Deflection of gravitational waves by astrophysical objects

ICCUB Winter meeting 2024

Helena Ubach

Virgo group at ICCUB

UNIVERSITAT DE BARCELONA

ICCUB

Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

EXCELENCIA MARÍA DE MAEZTU 2020-2023

A direct proof of General Relativity

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(A. Einstein, 1916)

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- mass/energy → gravity

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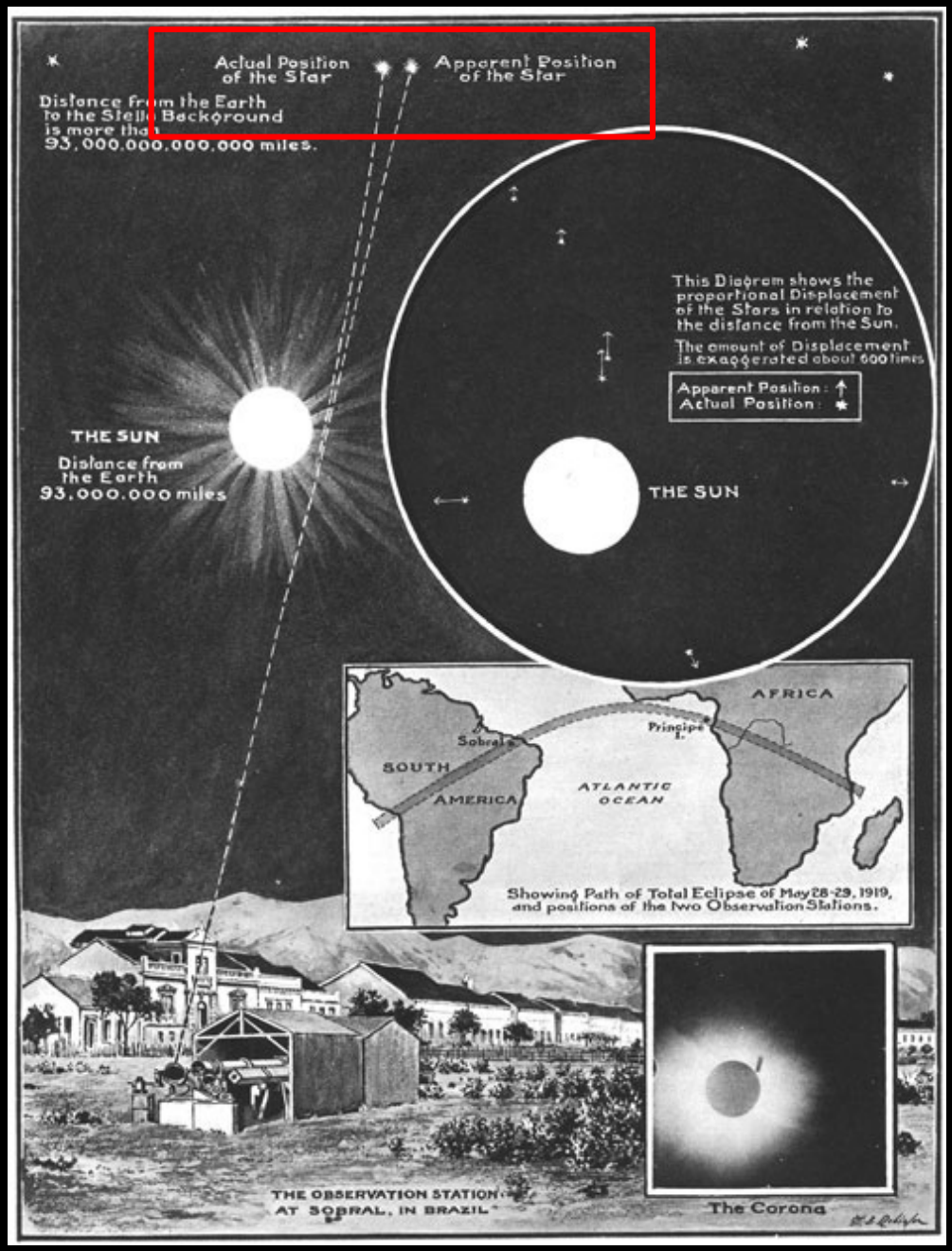
A. Eddington



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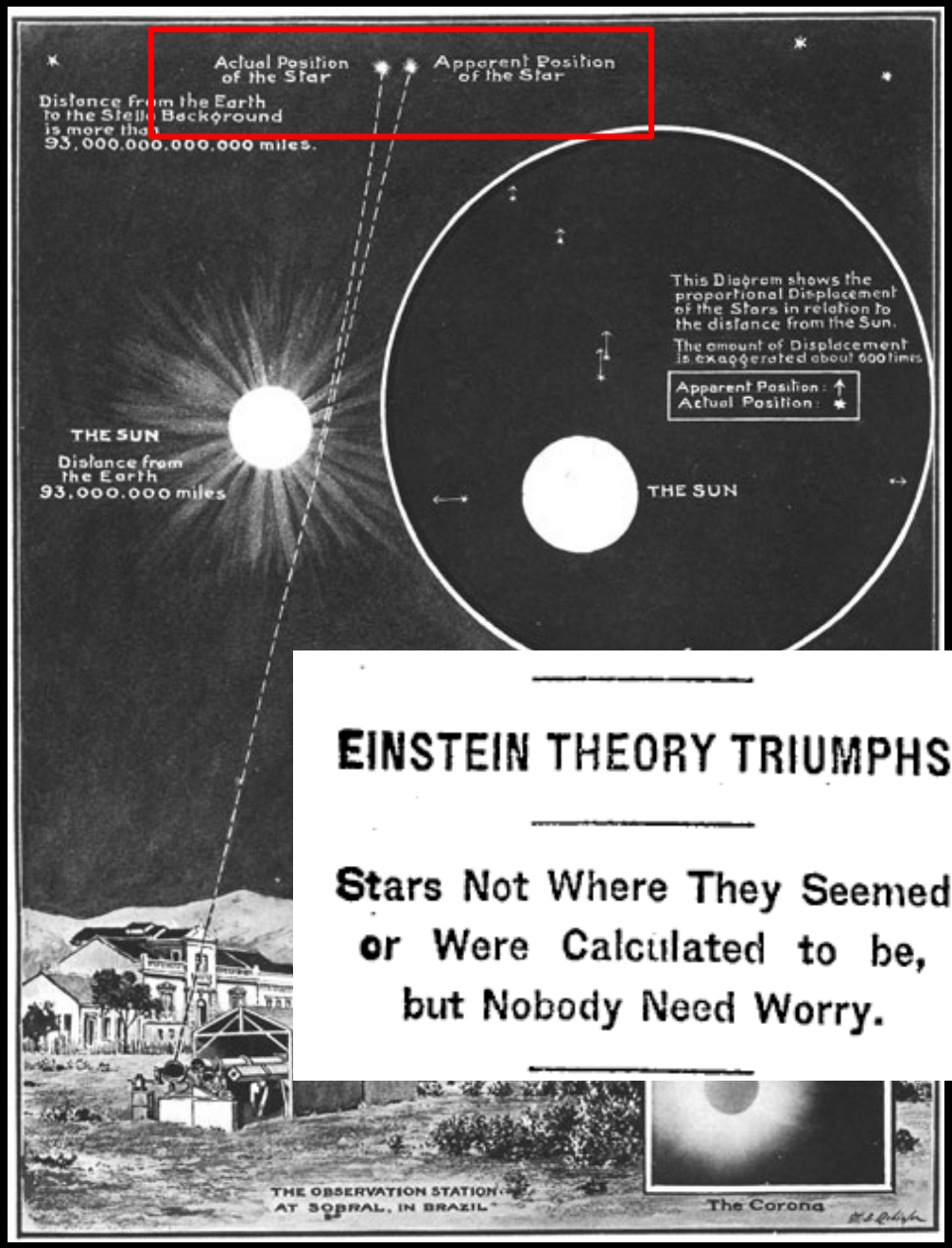
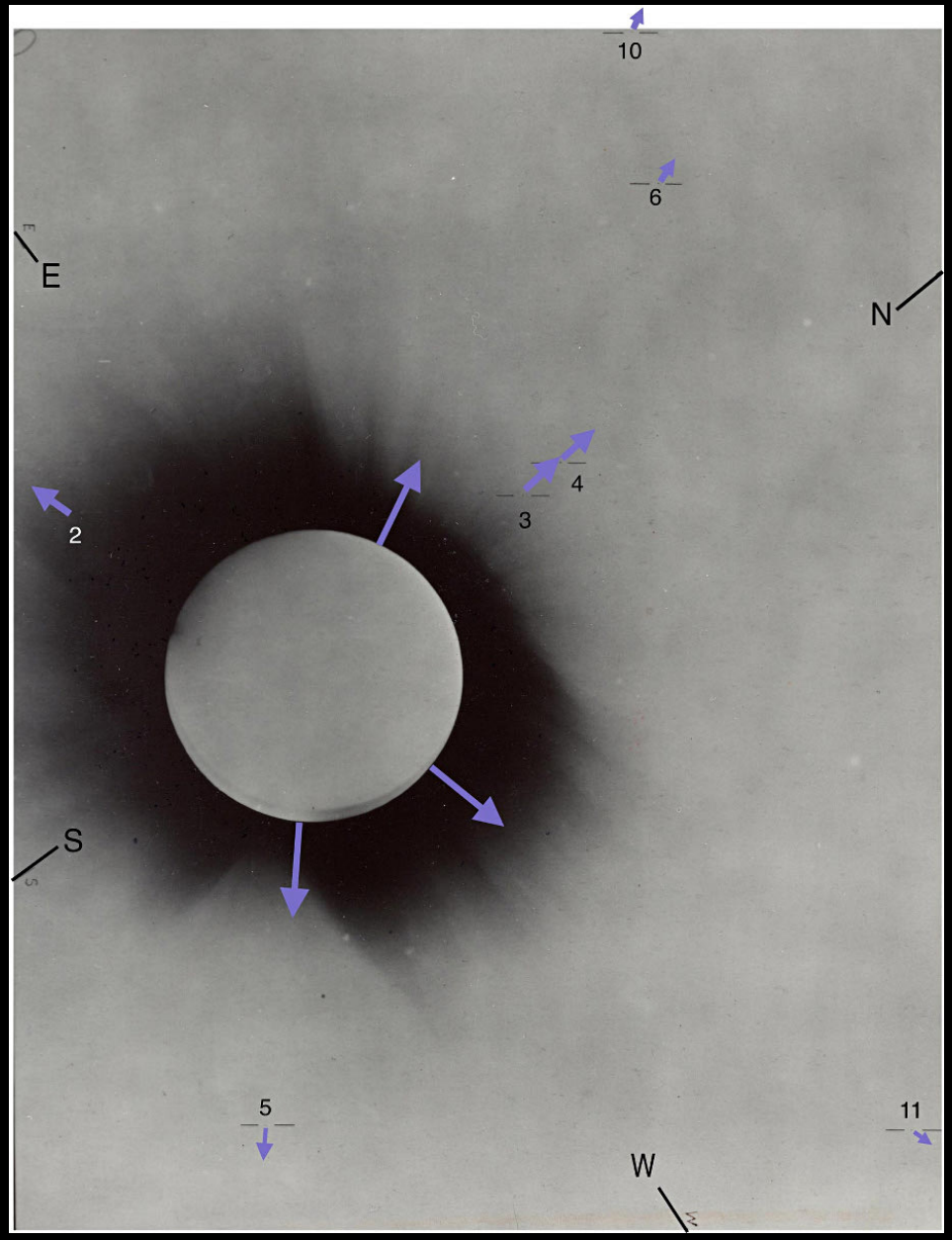
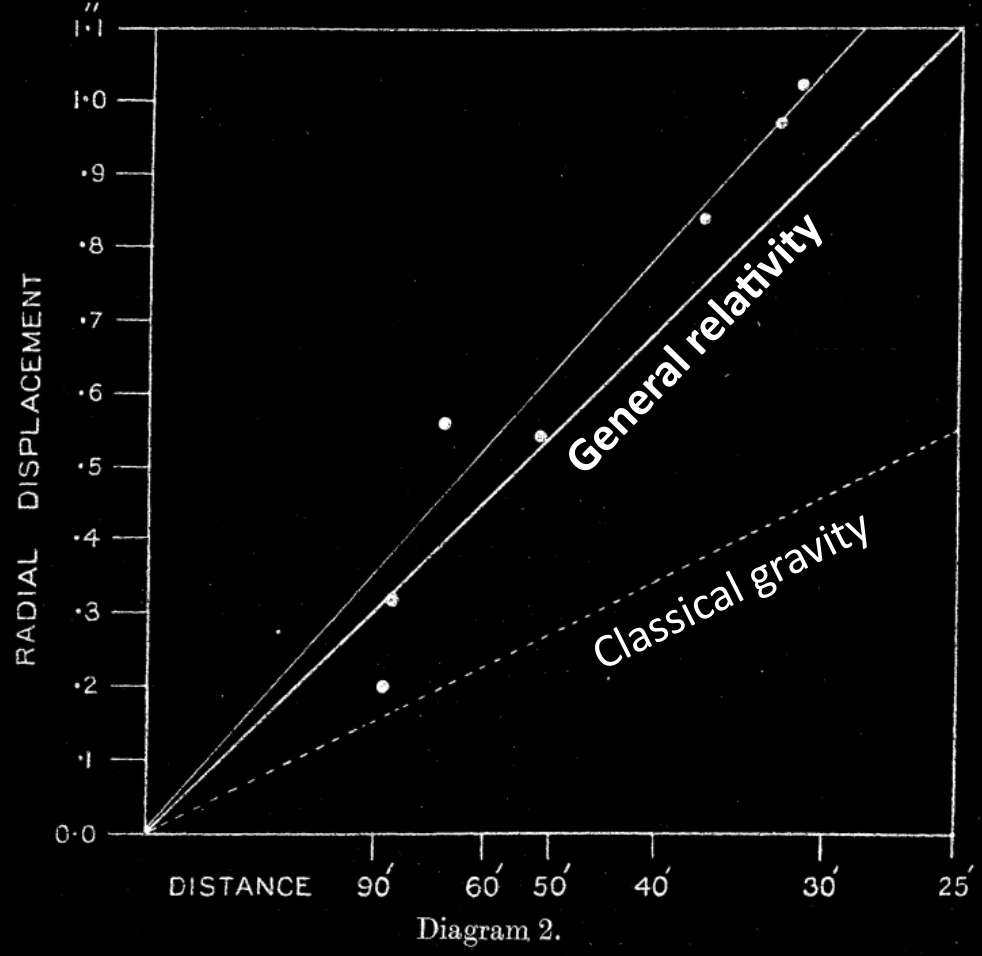


Gravitational lensing



Telescope plate [Image courtesy of Robin Catchpole]



Dyson, F. W. ; Eddington, A. S. ; Davidson, C. (1920)

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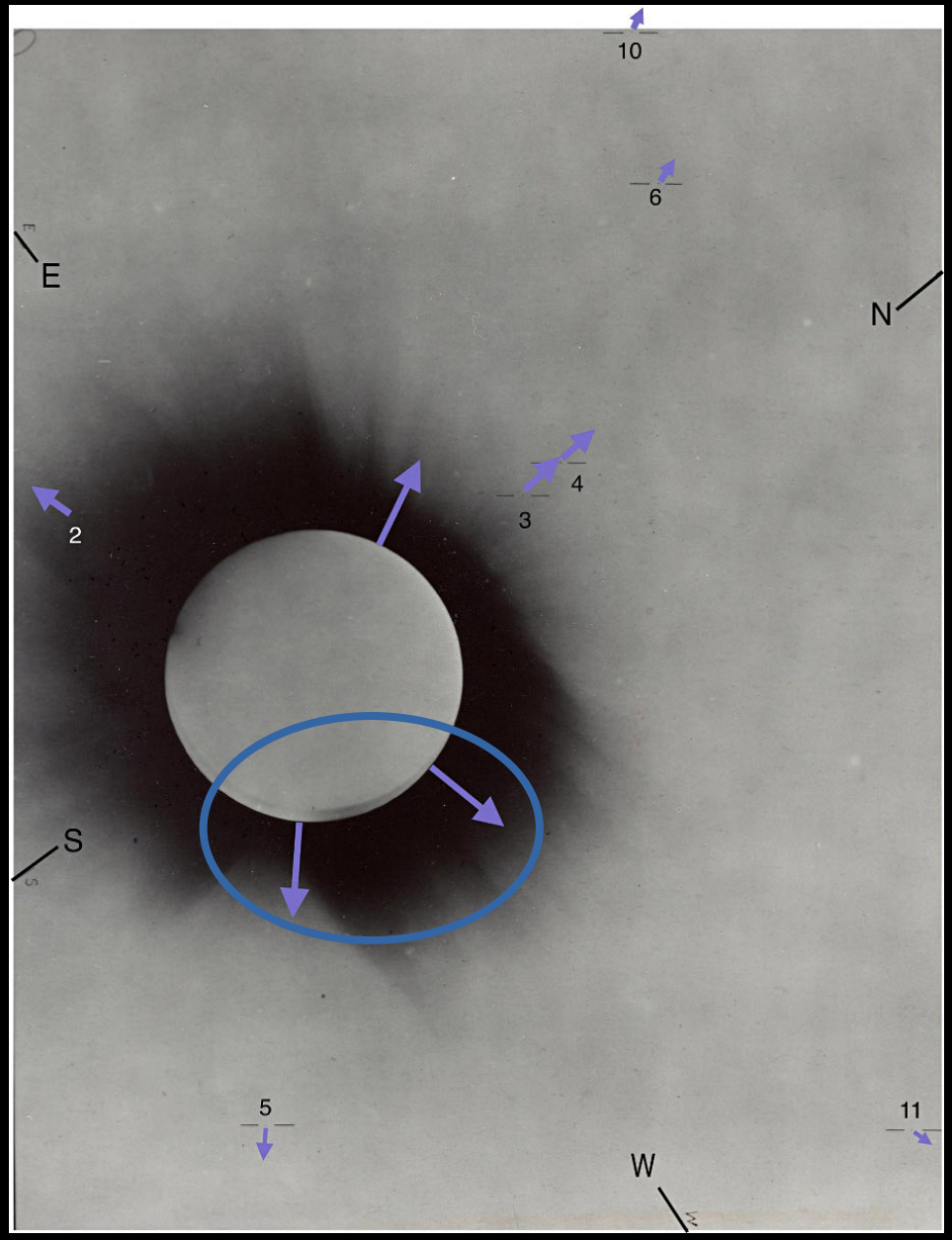

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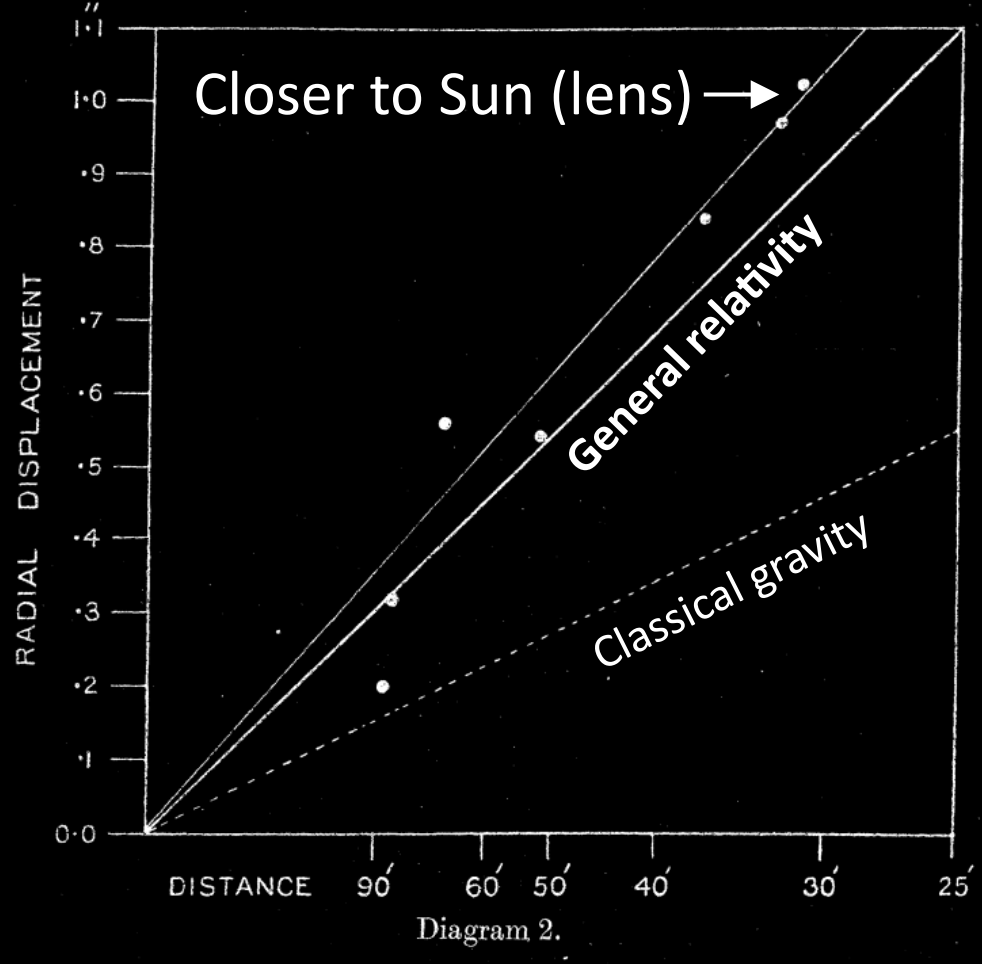

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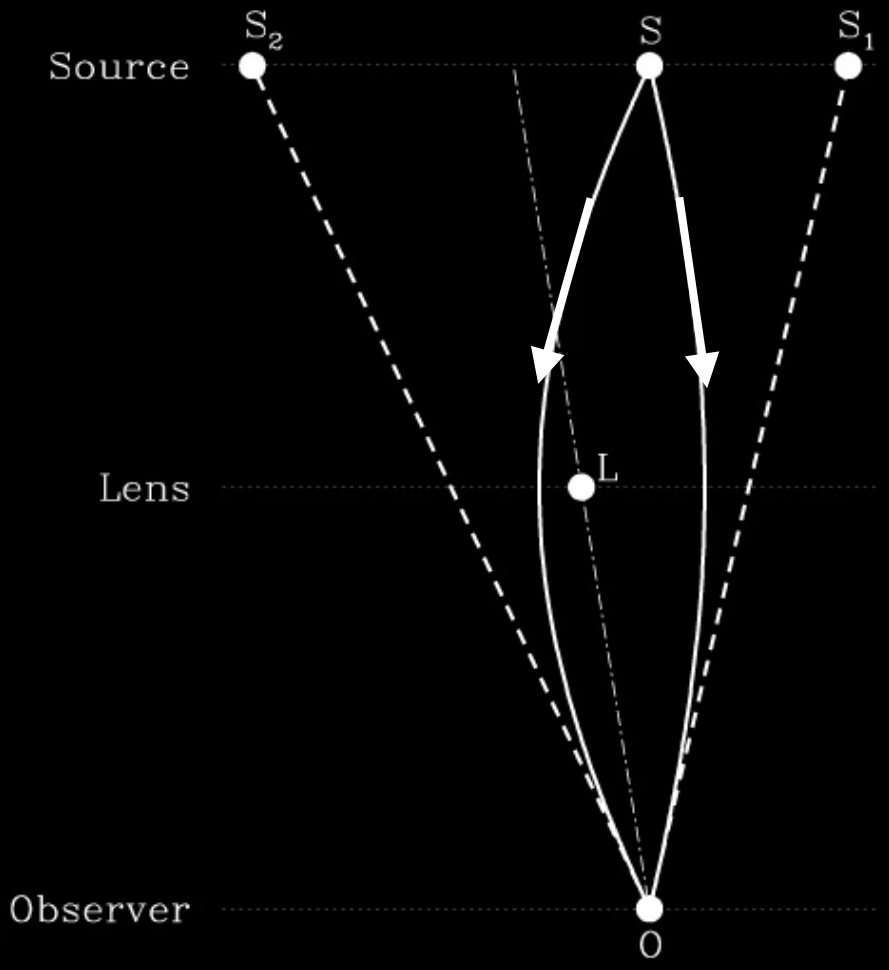
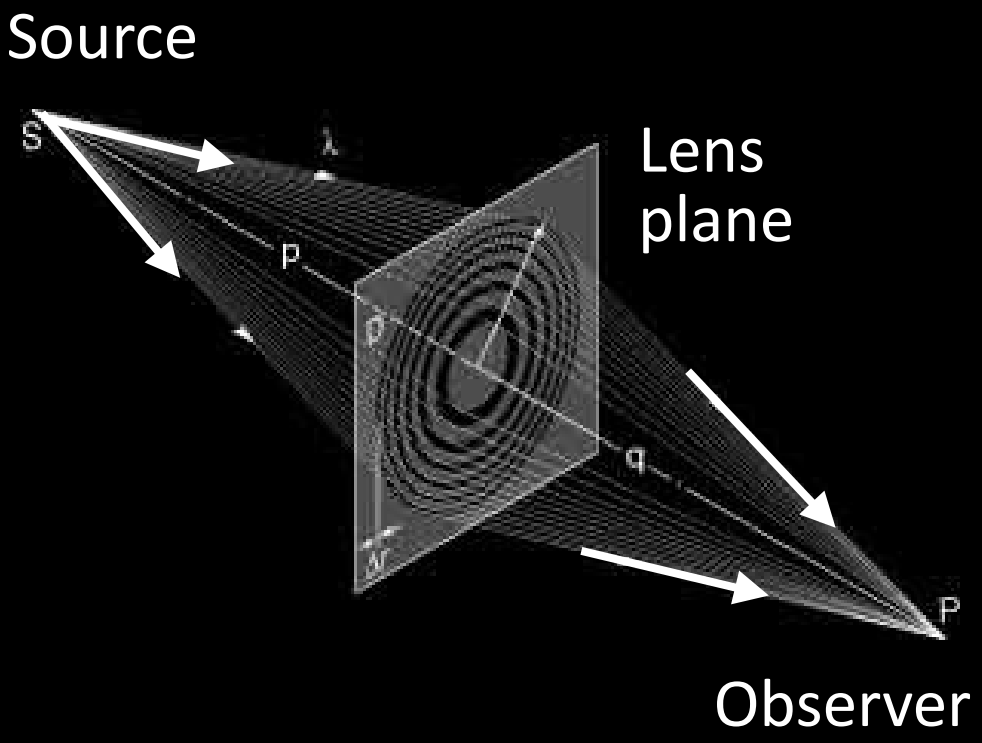


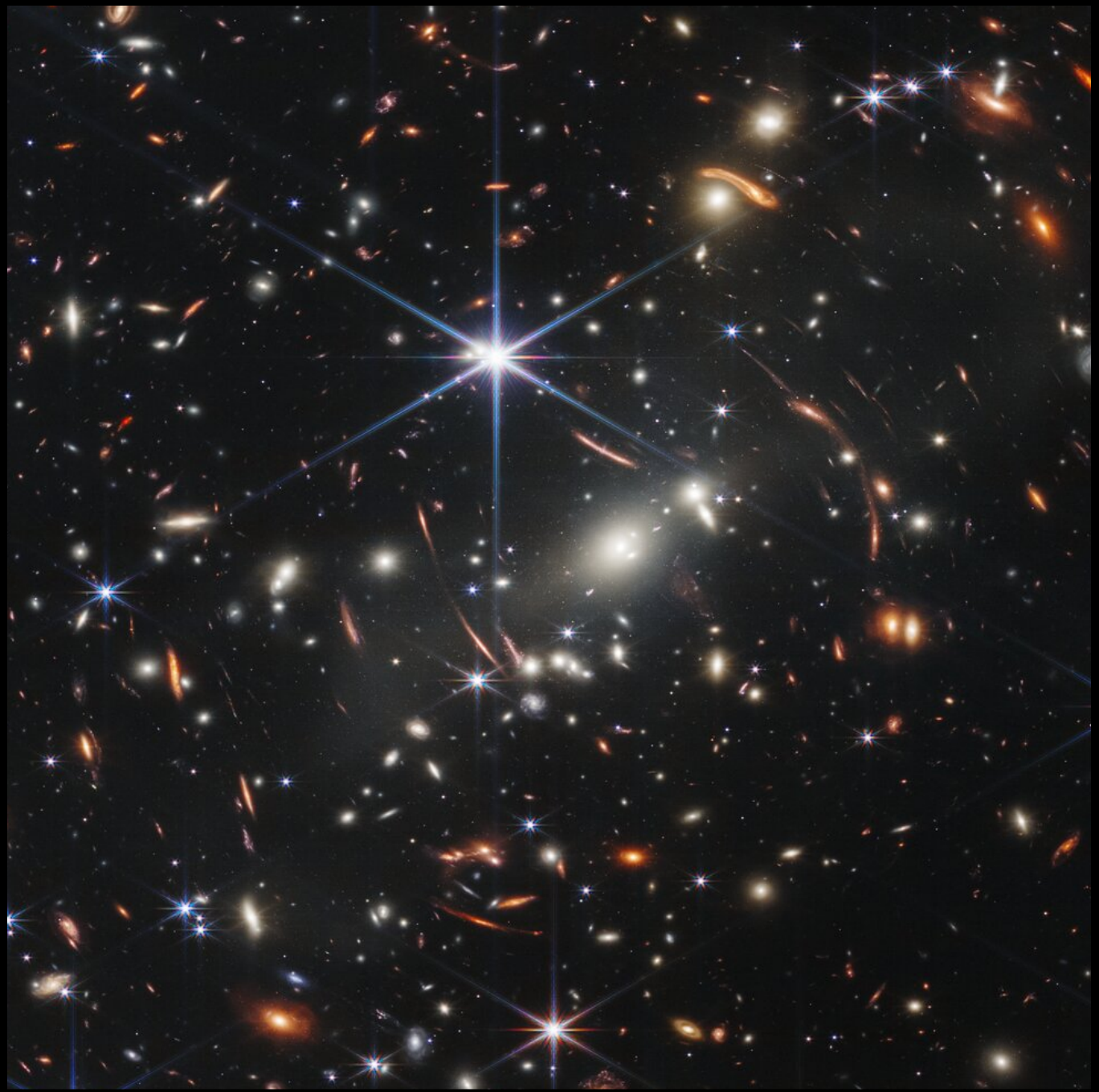
Joachim Wambsganss



D. T. Atwood

Gravitational lensing

...of light:



Webb Space Telescope (2023)

Gravitational lensing

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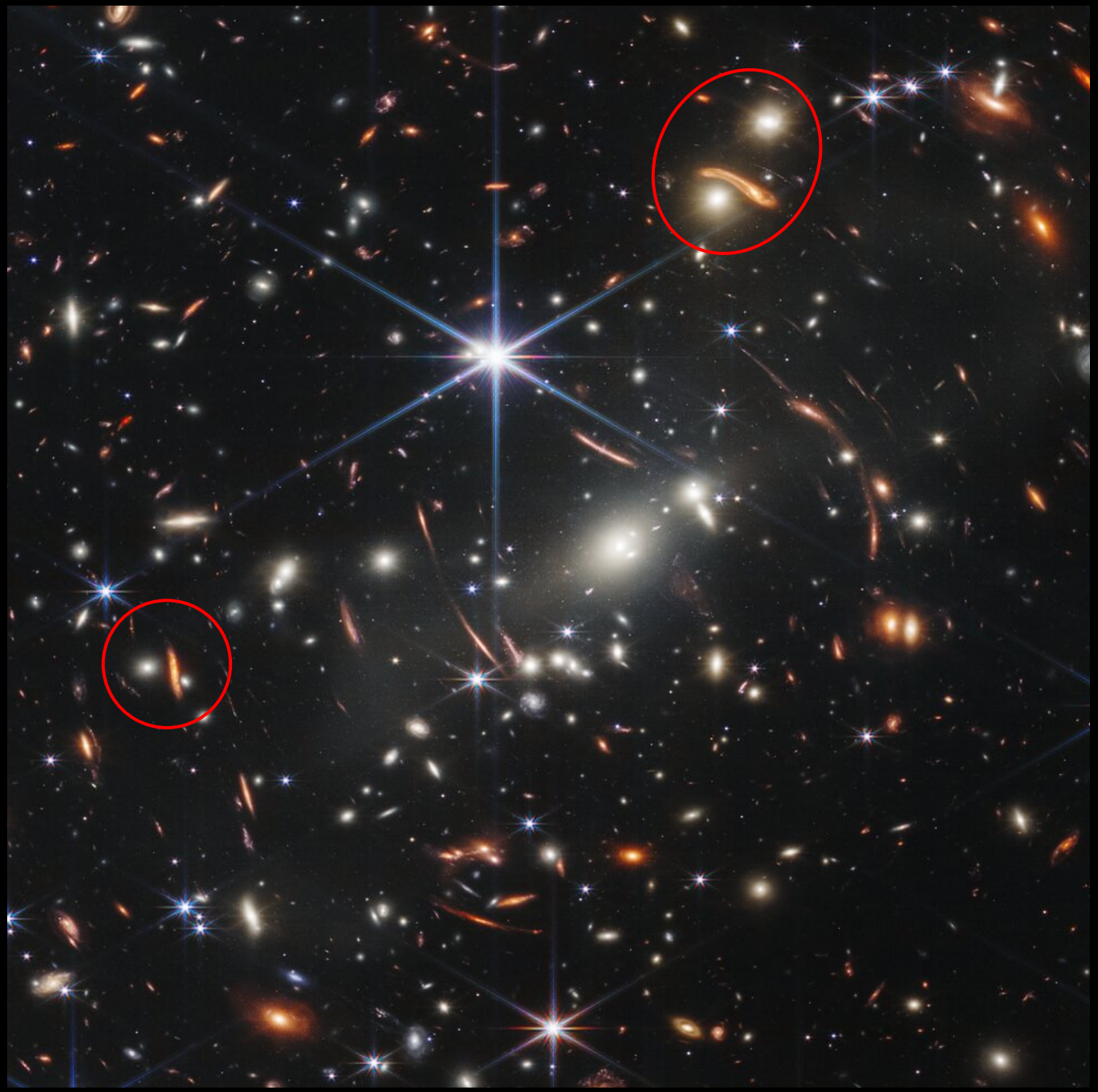

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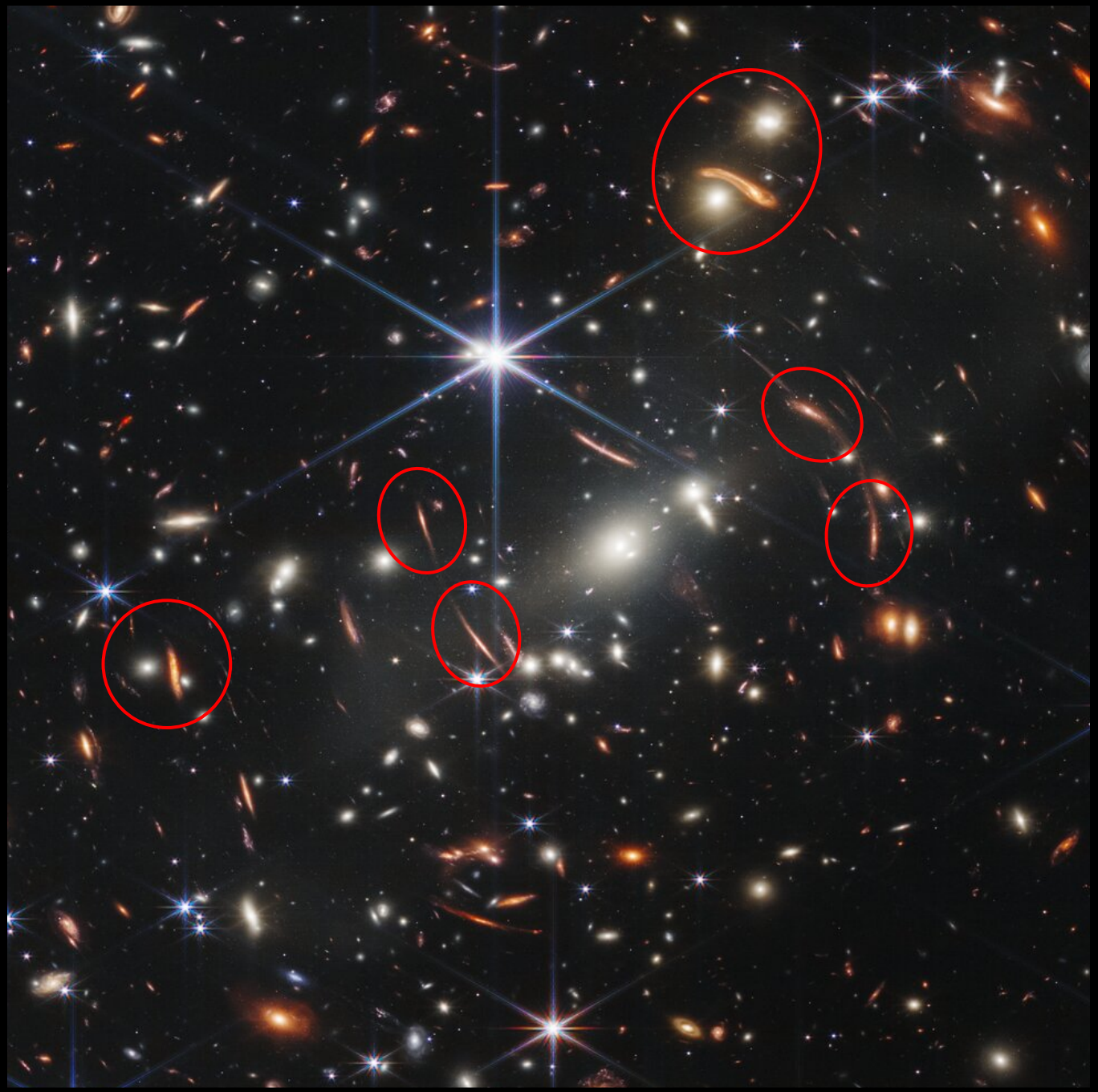

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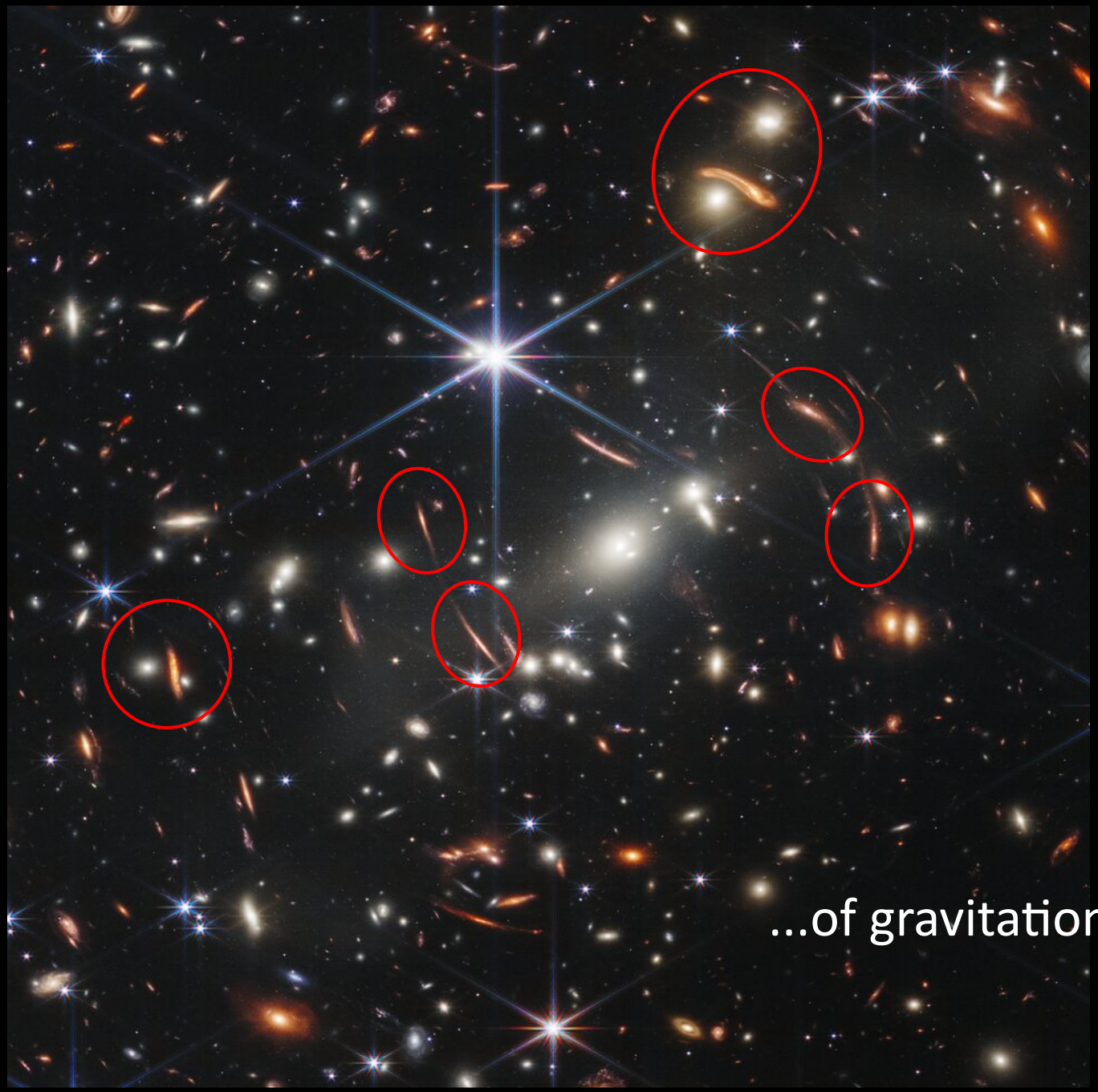

...of gravitational waves?

Webb Space Telescope (2023)

Gravitational lensing

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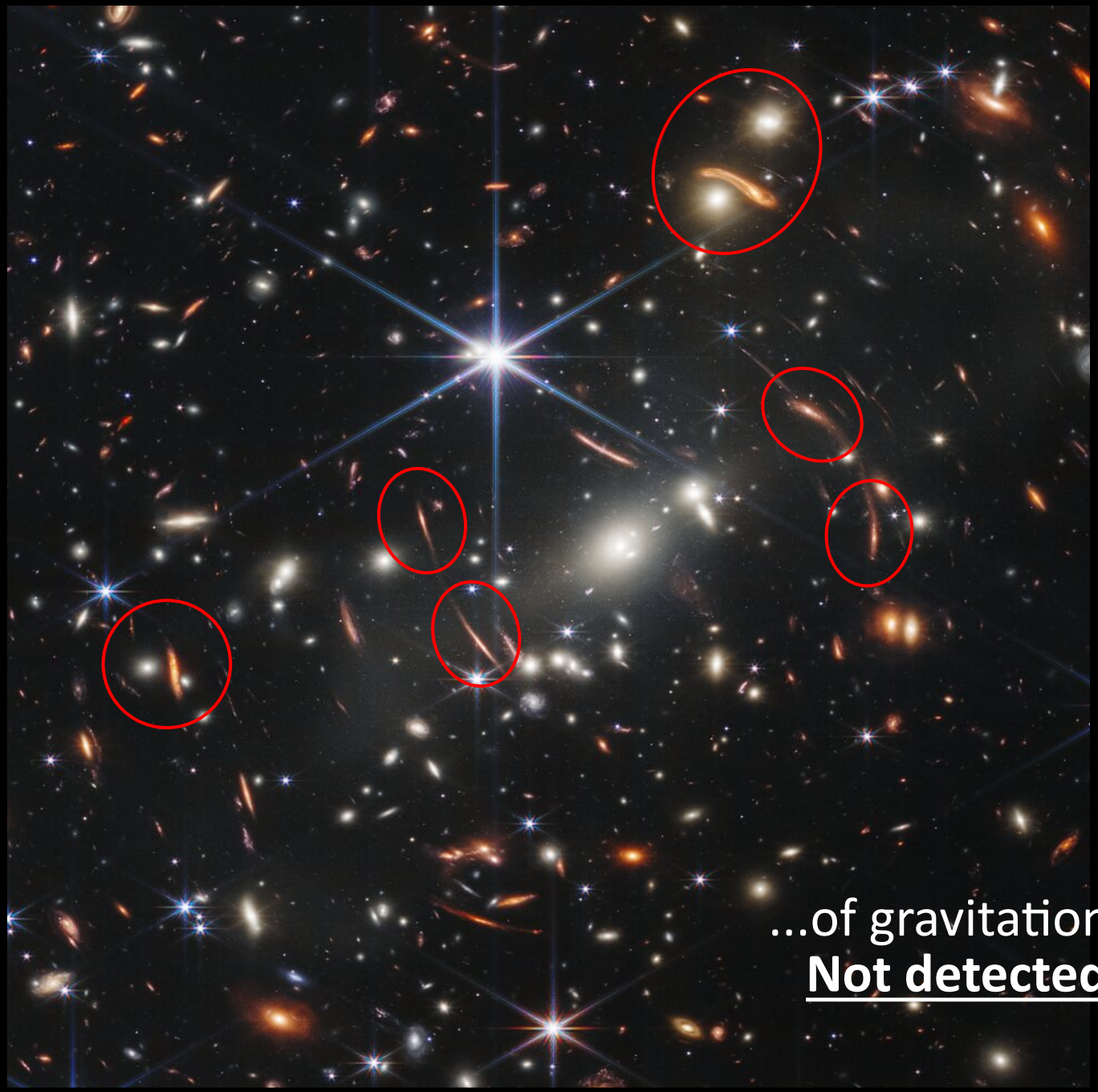
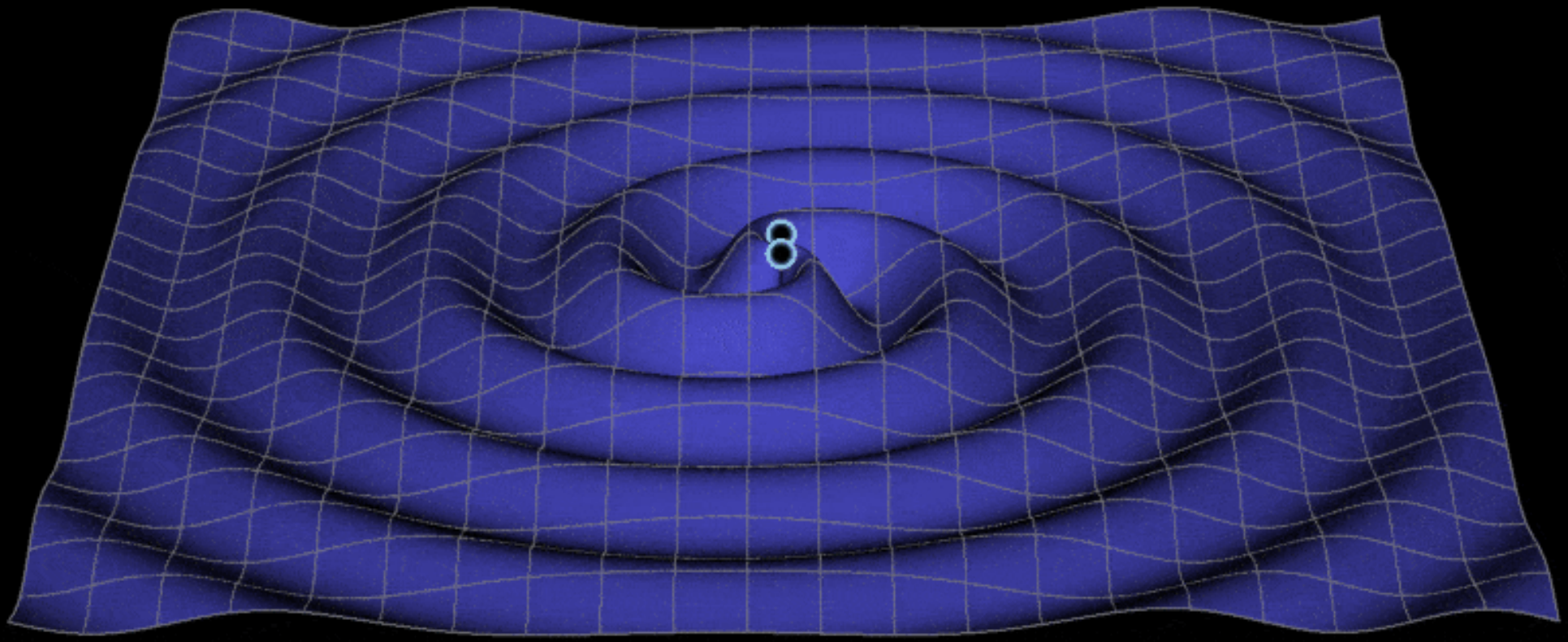


Webb Space Telescope (2023)

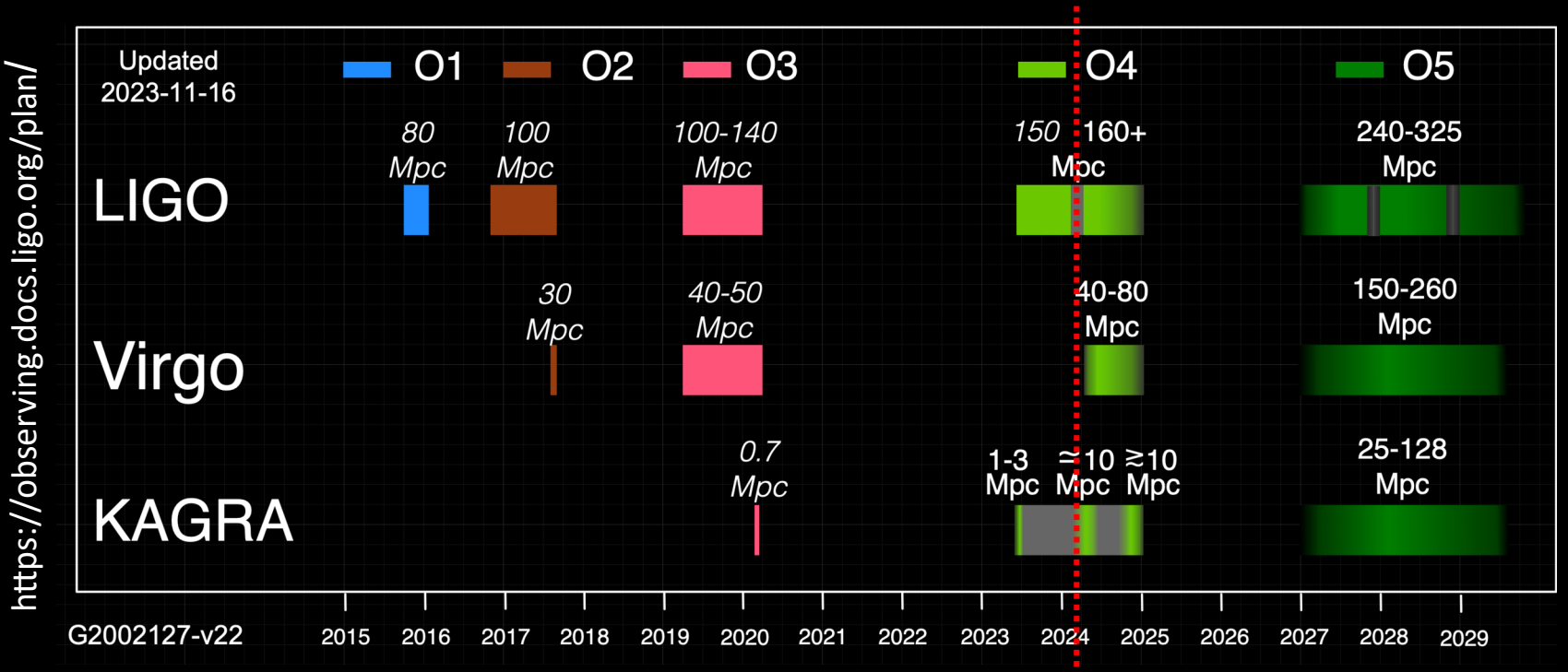
...of gravitational waves?
<u>Not detected</u> (yet!)

Gravitational waves

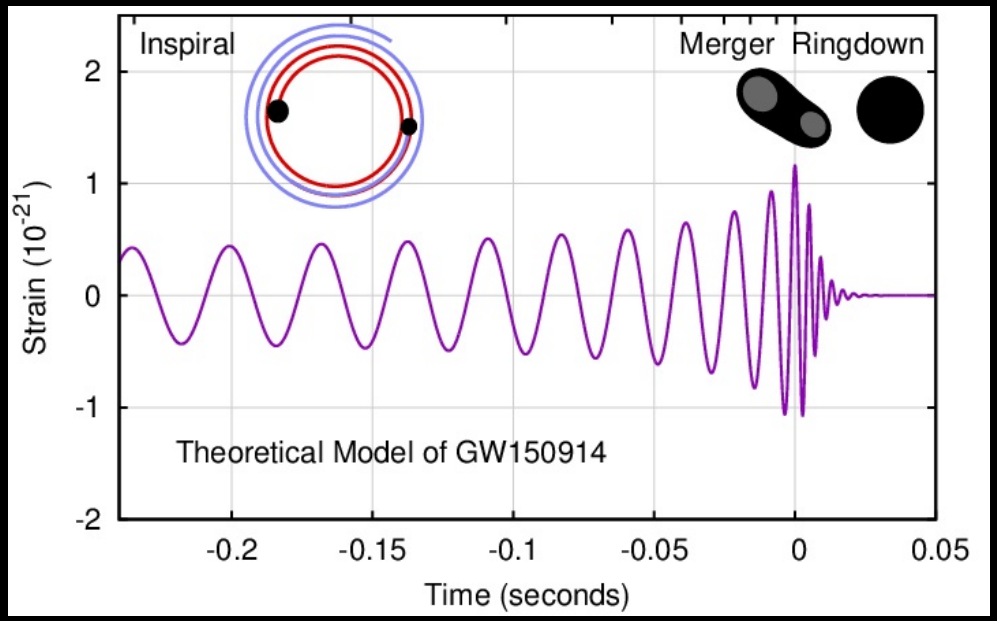


Jeff Bryant, Wolfram|Alpha, LLC

Gravitational waves: detections



2 merging neutron stars/ black holes



Rochester Institute of Technology

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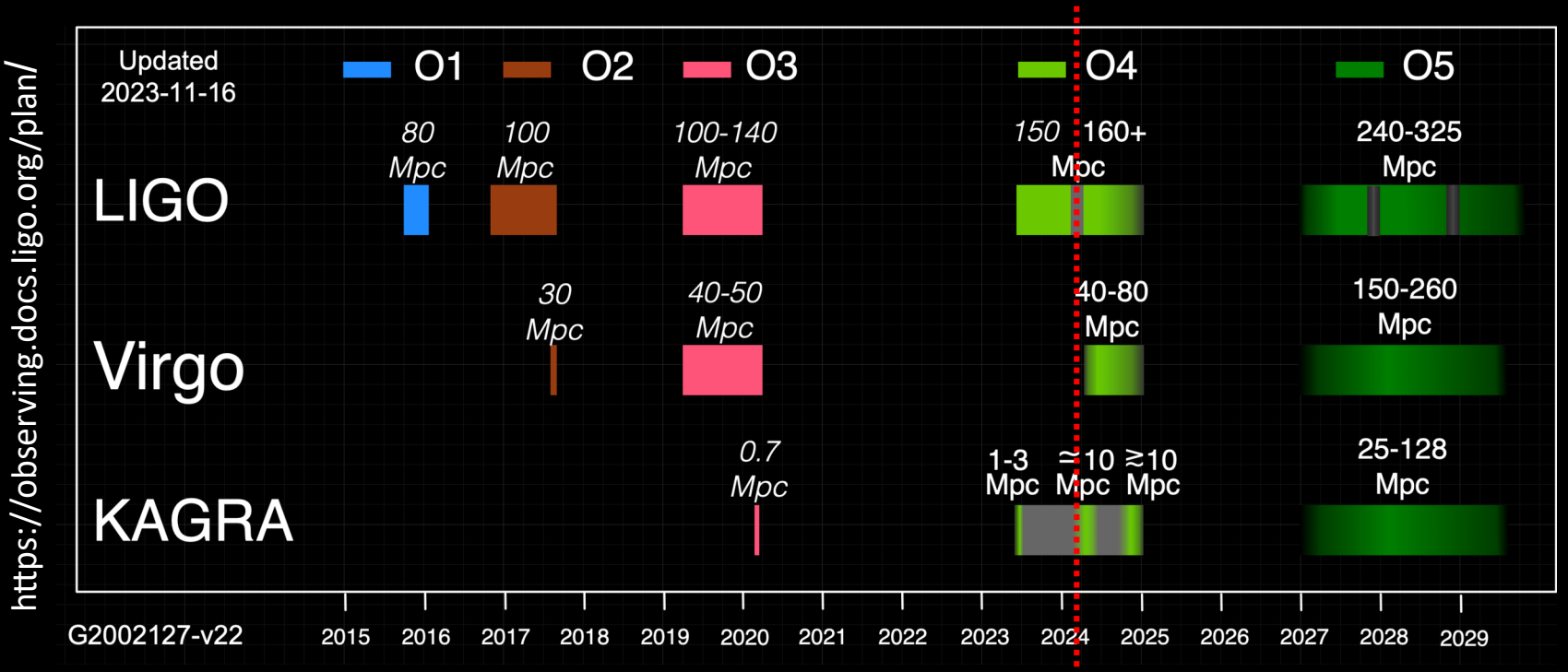

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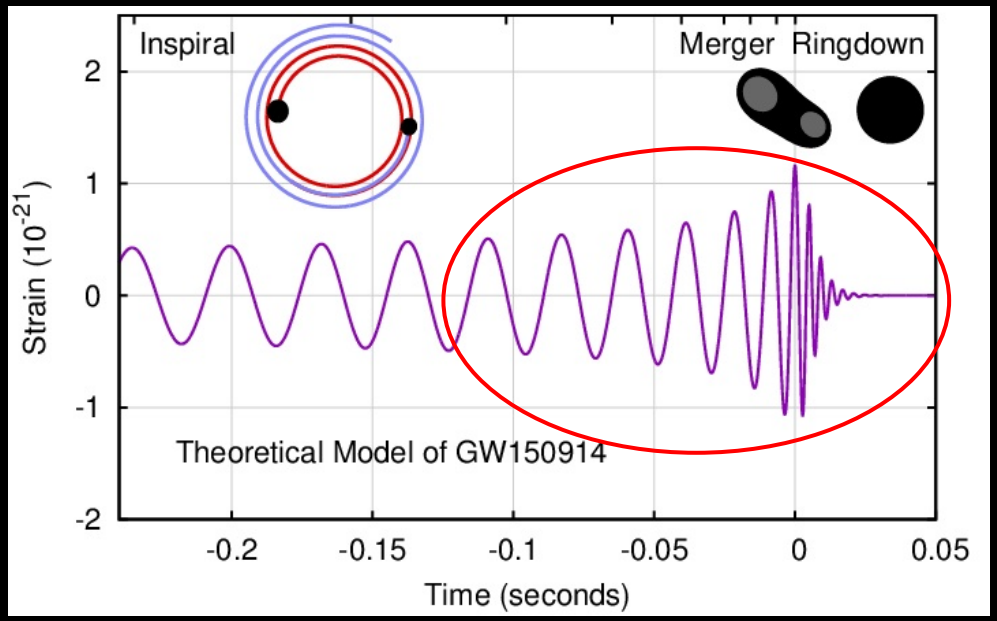


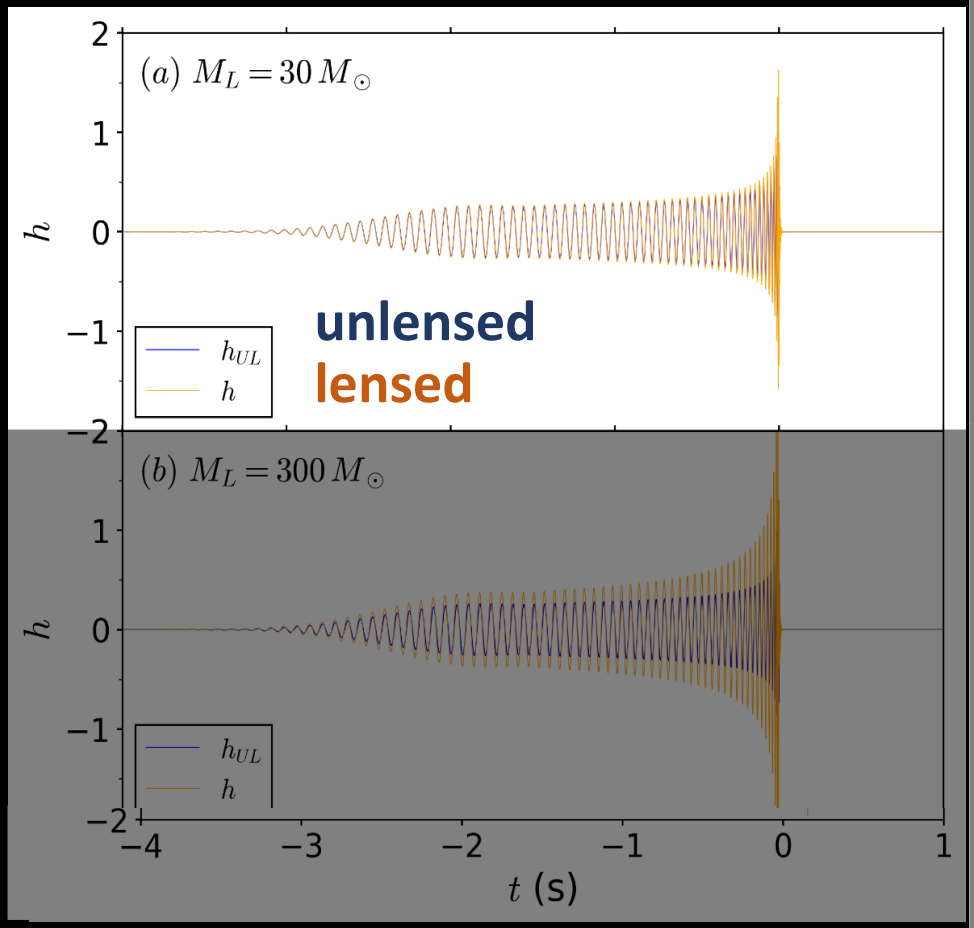
Last orbits

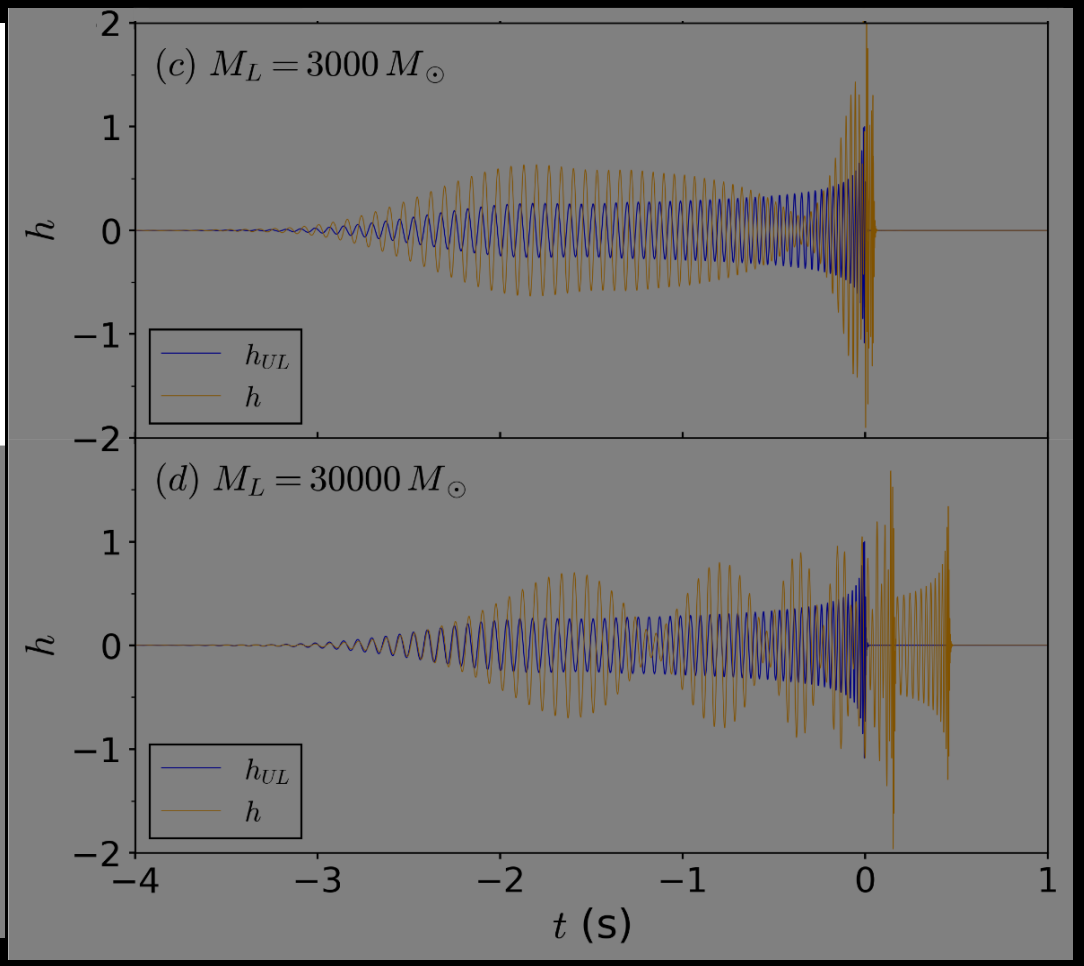
Rochester Institute of Technology

Lensing imprint

Lower lens mass / lower GW frequency

Point mass lens model





Bondarescu, Ubach, Bulashenko, Lundgren (2023)

Higher lens mass / higher GW frequency

Lensing imprint

Lower lens mass / lower GW frequency

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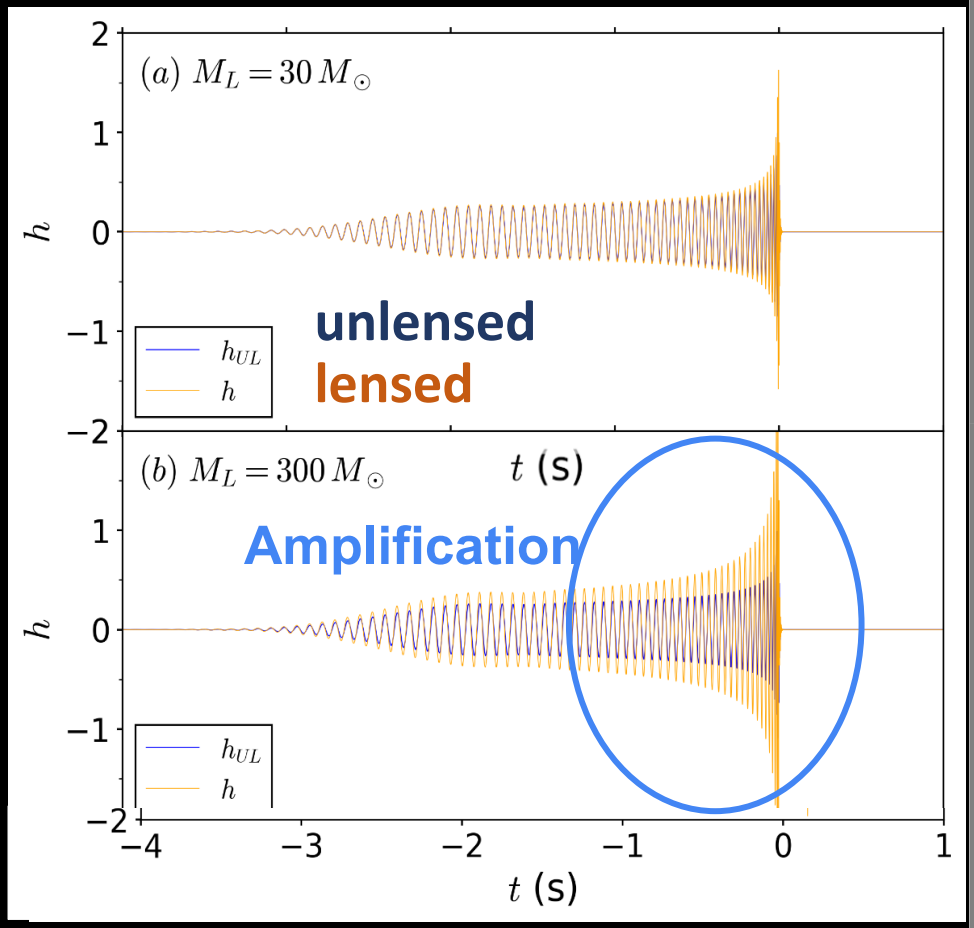

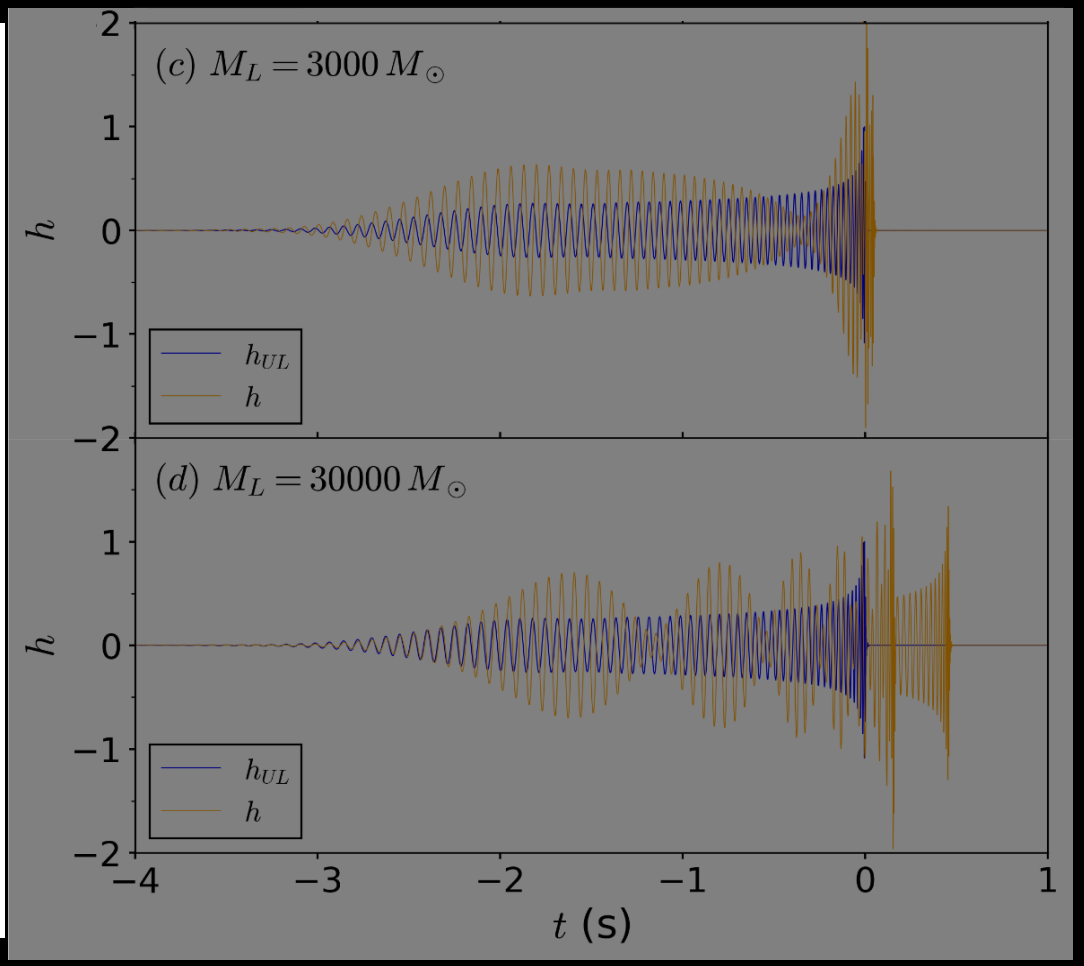

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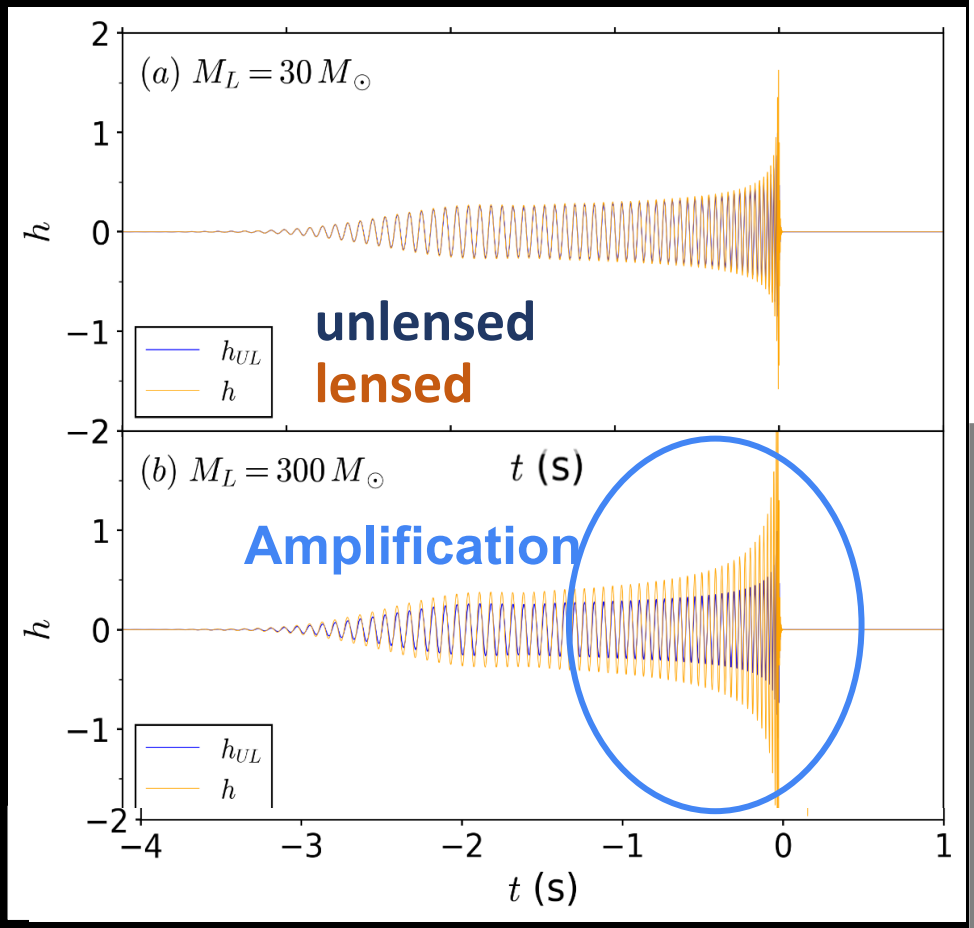

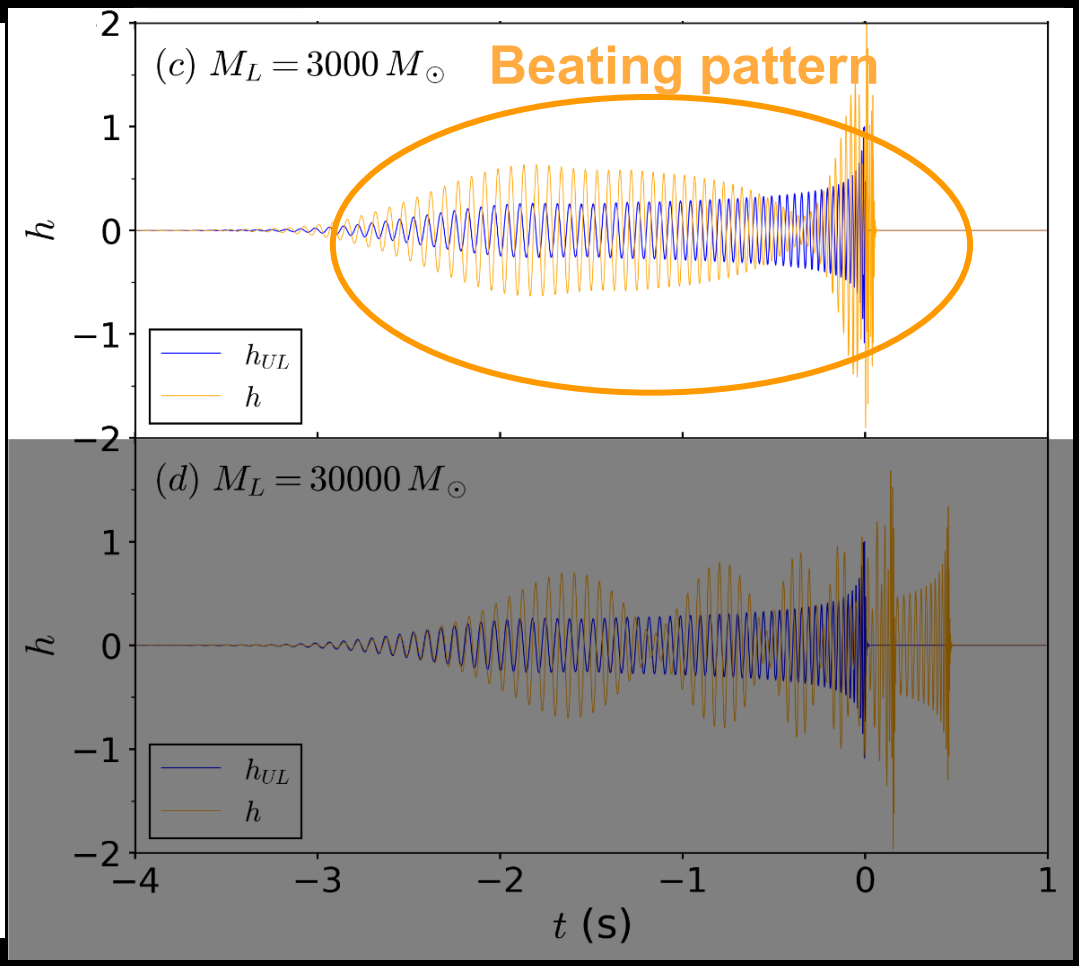

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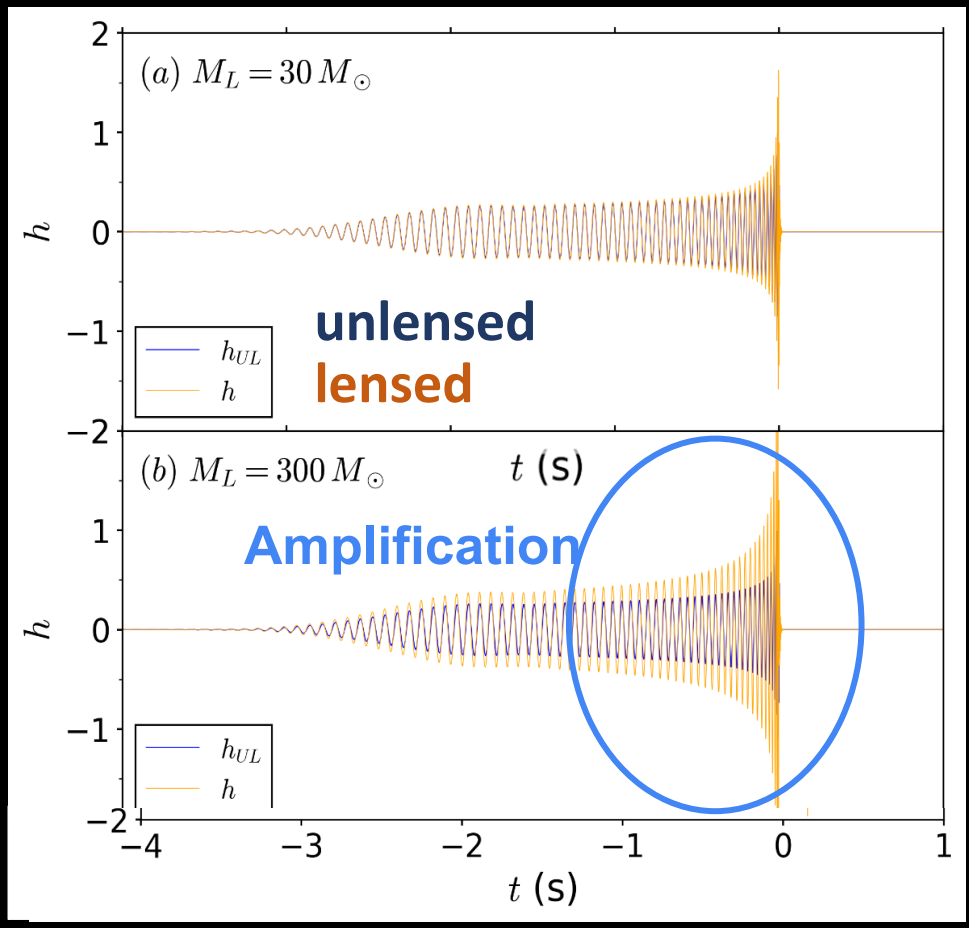

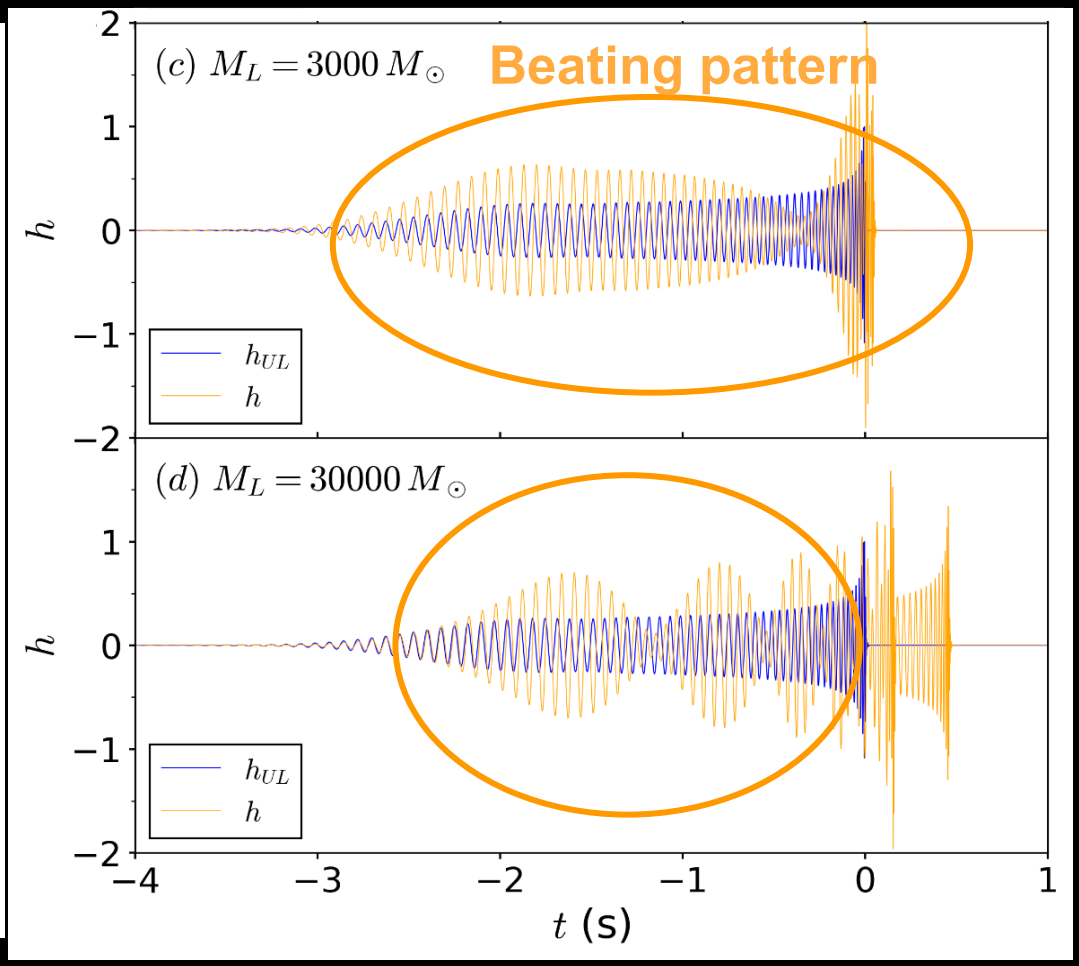

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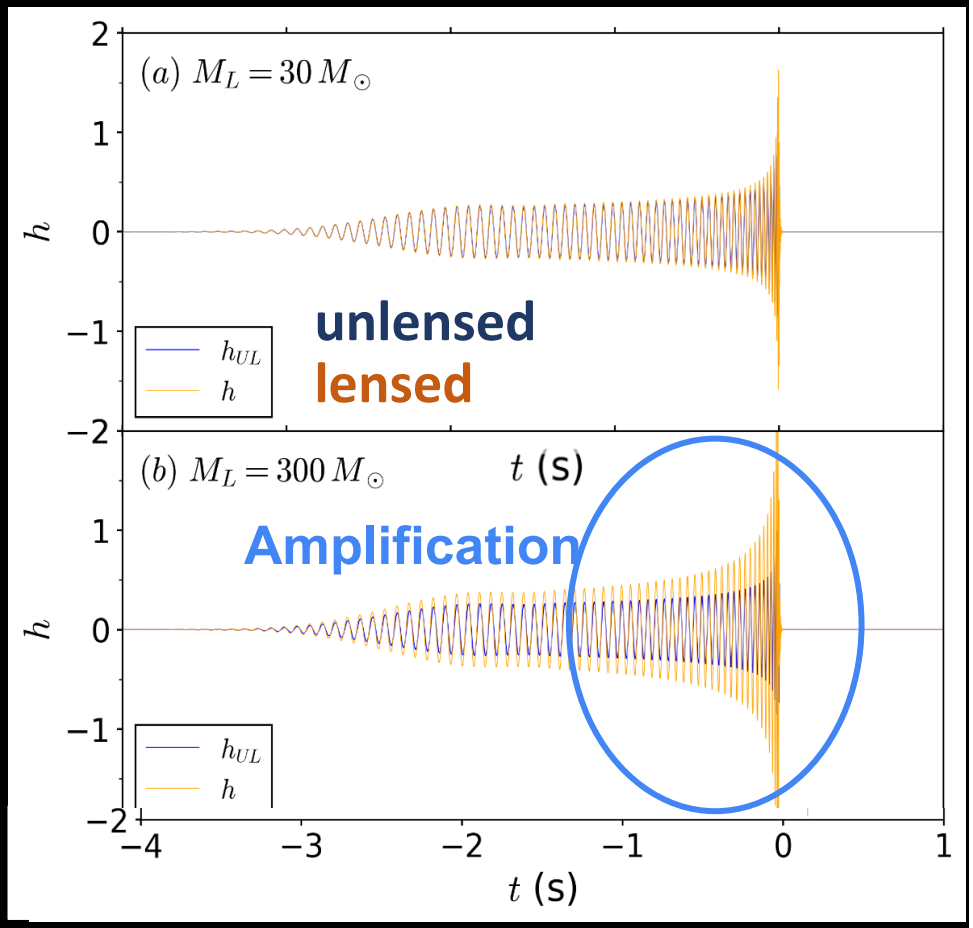
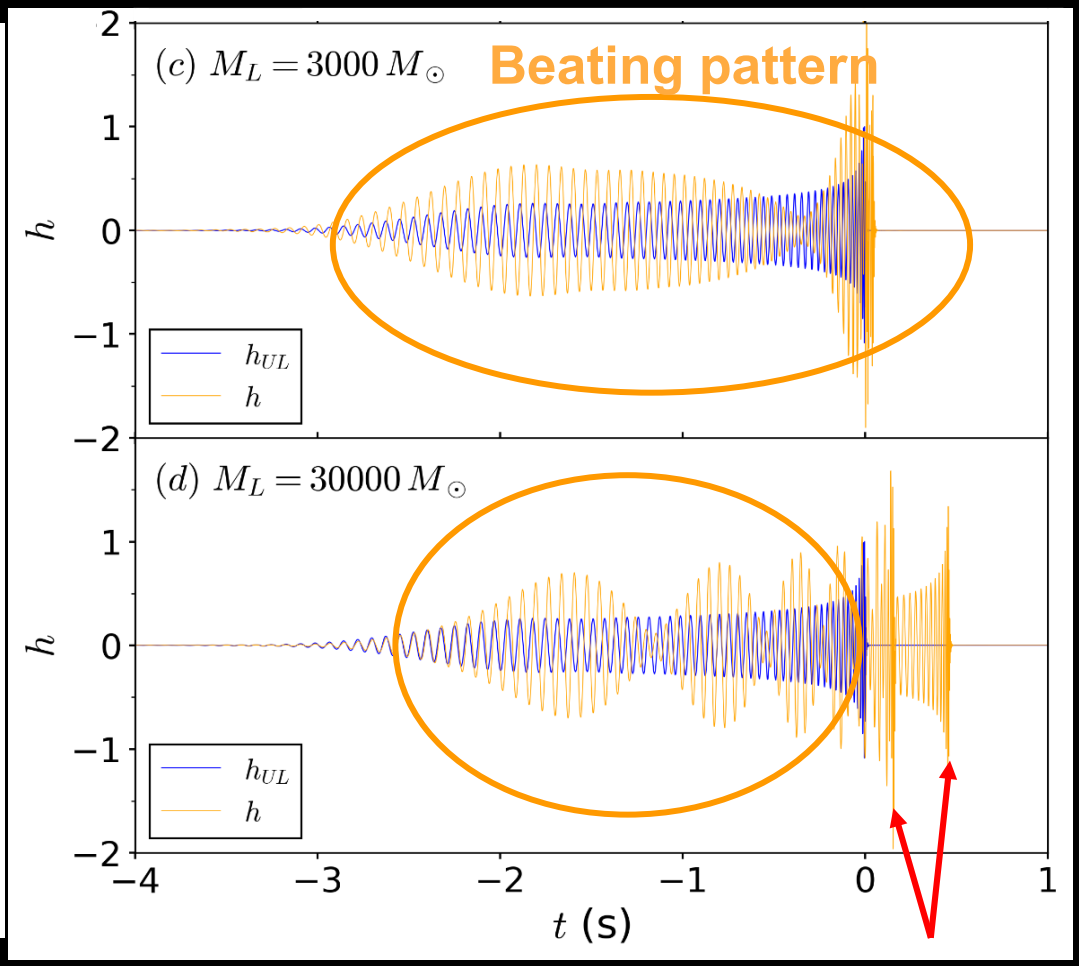




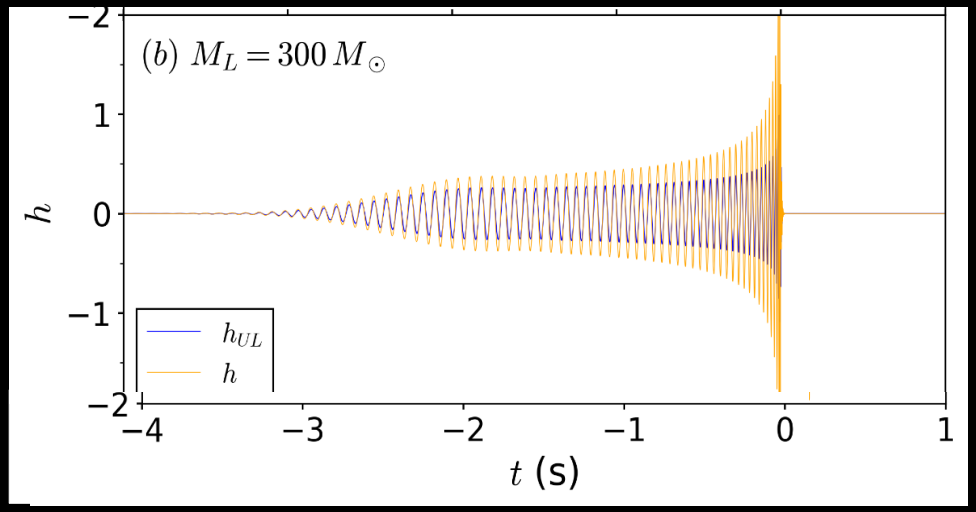
Bondarescu, Ubach, Bulashenko, Lundgren (2023)

Higher lens mass / higher GW frequency

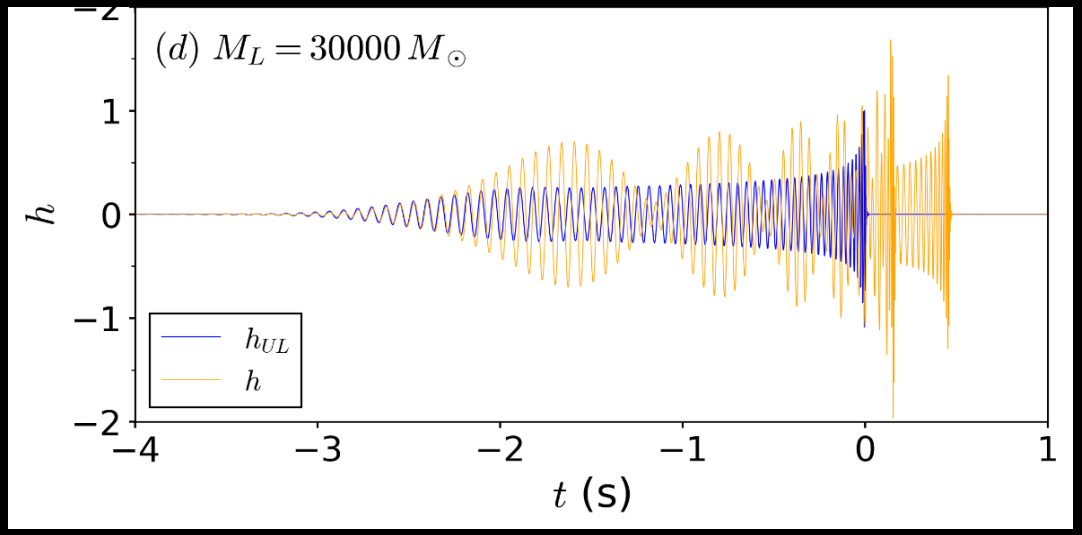
Two images

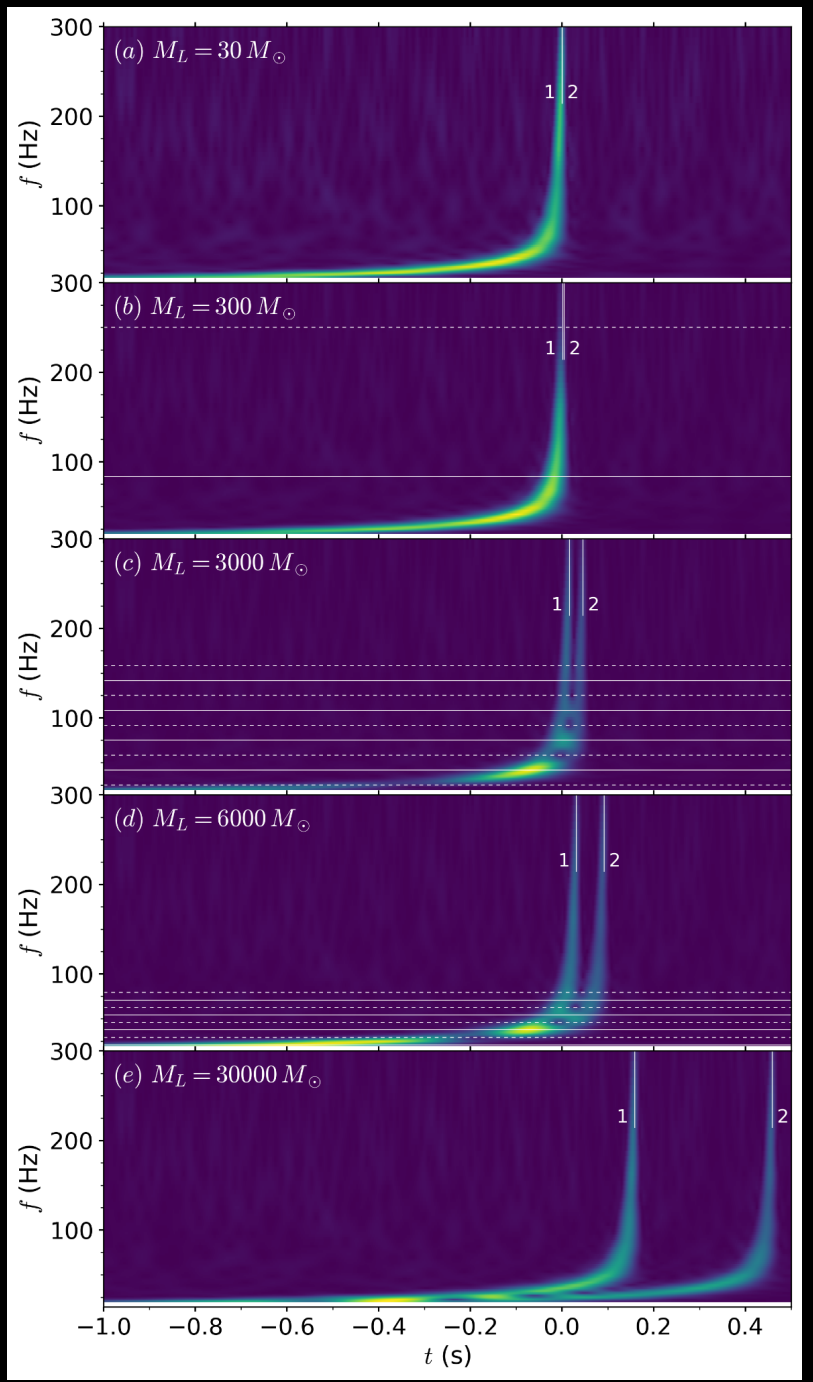
Lensing imprint

Amplification:



Beating pattern + multiple images:





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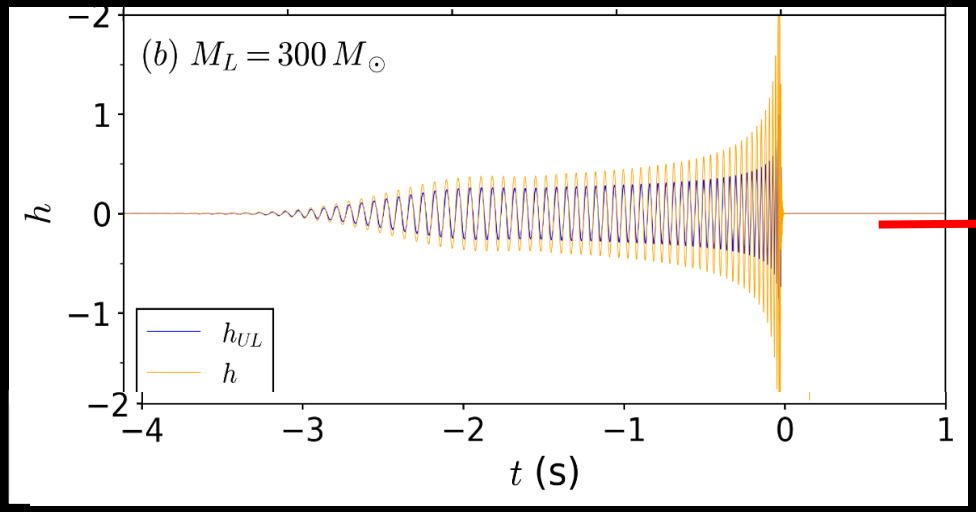

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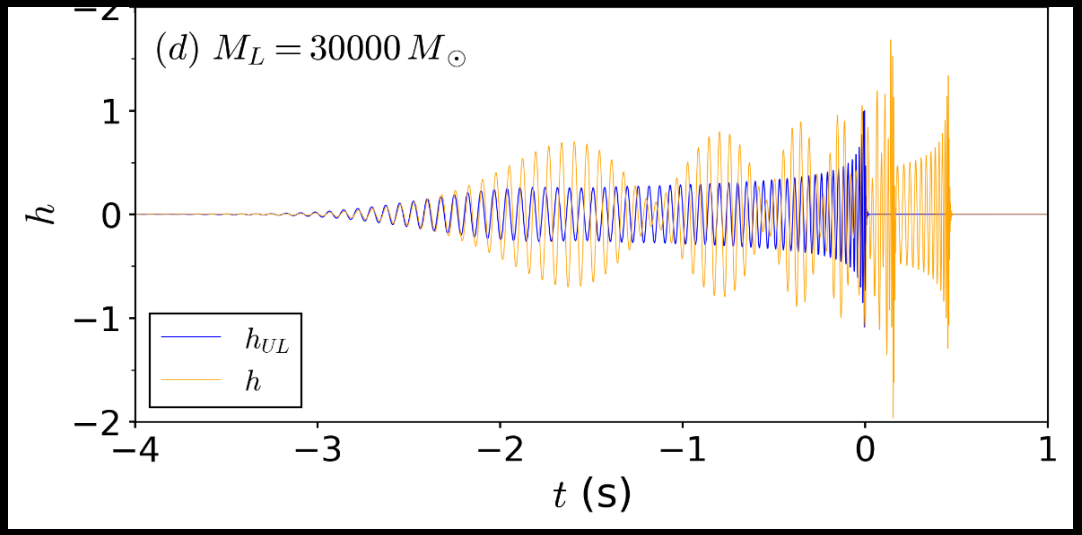

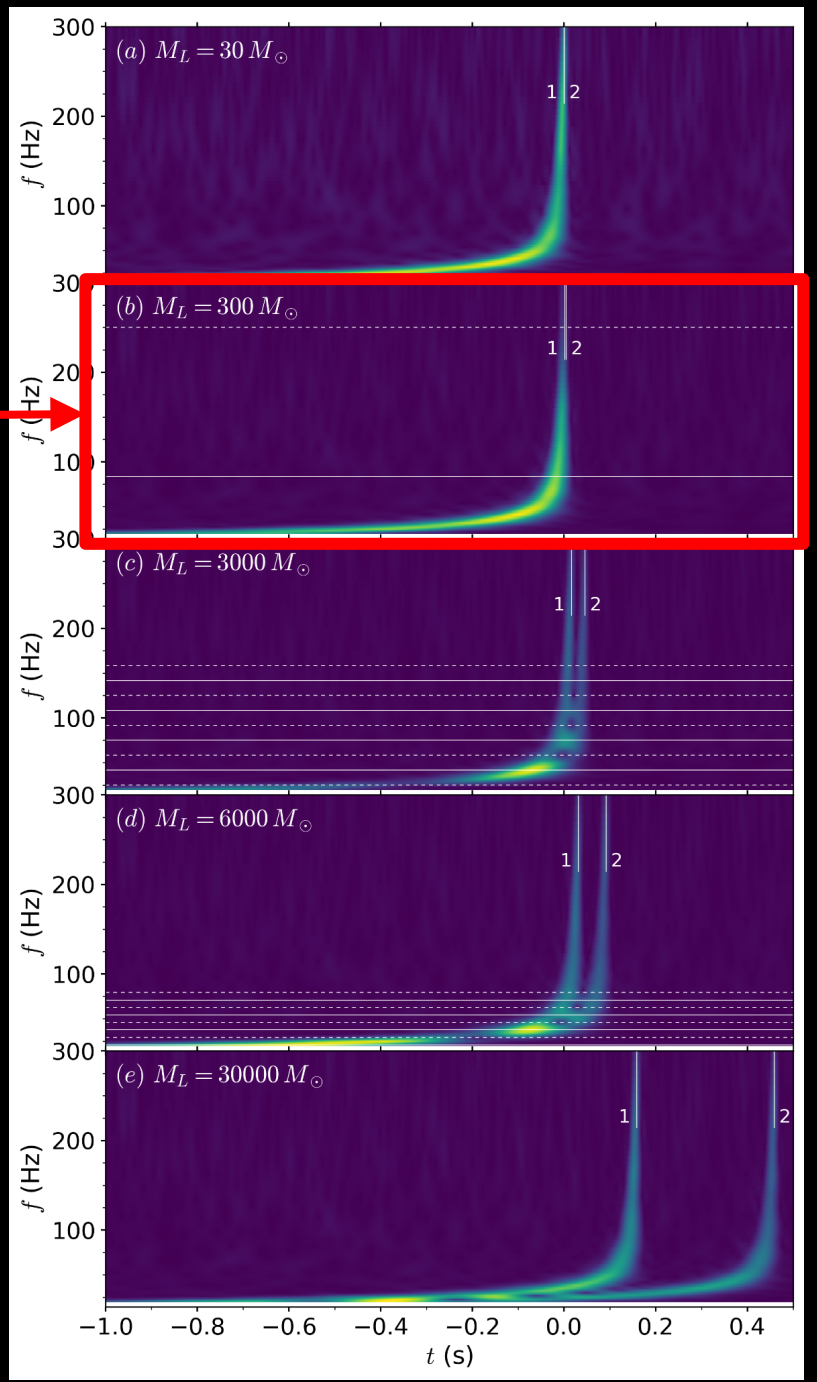

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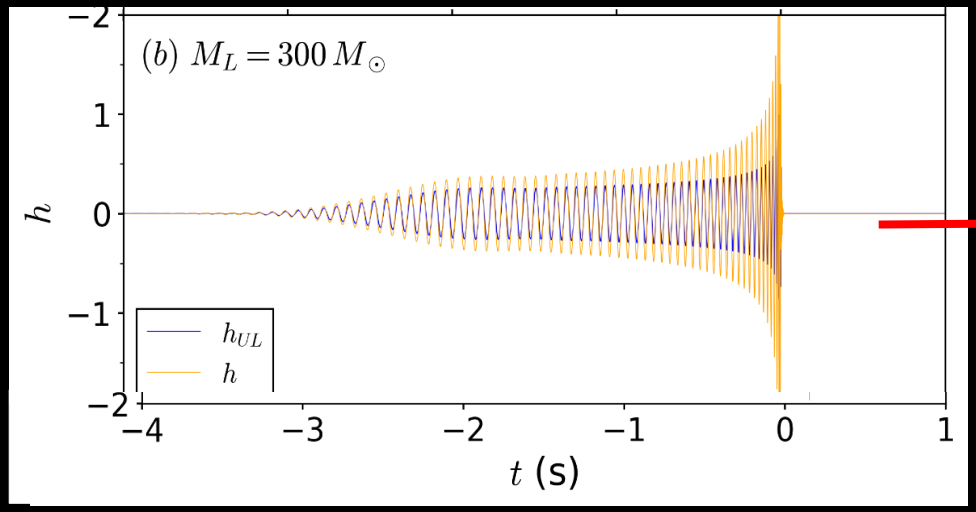

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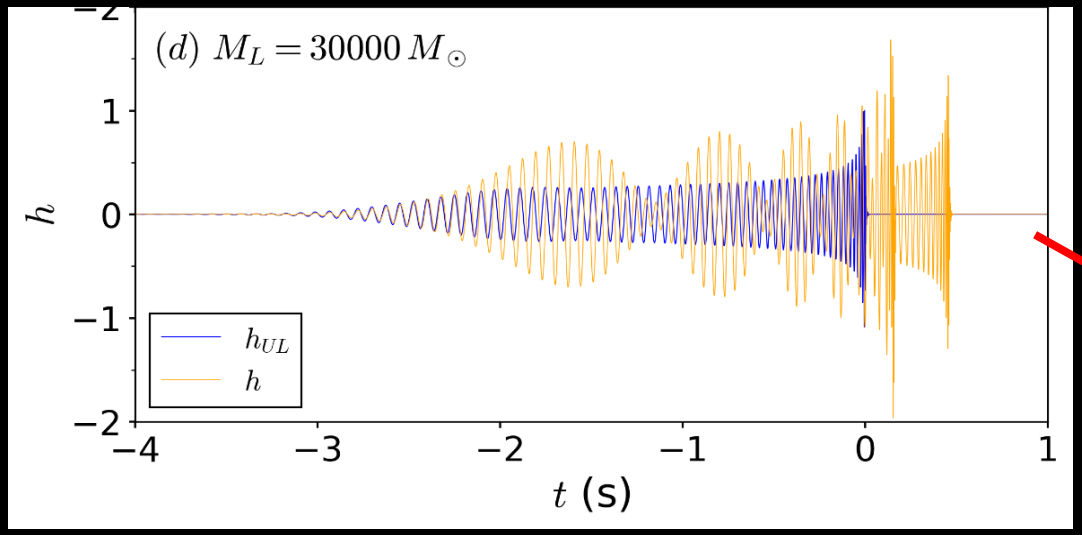
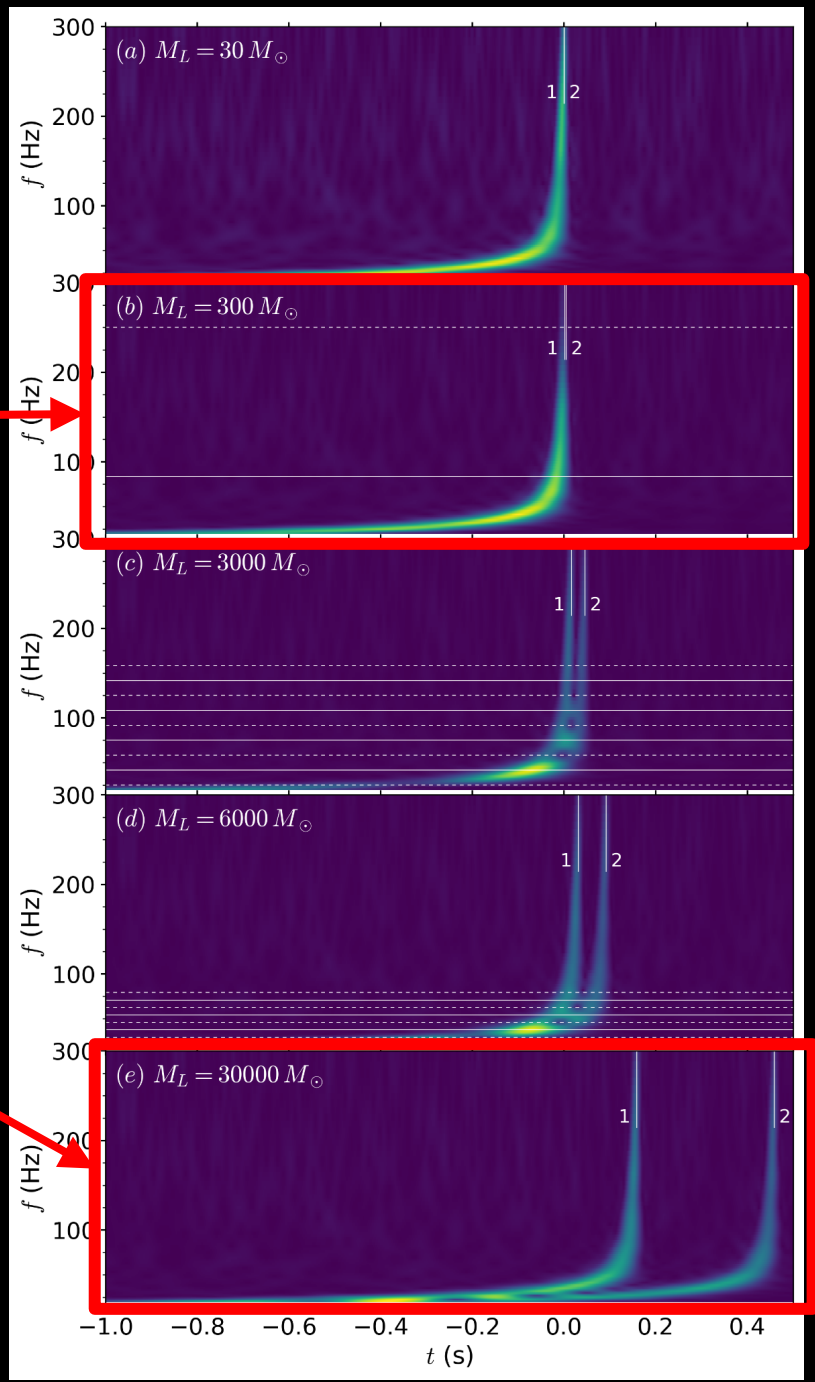
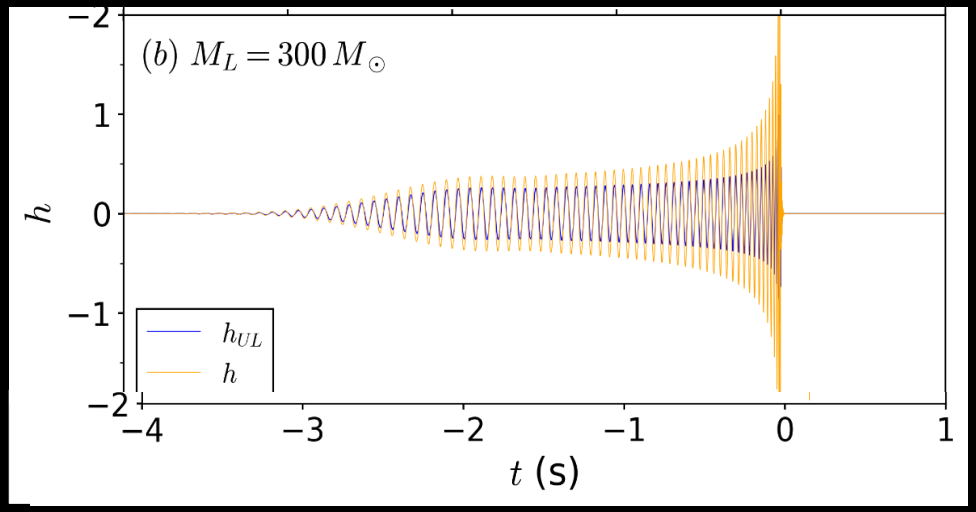
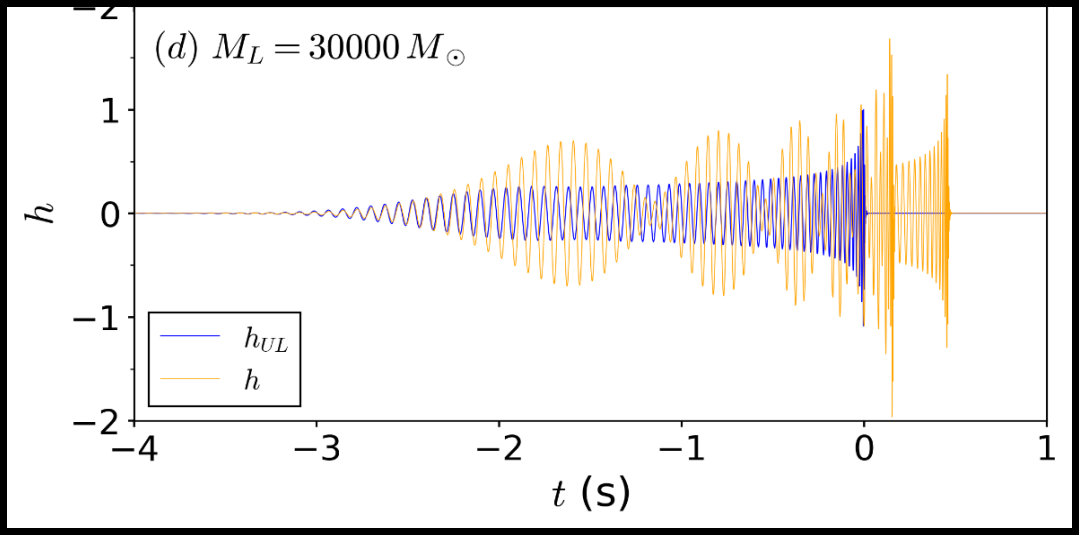




Artistic sonification

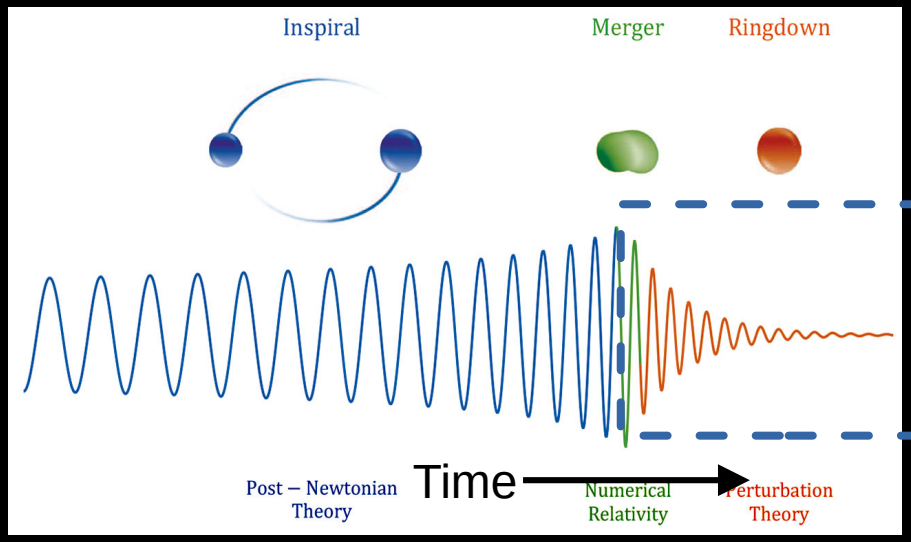
Amplification:



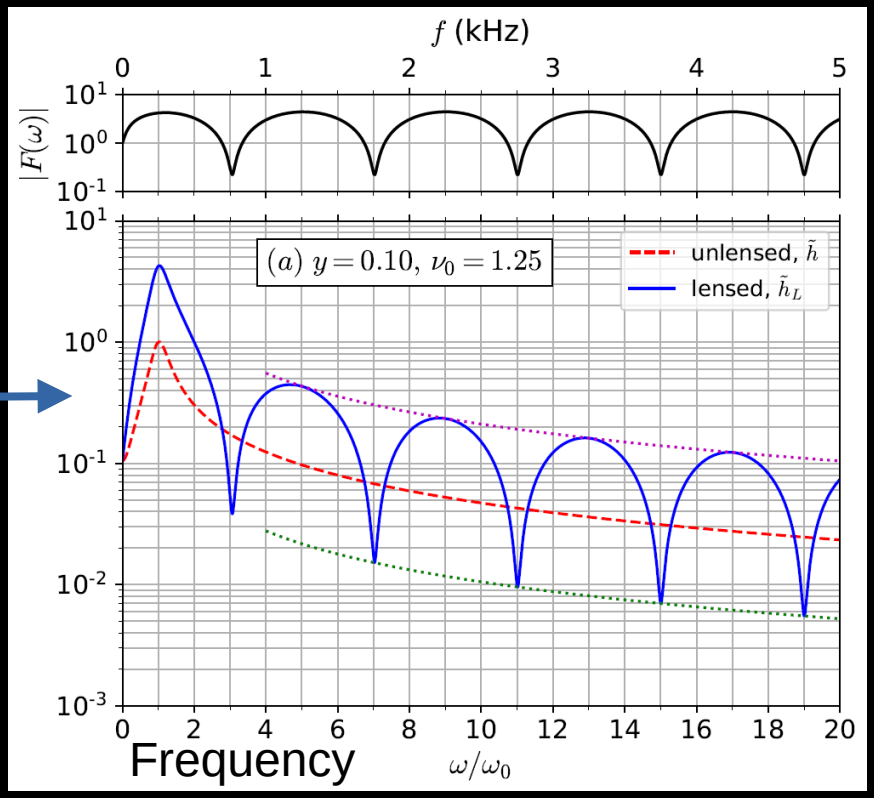


Sonification: Jordi Espuny
(work in progress)

Evenly spaced interference pattern

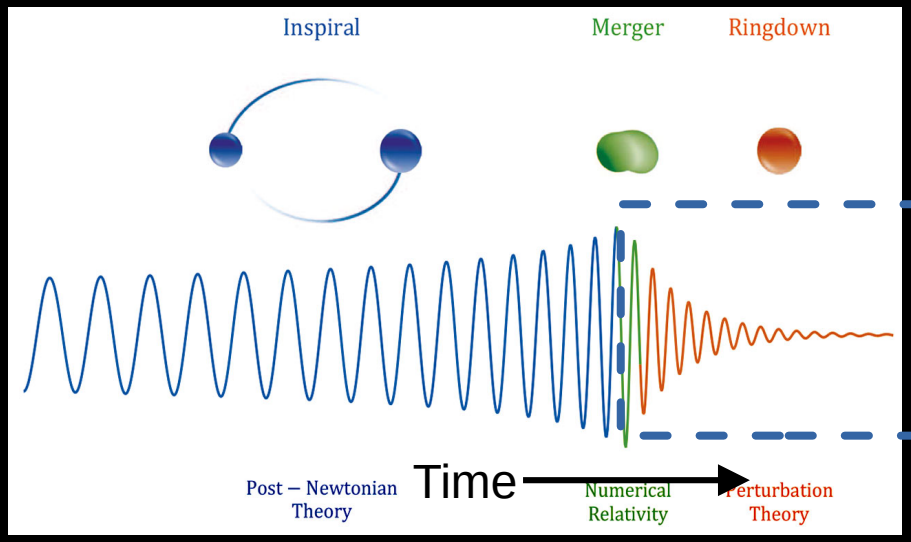


Antelis & Moreno (2017)



Bulashenko& Ubach (2022)

Evenly spaced interference pattern



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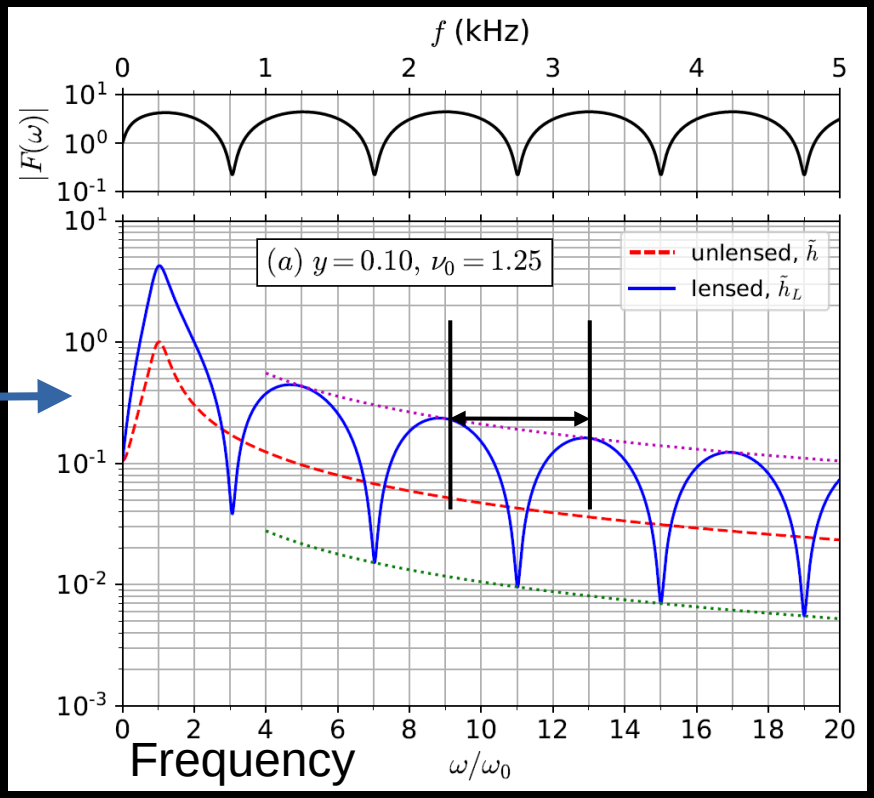

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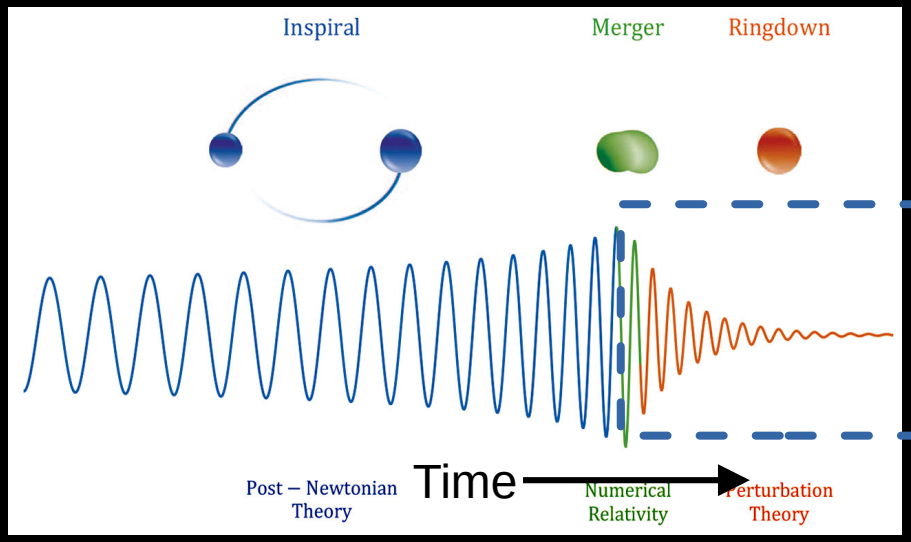

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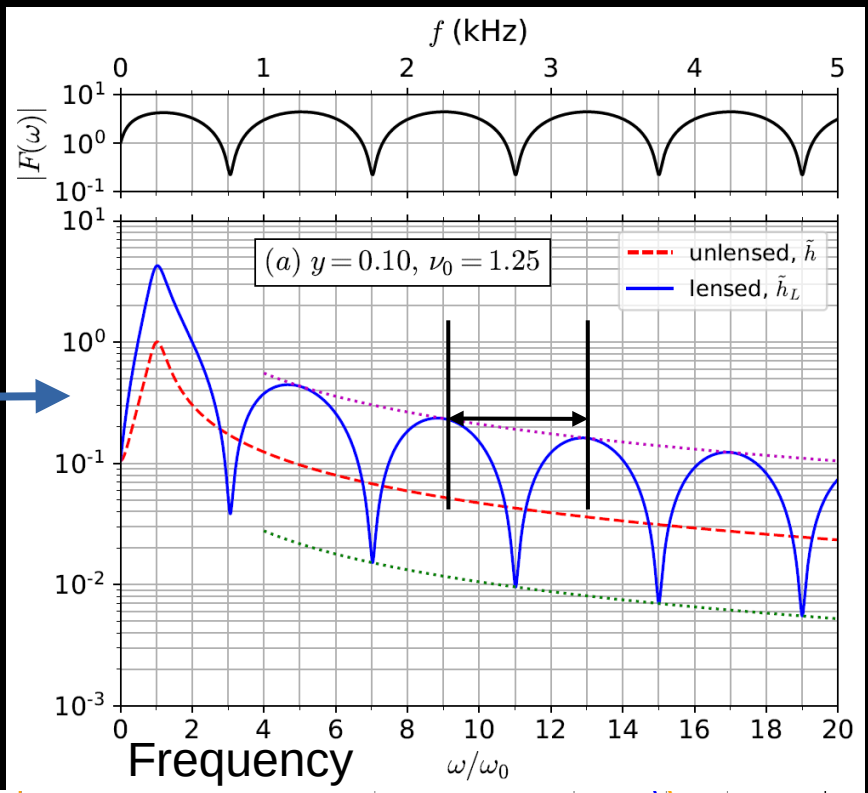

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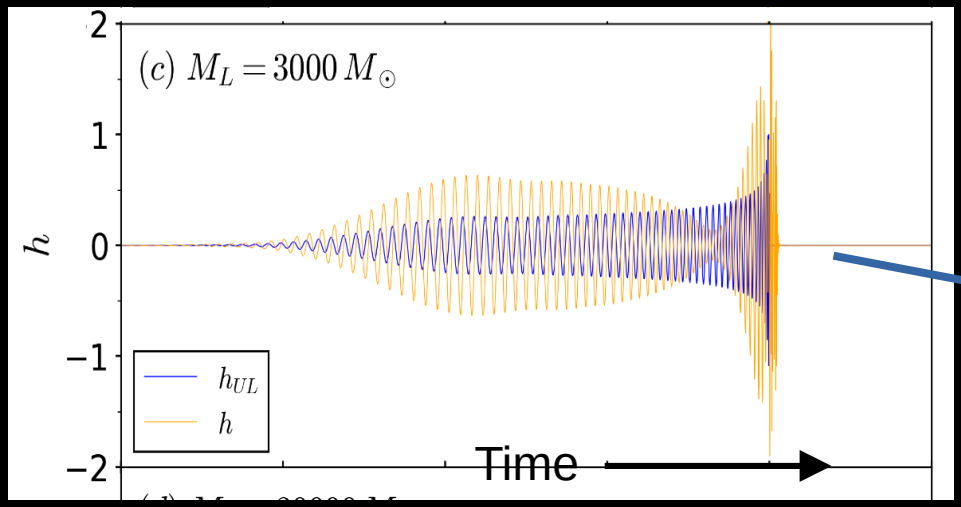
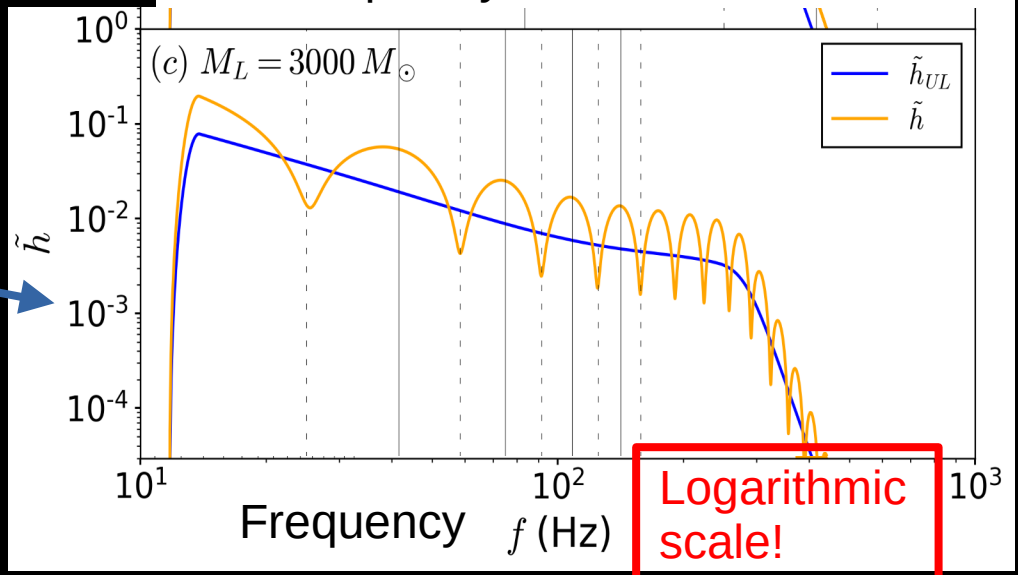
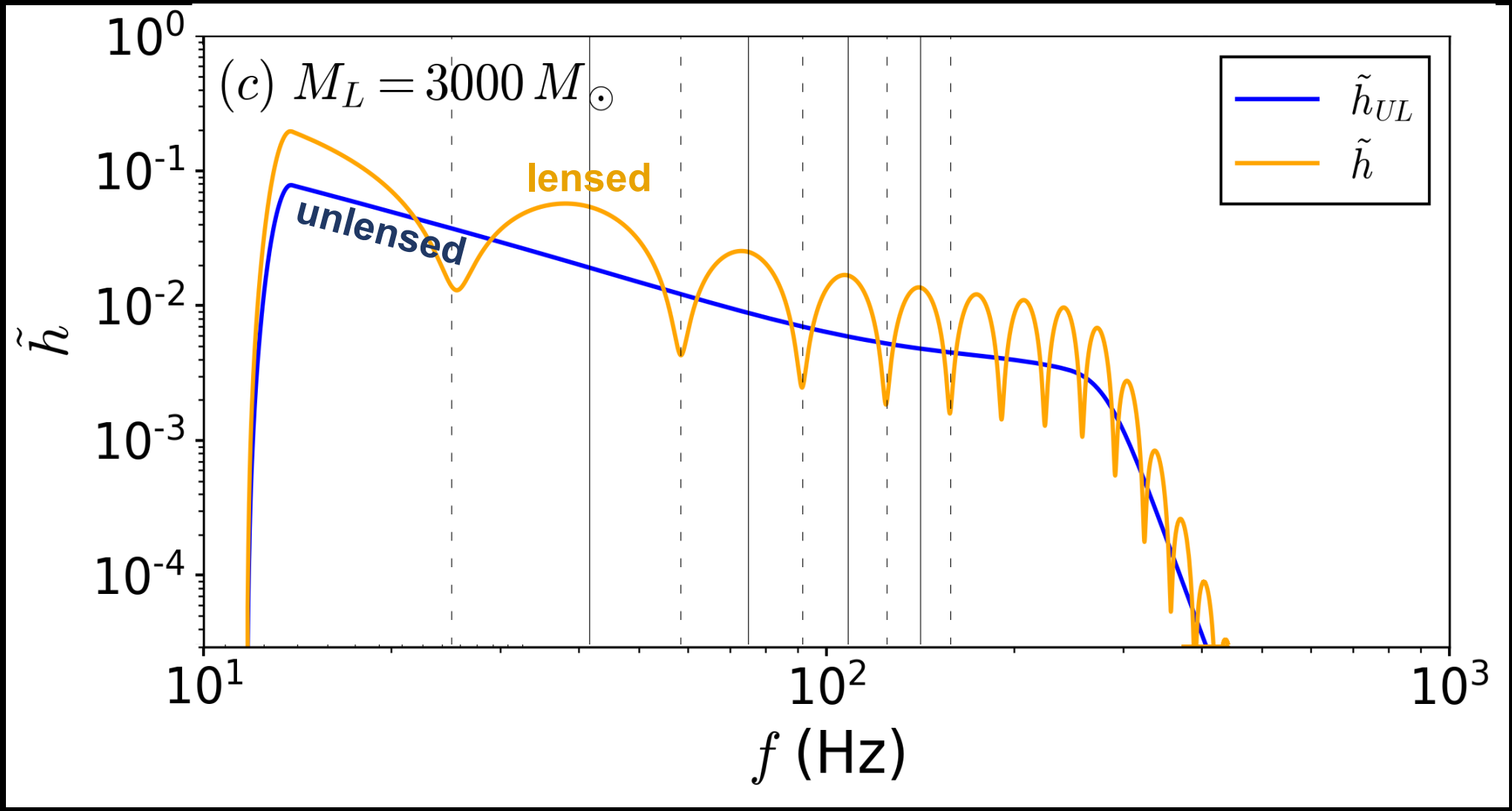




Bondarescu, Ubach, Bulashenko, Lundgren (2023)

Evenly spaced interference pattern



Bulashenko&
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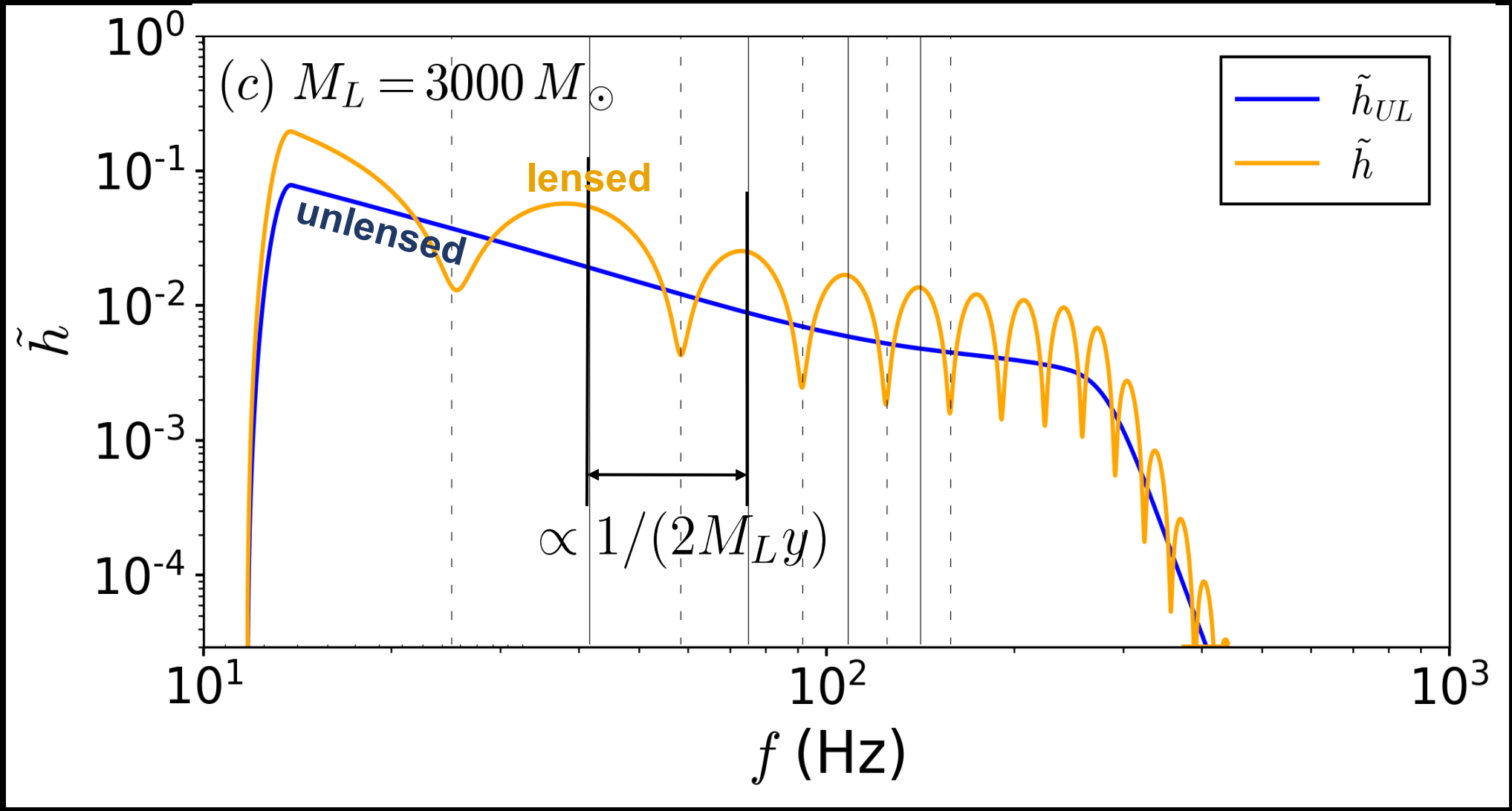

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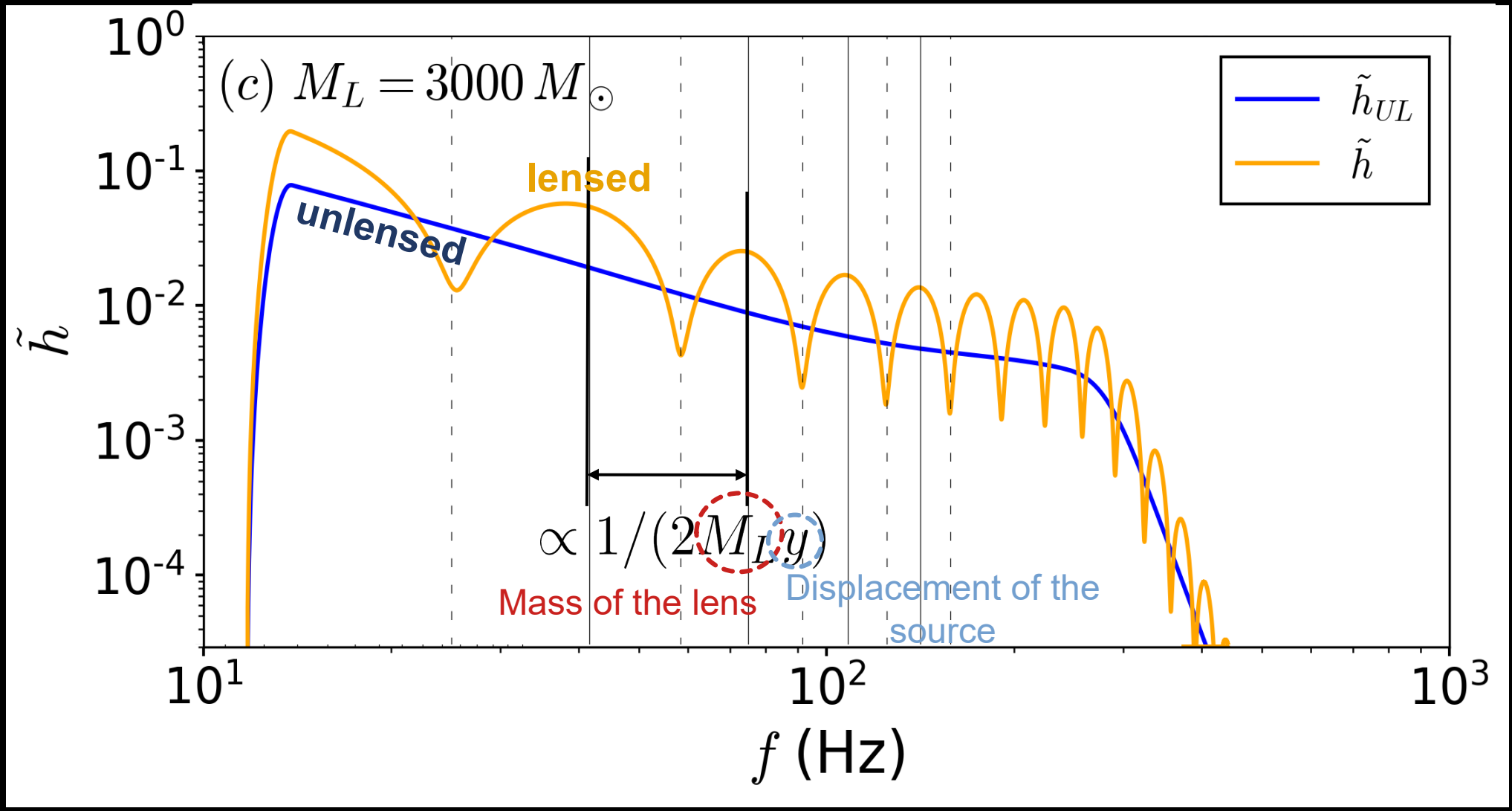

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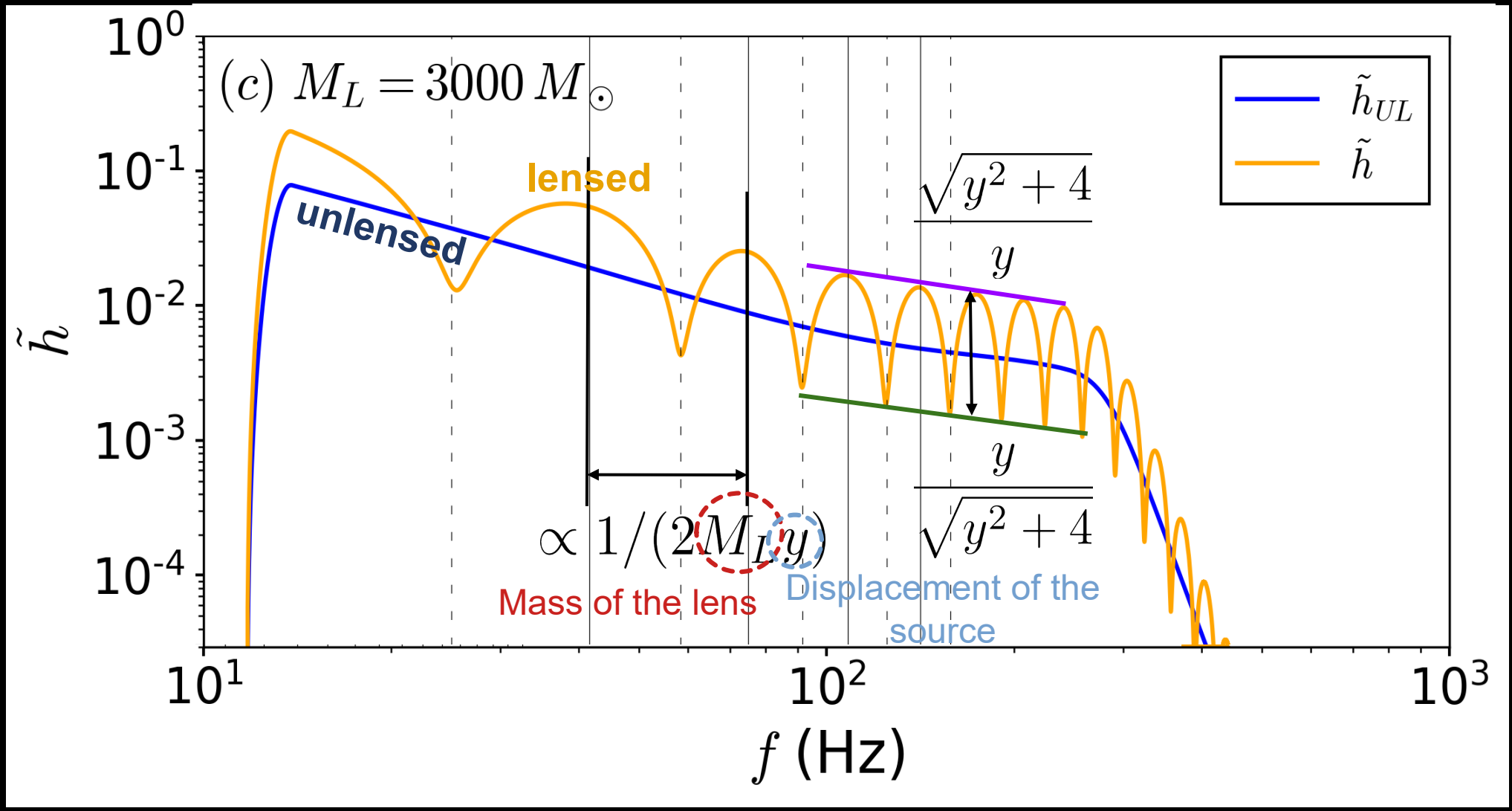
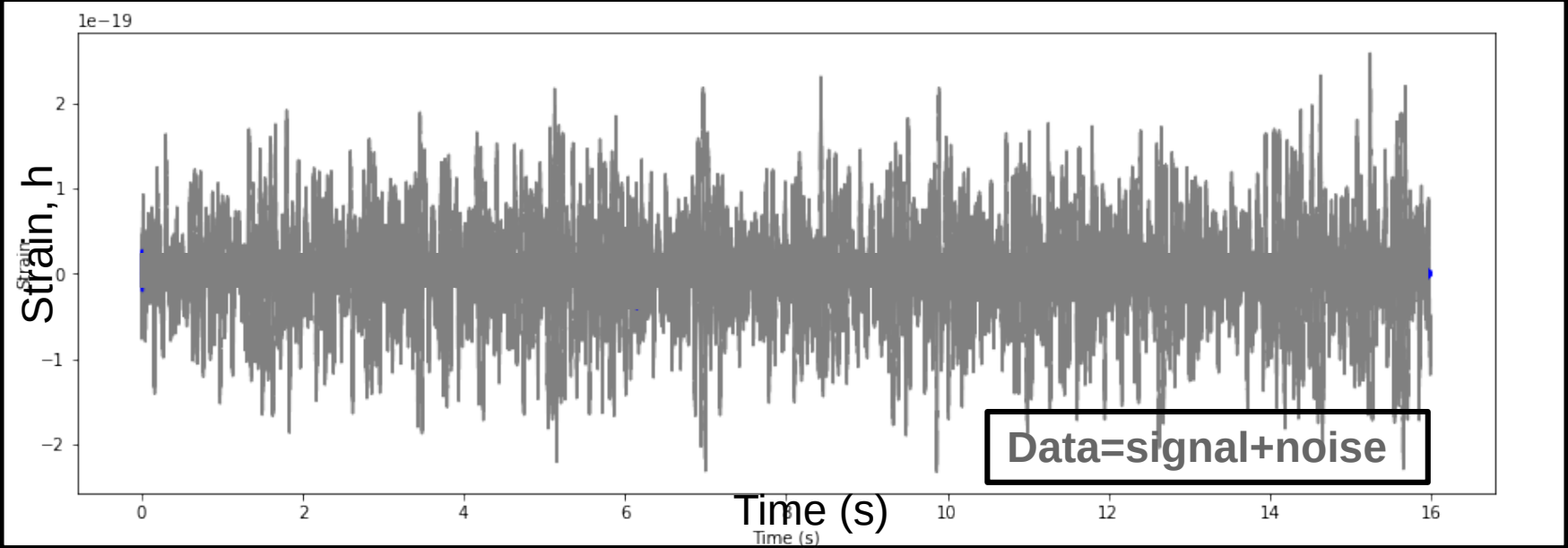


Bulashenko& Ubach (2022)

Bondarescu, Ubach, Bulashenko, Lundgren (2023)

Detection technique: matched filtering

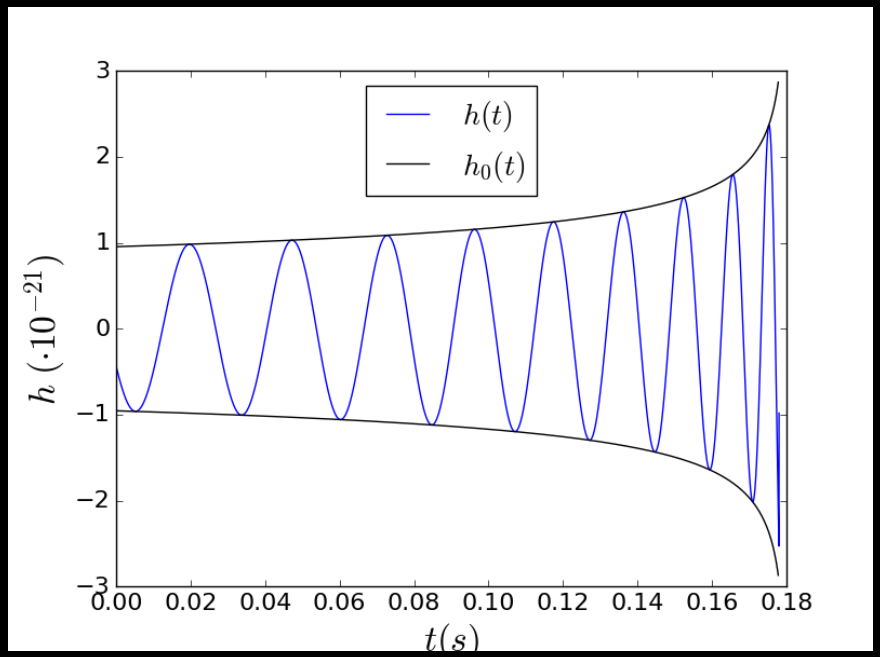
Recovery of an injected signal
(mock example)



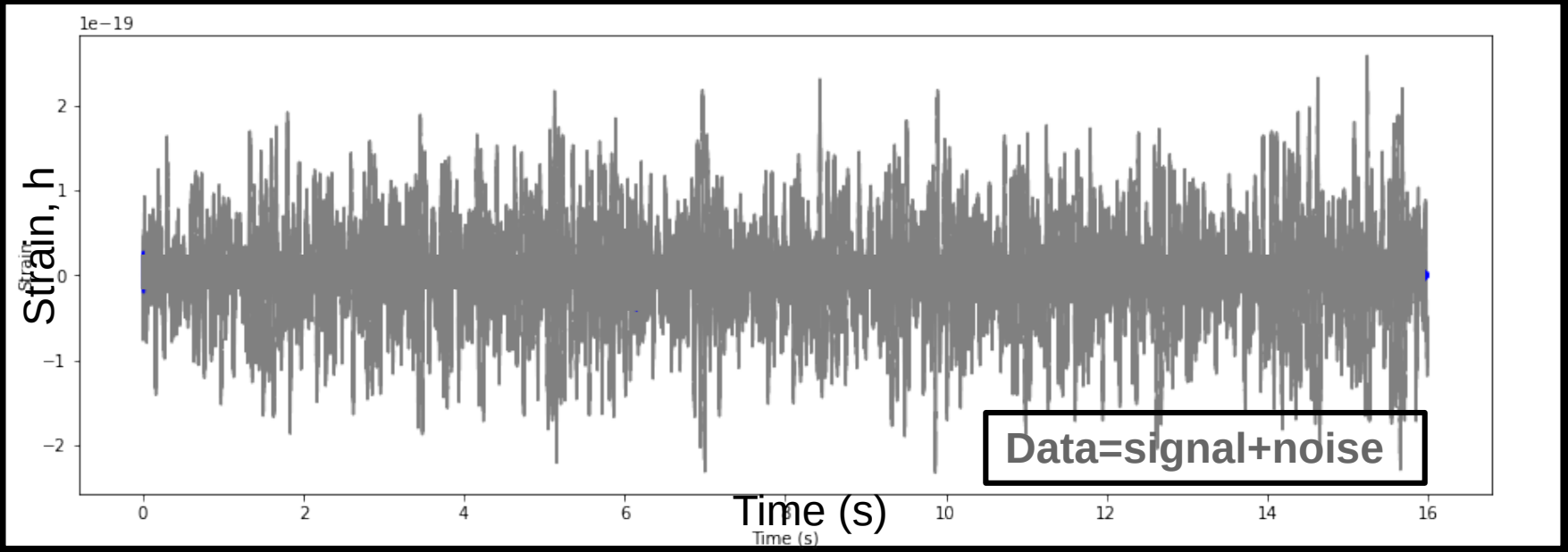
Using GWOSC code

Detection technique: matched filtering

Template:



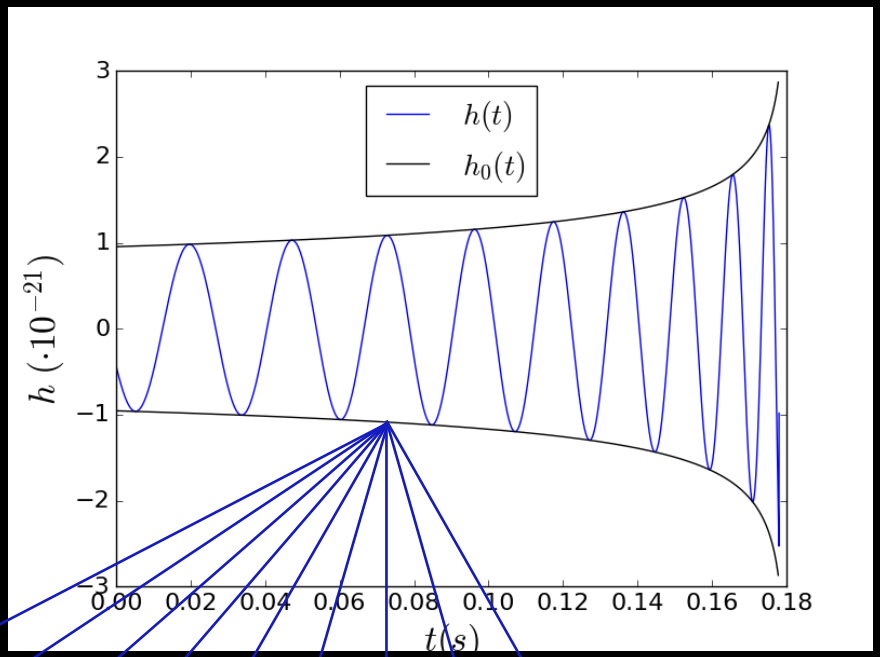
Recovery of an injected signal (mock example)



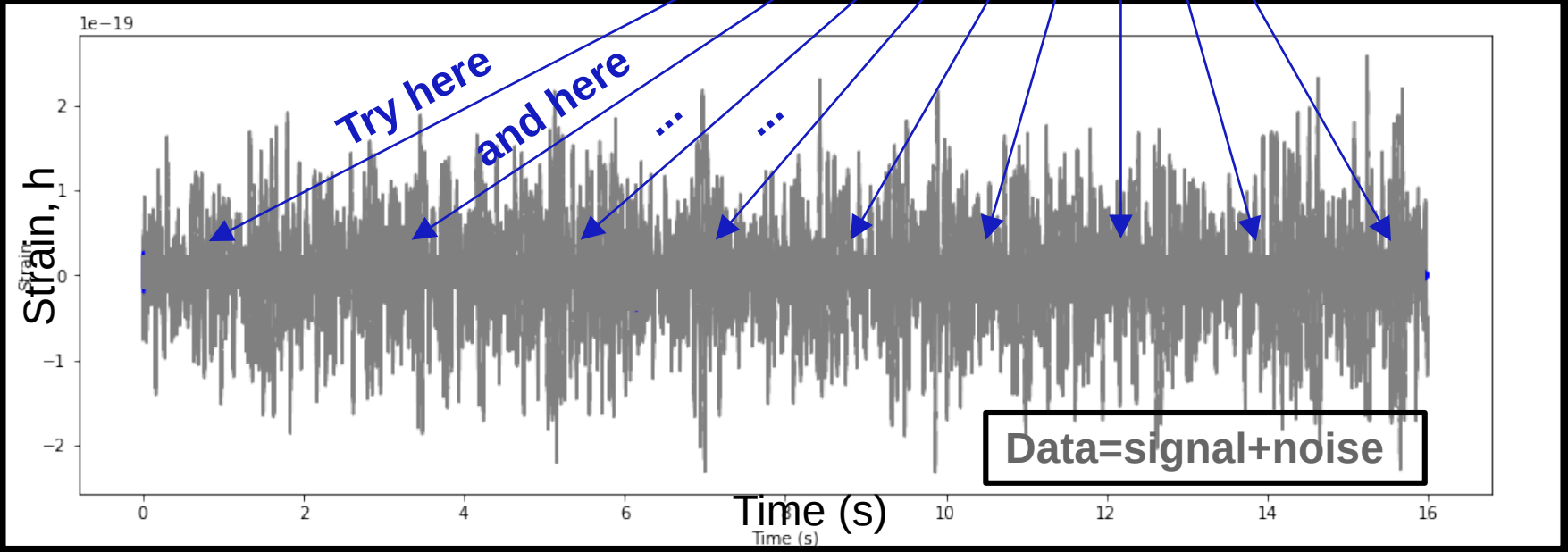
Using GWOSC code

Detection technique: matched filtering

Template:



Recovery of an injected signal
(mock example)



Using GWOSC code

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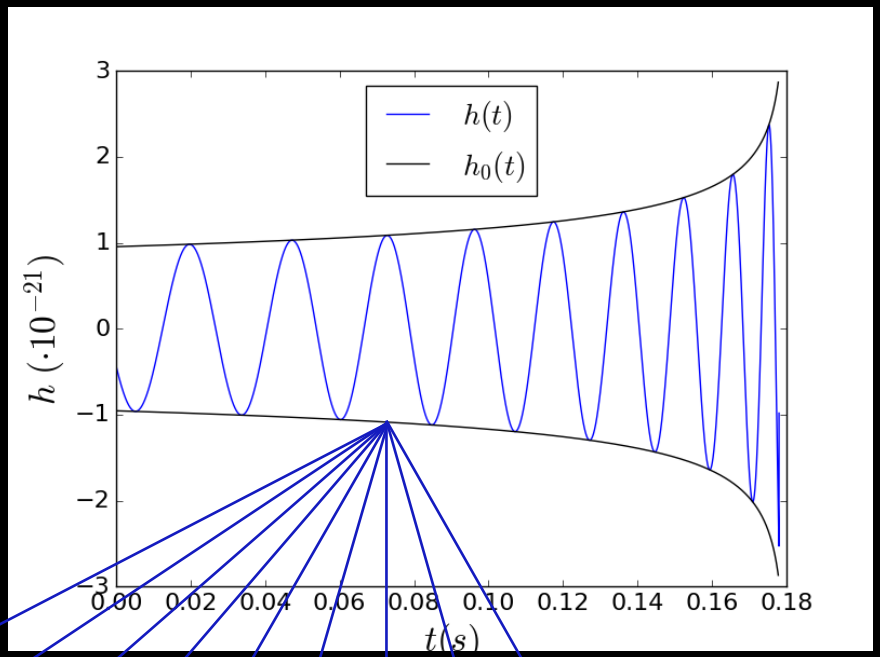
Template:



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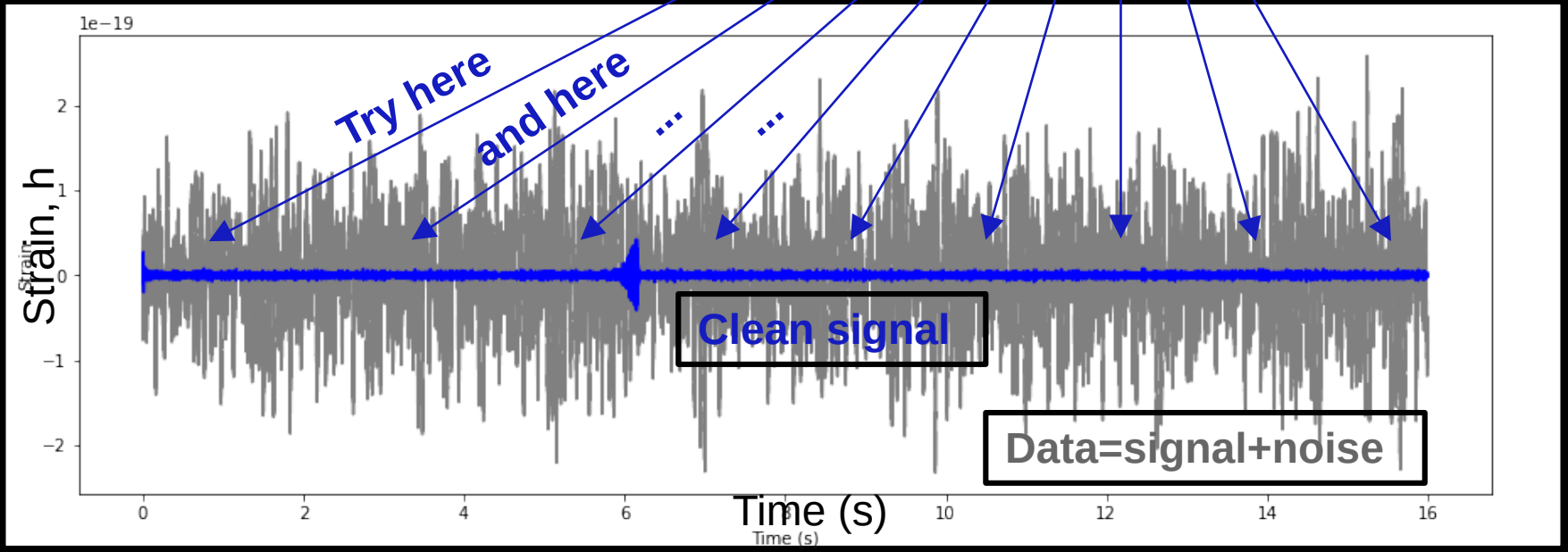
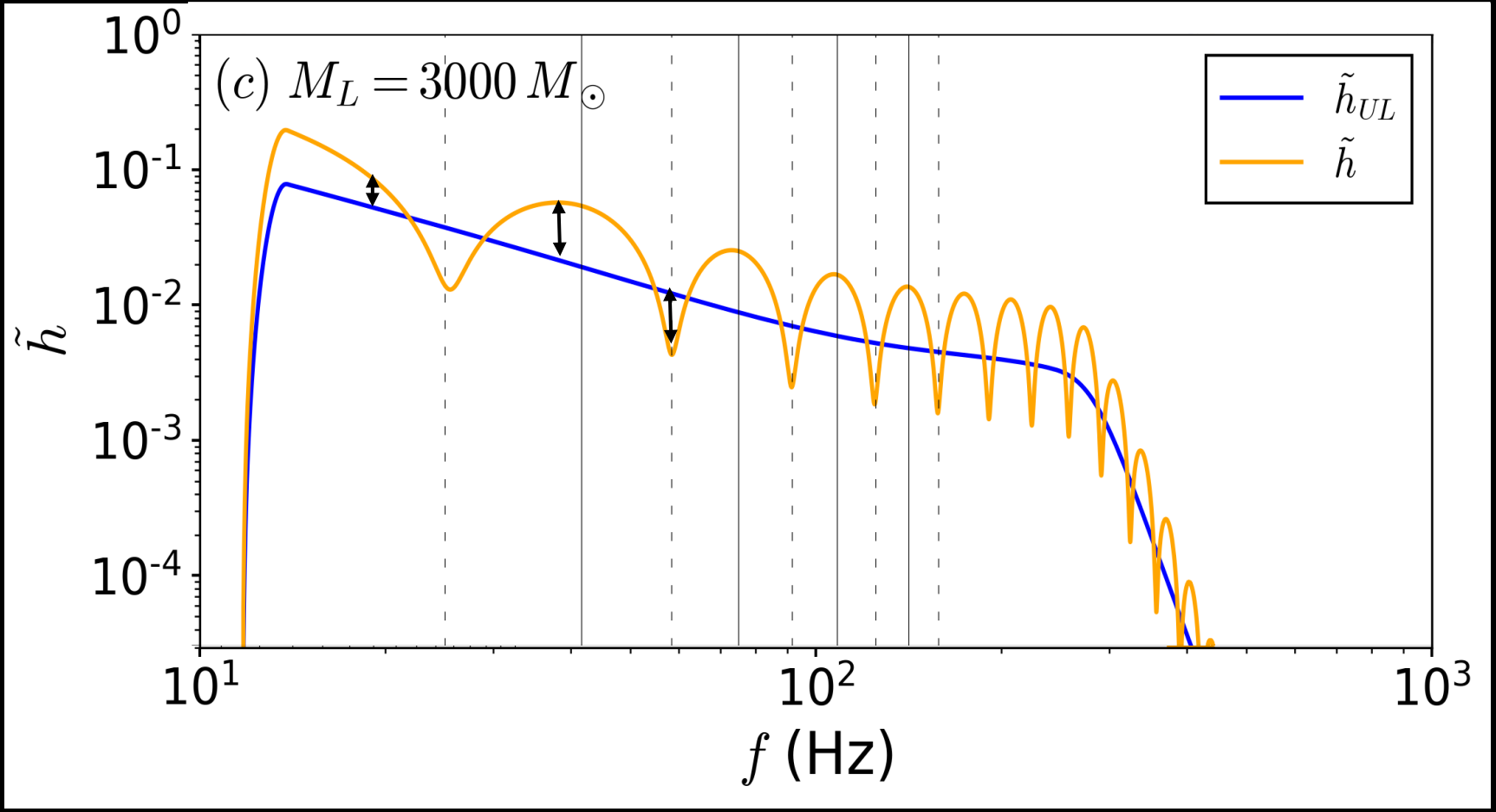


Using GWOSC code

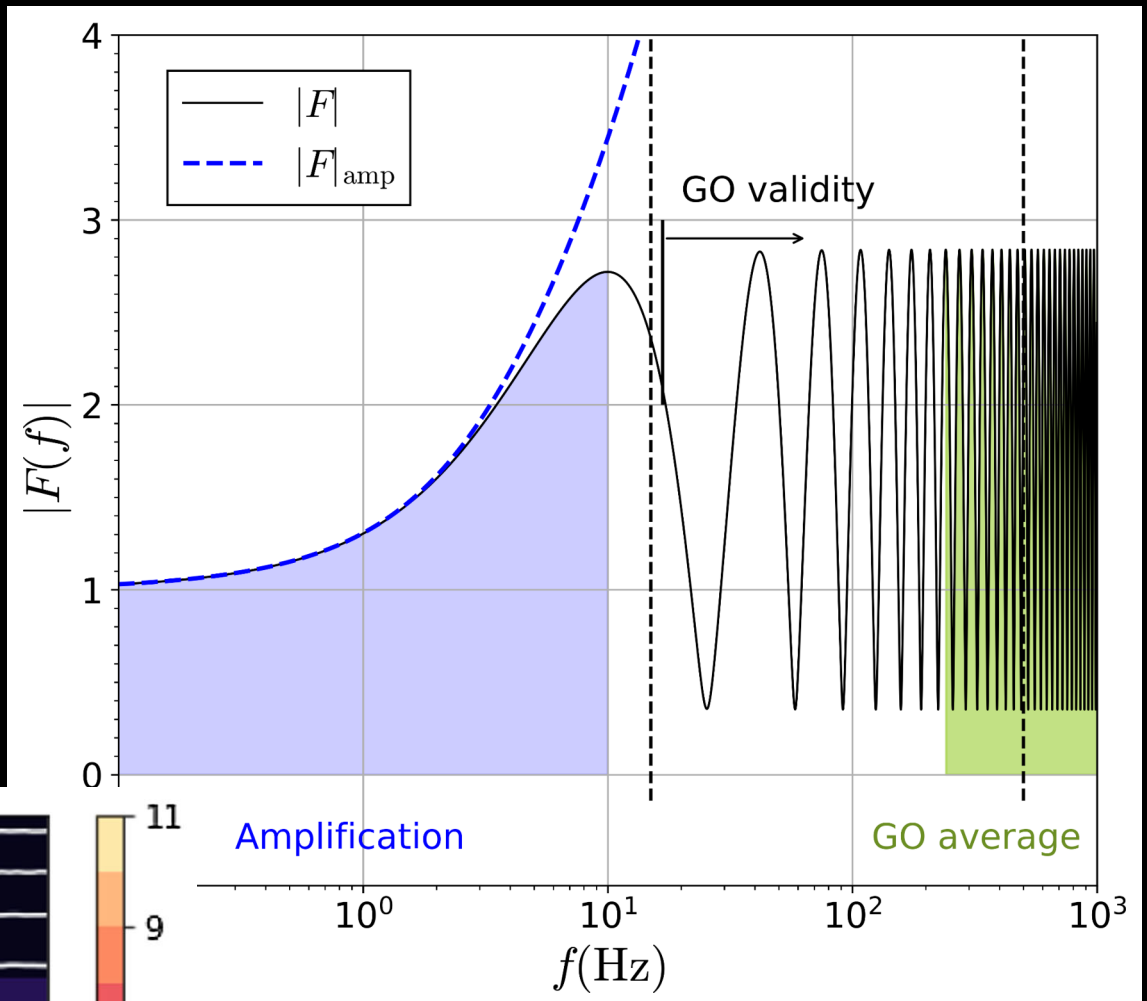
Match between templates

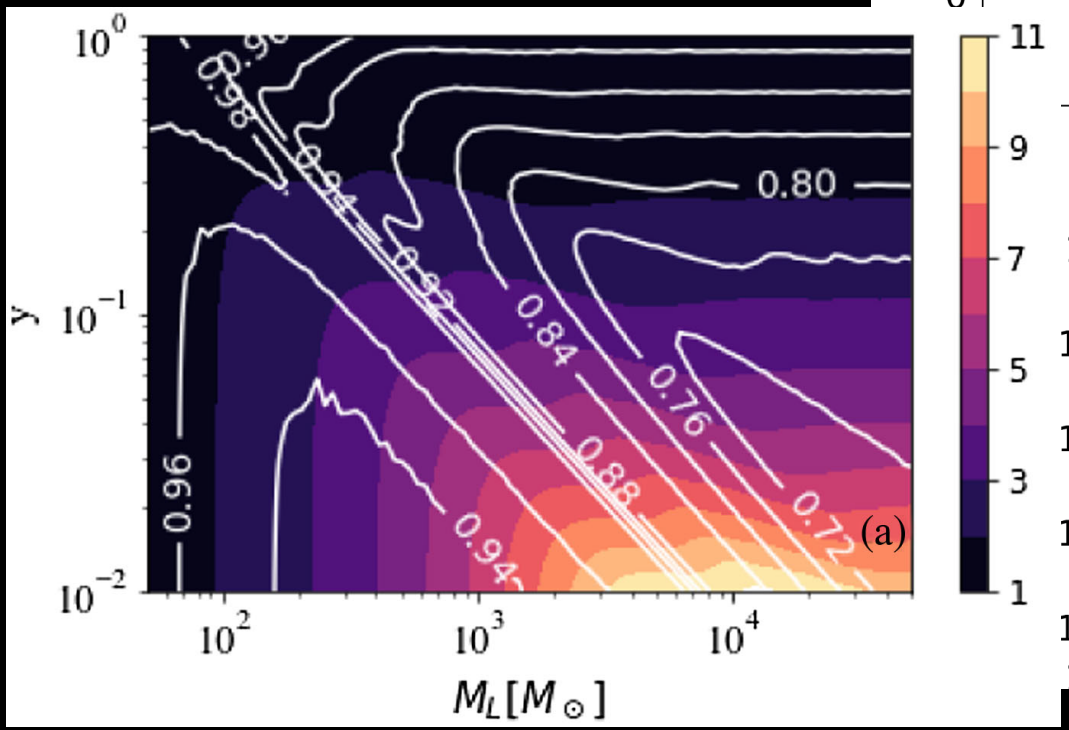
Deviation of data / **one template** from **another template**, over all frequencies
[Taking into account detector noise]

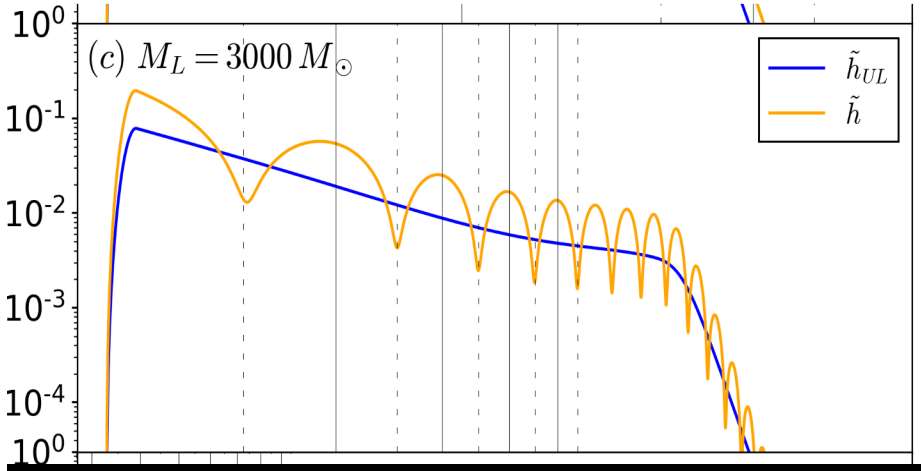


Match determines distortion from usual signal

Bondarescu, Ubach, Bulashenko, Lundgren (2023)







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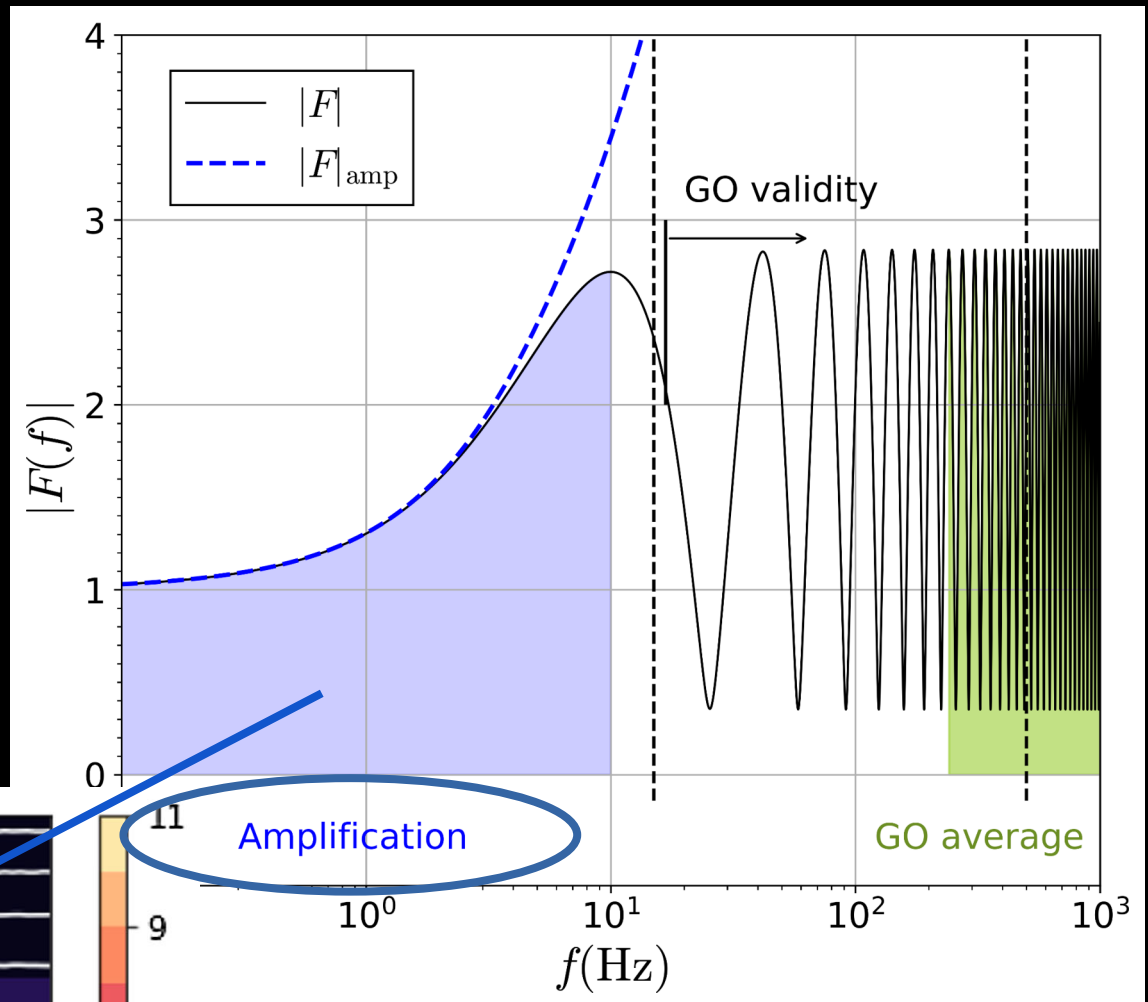

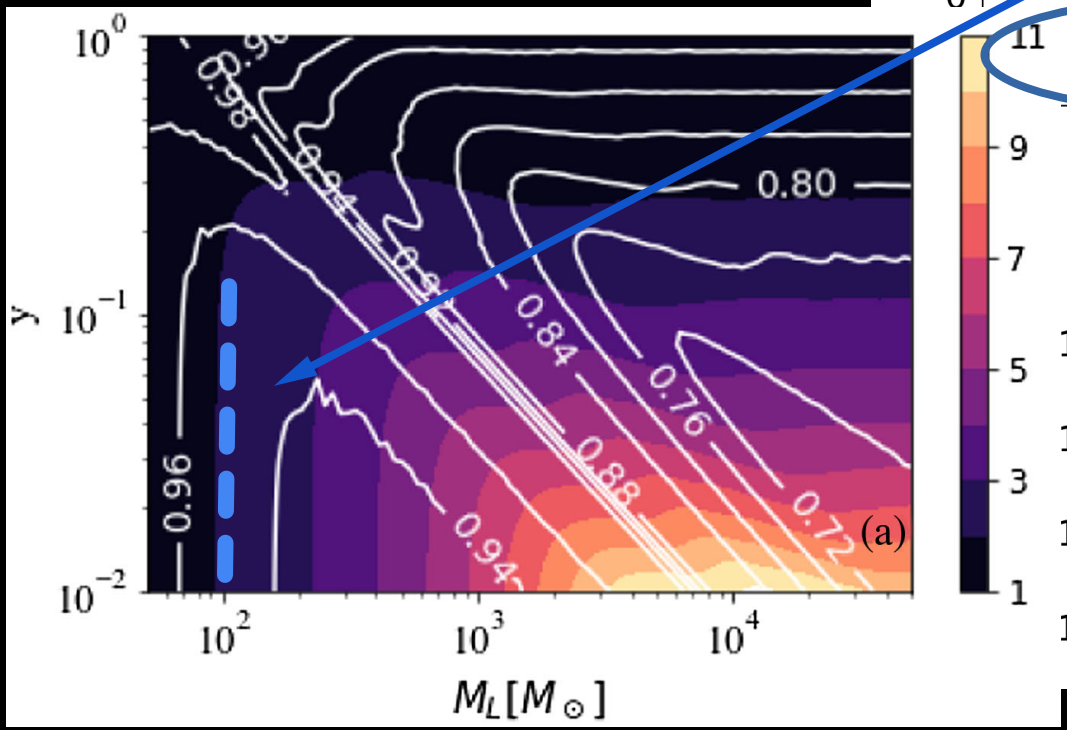

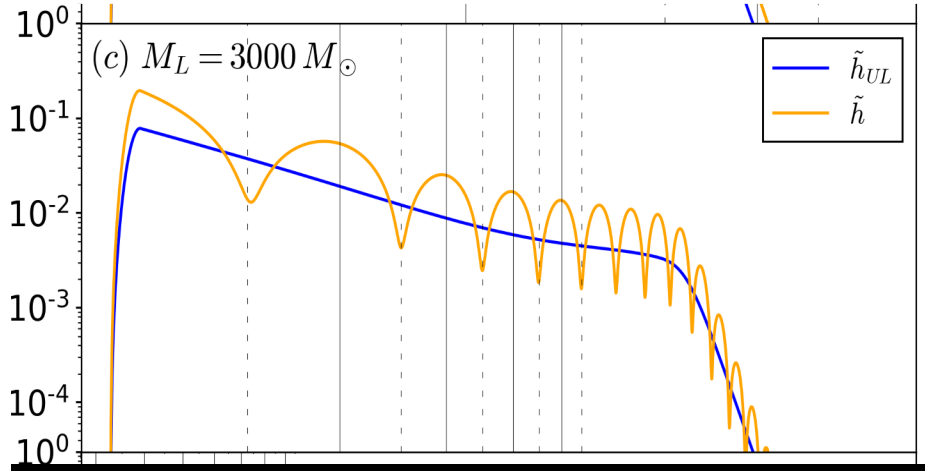

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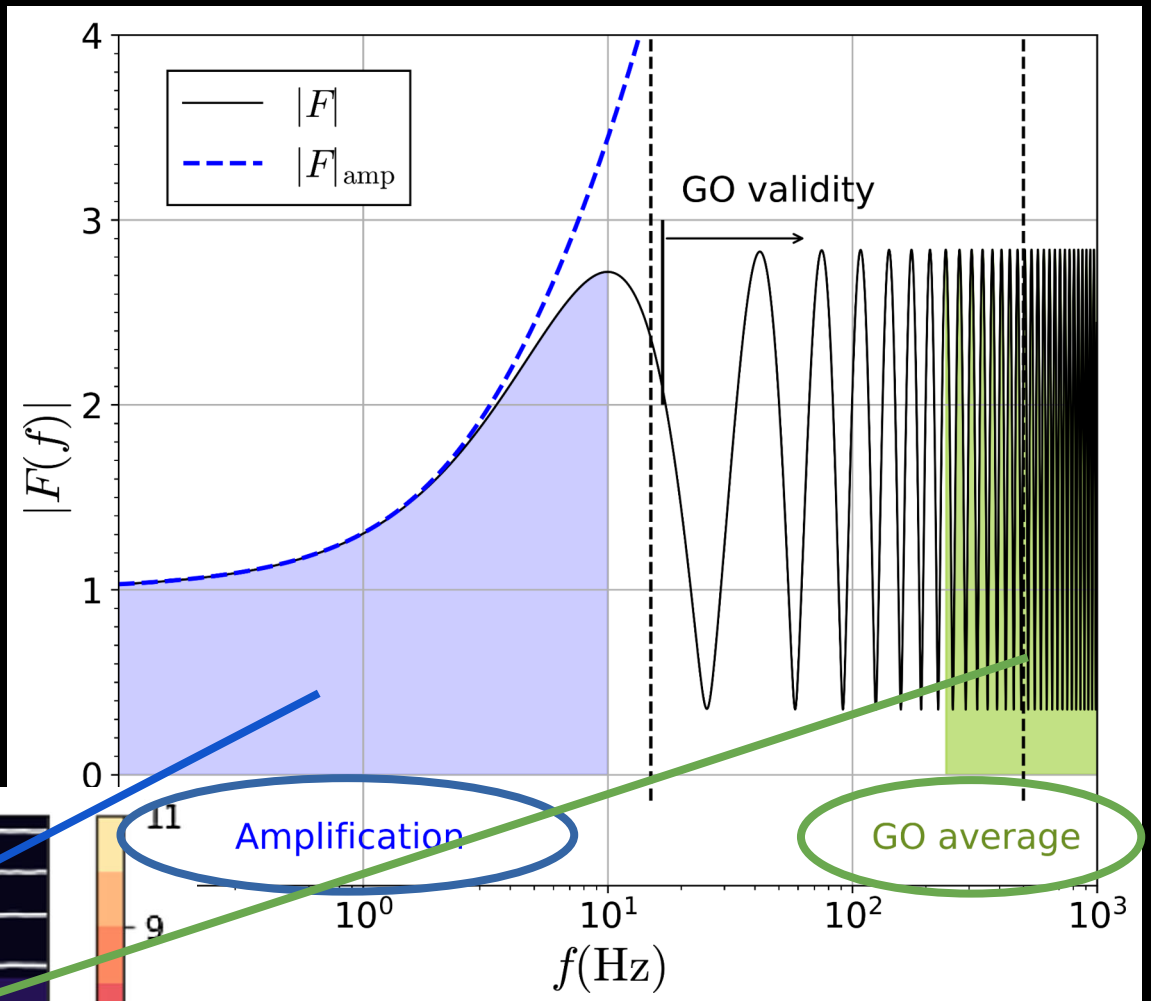

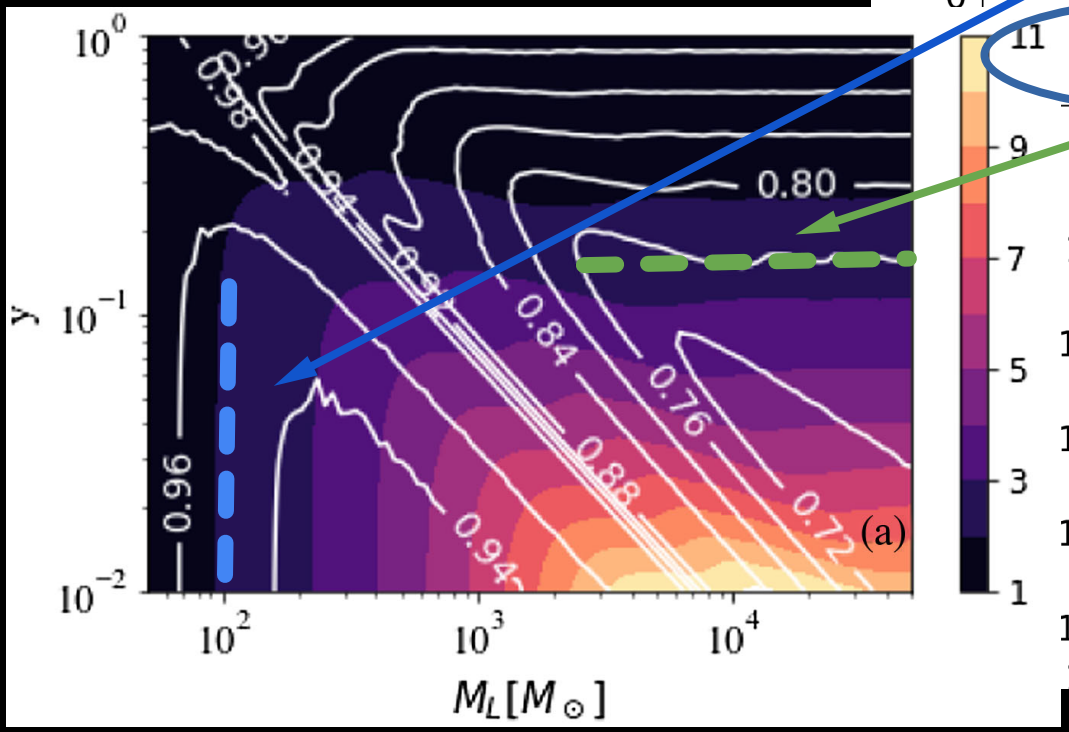

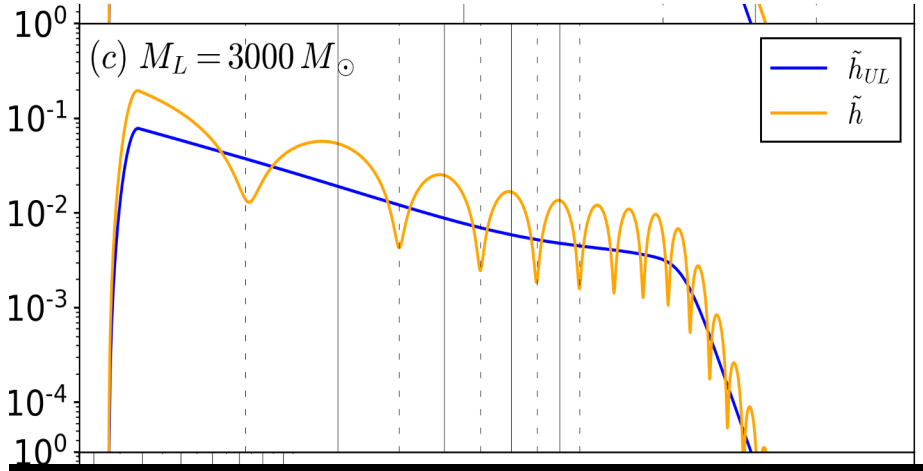

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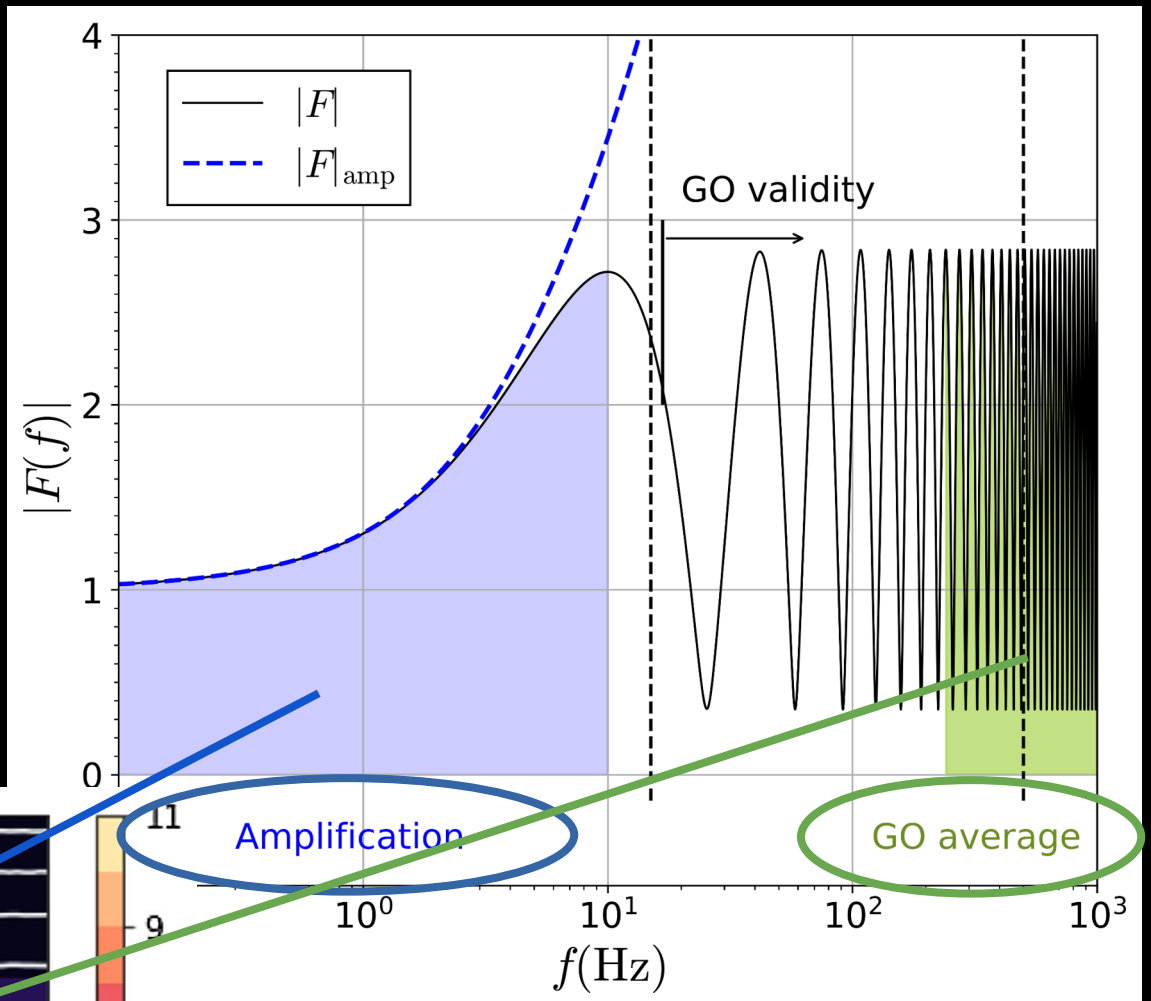
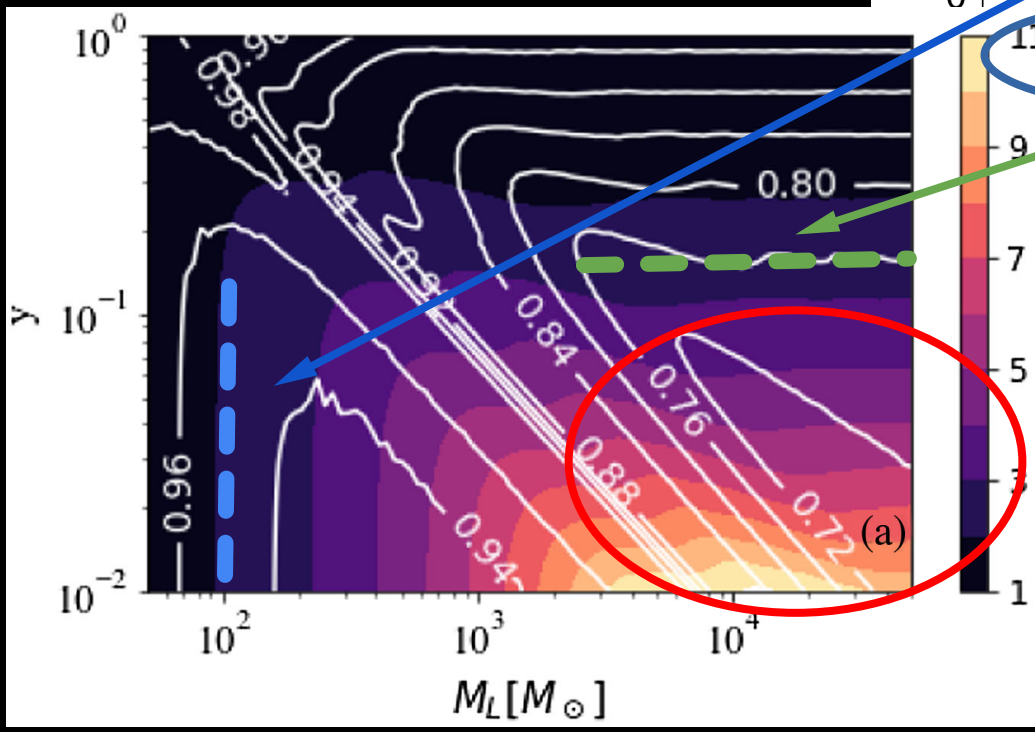
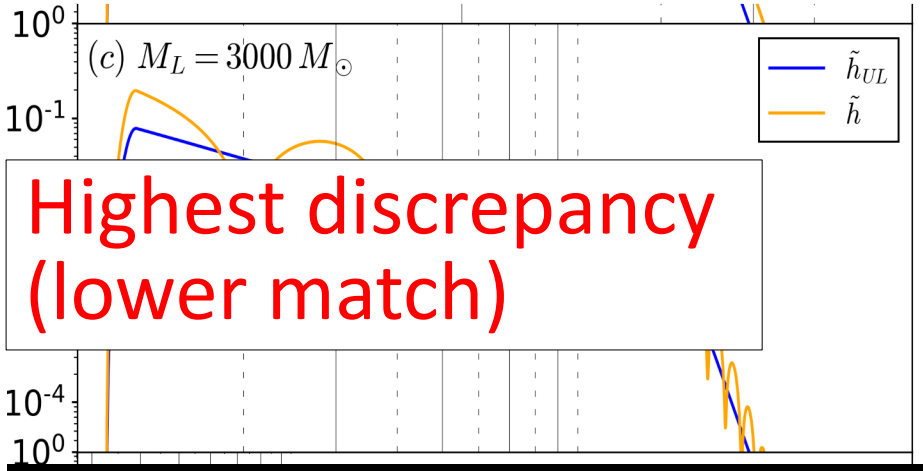




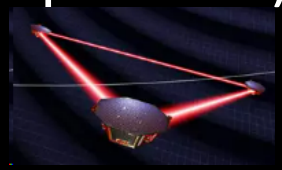


Highest discrepancy (lower match)

Extended lens

Ongoing work

- Galaxies / dark matter halos (as lens)
- Wave effects for either
  - low lens mass (in LIGO-Virgo-Kagra frequencies)
  - low frequency (e.g. LISA frequencies)



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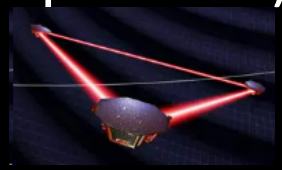
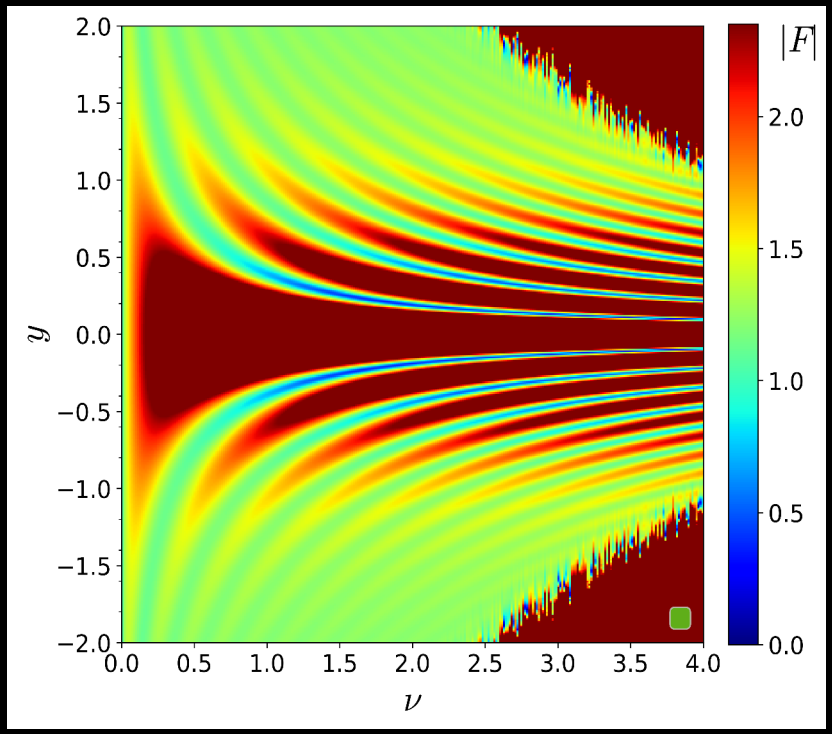


Wave effects, full formulation:



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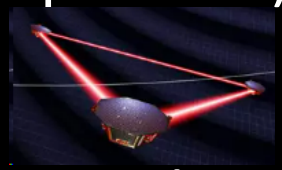
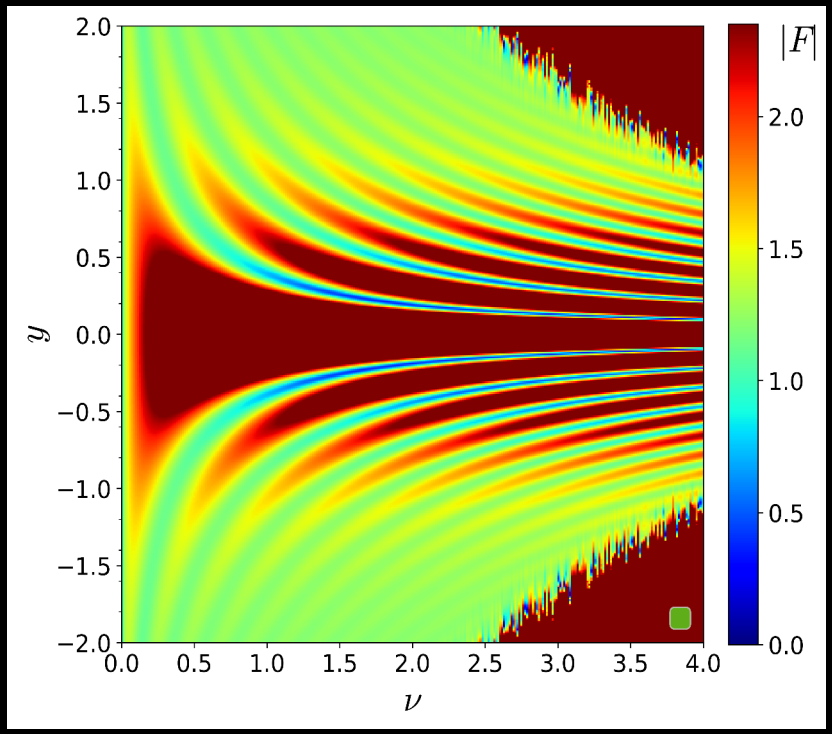
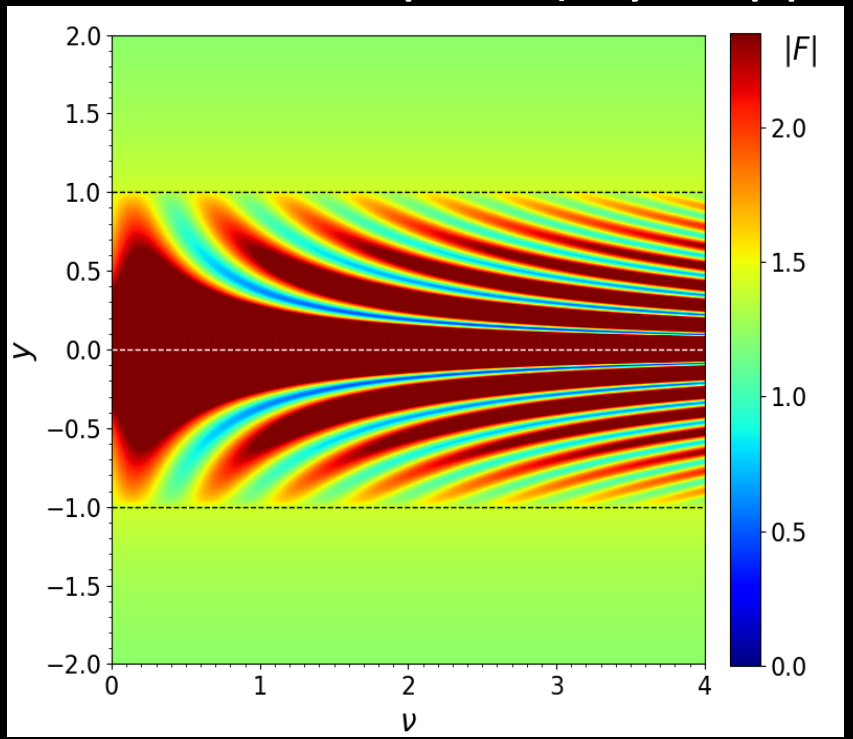


Wave effects, full formulation:



Geometrical Optics (rays, approx.):



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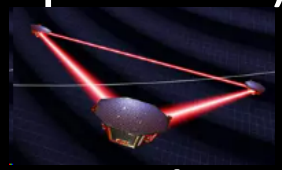

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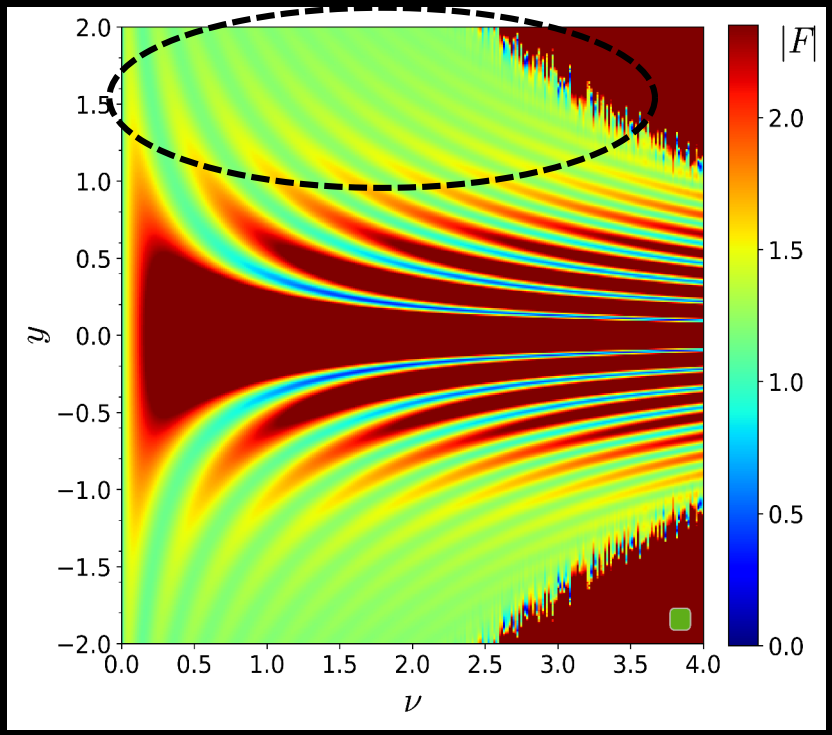

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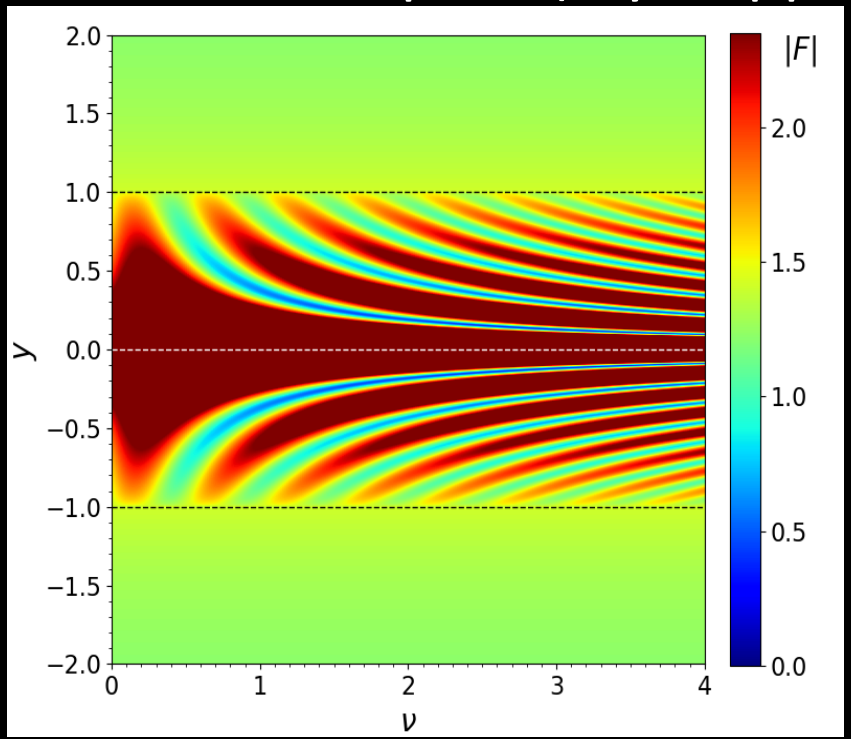
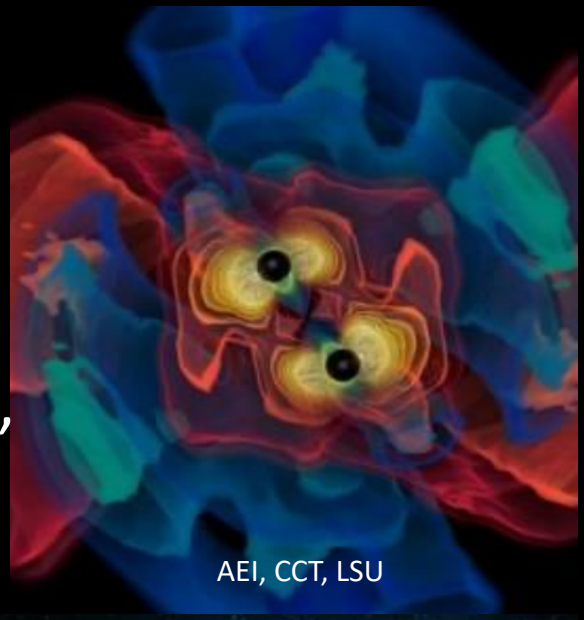


$+F_{\mathrm{diffract}}$

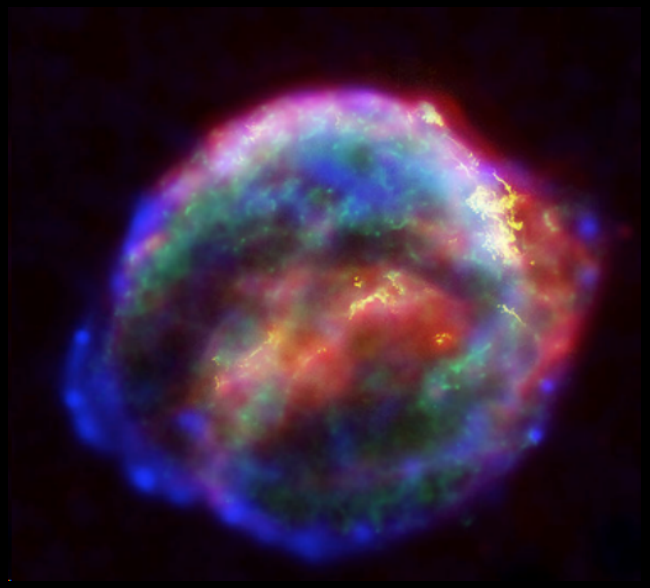
Gravitational waves and where to find them

Compact binary coalescence

(black holes, neutron stars...)



AEI, CCT, LSU

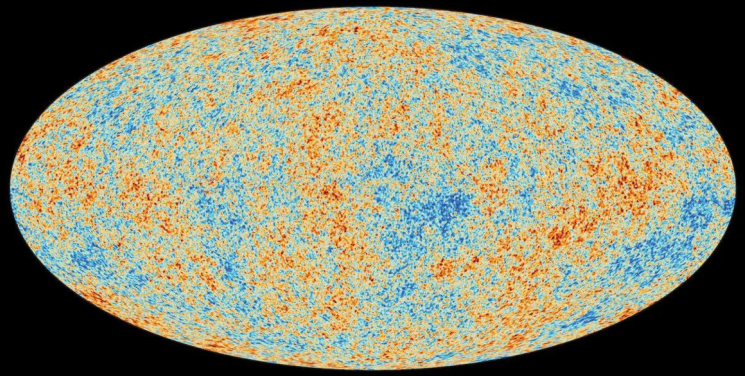


Chandra X-Ray Observatory

Core-collapse supernovae

Asymmetric rotating neutron stars

Casey Reed, Penn State



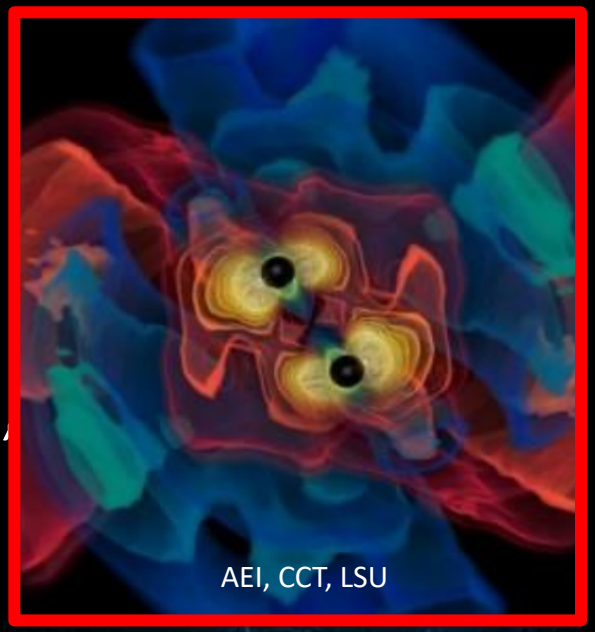
ESA/PLANCK COLLABORATION

Primordial gravitational wave background

Gravitational waves and where to find them

Compact binary coalescence

(black holes, neutron stars...)



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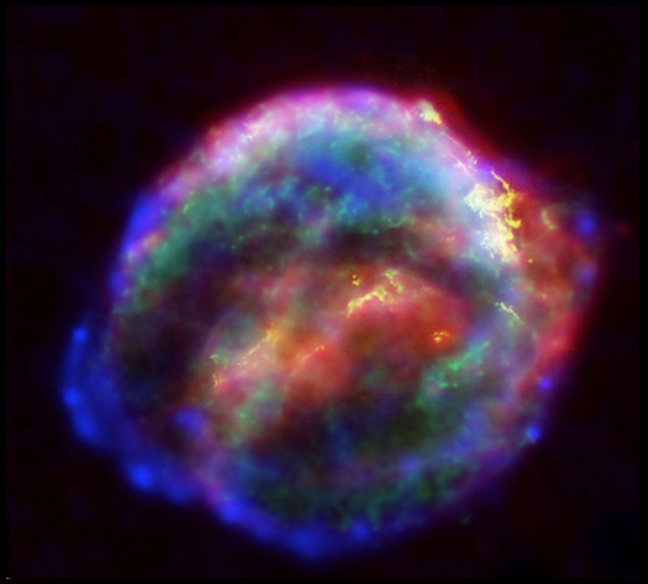

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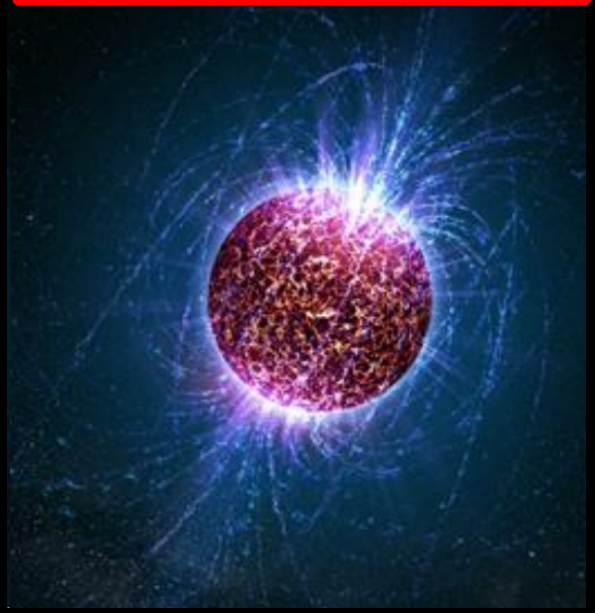

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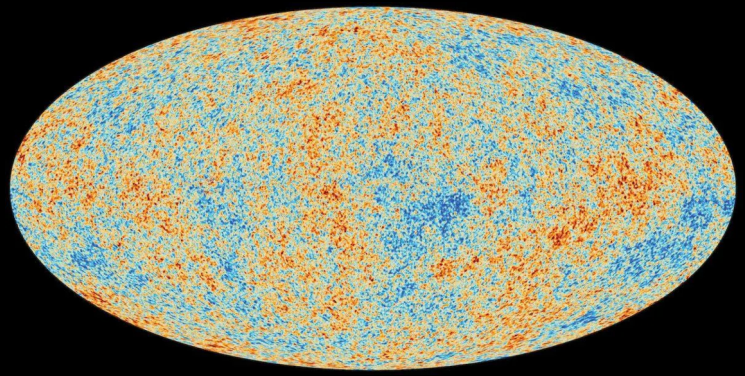


ESA/PLANCK COLLABORATION

Primordial gravitational wave background

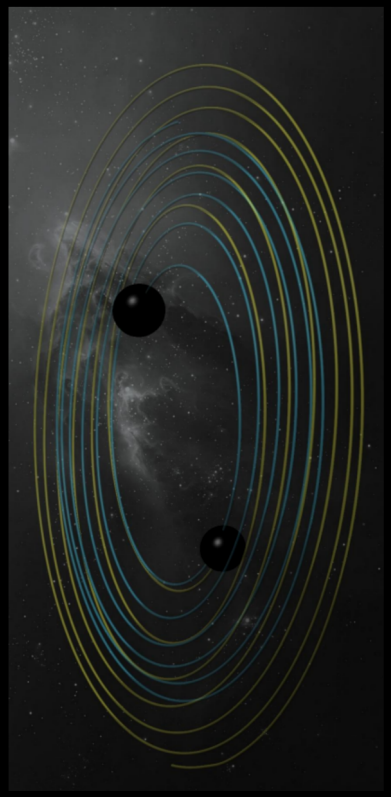
Gravitational waves and where to find them

Compact binary merger:

Gravitational waves and where to find them

Compact binary merger:

R. Jaume, S. Husa

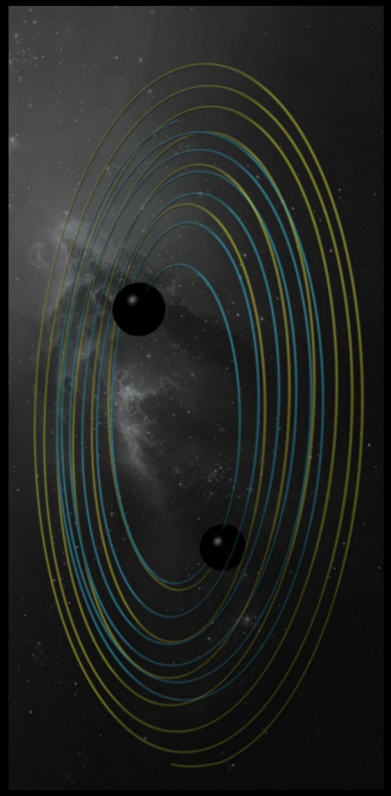


Isolated binary

Gravitational waves and where to find them

Compact binary merger:

R. Jaume, S. Husa



Isolated binary

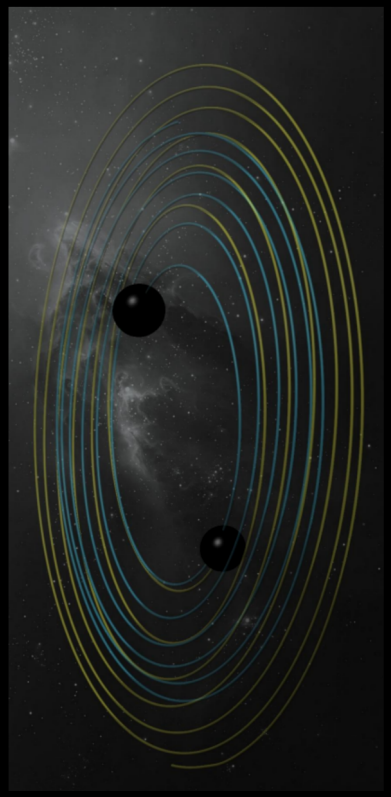
Hubble, NASA, ESA

Binary inside stellar clusters

Gravitational waves and where to find them

Compact binary merger:

R. Jaume, S. Husa



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NASA/ESA, Walter Jaffe/Leiden Observatory, Holland Ford/JHU/STScI

Binary inside Active Galactic Nuclei disks

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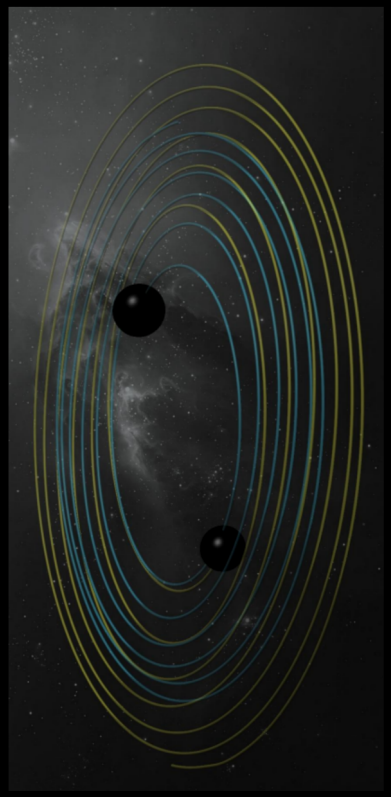

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Binary inside Active Galactic Nuclei disks

Ongoing work

Gravitational waves could be lensed by environment

Summary

Lensing of gravitational waves

- New information from lenses:
  - invisible astrophysical objects (black holes, dark matter)
  - hidden environments (centre of dense regions)
- Should exist & be detected (soon!)

Thank you for your attention!

Additional slides

How many lensed events can we expect?

Current detectors, O4 run:

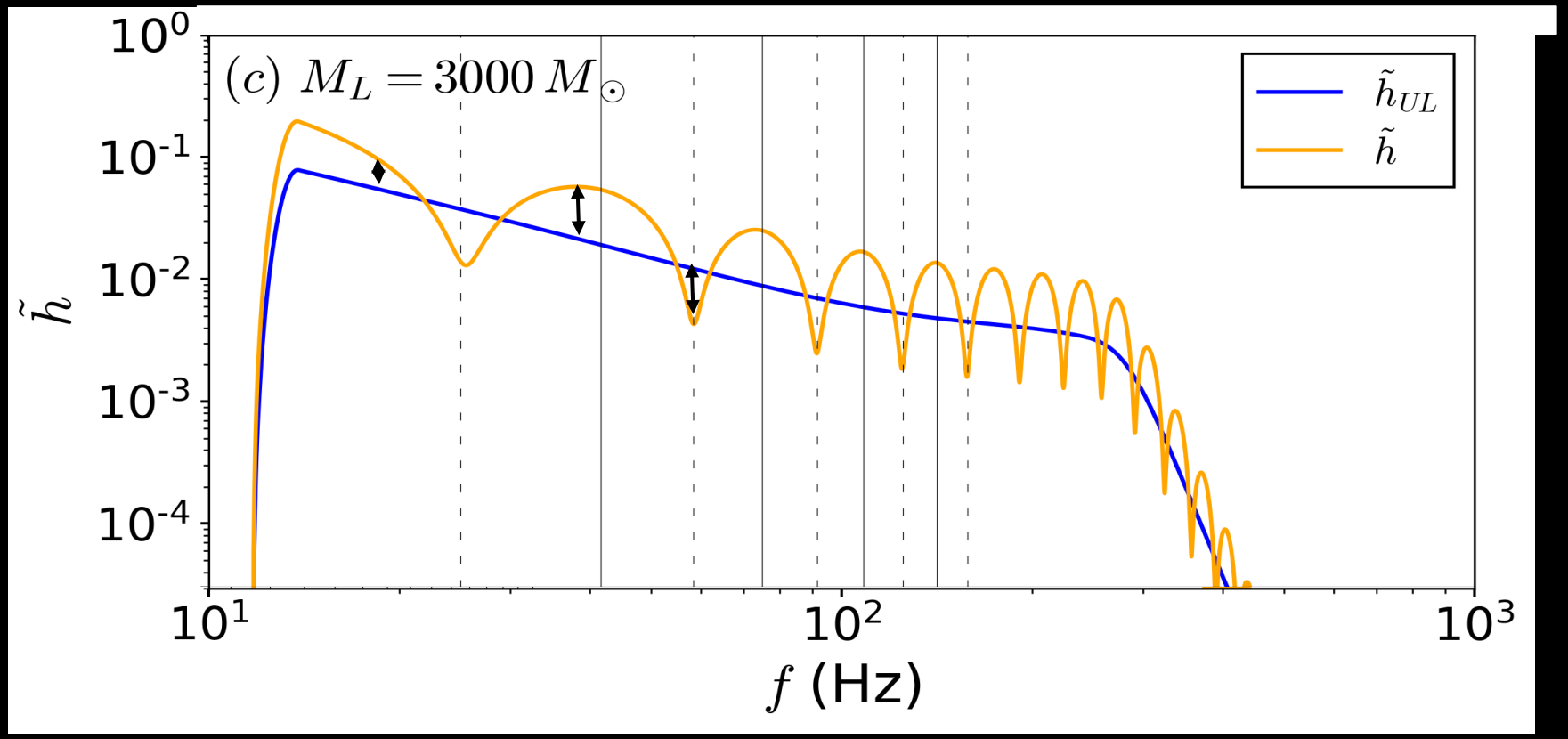
Multiple images: ~1 lensed event/year
(Ng et al. 2018, Xu et al. 2022, Yang et al. 2022...)

Wave effects: ~0.001-0.05 lensed events/year
(Yamamoto 2005, Choi et al. 2021, Lai et al. 2018...)

Match between templates

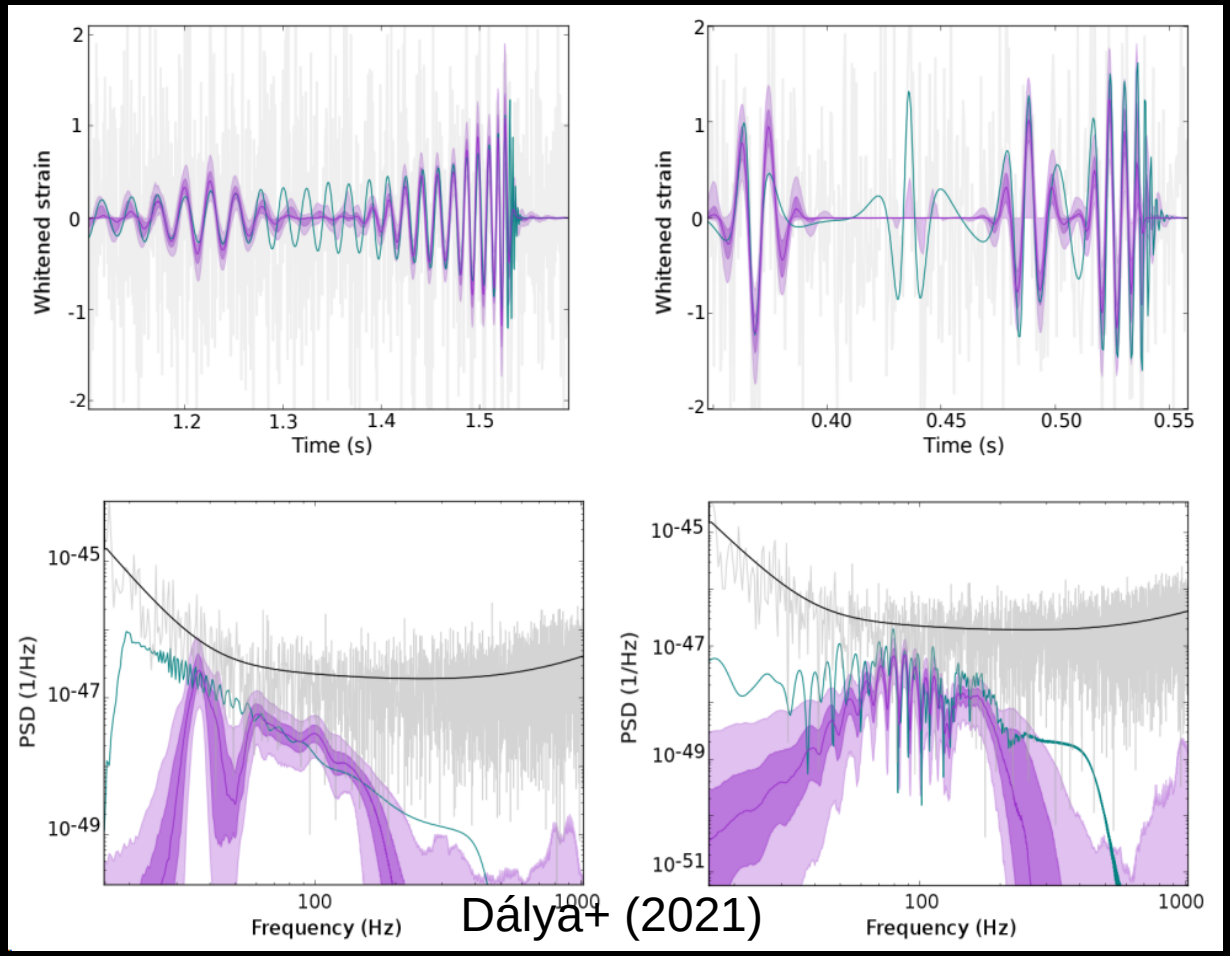
$$\mathcal{M}(a,b) = \max_{\varphi,t} \frac{\langle a(f), b(f;t,\varphi)\rangle}{\sqrt{\langle a,a\rangle\langle b,b\rangle}}$$

$$\langle a,b\rangle = 2\int_{f_{\min}}^{f_{\max}} \frac{\tilde{a}^*(f)\tilde{b}(f) + \tilde{a}(f)\tilde{b}^*(f)}{S_n(f)} df$$



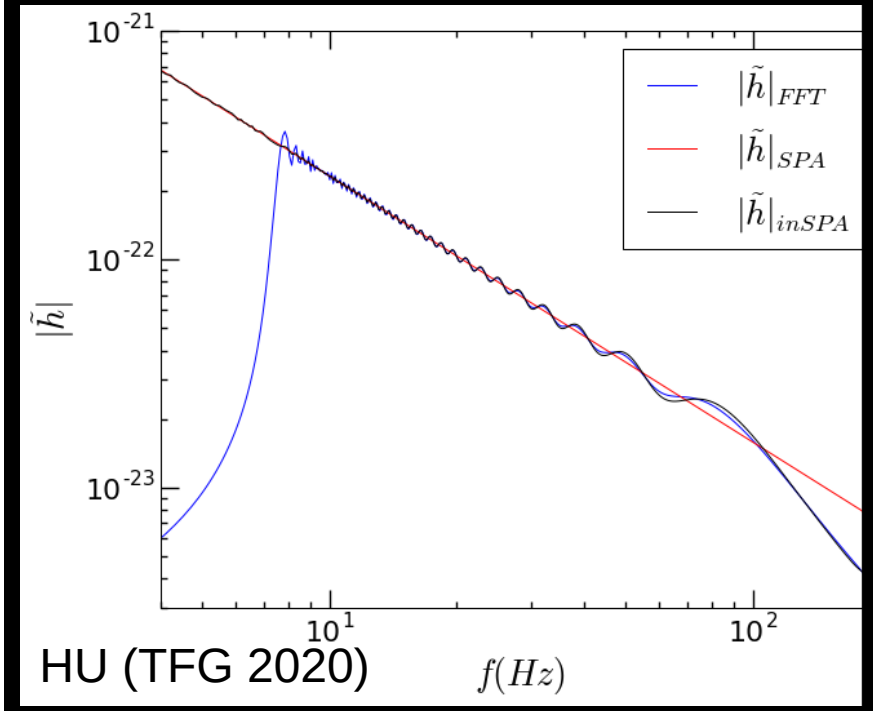
Other distortions in signals

Eccentricity, precession, spin misalignment...



Dálya+ (2021)

Template truncation, artifacts, noise...



HU (TFG 2020)

e.g. Damour+ (2000)

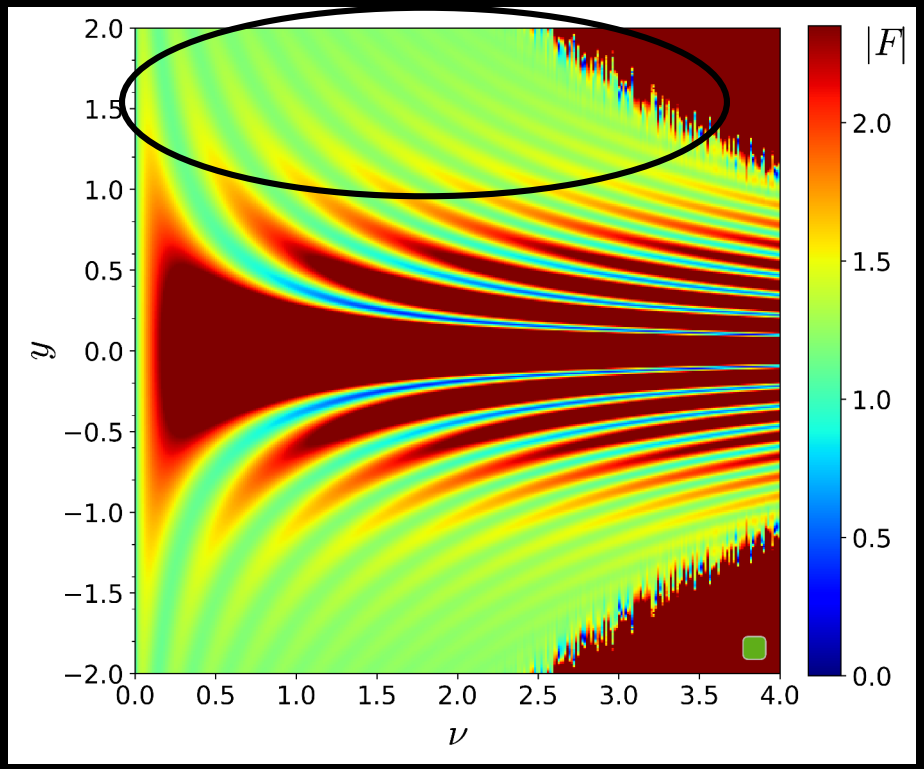
Extended lens (ongoing work)

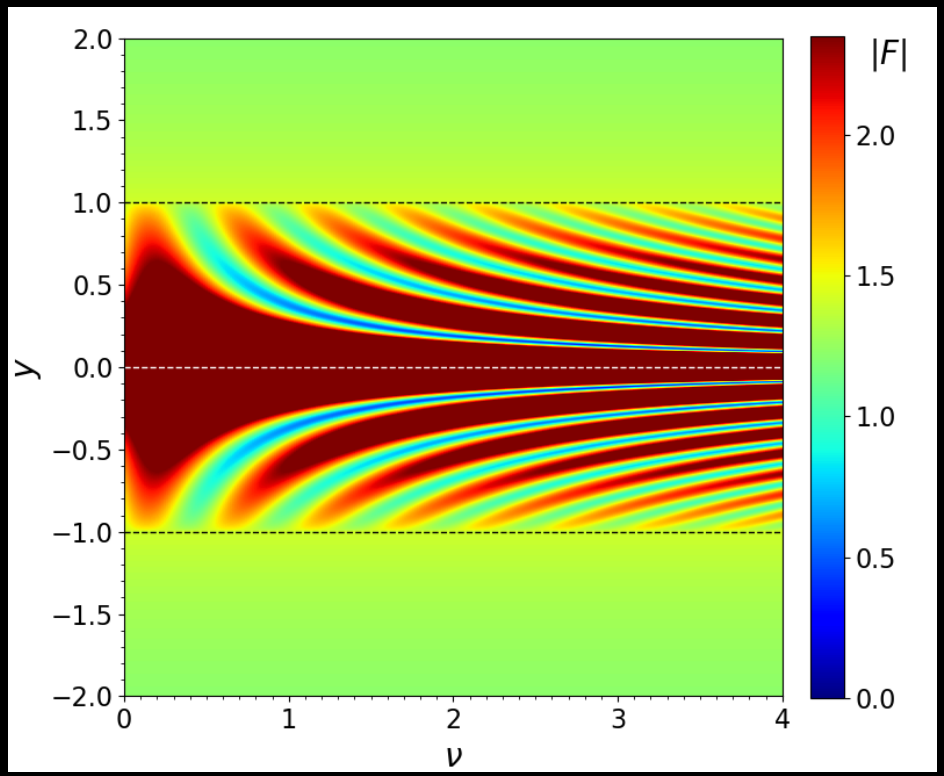
We want this...

$$F = -i\nu \iint_{lens} e^{i2\pi\nu\tau}$$

Current simple approximation: Geometrical Optics

$$F_{GO} = \sqrt{\frac{1+y}{y}} + \sqrt{\frac{1-y}{y}} e^{i4\pi\nu y - i\pi/2}$$





...simple approximation
...computed correctly at high $\nu \propto M_L f$
...computed faster

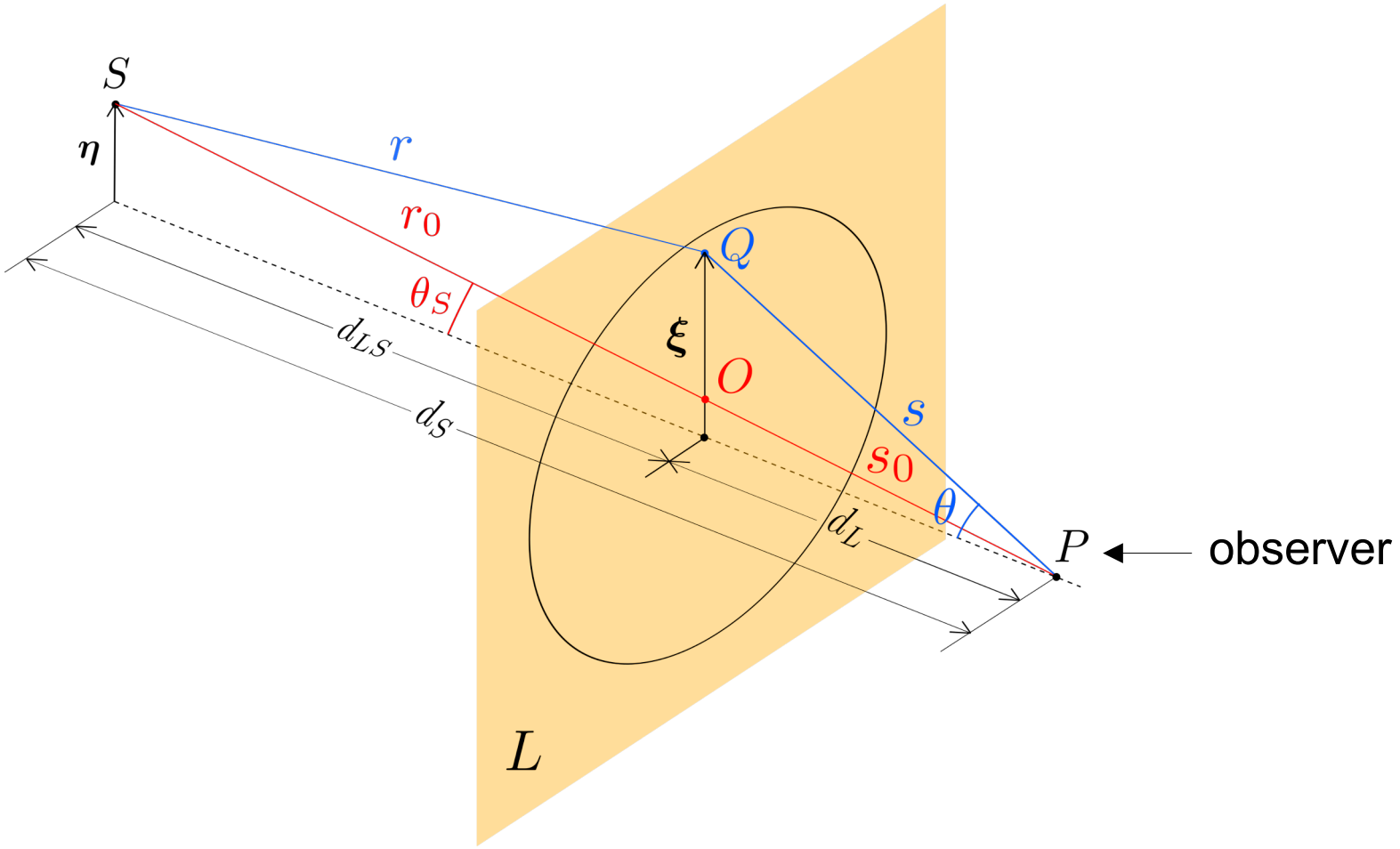
$+ F_{\text{diffract}}$

Add diffraction

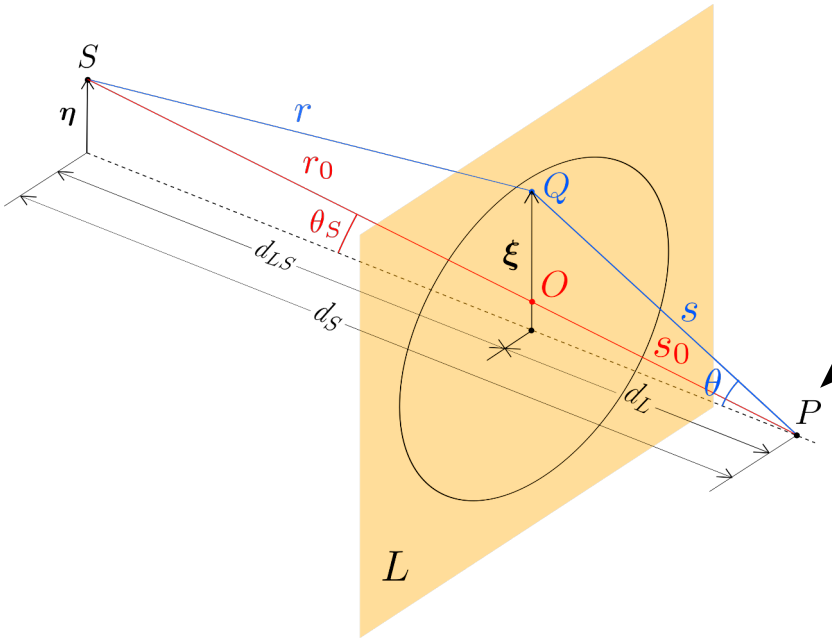
Thin-lens approximation

Propagation: $\left(\nabla^2 + k^2\right) \widetilde{\varphi} = \dfrac{4\,k^2}{c^2}\, U \widetilde{\varphi}$ (scalar wave equation)

Thin lens: lens size $\ll$ distances travelled (cosmological)



Fresnel-Kirchhoff integral



Huygens-Fresnel principle (wave optics)

Fresnel-Kirchhoff integral:

$$\widetilde{\varphi}(P) = \frac{A}{\mathrm{i}\lambda} \iint \frac{1}{rs}\, e^{\mathrm{i}[k(r+s)+\psi_L]}\, dS$$

Geometrical time delay

Gravitational time delay

superposition of partial waves at lens plane L

$$\psi_L(\boldsymbol{\xi}) = -\frac{4Gk}{c^2} \int \Sigma(\boldsymbol{\xi}')\, \ln|\boldsymbol{\xi} - \boldsymbol{\xi}'|\, d^2\boldsymbol{\xi}'$$

Σ depends on model, e.g. $\Sigma = M\,\delta^2(\boldsymbol{\xi}')$ for a point mass

Fresnel-Kirchhoff integral

Schwarzschild radius of the lens $R_S = \dfrac{2GM}{c^2}$

Dimensionless frequency: $\nu = \dfrac{2R_S}{\lambda}$

Wavelength of gravitational waves λ

$$\widetilde{\varphi}(P) = \frac{A}{\mathrm{i}\lambda} \iint \frac{1}{rs}\, e^{\mathrm{i}[k(r+s)+\psi_L]}\, dS$$

$$= -\mathrm{i}\nu \frac{A}{d_S} e^{\mathrm{i}k(r_0+s_0)} \iint \underbrace{e^{2\pi \mathrm{i}\,\nu\,\tau(\mathbf{x},\mathbf{y})}}_{\Phi(\mathbf{x})} d^2\mathbf{x}$$

Fresnel-Kirchhoff integral

$$\Phi(\mathbf{x}) = \arg\left(e^{2\pi \mathrm{i}\,\nu\,\tau(\mathbf{x},\mathbf{y})}\right)$$

$$\Phi(\mathbf{x}) \in [-\pi, \pi]$$

$$\tau(\mathbf{x},\mathbf{y}) = \frac{1}{2}(\mathbf{x}-\mathbf{y})^2 - \psi(\mathbf{x}) + \psi_0$$

$$\mathbf{x} = \frac{\boldsymbol{\theta}}{\theta_{\mathrm{E}}} = \frac{\boldsymbol{\xi}}{R_{\mathrm{E}}}, \qquad \mathbf{y} = \frac{\boldsymbol{\theta}_S}{\theta_{\mathrm{E}}} = \frac{d_L}{d_S}\frac{\boldsymbol{\eta}}{R_{\mathrm{E}}}.$$

$$\nu = \frac{2R_{\mathrm{S}}}{\lambda}$$

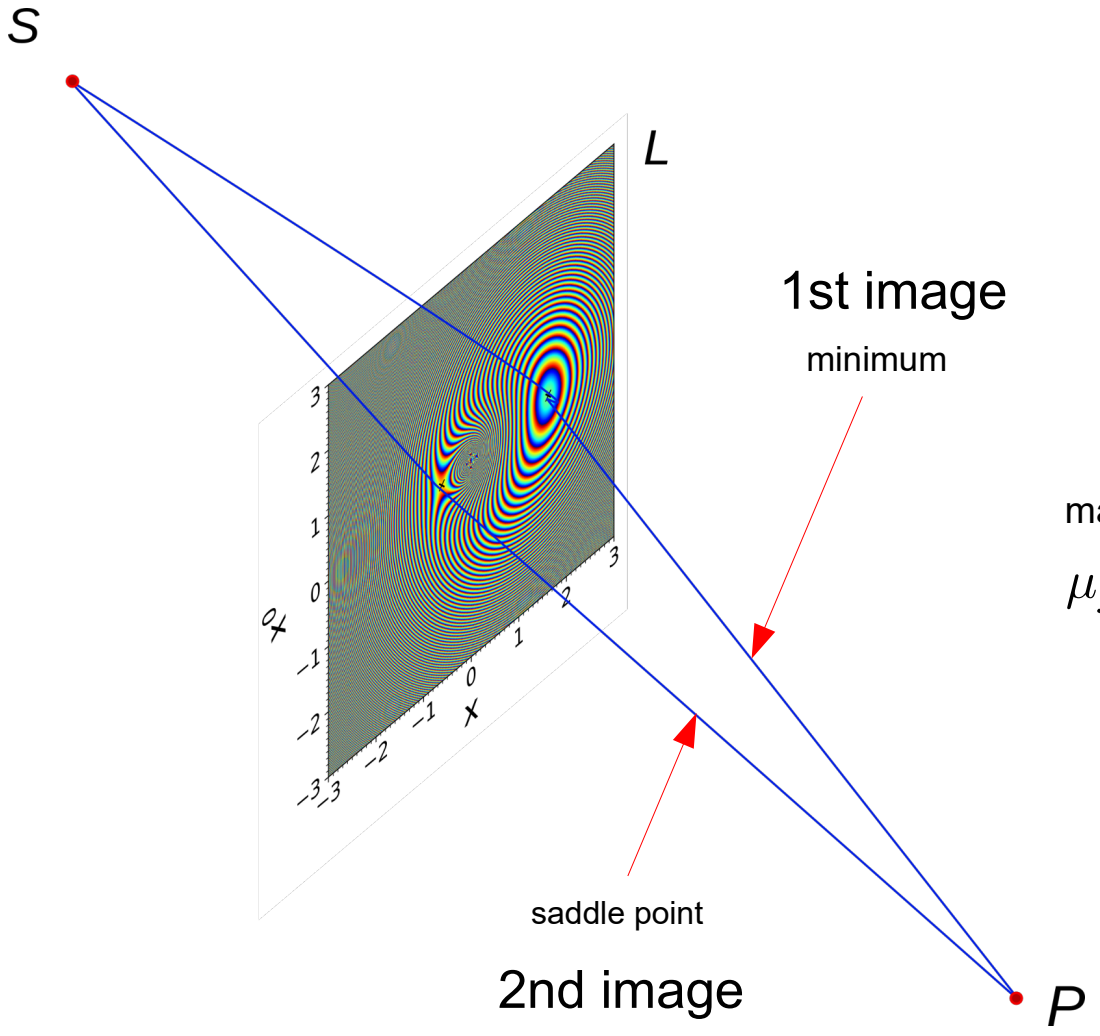
$\nu = 10$ $(\nu \gg 1,\ \lambda \ll R_{\mathrm{S}})$

Geometrical Optics limit

$\nu = 1$ $(\nu \sim 1, \lambda \sim R_{\mathrm{S}})$

O. Bulashenko and H. Ubach (2021) https://arxiv.org/abs/2112.10773 (preprint)

Geometrical Optics (GO) approximation



(2 images for the particular case of point mass lens, other models can have more images)

Solution to Fresnel-Kirchhoff integral:

$$\widetilde{\varphi}(P) \propto \sum_j |\mu_j|^{1/2} e^{i(2\pi\nu\,\tau(\mathbf{x}_j,\mathbf{y}) - n_j\pi/2)}$$

magnification of the j-th image

$$\mu_j = \left(\det\left(\partial\mathbf{y}/\partial\mathbf{x}_j\right)\right)^{-1}$$

$n_j = 0, 1, 2$

Depending on type of image

Wave effects in GO:

- Multiple images of source S
- Magnification/demagnification of images

Point mass lens

→ describes compact objects

Transmission factor: $F = \dfrac{\widetilde{\varphi}(P)}{\widetilde{\varphi}_0(P)}$

$F > 1$ Magnification

$F < 1$ Demagnification

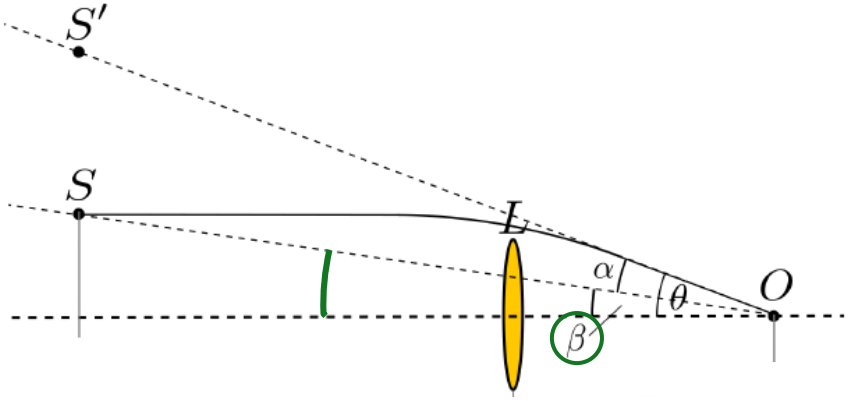
$$F = -\mathrm{i}\,\nu \iint e^{2\pi \mathrm{i}\,\nu\,\tau(\mathbf{x},\mathbf{y})}\, d^2\mathbf{x} \qquad \tau(\mathbf{x},\mathbf{y}) = \frac{1}{2}(\mathbf{x}-\mathbf{y})^2 - \psi(\mathbf{x}) + \psi_0$$

↓ analytical solution to Fresnel-Kirchhoff integral

↓

$$\psi(\mathbf{x}) = \ln|\mathbf{x}|$$

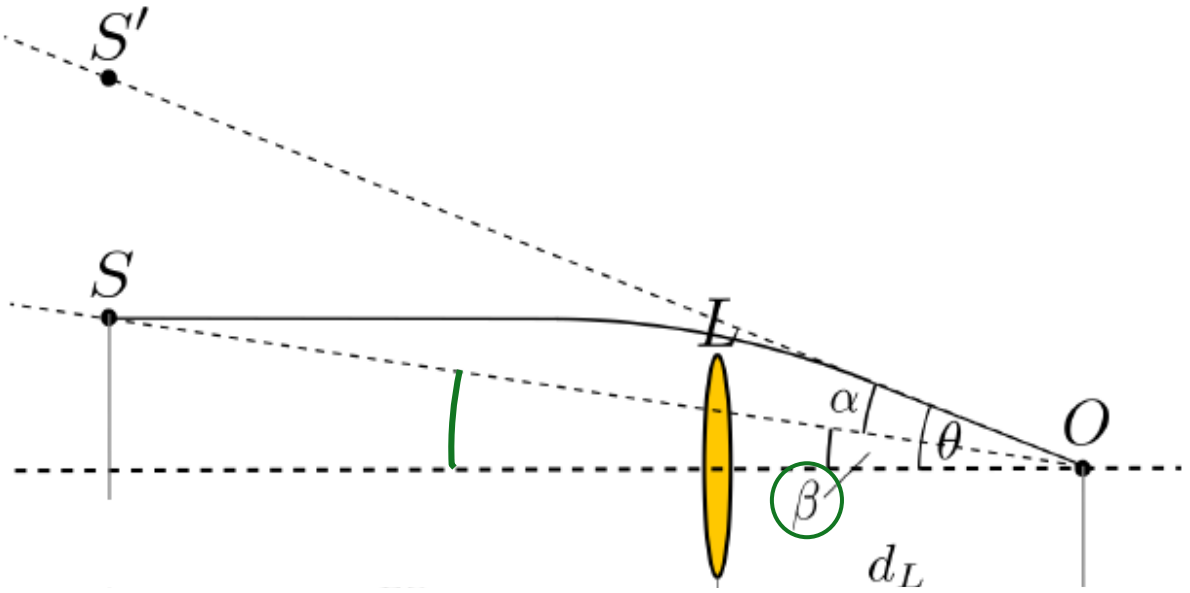
$$F(\nu, y) = e^{\frac{1}{2}\pi^2\nu} e^{\mathrm{i}\pi\nu\ln(\pi\nu)}\,\Gamma(1-\mathrm{i}\pi\nu)\,{}_1F_1(\mathrm{i}\pi\nu;\,1;\,\mathrm{i}\pi\nu y^2)$$



$$\nu = \frac{2R_\mathrm{S}}{\lambda} \qquad y = \frac{\beta}{\theta_\mathrm{E}}$$

$$\theta_\mathrm{E} = \sqrt{2R_\mathrm{S}\frac{d_{LS}}{d_L d_S}}$$

Point mass lens



$$\nu = \frac{2R_{\rm S}}{\lambda}$$

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$$\theta_{\rm E} = \sqrt{2R_{\rm S}\frac{d_{LS}}{d_L d_S}}$$

O. Bulashenko and H. Ubach (2021)
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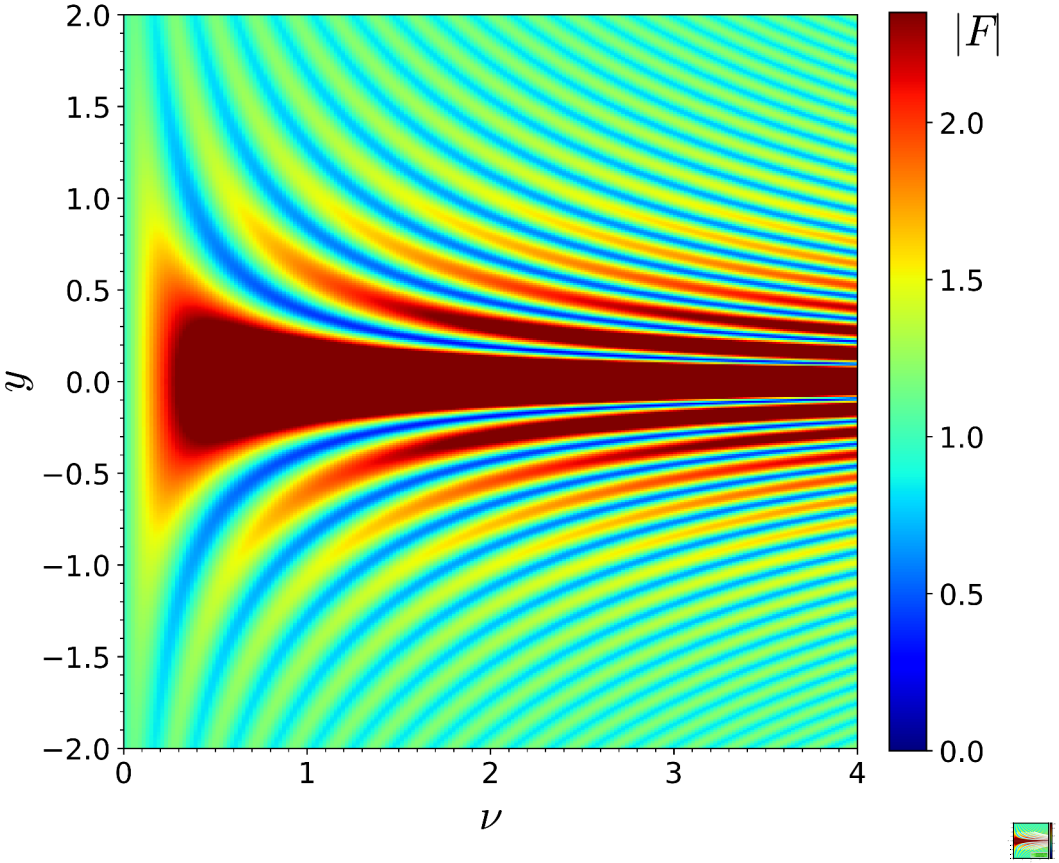
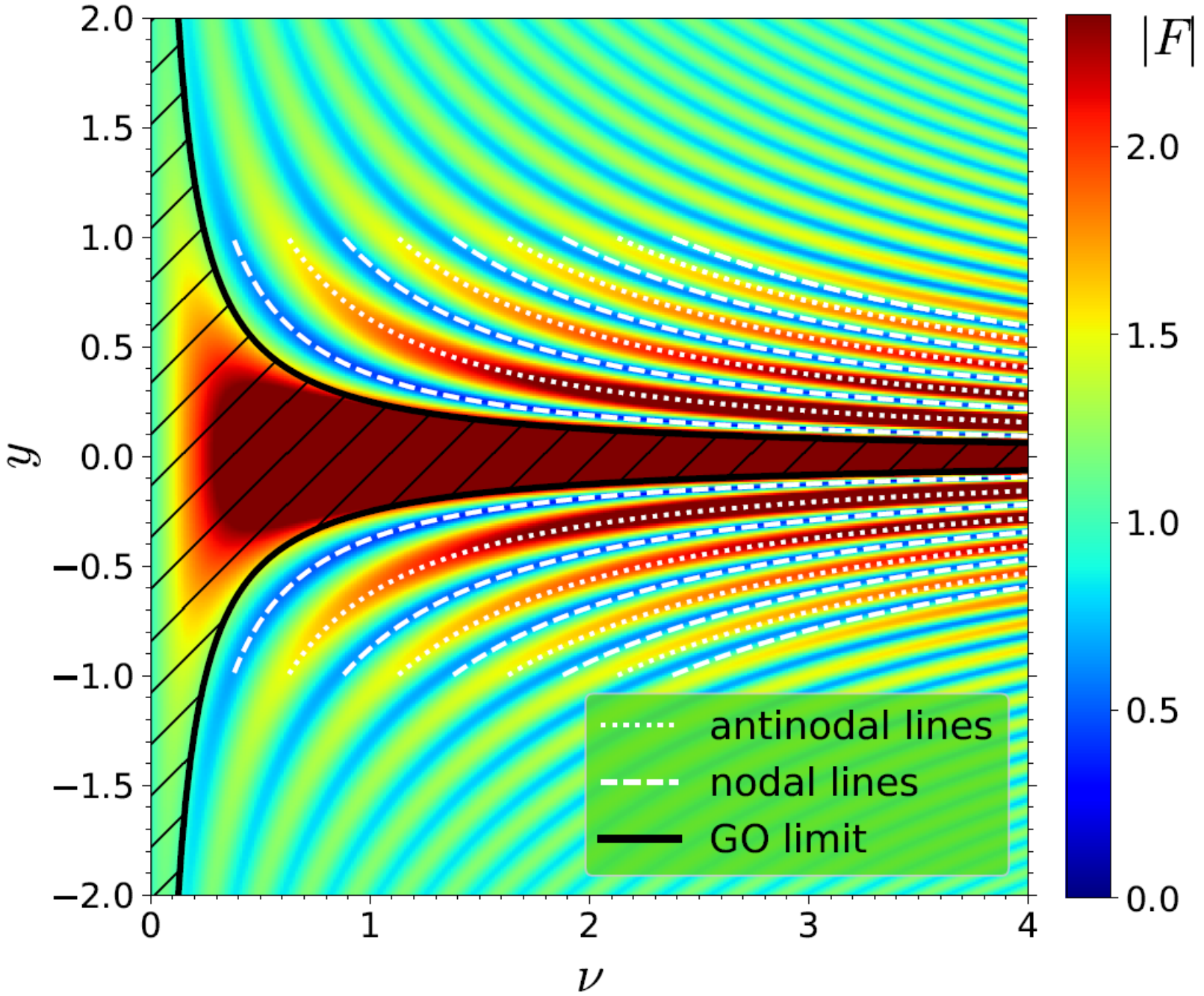


Full-wave

Geometrical Optics (GO)



O. Bulashenko and H. Ubach (2021)
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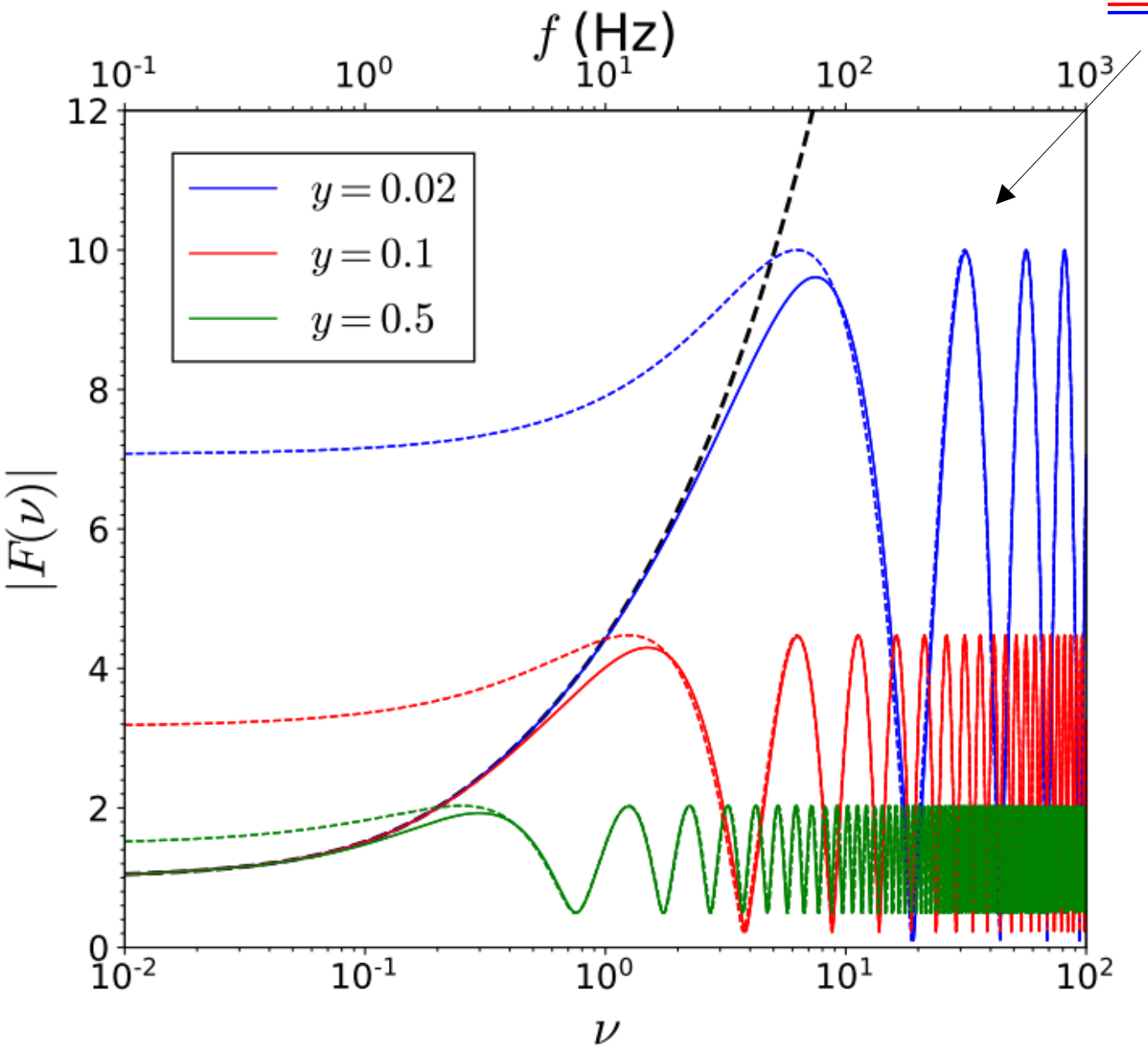
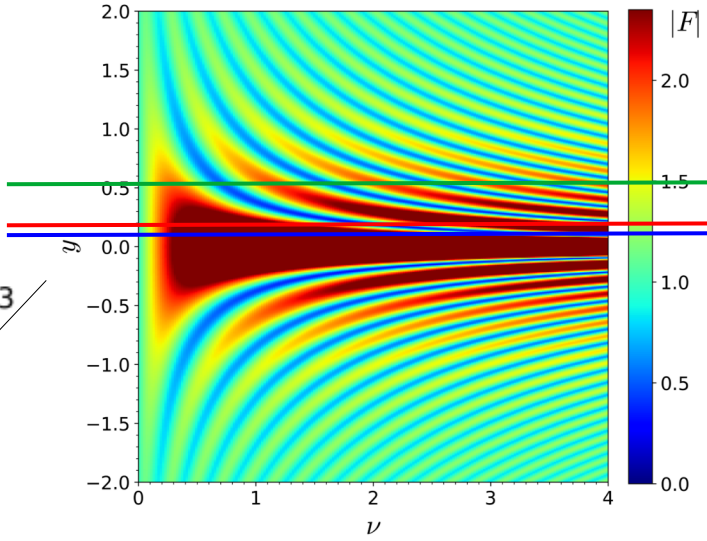
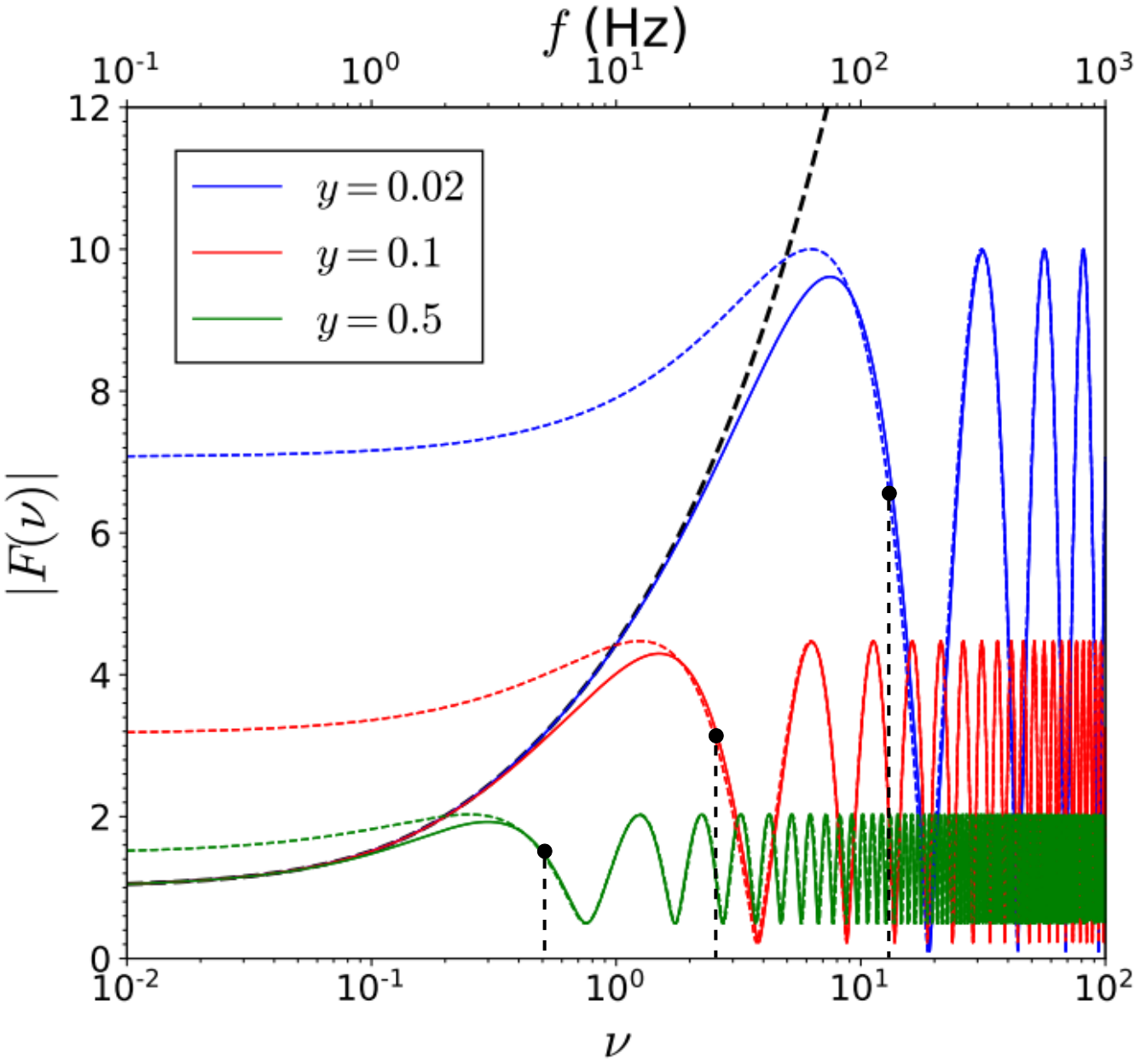




(horizontal sections)

- - - Geometrical Optics
—— Full-wave solution



GO approximation valid:

$$\nu \gtrsim \nu_G$$

- $\nu_G = \dfrac{1}{4y}$

Geometrical optics (GO) validity

GO approximation validity: $\nu \gtrsim \nu_G$

$$R_S = \frac{2GM}{c^2}$$

Before:

$$\nu \gg 1$$

$$\Downarrow$$

$$\lambda \ll R_{\rm S}$$

$$\lambda \lesssim 8 R_{\rm S} y$$

- Less restrictive condition
- Depends also on y

O. Bulashenko and H. Ubach (2021)
https://arxiv.org/abs/2112.10773 (preprint)

$$\widetilde{\varphi}(P) = \frac{A}{\mathrm{i}\lambda}\frac{e^{\mathrm{i}k(r_0+s_0)}}{d_L d_{LS}} \iint e^{\mathrm{i}[k\,\Delta l+\psi_L]}\,dS$$

$$F = \frac{\widetilde{\varphi}(P)}{\widetilde{\varphi}_0(P)} = \frac{1}{\mathrm{i}\lambda}\,\frac{d_S}{d_L d_{LS}} \iint e^{\mathrm{i}[k\,\Delta l+\psi_L]}\,dS.$$

$$\widetilde{\varphi}_0(P) = \frac{A}{d_S} e^{\mathrm{i}k(r_0+s_0)}$$

$$\Delta l\,(\boldsymbol{\xi},\boldsymbol{\eta}) = \frac{d_S}{2d_L d_{LS}}\left(\boldsymbol{\xi} - \frac{d_L}{d_S}\,\boldsymbol{\eta}\right)^2$$

$$\psi_L(\boldsymbol{\xi}) = -\frac{4Gk}{c^2}\int \Sigma(\boldsymbol{\xi}')\,\ln|\boldsymbol{\xi}-\boldsymbol{\xi}'|\,d^2\boldsymbol{\xi}'$$

$$k\Delta l + \psi_L = \omega\,\frac{2R_{\mathrm{S}}}{c}\left[\frac{1}{2}(\mathbf{x}-\mathbf{y})^2 - \psi(\mathbf{x}) + \psi_0\right]$$

$$|F| = \left(\frac{2\pi^2\nu}{1-e^{-2\pi^2\nu}}\right)^{1/2} \left|{}_1F_1(\mathrm{i}\pi\nu;\,1;\,\mathrm{i}\pi\nu y^2)\right|,$$

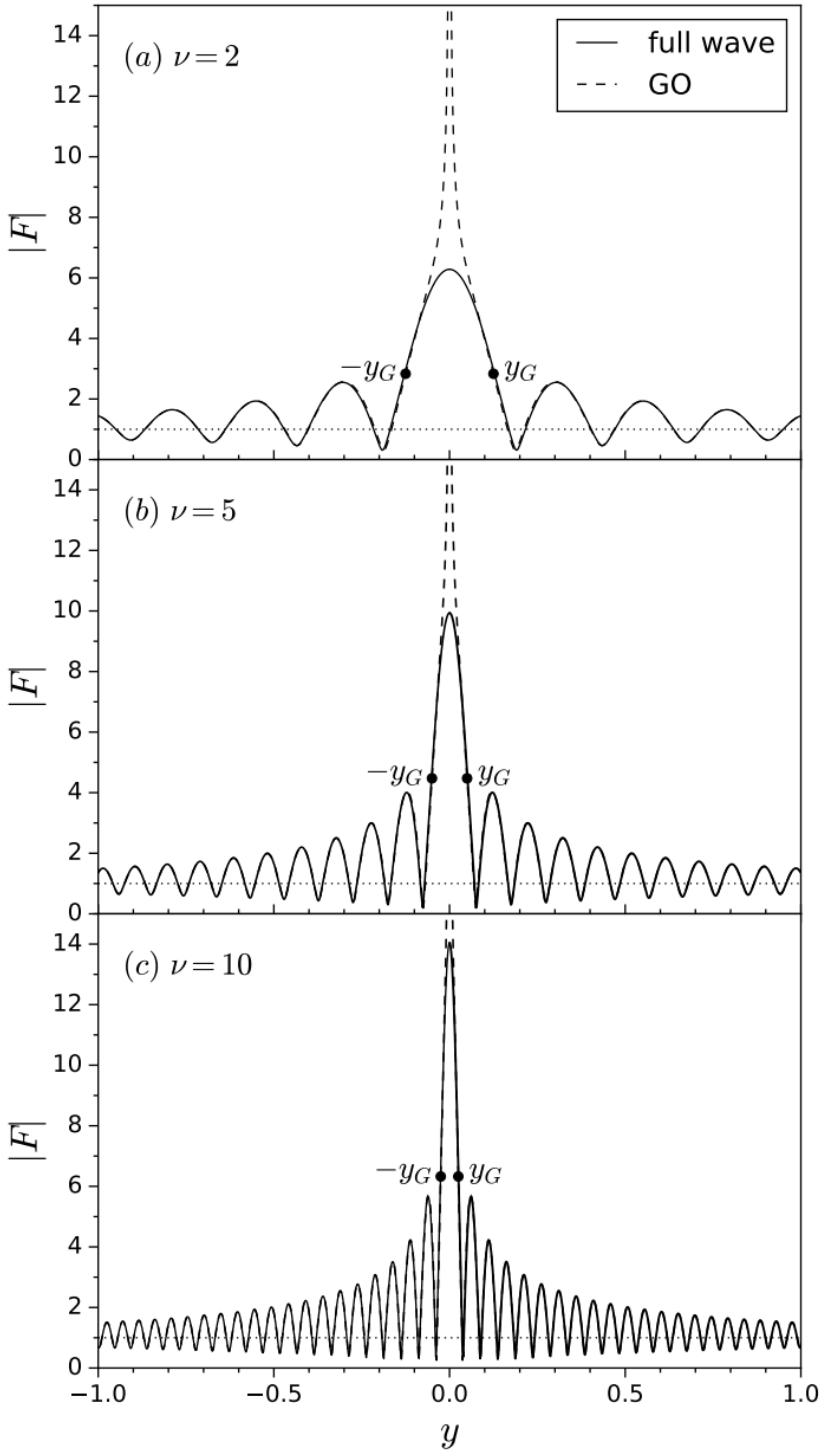
$$F_{GO} = \sum_j |\mu_j|^{1/2} e^{\mathrm{i}(2\pi\nu\,\tau(\mathbf{x}_j,\mathbf{y})-n_j\pi/2)},$$

Point mass lens:

$$x_{1,2} = \frac{1}{2}\left(y \pm \sqrt{y^2+4}\right) \qquad \mu_{1,2} = \frac{1}{4}\left(\frac{y}{\sqrt{y^2+4}} + \frac{\sqrt{y^2+4}}{y} \pm 2\right)$$

Hyperbolas:

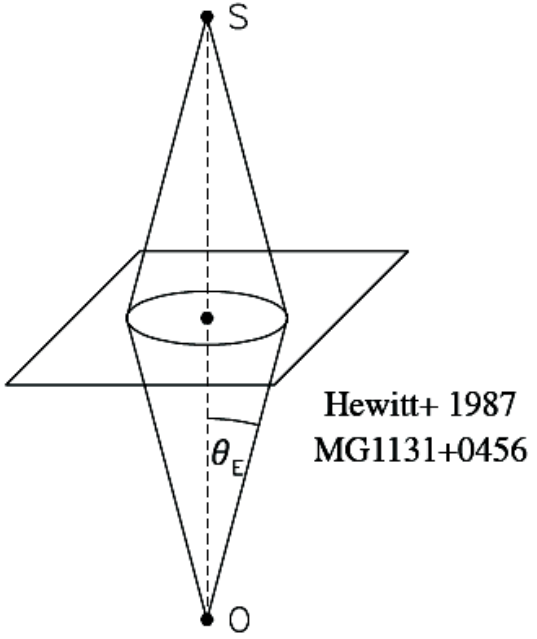
$$y_n = \frac{1}{2\nu} \cdot \begin{cases} \left(n+\frac{1}{4}\right), & \text{at maxima,} \\ \left(n+\frac{3}{4}\right), & \text{at minima,} \end{cases}$$


(a) $\nu = 2$
full wave
GO
$-y_G$
y_G
(b) $\nu = 5$
$-y_G$
y_G
(c) $\nu = 10$
$-y_G$
y_G
$|F|$
y
0
2
4
6
8
10
12
14
−1.0
−0.5
0.0
0.5
1.0

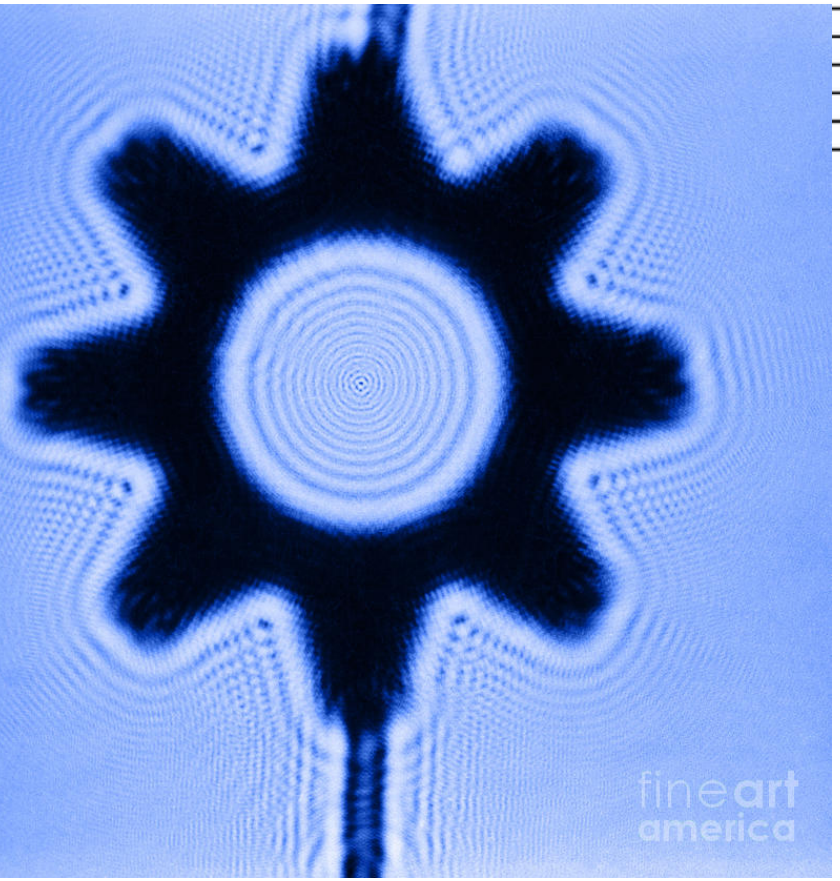
Characteristic angles

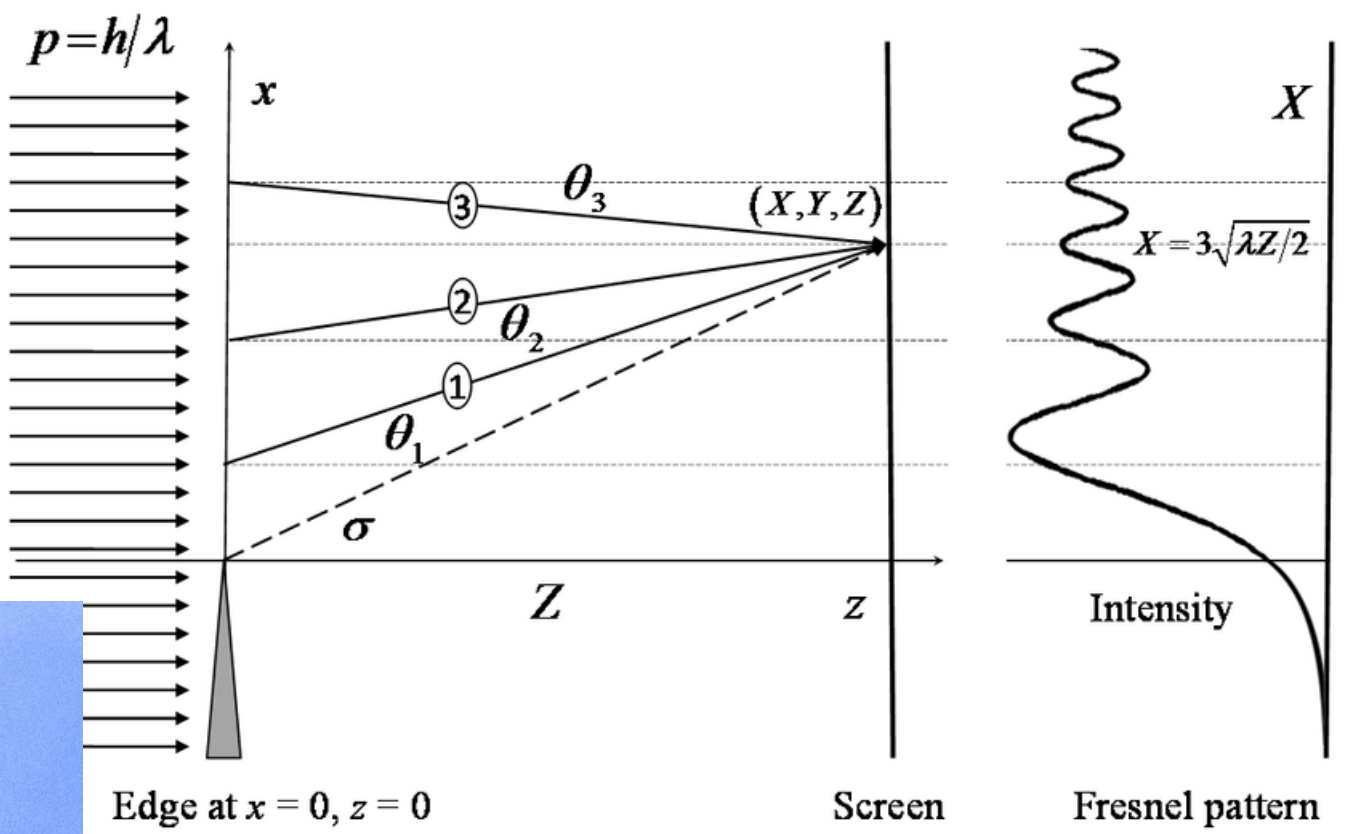
$$\theta_{\rm E} = (0.9\,{\rm mas}) \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D}{10\,{\rm kpc}}\right)^{-1/2},$$

$$\theta_{\rm E} = (0\farcs9) \left(\frac{M}{10^{11}\,M_\odot}\right)^{1/2} \left(\frac{D}{\rm Gpc}\right)^{-1/2}.$$



Hewitt+ 1987
MG1131+0456


fineart
america


p=h/λ
x
θ3
(X,Y,Z)
3
2
θ2
1
θ1
σ
Z
z
X
X=3√(λZ/2)
Intensity
Edge at x = 0, z = 0
Screen
Fresnel pattern
(a)
(b)