Neutron stars: from macroscopic to microscopic dynamics

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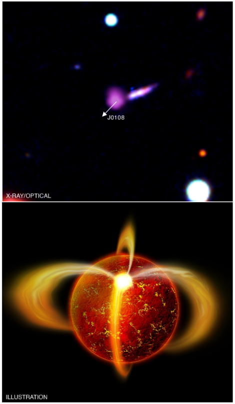
Overview



Pulsar PSR J0108-1431, located only 770 light years from us. X-ray: NASA/CXC/Penn State/G.Pavlov et al.; Optical: ESO/VLT/UCL/R.Mignani et al.; Illustration: NASA/CXC/M.Weiss.

Neutron stars are remnants of main-sequence stars that undergo core-collapse supernovae.

- Magnetars, radio pulsars, x-ray pulsars, magnetar+pulsar, radio-quiet neutron stars...
- Mass from 1 to 2 $M_\odot$ (typically) and radii of about 10 km.
- Second densest objects in the Universe $\rho \sim 5n_0 - 10n_0$ ($n_0 \sim 3 \times 10^{14}$ g/cm^3).
- Strongest magnets in the Universe, $B \sim 10^4 - 10^{11}$ T.
- "Cold stars", $T \sim 10^6$-10^{11} K.
- Rotational periods from milliseconds to seconds.

Relativistic fluid dynamics

Neutron stars are modeled using relativistic fluid dynamics (effective description of the dynamics of conserved quantities):

- Uncharged ideal fluids (equilibrium or absence of fluctuations), the energy-momentum tensor is given by

$$T^{\mu\nu}_{(0)} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu},$$

  ϵ (energy density), P (pressure), u^μ (fluid velocity, $u^\mu u_\mu = -1$), and $g^{\mu\nu}$ (metric tensor).

- Dissipative fluid dynamics (small deviations from equilibrium), here:

$$\langle T^{\mu\nu} \rangle = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + T^{\mu\nu}_{(2)} + \cdots ,$$

  the subscripts indicate the number of gradient terms. (e.g. the building blocks for $T^{\mu\nu}_{(1)}$ are $\nabla_\mu \ln \epsilon$ and $\nabla_\mu u_\nu$.)

- Fluids out of equilibrium (gradients of hydrodynamics fields are not small)

Hydrodynamics equations

The equations of motion are consequence of the conservation laws of the system (energy, momentum, and mass or particle number)

$$\nabla_\mu T^{\mu\nu}_{(0)} = 0, \quad \text{or} \quad u_\nu \nabla_\mu T^{\mu\nu}_{(0)} = 0, \quad \text{and} \quad \Delta^\rho_\nu \nabla_\mu T^{\mu\nu}_{(0)} = 0.$$

and we get the continuity equation and the relativistic Euler equation

$$D\epsilon + (\epsilon + P)\nabla^\perp_\mu u_\mu = 0, \quad (\epsilon + P) D u^\rho + c_s^2 \nabla^\rho_\perp \epsilon = 0,$$

respectively ($\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$, $D = u^\mu \nabla_\mu$, and $\nabla^\rho_\perp = \Delta^{\mu\rho}\nabla_\mu$).
Using the 1st correction we get the relativistic Navier-Stokes equations

$$\begin{aligned} D\epsilon + (\epsilon + P)\nabla^\perp_\lambda u^\lambda &= \frac{\eta}{2}\sigma^{\mu\nu}\sigma_{\mu\nu} + \zeta(\nabla^\perp_\lambda u^\lambda)^2, \\ (\epsilon + P) D u^\alpha + c_s^2 \nabla^\alpha_\perp \epsilon &= \Delta^\alpha_\nu \nabla_\mu (\eta \sigma^{\mu\nu} + \zeta \Delta^{\mu\nu} \nabla^\perp_\lambda u^\lambda). \end{aligned}$$

where $\sigma^{\mu\nu} = \nabla^\mu_\perp u^\nu + \nabla^\nu_\perp u^\mu - \frac{2}{d-1}\Delta^{\mu\nu}\nabla^\perp_\lambda u^\lambda$.
c_s, ζ, and η are the speed of sound, the shear and bulk viscosities, respectively.

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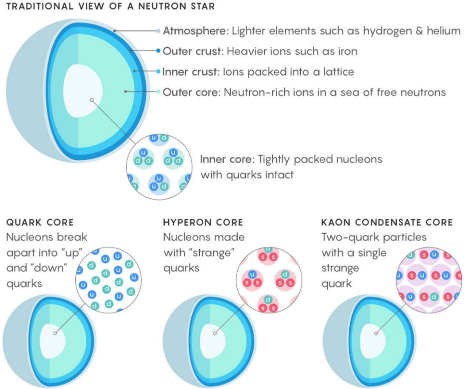
Transport coefficients

c_s, ζ, and η are transport coefficients (they describe the transport of physical quantities within the medium due to non-equilibrium processes)

arXiv:1712.05815

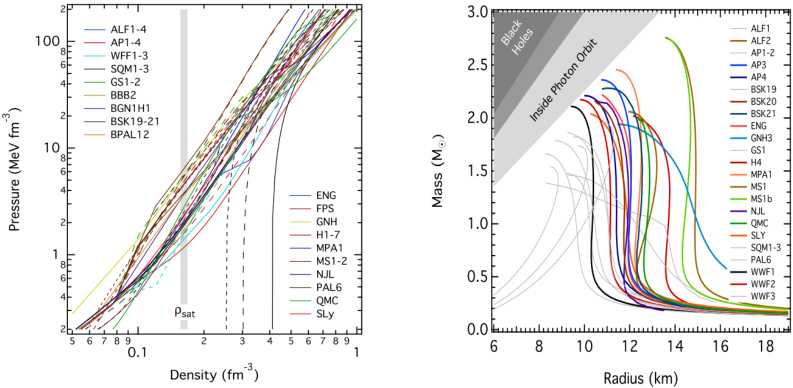
- Hydrodynamics only provides relations among them (Einstein relations).
- Strongly dependence on the microscopic physics.



Lucy Reading-Ikkanda/Quanta Magazine; Source: Feryal Özel.

Equation of state

The equation of state (EOS, the thermodynamic relation between the mass or energy density and pressure).

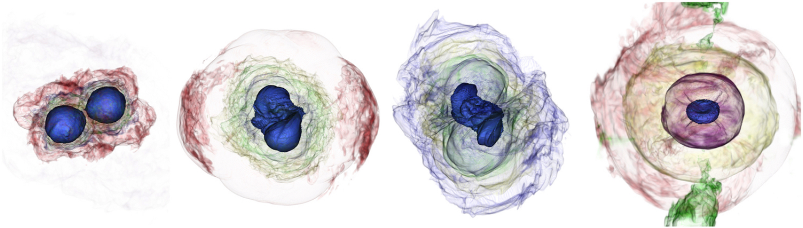


(Left) A large sample of proposed EOS calculated under different physical assumptions.
(Right) The mass-radius curves corresponding to the EOS in the left. arXiv:1603.02698

Relativistic numerical simulations

Macroscopic and microscopic physics are required for numerical simulations of rotating neutron stars and neutron star mergers.

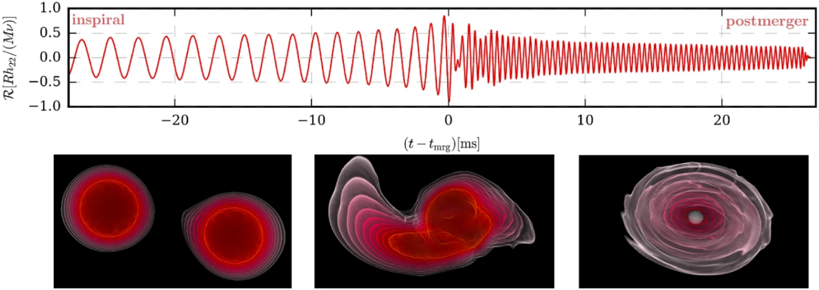
- Relativistic hydrodynamic equations, Einstein field equations, EOS and the Einstein relations.
- Complex implementation of numerical methods and computational techniques to solve nonlinear partial differential equations.



GRMHD simulations of the merging of two neutron stars from the moment before the neutron stars meet, when their mutual gravity stretches them into teardrop shapes, to the merger aftermath, when an accretion disk feeds the sole remaining star. arXiv:2206.03618

Bulk viscosity in mergers

Dissipation encoded in ζ can be relevant for neutron star mergers. e.g.
arXiv:1707.09475



Numerical simulations of a merger showing the gravitational-wave profile and the matter evolution. arXiv:2004.02527

e.g. hadronic matter (e^-, n, p)

$$\zeta = -\frac{\mathrm{Re}[\Pi]}{\theta}, \quad \theta = \partial_\mu u^\mu, \quad \Pi = \delta P(\delta n_e, \delta n_n, \delta n_p)\,\text{(out of equilibrium)},$$

deviations of the particle densities depends on the weak processes (Urca-type and modified Urca-type processes)

Thank you for your attention