Hyperfine splittings of heavy quarkonium hybrids

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Introduction

Heavy flavours

- Heavy quarks: $Q = c, b, t,\ m_Q \gg \Lambda_{\text{QCD}}$
- Heavy hadrons: hadrons containing at least a heavy quark: $Q = b, c$

In the hadron rest frame the heavy quark moves slowly $\Rightarrow$ use a **non-relativistic approximation**

- A universal way to encode it together with relativistic correction is using **Effective Field Theories**
  - **NRQCD**/HQET are the suitable ones
  - They imply heavy quark spin symmetry at leading order.

Exotic hardons

- Hadrons beyond mesons $q\bar{q}$ and baryons qqq

By QCD:

Any color singlet state made out of quarks and gluons may become a hadron

Using as heavy quark $Q = b,\, c$ and as light quark $q = u,\, d,\, s$, the hardons with two heavy quarks are the following:

- $QQ+$ light quarks and gluons
  - Double Heavy Baryons: QQq
  - Tetraquarks: $QQ\bar{q}\bar{q}$
  - Pentaquarks: $QQqq\bar{q}$
  - Hybrids: $QQqg$
  - ...
- $Q\bar{Q}+$ light quarks and gluons
  - **Heavy Quarkonium**: $Q\bar{Q}$
  - **Hybrids**: $Q\bar{Q}g$
  - Tetraquarks: $Q\bar{Q}q\bar{q}$
  - Pentaquarks: $Q\bar{Q}qqq$
  - ...

Heavy quarkonium

$Q\bar{Q}$ bound state, $m_Q >> \Lambda_{QCD}$, $\alpha_s(m_Q) << 1$

- Heavy quarks move slowly $v << 1$
- Non-relativistic system $\rightarrow$ multiscale problem
  - $m_Q \gg m_Q v$ (Relative momentum)
  - $m_Q v \gg m_Q v^2$ (Binding energy)
  - $m_Q \gg \Lambda_{QCD}$
- Some useful EFTs
  - NRQCD: $m_Q \gg m_Q v, m_Q v^2, \Lambda_{QCD}$
  - pNRQCD (weak coupling): $m_Q v \gg m_Q v^2, \Lambda_{QCD}$
  - pNRQCD (strong coupling): $m_Q v, \Lambda_{QCD} \gg m_Q v^2$

Strong coupling pNRQCD $\equiv$ **Born-Oppenheimer EFT**

Hybrid spectrum

$$\mathcal{L} = \mathrm{tr}\left(H^{i\dagger}\left(\delta_{ij} i\partial_0 - h_{Hij}\right) H_j\right)$$

$$h_{Hij} = \left(-\frac{\nabla^2}{m_Q} + V_{\Sigma_u^-}(r)\right)\delta_{ij} + \left(\delta_{ij} - \hat{r}_i\hat{r}_j\right)\left[V_{\Pi_u}(r) - V_{\Sigma_u^-}(r)\right]$$

- Hybrid:

$$\mathbf{H}(\mathbf{r}) = \frac{1}{r}\left(P_0^+(r)\mathbf{Y}_{00}^{L=0} + \sum_{J=1}^{\infty}\sum_{M=-J}^{J}\left[P_J^+(r)\mathbf{Y}_{JM}^{L=J+1} + P_J^0(r)\mathbf{Y}_{JM}^{L=J} + \right.\right.$$
$$\left.\left. + P_J^-(r)\mathbf{Y}_{JM}^{L=|J-1|}\right]\right)$$

- Potentials:[1]

$$V_{\Sigma_u^-}(r) = \frac{\sigma_p}{r} + \kappa_s r + E_s^{\bar{Q}Q}, \qquad V_{\Pi_u}(r) = \frac{\sigma_s}{r}\left(\frac{1 + b_1 r + b_2 r^2}{1 + a_1 r + a_2 r^2}\right) + \kappa_p r + E_p^{\bar{Q}Q}$$

[1] Ruben Oncala, JS, Phys. Rev. D 96, 014004 (2017)

Hybrid spectrum

$$\mathcal{L} = \mathrm{tr}\left(H^{i\dagger}\left(\delta_{ij} i\partial_0 - h_{Hij}\right) H_j\right)$$

$$h_{Hij} = \left(-\frac{\nabla^2}{m_Q} + V_{\Sigma_u^-}(r)\right)\delta_{ij} + \left(\delta_{ij} - \hat{r}_i\hat{r}_j\right)\left[V_{\Pi_u}(r) - V_{\Sigma_u^-}(r)\right]$$

- Equations for $J \neq 0$:

$$\left[-\frac{1}{m_Q}\frac{\partial^2}{\partial r^2} + \begin{pmatrix} \frac{(J+1)J}{m_Q r^2} & 0 \\ 0 & \frac{(J+1)(J+2)}{m_Q r^2} \end{pmatrix} + V_{\Sigma_u^-}(r) + V_q(r)\begin{pmatrix} \frac{J+1}{2J+1} & \frac{\sqrt{(J+1)J}}{2J+1} \\ \frac{\sqrt{(J+1)J}}{2J+1} & \frac{J}{2J+1} \end{pmatrix}\right]\begin{pmatrix} P_J^-(r) \\ P_J^+(r) \end{pmatrix} = E\begin{pmatrix} P_J^-(r) \\ P_J^+(r) \end{pmatrix}$$

$$\left(-\frac{1}{m_Q}\frac{\partial^2}{\partial r^2} + \frac{J(J+1)}{m_Q r^2} + V_{\Pi_u}(r)\right) P_J^0(r) = E P_J^0(r)$$

Where $V_q(r) = V_{\Pi_u}(r) - V_{\Sigma_u^-}(r)$

- Equations for $J = 0$:

$$\left(-\frac{1}{m_Q}\frac{\partial^2}{\partial r^2} + \frac{2}{m_Q r^2} + V_{\Pi_u}(r)\right) P_0^+(r) = E P_0^+(r)$$

Results for charm

NL_J	w-f	$c\bar{c}$	Hybrid	$\mathcal{J}^{PC}$ $(S=0)$	$\mathcal{J}^{PC}$ $(S=1)$	Λ_η^ϵ
$1s$	S	3068		0^{-+}	1^{--}	Σ_g^+
$2s$	S	3678		0^{-+}	1^{--}	Σ_g^+
$3s$	S	4131		0^{-+}	1^{--}	Σ_g^+
$1p_0$, (H_3)	P^+		4486	0^{++}	1^{+-}	Σ_u^-
$4s$	S	4512		0^{-+}	1^{--}	Σ_g^+
$2p_0$	P^+		4920	0^{++}	1^{+-}	Σ_u^-
$3p_0$	P^+		5299	0^{++}	1^{+-}	Σ_u^-
$4p_0$	P^+		5642	0^{++}	1^{+-}	Σ_u^-
$1p$	S	3494		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2p$	S	3968		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1(s/d)_1$, (H_1)	$P^\pm$		4011	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1p_1$, (H_2)	P^0		4145	1^{++}	$(0,1,2)^{+-}$	Π_u
$2(s/d)_1$	$P^\pm$		4355	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$3p$	S	4369		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2p_1$	P^0		4511	1^{++}	$(0,1,2)^{+-}$	Π_u
$3(s/d)_1$	$P^\pm$		4692	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4(s/d)_1$	$P^\pm$		4718	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4p$	S	4727		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$3p_1$	P^0		4863	1^{++}	$(0,1,2)^{+-}$	Π_u
$5(s/d)_1$	$P^\pm$		5043	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$5p$	S	5055		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1d$	S	3793		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2d$	S	4210		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$1(p/f)_2$, (H_4)	$P^\pm$		4231	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$1d_2$, (H_5)	P^0		4334	2^{--}	$(1,2,3)^{-+}$	Π_u
$2(p/f)_2$	$P^\pm$		4563	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$3d$	S	4579		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2d_2$	P^0		4693	2^{--}	$(1,2,3)^{-+}$	Π_u
$3(p/f)_2$	$P^\pm$		4886	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$4d$	S	4916		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$4(p/f)_2$	$P^\pm$		4923	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$

Results for bottom

NL_J	w-f	$b\bar{b}$	Hybrid	$\mathcal{J}^{PC}\ (S=0)$	$\mathcal{J}^{PC}\ (S=1)$	Λ_η^ϵ
1s	S	9442		0^{-+}	1^{--}	Σ_g^+
2s	S	10009		0^{-+}	1^{--}	Σ_g^+
3s	S	10356		0^{-+}	1^{--}	Σ_g^+
4s	S	10638		0^{-+}	1^{--}	Σ_g^+
$1p_0$, (H_3)	P^+		11011	0^{++}	1^{+-}	Σ_u^-
$2p_0$	P^+		11299	0^{++}	1^{+-}	Σ_u^-
$3p_0$	P^+		11551	0^{++}	1^{+-}	Σ_u^-
$4p_0$	P^+		11779	0^{++}	1^{+-}	Σ_u^-
1p	S	9908		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
2p	S	10265		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
3p	S	10553		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$1(s/d)_1$, (H_1)	$P^\pm$		10690	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$1p_1$, (H_2)	P^0		10761	1^{++}	$(0,1,2)^{+-}$	Π_u
4p	S	10806		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$2(s/d)_1$	$P^\pm$		10885	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$2p_1$	P^0		10970	1^{++}	$(0,1,2)^{+-}$	Π_u
5p	S	11035		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$3(s/d)_1$	$P^\pm$		11084	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$4(s/d)_1$	$P^\pm$		11156	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
$3p_1$	P^0		11175	1^{++}	$(0,1,2)^{+-}$	Π_u
6p	S	11247		1^{+-}	$(0,1,2)^{++}$	Σ_g^+
$5(s/d)_1$	$P^\pm$		11284	1^{--}	$(0,1,2)^{-+}$	$\Pi_u\Sigma_u^-$
1d	S	10155		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
2d	S	10454		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
3d	S	10712		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$1(p/f)_2$, (H_4)	$P^\pm$		10819	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$1d_2$, (H_5)	P^0		10870	2^{--}	$(1,2,3)^{-+}$	Π_u
$4d$	S	10947		2^{-+}	$(1,2,3)^{--}$	Σ_g^+
$2(p/f)_2$	$P^\pm$		11005	2^{++}	$(1,2,3)^{+-}$	$\Pi_u\Sigma_u^-$
$2d_2$	P^0		11074	2^{--}	$(1,2,3)^{-+}$	Π_u
5d	S	11163		2^{-+}	$(1,2,3)^{--}$	Σ_g^+

Hyperfine splitting

Hyperfine splitting

- The lower lying hybrid potentials correspond to $\kappa^p = 1^+$
- This leads to two spin projections on the direction Q-$\bar{Q}$, $\Lambda = 0, 1$
- We need hyperfine potentials

How to estimate them?

We use an **interpolation** between the short distance expressions and long distance ones calculated in the QCD effective string theory

- They appear at $\mathcal{O}(1/m_Q)$ ($\mathcal{O}(1/m_Q^2)$) in hybrids (quarkonium)

Hyperfine splitting

Before: $J = L_{\bar{Q}Q} + J_g$ **Now:** $\mathcal{J} = J + S_{\bar{Q}Q}$

- Spin-dependent terms:

$$\left[V^{(1)}_{\kappa^p}(\mathbf{r})\right]^{n'm';nm} = -2V_{hf}(r)\left(\delta^{n'm}\delta^{nm'} - \delta^{n'm'}\delta^{nm}\right) - 2V_{hf2}(r)\left(\hat{r}^i\hat{r}^j - \frac{\delta^{ij}}{3}\right)$$
$$\times\left(\delta^{jm}\delta^{n'i}\delta^{nm'} + \delta^{ni}\delta^{jm'}\delta^{n'm} - \delta^{jm'}\delta^{n'i}\delta^{nm} - \delta^{ni}\delta^{jm}\delta^{n'm'}\right)$$

- Hyperfine hybrid:

$$H^{ji}_{S=1}(\mathbf{r},t) = \frac{1}{r}\sum_{LJ\mathcal{J}\mathcal{M}} Y^{ijLJ}_{\mathcal{J}\mathcal{M}}(\hat{\mathbf{r}})P^{LJ}_{1\mathcal{J}\mathcal{M}}(r)e^{-iEt}$$

- States:
  - 5x5 subspace: $P^{--}_{1\mathcal{J}\mathcal{M}}$, $P^{+-}_{1\mathcal{J}\mathcal{M}}$, $P^{00}_{1\mathcal{J}\mathcal{M}}$, $P^{-+}_{1\mathcal{J}\mathcal{M}}$, $P^{++}_{1\mathcal{J}\mathcal{M}}$
  - 4x4 subspace: $P^{-0}_{1\mathcal{J}\mathcal{M}}$, $P^{0-}_{1\mathcal{J}\mathcal{M}}$, $P^{+0}_{1\mathcal{J}\mathcal{M}}$, $P^{0+}_{1\mathcal{J}\mathcal{M}}$

Hyperfine potentials

- Short distance behaviour: depend on two unknown non-perturbative parameters.

$$V_{hf}(r)/m_Q = A + \mathcal{O}(r^2), \quad A = c_F k_A/m_Q, \;\; k_A \sim \Lambda^2_{\mathrm{QCD}}$$
$$V_{hf2}(r)/m_Q = Br^2 + \mathcal{O}(r^4), \quad B = c_F k_A/m_Q, \;\; k_B \sim \Lambda^4_{\mathrm{QCD}}$$

- Long distance behaviour: computed using QCD effective string th.

$$V_{hf} = \frac{1}{6}V^{sa}_{1+11}(r) - \frac{1}{3}V^{sb}_{1+10}(r) \qquad \frac{V^{sa}_{1+11}}{m_Q} = -\frac{2c_F\pi^2 g\Lambda'''}{m_Q\kappa r^3} \equiv V^{sa}_{ld}(r)$$
$$V_{hf2} = -\frac{1}{2}\left(V^{sa}_{1+11}(r) + V^{sb}_{1+10}(r)\right) \qquad \frac{V^{sb}_{1+10}}{m_Q} = \mp\frac{c_F\pi^2 g\Lambda'}{m_Q\sqrt{\pi\kappa}r^2} \equiv V^{sb}_{ld}(r)$$

  - κ is the string tension $\sim \Lambda^2_{QCD}$
  - $g\Lambda'$, $g\Lambda''' \sim \Lambda_{QCD}$
  - They can be extracted from lattice calculations

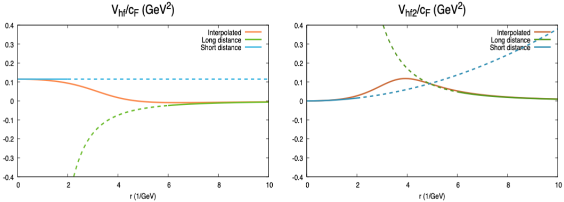
Hyperfine interpolations

- We use the following interpolation

$$\frac{V_{hf}(r)}{m_Q} = \frac{A + \left(\frac{r}{r_0}\right)^2 \left(\frac{1}{6} V_{ld}^{sa}(r_0) - \frac{r}{3r_0} V_{ld}^{sb}(r_0)\right)}{1 + \left(\frac{r}{r_0}\right)^5}$$

$$\frac{V_{hf2}(r)}{m_Q} = \frac{Br^2 - \left(\frac{r}{r_0}\right)^5 \left(\frac{r_0}{2r} V_{ld}^{sa}(r_0) + \frac{1}{2} V_{ld}^{sb}(r_0)\right)}{1 + \left(\frac{r}{r_0}\right)^7}$$

- $r_0 \simeq 3.96$ GeV$^{-1} \sim 1/\Lambda_{QCD}$



Results

Fitting the parameters

- We use lattice data of the HSC for the charmonium to fix A and B
- We focus on hyperfine splittings not on spin averages
- We have a 2 parameter fit and get $A = 0.115 \pm 0.034$ GeV, $B = 0.0038 \pm 0.0154$ GeV^3 with a $\chi^2/\mathrm{dof} \sim 0.64$
- Including long distance information from the QCD string improves the description of lattice data ($\chi^2/\mathrm{dof} \sim 1.193 \rightarrow \chi^2/\mathrm{dof} \sim 0.64$)
- Once A and B are fixed, we can predict the bottomonium hyperfine splittings

Charmonium Hybrids HFS

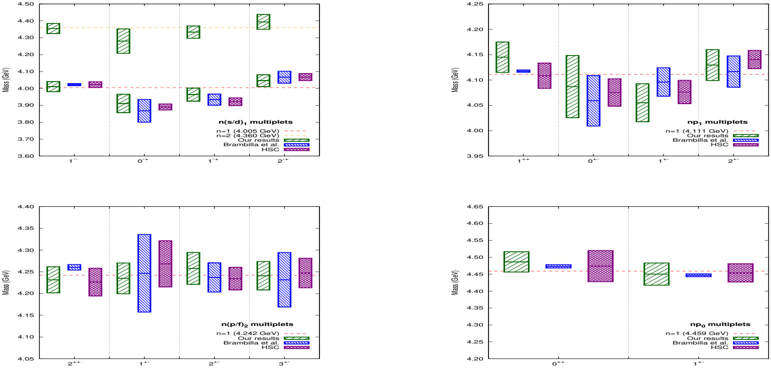


Figure 2: The spectrum of the lower-lying $n\,(s/d)_1$ (H_1), $n\,p_1$ (H_2), $n\,(p/f)_2$ (H_4) and $n\,p_0$ (H_3) charmonium hybrids

Bottomonium Hybrids HFS

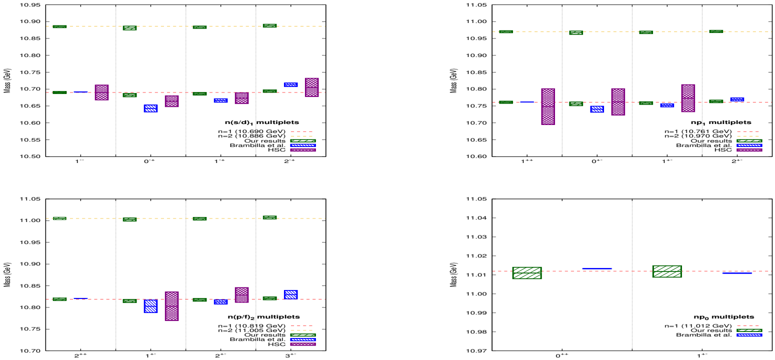


Figure 3: The spectrum of the lower-lying $n\,(s/d)_1$ (H_1), $n\,p_1$ (H_2), $n\,(p/f)_2$ (H_4) and $n\,p_0$ (H_3) bottomonium hybrids

Conclusions

- The BOEFT provides a QCD based framework to address doubly-heavy exotics systematically
- It requires non-perturbative potentials as an input
- When those potentials are not available, a Cornell-like approach of interpolating between the short distance QCD (pNRQCD) calculation and a long distance string calculation appears to be promising

Complementary information

NRQCD

$$m_Q \quad >> \quad m_Q v \quad , \quad m_Q v^2 \quad , \quad \Lambda_{QCD}$$

$$\mathcal{L}_\psi = \psi^\dagger \left\{ iD_0 + \frac{1}{2m_Q}\mathbf{D}^2 + \frac{1}{8m_Q^3}\mathbf{D}^4 + \frac{c_F}{2m_Q}\boldsymbol{\sigma}.g\mathbf{B} + \right.$$
$$\left. + \frac{c_D}{8m_Q^2}\left(\mathbf{D}.g\mathbf{E} - g\mathbf{E}.\mathbf{D}\right) + i\,\frac{c_S}{8m_Q^2}\boldsymbol{\sigma}.\left(\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times\mathbf{D}\right) \right\}\psi$$

c_F, c_D and c_S are short distance matching coefficients calculable from QCD in powers of α_s. They depend on m_Q and μ (factorization scale) but not on the lower energy scales.

[1]W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986).
G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51** (1995) 1125

Born-Oppenheimer EFT

$$L_{\text{pNRQCD}} = \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 \; S^\dagger \big(i\partial_0 - h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2)\big) S,$$

$$h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) = \frac{\mathbf{p}_1^2}{2m_Q} + \frac{\mathbf{p}_2^2}{2m_Q} + V_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2),$$

$$V_s = V_s^{(0)} + \frac{V_s^{(1)}}{m_Q} + \frac{V_s^{(2)}}{m_Q^2} + \cdots \, ,$$

All V_s's can be, and most of them have been, calculated on the lattice

- $V_s^{(0)}$ and $V_s^{(1)}$ are central $\Rightarrow$ Spin Symmetry holds
- $V_s^{(2)}$ contains spin and velocity dependent terms

Born-Oppenheimer EFT at LO and beyond

Born-Oppenheimer EFT at LO

- Matching to NRQCD in the static limit $\Rightarrow V_s^{(0)}$ is the ground state energy of two static color sources separated at a distance r
- Can be extracted from lattice calculations of the Wilson loop
- Well fitted by the Cornell potential

$$V_s^{(0)} = V_{\Sigma_g^+}(r) \approx -\frac{k_g}{r} + \kappa r + E_g^{Q\bar{Q}} \quad , \; k_g = 0.489 \quad , \kappa = 0.187 GeV^2$$

Born-Oppenheimer EFT beyond LO

- Ex. at $\mathcal{O}(1/m_Q^2)$: $V_{L_2S_1}^{(1,1)}$ spin-orbit potential (Eichten, Feinberg, 79)

$$V_{L_2S_1}^{(1,1)}(r) = -\frac{c_F}{r^2} i\mathbf{r} \cdot \lim_{T\to\infty} \int_0^T dt\, t\, \langle\langle g\mathbf{B}(t, \mathbf{r}/2) \times g\mathbf{E}(0, -\mathbf{r}/2)\rangle\rangle$$