Tensor and vector exchange contributions to $K\bar{K} \to K\bar{K}, D\bar{D} \to D\bar{D}$ and $\pi^+\pi^- \to \pi^+\pi^-$ reactions

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(With Jing Song, Pedro Brandão and Eulogio Oset, Eur.Phys.J.A **60**, 76 (2024))

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Motivation

- Meson-meson interactions $\rightarrow$ continues as a subject of intense debate
- Several mesons should not be interpreted as bound states obtained from effective quark models, but as dynamically generated
- A large number of meson states are broad resonances

  Examples:

- $\sigma/f_0(500)$ and $\kappa/K_0^*(700)$ $\rightarrow$ from $\pi\pi$ and $K\pi$ interactions
- $f_0(980)$ and $a_0(980)$ $\rightarrow$ from unitary coupled-channel ($\pi\pi, K\bar{K}, \eta\eta$ for $I=0$ and $\pi\eta, K\bar{K}$ for $I=1$)
- Tensor sector: $f_2(1270)$ $\rightarrow$ from VV interaction ($\rho\rho$: dominant)

  $\Downarrow$

  Local hidden gauge formalism: PPV, VVV and $VVVV$ structures

  (Molina, Nicmorus and Oset, PRD 78, 114018 (2008); Geng and Oset, PRD 79, 074009 (2009))
- Controversies on the nature of $f_2(1270)$: see EPJA **60**, 76 (2024)

- Relevant point: PPV vertex from the local hidden gauge formalism

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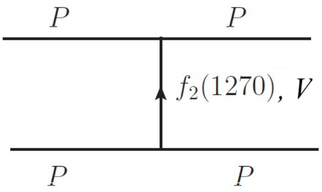
Tree-level pseudoscalar meson–pseudoscalar meson ($PP \to PP$) interactions with an intermediate vector-exchange



$$P = (K, \pi, D);\ V = \rho$$

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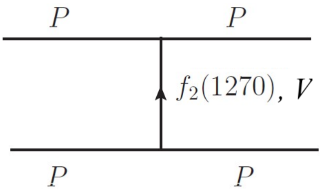

$$P = (K, \pi, D);\ V = \rho$$

Interesting questions:

What about the contribution coming from tensor-exchange mechanism for the scattering matrix?
Is it relevant with respect to the vector one?

- Ecker and Zauner, Eur. Phys. J. C **52**, 315-323 (2007): Tensor exchange contribution evaluated using ChPT_2
- $f_2(1270)$ as a $q\bar{q}$ state
- Lowest amplitudes:

$$T^{(V)} = \frac{-7}{2f_\pi^2} m_\pi^2; \quad T^{(T)} = -\frac{40}{f_\pi^4} L_3^{(T)} m_\pi^4;$$

- Ratio $R_{\pi\pi} = \frac{T^{(T)}}{T^{(V)}}$ to $\pi\pi$ scattering ($L_3^{(T)} = 0.16 \times 10^{-3}$):

$$R_{\pi\pi} = \frac{80}{7f_\pi^2} L_3^{(T)} m_\pi^2 = 4 \times 10^{-3}$$

- Also: $R_{K\bar{K}} = 7 \times 10^{-2}$

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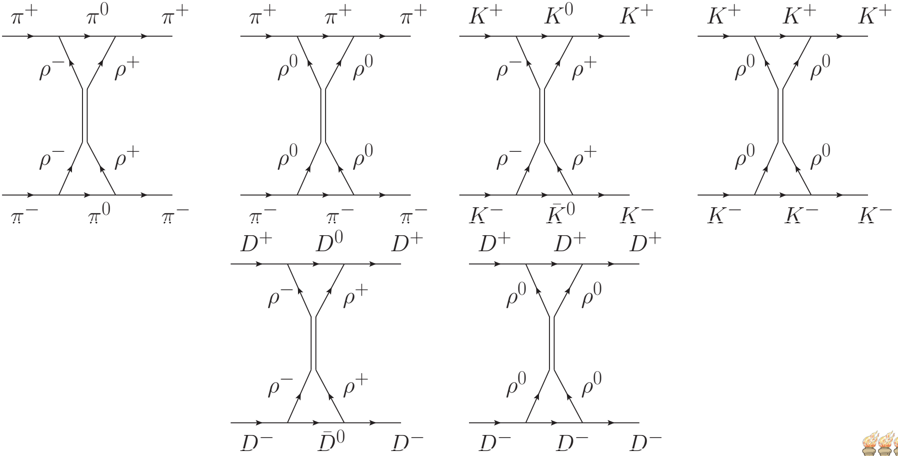
Our aim:

- Evaluation of the tensor and vector (ρ) contribution mechanisms to the elastic reactions $\pi^+\pi^-$, K^+K^- and D^+D^-, taking $f_2(1270)$ dynamically generated
- Comparison: light-heavy ($D\bar{D}$ scattering) and the light sectors

Formalism: elastic reactions $\pi^+\pi^-$, K^+K^- and D^+D^-

Tensor contribution mechanism:



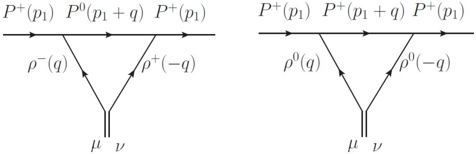
$f_2(1270)$: dynamically generated from the interaction of two intermediate ρ mesons, which generates two-loop amplitudes

- $f_2(1270)$: exchange contribution $\rightarrow$ $S-$wave $I=0, J=2$ channel of the $\rho\rho$ interaction

$$D(k) = \frac{g_{f_2}^2}{k^2 - m_{f_2}^2} P^{(2)},$$

$g_{f_2} = 10551$ MeV (Molina et al. PRD 78, 114018 (2008)); 10889 MeV (Geng and Oset, PRD 79, 074009 (2009))

- Loop contributions: Upper and lower vertices can be factorized



- PPV vertex [R. Molina, D. Nicmorus, and E. Oset, Phys. Rev. D **78**, 114018 (2008)]:

$$\mathcal{L}_{PPV} = -ig\langle[P, \partial_\mu P]V^\mu\rangle,$$

P, V: $q\bar{q}$ matrices in SU$_F(4)$ in terms of pseudoscalar or vector mesons;
$g = m_V/(2f_\pi)$ ($m_V = 800$ MeV, $f_\pi = 93$ MeV);

Loop:

$$-iV_{\mu\nu} = -C\frac{1}{\sqrt{6}} g_{f_2} g^2 \int \frac{d^4q}{(2\pi)^4} \left(\frac{1}{q^2 - m_\rho^2 + i\epsilon}\right)^2 \frac{1}{(p_1+q)^2 - m_P^2 + i\epsilon}$$
$$\times \left[-(2p_1+q)_\mu + \frac{1}{m_\rho^2}(2p_1+q)\cdot q q_\mu\right]\left[-(2p_1+q)_\nu + \frac{1}{m_\rho^2}(2p_1+q)\cdot q q_\nu\right],$$

$C = 3/2$ to $K\bar{K}$ and $D\bar{D}$; $C = 4$ to $\pi\pi$

- Cauchy integration over q^0 (poles: $q^0 = \pm\omega_\rho = \pm\sqrt{\vec{q}^2 + m_\rho^2}$)
- At the threshold of the pseudoscalar mesons:
  - $V_{0j}, V_{i0} \propto p_i = 0$
  - $V_{ij} = a\delta_{ij} + bp_ip_j = 0$
    (b does not contribute as $p_i = 0$; a vanishes when combined with $P^{(2)}$)
- Thus, only V_{00} contributes:

$$-iV_{00} = C'_j \frac{1}{\sqrt{6}} g_{f_2} g^2 i \frac{\partial}{\partial m_\rho^2} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_\rho \omega_P} \frac{\omega_\rho + \omega_P}{E_1^2 - (\omega_\rho + \omega_P)^2 + i\epsilon} \times \left(\frac{2E_1 \vec{q}^{\,2}}{m_\rho^2}\right)^2 \Theta(q_{\max} - |\vec{q}|) \left(\frac{\Lambda^2}{\Lambda^2 + \vec{q}^{\,2}}\right)^2$$

($C'_j = 3/2$ for $j = K, D$; $C'_j = 4$ for $j = \pi$)

- Combining all ingredients

$$-iT^{(T)}(k) = \frac{i}{k^2 - m_{f_2}^2} \left\{ \frac{1}{2} [(-i)V_{00}(-i)V_{00} + (-i)V_{00}(-i)V_{00}] - \frac{1}{3}(-i)V_{00}(-i)V_{00} \right\}$$

- At threshold:

$$T^{(T)}(0) = -\frac{2}{3m_{f_2}^2} [V_{00}]^2 . \quad (1)$$

- $T^{(T)}(0) \propto m_i^4, g^4, f^{-4}$

- Vector-exchange mechanism: PPV vertex gives the amplitude

$$-iT^{(V)}(k) \quad = \quad \frac{(-ig)}{\sqrt{2}}(2p_1+k)^{\mu}\frac{i}{k^2-m_\rho^2}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_\rho^2}\right)\frac{(ig)}{\sqrt{2}}(2p_1-k)^{\nu}$$

- At threshold:

$$T^{(V)}(0) = \begin{cases} -\frac{2g^2}{m_\rho^2}E_1^2 \ (\text{for} \quad K, \ D); \\ -\frac{8g^2}{m_\rho^2}E_1^2 \ (\text{for} \quad \pi). \end{cases}$$

- Next: evaluation of the ratio between the contributions coming from the tensor-exchange mechanism and the one with exchange of a vector meson:

$$R = \left|\frac{T^{(T)}(0)}{T^{(V)}(0)}\right|$$

Results for the evaluation of R

g_{f_2}[MeV]	$q_{\max}$[MeV]	$\pi^+\pi^- \to \pi^+\pi^-$	$K^+K^- \to K^+K^-$	$D^+D^- \to D^+D^-$
10551 (Molina (2008))	600	8.38×10^{-5}	2.960×10^{-4}	4.79×10^{-4}
	700	3.22×10^{-4}	1.29×10^{-3}	2.47×10^{-3}
	800	9.56×10^{-4}	4.22×10^{-3}	9.35×10^{-3}
	850	1.53×10^{-3}	7.00×10^{-3}	1.66×10^{-2}
	900	2.34×10^{-3}	1.11×10^{-2}	2.81×10^{-2}
10889 (Geng (2009))	600	8.93×10^{-5}	3.15×10^{-4}	5.10×10^{-4}
	700	3.43×10^{-4}	1.38×10^{-3}	2.63×10^{-3}
	800	1.02×10^{-3}	4.49×10^{-3}	9.96×10^{-3}
	850	1.63×10^{-3}	7.45×10^{-3}	1.77×10^{-2}
	900	2.49×10^{-3}	1.18×10^{-2}	2.99×10^{-2}

- g_{f_2} from Geng-Oset (2009) is about 3% bigger than the one from Molina et al. (2008): R greater than the former one by $6-7\%$
- R: increases as $q_{\max}$ increases
- Most importantly: $R \sim 10^{-2} - 10^{-5}$
  - $m_{f_2}^{-2}/m_\rho^{-2} \sim 0.4$
  - Small magnitude: mainly due to the vertex contribution V_{00}

Comparison to Ecker et al. (2007): we take $g_{f_2} = 10551$ MeV and $q_{\max} = 850$ MeV

- Our case: $R_{\pi\pi} \approx \frac{1}{2.6} R_{\pi\pi}^{(\text{Ecker})}$
  - (Our picture produces, with the small fine tuning, a good reproduction of $f_2(1270) \to \pi\pi$)
- Our case: $\frac{R_{K\bar{K}}}{R_{\pi\pi}} \approx 4.6$, while $\frac{R_{K\bar{K}}^{(\text{Ecker})}}{R_{\pi\pi}^{(\text{Ecker})}} \sim 17$
  - SU(3) symmetry: $\frac{R_{K\bar{K}}}{R_{\pi\pi}} \approx m_K^2/m_\pi^2 \approx 13$ (not far from 17)
  - Present case: loops employed to dynamically generate the $f_2(1270)$

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$$[(\frac{C_K}{C_\pi})^2 \frac{1}{4}]^{-1} = [(\frac{8}{3})^2 \frac{1}{4}]^{-1} = 0.56 \to 13 \times 0.56 \sim 7.3$$

(not far from 4.6; extra reduction: from V_{00} [w_i in the denominator])

- Tensor exchange with the $f_2(1270)$ dynamically generated: visible effect, reducing what one finds with the SU(3) symmetry
- $R_{D\bar{D}} \sim 1 - 2\%$ but still small
  (large mass of the D meson in V_{00} stabilizes R)

Conclusions

- Analysis of the tensor and vector-contribution mechanisms to the elastic reactions involving $\pi^+\pi^-$, K^+K^- and D^+D^-
- Tensor-exchange contribution: $f_2(1270)$ is dynamically generated from the $\rho\rho$ interaction with the use of a pole approximation (two-loop calculation with triangle loops factorized)
- Tensor-exchange mechanisms: small when compared with those from the vector-exchange processes
- $\pi^+\pi^-$ scattering: results are more ressemblant with SU(3) picture assumed in Ecker et al. (2007)
- Our picture: tensor contribution for $K\bar{K}$ is stabilized: only about 4.6 times bigger than the one of $\pi^+\pi^-$ (SU(3): 17 times larger)
- Our picture: ratio of tensor to vector exchange for the case of D mesons of the order of $1-2\%$

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