_A, \quad \Sigma_A(1 \ldots A, 1' \ldots A', z_A) = \frac{\delta \Phi}{\delta G_A(1 \ldots A, 1' \ldots A', z_A)}$$

Stationarity of the thermodynamical potential is implied

$$\frac{\delta \Omega}{\delta G_A(1 \ldots A, 1' \ldots A', z_A)} = 0.$$ 

Cluster virial expansion follows for this $\Phi$– functional

$$\Phi = \begin{array}{c} \{i \} \hline \{i+j \} \\ \{j \} \end{array}$$

$\equiv$

$$\begin{array}{c} \{i \} \hline \{j \} \\ \{i+j \} \end{array}$$

Figure: The $\Phi$ functional for $A$–particle correlations with bipartitions $A = i + j$. 
Unified approach to quark-nuclear matter

Green’s function and T-matrix, separable approx.

The $T_A$ matrix fulfills the Bethe-Salpeter equation in ladder approximation

$$T_{i+j}(1, 2, \ldots, A; 1', 2', \ldots A'; z) = V_{i+j} + V_{i+j} G_{i+j}^{(0)} T_{i+j},$$

which in the separable approximation for the interaction potential,

$$V_{i+j} = \Gamma_{i+j}(1, 2, \ldots, i; i+1, i+2, \ldots, i+j) \Gamma_{i+j}(1', 2', \ldots, i'; (i+1)', (i+2)', \ldots, (i+j)'),$$

leads to the closed expression for the $T_A$ matrix

$$T_{i+j}(1, 2, \ldots, i+j; 1', 2', \ldots (i+j)'; z) = V_{i+j} \left\{1 - \Pi_{i+j}\right\}^{-1},$$

with the generalized polarization function

$$\Pi_{i+j} = \text{Tr} \left\{ \Gamma_{i+j} G_i^{(0)} \Gamma_{i+j} G_j^{(0)} \right\}$$

The one-frequency free $i$–particle Green’s function is defined by the $(i - 1)$-fold Matsubara sum

$$G_i^{(0)}(1, 2, \ldots, i; \Omega_i) = \sum_{\omega_1 \ldots \omega_{i-1}} \frac{1}{\omega_1 - E(1)} \frac{1}{\omega_2 - E(2)} \cdots \frac{1}{\Omega_i - (\omega_1 + \ldots + \omega_{i-1}) - E(i)} \frac{(1-f_1)(1-f_2)\ldots(1-f_i)\cdots(-)^{f_1f_2\ldots f_i}}{\Omega_i - E(1) - E(2) - \ldots - E(i)}.$$
Unified approach to quark-nuclear matter

Useful relationships for many-particle functions

\[ G_{i+j}^{(0)} = G_{i+j}^{(0)}(1, 2, \ldots, i+j; \Omega_{i+j}) = \sum_{\Omega_i} G_i^{(0)}(1, 2, \ldots, i; \Omega_i) G_j^{(0)}(i+1, i+2, \ldots, i+j; \Omega_j). \]

Another set of useful relationships follows from the fact that in the ladder approximation both, the full two-cluster \((i+j \text{ particle})\) T matrix and the corresponding Greens' function

\[ G_{i+j} = G_{i+j}^{(0)} \left\{ 1 - \Pi_{i+j} \right\}^{-1} \]

have similar analytic properties determined by the \(i+j\) cluster polarization loop integral and are related by the identity

\[ T_{i+j} G_{i+j}^{(0)} = V_{i+j} G_{i+j}. \]

which is straightforwardly proven by multiplying Equation for the \(T_{i+j}\) matrix with \(G_{i+j}^{(0)}\) and using Equation (1). Since these two equivalent expressions in Equation (2) are at the same time equivalent to the two-cluster irreducible \(\Phi\) functional these functional relations follow

\[ T_{i+j} = \delta \Phi / \delta G_{i+j}^{(0)}, \]

\[ V_{i+j} = \delta \Phi / \delta G_{i+j}. \]
Unified approach to quark-nuclear matter

Generalized Beth-Uhlenbeck EOS from $\Phi$-deriv.

Consider the partial density of the $A-$particle state defined as

$$n_A(T, \mu) = -\frac{\partial \Omega_A}{\partial \mu} = -\frac{\partial}{\partial \mu} d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left[ \ln \left( -G_A^{-1} \right) + \text{Tr} \left( \Sigma_A G_A \right) \right] + \sum_{i,j} \Phi[G_i, G_j, G_{i+j}] .$$

Using spectral representation for $F(\omega)$ and Matsubara summation

$$F(i\omega_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\text{Im} F(\omega)}{\omega - i\omega_n}, \quad \sum_{\omega_n} \frac{c_A}{\omega - i\omega_n} = f_A(\omega) = \frac{1}{\exp[(\omega - \mu)/T] - (-1)^A}$$

with the relation $\partial f_A(\omega)/\partial \mu = -\partial f_A(\omega)/\partial \omega$ we get for Equation (3) now

$$n_A(T, \mu) = -d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_A(\omega) \frac{\partial}{\partial \omega} \left[ \ln \left( -G_A^{-1} \right) + \text{Im} \left( \Sigma_A G_A \right) \right] + \sum_{i,j} \frac{\partial \Phi[G_i, G_j, G_{i+j}]}{\partial \mu} ,$$

where a partial integration over $\omega$ has been performed. For two-loop diagrams of the sunset type holds a cancellation which generalize here for cluster states

$$d_A \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_A(\omega) \frac{\partial}{\partial \omega} \left( \text{Re} \Sigma_A \text{Im} G_A \right) - \sum_{i,j} \frac{\partial \Phi[G_i, G_j, G_{i+j}]}{\partial \mu} = 0 .$$

Using generalized optical theorems we can show that ($G_A = |G_A| \exp(i\delta_A)$)

$$\frac{\partial}{\partial \omega} \left[ \text{Im} \ln \left( -G_A^{-1} \right) + \text{Im} \Sigma_A \text{Re} G_A \right] = 2 \text{Im} \left[ G_A \text{Im} \Sigma_A \frac{\partial}{\partial \omega} G_A^* \text{Im} \Sigma_A \right] = -2\sin^2 \delta_A \frac{\partial \delta_A}{\partial \omega} .$$

The density in the form of a generalized Beth-Uhlenbeck EoS follows

$$n(T, \mu) = \sum_{i=1}^{A} n_i(T, \mu) = \sum_{i=1}^{A} d_i \int \frac{d^3 q}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_i(\omega) 2 \sin^2 \delta_i \frac{\partial \delta_i}{\partial \omega} .$$

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Unified approach to quark-nuclear matter

Example: deuterons in nuclear matter

The $\Phi$–derivable thermodynamical potential for the nucleon-deuteron system reads

$$\Omega = -\text{Tr} \{ \ln(-G_1) \} - \text{Tr} \{ \Sigma_1 G_1 \} + \text{Tr} \{ \ln(-G_2) \} + \text{Tr} \{ \Sigma_2 G_2 \} + \Phi[G_1, G_2] ,$$

where the full propagators obey the Dyson-Schwinger equations

$$G_1^{-1}(1, z) = z - E_1(p_1) - \Sigma_1(1, z); \quad G_2^{-1}(12, 1'2', z) = z - E_1(p_1) - E_2(p_2) - \Sigma_2(12, 1'2', z) ,$$

with selfenergies and $\Phi$ functional

$$\Sigma_1(1, 1') = \frac{\delta \Phi}{\delta G_1(1, 1')} ; \quad \Sigma_2(12, 1'2', z) = \frac{\delta \Phi}{\delta G_2(12, 1'2', z)} , \quad \Phi = \begin{array}{c}
\end{array},$$

fulfilling stationarity of the thermodynamic potential $\partial \Omega / \partial G_1 = \partial \Omega / \partial G_2 = 0$.

For the density we obtain the cluster virial expansion

$$n = \frac{1}{V} \frac{\partial \Omega}{\partial \mu} = n_{\text{qu}}(\mu, T) + 2n_{\text{corr}}(\mu, T) ,$$

with the correlation density in the generalized Beth-Uhlenbeck form

$$n_{\text{corr}} = \int \frac{dE}{2\pi} g(E) 2 \sin^2 \delta(E) \frac{d\delta(E)}{dE} .$$
Unified approach to quark-nuclear matter
Cluster virial expansion for quark-hadron matter

\[
\Omega = \sum_{i=Q,M,D,B} c_i \left[ \text{Tr} \ln \left(-G_i^{-1}\right) + \text{Tr} \left(\Sigma_i G_i\right) \right] + \Phi [G_Q, G_M, G_D, G_B],
\]

When \(\Phi\) functional for the system is given by 2-loop diagrams holds

\[
n = -\frac{\partial \Omega}{\partial \mu} = \sum_a a n_a(T, \mu)
\]

\[
= \sum_a a d_a \int \frac{d\omega}{\pi} \int \frac{d^3q}{(2\pi)^3} \left\{ f_{\phi,(a),+} - \left[f_{\phi,(a),-}\right]^* \right\} 2 \sin^2 \delta_a(\omega, q) \frac{\partial \delta_a(\omega, q)}{\partial \omega},
\]

Analogous for the entropy density \(s = -\frac{\partial \Omega}{\partial T}\).
Unified approach to quark-nuclear matter
Cluster virial expansion for quark-hadron matter

The cluster decomposition of the thermodynamic potential is given as

\[ \Omega_{\text{total}}(T, \mu, \phi, \bar{\phi}) = \Omega_{\text{PNJL}}(T, \mu, \phi, \bar{\phi}) + \Omega_{\text{pert}}(T, \mu, \phi, \bar{\phi}) + \Omega_{\text{MHRG}}(T, \mu, \phi, \bar{\phi}), \]

where the first two terms describe the quark and gluon degrees of freedom via the mean-field thermodynamic potential for quark matter in a gluon background field \( \mathcal{U} \)

\[ \Omega_{\text{PNJL}}(T, \mu, \phi, \bar{\phi}) = \Omega_Q(T, \mu, \phi, \bar{\phi}) + \mathcal{U}(T, \phi, \bar{\phi}) \]

with a perturbative correction \( \Omega_{\text{pert}}(T, \mu, \phi, \bar{\phi}) \).

The Mott-Hadron-Resonance-Gas (MHRG) part for the multi-quark clusters is

\[ \Omega_{\text{MHRG}}(T, \mu, \phi, \bar{\phi}) = \sum_{i=M,B,...} \Omega_i(T, \mu, \phi, \bar{\phi}), \]

where the multi-quark states are described by the GBU formula:

\[ n = - \frac{\partial \Omega}{\partial \mu} = \sum_a a n_a(T, \mu) \]

\[ = \sum_a d_a \int \frac{d\omega}{\pi} \int \frac{d^3q}{(2\pi)^3} \left\{ f_\phi^{(a),+} - \left[ f_\phi^{(a),-}\right]^* \right\} 2 \sin^2 \delta_a(\omega, q) \frac{\partial \delta_a(\omega, q)}{\partial \omega}, \]

where \( d_i \) is the degeneracy factor, \( a \) is the number of valence quarks in the cluster an \( f_\phi^{(a),+}, \left[ f_\phi^{(a),-}\right]^* \) are the Polyakov-loop modified distribution functions.

Analogous for the entropy density \( s = - \partial \Omega / \partial T \).
Unified approach to quark-nuclear matter

Polyakov-loop modified distribution functions

For multiquark clusters with net number $a$ of valence quarks holds

\[ f^{(a),\pm}_\phi \quad (a \text{ even}) \quad \frac{(\phi - 2\phi y_a^\pm) y_a^\pm + y_a^{\pm 3}}{1 - 3(\phi - \phi y_a^\pm) y_a^\pm - y_a^{\pm 3}}, \]

\[ f^{(a),\pm}_\phi \quad (a \text{ odd}) \quad \frac{(\phi + 2\phi y_a^\pm) y_a^\pm + y_a^{\pm 3}}{1 + 3(\phi + \phi y_a^\pm) y_a^\pm + y_a^{\pm 3}}, \]

where $y_a^\pm = e^{-(E_p \mp a\mu)/T}$ and $E_p = \sqrt{\vec{p}^2 + M^2}$.

It is instructive to consider the two limits $\phi = \bar{\phi} = 1$ (deconfinement)

\[ f^{(a=0,2,4,\ldots),\pm}_\phi = \frac{y_a^\pm}{1 - y_a^\pm}, \quad f^{(a=1,3,5,\ldots),\pm}_\phi = \frac{y_a^\pm}{1 + y_a^\pm}, \]

and $\phi = \bar{\phi} = 0$ (confinement),

\[ f^{(a=0,2,4,\ldots),\pm}_\phi = \frac{y_a^{\pm 3}}{1 - y_a^{\pm 3}}, \quad f^{(a=1,3,5,\ldots),\pm}_\phi = \frac{y_a^{\pm 3}}{1 + y_a^{\pm 3}}. \]
Unified approach to quark-hadron matter
Inputs: mass spectrum & phase shifts (models)

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14
Unified approach to quark-hadron matter

Inputs: mass spectrum (Particle Data Tables)

<table>
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<tr>
<th>PDG mesons</th>
<th>$d_i$</th>
<th>$M_{PDG}$ [MeV]</th>
<th>$M_i$ [MeV]</th>
<th>$M_{th,i}^&lt;$ [MeV]</th>
<th>$M_{th,i}^&gt;$ [MeV]</th>
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**Mesons**

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**Baryons**

... and colored clusters (model)!

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14
Unified approach to quark-hadron matter
Inputs for the phase shifts (models)

Step-up (SU) model → Hadron Resonance Gas
Step-up-step-down model → Mott Hadron Resonance Gas (MHRG)

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14
Unified approach to quark-hadron matter
Results for Mott-Hadron Resonance Gas (MHRG)

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14
Unified approach to quark-hadron matter

Entropy for MHRG: role of the $\sin^2$-term

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14
Unified approach to quark-hadron matter

Results for the entropy density

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14
Unified approach to quark-hadron matter

Results for the entropy density & composition

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14
Unified approach to quark-hadron matter

Results for pressure=thermodynamic potential

\[ \mu_B/T = 0 \]

Borsányi et al., PRL (2021)
Borsányi et al., PLB (2014)

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14
Unified approach to quark-hadron matter
Chiral condensate & baryon susceptibilities

D.B., O. Ivanytskyi and G. Röpke, in preparation (2024)
**JWST results – primordial black holes!**

Primordial BHs

First stars

Talk at University of Wroclaw by Günther Hasinger, Founding director of the German Centre for Astrophysics in Görlitz:

**Key role plays the QCD hadronization transition!**

Different peaks correspond to different particles created at the early universe phase transitions and the corresponding reduction in the sound velocity.

BH mass corresponds to the horizon size at each time.

Only requirement is enough fluctuation power in a volume fraction of $10^{-9}$ of the early Universe.

*Carr, Clesse, García-Bellido 2019*
JWST results – primordial black holes!

Masses in the Stellar Graveyard

OGLE/GAIA/HST events

... the plot thickens!

Courtesy: Günther Hasinger (Karpacz, 2024)
JWST results – primordial black holes!

Lepton Flavor Asymmetries

Baryon asymmetry is roughly $10^{-11}$. Lepton flavor asymmetry could be as large as $10^{-2}$. This has significant consequences for the QCD phase transition!


Deutsches Zentrum für Astrophysik

Courtesy: Günther Hasinger (Karpacz 2024)
JWST results – primordial black holes!

New constraints on PBH mass function


New 20-yr OGLE microlensing constraints (Mroz, P. et al., arXiv 2403.02386).


→ Just about fits!

Courtesy: Günther Hasinger (Karpacz 2024)
Many-particle systems under extreme conditions

https://events.hifis.net/event/1076