Pole trajectories of the Λ(1405) helps establish its dynamical nature

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Z. Zhuang, R. M., J.-X. Lu, & L.-S. Geng, arXiv: 2405.07686

11.07.2024

Outline

- Introduction
- Framework
  - Chiral unitary approach
  - Quak mass dependence of the hadron masses
  - Analysis of the LQCD data
- Fitting results and discussion

Introduction

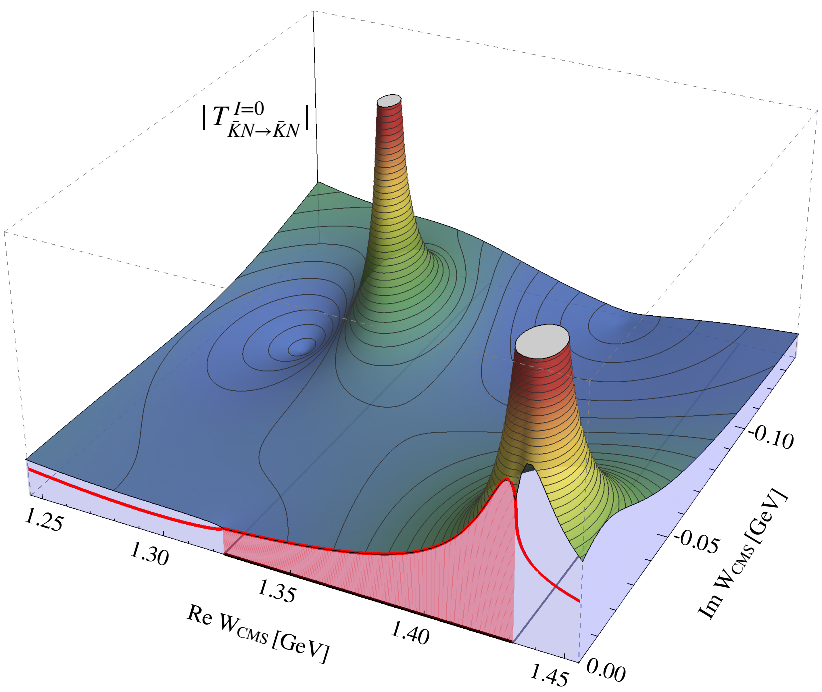
- The resonance first appeared in bubble chamber experimens

R. H. Dalitz and S. F. Tuan, PRL 2, 425(1959)

R. H. Dalitz and S. F. Tuan, Annals Phys, 10, 307

- $\Lambda(1405)$: $J^P = \frac{1}{2}^-, I = 0$ Status: ****
- Mass: $1405.1^{+1.3}_{-1}$ MeV, width: 50.5 ± 2 MeV
- Two pole structures of Λ(1405)
  - Coupled-channel framework
  - Chiral unitary approach …
- PDG added Λ(1380) with **

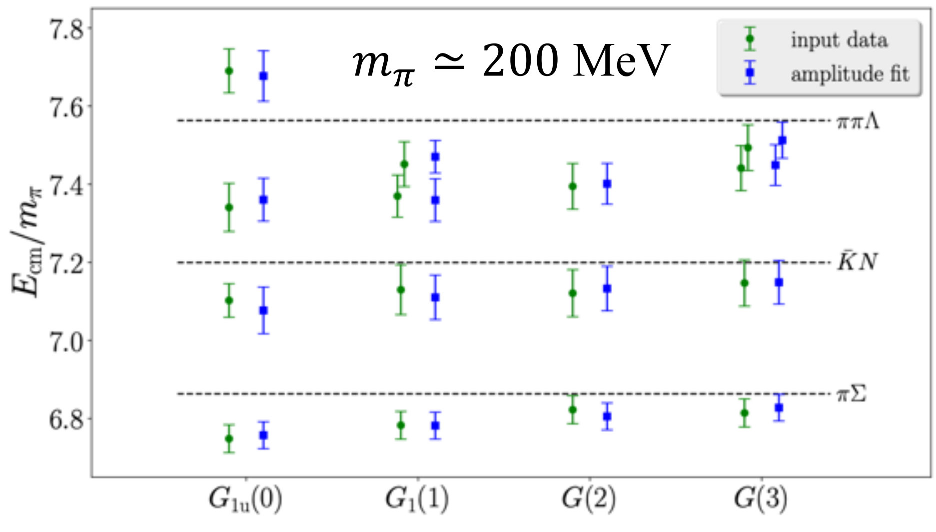
PDG, Prog. Theor. Exp. Phys. 2022, 083C01 (2022)



Maxim Mai, EPJST 230 (2021) 6, 1593-1607

Λ(1405) in a recent LQCD simulation

Finite-volume spectrum (error bars in green)



J. Bulava et al, (BaSc), PRD 109, 014511 (2024)

J. Bulava et al, (BaSc), PRL 132, 051901 (2024)

- The coupled $\pi\Sigma - \bar{K}N$ scattering
- Type of trajectory: $\mathrm{Tr}[M] = \mathrm{cte}$
  - $M = \mathrm{diag}(m_u, m_d, m_s)$
- $G_i(d^2)$: the little group of $\boldsymbol{P}^2 = \left(\frac{2\pi}{L}\right)^2 \boldsymbol{d}^2$
  - $\boldsymbol{d} \in \{[000],\ [001], [011], [111]\}$
- K-matrix parameterization for the $\Lambda(1405)$
- Pole positions $E_i = \left(M, \frac{\Gamma}{2}\right)$ [MeV]
  - $(1392(9)(2)(16), 0)$ virtual state
  - $(1455(13)(2)(17), 11.5(4.4)(4)(0.1))$

- A chiral extrapolation to the physical point from the analyses of the energy levels has not been done
- The poles over the $\mathrm{Tr}[M] = \mathrm{cte}$ trajectory towards the SU(3) symmetric limit has not been studied

Motivation

Chiral unitary approach

- The interaction kernel up to NLO $\quad a,b \in \{N,\Lambda,\Sigma,\Xi\}$, $\; i,j \in \{\pi,K,\eta\}$ $\qquad N_a = \sqrt{\frac{M_a+E_a}{2M_a}}$

$$V_{jb,ia} = -\frac{N_b N_a}{f^2}\left[C_{jb,ia}(2\sqrt{s}-M_b-M_a) - 4(D_{jb,ia} - 2k_\mu k'^\mu L_{jb,ia})\right]$$

Weinberg-Tomozawa term (WT) NLO term (S-wave)

- $f = (f_\pi(m_\pi) + f_K(m_\pi) + f_\eta(m_\pi))\;/\;3$

$$C_{jb,ia} = \begin{bmatrix} 4 & -\sqrt{\frac{3}{2}} & 0 & \sqrt{\frac{3}{2}} \\ & 3 & \frac{3}{\sqrt{2}} & 0 \\ & & 0 & -\frac{3}{\sqrt{2}} \\ & & & 3 \end{bmatrix}$$

$$D_{jb,ia} = \begin{bmatrix} 4(b_0+b_D)m_\pi^2 & -\sqrt{\frac{3}{2}}(b_D-b_F)\mu_1^2 & -\frac{4b_D m_\pi^2}{\sqrt{3}} & \sqrt{\frac{3}{2}}(b_D+b_F)\mu_1^2 \\ & (2b_0+3b_D+b_F)m_K^2 & \frac{(b_D+3b_F)\mu_2^2}{3\sqrt{2}} & 0 \\ & & \frac{4}{9}(3b_0\mu_3^2+b_D\mu_4^2) & -\frac{(b_D-3b_F)\mu_2^2}{3\sqrt{3}} \\ & & & 2(2b_0+3b_D-b_F)m_K^2 \end{bmatrix}$$

$$L_{jb,ia} = \begin{bmatrix} -4d_2+4d_3+2d_4 & \sqrt{\frac{3}{2}}(d_1+d_2-2d_3) & -\sqrt{3}d_3 & \sqrt{\frac{3}{2}}(d_1-d_2+2d_3) \\ & d_1+3d_2+2(d_3+d_4) & \frac{d_1-3d_2+2d_3}{\sqrt{2}} & 6d_2-3d_3 \\ & & 2(d_3+d_4) & \frac{d_1+3d_2-2d_3}{\sqrt{2}} \\ & & & -d_1+3d_2+2(d_3+d_4) \end{bmatrix}$$

Interaction in the SU(3) limit

➢ Projecting the two-particle isoscalar states to the relevant multiplets

$$\begin{pmatrix} |\pi\Sigma\rangle \\ |\bar{K}N\rangle \\ |\eta\Lambda\rangle \\ |K\Xi\rangle \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} \sqrt{15} & -\sqrt{24} & 0 & -1 \\ -\sqrt{10} & -2 & \sqrt{20} & -\sqrt{6} \\ -\sqrt{5} & -\sqrt{8} & 0 & 3\sqrt{3} \\ \sqrt{10} & 2 & 2\sqrt{5} & \sqrt{6} \end{pmatrix} \begin{pmatrix} |1\rangle \\ |8\rangle \\ |8'\rangle \\ |27\rangle \end{pmatrix}$$

➢ Example: $C_{jb,ia}$ in irreps. representation

$$C_{\alpha\beta} = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad \text{for} \quad \alpha, \beta \in \{1, 8, 8', 27\}$$

6 → $\Lambda(1380)$

3 → $\Lambda(1405)$

3 → $\Lambda(1680)$

1: singlet

8: symmetric octet representation

$8'$: antisymmetric octet representation

Model extrapolation to unphysical point

- Bethe-Salpeter equation

$$T_{ij} = V_{ij} + V_{ik} G_k^{DR} T_{kj}$$

- Loop function $G_k^{DR}(s, a(\mu))$

$$G_k^{DR}(s, a(\mu)) = \frac{2M_k}{16\pi^2}\left\{a_k(\mu) + \ln\frac{m_k^2}{\mu^2}\frac{M_k^2 - m_k^2 + s}{2s}\ln\frac{M_k^2}{m_k^2} + \text{log terms}\right\}$$

- Framework of the subtraction constant $a(\mu)$

$\mu = 630\text{ MeV}$

J. A. Oller, Prog. Part. Nucl. Phys. 110, 103728 (2020)

E. Oset et al, Nucl. Phys. A 635, 99 (1998)

$$a(\mu) = -\frac{2}{m_1 + m_2}\left[m_1 \log\left(1 + \sqrt{1 + \frac{m_1^2}{q_{\max}'^2}}\right) + m_2 \log\left(1 + \sqrt{1 + \frac{m_2^2}{q_{\max}'^2}}\right)\right] + 2\log\frac{\mu}{q'_{\max}}$$

- $q'_{\max}$ as free parameter fixed by LQCD energy levels

the cutoff: $q'_{\max}$

Model extrapolation to unphysical point

➢ Masses of the NG bosons

$$m_\pi^2 = M_{0\pi}^2 \left[1 + \mu_\pi - \frac{\mu_\eta}{3} + \frac{16 M_{0K}^2}{f_0^2}(2L_6^r - L_4^r)\right] + \frac{8 M_{0\pi}^2}{f_0^2}(2L_6^r + 2L_8^r - L_4^r - L_5^r) ,$$

$$m_K^2 = M_{0K}^2 \left[1 + \frac{2\mu_\eta}{3} + \frac{8 M_{0\pi}^2}{f_0^2}(2L_6^r - L_4^r)\right] + \frac{8 M_{0K}^2}{f_0^2}(4L_6^r + 2L_8^r - 2L_4^r - L_5^r) ,$$

$$m_\eta^2 = M_{0\eta}^2 \left[1 + 2\mu_K - \frac{4}{3}\mu_\eta + \frac{8 M_{0\eta}^2}{f_0^2}(2L_8^r - L_5^r) + \frac{8}{f_0^2}(2M_{0K}^2 + M_{0\pi}^2)(2L_6^r - L_4^r)\right]$$
$$+ M_{0\pi}^2 \left[-\mu_\pi + \frac{2}{3}\mu_K + \frac{1}{3}\mu_\eta\right] + \frac{128}{9 f_0^2}(M_{0K}^2 - M_{0\pi}^2)^2 (3L_7 + L_8^r) ,$$

Gell-Mann-Okubo relation
$4M_{0K}^2 = 3M_{0\eta}^2 + M_{0\pi}^2$

$$\mu_P = \frac{M_{0P}^2}{32\pi^2 f_0^2} \log \frac{M_{0P}^2}{\mu_r^2}, \qquad P = \pi, K, \eta$$

$\mu_r = 770$ MeV, $f_0 = 80$ MeV, and the LECs L_i^r from R. M. et al, JHEP 11, 017 (2020)

Model extrapolation to unphysical point

- Pion mass dependence of the decay constants

$$f_\pi = f_0 \left[1 - 2\mu_\pi - \mu_K + \frac{4M_{0\,\pi}^2}{f_0^2}(L_4^r + L_5^r) + \frac{8M_{0\,K}^2}{f_0^2}L_4^r\right]$$

$$f_K = f_0 \left[1 - \frac{3\mu_\pi}{4} - \frac{3\mu_K}{2} - \frac{3\mu_\eta}{4} + \frac{4M_{0\,\pi}^2}{f_0^2}L_4^r + \frac{4M_{0\,K}^2}{f_0^2}(2L_4^r + L_5^r)\right],$$

$$f_\eta = f_0 \left[1 - 3\mu_K + \frac{4L_4^r}{f_0^2}(M_{0\,\pi}^2 + 2M_{0\,K}^2) + \frac{4M_{0\,\eta}^2}{f_0^2}L_5^r\right]$$

R. M. et al, JHEP 11, 017 (2020)

Model extrapolation to unphysical point

- The octet baryon masses using one-loop NLO covariant baryon ChPT

$$m_B = m_0 + m_B^{(2)} + m_B^{(3)}$$

- m_0 is the baryon mass in the chiral limit
- $m_B^{(2)} = -\sum_{\phi=\pi,K} \xi_{B,\phi}^{(a)} m_\phi^2$

$\xi_{B,\phi}^{(a)} = \xi_{B,\phi}^{(a)}(b_0, b_D, b_F)$

- $m_B^{(3)} = \sum_{\phi=\pi,K,\eta} \frac{1}{(4\pi f_\phi)^2} \xi_{B,\phi}^{(b)} H_B^{(b)}(m_\phi)$

$\xi_{B,\phi}^{(b)} = \xi_{B,\phi}^{(b)}(D, F)$

$D = 0.8, F = 0.46$

B. Borasoy, PRD 59, 054021 (1999)

$H_B^{(b)}(m_\phi)$ is EOMS loop-function

- m_0, b_0, b_D, and b_F as the free parameters fitting to baryon masses from CLS ensemble

G. S. Bali et al, JHEP 05, 035 (2023)

Finite volume effect

- Bethe-Salpeter equation in a finite volume (FV)

$$\tilde{T} = \frac{V}{I - V\tilde{\mathcal{G}}}$$

- E^{FV} in each irrep as a solution of the eigenvalue problem

$$\det\left[I - V\tilde{\mathcal{G}}\right] = 0$$

- $\tilde{\mathcal{G}}$ in the dimensional regularization scheme

A. M. T. et al, PRD 85, 014027 (2012)

$$\tilde{\mathcal{G}} = G^{DR}(s, a(\mu)) - \lim_{q_{\max}\to\infty}\left(\tilde{G}(P, q_{\max}) - G^{\text{cutoff}}(s, q_{\max})\right)$$

$$\tilde{G}(P, q_{\max}) = \frac{1}{L^3}\sum_{n}^{q_{\max}} \frac{E}{P_0} I(q^*)$$

$$I(\vec{q}\,) = \frac{\omega_1(q) + \omega_2(q)}{2\omega_1(q)\omega_2(q)\left[P_0^2 - (\omega_1(q) + \omega_2(q))^2 + i\epsilon\right]}$$

$$\omega_i = \sqrt{q^2 + m_i^2}, \text{ and } q = |\vec{q}\,|$$

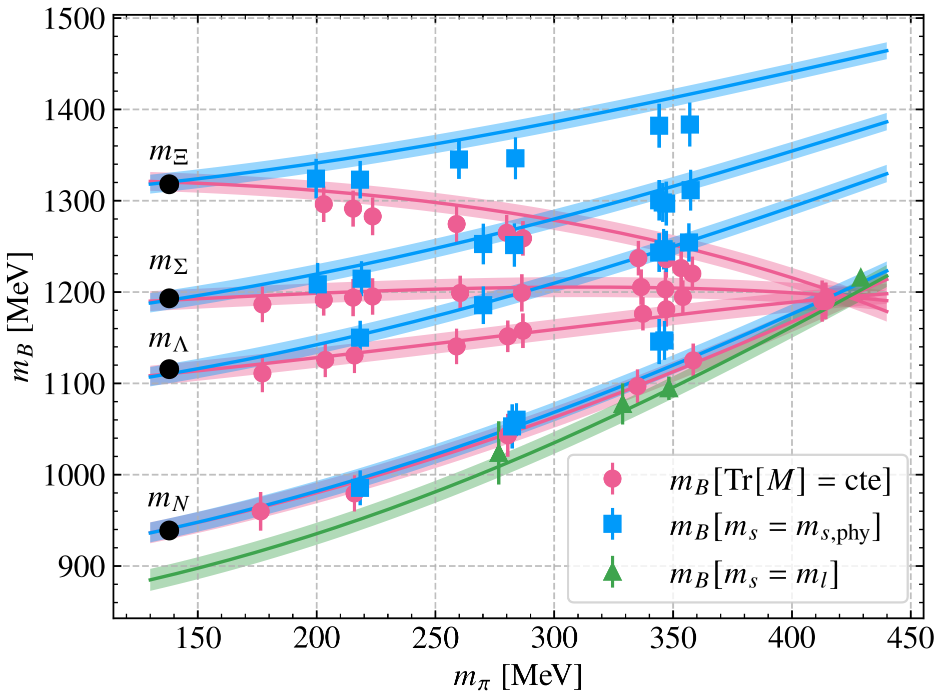
q^* the center-of-mass (CM) frame momentum

Fit to the baryon masses in LQCD

- Global fit to the baryon masses with three types of the trajectories

$\mathrm{Tr}[M] = \mathrm{cte}$, $m_s = m_{s,\mathrm{phy}}$, and $m_s = m_l$ $(l = u, d)$

$M = \mathrm{diag}(m_u, m_d, m_s)$



Data from G. S. Bali et al, JHEP 05, 035 (2023)

- LECs $[\mathrm{GeV}^{-1}]$

$b_0 = -0.65(1)$, $b_D = 0.070(1)$

$b_F = -0.380(1)$

- Baryon mass in the chiral limit

$m_0 = 840 \pm 12$ MeV

- SU(3) limit: $m_\pi = 423$ MeV

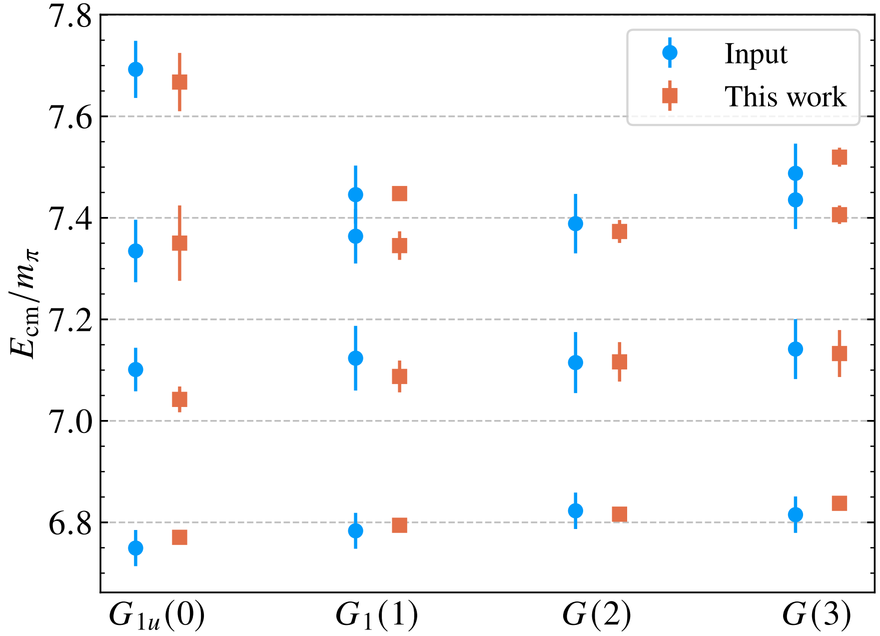
Fit to the energy levels

@LO+NLO (S-wave)
@Coupled $\pi\Sigma - \bar{K}N$ scattering

- Finite-volume spectrum used input to constrain the d_i and the $q'_{\max}$

$i = 1,4$

The best fit: $\chi^2_{\text{d.o.f}} = 2.2$



J. Bulava et al, (BaSc), PRD 109, 014511 (2024)
J. Bulava et al, (BaSc), PRL 132, 051901 (2024)

- $G_i(d^2)$: irrep. representation

$G_{1u}(0)$: [000], $G_1(1)$: [001]

$G(2)$: [011], $G(3)$: [111]

- $\chi^2 = \Delta E^T C^{-1} \Delta E$

$\Delta E = E_{\text{lattice}} - E_{\text{th}}$

C is the covariance matrix

d_1 [GeV^{-1}]	d_2 [GeV^{-1}]	d_3 [GeV^{-1}]	d_4 [GeV^{-1}]	$q'_{\max}$ [MeV]
-0.4 ± 0.7	0.024 ± 0.001	-0.1 ± 0.4	-0.6 ± 0.7	711 ± 90

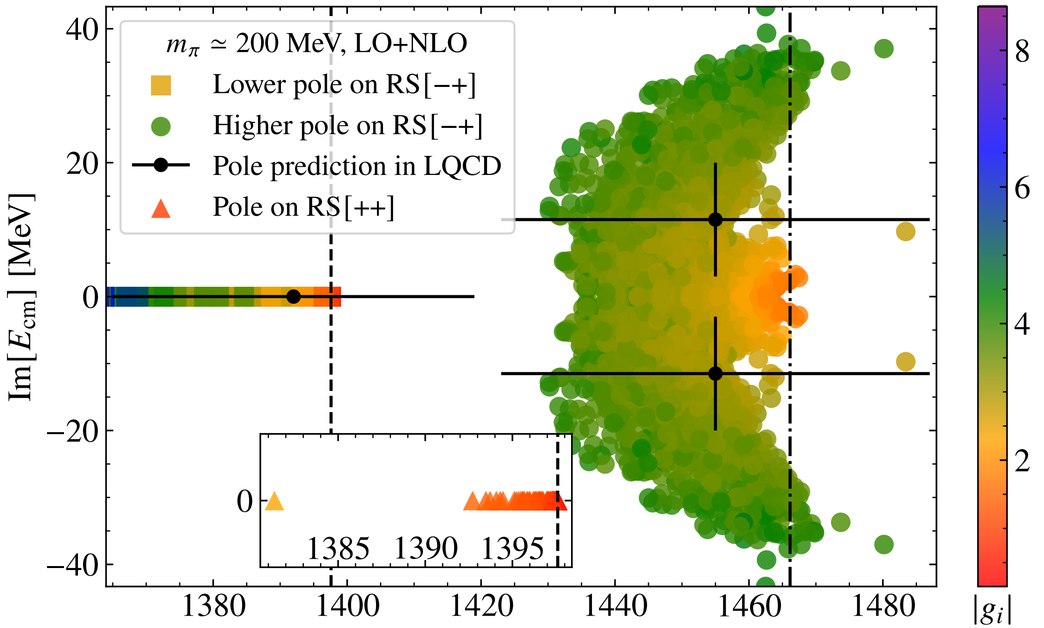
Pole positions $@m_\pi \simeq 200$ MeV

RS: Riemann sheet

$+$: Physical sheet

$-$: Unphysical sheet

1-σ(68%) confidence level

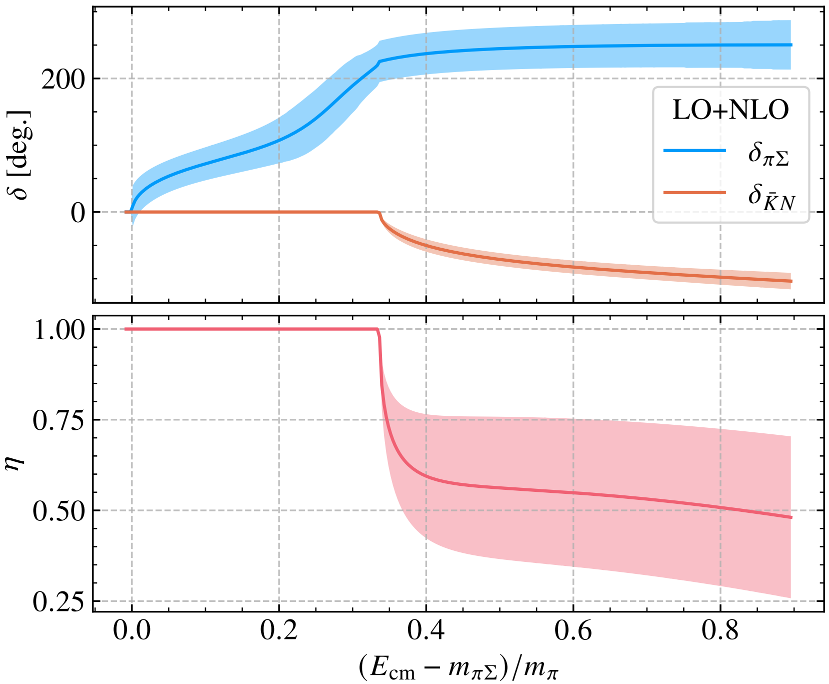


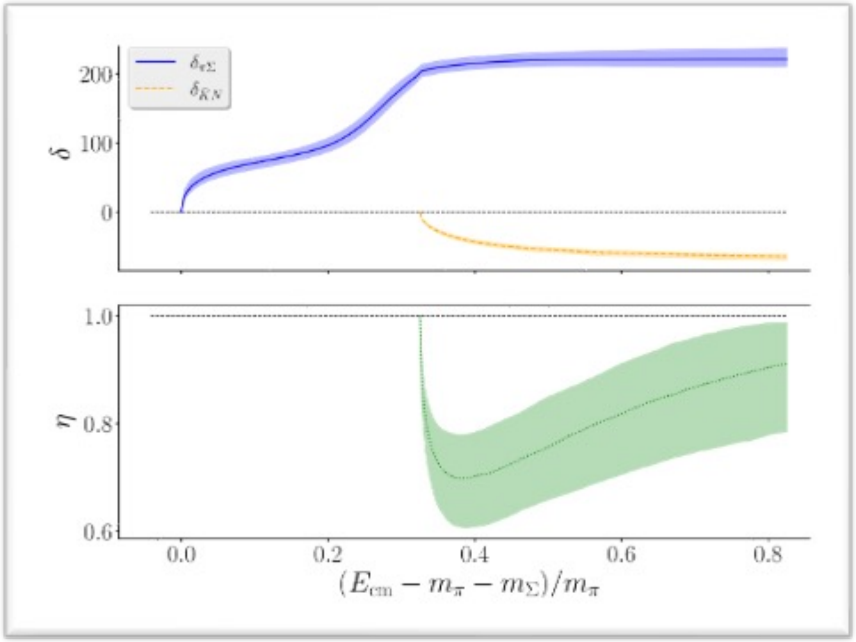
$|g_i| = |g_{\pi\Sigma}|$ or $|g_{\bar{K}N}|$

- $a_{\pi\Sigma} = -2.3, a_{\bar{K}N} = -2.1$
- The pole positions $(M, \Gamma/2)$ and the ratios of the couplings

	z_1	z_2
Pole [MeV]	$(1378 \pm 17, 0)$	$(1454 \pm 8, 12 \pm 11)$
$\lvert g_{\pi\Sigma} / g_{\bar{K}N} \rvert$	2.5 ± 1.8	0.4 ± 0.1

Phase shift @LO+NLO (S-wave)





Our result VS

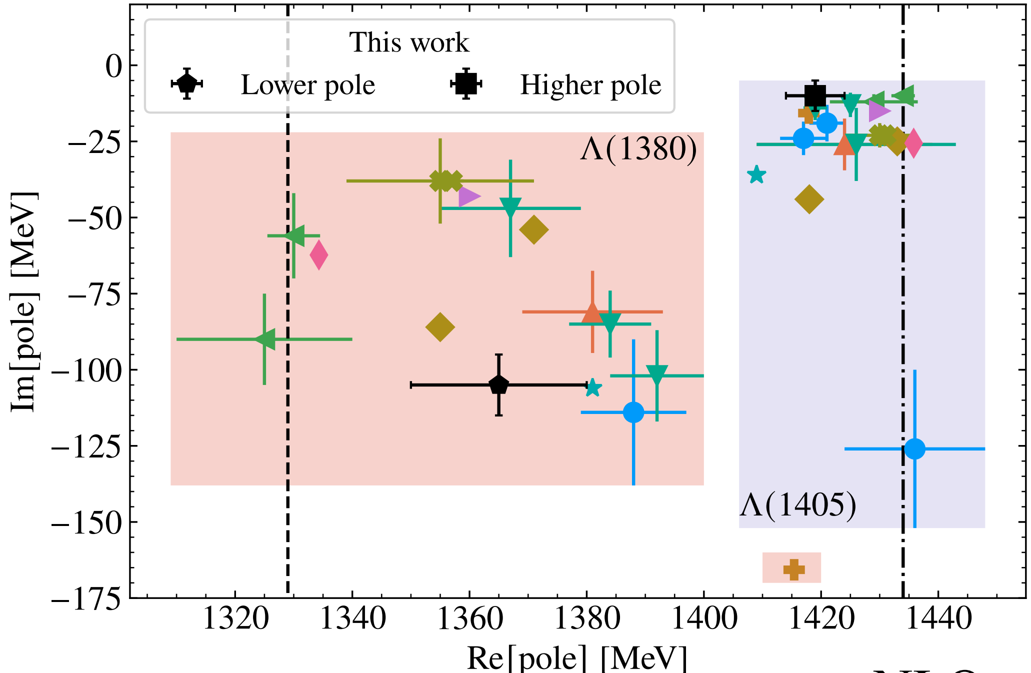
J. Bulava et al, (BaSc), PRD 109, 014511 (2024)

J. Bulava et al, (BaSc), PRL 132, 051901 (2024)

➢ Inelasticity η and phase shift $\delta_{\pi\Sigma}$ and $\delta_{\bar{K}N}$ against center-of mass energy difference to the $\pi\Sigma$ threshold

Pole positions $@m_{\pi,\text{phy}}$

Two-coupled channel
Up to NLO (S-wave)



[11] Guo et al, PRC 87, 035202 (2013)
[10] Y. Ikeda et al, NPA 881, 98 (2012)
[23] M. Mai et al, EPJA 51, 30 (2015)
[14] D. Sadasivan et al, PLB 789, 329 (2019)
[72] A. Cieply et al, NPA 881, 115 (2012)
[73] N. V. Shevchenko, PRC 85, 034001 (2012)
[74] J. Haidenbauer et al, EPJA 47, 18 (2011)
[55] F.-K. Guo et al, PLB 846, 138264 (2023)
[75] D. S. et al, Front. Phys 11, 1139236 (2023)
[35] J.-X. Lu et al, PRL 130, 071902 (2023)

NLO: [11] [10] [23] [14] [72] [73] [74] [55] [75]
NNLO: [35]

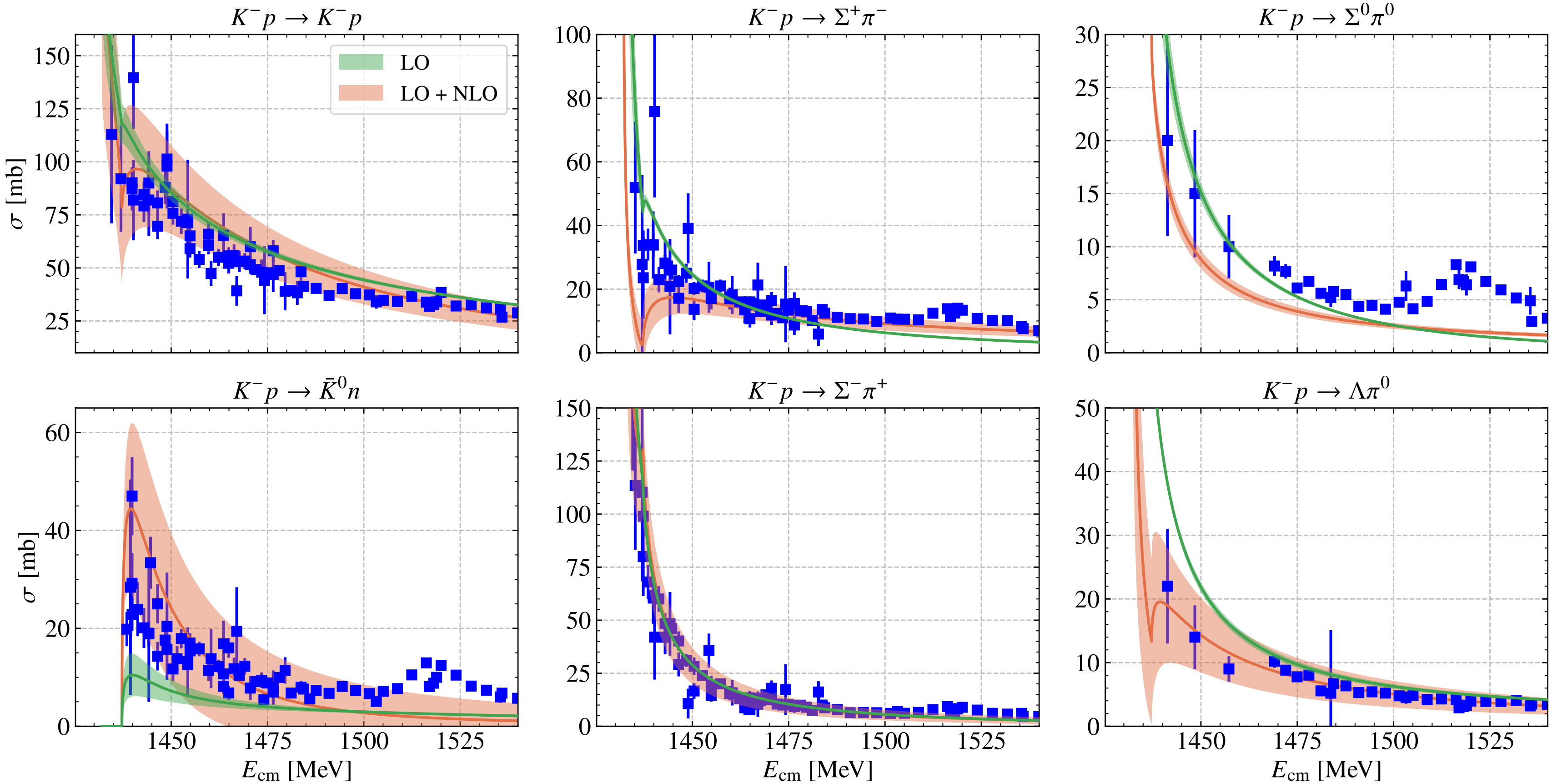
- $a_{\pi\Sigma} = -2.33$, $a_{\bar{K}N} = -2.07$

	z_1	z_2
Pole [MeV]	$(1365 \pm 15, 105 \pm 10)$	$(1419 \pm 5, 10 \pm 5)$
$\lvert g_{\pi\Sigma}/g_{\bar{K}N}\rvert$	1.5 ± 0.6	0.3 ± 0.1

- The positions of the poles agree remarkable well with prevoius experimental data analysis

Compatible with experimental data?

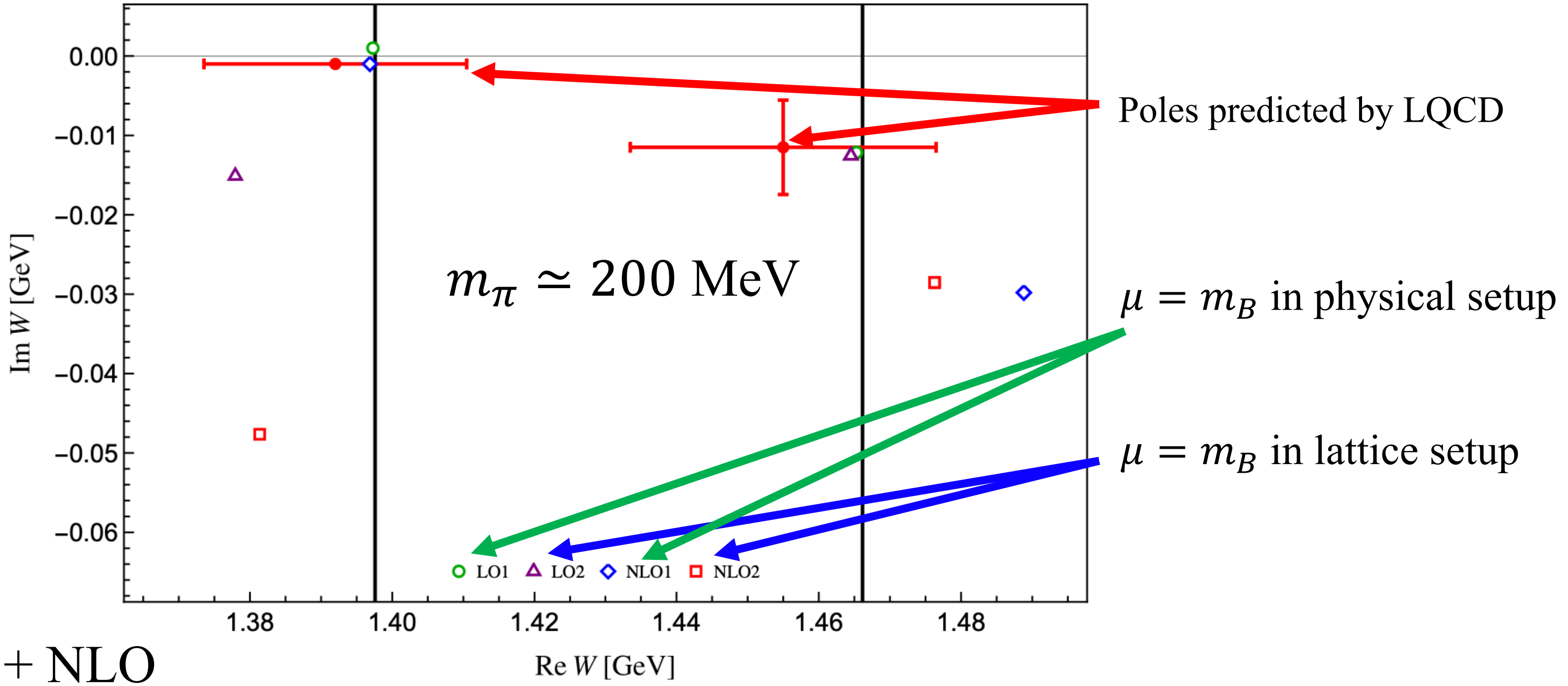
@WT term + NLO (S-wave)



- ➢ The LECs b_0, b_D, b_F, d_i and the $q'_{\max}$ are fixed by **the LQCD**
- ➢ Up to NLO, the model is compatible with the experimental data well

Example I: motion of the poles

F.-K Guo et al, PLB 846, 138264 (2023)



- WT term + NLO

The LECs fixed by experimental data

- $G_{DR}(s = m_N^2, a(m_B)) = 0 \Rightarrow a$

$B \in \{N, \Lambda, \Sigma, \Xi\}$

M.F.M. Lutz et al, Nucl. Phys. A 700(2002) 193-308

T. H. et al, PRC 78, 025203 (2008)

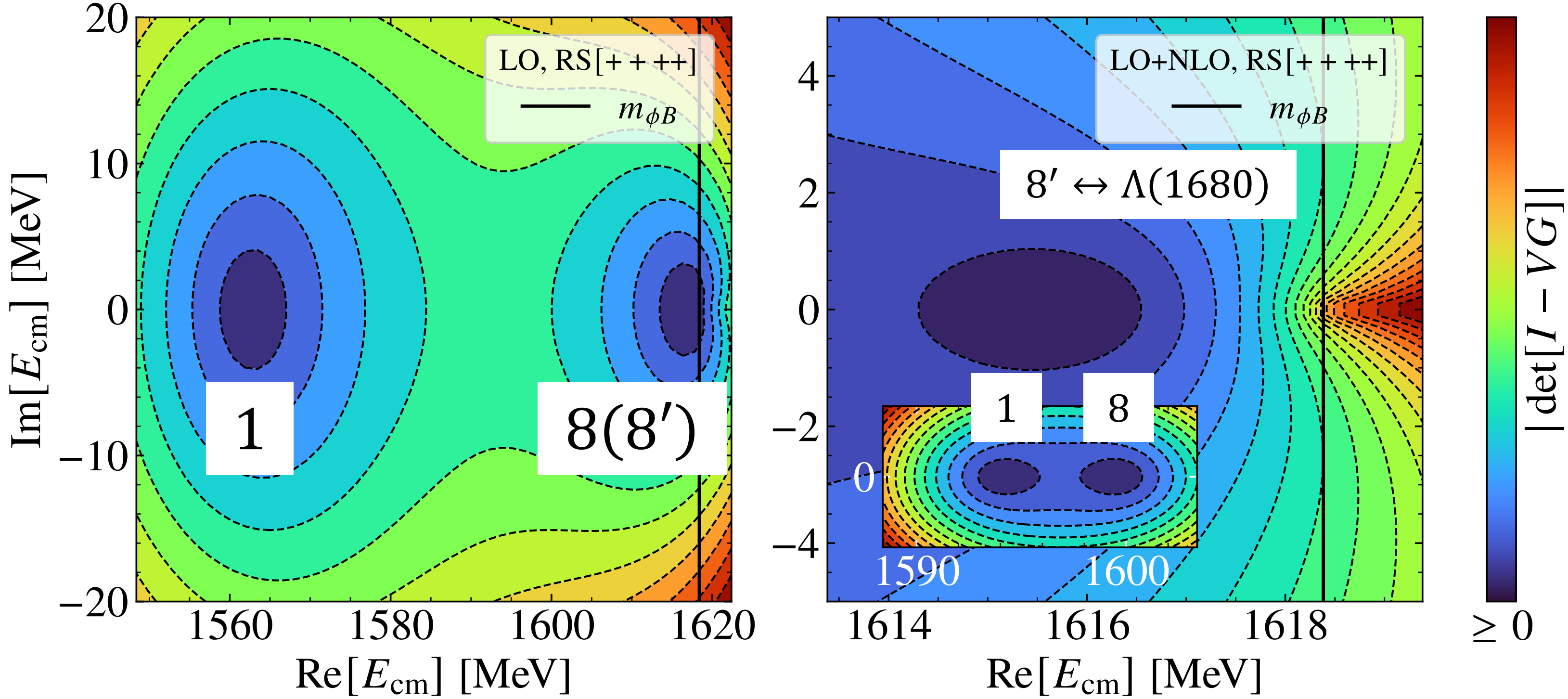
- $a(\mu)$ with $m_\pi = 138\ (200)$ MeV

$a_{\pi\Sigma}$	$a_{\bar{K}N}$	$a_{\eta\Lambda}$	$a_{K\Xi}$
$-0.7(-0.73)$	$-1.15(-1.09)$	$-1.21(-1.19)$	$-1.13(-1.1)$

- $a(\mu)$ are away from the natural value -2

Y. Ikeda et al, PLB 706, 63 (2011)

Pole structures in the SU(3) limits @four-coupled channel



- Two poles on the real axis at LO [MeV]

$E^{(1)}$	$E^{(8(8'))}$
1563 ± 13	1618 ± 2

- The lower pole connected to the 1

- Three poles on the real axis up to NLO [MeV]

$E^{(1)}$	$E^{(8)}$	$E^{(8')}$
1595 ± 8	1600 ± 4	1616 ± 4

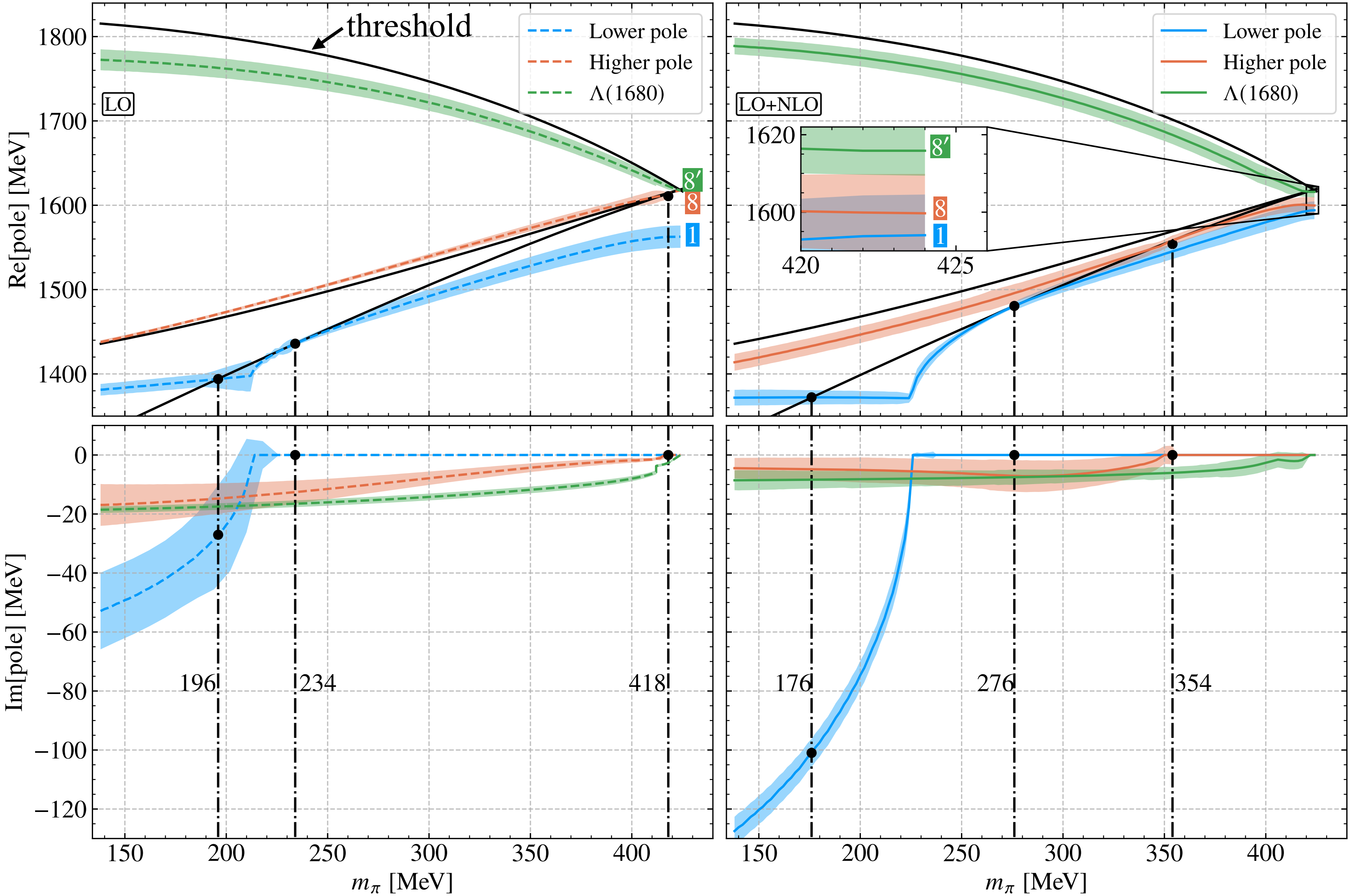
- The higher pole connected to the 8

Summary @LO + NLO (S-wave)

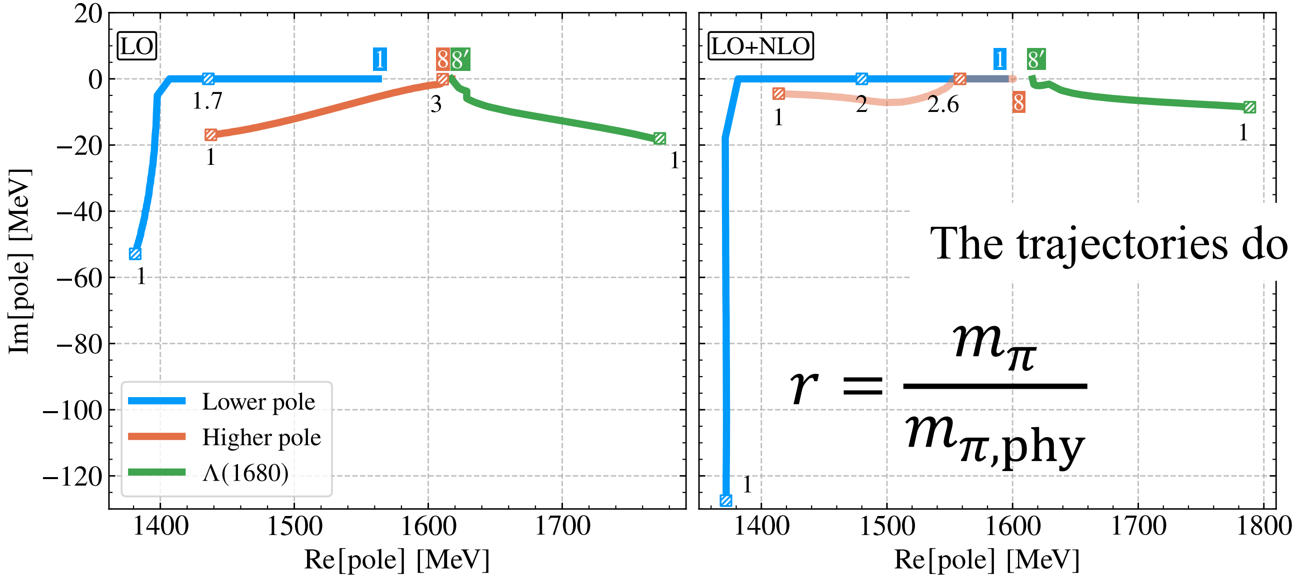
m_π [MeV]	138		200		423		
	z_1	z_2	z_1	z_2	z_1	z_2	z_3
Pole [MeV]	$1365 \pm 15 - i(105 \pm 10)$	$1419 \pm 5 - i(10 \pm 5)$	1378 ± 17	$1454 \pm 8 - i(12 \pm 11)$	1595 ± 8	1600 ± 4	1616 ± 4
$\lvert g_{\pi\Sigma}\rvert$	2.8 ± 0.3	1.0 ± 0.4	4.1 ± 1.3	1.3 ± 0.6	1.7 ± 0.4	0.6 ± 0.3	1.2 ± 0.3
$\lvert g_{\bar{K}N}\rvert$	1.9 ± 0.7	3.0 ± 0.6	1.7 ± 0.8	3.0 ± 0.5	1.4 ± 0.4	2.0 ± 0.4	0.3 ± 0.4
$\left\lvert \frac{g_{\pi\Sigma}}{g_{\bar{K}N}} \right\rvert$	1.5 ± 0.6	0.3 ± 0.1	2.5 ± 1.8	0.4 ± 0.1	1.2 ± 0.4	0.3 ± 0.7	4.0 ± 1.2

- At $m_\pi = 138$ MeV, both of two poles of the $\Lambda(1405)$ are the resonances
- At $m_\pi \simeq 200$ MeV, the lower pole is a virtual state and the higher pole is a resonance
- In the SU(3) limit, three poles are found on the physical sheet in four-coupled scattering

Trajectories of the poles @four-coupled channel

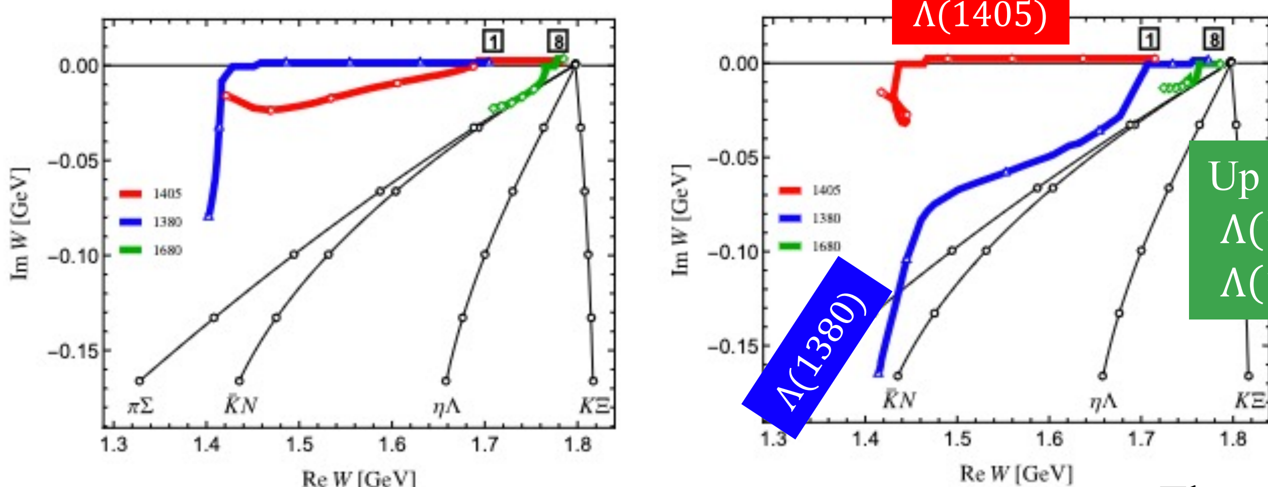


Example II: motion of the poles



The trajectories do not change

$$r = \frac{m_\pi}{m_{\pi,\text{phy}}}$$



Λ(1405)

Λ(1380)

Up to NLO:
$\Lambda(1405) \to E^{(1)}$
$\Lambda(1380) \to E^{(8)}$

F.-K Guo et al, PLB 846, 138264 (2023)

The trajectories change

Conclusion

- In this work, we have conducted an analysis of the LQCD data on $\pi\Sigma - \bar{K}N$ scattering for $I = 0$
- The LECs b_0, b_D, and b_F are fixed by the quark mass dependence of the octet baryon masses
- The result of two poles of the $\Lambda(1405)$ is consistent with LQCD pole extraction at $m_\pi \simeq 200$ MeV and the experimental data analyses at physical pion mass
- At the SU(3) limit, the lower pole belongs to the singlet 1, and the higher pole is connected the octet representation

Thank you for your attention!!!