Dust column densities: errors, estimates, and error estimates

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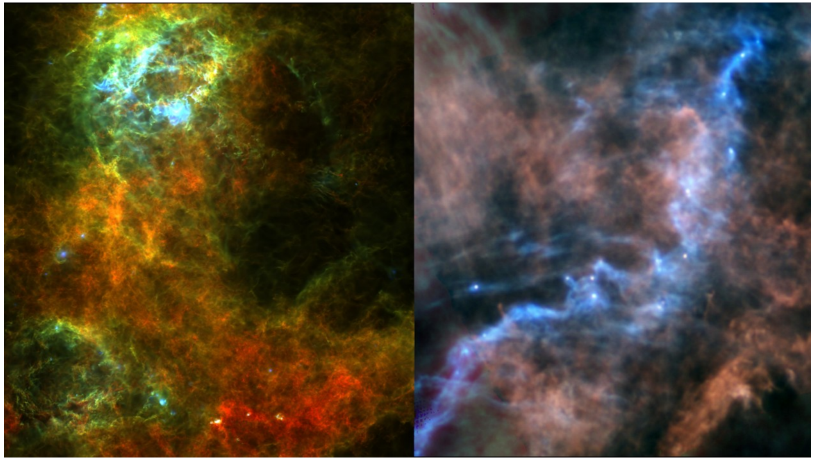
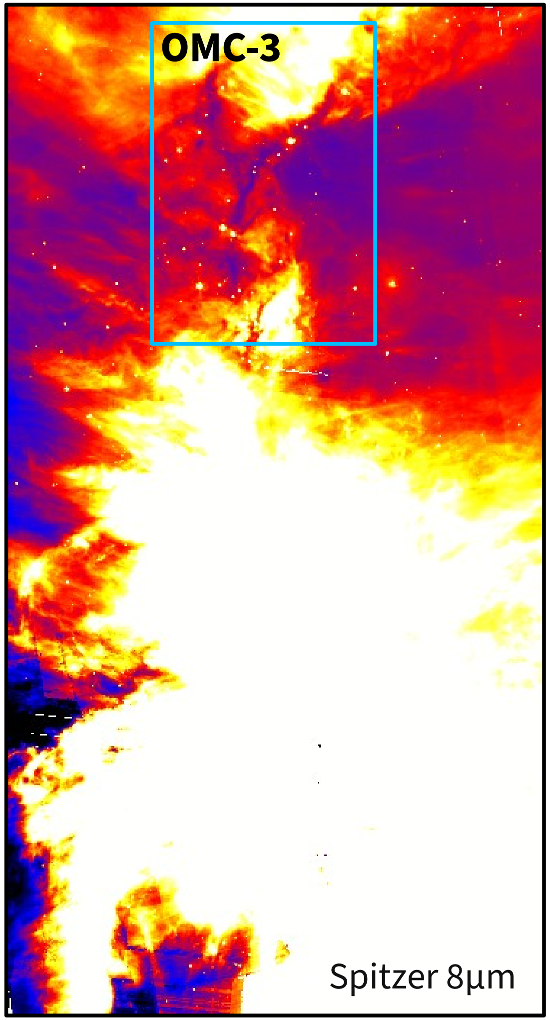
Analysis of FIR dust emission

- **problem** of multiple temperatures
- **answer**: multi-component modified blackbody (MBB) models ?

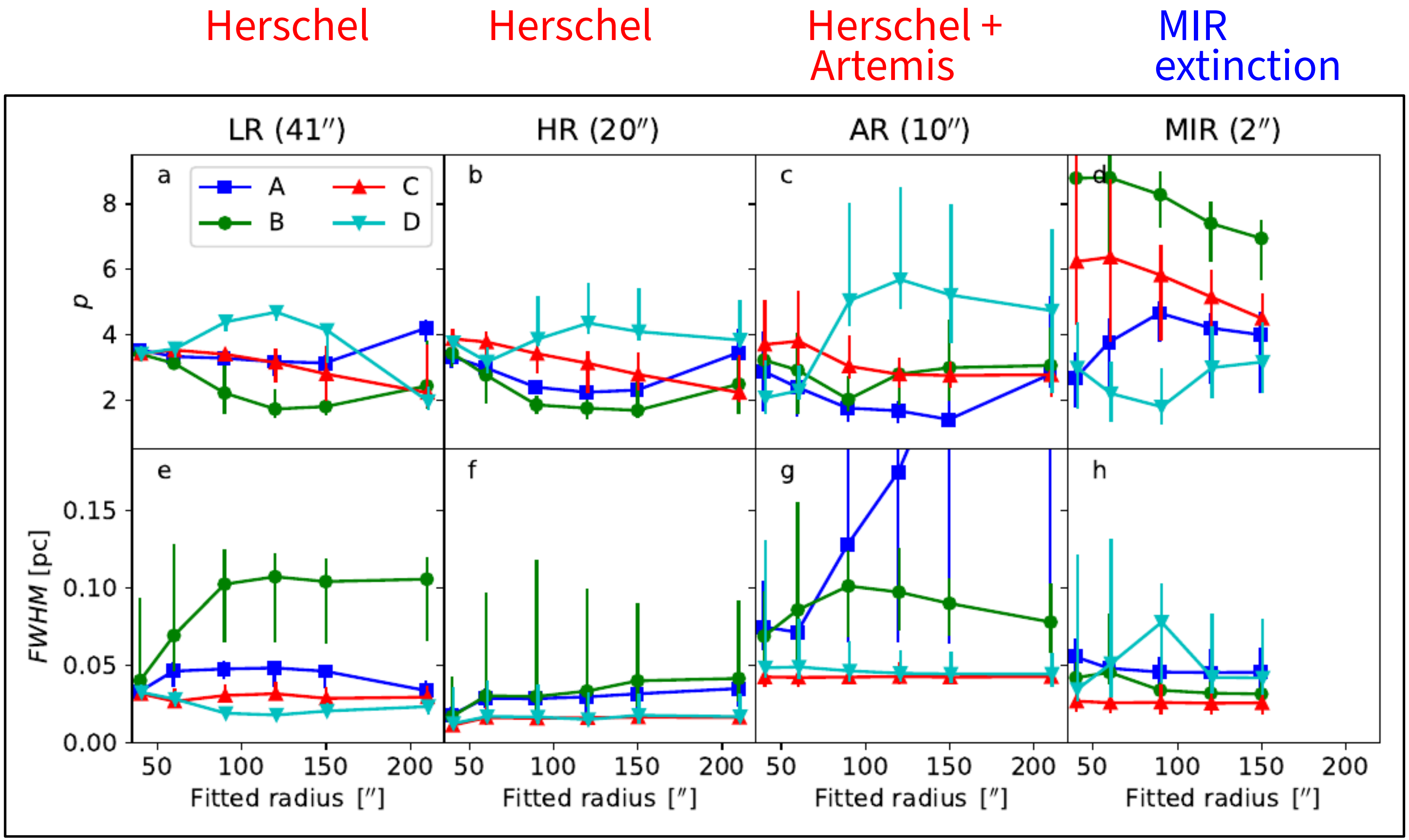


column density

- **extinction**
  - **NIR** background stars with low angular resolution – or very high resolution but sparse sampling
    - Malinen et al. 2013: TMC-1N
  - **MIR** extinction, as used in connection with infrared dark clouds (IRDCs)
    - Butler & Tan 2012; Butler, Tan, Kainulainen 2014; Kainulainen et al. 2017; Mattern et al. 2018
    - requires high column density, bright background, and low local emission
  - **independent of temperature**
- **dust emission**
  - **sensitive to T variations in the beam**



- **parameters vs. size of fitted area, FIR vs. MIR data**

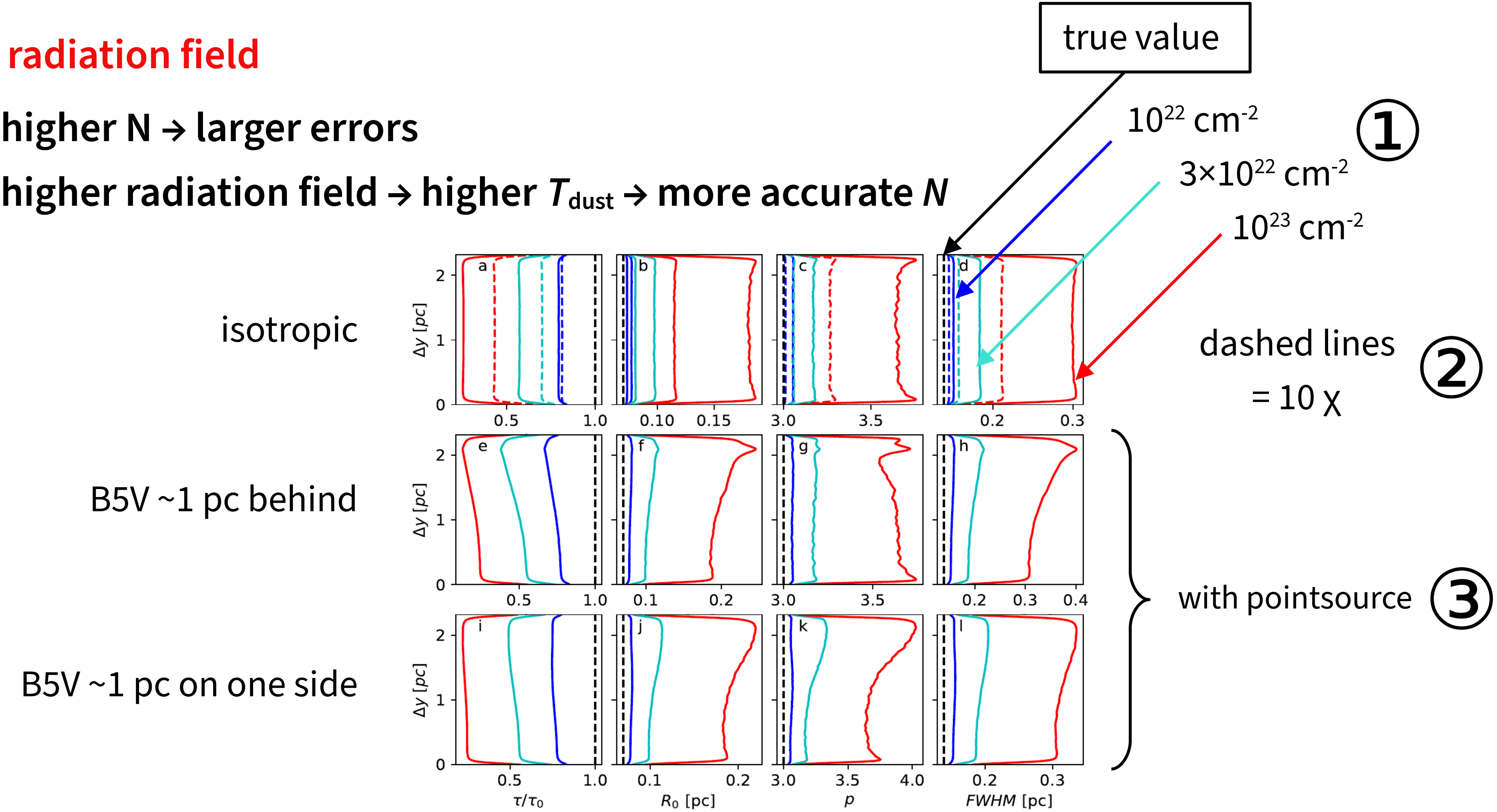


Examination of uncertainties

- **radiative transfer modelling of Plummer-profiled cylinders**
  - FIR: variable **radiation field ⇒ temperature variations ⇒ τ bias**
    - weak vs. strong field, isotropic or with a point source
  - FIR: **dust populations**
    - effective optical depth → temperature contrast → N bias
  - MIR: **local cattering**
    - effects of isotropic background vs. point source (in front, behind, towards one side)
  - MIR: **local dust mission**
    - stochastically heated grains; effects of isotropic background vs. point source

FIR: radiation field

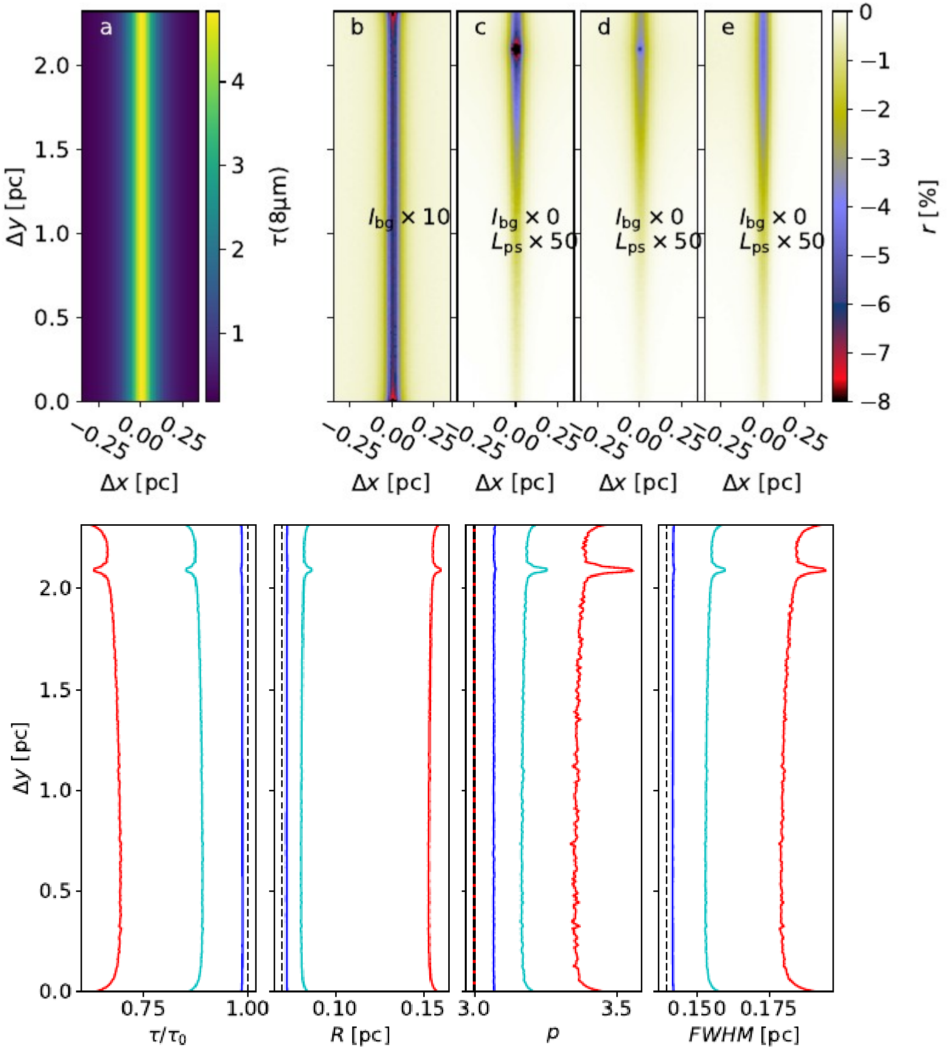
- higher N → larger errors
- higher radiation field → higher T_{dust} → more accurate N



MIR: Scattered light

- **small effect at 8μm**
  - unless *N* and χ high, I^{bg} low
  - or scattering efficiency very high
  - increases FWHM

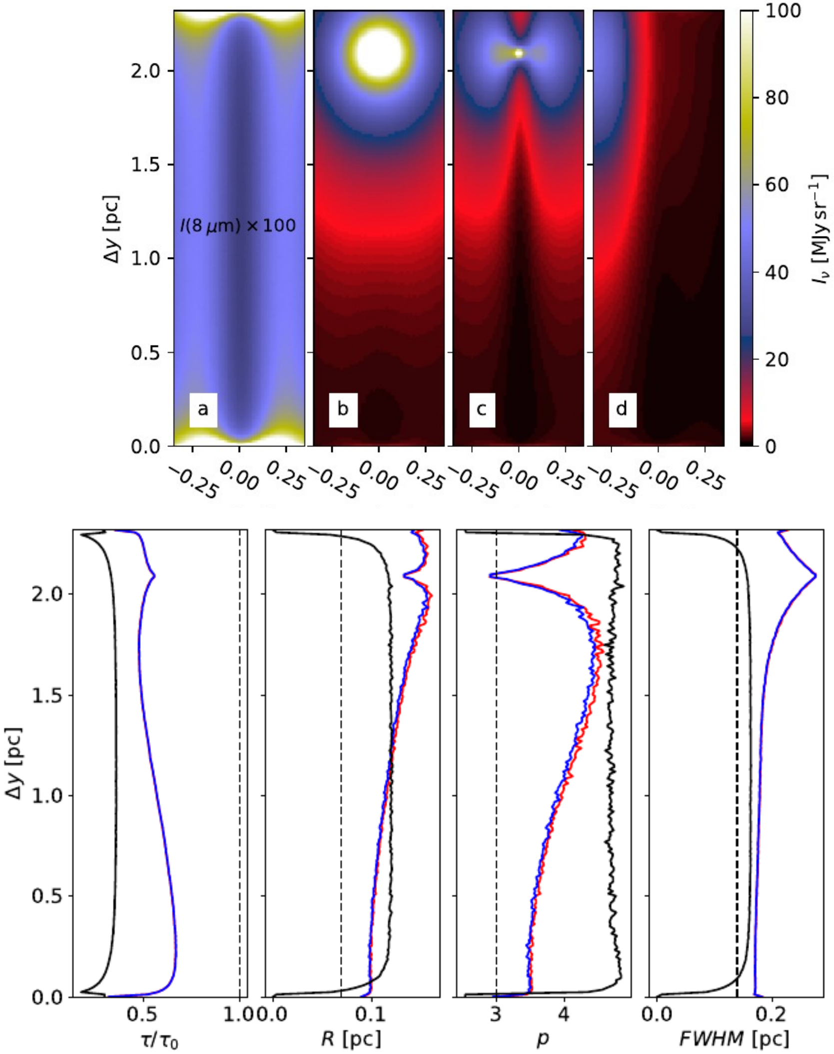
- **example N=10^{24} cm^{-2}** ☞
  - χ=1 + B5V
  - field ×20
  - field ×100



MIR: Dust emission

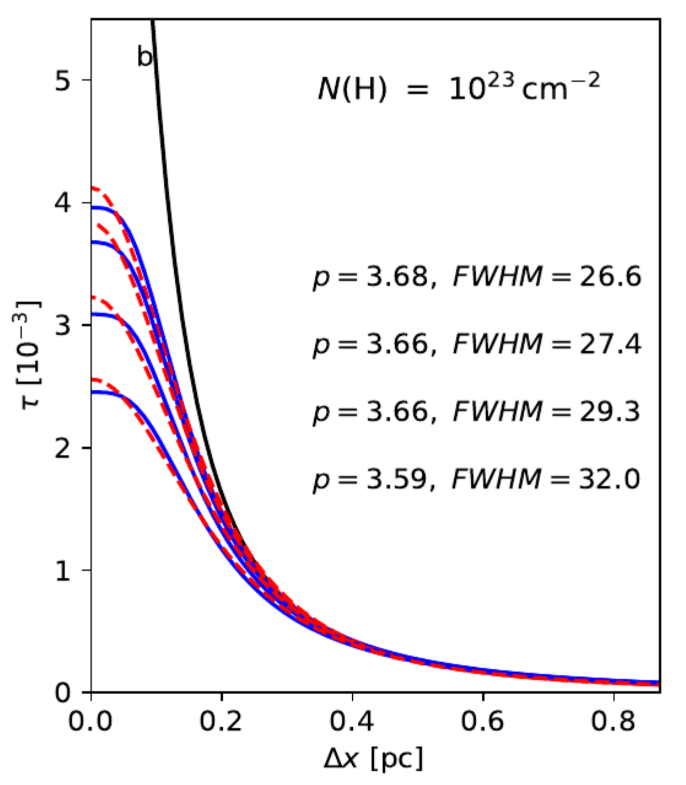
- **can be significant in case of high radiation fields**
  - increases p estimates

- **example**
  - $N\,(H_2) = 3\times10^{23}$ cm^{-2}
  - χ =1 and B5V
    - correct $I^{\,bg}$
    - $I^{\,bg}$ modified by dust emission
  - χ =10 radiation field (black curves)



OMC-3: Conclusions

- **OMC-3 filament FWHM ~ 0.05 pc**
  - MIR extinction ~ FIR dust emission
  - no strong dependence on **data resolution**
- **some bias always to be expected**
  - scattering has limited impact in the MIR
  - dust emission can affect MIR results: FWHM and *p* overestimated
  - **FIR overestimates FWHM**
    - effects larger at high column densities and in a weak radiation field



FIR: Modified blackbody functions (MBBs) and column density

$$I_\nu \approx \int_0^{\tau_\nu} B_\nu(T_{\mathrm{dust}})\, e^{-\tau'_\nu}\, d\tau'_\nu$$

$$\approx \sum_{i=1}^{N} B_\nu(T_{\mathrm{dust},i})\, \left(1 - e^{-\tau_{\nu,i}}\right)$$

$$\tau_\nu = \kappa_\nu \Sigma_{\mathrm{mass}} \approx \kappa_{\nu_0} (\nu / \nu_0)^\beta \Sigma_{\mathrm{mass}}$$

$$I_\nu \approx \sum_{i=1}^{N} \Sigma_{\mathrm{mass}} B_\nu(T_{\mathrm{dust}})\, \kappa_{\nu_0} (\nu / \nu_0)^\beta$$

$I_\nu = \Sigma_{\mathrm{mass}} \times B_\nu(T_{\mathrm{dust}}) \times \kappa(\nu)$

- opacity **κ** has powerlaw index **β**, but $\boldsymbol{T_{\mathrm{dust}}}$ and **β** are degenerate

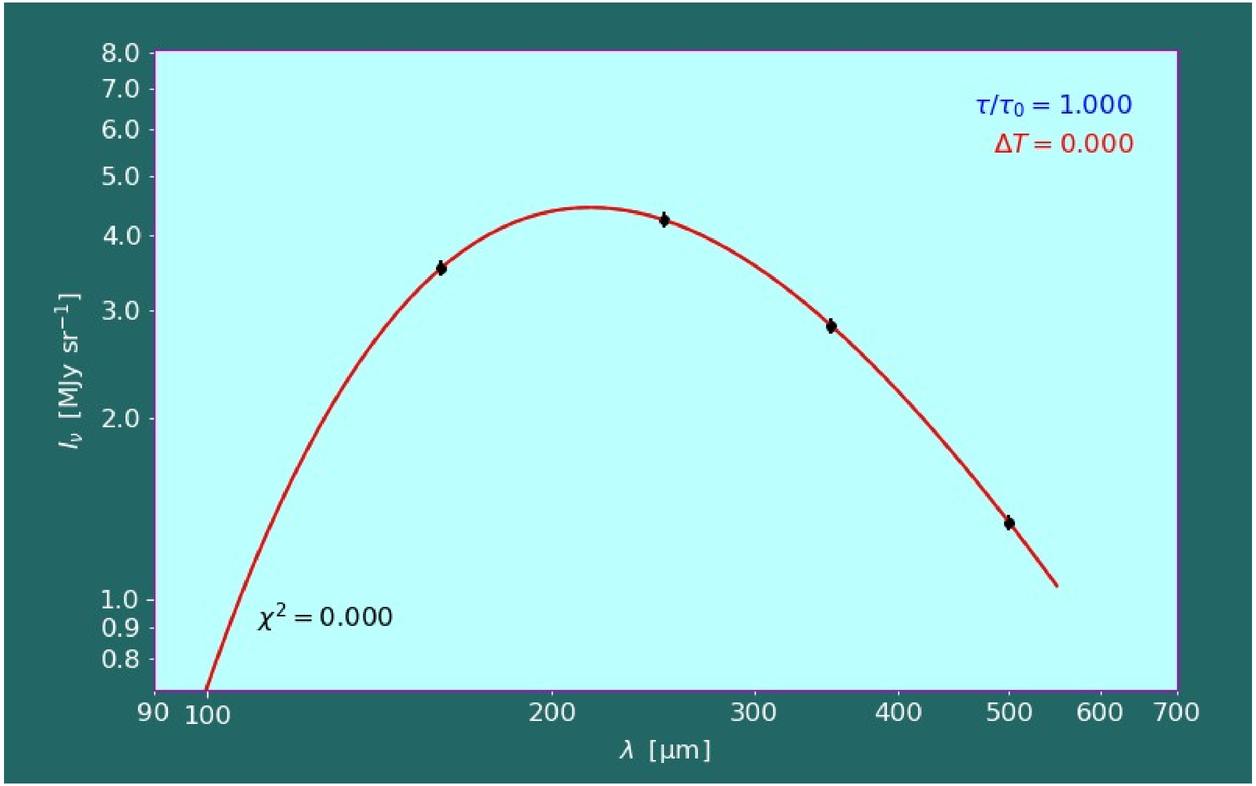
  ⇛

  keep free parameters
  (1) temperature $\boldsymbol{T_{\mathrm{dust}}}$
  (2) $\boldsymbol{\Sigma_{\mathrm{mass}}}$ or intensity scaling $\boldsymbol{I_0}$

Line-of-sight temperature variation needs to be taken into account ⇛ *N*>1 !

Single MBB

- **parameters:** I_0, T_{dust}
- **example:**
  - 160, 250, 350, and 500 µm observations
- ***no* observational noise**



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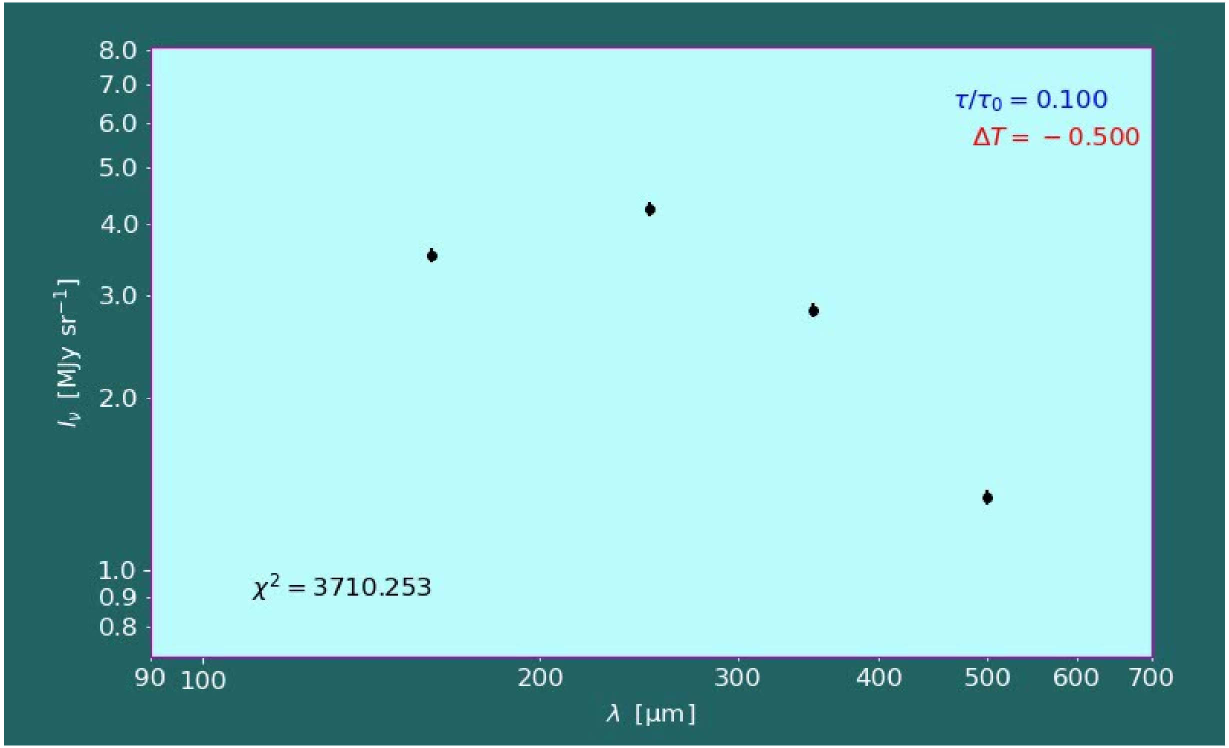

Single MBB

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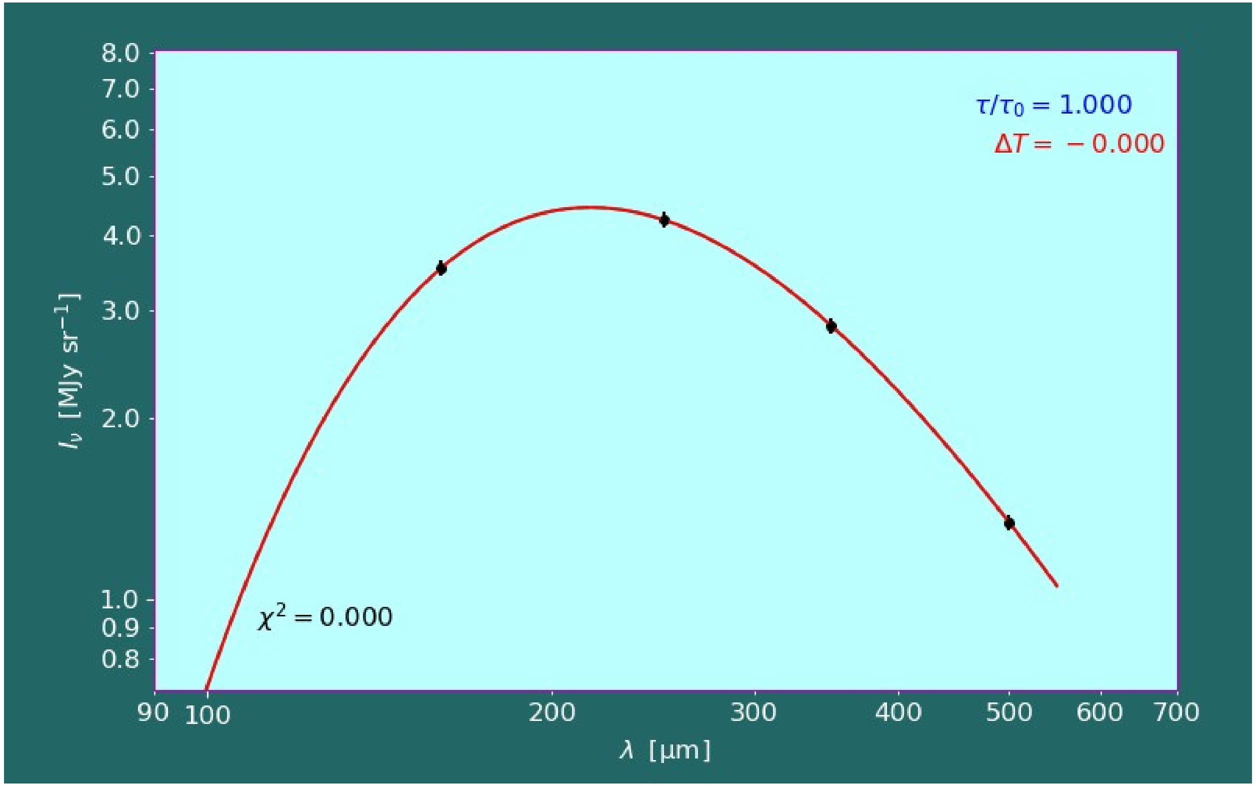
fit ok, χ^2 close to zero



Two components

- **parameters: I_1, T_1, I_2, T_2**
  - $T = 14,\ 17$ K and $\tau_1 = \tau_2$
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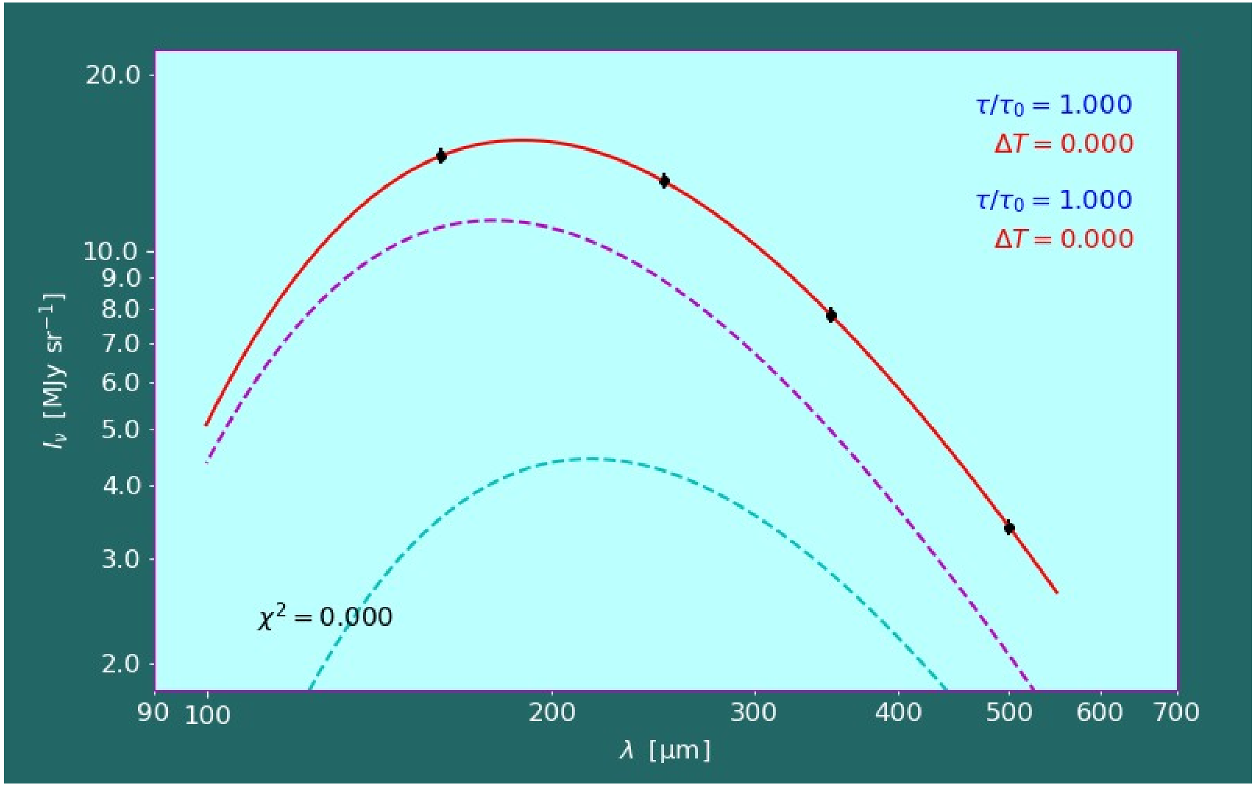

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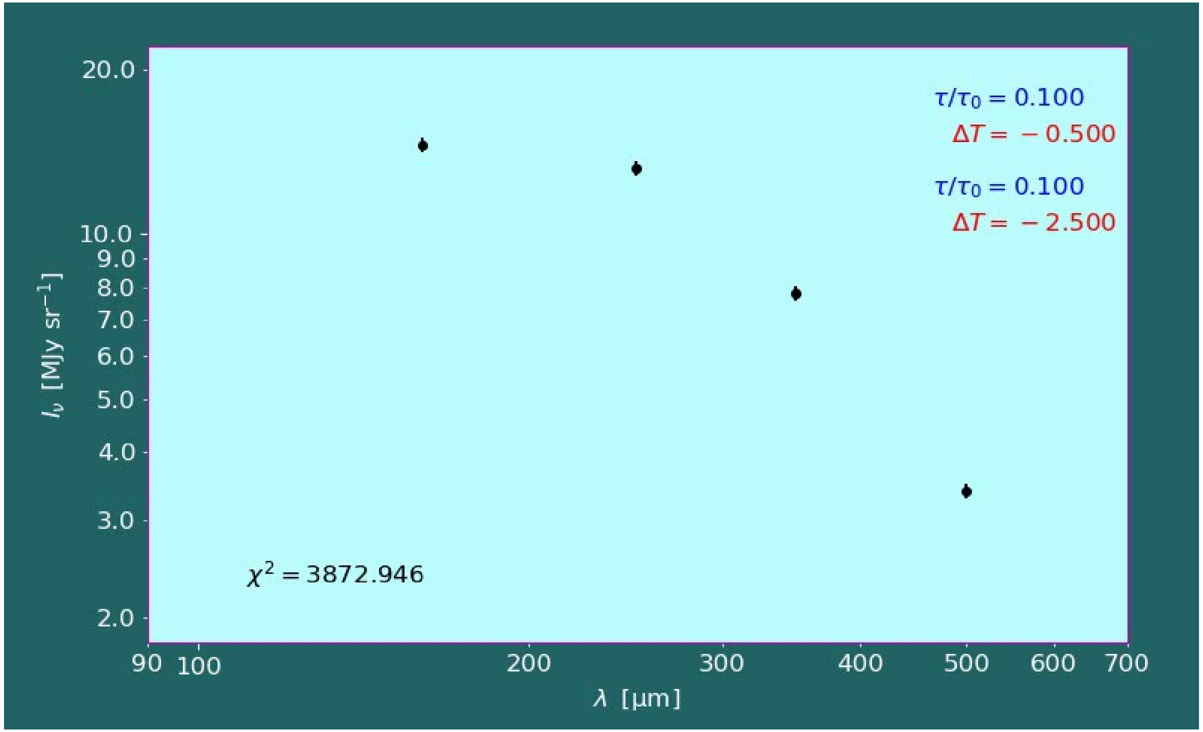

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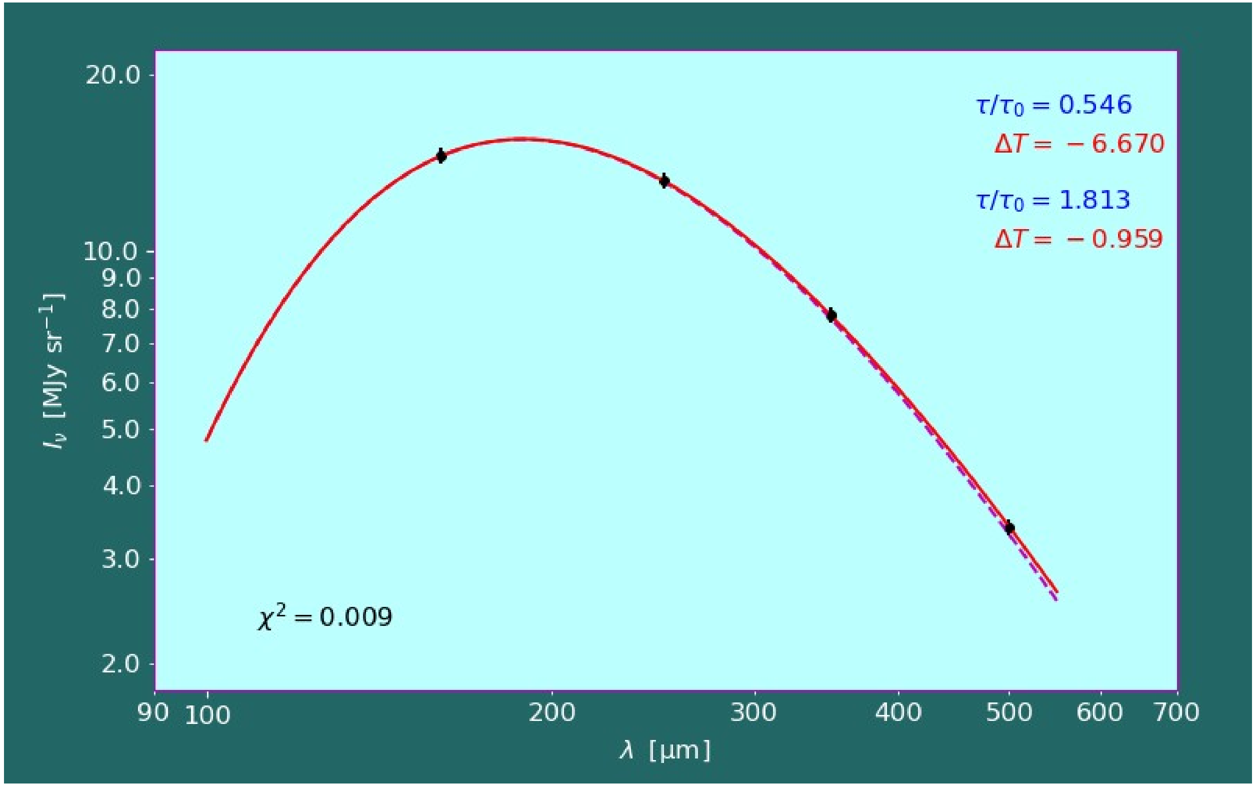
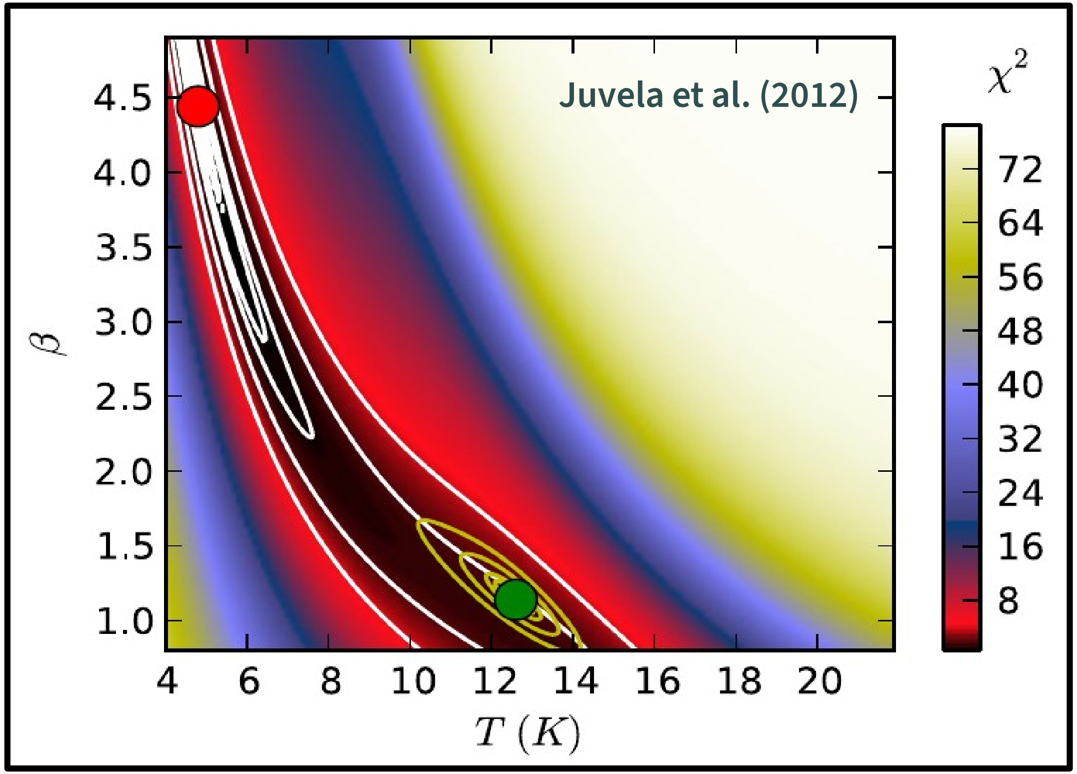
⇒ not quite ok !

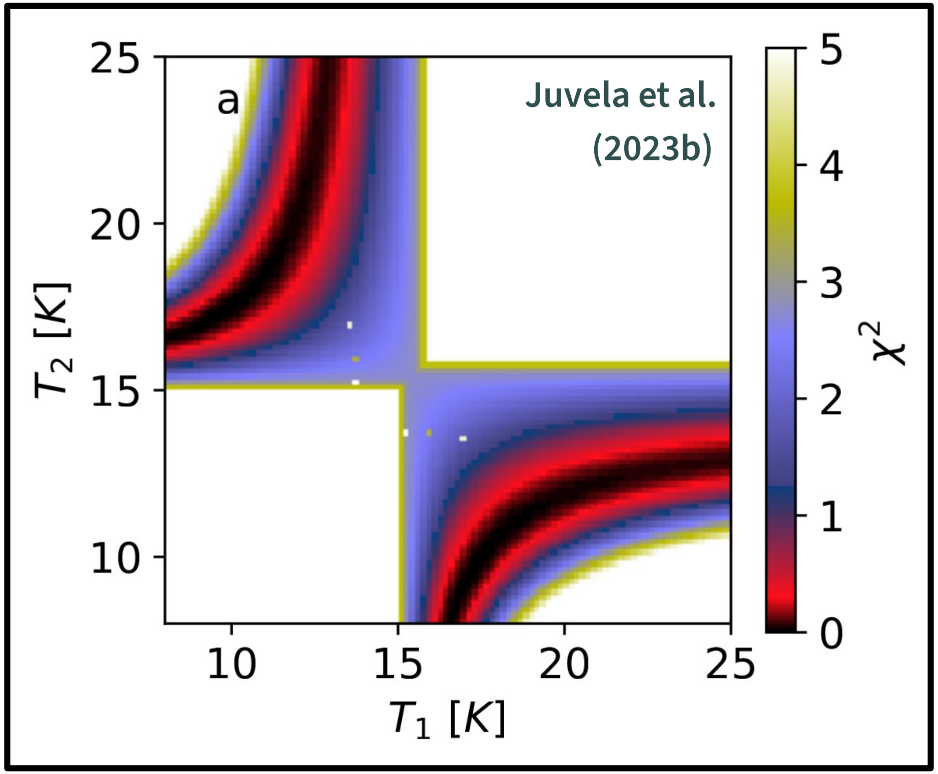


Degeneracies

T vs. *β*



T₁ vs. *T₂*



fundmental problem of
parameter degeneracy

practical / computational
problem of beam sizes

Angular resolution

- **FWHM changes between bands** ⇒

  1) ***convolve*** data to common resolution ⇒ each pixel an **independent** MBB(s) fit

  2) data at ***original*** resolution: **observation = model ⊗ beam**

     - an optimisation problem with **millions of parameters**
     - each step includes convolution of model-predicted maps

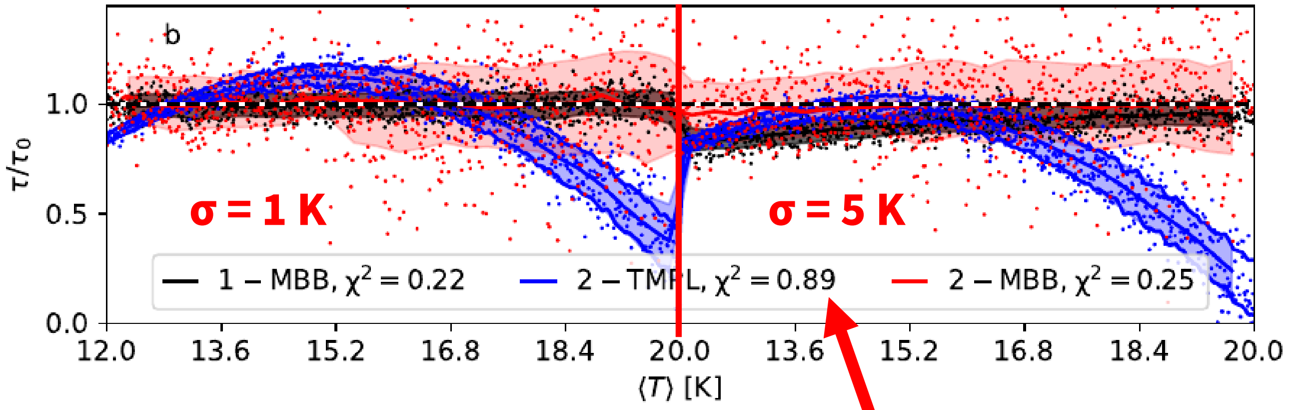
  – **PPMAP** (Marsh et al. 2015) is one widely used method… but run time for **128×128** pixel map and a few MBB components seems to be over **XX hours** !

     - minimum number of points in phase space to reach $\chi^2 \sim 1$
     - *"Direct maximisation of the a posteriori probability would involve searching a prohibitively large parameter space"*

Direct optimisation – Basis functions

First test: simulated data with $T_{dust} \sim N(\langle T \rangle, \sigma)$ – no convolutions

- fitted model is the sum of MBBs with
  - *N*-**TMPL**: fixed temperatures T_C + intensity scaling
  - *N*-**MBB**: free temperatures T_i + intensity scaling

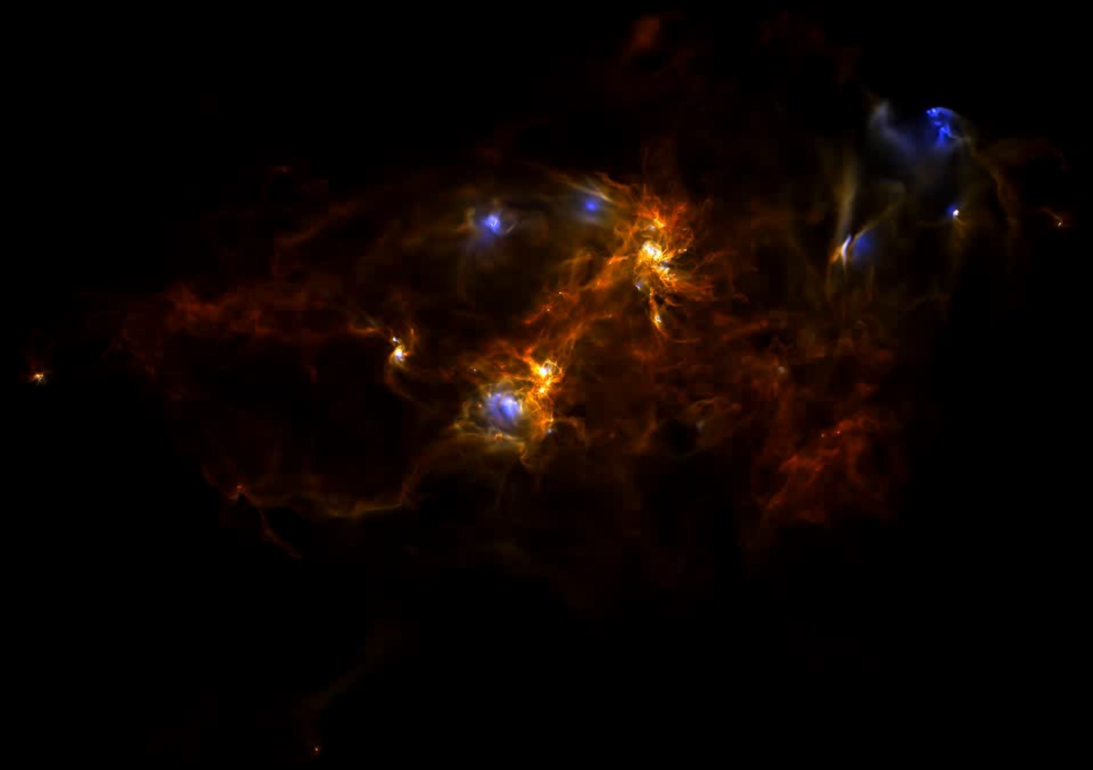


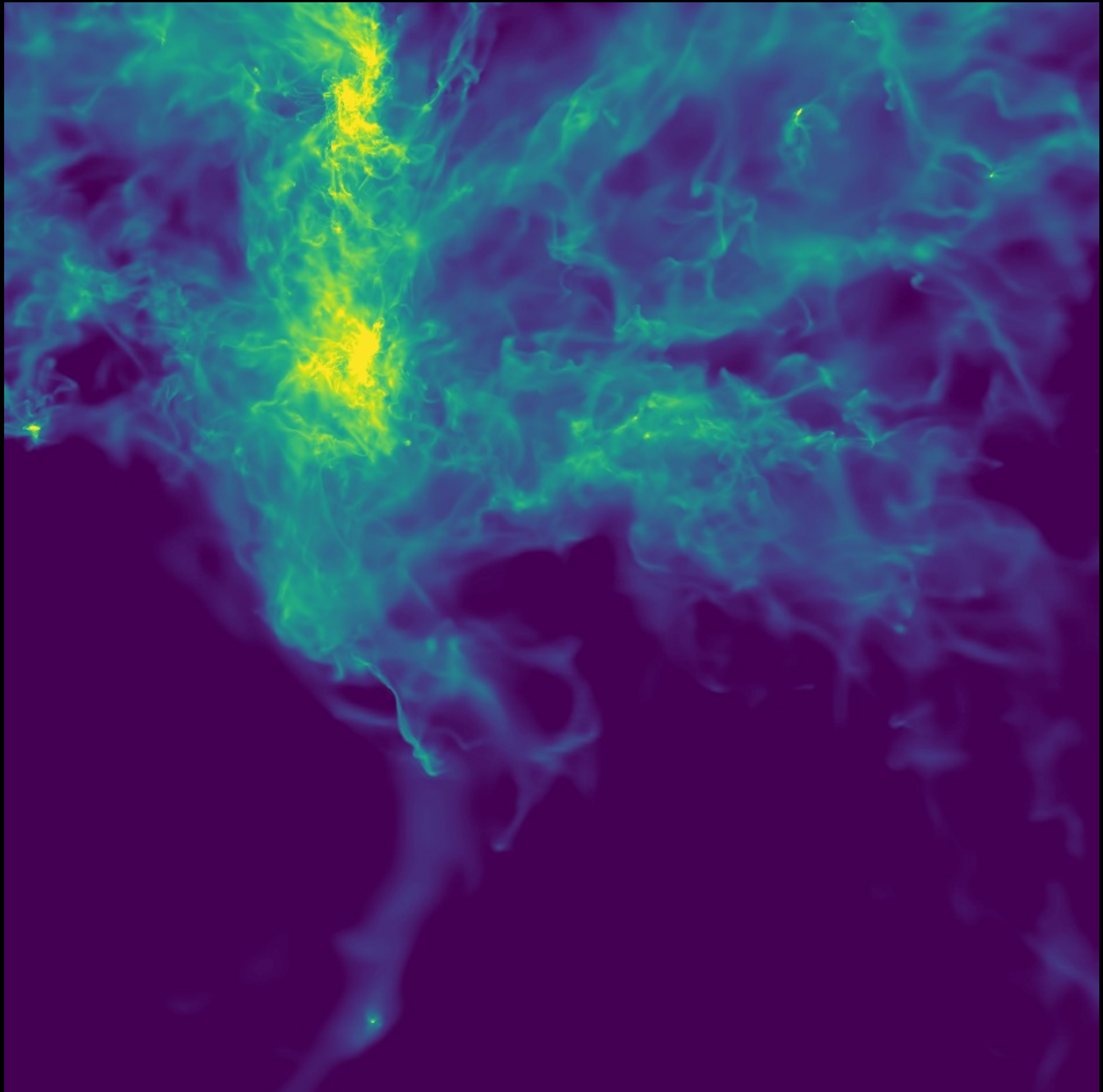
1-MBB (2 free parameters)

2-MBB (4 free parameters)

2-TMPL (2 free; T_C=13, 17 K)

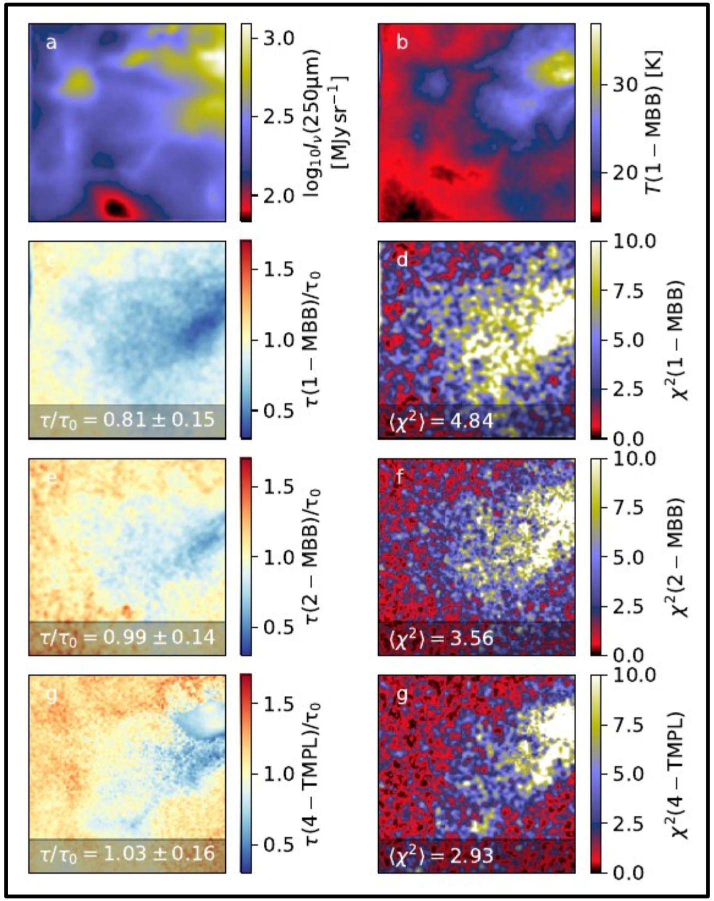
accurate match to observations ≠ accurate estimates of column density





Synthetic observations: Multi-component + multi-beam

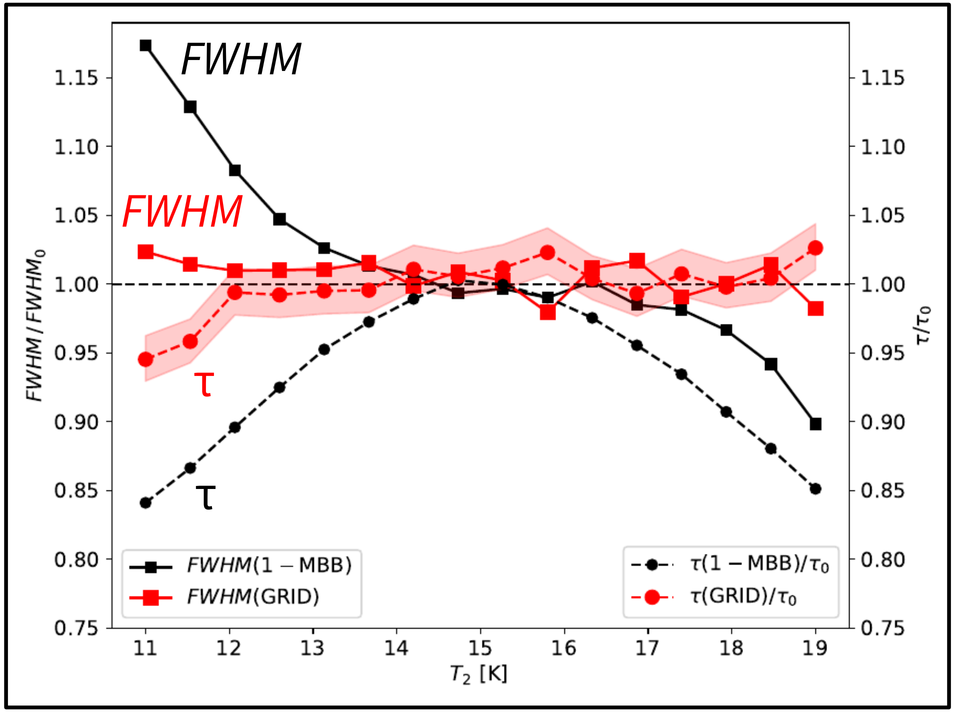
- **1875×1875 pixel maps of a MHD simulation of a star-forming cloud**
  - T_{dust} varies from ~6 K to over 100 K
- ***N*-TMPL and *N*-MBB very *slightly* better than 1-MBB**
  - bias reduced
  - higher angular resolution (×2)
- **run times ~ ½ hours (CG+GPU)**
  - *N*-TMPL in Fourier space ~ **one minute**



Markov chain Monte Carlo

- even MCMC feasible, using SED templates for temperature *distributions*
  - $T \sim N(T_0, \sigma_T)$
- parallel processing of patches with separation larger than the beams
- 320×320 pixels, run time one hour
- accuracy of (1) estimated τ ☞
  (2) filament FWHM ☞

Filaments: temperature gradients from T_1=15 K on the outside to T_2 = 11 K or 19 K in the centre.



N-MBB – or 1-MBB with corrections?

1) **Try to apply more complex models to the few observed data, in the hope that the model happens to be realistic enough to make good predictions of *N***

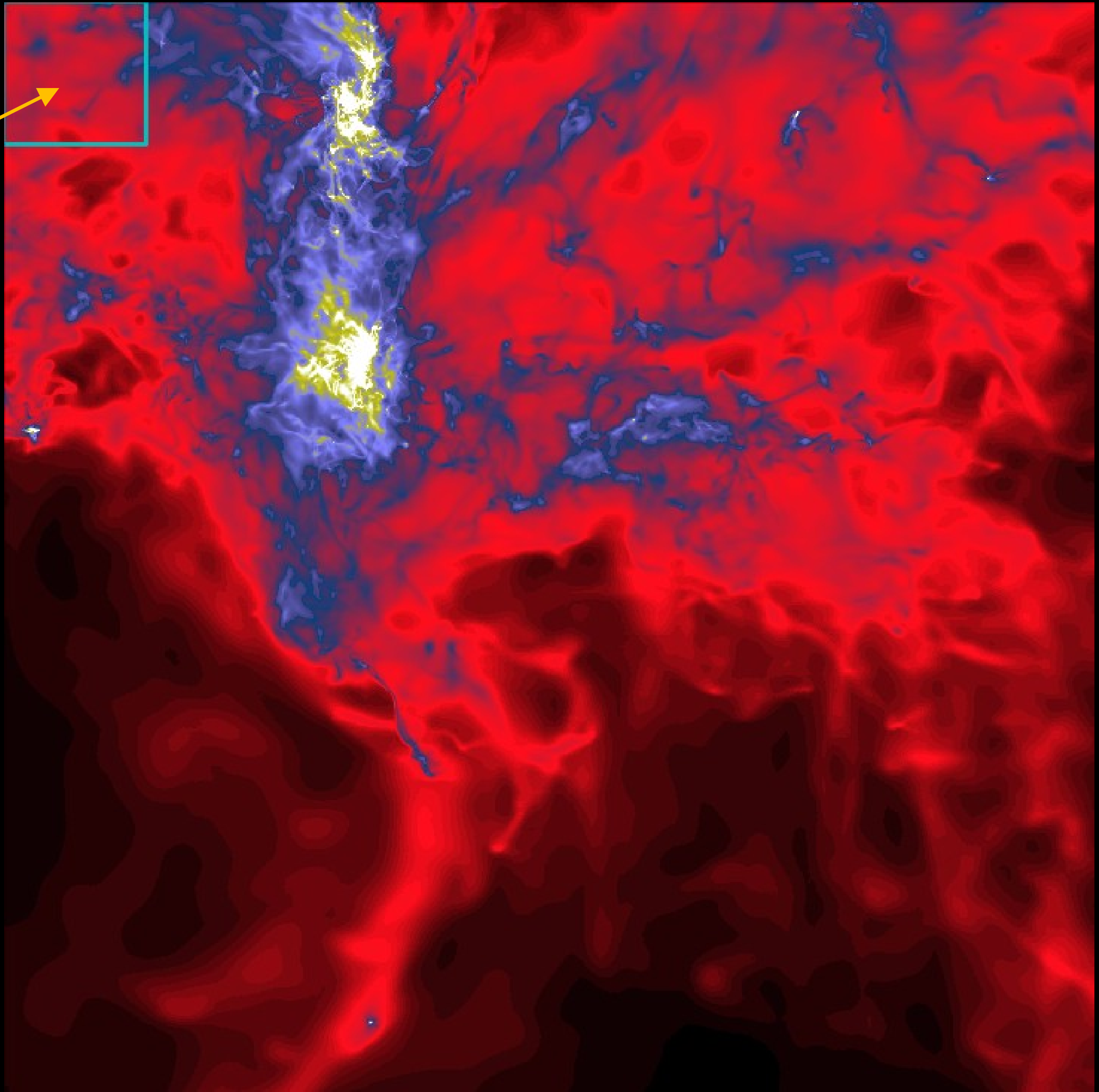
2) **Use a simple model (e.g. 1-MBB), combined with empirical corrections**

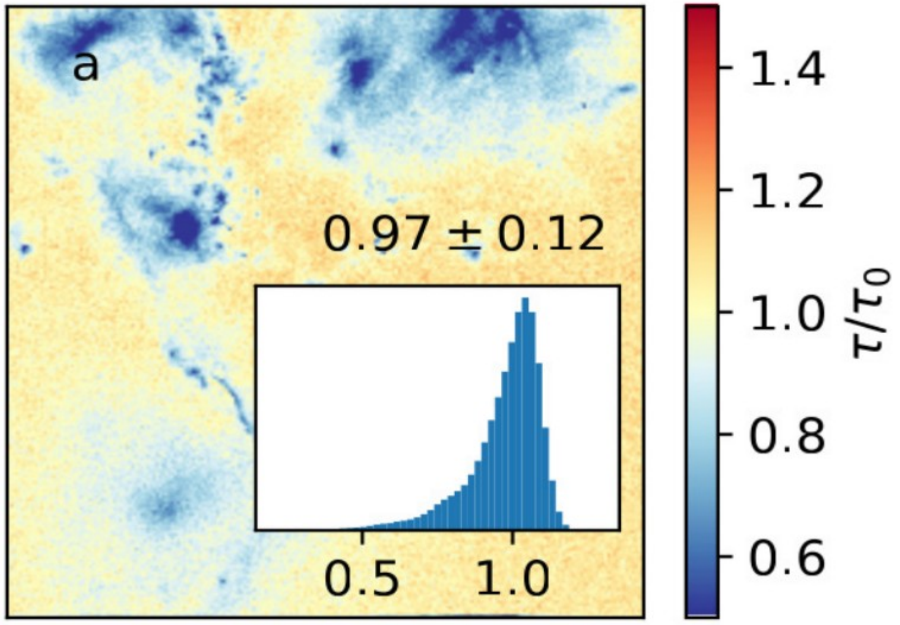
- **The latter option was tested with the MHD model**
  - the most challending case, with diffuse regions, and hot and cold cores (6-100K)
  - as the first test, one can just look at the **width of the SED**
    - narrow T_{dust} distribution ⇒ SED is a single MBB
    - broad distribution of T_{dust} values ⇒ SED is **wider compared to best-fit MBB**

$$\Sigma(\Delta I_\nu / I_\nu) = r(160\,\mu\text{m}) - r(250\,\mu\text{m}) - r(350\,\mu\text{m}) + r(500\,\mu\text{m})$$

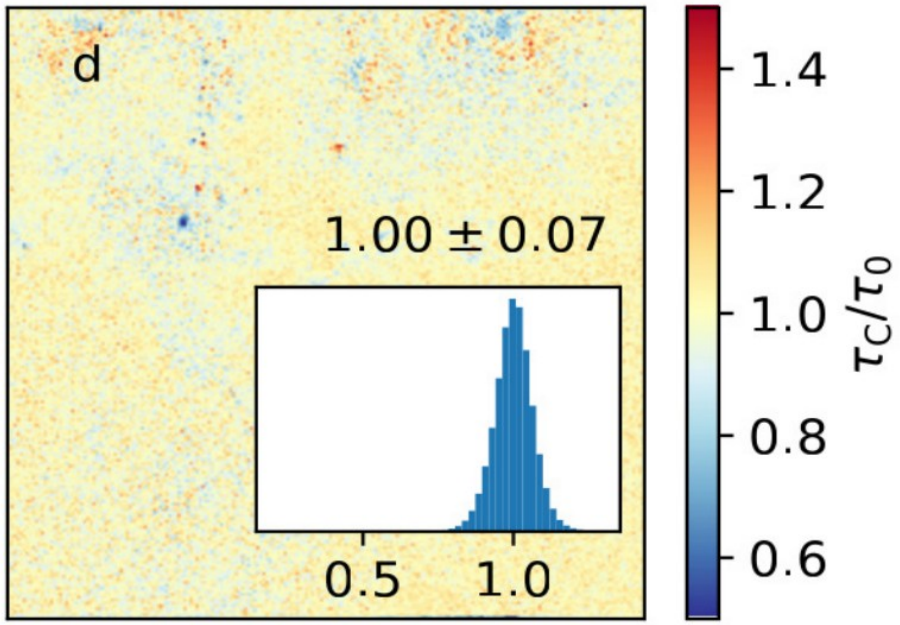
– r = ratio of observed intensity and the 1-MBB fitted value

size of the area
used for training





1-MBB fit



1-MBB + empirical correction as good as multi-MBB models

Conclusions

- **One *can* fit multi-MBB models using**
  - **full data resolution**
  - **direct optimisation**
  - even **MCMC**
- ***Should one*?**
  - degenerate basis functions, large model errors (0...∞)
    - fit quality ≈ low χ^2 ≠ accuracy of model prediction
  - depends on the chosen **priors**
  - **parameterised temperature distributions better than discrete MBBs ?**
- **However, a single MBB is fast – and reliably biased**
  - empirical corrections may provide reasonable accuracy