INTRODUCTION

- Perturbation theory cannot always be applied to compute observables in QCD.
- Coupling constant becomes large in the low-energy regime (quark confinement).
- We employ the variational method, a rigorous, non-perturbative approach which provides variational upper bounds on the ground state energy.
- We study the viability of employing a neural network as our variational ansatz.
- We study scalar field theories, which serve as a toy model for Yang-Mills theories.

2 METHODS

- Monte Carlo integration with importance sampling is used to compute the energy:
  \[ E = \frac{1}{N} \sum_{\phi} \left( \frac{1}{2} \sum_{x} \phi(x)^2 - U(\phi) \right) \]

\[ \phi(x) \rightarrow \Phi_{\text{MC}}(\phi(x)) \]

\[ \text{Gradient descent optimizes the variational parameters } \alpha. \]

3 QUANTUM MECHANICS

- Harmonic oscillator and anharmonic oscillators (with different values of \( \lambda \)) are studied.
- Variational upper bounds saturated for the harmonic oscillator.
- Variational result for the anharmonic oscillator are compared with the ones of perturbation theory.

4 \( \phi^4 \) FIELD THEORY

- We are now studying the variational wavefunctional.
- The lattice regularized Hamiltonian is:
  \[ H = \sum_{\langle x,y \rangle} \left( \frac{\phi(x)^2 + \phi(y)^2}{2m^2} - \frac{1}{2} m^2 \phi(x)^2 \phi(y)^2 - \frac{1}{4} \lambda \phi(x)^4 \right) \]

\[ \text{Error in energy } H_x = \sum_{\langle x,y \rangle} \left( \frac{\phi(x)^2 + \phi(y)^2}{2m^2} - \frac{1}{2} m^2 \phi(x)^2 \phi(y)^2 - \frac{1}{4} \lambda \phi(x)^4 \right) \]

\[ \text{Error in energy } H_y = \sum_{\langle x,y \rangle} \left( \frac{\phi(x)^2 + \phi(y)^2}{2m^2} - \frac{1}{2} m^2 \phi(x)^2 \phi(y)^2 - \frac{1}{4} \lambda \phi(x)^4 \right) \]

\[ \text{Error in energy } H_z = \sum_{\langle x,y \rangle} \left( \frac{\phi(x)^2 + \phi(y)^2}{2m^2} - \frac{1}{2} m^2 \phi(x)^2 \phi(y)^2 - \frac{1}{4} \lambda \phi(x)^4 \right) \]

\[ \text{Error in energy } H_{xy} = \sum_{\langle x,y \rangle} \left( \frac{\phi(x)^2 + \phi(y)^2}{2m^2} - \frac{1}{2} m^2 \phi(x)^2 \phi(y)^2 - \frac{1}{4} \lambda \phi(x)^4 \right) \]

\[ \text{Error in energy } H_{xz} = \sum_{\langle x,y \rangle} \left( \frac{\phi(x)^2 + \phi(y)^2}{2m^2} - \frac{1}{2} m^2 \phi(x)^2 \phi(y)^2 - \frac{1}{4} \lambda \phi(x)^4 \right) \]

\[ \text{Error in energy } H_{yz} = \sum_{\langle x,y \rangle} \left( \frac{\phi(x)^2 + \phi(y)^2}{2m^2} - \frac{1}{2} m^2 \phi(x)^2 \phi(y)^2 - \frac{1}{4} \lambda \phi(x)^4 \right) \]

\[ \text{Error in energy } H_{xyz} = \sum_{\langle x,y \rangle} \left( \frac{\phi(x)^2 + \phi(y)^2}{2m^2} - \frac{1}{2} m^2 \phi(x)^2 \phi(y)^2 - \frac{1}{4} \lambda \phi(x)^4 \right) \]

5 CONCLUSIONS

- The algorithm seems to achieve convergence and reproduce well the harmonic and anharmonic oscillators in one dimension when a neural network is used as ansatz. It also gives satisfactory upper bounds to the ground energy of these systems. That is also the case for the free field and the interacting field in three dimensions.
- For the anharmonic oscillator, different values of \( \lambda \) are explored. Its variational results are compared with those obtained in perturbation theory. An agreement between both is observed.
- In all cases, the ansatz succeeds in reproducing the true ground state, with the optimal settings. It takes around 10-15 iterations to achieve convergence.
- For the field theories, round states are computed analytically in a 4\(^{th}\) lattice. The values are:
  - at\( \theta = 0 \) for a free field,
  - at\( \theta = 0.15 \) = 52.98 for a \( \phi^4 \) theory.

6 FUTURE RESEARCH

Focus on the optimization of the algorithm. As it takes \( \approx 80 \text{ min/iteration for the field theory, in comparison with the 43 s/iteration of HO.} \]

Different values of \( \lambda \) for the interacting field theory could be explored and compared with perturbation theory.

In both, quantum mechanics and quantum field systems, operator projection could be presented, in order to compute excited states.

Going beyond scalar field theory, like QCD. This would require improving the code and changing the \( \phi^4 \) Lagrangian for the QCD one.