Chemodynamical models of the Milky Way

Eugene Vasiliev

based on:

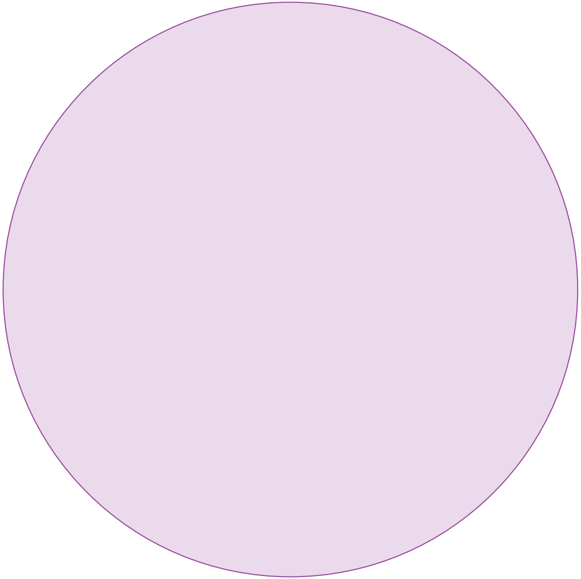
$\mathcal{P}$1: Binney&Vasiliev, 2206.03523

$\mathcal{P}$2: Binney&Vasiliev, 2306.11602

MW-Gaia workshop

Barcelona, 6 September 2023

Part 1: inventory



Gaia 5d astrometric catalogue: 1.5×10^9

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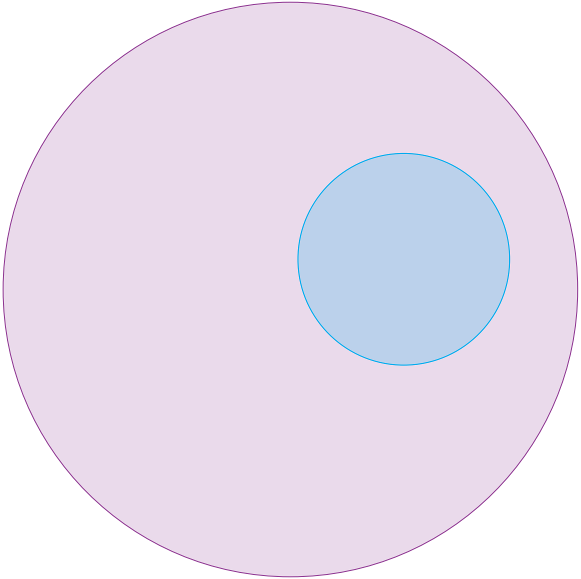

Gaia 5d astrometric catalogue: 1.5×10^9

$\varpi/\epsilon_\varpi > 5$: 2×10^8

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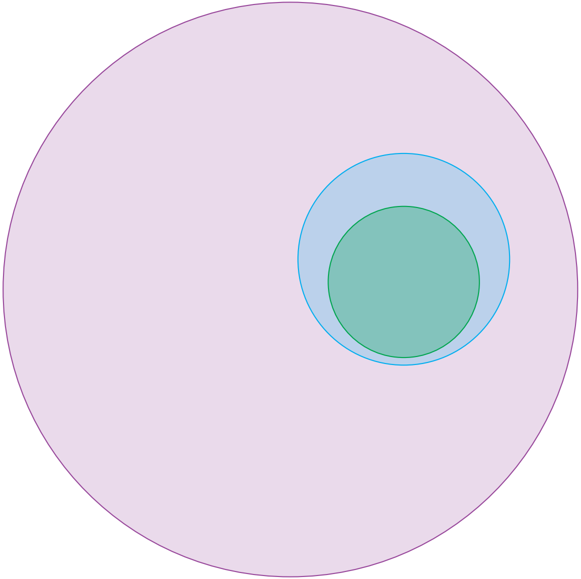

Gaia 5d astrometric catalogue: 1.5×10^9

$\varpi/\epsilon_\varpi > 5$: 2×10^8

$\varpi/\epsilon_\varpi > 10$: 1×10^8

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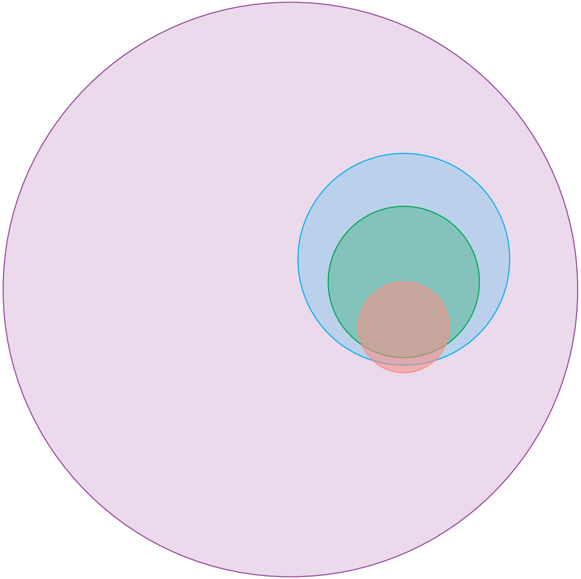

Gaia 5d astrometric catalogue: 1.5×10^9

$\varpi/\epsilon_\varpi > 5$: 2×10^8

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Gaia RVS sample: 3×10^7

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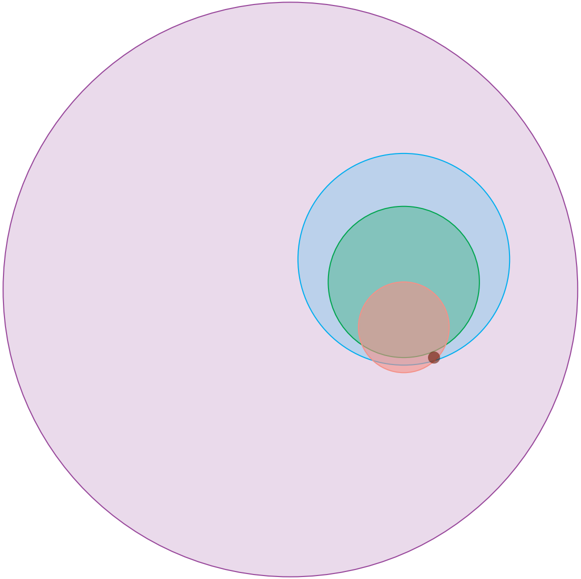

Gaia 5d astrometric catalogue: 1.5×10^9

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Gaia RVS sample: 3×10^7

APOGEE DR17: 6×10^5

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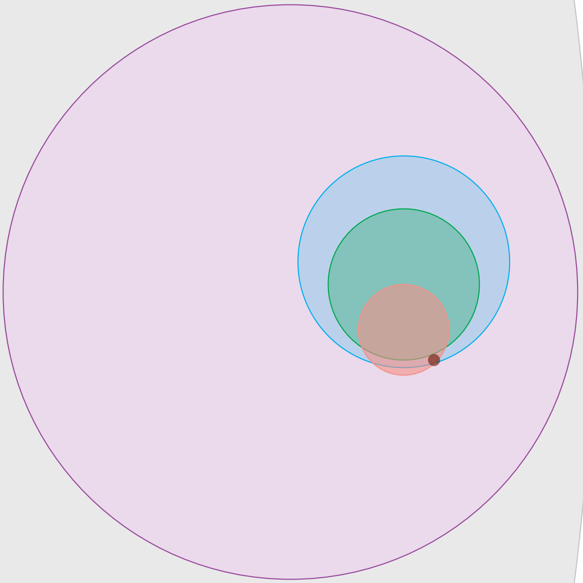


entire Milky Way: 10^{11}

Gaia 5d astrometric catalogue: 1.5×10^9

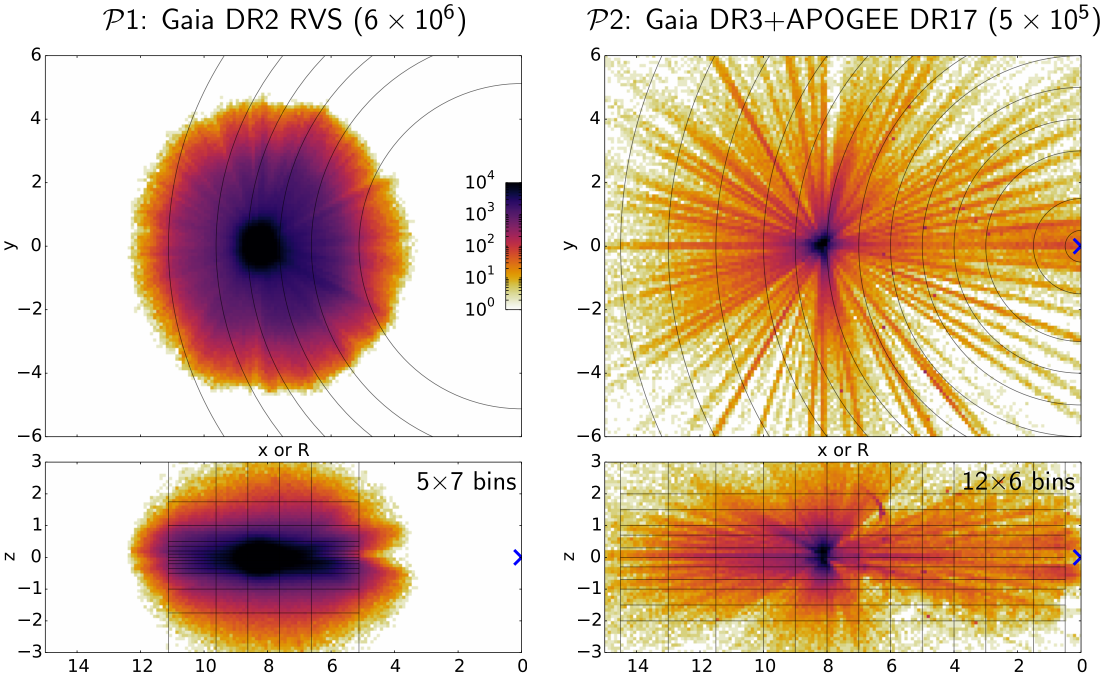
$\varpi/\epsilon_\varpi > 5$: 2×10^8

$\varpi/\epsilon_\varpi > 10$: 1×10^8

Gaia RVS sample: 3×10^7

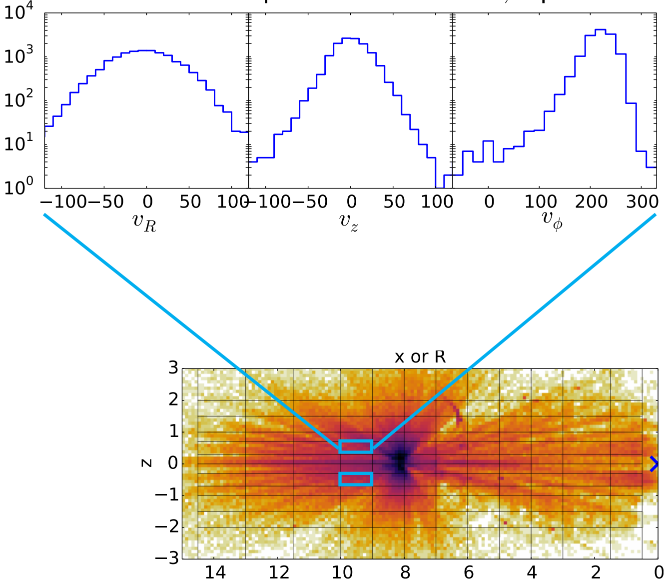
APOGEE DR17: 6×10^5

Input data for models: 6d kinematic catalogues

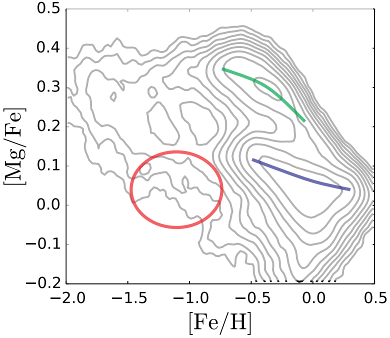


Input data for models: 6d kinematic catalogues

1d histograms of velocity distributions in a few dozen spatial bins across R, z plane



Chemo-kinematic components in the Milky Way



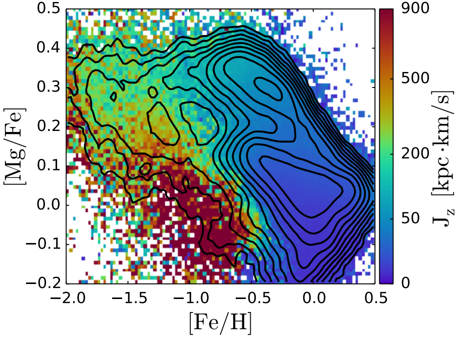
α-rich disc

α-poor disc

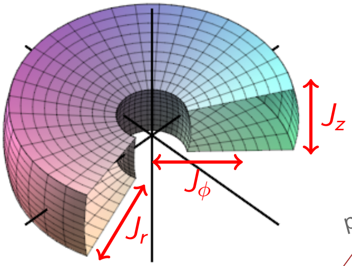
halo

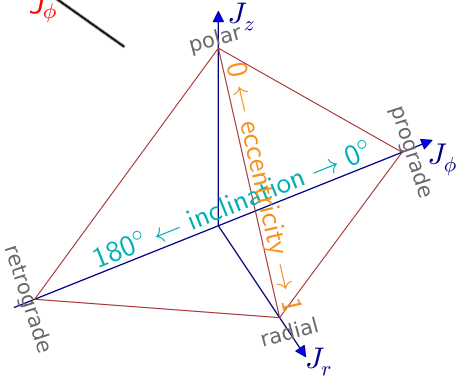
based on APOGEE DR17

Chemo-kinematic components in the Milky Way



split by angular momentum J_ϕ
and vertical action J_z





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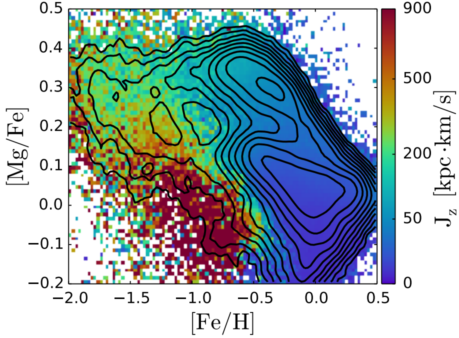

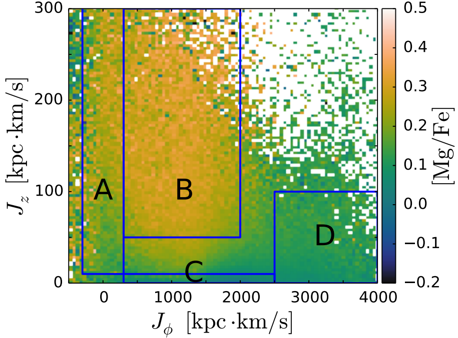

split by angular momentum J_ϕ
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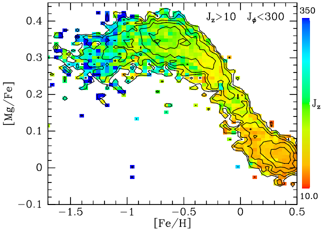
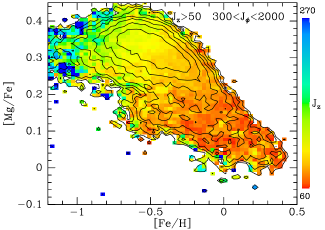
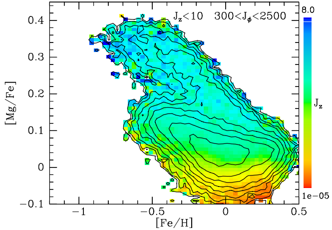
A: bulge+halo



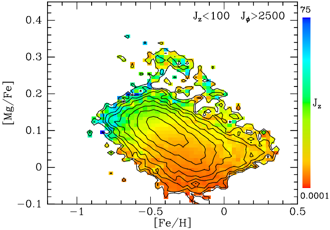
B: thick, high-α disc



C: inner thin, low-α disc



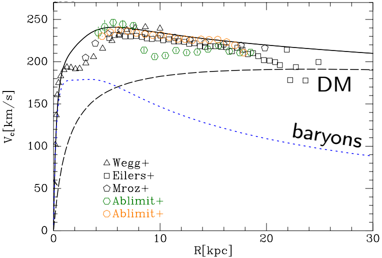
D: outer low-α disc



Part 2: global [chemo]-dynamical models of the Galaxy

- Present a bird's-eye view on the Milky Way (ignore details).
- Synthesize a coherent picture from a large diversity of observational data.
- Ensure dynamical self-consistency
  (stars + DM are responsible for the total gravitational potential).
- Provide distribution functions for different Galactic populations.
- Allow one to infer [missing] attributes for individual objects
  or to construct of mock datasets by sampling from the model.

example of model deliverables:
circular-velocity curve,
fractional contribution of DM



Iterative construction of self-consistent dynamical models

A given population of stars k (e.g., α-rich disc) is fully described by the distribution function in the 6d phase space $f_k(\mathbf{x}, \mathbf{v})$.

In particular, the density is $\rho_k(\mathbf{x}) = \iiint f_k(\mathbf{x}, \mathbf{v})\; d^3\mathbf{v}$.

In a steady state, the DF must depend only on the integrals of motion $\mathcal{I}(\mathbf{x}, \mathbf{v};\;\; \Phi)$ (Jeans's theorem), which depend on the potential Φ (e.g., energy $E = \Phi(\mathbf{x}) + \frac{1}{2}|\mathbf{v}|^2$).

The potential, in turn, is linked to density by the Poisson equation $\nabla^2\Phi = 4\pi G \sum_k \rho_k$.

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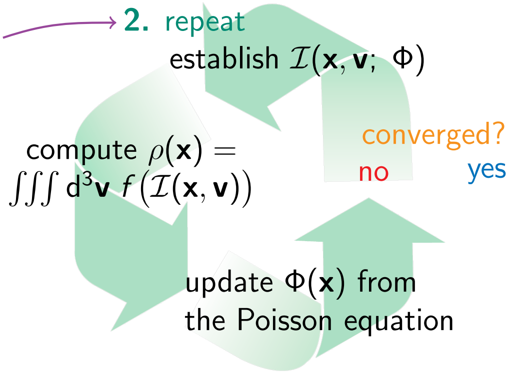
1. assume $f(\mathcal{I})$ and an initial guess for Φ

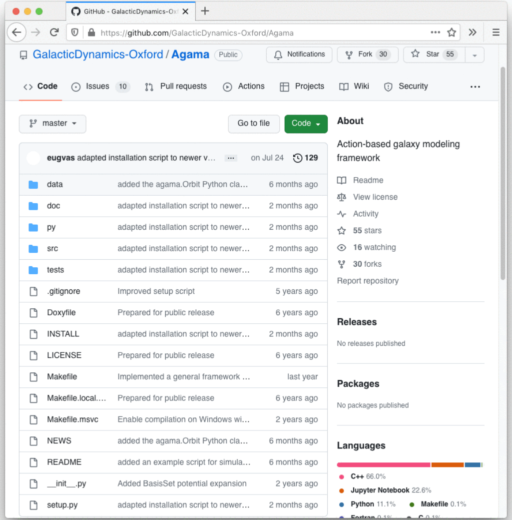


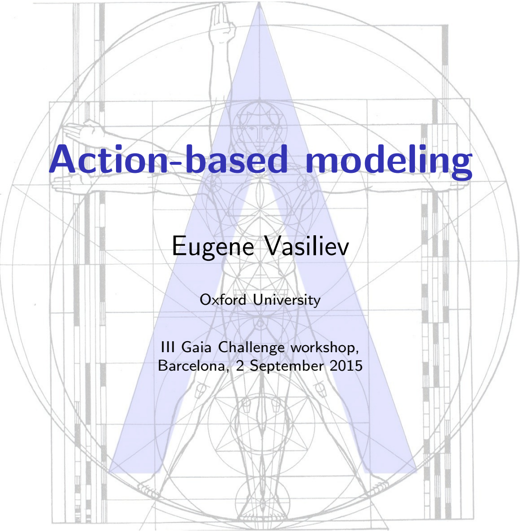
3. enjoy!

Dynamical modelling with AGAMA

- Solving the Poisson equation for an arbitrary density profile $\rho(\mathbf{x}) \Longrightarrow$ flexible `Multipole`, `BasisSet` and `CylSpline` potential expansions.
- Computing the [approximate] integrals of motion in an arbitrary potential $\Longrightarrow$ `Stäckel fudge` action finder [axisymmetric].
- Distribution functions for discy and spheroidal populations $\Longrightarrow$ `QuasiIsothermal`, `Exponential`, `DoublePowerLaw` DF families.
- Computation of DF moments ($\overline{v}$, σ), velocity distributions, etc., generation of samples from the DF (e.g., particle snapshots for N-body simulations).
- Iterative construction of self-consistent models specified by DFs.
- Orbit integration in the given potential.
- Schwarzschild orbit-superposition modelling.
- Interfaces to other stellar-dynamical packages: `gala`, `galpy`, `nemo`, `amuse`.

Agama – all-purpose galaxy modelling framework

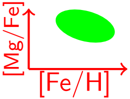




see also a presentation at the upcoming ChaICA virtual workshop organized by IAU (*Challenges and innovations in computational astrophysics*, part V), 7–9 November 2023, `https://dias.ie/chaica5/`

Part 3: Model fitting

- Choose suitable DF families $f_k(\mathbf{J};\ \boldsymbol{\xi})$ for all galactic components (several discs, bulge, stellar and dark halo) with 6–10 free parameters $\boldsymbol{\xi}$ per component k.
- [$\mathcal{P}$2 only]: assign a chemical DF $P_k(\mathbf{c} \mid \mathbf{J};\ \boldsymbol{\eta})$ for each stellar component ($\mathbf{c} \equiv$ [Fe/H] and [Mg/Fe], $\boldsymbol{\eta}$ are ~ 10 chemical parameters).



- For each choice of parameters $\boldsymbol{\xi}, \boldsymbol{\eta}$:
  - Construct a self-consistent dynamical model ($\sim$ a few minutes);
  - Compute velocity distributions $f(v_R)$, $f(v_z)$, $f(v_\phi)$ in a few dozen spatial bins;
  - [$\mathcal{P}$2 only]: Compute chemical distributions in a few dozen bins in action space;
  - Compare with observed histograms, *ignoring (freely adjusting) the overall normalization in each bin*, compute the [quasi-]likelihood $\mathcal{L}$.
- Adjust parameters and repeat (try to find the maximum-likelihood solution)...

Inferring the potential from kinematic data

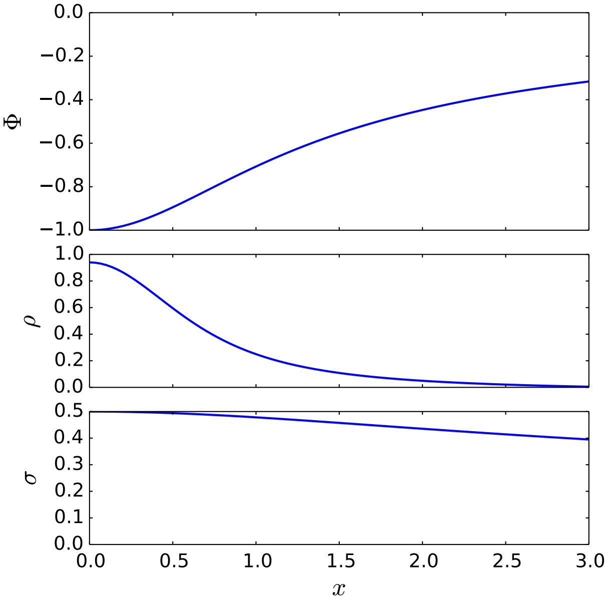
Jeans equation(s):

$$\rho(x),\ \sigma(x)\ \Rightarrow\ \Phi(x).$$

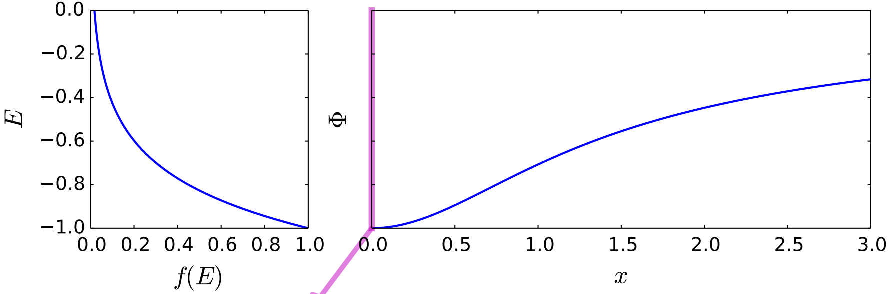
1d example:

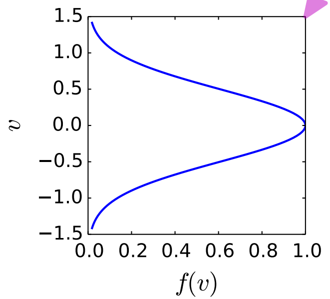
$$\underbrace{\frac{\mathrm{d}\left(\rho\,\sigma^2\right)}{\mathrm{d}x}}_{\text{pressure gradient}} + \rho\,\underbrace{\frac{\mathrm{d}\Phi}{\mathrm{d}x}}_{\text{gravitational force}} = 0$$

(hydrostatic equilibrium).



Inferring the potential from kinematic data





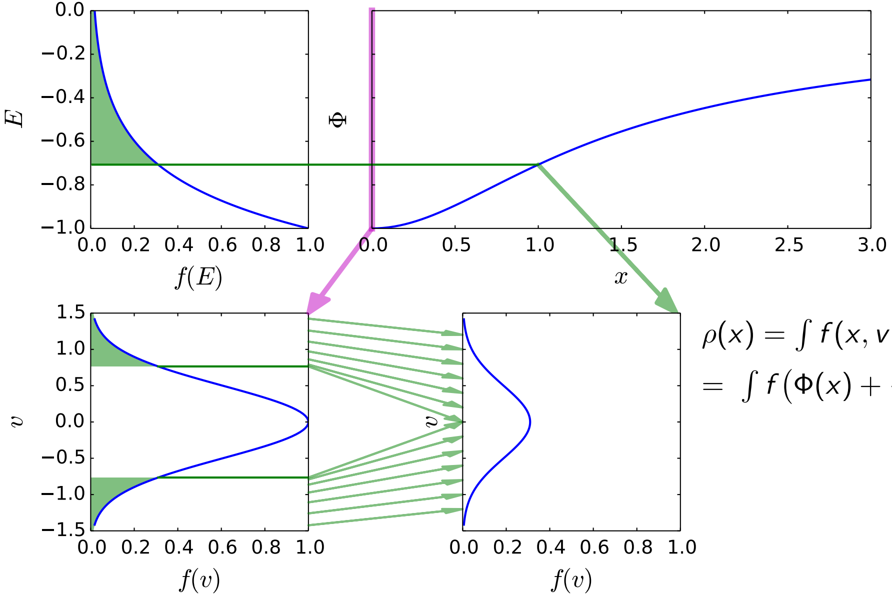
$$f(x, v) \;\Leftrightarrow\; f\left(E = \Phi(x) + \tfrac{1}{2} v^2\right)$$

for example,

$$f(E) \;=\; f_0 \exp\left(-\frac{E}{\sigma^2}\right),$$

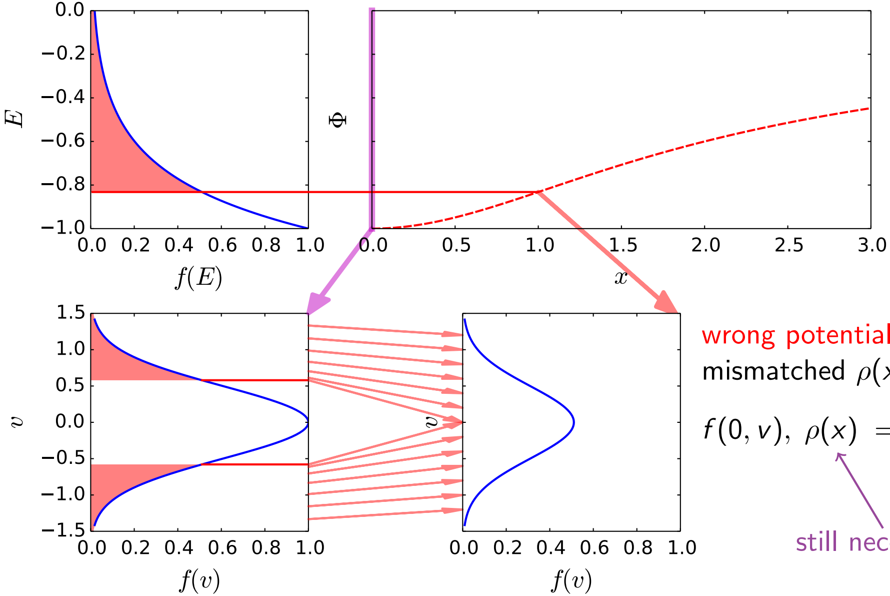
$$f(x, v) \;=\; f_0 \exp\left(-\frac{\Phi(x)}{\sigma^2}\right) \exp\left(-\frac{1}{2}\frac{v^2}{\sigma^2}\right)$$

Inferring the potential from kinematic data



$$\rho(x) = \int f(x, v)\, \mathrm{d}v$$
$$= \int f\left(\Phi(x) + \tfrac{1}{2}v^2\right)\, \mathrm{d}v$$

Inferring the potential from kinematic data

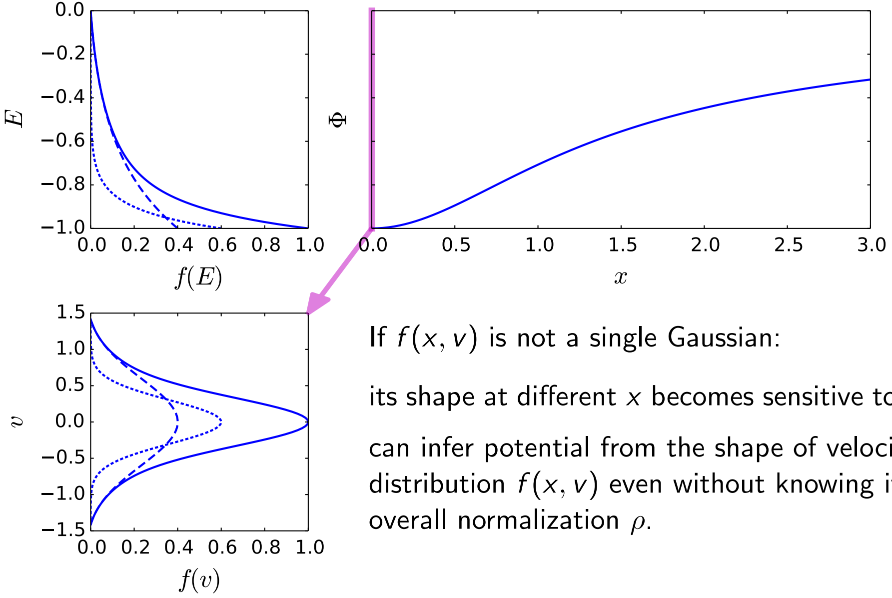


wrong potential $\Rightarrow$ mismatched $\rho(x)$.

$f(0, v),\ \rho(x) \ \Rightarrow\ \Phi(x)$.

still necessary!

Inferring the potential from kinematic data

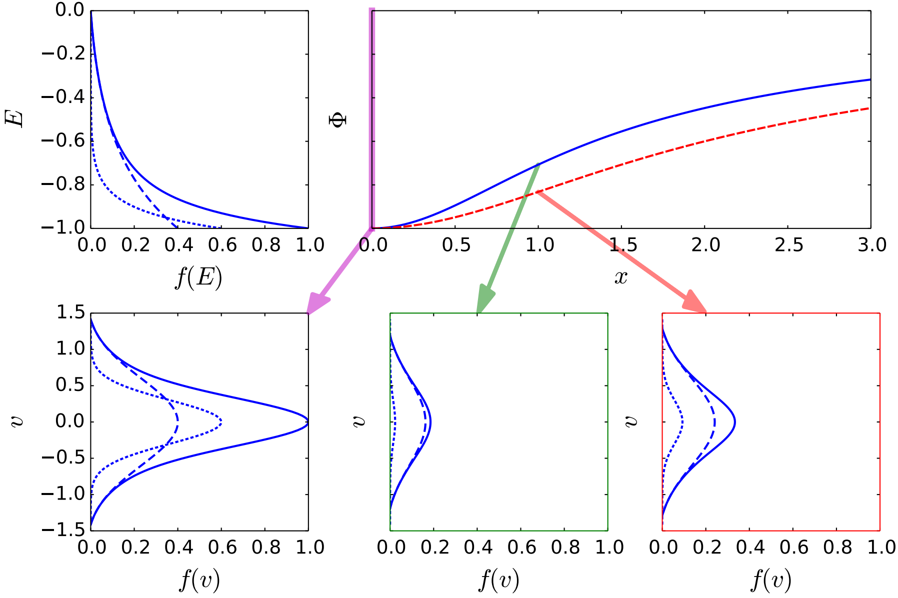


If $f(x, v)$ is not a single Gaussian:

its shape at different x becomes sensitive to Φ $\Rightarrow$

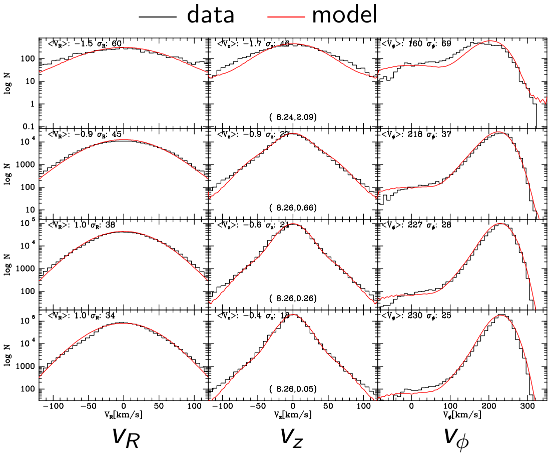
can infer potential from the shape of velocity distribution $f(x, v)$ even without knowing its overall normalization ρ.

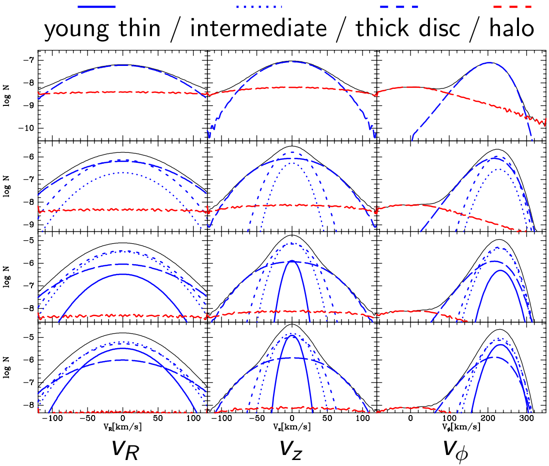
Inferring the potential from kinematic data



Model–data comparison: kinematics

significantly non-Gaussian velocity distributions are produced by a superposition of several components (thin disc with a certain age–velocity dispersion relation discretized into three parts, thick disc and stellar halo).





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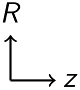
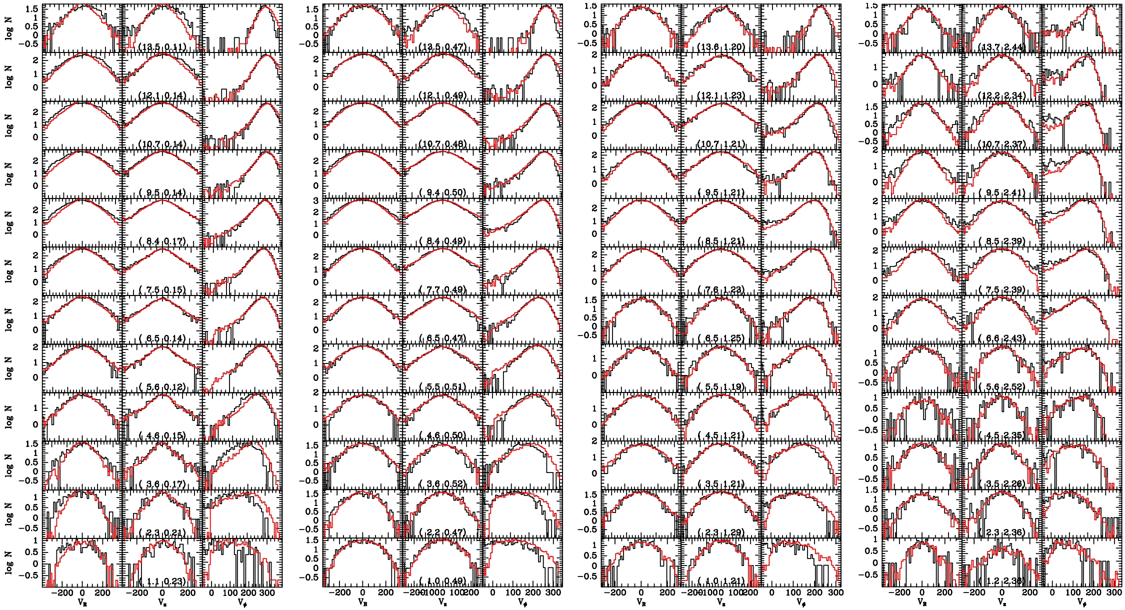
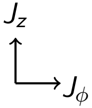


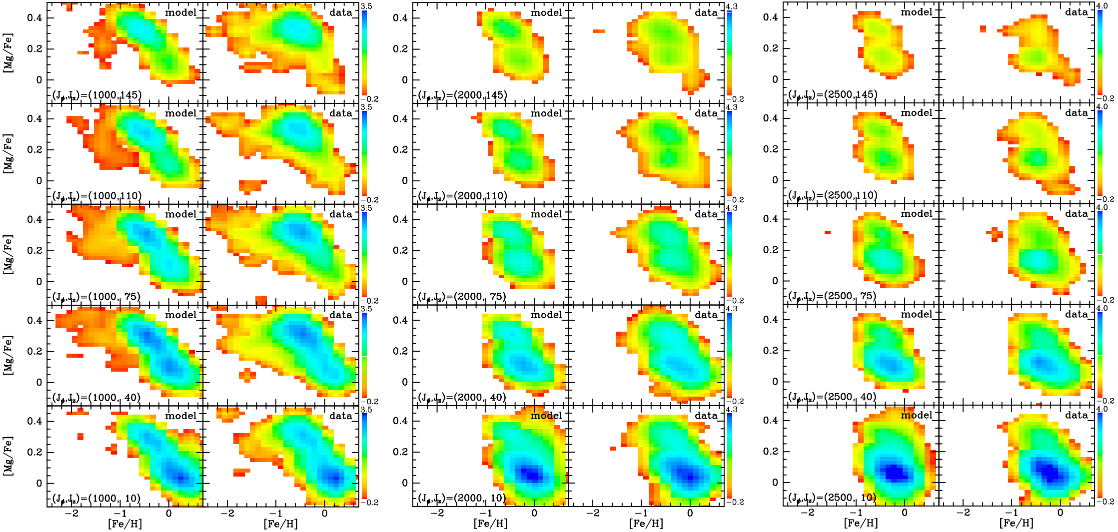
fits to velocity histograms across the entire disc: not perfect, but reasonable



Model–data comparison: chemistry

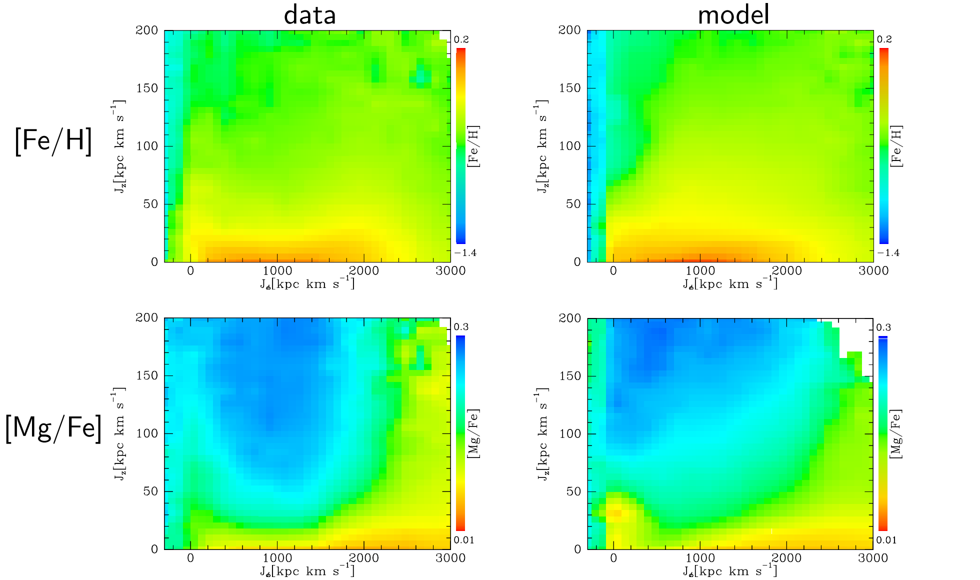


fits to chemical histograms [Fe/H] vs. [Mg/Fe] in 30 bins
in the J_ϕ–J_z space: qualitatively reproduce the main features
(e.g., α-poor becoming geometrically thick outside $R_\odot$)



Model–data comparison: chemistry

chemical gradients in the action space are still steeper in the data; the model struggles to reproduce the sharp transition to the outer α-poor but vertically thick disc at $R \gtrsim R_\odot$.



Caveats and limitations of the model

- axisymmetry (needed for action computation) $\Rightarrow$
  the bar region is not adequately represented.
- equilibrium (precondition for the Jeans theorem) $\Rightarrow$
  features such as the *Gaia* snail or spiral arms are ignored in the baseline model, but can be considered in the perturbation theory.
- only fit the mean abundances
  (although the underlying chemical model provides a full abundance distribution).
- stellar ages are ignored (the age-σ relation is imposed implicitly):
  the age distribution may be treated similarly to the chemical one.
- no built-in chemical evolution or radial migration model.
- presented one plausible model, but cannot claim to have found the global maximum-likelihood solution $\Rightarrow$
  model fitting in the 100-dimensional parameter space is a nightmare, need better optimisation methods.

MORE WORK IS NEEDED

- axisymmetry (needed for action computation) ⇒
  the bar region is not adequately represented.
- equilibrium (precondition for the Jeans theorem) ⇒
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Joan Ponç – Física