

Pole trajectories of the $\Lambda(1405)$ helps establish its dynamical nature

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Z. Zhuang, R. M., J.-X. Lu, & L.-S. Geng, arXiv: 2405.07686

11.07.2024

Outline

➤ Introduction

➤ Framework

- Chiral unitary approach
- Quark mass dependence of the hadron masses
- Analysis of the LQCD data

➤ Fitting results and discussion

Introduction

➤ The resonance first appeared in bubble chamber experiments

➤ $\Lambda(1405): J^P = \frac{1}{2}^-, I = 0$ Status: ****

R. H. Dalitz and S. F. Tuan, PRL 2, 425(1959)

R. H. Dalitz and S. F. Tuan, Annals Phys, 10, 307

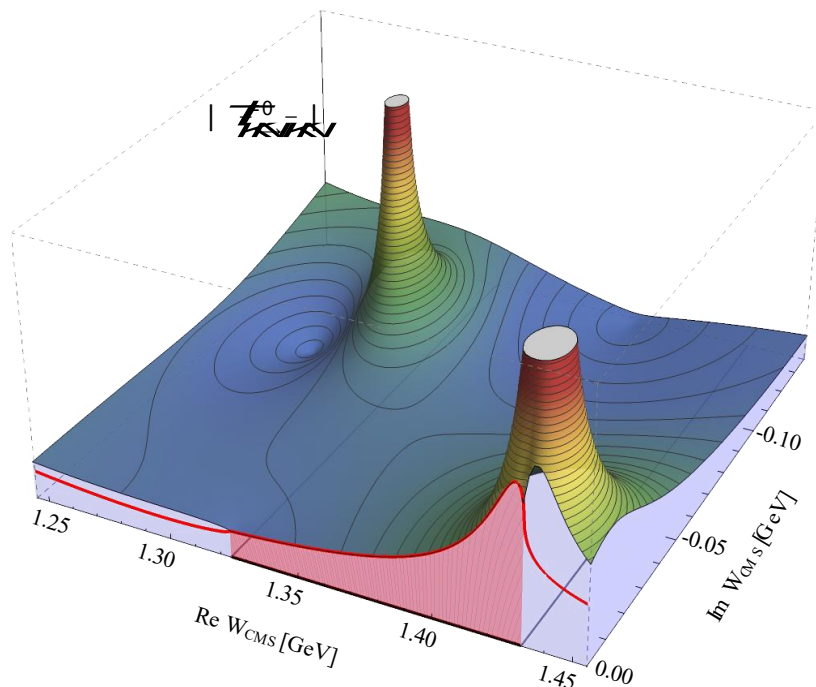
➤ Mass: $1405.1^{+1.3}_{-1}$ MeV, width: 50.5 ± 2 MeV

➤ Two pole structures of $\Lambda(1405)$

- Coupled-channel framework
- Chiral unitary approach ...

➤ PDG added $\Lambda(1380)$ with **

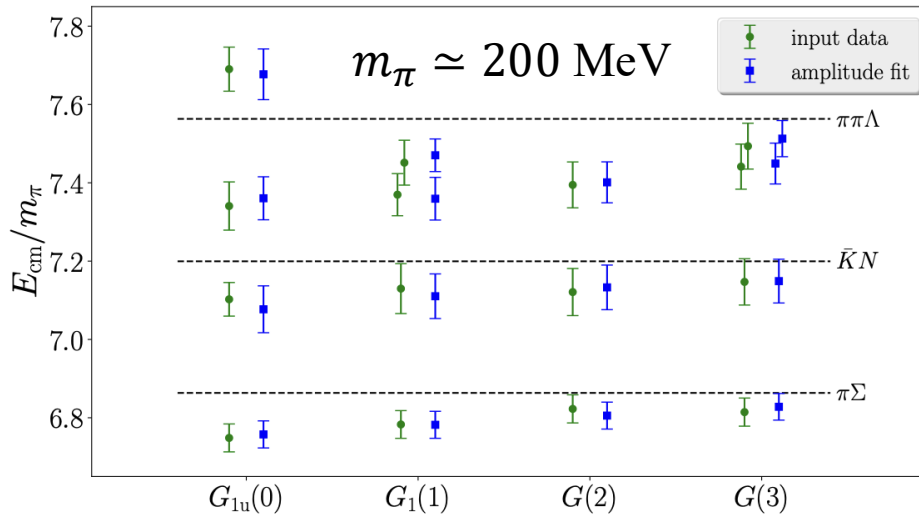
PDG, Prog. Theor. Exp. Phys. 2022, 083C01 (2022)



Maxim Mai, EPJST 230 (2021) 6, 1593-1607

$\Lambda(1405)$ in a recent LQCD simulation

Finite-volume spectrum (error bars in green)



J. Bulava et al, (BaSc), PRD 109, 014511 (2024)

J. Bulava et al, (BaSc), PRL 132, 051901 (2024)

- A chiral extrapolation to the physical point from the analyses of the energy levels has not been done
- The poles over the $\text{Tr}[M] = \text{cte}$ trajectory towards the SU(3) symmetric limit has not been studied

➤ The coupled $\pi\Sigma - \bar{K}N$ scattering

➤ Type of trajectory: $\text{Tr}[M] = \text{cte}$

- $M = \text{diag}(m_u, m_d, m_s)$

➤ $G_i(d^2)$: the little group of $\mathbf{P}^2 = \left(\frac{2\pi}{L}\right)^2 \mathbf{d}^2$

- $\mathbf{d} \in \{[000], [001], [011], [111]\}$

➤ K -matrix parameterization for the $\Lambda(1405)$

➤ Pole positions $E_i = \left(M, \frac{\Gamma}{2}\right)$ [MeV]

- $(1392(9)(2)(16), 0)$ **virtual state**

- $(1455(13)(2)(17), 11.5(4.4)(4)(0.1))$

Motivation

Chiral unitary approach

- The interaction kernel up to NLO $a, b \in \{N, \Lambda, \Sigma, \Xi\}$ $i, j \in \{\pi, K, \eta\}$ $N_a = \sqrt{\frac{M_a + E_a}{2M_a}}$

$$V_{jb,ia} = -\frac{N_b N_a}{f^2} [C_{jb,ia}(2\sqrt{s} - M_b - M_a) - 4(D_{jb,ia} - 2k_\mu k'^\mu L_{jb,ia})]$$

Weinberg-Tomozawa term (WT) NLO term (S-wave)

- $f = (f_\pi(m_\pi) + f_K(m_\pi) + f_\eta(m_\pi)) / 3$

$$C_{jb,ia} = \begin{bmatrix} 4 & -\sqrt{\frac{3}{2}} & 0 & \sqrt{\frac{3}{2}} \\ & 3 & \frac{3}{\sqrt{2}} & 0 \\ & & 0 & -\frac{3}{\sqrt{2}} \\ & & & 3 \end{bmatrix} \quad D_{jb,ia} = \begin{bmatrix} 4(b_0 + b_D)m_\pi^2 & -\sqrt{\frac{3}{2}}(b_D - b_F)\mu_1^2 & -\frac{4b_D m_\pi^2}{\sqrt{3}} & \sqrt{\frac{3}{2}}(b_D + b_F)\mu_1^2 \\ & (2b_0 + 3b_D + b_F)m_K^2 & \frac{(b_D + 3b_F)\mu_2^2}{3\sqrt{2}} & 0 \\ & & \frac{4}{9}(3b_0\mu_3^2 + b_D\mu_4^2) & -\frac{(b_D - 3b_F)\mu_2^2}{3\sqrt{3}} \\ & & & 2(2b_0 + 3b_D - b_F)m_K^2 \end{bmatrix}$$

$$L_{jb,ia} = \begin{bmatrix} -4d_2 + 4d_3 + 2d_4 & \sqrt{\frac{3}{2}}(d_1 + d_2 - 2d_3) & -\sqrt{3}d_3 & \sqrt{\frac{3}{2}}(d_1 - d_2 + 2d_3) \\ & d_1 + 3d_2 + 2(d_3 + d_4) & \frac{d_1 - 3d_2 + 2d_3}{\sqrt{2}} & 6d_2 - 3d_3 \\ & & 2(d_3 + d_4) & \frac{d_1 + 3d_2 - 2d_3}{\sqrt{2}} \\ & & & -d_1 + 3d_2 + 2(d_3 + d_4) \end{bmatrix}$$

Interaction in the SU(3) limit

- Projecting the two-particle isoscalar states to the relevant multiplets

$$\begin{pmatrix} |\pi\Sigma\rangle \\ |\bar{K}N\rangle \\ |\eta\Lambda\rangle \\ |K\Xi\rangle \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} \sqrt{15} & -\sqrt{24} & 0 & -1 \\ -\sqrt{10} & -2 & \sqrt{20} & -\sqrt{6} \\ -\sqrt{5} & -\sqrt{8} & 0 & 3\sqrt{3} \\ \sqrt{10} & 2 & 2\sqrt{5} & \sqrt{6} \end{pmatrix} \begin{pmatrix} |1\rangle \\ |8\rangle \\ |8'\rangle \\ |27\rangle \end{pmatrix}$$

- Example: $C_{jb,ia}$ in irreps. representation

$$C_{\alpha\beta} = \begin{pmatrix} \textcircled{6} & 0 & 0 & 0 \\ 0 & \textcircled{3} & 0 & 0 \\ 0 & 0 & \textcircled{3} & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad \text{for } \alpha, \beta \in \{1, 8, 8', 27\}$$

$\nearrow \Lambda(1380)$
 $\nearrow \Lambda(1405)$
 $\nearrow \Lambda(1680)$

1: singlet

8: symmetric octet representation

8': antisymmetric octet representation

Model extrapolation to unphysical point

- Bethe-Salpeter equation

$$T_{ij} = V_{ij} + V_{ik} G_k^{DR} T_{kj}$$

- Loop function $G_k^{DR}(s, a(\mu))$

$$G_k^{DR}(s, a(\mu)) = \frac{2M_k}{16\pi^2} \left\{ a_k(\mu) + \ln \frac{m_k^2}{\mu^2} \frac{M_k^2 - m_k^2 + s}{2s} \ln \frac{M_k^2}{m_k^2} + \log \text{ terms} \right\}$$

- Framework of the subtraction constant $a(\mu)$

$$\mu = 630 \text{ MeV}$$

J. A. Oller, Prog. Part. Nucl. Phys. 110, 103728 (2020)

E. Oset et al, Nucl. Phys. A 635, 99 (1998)

$$a(\mu) = -\frac{2}{m_1 + m_2} \left[m_1 \log \left(1 + \sqrt{1 + \frac{m_1^2}{q_{\max}'^2}} \right) + m_2 \log \left(1 + \sqrt{1 + \frac{m_2^2}{q_{\max}'^2}} \right) \right] + 2 \log \frac{\mu}{q_{\max}'}$$

- $q'_{\max}$ as free parameter fixed by LQCD energy levels

the cutoff: $q'_{\max}$

Model extrapolation to unphysical point

➤ Masses of the NG bosons

$$m_\pi^2 = M_{0\pi}^2 \left[1 + \mu_\pi - \frac{\mu_\eta}{3} + \frac{16M_{0K}^2}{f_0^2} (2L_6^r - L_4^r) \right] + \frac{8M_{0\pi}^2}{f_0^2} (2L_6^r + 2L_8^r - L_4^r - L_5^r) ,$$

$$m_K^2 = M_{0K}^2 \left[1 + \frac{2\mu_\eta}{3} + \frac{8M_{0\pi}^2}{f_0^2} (2L_6^r - L_4^r) \right] + \frac{8M_{0K}^2}{f_0^2} (4L_6^r + 2L_8^r - 2L_4^r - L_5^r) ,$$

$$m_\eta^2 = M_{0\eta}^2 \left[1 + 2\mu_K - \frac{4}{3}\mu_\eta + \frac{8M_{0\eta}^2}{f_0^2} (2L_8^r - L_5^r) + \frac{8}{f_0^2} (2M_{0K}^2 + M_{0\pi}^2) (2L_6^r - L_4^r) \right] \\ + M_{0\pi}^2 \left[-\mu_\pi + \frac{2}{3}\mu_K + \frac{1}{3}\mu_\eta \right] + \frac{128}{9f_0^2} (M_{0K}^2 - M_{0\pi}^2)^2 (3L_7^r + L_8^r) ,$$

Gell-Mann-Okubo relation

$$4M_{0K}^2 = 3M_{0\eta}^2 + M_{0\pi}^2$$

$$\mu_P = \frac{M_{0P}^2}{32\pi^2 f_0^2} \log \frac{M_{0P}^2}{\mu_r^2}, \quad P = \pi, K, \eta$$

$\mu_r = 770 \text{ MeV}$, $f_0 = 80 \text{ MeV}$, and the LECs L_i^r from R. M. et al, JHEP 11, 017 (2020)

Model extrapolation to unphysical point

➤ Pion mass dependence of the decay constants

$$f_\pi = f_0 \left[1 - 2\mu_\pi - \mu_K + \frac{4M_{0\pi}^2}{f_0^2} (L_4^r + L_5^r) + \frac{8M_{0K}^2}{f_0^2} L_4^r \right]$$

$$f_K = f_0 \left[1 - \frac{3\mu_\pi}{4} - \frac{3\mu_K}{2} - \frac{3\mu_\eta}{4} + \frac{4M_{0\pi}^2}{f_0^2} L_4^r + \frac{4M_{0K}^2}{f_0^2} (2L_4^r + L_5^r) \right],$$

$$f_\eta = f_0 \left[1 - 3\mu_K + \frac{4L_4^r}{f_0^2} (M_{0\pi}^2 + 2M_{0K}^2) + \frac{4M_{0\eta}^2}{f_0^2} L_5^r \right]$$

R. M. et al, JHEP 11, 017 (2020)

Model extrapolation to unphysical point

- The octet baryon masses using one-loop NLO covariant baryon ChPT

$$m_B = m_0 + m_B^{(2)} + m_B^{(3)}$$

- m_0 is the baryon mass in the chiral limit

- $m_B^{(2)} = -\sum_{\phi=\pi,K} \xi_{B,\phi}^{(a)} m_\phi^2$ $\xi_{B,\phi}^{(a)} = \xi_{B,\phi}^{(a)}(b_0, b_D, b_F)$

- $m_B^{(3)} = \sum_{\phi=\pi,K,\eta} \frac{1}{(4\pi f_\phi)^2} \xi_{B,\phi}^{(b)} H_B^{(b)}(m_\phi)$ $\xi_{B,\phi}^{(b)} = \xi_{B,\phi}^{(b)}(D, F)$

$$D = 0.8, F = 0.46$$

$$H_B^{(b)}(m_\phi) \text{ is EOMS loop-function}$$

B. Borasoy, PRD 59, 054021 (1999)

- m_0, b_0, b_D , and b_F as the **free parameters** fitting to baryon masses from CLS ensemble

G. S. Bali et al, JHEP 05, 035 (2023)

Finite volume effect

- Bethe-Salpeter equation in a finite volume (FV)

$$\tilde{T} = \frac{V}{I - V\tilde{G}}$$

- E^{FV} in each irrep as a solution of the eigenvalue problem

$$\det[I - V\tilde{G}] = 0$$

- $\tilde{G}$ in the dimensional regularization scheme

A. M. T. et al, PRD 85, 014027 (2012)

$$\tilde{G} = G^{DR}(s, a(\mu)) - \lim_{q_{\max} \rightarrow \infty} (\tilde{G}(P, q_{\max}) - G^{\text{cutoff}}(s, q_{\max}))$$

$$\tilde{G}(P, q_{\max}) = \frac{1}{L^3} \sum_n^{q_{\max}} \frac{E}{P_0} I(q^*)$$

$$I(\vec{q}) = \frac{\omega_1(q) + \omega_2(q)}{2\omega_1(q)\omega_2(q) [P_0^2 - (\omega_1(q) + \omega_2(q))^2 + i\epsilon]}$$

$$\omega_i = \sqrt{q^2 + m_i^2}, \text{ and } q = |\vec{q}|$$

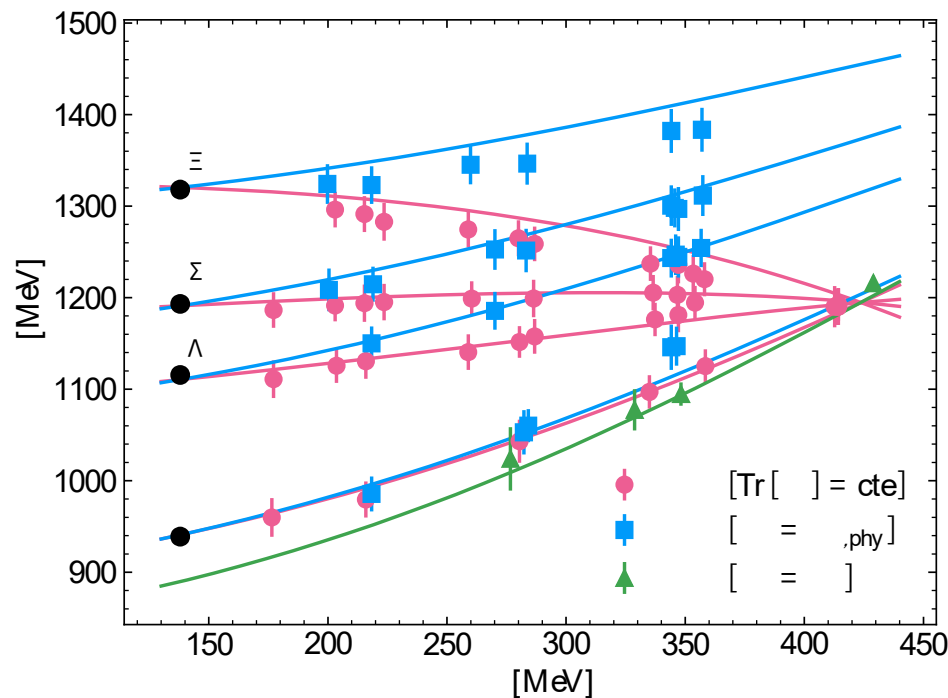
q^* the center-of-mass (CM) frame momentum

Fit to the baryon masses in LQCD

- Global fit to the baryon masses with three types of the trajectories

$$\text{Tr}[M] = \text{cte}, m_s = m_{s,\text{phy}}, \text{ and } m_s = m_l \ (l = u, d)$$

$$M = \text{diag}(m_u, m_d, m_s)$$



Data from G. S. Bali et al, JHEP 05, 035 (2023)

- LECs [GeV^{-1}]

$$b_0 = -0.65(1), b_D = 0.070(1)$$

$$b_F = -0.380(1)$$

- Baryon mass in the chiral limit

$$m_0 = 840 \pm 12 \text{ MeV}$$

- SU(3) limit: $m_\pi = 423 \text{ MeV}$

Fit to the energy levels

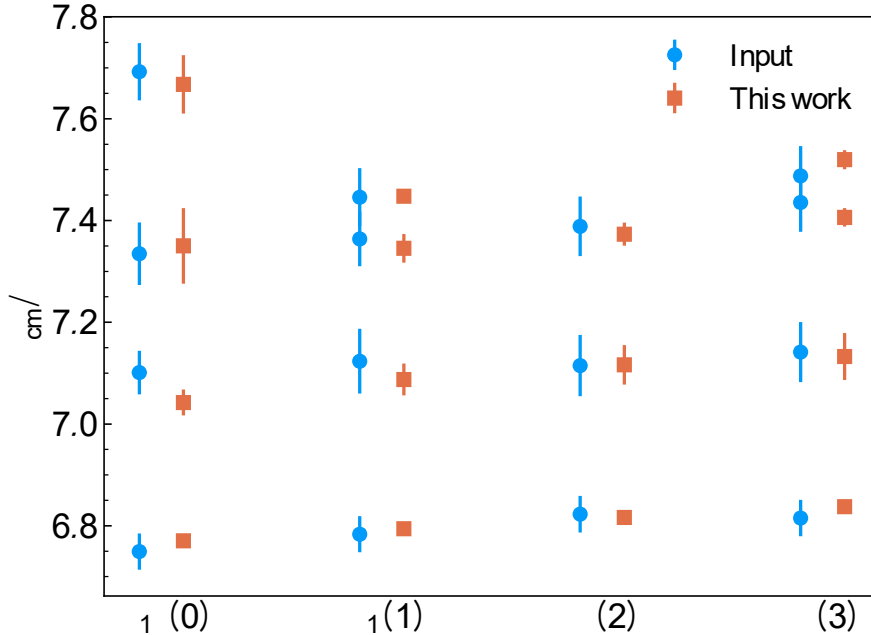
@LO+NLO (S -wave)

@Coupled $\pi\Sigma - \bar{K}N$ scattering

- Finite-volume spectrum used input to constrain the d_i and the $q'_{\max}$

The best fit: $\chi^2_{\text{d.o.f}} = 2.2$

$i = 1, 4$



J. Bulava et al, (BaSc), PRD 109, 014511 (2024)

J. Bulava et al, (BaSc), PRL 132, 051901 (2024)

- $G_i(d^2)$: irrep. representation

$G_{1u}(0)$: [000], $G_1(1)$: [001]

$G(2)$: [011], $G(3)$: [111]

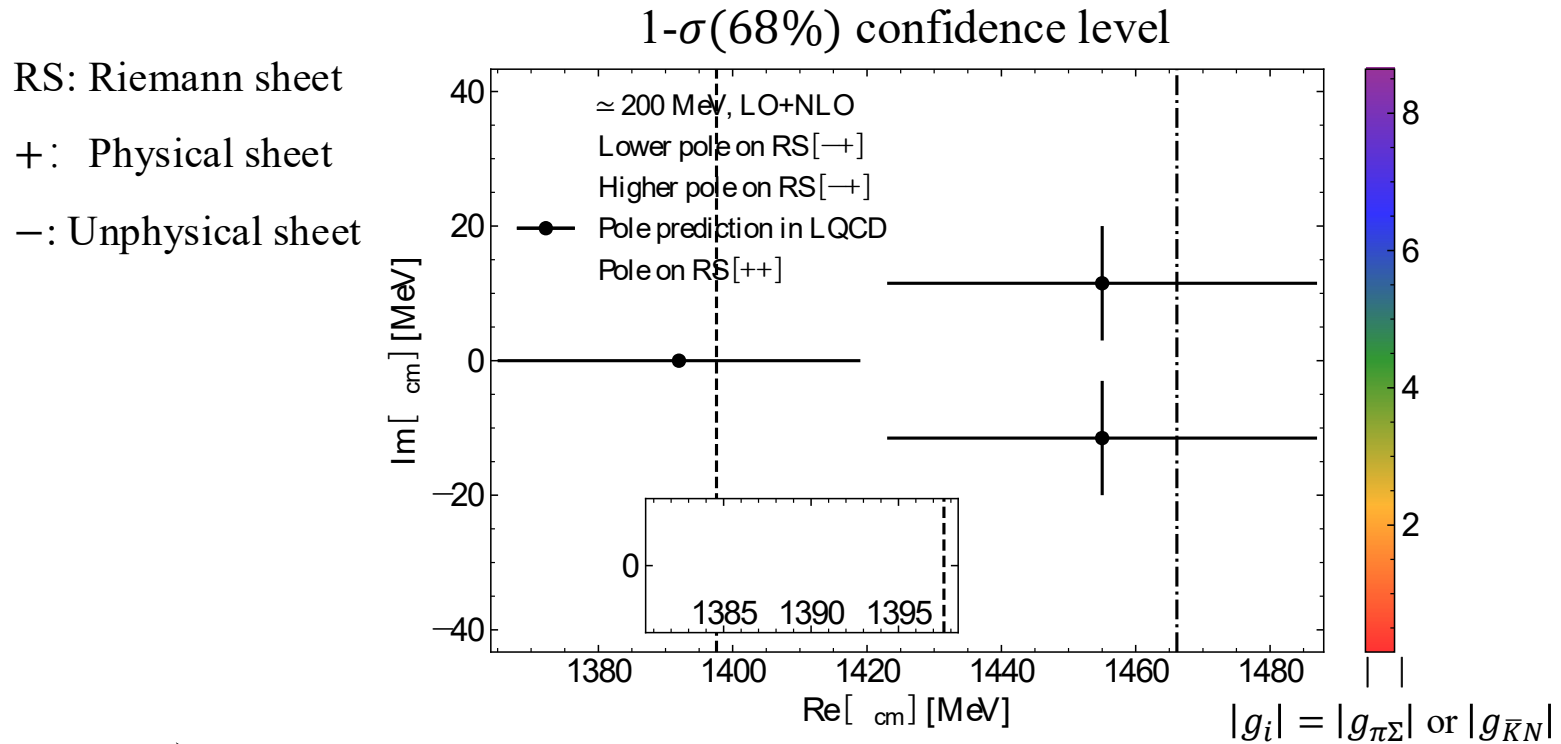
- $\chi^2 = \Delta E^T C^{-1} \Delta E$

$$\Delta E = E_{\text{lattice}} - E_{\text{th}}$$

C is the covariance matrix

d_1 [GeV $^{-1}$]	d_2 [GeV $^{-1}$]	d_3 [GeV $^{-1}$]	d_4 [GeV $^{-1}$]	$q'_{\max}$ [MeV]
-0.4 ± 0.7	0.024 ± 0.001	-0.1 ± 0.4	-0.6 ± 0.7	711 ± 90

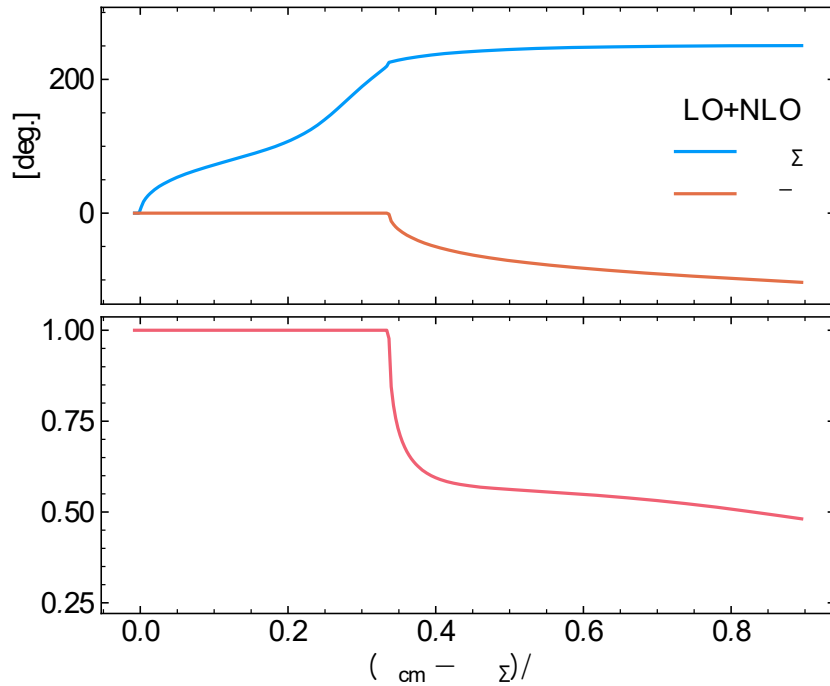
Pole positions @ $m_\pi \simeq 200$ MeV



- $a_{\pi\Sigma} = -2.3, a_{\bar{K}N} = -2.1$
- The pole positions ($M, \Gamma/2$) and the ratios of the couplings

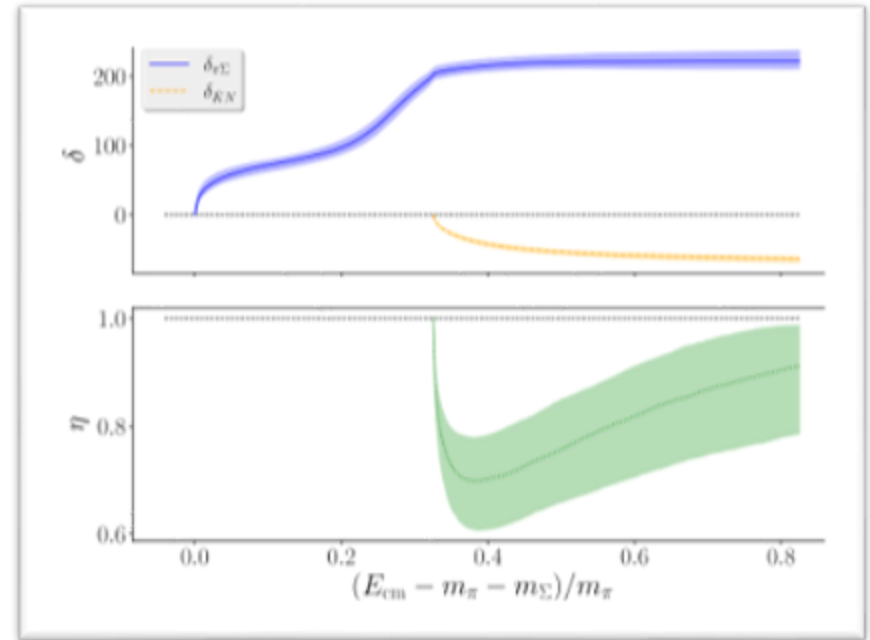
	z_1	z_2
Pole [MeV]	$(1378 \pm 17, 0)$	$(1454 \pm 8, 12 \pm 11)$
$ g_{\pi\Sigma} / g_{\bar{K}N} $	2.5 ± 1.8	0.4 ± 0.1

Phase shift @LO+NLO (S -wave)



Our result

VS



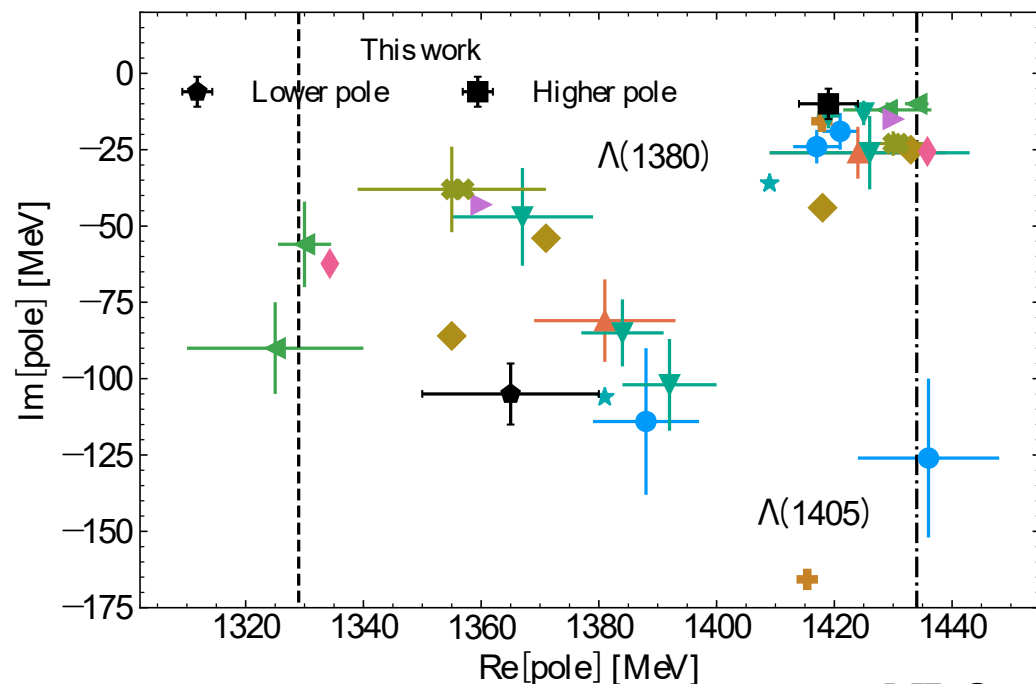
J. Bulava et al, (BaSc), PRD 109, 014511 (2024)

J. Bulava et al, (BaSc), PRL 132, 051901 (2024)

➤ Inelasticity η and phase shift $\delta_{\pi\Sigma}$ and $\delta_{\bar{K}N}$ against center-of mass energy difference to the $\pi\Sigma$ threshold

Pole positions @ $m_{\pi,\text{phy}}$

Two-coupled channel
Up to NLO (S-wave)



- [11] Guo et al, PRC 87, 035202 (2013)
- [10] Y. Ikeda et al, NPA 881, 98 (2012)
- [23] M. Mai et al, EPJA 51, 30 (2015)
- [14] D. Sadasivan et al, PLB 789, 329 (2019)
- [72] A. Cieply et al, NPA 881, 115 (2012)
- [73] N. V. Shevchenko, PRC 85, 034001 (2012)
- [74] J. Haidenbauer et al, EPJA 47, 18 (2011)
- [55] F.-K. Guo et al, PLB 846, 138264 (2023)
- [75] D. S. et al, Front. Phys 11, 1139236 (2023)
- [35] J.-X. Lu et al, PRL 130, 071902 (2023)

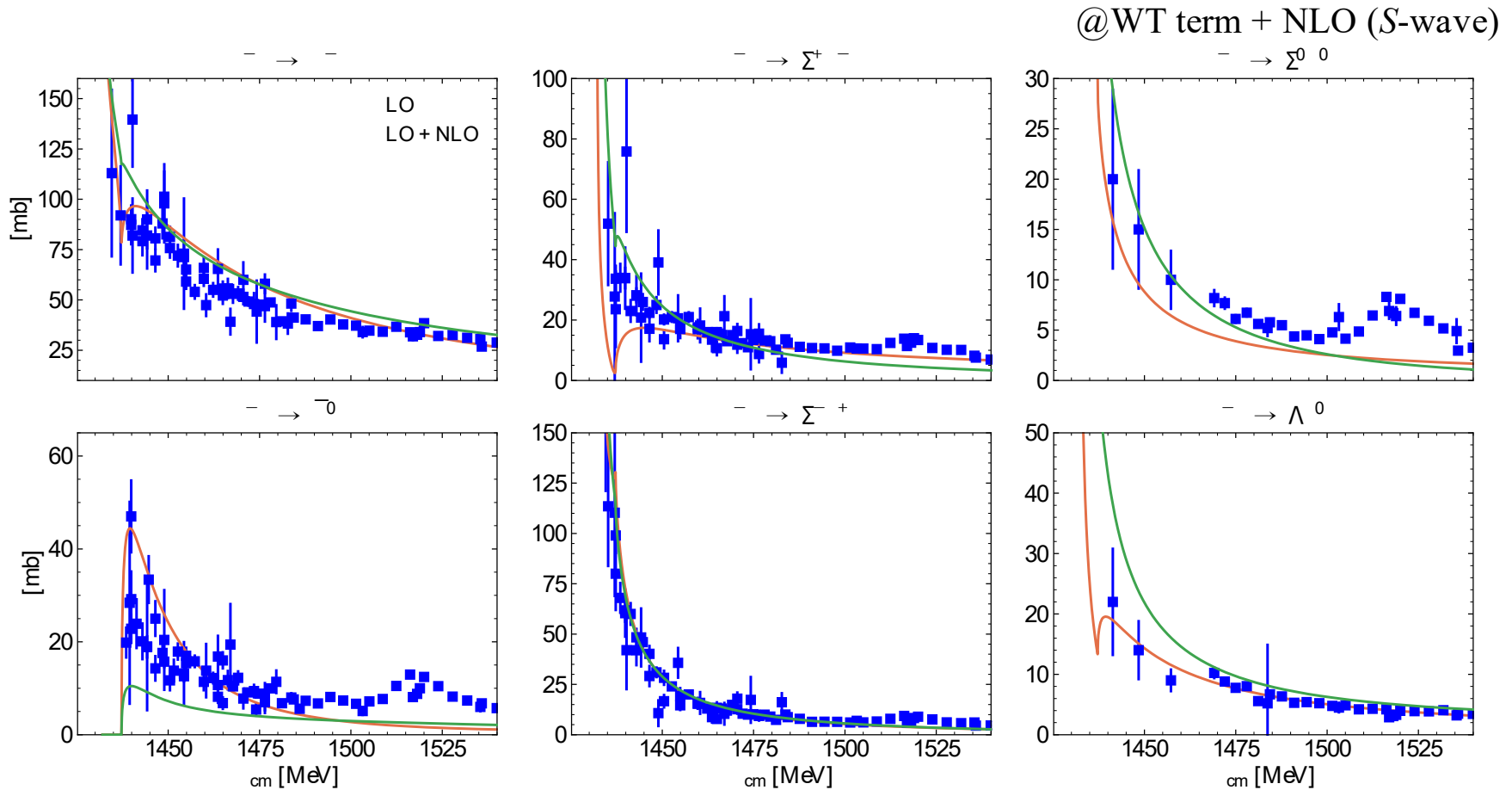
➤ $a_{\pi\Sigma} = -2.33, a_{\bar{K}N} = -2.07$

NLO: [11] [10] [23] [14] [72] [73] [74] [55] [75]
 NNLO: [35]

	Z_1	Z_2
Pole [MeV]	$(1365 \pm 15, 105 \pm 10)$	$(1419 \pm 5, 10 \pm 5)$
$ g_{\pi\Sigma}/g_{\bar{K}N} $	1.5 ± 0.6	0.3 ± 0.1

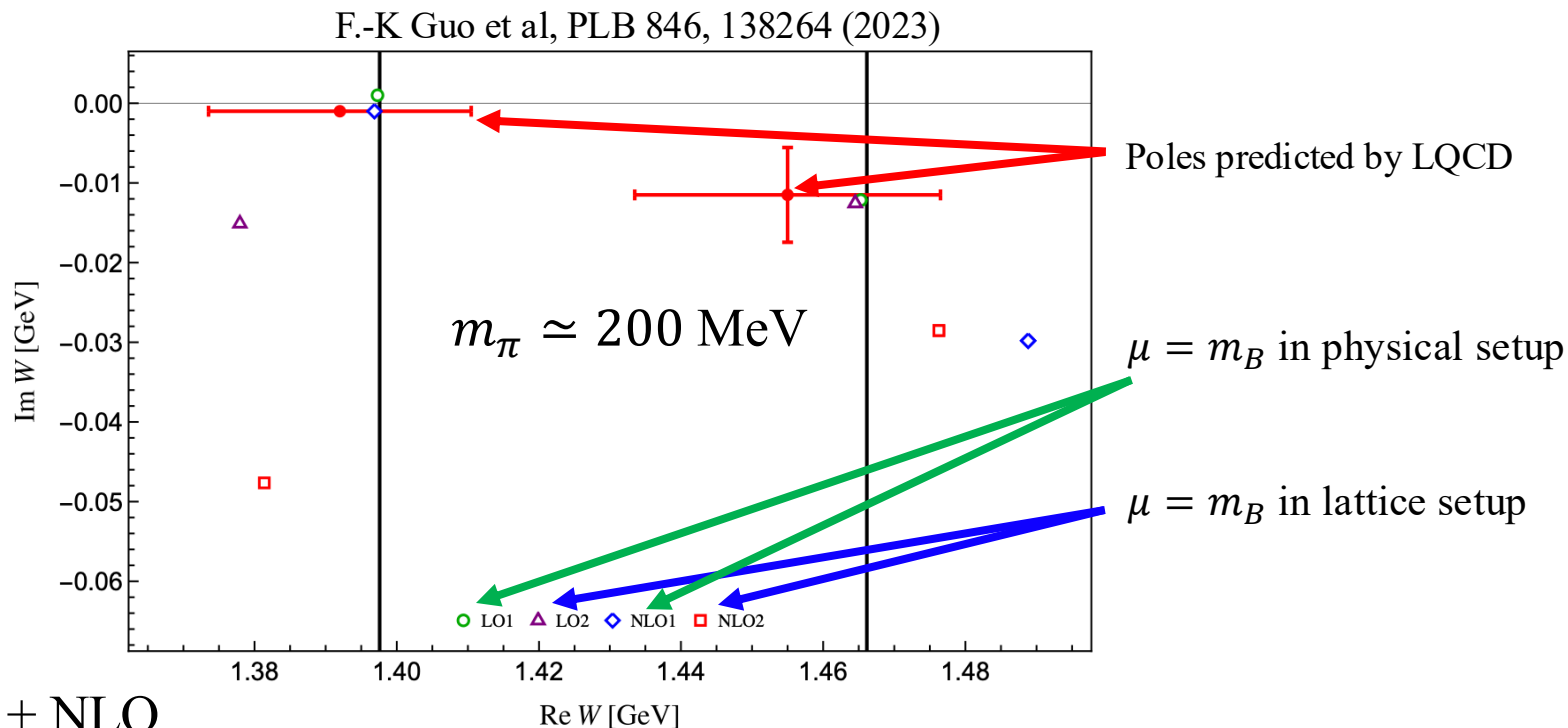
➤ The positions of the poles agree remarkable well with prevoius experimental data analysis

Compatible with experimental data?



- The LECs b_0, b_D, b_F, d_i and the $q'_{\max}$ are fixed by **the LQCD**
- Up to NLO, the model is compatible with the experimental data well

Example I: motion of the poles



➤ WT term + NLO

The LECs fixed by experimental data

➤ $G_{DR}(s = m_N^2, a(m_B)) = 0 \Rightarrow a$
 $B \in \{N, \Lambda, \Sigma, \Xi\}$

M.F.M. Lutz et al, Nucl. Phys. A 700(2002) 193-308

T. H. et al, PRC 78, 025203 (2008)

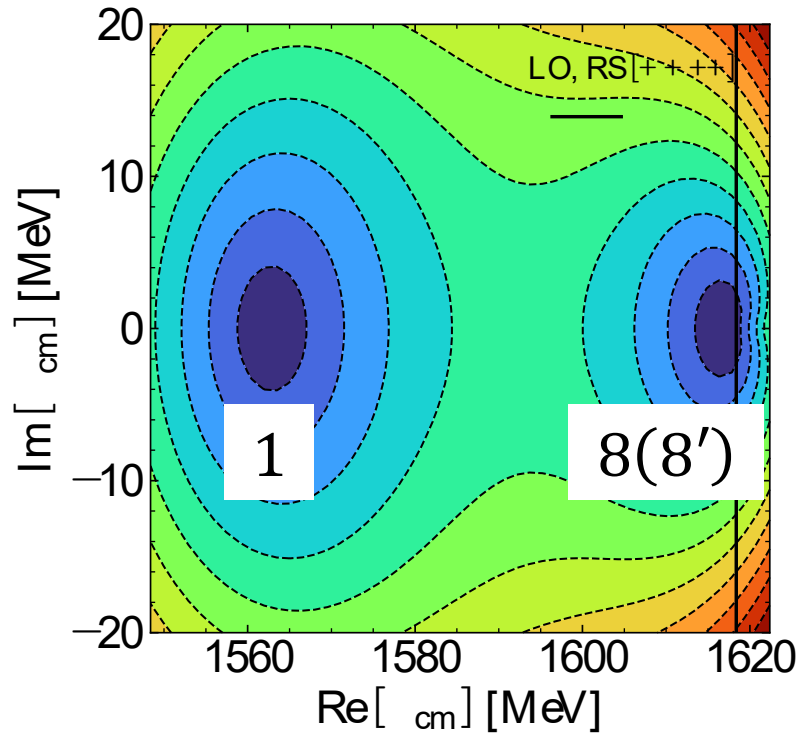
➤ $a(\mu)$ with $m_\pi = 138$ (200) MeV

$a_{\pi\Sigma}$	$a_{\bar{K}N}$	$a_{\eta\Lambda}$	$a_{K\Xi}$
-0.7(-0.73)	-1.15(-1.09)	-1.21(-1.19)	-1.13(-1.1)

➤ $a(\mu)$ are away from the natural value -2

Y. Ikeda et al, PLB 706, 63 (2011)

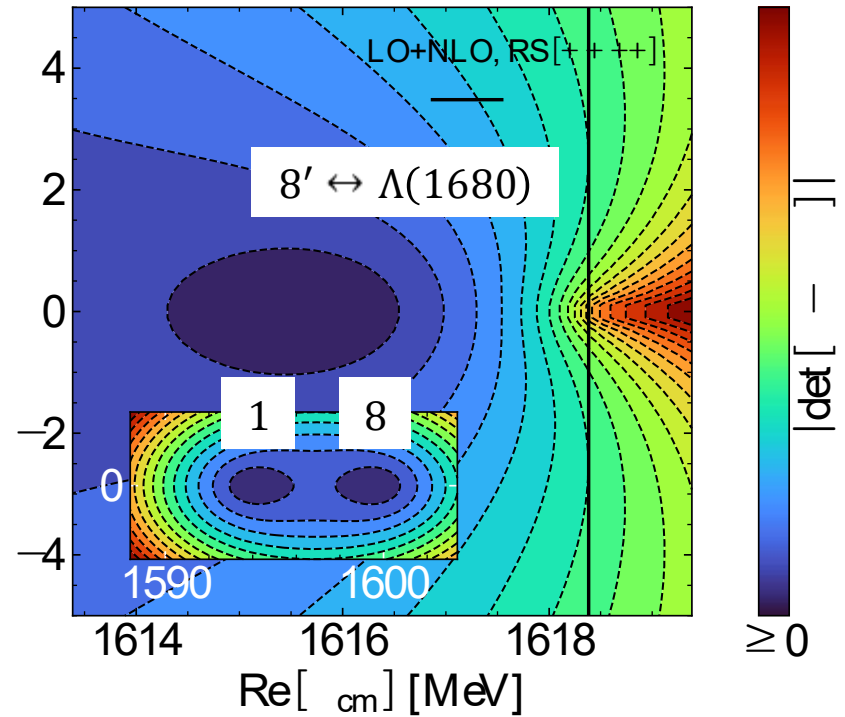
Pole structures in the SU(3) limits @four-coupled channel



- Two poles on the real axis at LO [MeV]

$E^{(1)}$	$E^{(8(8'))}$
1563 ± 13	1618 ± 2

- The lower pole connected to the 1



- Three poles on the real axis up to NLO [MeV]

$E^{(1)}$	$E^{(8)}$	$E^{(8')}$
1595 ± 8	1600 ± 4	1616 ± 4

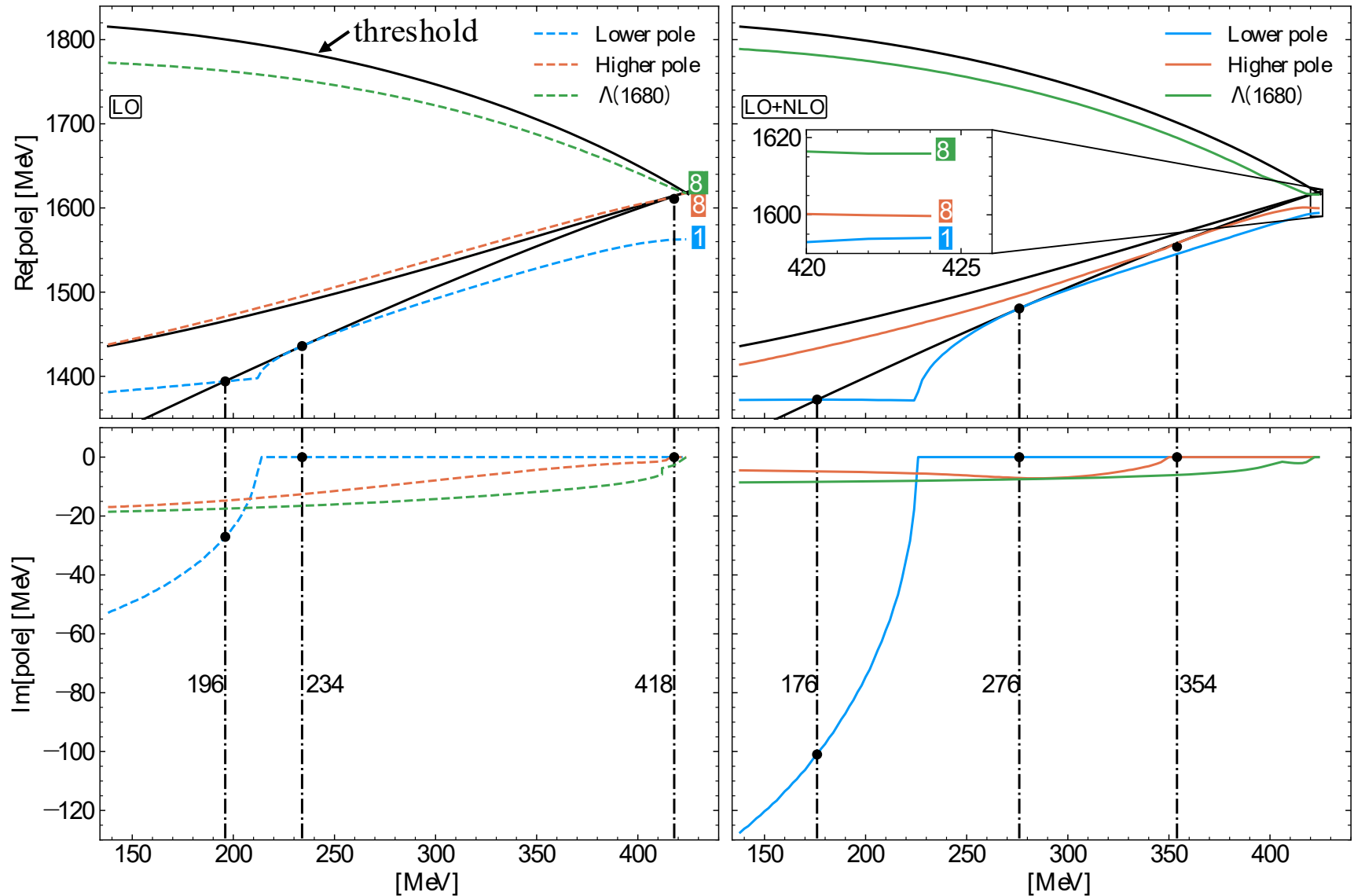
- The higher pole connected to the 8

Summary @LO + NLO (S -wave)

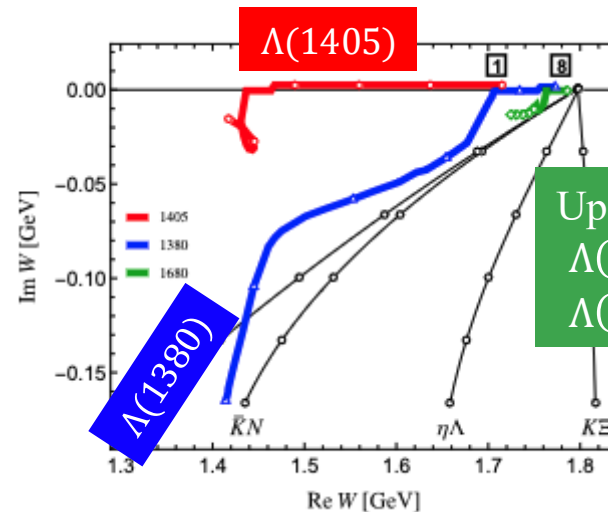
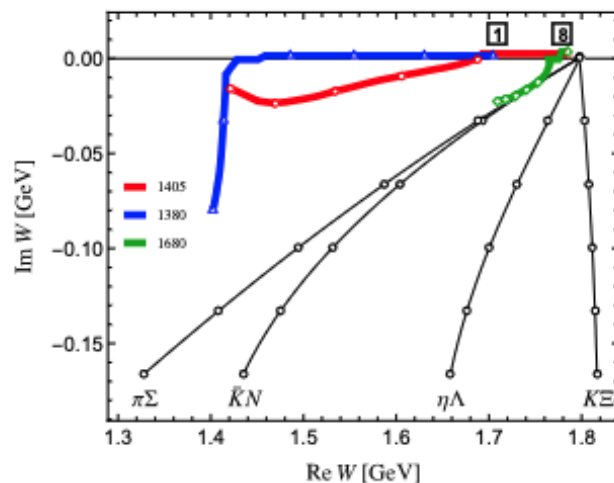
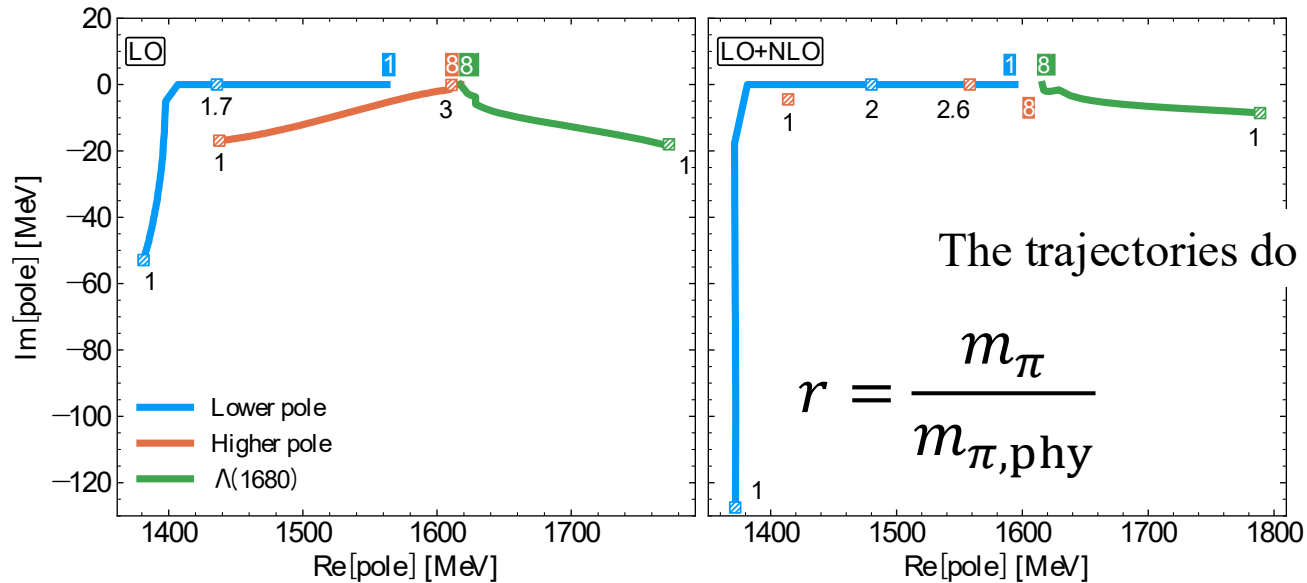
m_π [MeV]	138		200		423		
	z_1	z_2	z_1	z_2	z_1	z_2	z_3
Pole [MeV]	$1365 \pm 15 - i(105 \pm 10)$	$1419 \pm 5 - i(10 \pm 5)$	1378 ± 17	$1454 \pm 8 - i(12 \pm 11)$	1595 ± 8	1600 ± 4	1616 ± 4
$ g_{\pi\Sigma} $	2.8 ± 0.3	1.0 ± 0.4	4.1 ± 1.3	1.3 ± 0.6	1.7 ± 0.4	0.6 ± 0.3	1.2 ± 0.3
$ g_{\bar{K}N} $	1.9 ± 0.7	3.0 ± 0.6	1.7 ± 0.8	3.0 ± 0.5	1.4 ± 0.4	2.0 ± 0.4	0.3 ± 0.4
$\left \frac{g_{\pi\Sigma}}{g_{\bar{K}N}} \right $	1.5 ± 0.6	0.3 ± 0.1	2.5 ± 1.8	0.4 ± 0.1	1.2 ± 0.4	0.3 ± 0.7	4.0 ± 1.2

- At $m_\pi = 138$ MeV, both of two poles of the $\Lambda(1405)$ are the resonances
- At $m_\pi \simeq 200$ MeV, the lower pole is a virtual state and the higher pole is a resonance
- In the SU(3) limit, three poles are found on the physical sheet in four-coupled scattering

Trajectories of the poles @four-coupled channel



Example II: motion of the poles



F.-K Guo et al, PLB 846, 138264 (2023)

The trajectories change

Conclusion

- In this work, we have conducted an analysis of the LQCD data on $\pi\Sigma - \bar{K}N$ scattering for $I = 0$
- The LECs b_0, b_D , and b_F are fixed by the quark mass dependence of the octet baryon masses
- The result of two poles of the $\Lambda(1405)$ is consistent with LQCD pole extraction at $m_\pi \simeq 200$ MeV and the experimental data analyses at physical pion mass
- At the SU(3) limit, the lower pole belongs to the **singlet 1**, and the higher pole is connected the **octet representation**

Thank you for your attention!!!