

Exotics: Structure and Production in Heavy Ion Collisions

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- Introduction
- Exotics: questions and answers in the quark model
- The structure of $X(3872)$ and $T_{cc}(3875)$ and production in Heavy Ion Collision
- Final Thoughts

Acknowledgments:

Yonsei group : [W. Park](#), [A. Park](#), [J. Hong](#), [S. Noh](#), [H. Yoon](#), [D. Park](#),

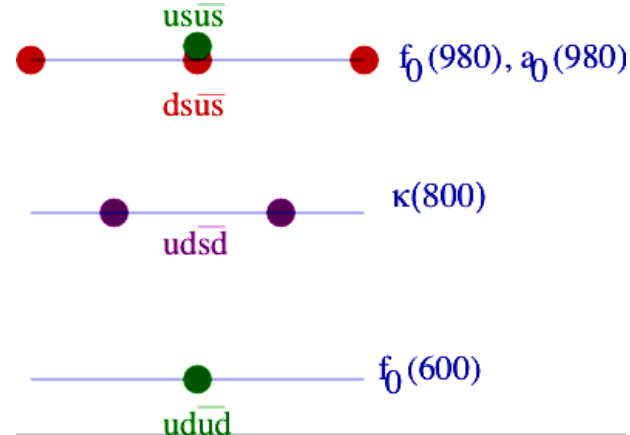
External collaborators: [Che-Ming Ko](#), [Sungtae Cho](#), [Sanghoon Lim](#), [Yongsun Kim](#) + other ExHIC collaboration

Recent findings of Exotics: Solution to an old topic

☞ Tetraquark:

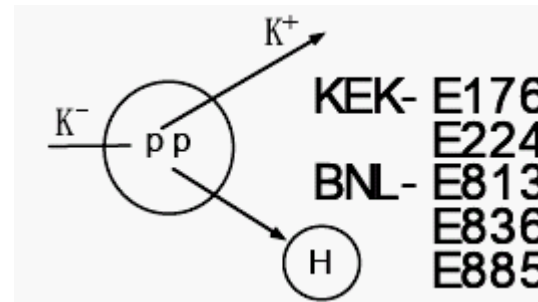
- scalar tetraquark (Jaffe 76)
- Still controversial

But ALICE(Junlee Kim) analysis suggests f_0 is most likely a $(\bar{q}q)$ without $(\bar{s}s)$



☞ Dibaryon

- H (ududss) dibaryon (Jaffe 77):
- experimentally not found

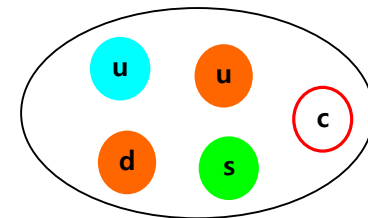


☞ Pentaquark

- $P_{c\bar{s}}$ (Gignoux, Silvestre-Brac, Richard 87)
- $P_{c\bar{s}}$ (udusc \bar{c}) (Lipkin 87)

→ Fermilab E791 : not found

$$P_{c\bar{s}}^0 \rightarrow K^{*0} K^- p$$



- Θ^+ (Diakonov, Petrov, Polyakov 97)

→ LEPS 2003 but not confirmed

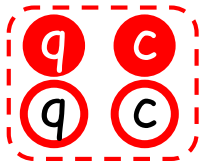
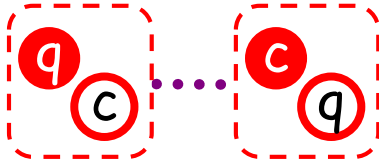
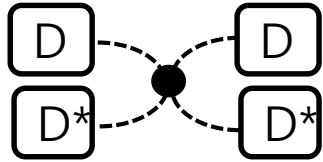
Few examples of recent findings that could be probed in HIC

Bound
Near Threshold

Above
Threshold

Tetraquark	Mass	Quark content	2-body Threshold	Observed mode	Exp
$\chi_{c1}(3872)$ $X(3872)$	3871.65	$[c\bar{c}q\bar{q}]$	$\bar{D}^0 D^{*0}$ (3871.69) $D^- D^{*+}$ (3879.92)	$J/\psi\pi^-\pi^+$	Belle ..
$T_{cc}(3875)$	3875	$[c\bar{u}c\bar{d}]$	$D^0 D^{*+}$ (3875.26) $D^+ D^{*0}$ (3876.51)	$D^0 D^0 \pi^+$	LHCb
$T_{\psi s1}^\theta(4000)$ $Z_{cs}(3872)$	4003+i(131)	$[c\bar{c}u\bar{s}]$	$\bar{D}^0 D_s^{*+}$ (3977) $J/\psi K^+$ (3590.58)	$J/\psi K^+$	LHCb (BES?)
$X(5568)$	5568+i(21.9)	$[b\bar{d}u\bar{s}]$	$B^0 K^+$ (5773) $B_s^0 \pi^\pm$ (5506.49)	$B_s^0 \pi^\pm$	D0
$T_{c\bar{s}0}^a(2900)$	2908+i(136)	$[c\bar{s}u\bar{d}]$	2251.77	$D_s^+ \pi^+$	LHCb
$X(6600)$ $X(6900)$		$[c\bar{c}c\bar{c}]$	6193.8 MeV	$J/\psi J/\psi$	CMS LHCb

Types of Exotic particles

	Compact multiquark	Molecule	Resonance
Picture			
Size Threshold width	$\langle r \rangle < 0.6 \text{ fm}$ Near threshold or other small	$\langle r \rangle > 2 \text{ fm}$ Near threshold small	$\langle r \rangle \sim 1 \text{ fm}$ Above threshold or other large
Typical model used	Quark Model	Meson exchange models	Unitary approach Quark model
	Effective field theory: constants QCD sum rules: uncertainty		

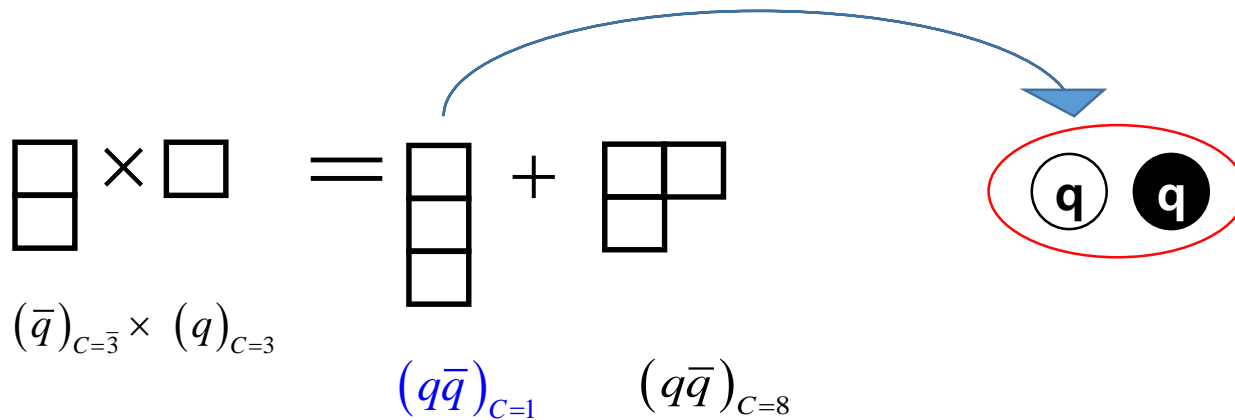
- In some cases, two pictures seem possible. Compact and Molecular
- Yet, there are common features to exotics not seen in usual hadrons

Why are exotics interesting?

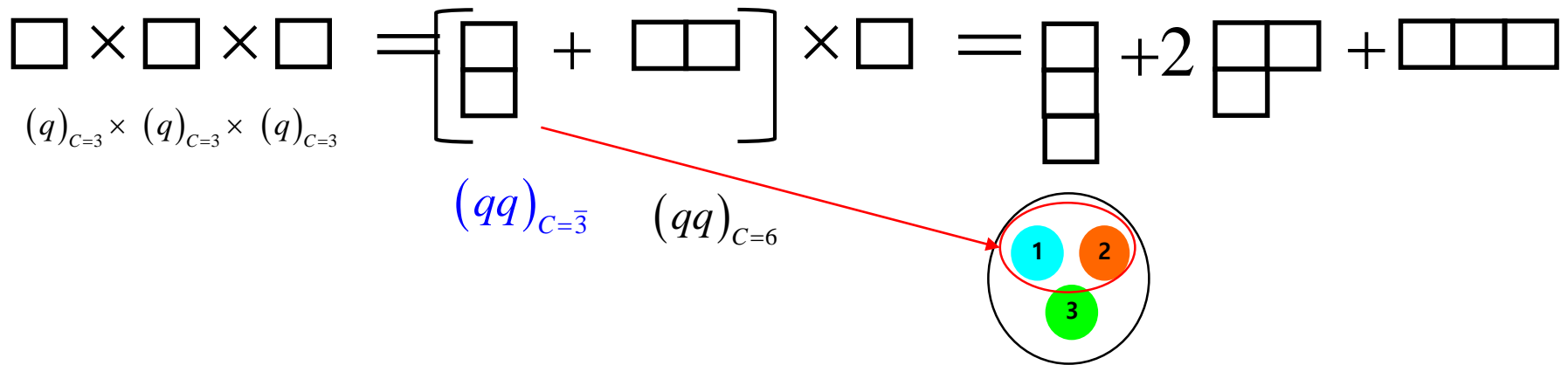
- A New color configuration
- Quark-Gluon Plasma

So far only $(q\bar{q})_{C=1}$, $(qq)_{C=3}$ are seen

☞ Meson: $\bar{3} \times 3 = 1 + 8$



☞ Baryon: $3 \times 3 \times 3 = (\bar{3} + 6) \times 3 = 1 + 2 \cdot 8 + 10$



A new color configuration of SU(3)

➡ Usual ground state hadron $(q\bar{q})_{C=1}$ $(qq)_{C=\bar{3}}$ or $(\bar{q}\bar{q})_{C=3}$

➡ But Exotics contain additional color configurations with higher degeneracy
For example: Tetraquark state

$$3 \times 3 \times \bar{3} \times \bar{3} = (\bar{3} + 6) \times (3 + \bar{6}) = 3 \times \bar{3} + 6 \times \bar{6} + \dots$$

$$(qq)_{C=\bar{3}} \otimes (\bar{q}\bar{q})_{C=3} \quad \text{and} \quad (qq)_{C=6} \otimes (\bar{q}\bar{q})_{C=\bar{6}}$$

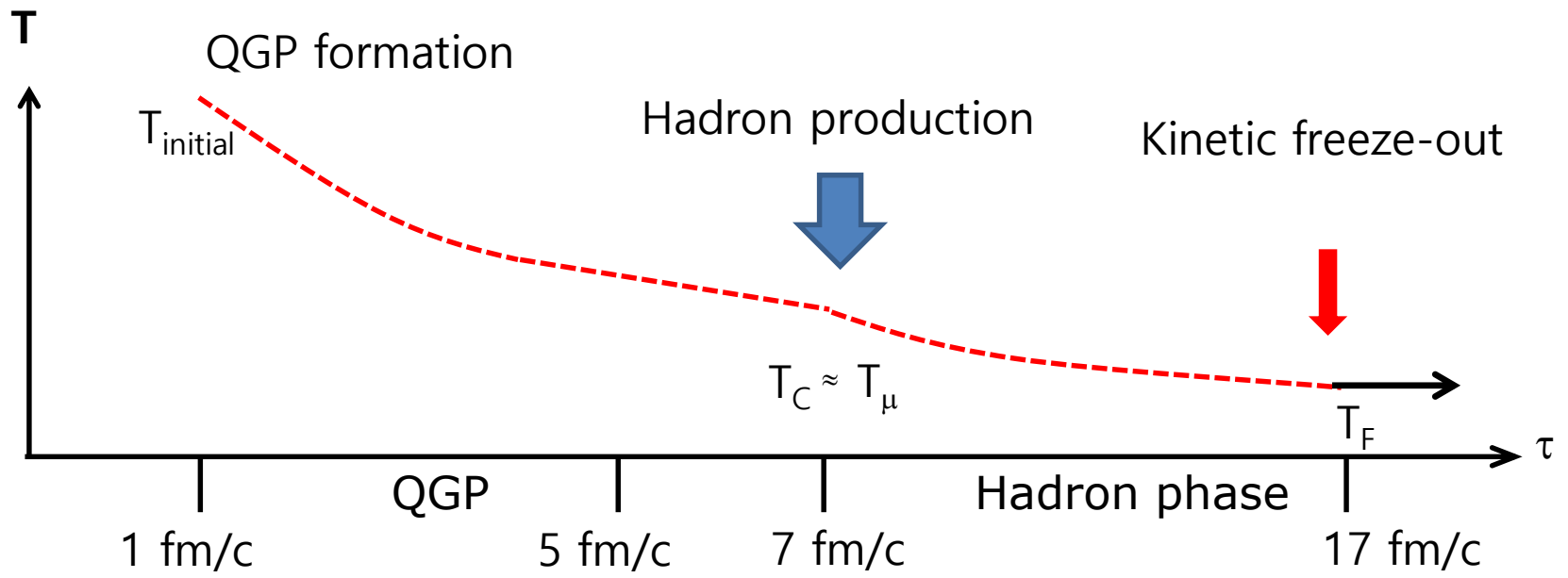
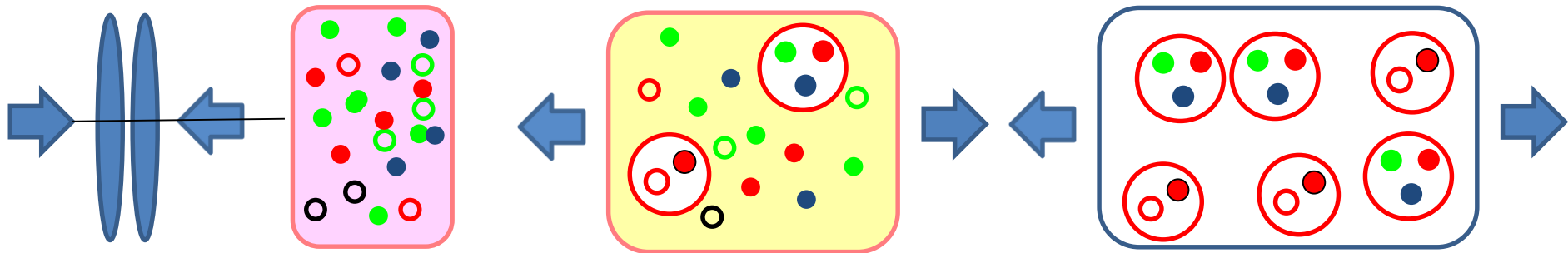
degeneracy: 3×3 and 6×6

$$3 \times \bar{3} \times 3 \times \bar{3} = (1 + 8) \times (1 + 8) = 1 \times 1 + 8 \times 8 + \dots$$

$$(q\bar{q})_{C=1} \otimes (q\bar{q})_{C=1} \quad \text{and} \quad (q\bar{q})_{C=8} \otimes (q\bar{q})_{C=8}$$

degeneracy: 1×1 and 8×8

QGP contains all configurations and correlations \rightarrow Exotics



What does the quark model tell us about compact configurations?

- Two-body quark force: color-color and color-spin interaction
- Three-body force: from meson to baryon

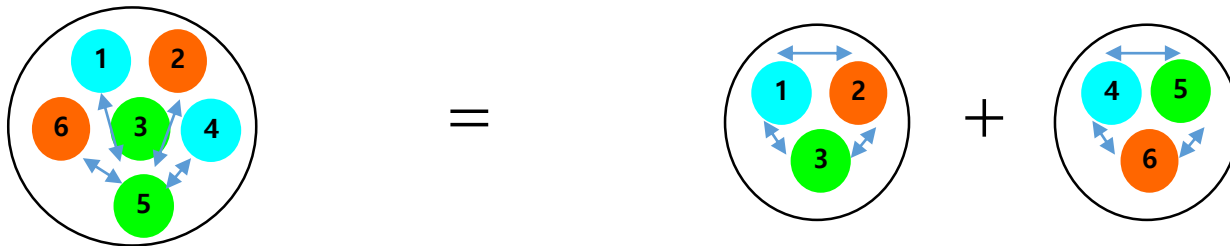
☞ When brought together need to overcome **Additional Kinetic energy >100 MeV**

$$H = \sum_{i=1}^n \left(m_i + \frac{p_i^2}{2m_i} \right) - \sum_{i<j}^n \left(\lambda_i^c \lambda_j^c \right) V_{ij}^C (r_{ij}) - \sum_{i<j}^n \frac{(\lambda_i^c \lambda_j^c)(\sigma_i \sigma_j)}{m_i m_j} V_{ij}^{SS} (r_{ij})$$

☞ **Color-Color** interaction is not important for short range N-N interaction

$$\sum_{i<j}^N (\lambda_i^c \lambda_j^c) = \frac{1}{2} \left[(\lambda_1^c + \dots + \lambda_N^c)^2 - \lambda_1^2 - \dots - \lambda_N^2 \right] \quad N = N_{B_1} + N_{B_2}$$

$$= 0 - \frac{8}{3} (N_{B_1} + N_{B_2}) = \sum_{i<j}^{N_{B_1}} (\lambda_i^c \lambda_j^c) + \sum_{i<j}^{N_{B_2}} (\lambda_i^c \lambda_j^c)$$



$$H = \sum_{i=1}^n \left(m_i + \frac{p_i^2}{2m_i} \right) - \sum_{i<j}^n (\lambda_i^c \lambda_j^c) V_{ij}^C (r_{ij}) - \sum_{i<j}^n \frac{(\lambda_i^c \lambda_j^c)(\sigma_i \sigma_j)}{m_i m_j} V_{ij}^{SS} (r_{ij})$$

Color-spin interaction for 2 body:

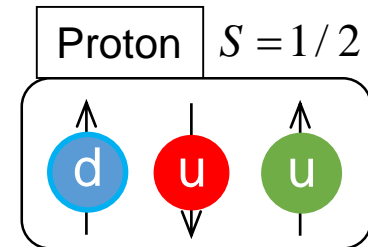
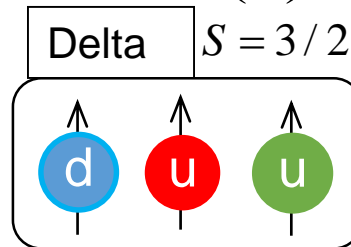
	Q-Q				Q-Q̄			
Color	A	S	A	S	1	8	1	8
Flavor	A	A	S	S				
Spin	A(0)	S(1)	S(1)	A(0)	0	0	1	1
<i>K</i>	-8	-4/3	8/3	4	-16	2	16/3	-2/3

$$K = - \sum_{i<j}^N (\lambda_i^c \lambda_j^c)(\sigma_i^s \sigma_j^s) \longrightarrow$$

$K < 0$ attraction; $K > 0$ repulsion

$M_\Delta - M_P \approx 290 \text{ MeV} \rightarrow K \text{ factors } 3 \times \left(\frac{8}{3} \right) - (-8) = 16$

K factor of 1 \rightarrow 18 MeV

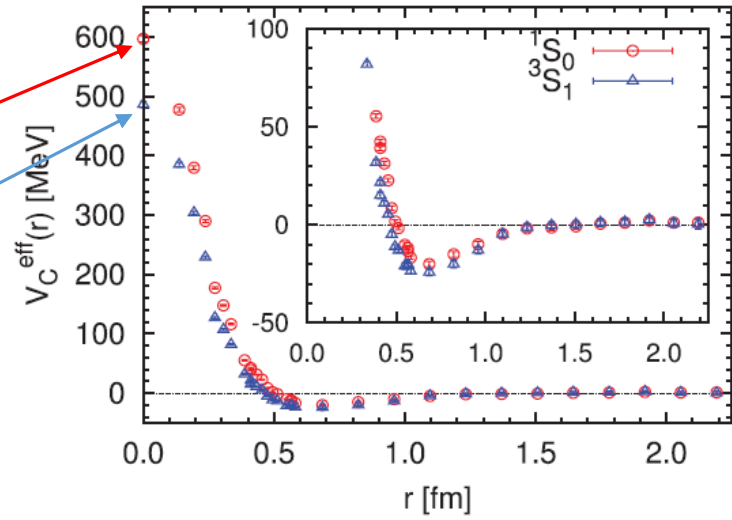


👉 NN force in SU(2) spin 1 vs spin 0 channel: comparison to lattice

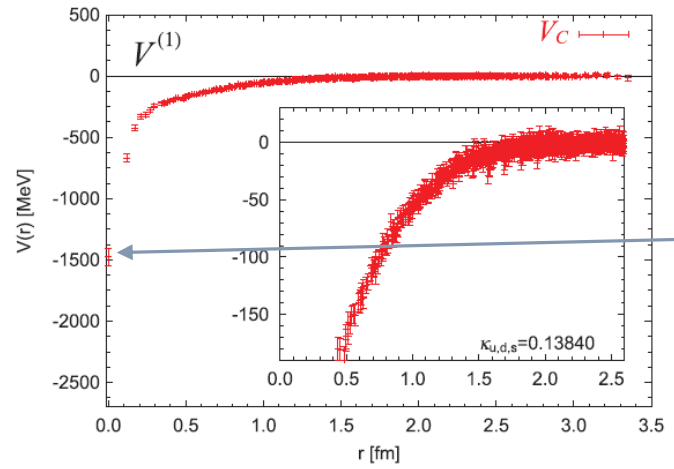
$$K_{2-N} = K_{6-quark} - (K_{1N} + K_{1N})$$

$$\frac{K_{2-N}^{S=0}}{K_{2-N}^{S=1}} = 1.29 \rightarrow \text{comparison}$$

QCD HAL collaboration

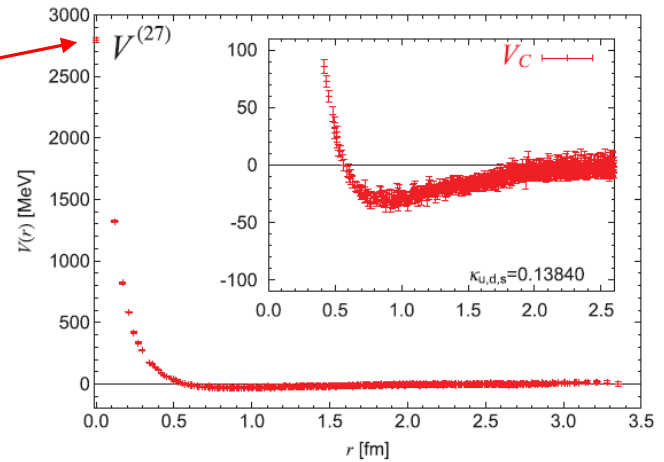


👉 H dibaryon channel: Flavor 1 vs Flavor 27



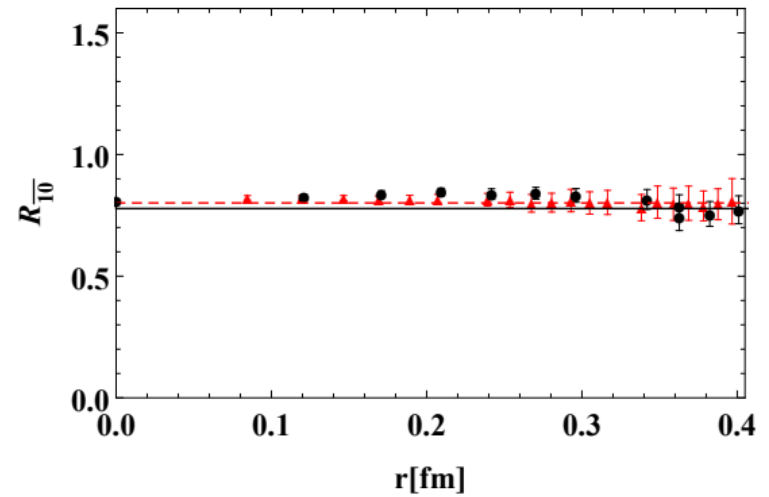
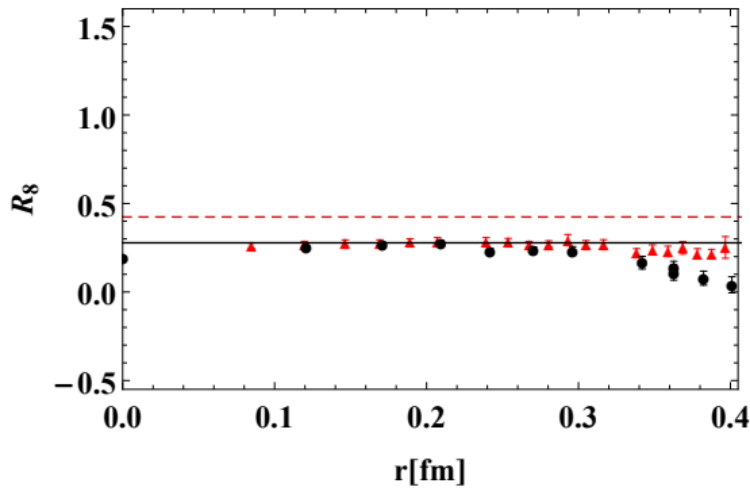
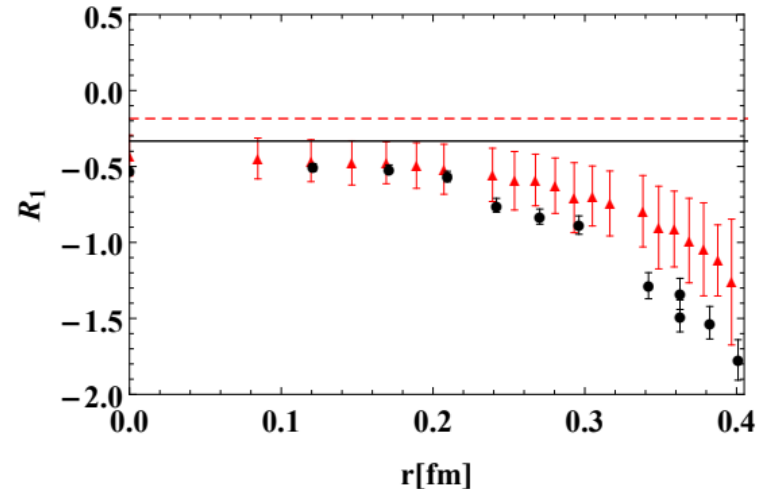
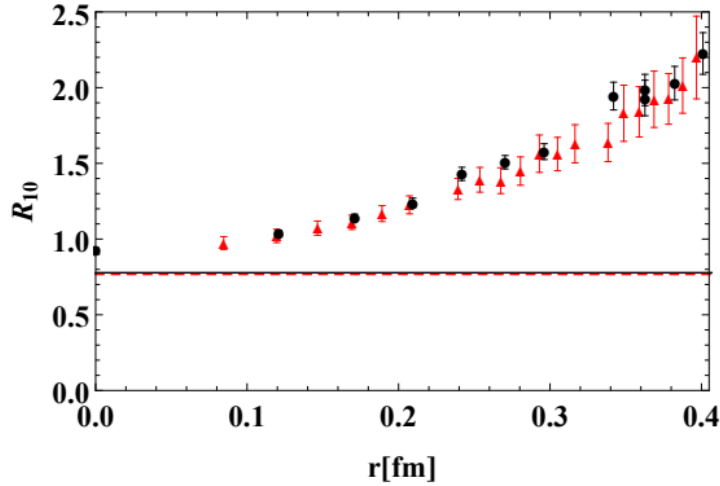
$$\frac{K_{2-N}^{F=27}}{K_{2-N}^{F=1}} = -3$$

(HAL QCD Collaboration)



$$\mathcal{R}_\ell^{\text{CQM}} = \frac{V_{\text{CQM}}(F_\ell)}{V_{\text{CQM}}(F_{27})}$$

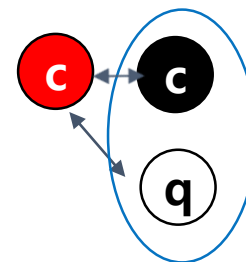
$$\mathcal{R}_\ell^{\text{LQCD}} = \frac{V_{\text{LQCD}}(F_\ell)}{V_{\text{LQCD}}(F_{27})}$$



Why Heavy quarks are needed for multiquark configuration

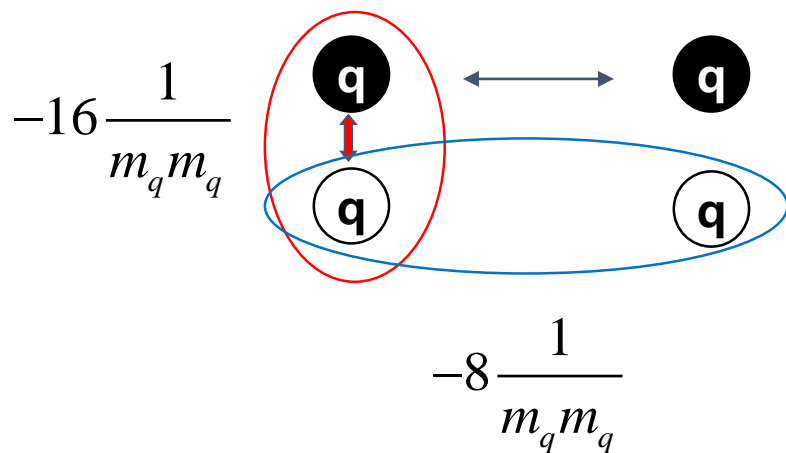
Color-color interaction becomes stronger (Karlner Rosner)

$$H_{cc} = \dots + \lambda_i^c \lambda_j^c \left(\frac{g^2}{r_{ij}} \right) + \dots \quad r \approx \frac{1}{mg^2}, \quad E_C \approx -mg^4$$

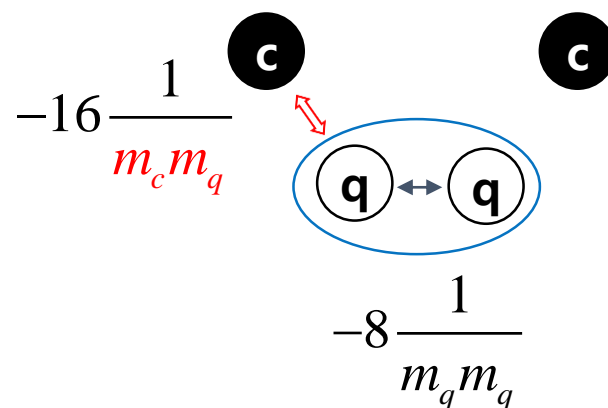


Color-spin interaction becomes weaker with heavy quarks

When all light quarks
Fall apart into two mesons



When heavy quarks,
could be compact (Tcc)



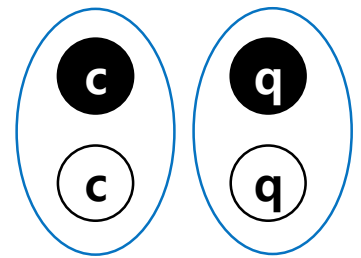
Compact multiquarks or loosely bound molecules

Will Look at X(3872) and Tcc(3875)

Can they be compact?

X(3872)

$$I^G (J^{PC}) = 0^+ (1^{++})$$



👉 **Color-spin** (C=color, S=spin)

$$K_{X(3872)} - K_D - K_{D^*} = \begin{pmatrix} \frac{16}{3} \frac{1}{m_c^2} + \frac{16}{3} \frac{1}{m_q^2} + \frac{32}{3} \frac{1}{m_c m_q} & 0 \\ 0 & -\frac{2}{3} \frac{1}{m_c^2} - \frac{2}{3} \frac{1}{m_q^2} - \frac{4}{3} \frac{1}{m_c m_q} \end{pmatrix} \begin{matrix} (c\bar{c})_{S=1}^{C=1} \otimes (q\bar{q})_{S=1}^{C=1} \\ (c\bar{c})_{S=1}^{C=8} \otimes (q\bar{q})_{S=1}^{C=8} \end{matrix}$$

↗ $\sim +140 \text{ MeV}$
↘ $\sim -20 \text{ MeV}$

👉 **Color-color interaction of $(c\bar{c})_{S=1}^{C=8} \otimes (q\bar{q})_{S=1}^{C=8}$ is repulsive**

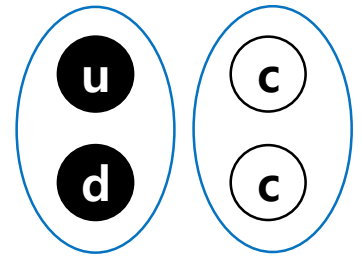
To overcome additional kinetic term attraction has to be $>100 \text{ MeV}$

Full quark model calculation \rightarrow Fall apart to two mesons

(W. Park, SHL, NPA925 (2014) 161)

Tcc(3875)

$$I^G (J^P) = 0^+ (1^+)$$



Color-spin

$$K_{T_{cc}(3875)} - K_D - K_{D^*} = \left(\begin{array}{cc} \boxed{-8 \frac{1}{m_q^2} + \frac{8}{3} \frac{1}{m_c^2} + \frac{32}{3} \frac{1}{m_c m_q}} & -8\sqrt{2} \frac{1}{m_c m_q} \\ -8\sqrt{2} \frac{1}{m_c m_q} & \boxed{-\frac{4}{3} \frac{1}{m_q^2} + 4 \frac{1}{m_c^2} + \frac{32}{3} \frac{1}{m_c m_q}} \end{array} \right) \begin{array}{l} (ud)_{S=0}^{C=\bar{3}} \otimes (\bar{c}c)_{S=1}^{C=3} \\ (ud)_{S=1}^{C=6} \otimes (\bar{c}c)_{S=0}^{C=\bar{6}} \end{array}$$

↖ ~ -100 MeV

↘ ~ +17 MeV

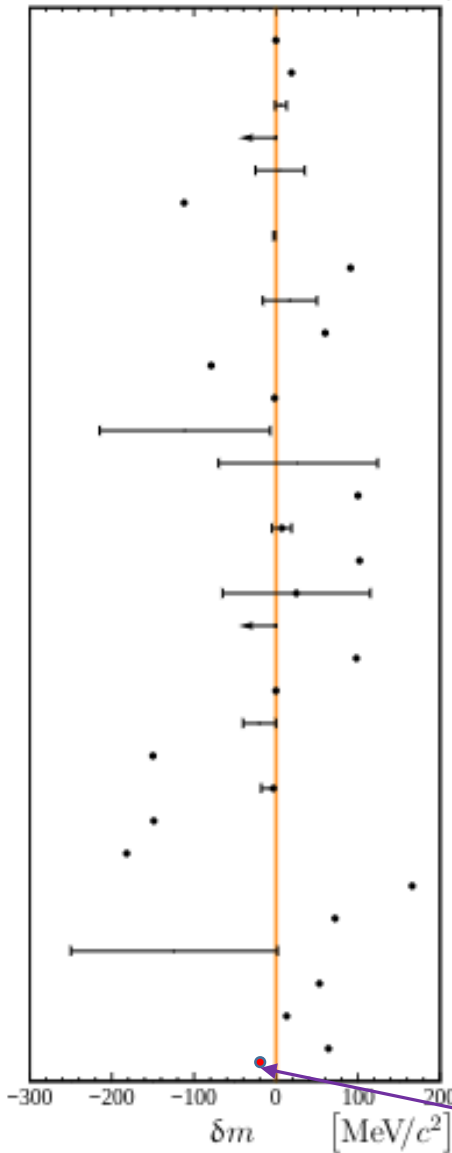
Color-color interaction of $(ud)_{S=0}^{C=\bar{3}} \otimes (\bar{c}c)_{S=1}^{C=3}$ is attractive

Full quark model calculation → Could be compact

-2021- $T_{cc}(3875)$ LHCb coll.

There is a strong short range attraction for $T_{cc} \rightarrow$ Could be compact, but depends sensitively on parameters:

The short range attraction for $X(3872)$ is very weak \rightarrow Can not be compact



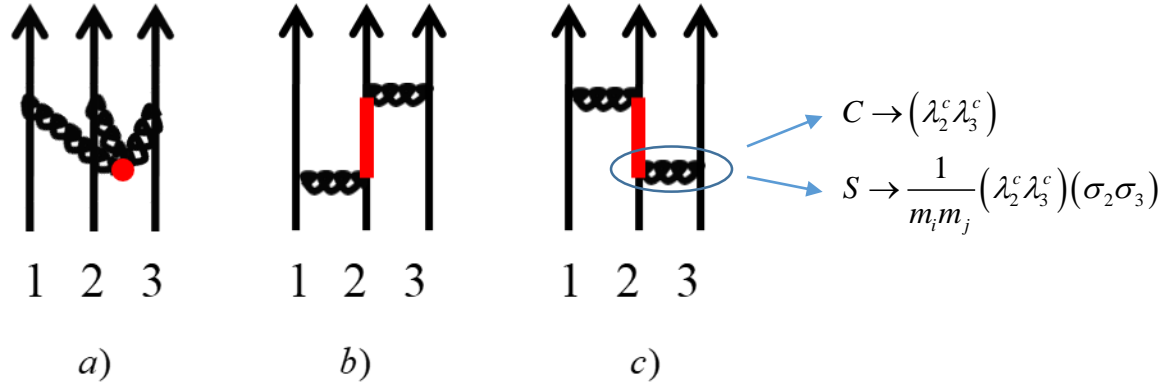
J. Carlson <i>et al.</i>	1987	[20]
B. Silvestre-Brac and C. Semay	1993	[21]
C. Semay and B. Silvestre-Brac	1994	[22]
S. Pepin <i>et al.</i>	1996	[23]
B. A. Gelman and S. Nussinov	2003	[24]
J. Vijande <i>et al.</i>	2003	[25]
D. Janc and M. Rosina	2004	[26]
F. Navarra <i>et al.</i>	2007	[27]
J. Vijande <i>et al.</i>	2007	[28]
D. Ebert <i>et al.</i>	2007	[29]
S. H. Lee and S. Yasui	2009	[30]
Y. Yang <i>et al.</i>	2009	[31]
G.-Q. Feng <i>et al.</i>	2013	[32]
Y. Ikeda <i>et al.</i>	2013	[33]
S.-Q. Luo <i>et al.</i>	2017	[34]
M. Karliner and J. Rosner	2017	[35]
E. J. Eichten and C. Quigg	2017	[36]
Z. G. Wang	2017	[37]
G. K. C. Cheung <i>et al.</i>	2017	[38]
W. Park <i>et al.</i>	2018	[39]
A. Francis <i>et al.</i>	2018	[40]
P. Junnarkar <i>et al.</i>	2018	[41]
C. Deng <i>et al.</i>	2018	[42]
M.-Z. Liu <i>et al.</i>	2019	[43]
G. Yang <i>et al.</i>	2019	[44]
Y. Tan <i>et al.</i>	2020	[45]
Q.-F. Lü <i>et al.</i>	2020	[46]
E. Braaten <i>et al.</i>	2020	[47]
D. Gao <i>et al.</i>	2020	[48]
J.-B. Cheng <i>et al.</i>	2020	[49]
S. Noh <i>et al.</i>	2021	[50]
R. N. Faustov <i>et al.</i>	2021	[51]

S. Noh, Park, PRD 2023

What does the quark model tell us

- Three-body force: from meson to baryon and tetraquark

👉 Origin could be similar to Nuclear-Three-body force



$$L_{123}^{C-C} = \frac{4}{3} \left(\frac{\lambda_2^c \lambda_3^c}{m_1} + \frac{\lambda_1^c \lambda_3^c}{m_2} + \frac{\lambda_1^c \lambda_2^c}{m_3} \right) + 2d^{abc} (\lambda_1^a \lambda_2^b \lambda_3^c) \left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} \right)$$

$$L_{123}^{S-S} = \frac{1}{m_1 m_2 m_3} \left[\frac{4}{3} \left(\frac{(\sigma_2 \cdot \sigma_3) (\lambda_2^c \lambda_3^c)}{m_1^2} + \frac{(\sigma_1 \cdot \sigma_3) (\lambda_1^c \lambda_3^c)}{m_2^2} + \frac{(\sigma_1 \cdot \sigma_2) (\lambda_1^c \lambda_2^c)}{m_3^2} \right) + 2d^{abc} (\lambda_1^a \lambda_2^b \lambda_3^c) \left(\frac{\sigma_2 \cdot \sigma_3}{m_1^2} + \frac{\sigma_1 \cdot \sigma_3}{m_2^2} + \frac{\sigma_1 \cdot \sigma_2}{m_3^2} \right) - 2\epsilon_{ijk} \sigma_1^i \sigma_2^j \sigma_3^k f^{abc} \lambda_1^a \lambda_2^b \lambda_3^c \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{1}{m_3^2} \right) \right]$$

$$L_{123}^{C-S} = \frac{4}{3} \left[\frac{(\lambda_1^c \lambda_3^c)}{m_2} \left(\frac{\sigma_2 \cdot \sigma_3}{m_2 m_3} + \frac{\sigma_1 \cdot \sigma_2}{m_2 m_1} \right) + \frac{(\lambda_1^c \lambda_2^c)}{m_3} \left(\frac{\sigma_3 \cdot \sigma_2}{m_2 m_3} + \frac{\sigma_1 \cdot \sigma_3}{m_3 m_1} \right) + \frac{(\lambda_2^c \lambda_3^c)}{m_1} \left(\frac{\sigma_1 \cdot \sigma_3}{m_1 m_3} + \frac{\sigma_1 \cdot \sigma_2}{m_2 m_1} \right) \right] + 2d_{abc} (\lambda_1^a \lambda_2^b \lambda_3^c) \left[\frac{1}{m_2} \left(\frac{\sigma_2 \cdot \sigma_3}{m_2 m_3} + \frac{\sigma_1 \cdot \sigma_2}{m_2 m_1} \right) + \frac{1}{m_3} \left(\frac{\sigma_3 \cdot \sigma_2}{m_2 m_3} + \frac{\sigma_1 \cdot \sigma_3}{m_3 m_1} \right) + \frac{1}{m_1} \left(\frac{\sigma_1 \cdot \sigma_3}{m_1 m_3} + \frac{\sigma_1 \cdot \sigma_2}{m_2 m_1} \right) \right]$$

👉 Quark-three-body force: from meson to baryon

$$H_{\text{Total}} = H_{2\text{-body}} + A \cdot L^{C-C} + B \cdot L^{S-S} + C \cdot L^{C-S}$$

$$H_{3\text{-body}} = \begin{cases} A \frac{128}{3m_q} - \left(B \frac{128}{3m_q^5} + C \frac{256}{3m_q^3} \right) & \text{For N} \\ A \frac{128}{3m_q} + \left(B \frac{128}{3m_q^5} + C \frac{256}{3m_q^3} \right) & \text{For } \Delta \end{cases}$$

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameter (fm ⁻²)
η_c	2983.6	2996.9	$a = 13.1$
$J\Psi$	3096.9	3089.6	$a = 11.1$
D	1864.8	1864.1	$a = 4.5$
D^*	2010.3	2010.7	$a = 3.7$
π	139.57	139.39	$a = 4.6$
ρ	775.11	775.49	$a = 2.2$
K	493.68	494.62	$a = 4.6$
K^*	891.66	888.82	$a = 2.8$

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameters (fm ⁻²)
Λ_c	2286.5	2266.7 (2281.6)	$a_1 = 2.9, a_2 = 3.7$
Σ_c	2452.9	2441.6 (2480.9)	$a_1 = 2.1, a_2 = 3.8$
Λ	1115.7	1113.6 (1134.1)	$a_1 = 2.8, a_2 = 2.7$
Σ	1192.6	1196.5 (1231.6)	$a_1 = 2.1, a_2 = 3.1$
Σ_c^*	2518.5	2522.9 (2567.7)	$a_1 = 2.0, a_2 = 3.4$
Σ^*	1383.7	1398.9 (1455.2)	$a_1 = 1.9, a_2 = 2.4$
p	938.27	980.47 (1005.3)	$a_1 = 2.4, a_2 = 2.4$
Δ	1232	1272.1 (1346.8)	$a_1 = 1.8, a_2 = 1.8$

The standard deviation is $\sigma = 5.86$.



$\sigma = 24.44(63.10)$

With quark-three-body force

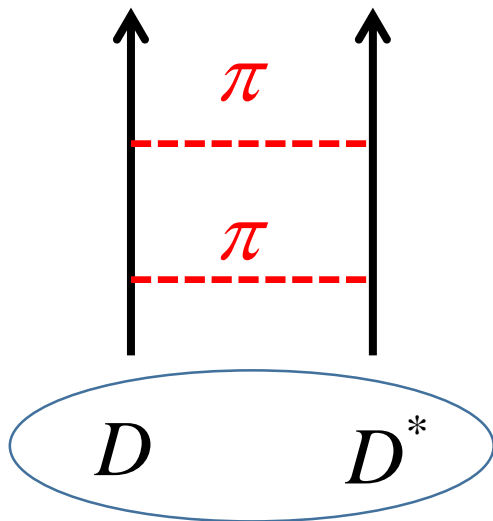
➡ Effect of 3-quark-interaction on Tetraquarks : Repulsive

$$H_{\text{Total}} = H_{\text{2-body}} + A \cdot L^{C-C} + B \cdot L^{S-S} + C \cdot L^{C-S}$$

Particle	Measured mass (MeV)	$\sum_{i<j<k} L_{ijk}^{C-C}$	$\sum_{i<j<k} L_{ijk}^{S-S}$	$\sum_{i<j<k} L_{ijk}^{C-S}$
T_{cc}	3875	-4.84236	0.0319013	20.9444
$X(3872)$	3872	19.3694	0.0427164	-1.36541

Can $X(3872)$ be $D-\bar{D}^*$ and T_{cc} be $D-D^*$ Molecules?

Perspectives from the π -exchange



$M(J_M, I_M)$

Especially important when

$J_M \neq 0$ Mixing with D-wave

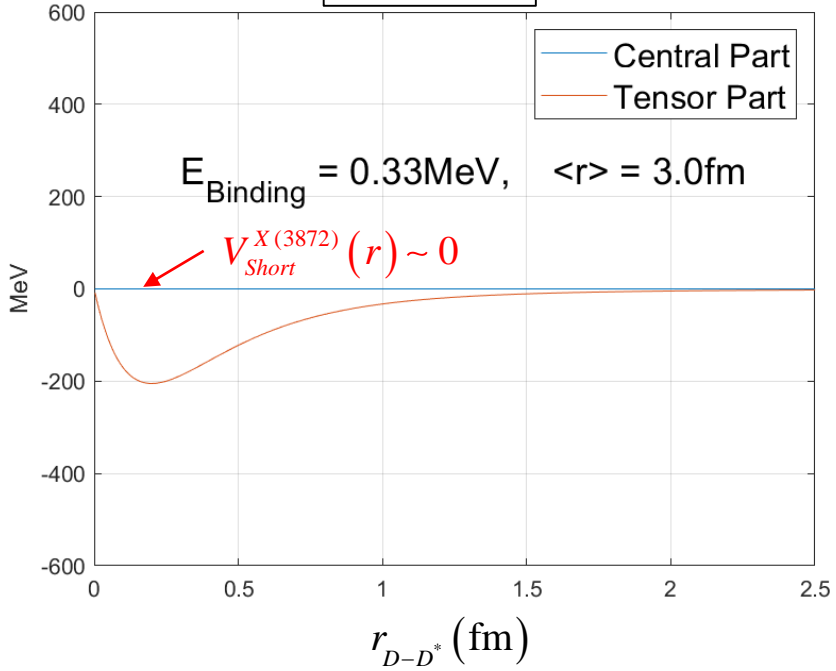
and

$I_M < (I_D + I_{D^*})$ Mixing is strong

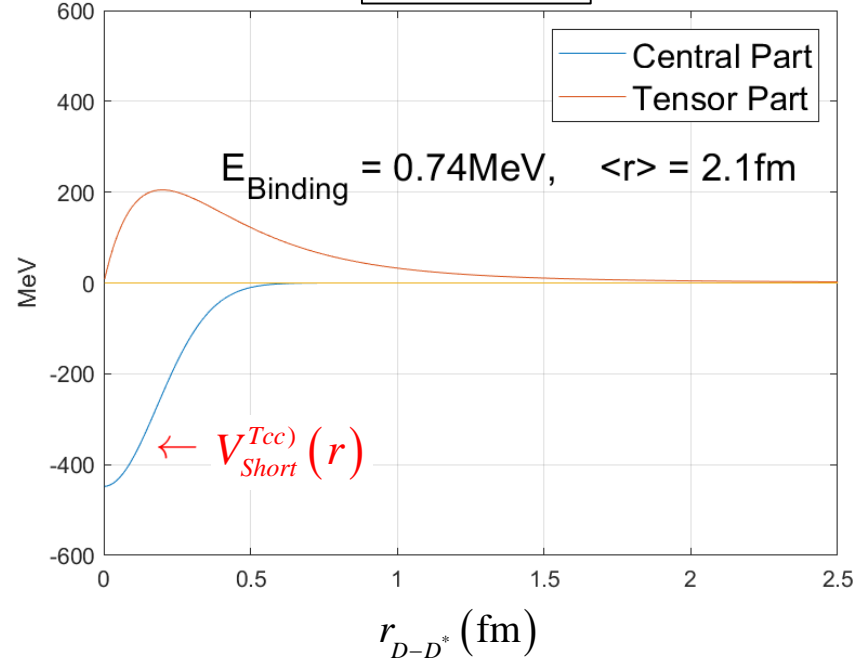
$$\begin{matrix} -:X(3872): D-\bar{D}^* \\ +:T_{cc}: D-\bar{D}^* \end{matrix} V(r) = V_{Short}(r) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mp 3V_0 \left[\begin{pmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix} T_\pi(r) \right]$$

Central Part = $V_{Short}(r)$ — ; Tensor Part = $\pm T_\pi(r)$ —

X(3872)

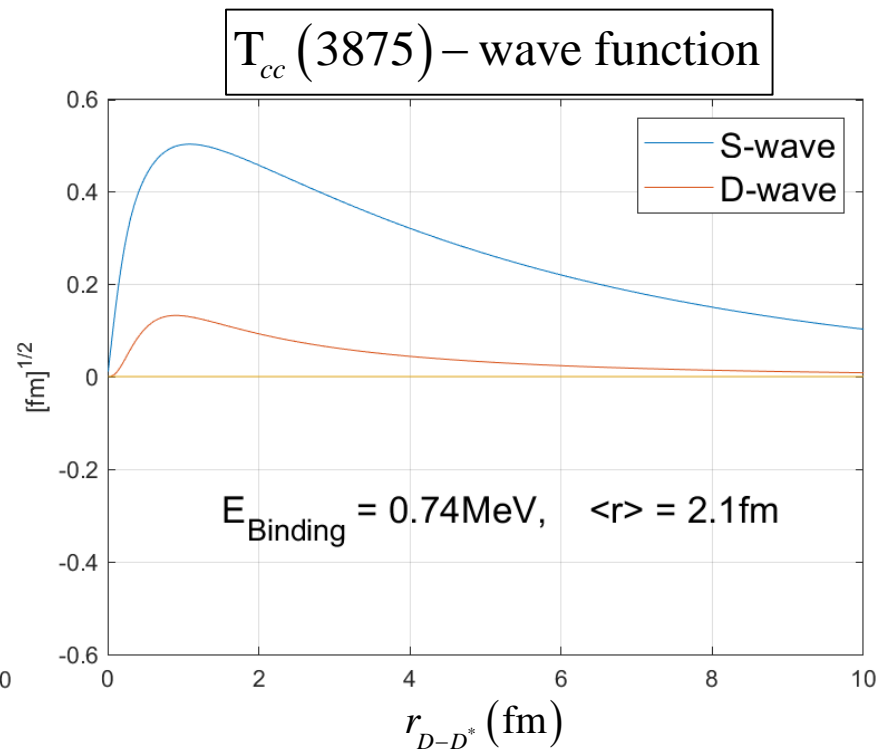
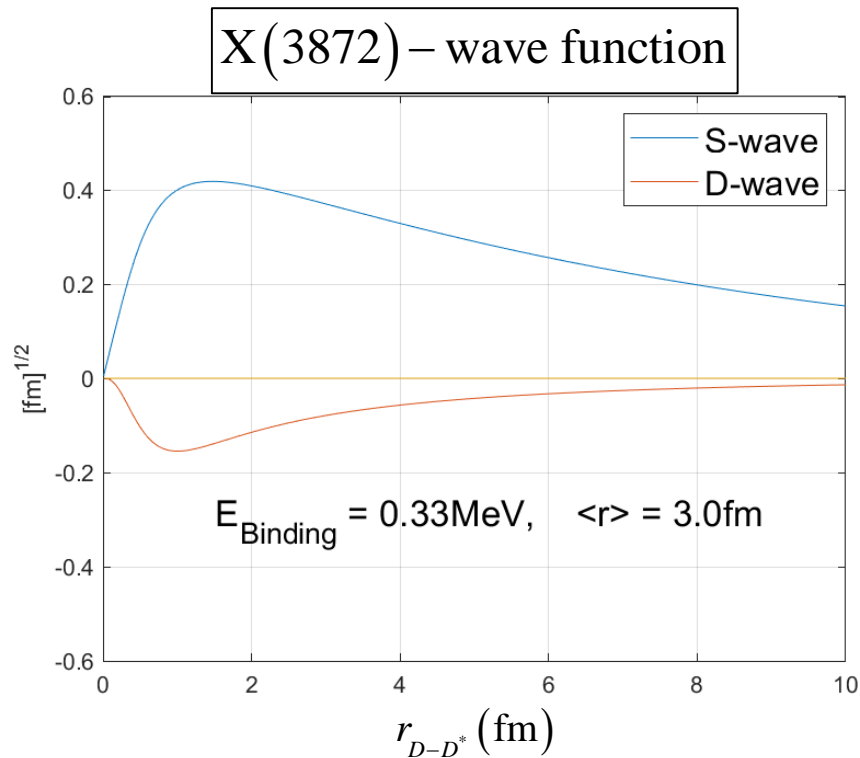
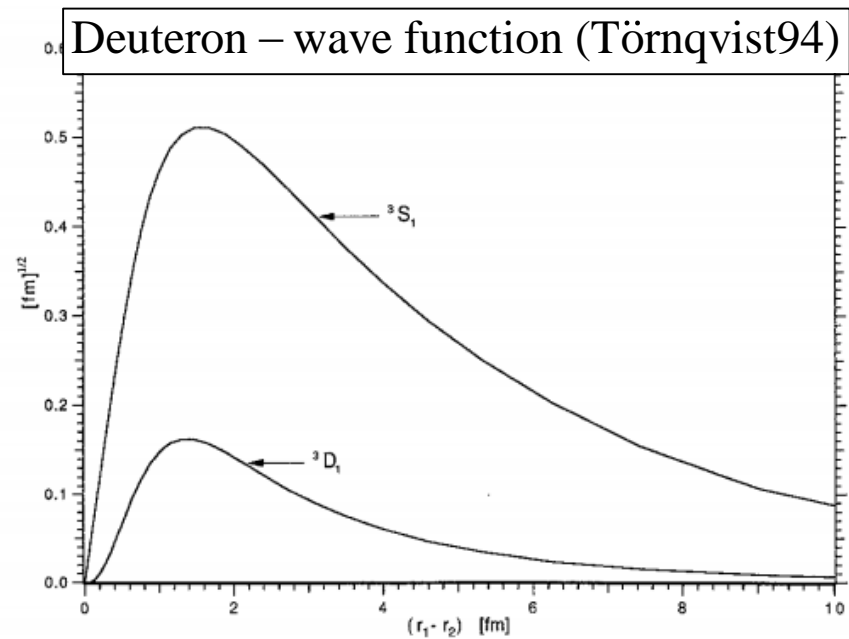


T_{cc}(3875)



👉 Wave functions:
Similar to that of Deuteron

👉 Both X(3872) : D and Tcc(3875)
could be a large molecular configuration



II: Measuring Exotics in Heavy Ion Collision:

X(3872) must be dominantly molecular
Tcc(3875) could be compact or molecules

IOPscience

Heavy-ion collisions at the LHC—Last call for predictions

N Armesto¹, N Borghini², S Jeon³, U A Wiedemann⁴, S Abreu⁵, S V Akkelin⁶, J Alam⁷, J L Albacete⁸, A Andronic⁹, D Antonov¹⁰ [+ Show full author list](#)

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Citation N Armesto *et al* 2008 *J. Phys. G: Nucl. Part. Phys.* **35** 054001

10.3. Charmed exotics from heavy-ion collision

S H Lee, S Yasui, W Liu and C M Ko

Our contribution to the volume

We discuss why charmed multiquark hadrons are likely to exist and explore the possibility of observing such states in heavy-ion reactions at the LHC.

Multiquark hadronic states are usually unstable as their quark configurations are energetically above those of combined meson and/or baryon states. However, constituent quark model calculations suggest that multiquark states might become stable when some of the light quarks are replaced by heavy quarks. Two possible states that could be realistically observed in heavy-ion collisions at LHC are the tetraquark $T_{cc}(ud\bar{c}\bar{c})$ [385] and the pentaquark

nature
physics

Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration*

Theory prediction

Identifying Multiquark Hadrons from Heavy Ion Collisions

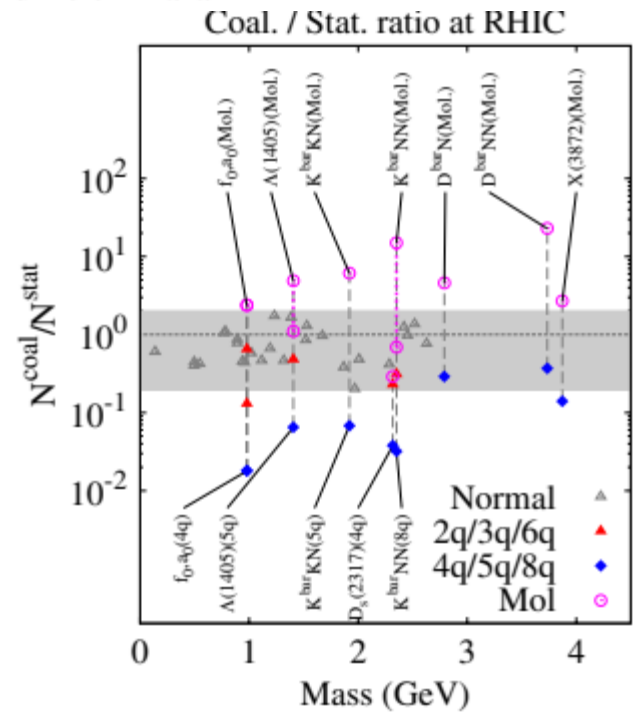
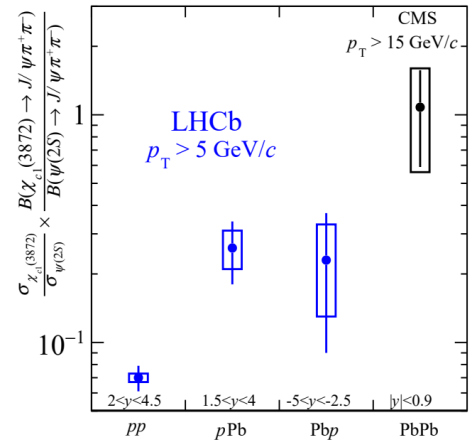
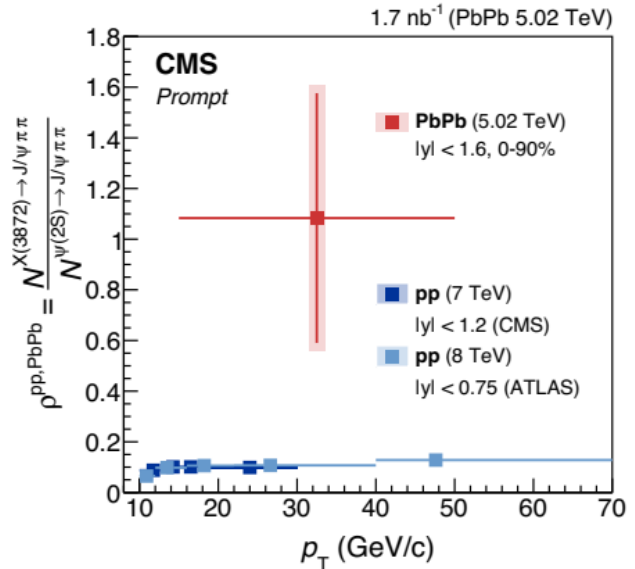
Sungtae Cho,¹ Takenori Furumoto,^{2,3} Tetsuo Hyodo,⁴ Daisuke Jido,² Che Ming Ko,⁵ Su Houng Lee,^{1,2}
Marina Nielsen,⁶ Akira Ohnishi,² Takayasu Sekihara,^{2,7} Shigehiro Yasui,⁸ and Koichi Yazaki^{2,3}

(ExHIC Collaboration)

Experiment

Evidence for X(3872) in Pb-Pb Collisions and Studies of its Prompt Production at $\sqrt{s_{NN}} = 5.02$ TeV

A. M. Sirunyan *et al.*
CMS Collaboration



$$\frac{dN_X}{dp_X} = C \int dx_1 dx_2 dp_1 dp_2 \frac{dN_1}{dp_1} \frac{dN_2}{V dp_2} W(x_1, x_2, p_1, p_2) \delta(p_X - p_1 - p_2)$$

⊙ Normalization conditions $\int dx_i dp_i \frac{dN_i}{V dp_i} = N_i$ $\int dx dp W(x, p) = (2\pi)^n$

⊙ Wigner function $W(x, p) = (2)^n \exp\left[-\frac{x^2}{\sigma^2} - \sigma^2 p^2\right]$

Should use x, p in CM frame S. Cho, K.J. Sun, C.M. Ko, SH Lee, Y. Oh, PRC101(20)024909

⊙ $\sigma \rightarrow$ infinity limit

$$\frac{dN_X}{dp_X} = C \left(\frac{\gamma}{V} \right) \frac{dN_1}{dp_1} \Big|_{p_1 = \frac{p_X}{2}} \frac{dN_2}{dp_2} \Big|_{p_2 = \frac{p_X}{2}}$$

- Coalescence probability is suppressed for smaller object when

$$\frac{dN_i}{Vdp_i} \propto \exp\left[-\frac{p_i^2}{2mT}\right] \quad W(x, p) = (2)^n \exp\left[-\frac{x^2}{\sigma^2} - \sigma^2 p^2\right]$$

$$\frac{dN_X}{dp_X} = \frac{1}{\left(1 + \frac{1}{mT\sigma^2}\right)^{n/2}} C\left(\frac{\gamma}{V}\right) \frac{dN_1}{dp_1} \Big|_{p_1 = \frac{p_X}{2}} \frac{dN_2}{dp_2} \Big|_{p_2 = \frac{p_X}{2}}$$

correction becomes visible when $\sigma < 0.5$ fm

- Deuteron Pt distribution should be determined by that of proton

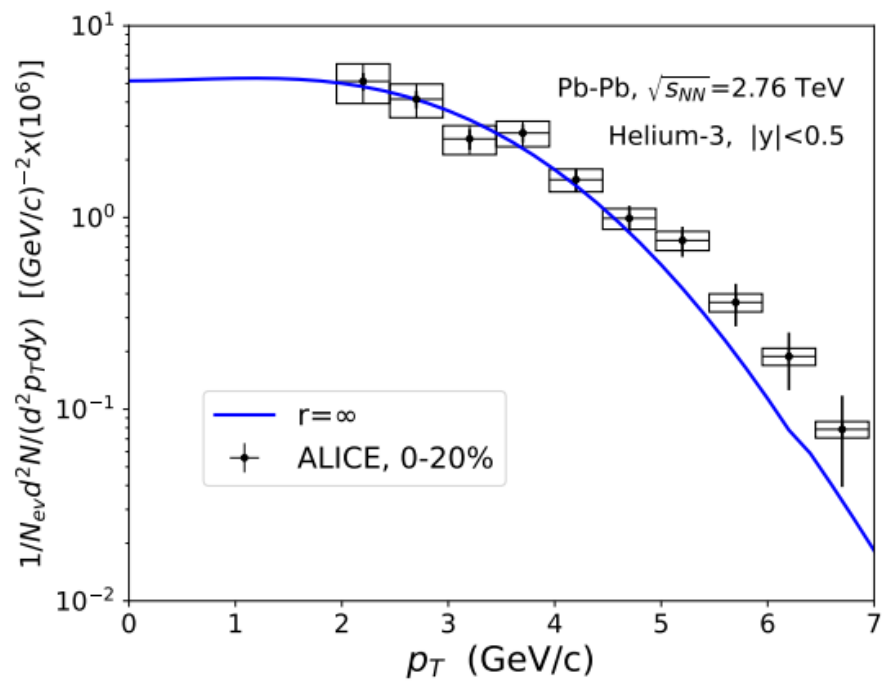
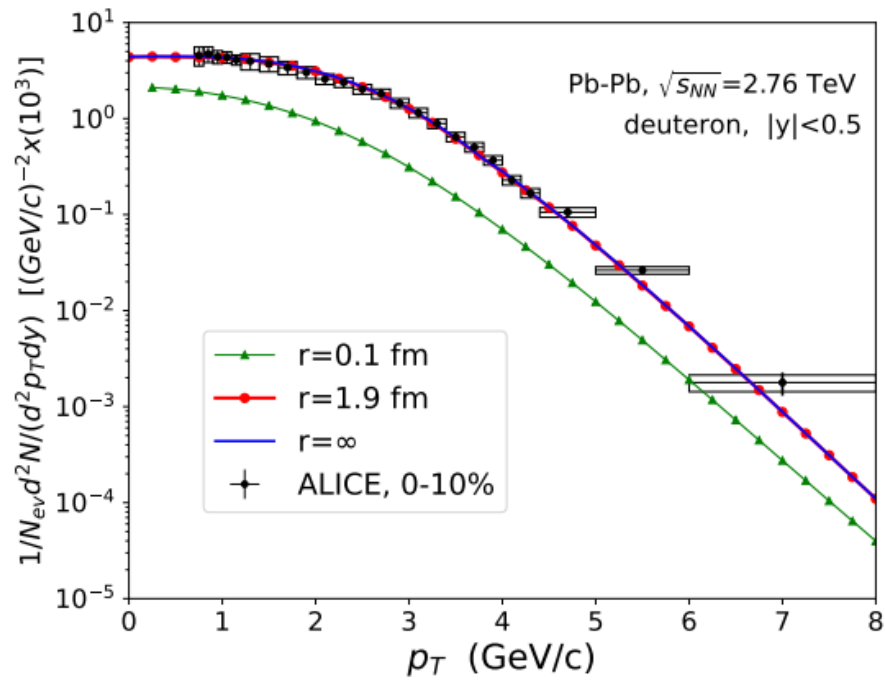
- Use $\frac{dN_i}{dp} = R_b \frac{dN_{\text{Proton}}}{dp} \Big|_{\text{Measured}}$

$$\frac{d^2 N_{\text{deuteron}}}{d^2 p_T} = \frac{g_d}{g_1 g_2} (2\pi)^2 \gamma \frac{R_b^2}{V} \frac{d^2 N_{\text{Proton}}}{d^2 p_1} \Big|_{p_1 = \frac{p_T}{2}} \frac{d^2 N_{\text{Proton}}}{d^2 p_2} \Big|_{p_2 = \frac{p_T}{2}}$$

$$\frac{d^2 N_{^3\text{He}}}{d^2 p_T} = \frac{g_h}{g_1 g_2 g_3} (2\pi)^4 \gamma^2 \frac{R_b^3}{V^2} \frac{d^2 N_{\text{Proton}}}{d^2 p_1} \Big|_{p_1 = \frac{p_T}{3}} \frac{d^2 N_{\text{Proton}}}{d^2 p_2} \Big|_{p_2 = \frac{p_T}{3}} \frac{d^2 N_{\text{Proton}}}{d^2 p_3} \Big|_{p_3 = \frac{p_T}{3}}$$

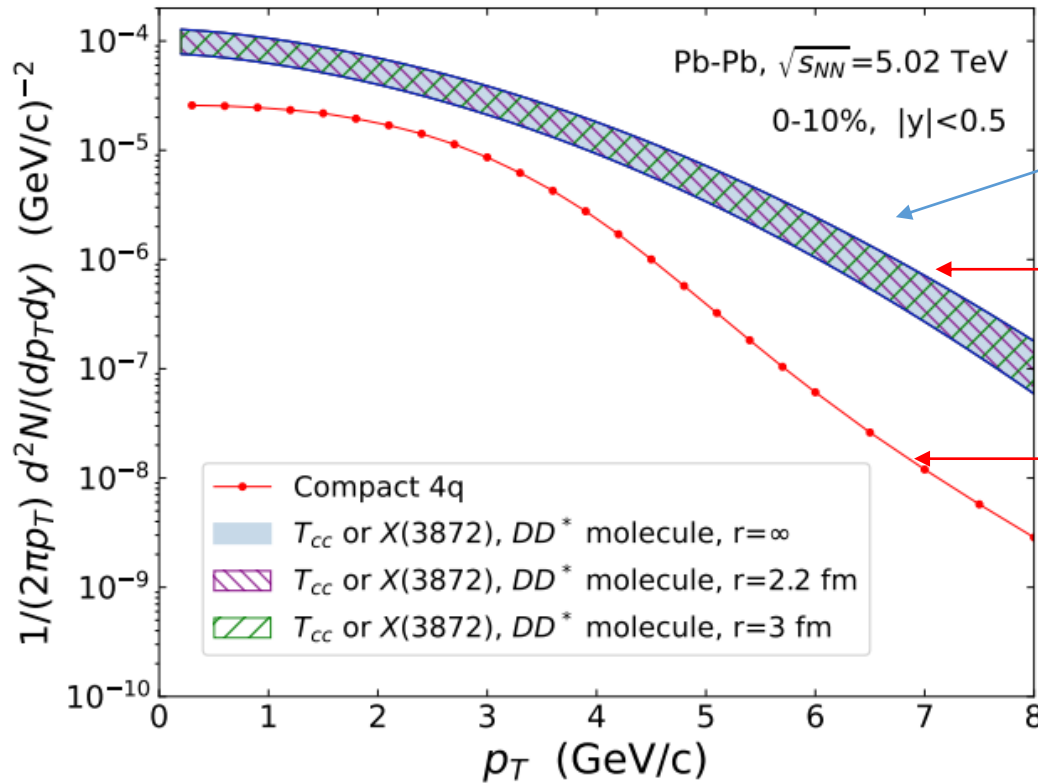
A simple fit to Deuteron and ^3He using (R_b, V) - II

1. For $r > 1.9$ fm result are similar to $\sigma \rightarrow$ infinity result
2. Both can be fit by choosing $R_b = 0.36 \rightarrow$ similar to feed-down effects SHM
3. $V(2\text{-dim}) = 608 \text{ fm}^2$



Expectation for Molecular configuration of X(3872) and Tcc

1. Use measured D and D* Pt distribution
2. Use $R_b=0.31$ from feed-down effects SHM
3. Use same $V(2\text{-dim})=608 \text{ fm}^2$



X(3872)

Yun, Cho, SHL, PRC107 (2023) 014906

Tcc if Molecular structure

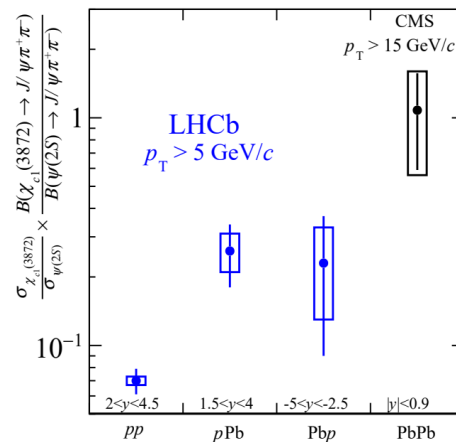
Tcc if Compact multiquark

S. Cho, SHL, PRC101 (2020) 024902

Large molecules and compact quark states seem to have different P_T dependence

Final Thoughts:

If $X(3872)$ is a $\bar{D} D^*$ S-wave molecule (with S. Noh, A. Park)

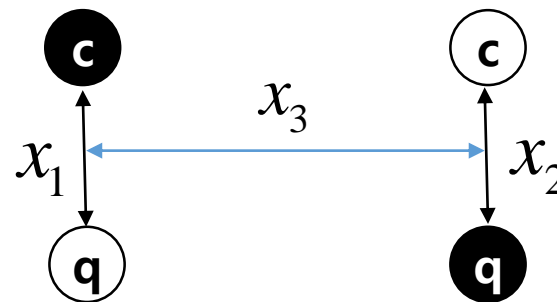


Quark Spatial wave function of X(3872) : $D\bar{D}^*$ molecule

☞ S-wave in $(q\bar{c}), (c\bar{q})$ basis $D - \bar{D}^*$

$$\psi_1^{Spatial} \propto \exp\left[-a_1 x_1^2 - a_2 x_2^2 - a_3 x_3^2\right]$$

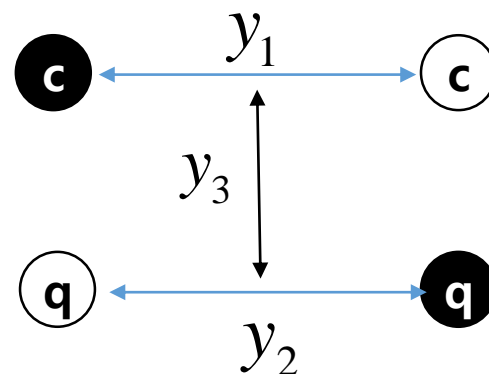
$$R_{D \text{ or } D^*} \sim 0.55 \text{ fm}, \quad R_{D-\bar{D}^*} \sim 4 \text{ fm}$$



☞ Transformation into $(c\bar{c}), (q\bar{q})$ basis

$$\psi_1^{Spatial} \propto \exp\left[-b_1 y_1^2 - b_{12} y_1 \cdot y_2 - b_2 y_2^2 - b_3 y_3^2\right]$$

$$R_{(c\bar{c})} \sim 4.01 \text{ fm}, \quad R_{(q\bar{q})} \sim 4.06 \text{ fm}, \quad R_{(c\bar{c})-(q\bar{q})} \sim 0.394 \text{ fm}$$

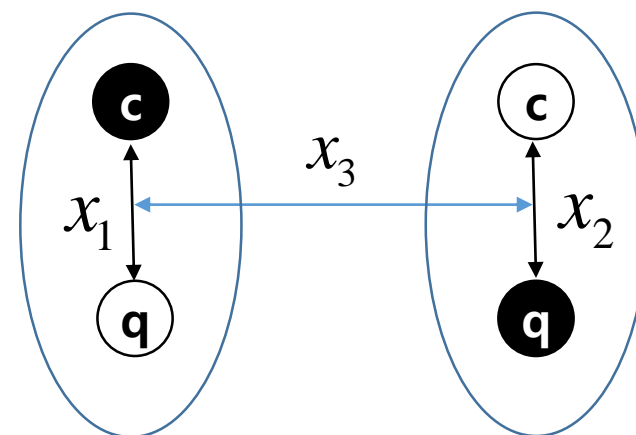


$$I^G(J^{PC}) = 0^+(1^{++})$$

☞ In $(q\bar{c}), (c\bar{q})$ basis

$$|1'\rangle = (q\bar{c})_{S=0}^{C=1} \otimes (c\bar{q})_{S=1}^{C=1} \rightarrow D - \bar{D}^*$$

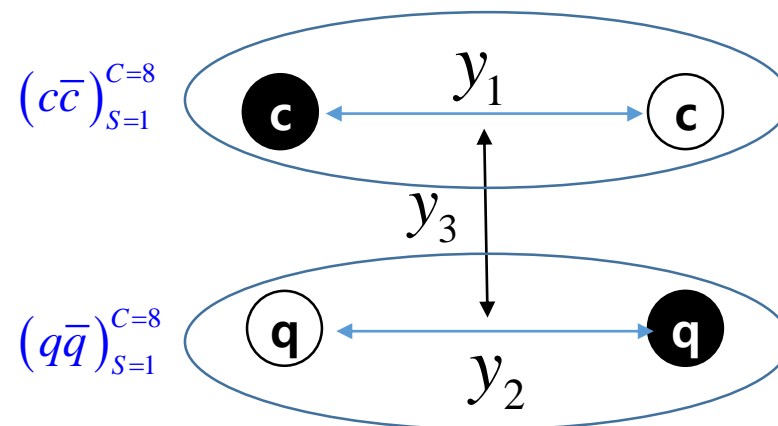
$$|2'\rangle = (q\bar{c})_{S=0}^{C=8} \otimes (c\bar{q})_{S=1}^{C=8}$$



☞ Transformation into $(c\bar{c}), (q\bar{q})$ basis

$$|1\rangle = (c\bar{c})_{S=1}^{C=8} \otimes (q\bar{q})_{S=1}^{C=8}$$

$$|2\rangle = (c\bar{c})_{S=1}^{C=1} \otimes (q\bar{q})_{S=1}^{C=1}$$



➔

$$|1'\rangle = \frac{2\sqrt{2}}{3}|1\rangle + \frac{1}{3}|2\rangle$$

$$|2'\rangle = -\frac{1}{3}|1\rangle + \frac{2\sqrt{2}}{3}|2\rangle$$

$D\bar{D}^*$ is mostly composed of $(c\bar{c})_{S=1}^{C=8} \otimes (q\bar{q})_{S=1}^{C=8}$

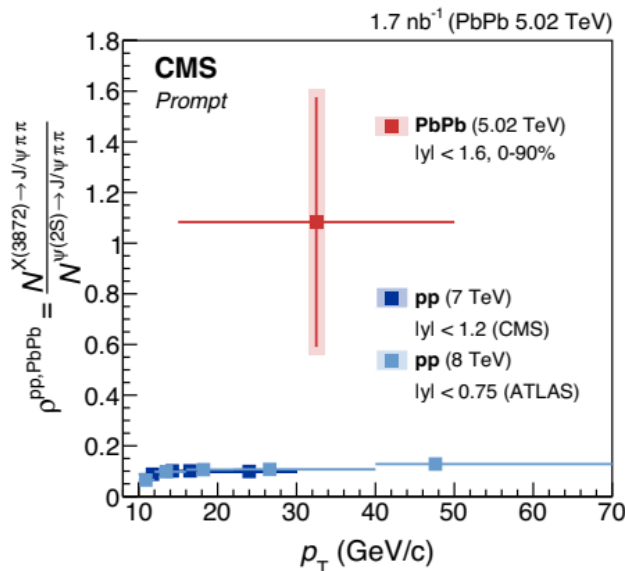
Possible explanation of abundant production of X(3872)

➡ $D\bar{D}^*$ is mostly composed of $(c\bar{c})_{S=1}^{C=8} \otimes (q\bar{q})_{S=1}^{C=8}$

➡ $R_{(c\bar{c})} \sim 4.01 \text{ fm}$, $R_{(q\bar{q})} \sim 4.06 \text{ fm}$, $R_{(c\bar{c})-(q\bar{q})} \sim 0.394 \text{ fm}$

➡ $\psi(2S)$ production at high Pt is dominated by $(c\bar{c})_{S=1}^{C=8}$ but has to be multiplied by a small overlap into color singlets: NRQCD

➡ X(3872) production at high Pt might be a direct recombination of $(c\bar{c})_{S=1}^{C=8}$ with $(q\bar{q})_{S=1}^{C=8}$ in QGP



Summary

- ⊙ Most exotics have multiple heavy quark: HIC is an excellent factory

 - ⊙ Production characteristics can reveal the structure of exotics:
compact vs molecule

 - ⊙ Enhanced productions always are linked to strong correlation and/or resonance: For example, Hoyle state
- Measurement of exotics in heavy ion collision will reveal new color configurations, strong correlations and possible resonance structures of quarks and gluons

Baryon 2025

Sep 8-12, Jeju Korea



Visa-free entry to Jeju Island: using direct flight to Jeju

https://overseas.mofa.go.kr/sg-en/brd/m_2435/view.do?seq=761394&page=1

Indico

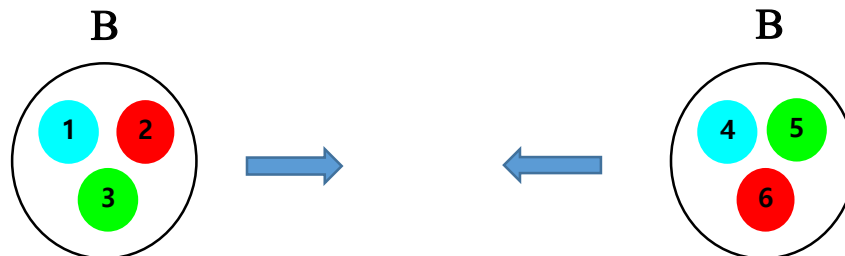
<https://indico.cern.ch/event/1339154/>

$$H = \sum_{i=1}^n \left(\underline{m_i + \frac{p_i^2}{2m_i}} \right) - \sum_{i<j}^n (\lambda_i^c \lambda_j^c) V_{ij}^C (r_{ij}) - \sum_{i<j}^n \frac{(\lambda_i^c \lambda_j^c)(\sigma_i \sigma_j)}{m_i m_j} V_{ij}^{SS} (r_{ij})$$

$$m_q = 300 \text{ MeV}, \quad m_s = 500 \text{ MeV}, \quad m_c = 1500 \text{ MeV}.$$

☞ When brought together need to overcome **Additional Kinetic energy**

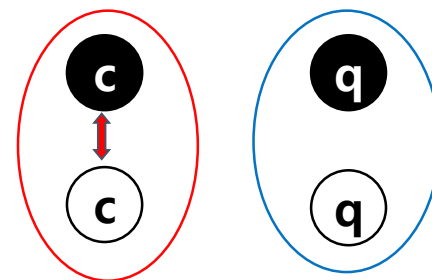
$$\frac{p_{BB}^2}{2\mu_{BB}} \approx \frac{1}{2\mu_{BB}} \frac{1}{(0.6\text{fm})^2} \sim 100\text{MeV}$$



→ To have a compact configuration, short range attraction should be larger than 100 MeV

$$X(3872) \begin{cases} (c\bar{c}) \rightarrow (C=8, S=1) \\ (q\bar{q}) \rightarrow (C=8, S=1) \end{cases}$$

$$H_{cc} = \lambda_c^a \left(\lambda_c^a \frac{g}{r_{cc}} \right) ?$$



Color-Color (X(3872))

$$\lambda_c^a (\lambda_c^a) = \frac{1}{2} [(\lambda_c^a + \lambda_c^a)^2 - \lambda_c^2 - (\lambda_c^a)^2]$$

$$\frac{1}{4} \lambda^2 = C = \frac{1}{3} (p^2 + q^2 + pq + 3(p+q)) \quad C(p=1, q=1) = 3, \quad C_f(p=1, q=0) = \frac{4}{3}$$

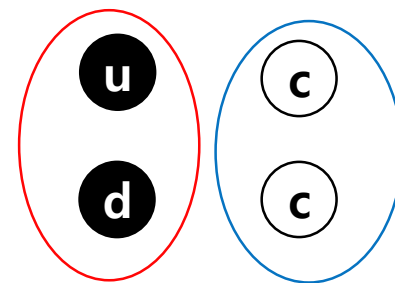
If cc is in $(C=8, S=1)$

$$\lambda_c^a (\lambda_c^a) = \frac{4}{2} \left[3 - 2 \frac{4}{3} \right] = \frac{2}{3} > 0$$

No additional attraction from color-color interaction

→ X(3872) can not be compact multiquark state

$$T_{cc}(3875) \begin{cases} (ud) \rightarrow (C = \bar{3}, S = 0) \\ (\bar{c}\bar{c}) \rightarrow (C = 3, S = 1) \end{cases} \quad H_{cc} = \lambda_c^a \left(\lambda_c^a \frac{g}{r_{cc}} \right) ?$$



Color-Color (Tcc)

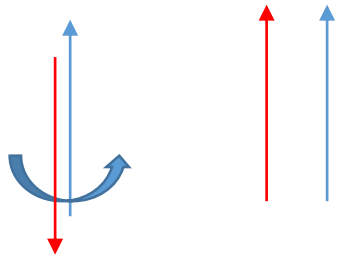
$$\lambda_c^a(\lambda_c^a) = \frac{1}{2} \left[(\lambda_c^a + \lambda_c^a)^2 - \lambda_c^2 - (\lambda_c^a)^2 \right]$$

$$\frac{1}{4} \lambda^2 = C = \frac{1}{3} (p^2 + q^2 + pq + 3(p+q)) \quad C(p=0, q=1) = \frac{4}{3}, \quad C(p=1, q=0) = \frac{4}{3}$$

$$\text{If } \bar{c}\bar{c} \text{ is in } (C = 3, S = 1) \quad \lambda_c^a(\lambda_c^a) = \frac{4}{2} \left[\frac{4}{3} - 2 \frac{4}{3} \right] = -\frac{8}{3} < 0$$

Hence there is additional attraction

→ Tcc(3875) could be a compact multiquark state



when two spin is pointing in the same direction
 outside $H = -\mu_2 \cdot B_1 = s_2 \cdot B_1 \approx s_2 \cdot s_1 > 0$ Hence repulsive
 inside $H = -\mu_2 \cdot B_1 = s_2 \cdot B_1 \approx -s_2 \cdot s_1 < 0$ Hence attractive

Color-spin interaction for 2 body:

$$K = -\sum_{i < j}^N (\lambda_i^c \lambda_j^c) (\sigma_i^s \sigma_j^s) \longrightarrow$$

	Q-Q				Q-Q̄			
Color	A	S	A	S	1	8	1	8
Flavor	A	A	S	S				
Spin	A(0)	S(1)	S(1)	A(0)	0	0	1	1
<i>K</i>	-8	-4/3	8/3	4	-16	2	16/3	-2/3

$K < 0$ attraction; $K > 0$ repulsion

when two spin is pointing in opposite direction

outside $H = -\mu_2 \cdot B_1 = s_2 \cdot B_1 \approx s_2 \cdot s_1 < 0$ Hence attractive

inside $H = -\mu_2 \cdot B_1 = s_2 \cdot B_1 \approx -s_2 \cdot s_1 > 0$ Hence repulsive

➡ For Deuteron and ^3He , results are similar SHM

Nucleus	$N_{SHM}^{Nucleus} / N_{SHM}^p$	$N_{coal}^{Nucleus} / N_{SHM}^p$
d	9.07×10^{-3}	8.84×10^{-3}
^3He	2.68×10^{-5}	2.03×10^{-5}

TABLE II. The yield ratio of light nucleus with proton in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. For deuteron and ^3He the centralities are 0–10 % and 0–20 %, respectively.

➡ For X(3872) and Tcc, yields for molecular configurations are larger

Tetraquark	dN_{coal}/dy	$N_{coal}/N_{SHMc}^{X(3872)}$	$N_{coal}/N_{SHMc}^{\psi(2S)}$
DD^* molecule	$(2.45 \pm 0.71) \times 10^{-3}$	2.47 ± 0.716	0.806 ± 0.234
<i>Compact 4q</i>	6.2×10^{-4}	6.25×10^{-1}	0.204

no feed down for D^*

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} N_{SHMc}^{X(3872)} / N_{SHMc}^{\psi(2S)} = 0.326$$

TABLE III. The first column shows the total yield of the tetraquark depending on its structure calculated by the coalescence model in Pb-Pb collisions at $\sqrt{s_{NN}}=5.02$ TeV at 0-10% centrality.. The remaining columns show their ratios to the statistical hadronization model with charm (SHMc)[28]. Here we used $dN_{\psi(2S)}/dy = 3.04 \times 10^{-3}$ and $N_{X(3872)}/N_{\psi(2S)} = 0.326$ obtained in SHMc.