$b \to c\tau^- \bar{\nu}_\tau$ semileptonic decays: visible distributions and tests of Lepton Flavour Universality

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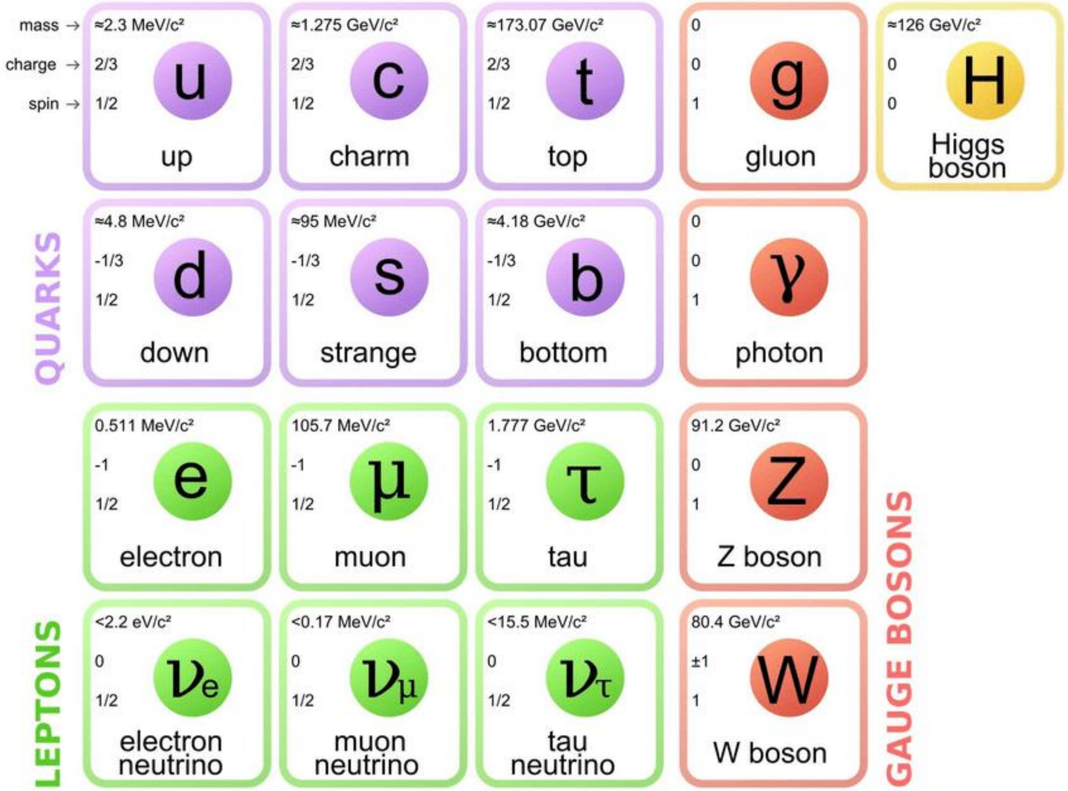
Outline

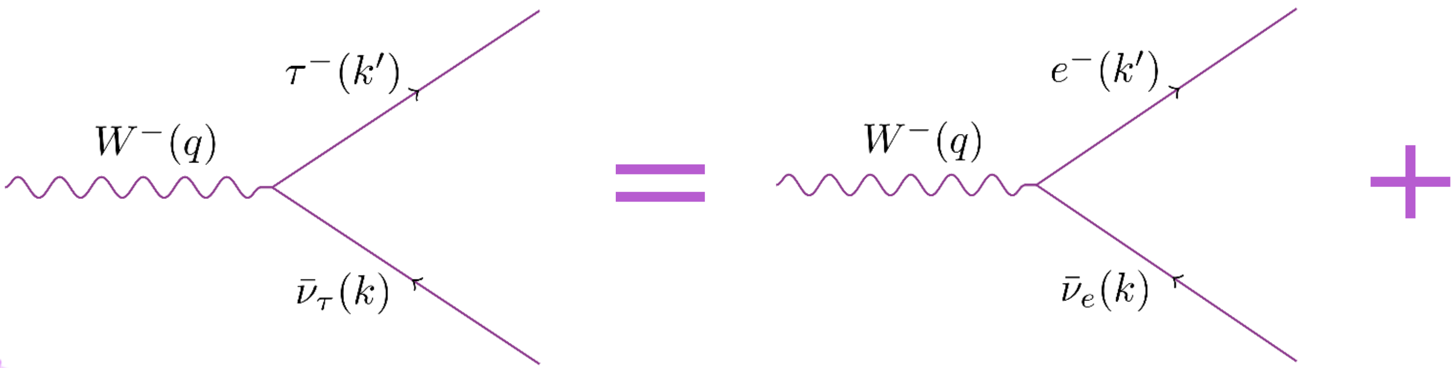
Standard model

- Describes all known fundamental particles in nature and their interactions.
- Elementary particles
  - Fermions:
    - Three families of quarks and leptons
  - Bosons
    - Gluons (strong force mediators)
    - Photon (electromagnetism mediator)
    - Z and W (weak force mediators)
    - Higgs boson



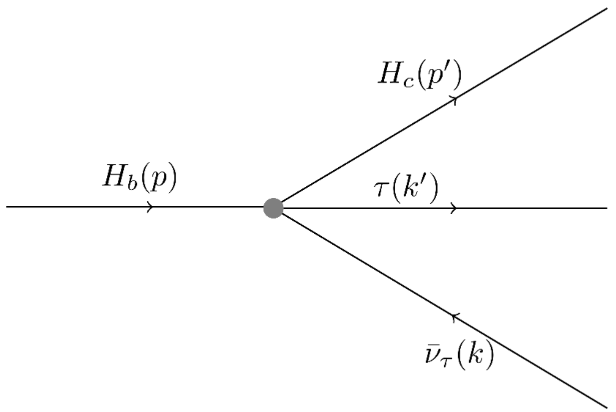
Lepton Flavour Universality

- Lepton Flavour Universality:
  - The coupling of the gauge bosons to the leptons is flavour independent.
  - The SM predictions should be the same for all 3 families of leptons except for mass effects.



Mass corrections

LFU Violation in $b \to c\tau\bar{\nu}_\tau$ decays?

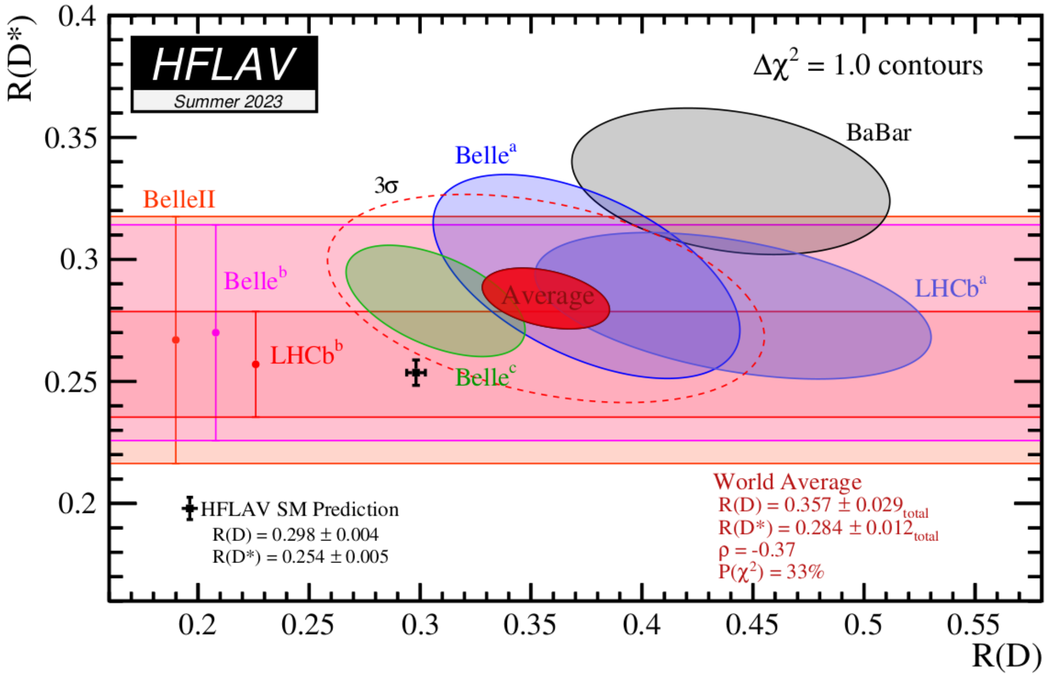


$$\mathcal{R}(H_c) = \frac{\Gamma(H_b \to H_c\tau\bar{\nu}_\tau)}{\Gamma(H_b \to H_c\ell\bar{\nu}_\ell)} = 1 + \text{mass corrections};$$

$$H_b \to H_c = \bar{B} \to D,\ \bar{B} \to D^*,\ \Lambda_b \to \Lambda_c\ldots$$
$$\ell = e, \mu$$

- Combined results for $\mathcal{R}(D), \mathcal{R}(D^*)$ show a $\sim 3\sigma$ deviation from the SM.[1]
- Deviations also in $\mathcal{R}(\Lambda_c)$ and other observables as $P_\tau(D^*)$ and $F_L^{D^*}$.

(see Alessandra's talk from yesterday)



Effective Hamiltonian

One way of adding NP effects is considering the most general effective Hamiltonian:

$$H_{\text{eff}} = \frac{4G_F V_{cb}}{\sqrt{2}} \Big[(1 + \underbrace{C^V_{LL})\mathcal{O}^V_{LL} + C^V_{RL}\mathcal{O}^V_{RL}}_{\text{(axial}-\text{)vector}} + \underbrace{C^S_{LL}\mathcal{O}^S_{LL} + C^S_{RL}\mathcal{O}^S_{RL}}_{\text{(pseudo}-\text{)scalar}} + \underbrace{C^T_{LL}\mathcal{O}^T_{LL}}_{\text{tensor}}$$

$$+ \underbrace{C^V_{LR}\mathcal{O}^V_{LR} + C^V_{RR}\mathcal{O}^V_{RR} + C^S_{LR}\mathcal{O}^S_{LR} + C^S_{RR}\mathcal{O}^S_{RR} + C^T_{RR}\mathcal{O}^T_{RR}}_{\text{right}-\text{handed neutrinos}} \Big] + h.c.,$$

- $\mathcal{R}(D), \mathcal{R}(D^*)$ expressions depend on these C^Γ_{ij}.
- Different sets of C^Γ_{ij} (NP models) are fitted to the anomalies observed in the semileptonic B-meson decays.
- We need more observables/measures to distinguish among these models.

Objectives

- Developing a general formalism for studying $b \to c$ semileptonic decays.

- Applying the formalism to several hadronic transitions both in the meson and in the baryon sectors.

- Looking for observables that distinguish among different NP models/fits that provide same values for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$. We'll show some predictions using fits from several works:
  - Murgui et al. JHEP 09 (2019) 103
  - Mandal et al. JHEP 08 (2020) 022
  - Shi et al., JHEP 12 (2019) 065

Tensor formalism

- General formalism: the hadronic tensors
- General expressions

- We want to compute decay widths and other related observables.
  ⇒ Computing squared amplitudes.

- With the tensor formalism it can be done in a general way
  - It works for any hadronic decay
  - Can be applied to other quark transitions ($s \to u\, \ell^- \overline{\nu}_\ell, c \to d\, \ell^+ \nu_\ell$, $\bar{b} \to \bar{c}\, \ell^+ \nu_\ell$)
  - Sums over the hadron spins but considers the most general polarization state of the τ

NP, Hernández, Nieves. *Phys.Rev.D* 100 (2019) 11, 113007 ; *Phys.Rev.D* 101 (2020) 11, 113004; *JHEP* 10 (2021) 122

Squared amplitude

Using the previous Hamiltonian the amplitude has the form:

$$\mathcal{M} = \left(J_H^{\alpha} J_{\alpha}^{L} + J_H J^L + J_H^{\alpha\beta} J_{\alpha\beta}^{L}\right)_{\bar{\nu}_{\ell L}} + \left(J_H^{\alpha} J_{\alpha}^{L} + J_H J^L + J_H^{\alpha\beta} J_{\alpha\beta}^{L}\right)_{\bar{\nu}_{\ell R}}$$

Hadronic matrix elements

- Depend on the considered hadrons
- Difficult to compute: Form Factor parametrization

Leptonic currents

- Have into account the lepton polarization
- Easy to compute

The expressions for $|\mathcal{M}|^2$ and the observables depend on the form factors and are different for each hadronic transition.

Hadronic tensors

What we do: Separating the squared amplitude in leptonic and hadronic tensors

$$\overline{\sum}|\mathcal{M}|^2 = \sum_{\chi=L,R}\left[\sum_{(\alpha\beta)(\rho\lambda)} L_{(\alpha\beta)(\rho\lambda)}(k,k',h_\chi)W_\chi^{(\alpha\beta)(\rho\lambda)}(p,q)\right]$$

The hadronic tensors can be decomposed in linear combinations of Lorentz structures using Lorentz, parity and time-reversal transformations.

- This decomposition is general for all decays.
- At most quadratic in q and p.
- The coefficients multiplying the Lorentz tensors are scalars $(W_i(q^2))$ called structure functions (SFs).
- There are 16 SFs for each neutrino chirality and are functions of q^2 or ω and the WCs.

The observables depend on the structure functions and are general.

An example: The Standard Model

The SM hadronic tensor is given by:

$$W^{\mu\nu}(p,q) = -g^{\mu\nu}W_1 + \frac{p^\mu p^\nu}{M^2}W_2 + i\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta \frac{W_3}{2M^2} + \frac{q^\mu q^\nu}{M^2}W_4 + \frac{p^\mu q^\nu + p^\nu q^\mu}{2M^2}W_5,$$

An therefore the square matrix element:

$$\frac{2\overline{\sum}|\mathcal{M}|^2}{M^2} = \frac{1}{2}\left[\mathcal{A}(\omega) + \mathcal{B}(\omega)\frac{(k\cdot p)}{M^2} + \mathcal{C}(\omega)\frac{(k\cdot p)^2}{M^4}\right]$$

$$\mathcal{A}(\omega) = \frac{q^2 - m_\ell^2}{M^2}\left\{2W_1 - W_2 + \frac{M_\omega}{M}W_3 + \frac{m_\ell^2}{M^2}W_4\right\},$$

$$\mathcal{B}(\omega) = -\frac{2q^2}{M^2}W_3 + \frac{4M_\omega}{M}W_2 + \frac{2m_\ell^2}{M^2}W_5,$$

$$\mathcal{C}(\omega) = -4W_2.$$

General expressions

In general:

- S^μ defines the τ polarization.
- $h = \pm 1$

$$\frac{2\overline{\sum}|\mathcal{M}|^2}{M^2} \simeq \mathcal{N}(\omega, p\cdot k) + h\left\{\frac{(p\cdot S)}{M}\mathcal{N}_{\mathcal{H}_1}(\omega, p\cdot k) + \frac{(q\cdot S)}{M}\mathcal{N}_{\mathcal{H}_2}(\omega, p\cdot k) + \frac{\epsilon^{Sk'qp}}{M^3}\mathcal{N}_{\mathcal{H}_3}(\omega, p\cdot k)\right\},$$

There are 10 combinations of the SFs.

Non-polarized case

$$\mathcal{N}(\omega, k\cdot p) = \frac{1}{2}\left[\mathcal{A}(\omega) + \mathcal{B}(\omega)\frac{(k\cdot p)}{M^2} + \mathcal{C}(\omega)\frac{(k\cdot p)^2}{M^4}\right],$$

Defined τ polarization

$$\mathcal{N}_{\mathcal{H}_1}(\omega, k\cdot p) = \mathcal{A}_{\mathcal{H}}(\omega) + \mathcal{C}_{\mathcal{H}}(\omega)\frac{(k\cdot p)}{M^2},$$

$$\mathcal{N}_{\mathcal{H}_2}(\omega, k\cdot p) = \mathcal{B}_{\mathcal{H}}(\omega) + \mathcal{D}_{\mathcal{H}}(\omega)\frac{(k\cdot p)}{M^2} + \mathcal{E}_{\mathcal{H}}(\omega)\frac{(k\cdot p)^2}{M^4},$$

$$\mathcal{N}_{\mathcal{H}_3}(\omega, k\cdot p) = \mathcal{F}_{\mathcal{H}}(\omega) + \mathcal{G}_{\mathcal{H}}(\omega)\frac{(k\cdot p)}{M^2}.$$

Examples

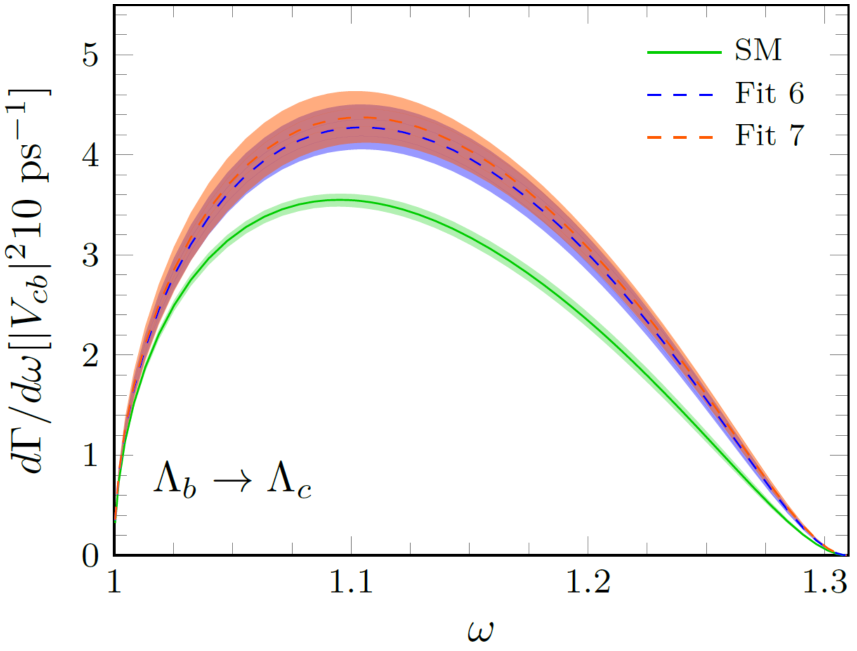
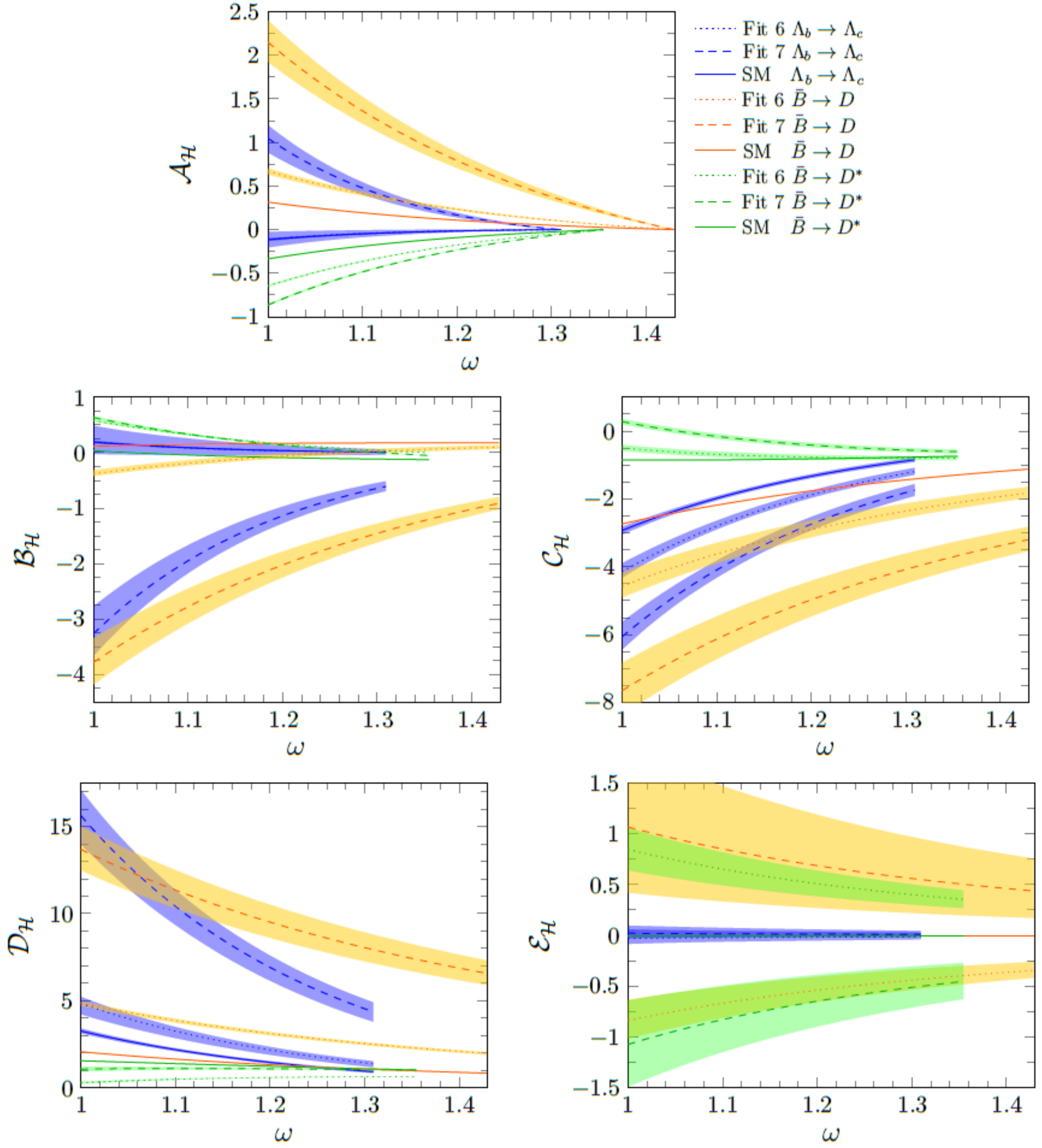


Figure: Differential $d\Gamma/d\omega$ distributions for the unpolarized $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ semileptonic decay.



Observables

- Tau kinematics
- Visible kinematics

- Things that can be measured:
  - Decay widths and angular or energy distributions
  - Angular and spin asymmetries
- They will depend on the particles'
  - Energy
  - Direction
  - Polarization

For example, one can measure

$$\frac{d^2\Gamma}{d\omega ds_{13}} = \Gamma_0 \overline{\sum} |\mathcal{M}|^2, \qquad \Gamma_0 = \frac{G_F^2 |V_{cb}|^2 M'^2}{(2\pi)^3 M},$$

$$\omega = \frac{M^2 + M'^2 - q^2}{2MM'}, \quad q^2 = (p - p')^2, \quad s_{13} = (p - k)^2$$

τ kinematics

CM frame	LAB frame

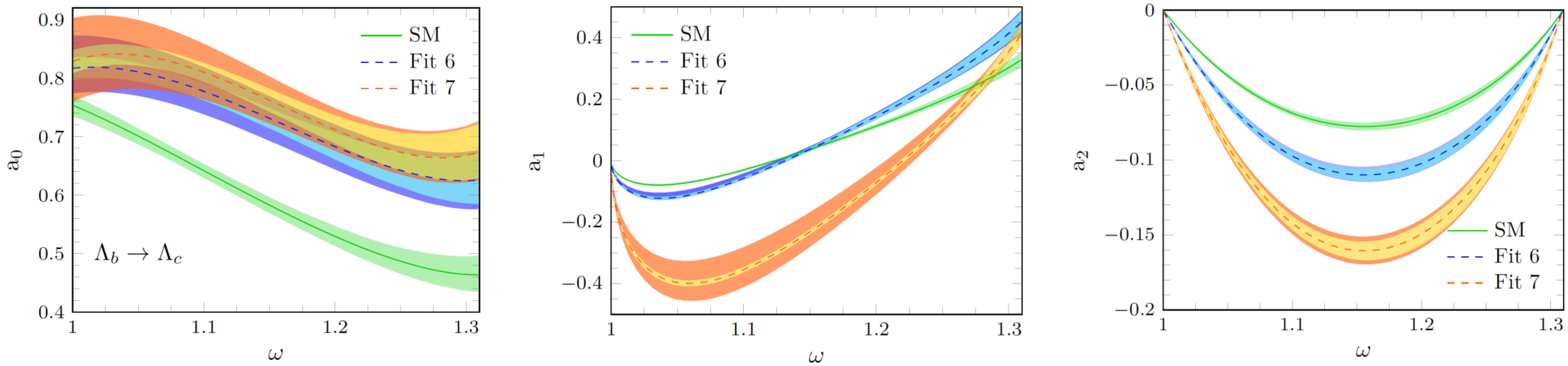
• $\vec{p} - \vec{p}\,' = 0$
• s_{13} depends on $\cos\theta_\ell$
• Angular distribution:
  • 3 non-polarized functions
  • 3 polarized functions
	• $\vec{p} = 0$
	• s_{13} depends on E_ℓ
	• Energy distribution
	  • 3 non-polarized functions
	  • 4 polarized functions

$$\frac{d\Gamma}{d\omega ds_{13}}$$

CM ↓

$$\frac{d^2\Gamma}{d\omega d\cos\theta_\ell} = \frac{\Gamma_0 M^3 M'}{2}\sqrt{\omega^2-1}\left(1-\frac{m_\ell^2}{q^2}\right)^2 \left\{a_0(\omega,h) + a_1(\omega,h)\cos\theta_\ell + a_2(\omega,h)\cos^2\theta_\ell\right\},$$

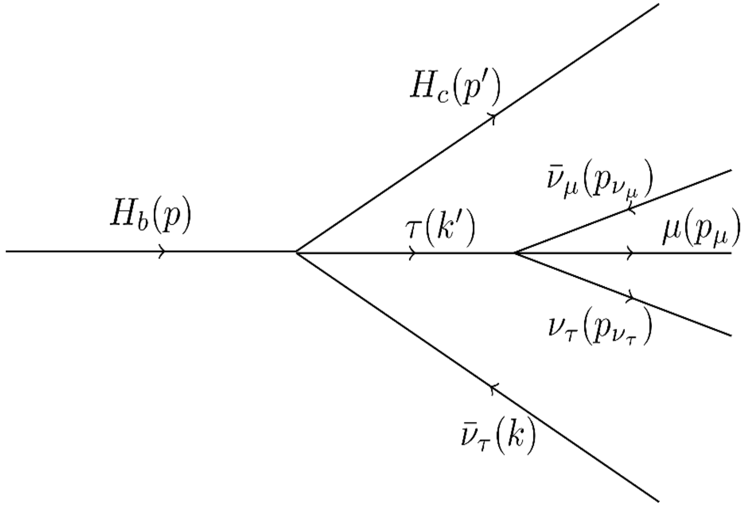
CM (of $\ell\bar{\nu}_\ell$) distributions

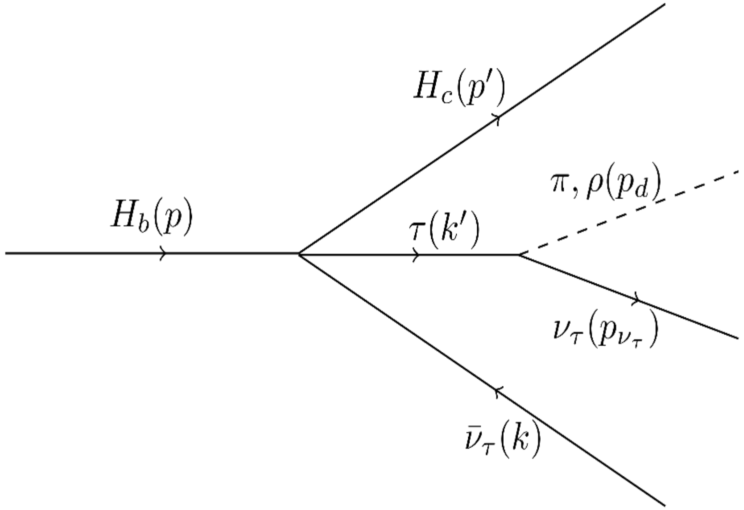


	Unpolarized	**Polarized**
CM frame	a_0, a_1, a_2	$a_0(h=-1), a_1(h=-1),$ $a_2(h=-1)$
LAB frame	c_0, c_1, c_2	$\hat{c}_0, \hat{c}_1, \hat{c}_2, \hat{c}_3$
Total	$\mathcal{A}, \mathcal{B}, \mathcal{C}$	$\mathcal{A}_\mathcal{H}, \mathcal{B}_\mathcal{H}, \mathcal{C}_\mathcal{H}, \mathcal{D}_\mathcal{H}, \mathcal{E}_\mathcal{H}$

Visible kinematics

- One would need to measure the outgoing τ 4-momentum and polarization state.
- The τ does not travel far and in any case determining the polarization is challenging.
- Its decay involves at least one more neutrino → Difficult to reconstruct.
- Solution: relying on the variables of the τ decay charged products.





Visible kinematics in the CM frame

We rely on the kinematical variables of the visible products (μ, π and ρ)

$$\frac{d^3\Gamma_d}{d\omega d\xi_d d\cos\theta_d}$$

$\cos\theta_d$: angle between the charged particle and the final hadron

ξ_d: proportional to the energy of the charged particle in the CM frame

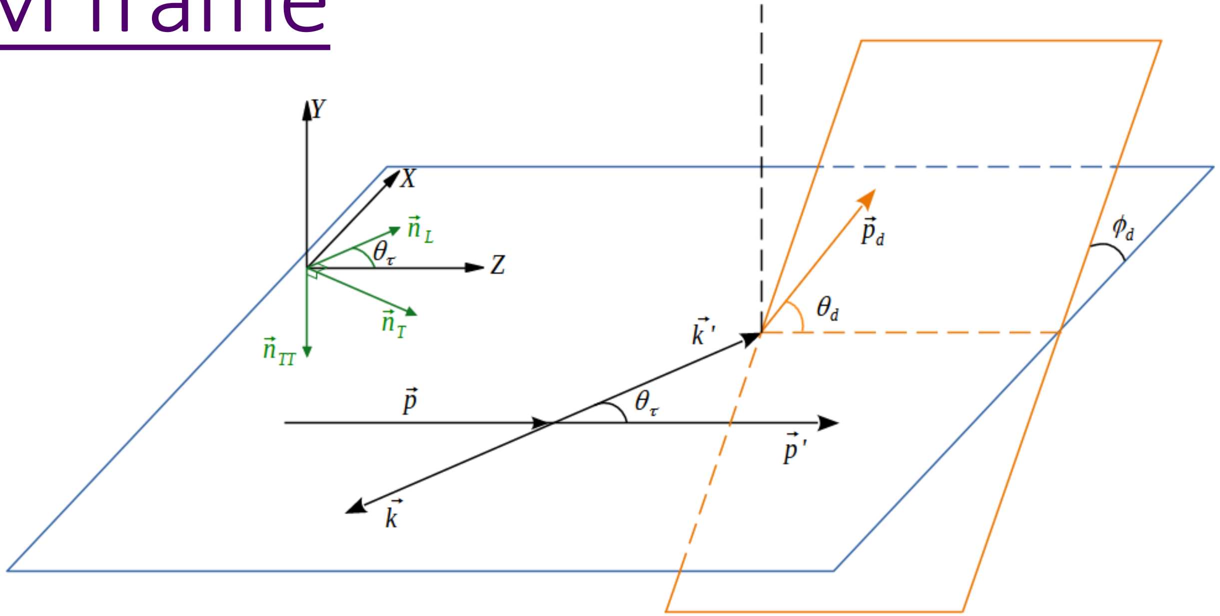


Figure: Kinematics in the $\tau^-\bar{\nu}_\tau$ CM reference system and the unit vectors $(\overrightarrow{n_L}, \overrightarrow{n_T}$ and $\overrightarrow{n_{TT}})$.

8 of the 10 ($\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{A}_\mathcal{H}, \ldots$) observables can be accessed from this differential decay width.

We loose the information on the CP violating asymmetries when we integrate on ϕ_d.

4(5)-body distribution

The $H_b \to H_c \tau(\to d\nu_\tau)\bar{\nu}_\tau$ differential decay rate:

$$\frac{d^3\Gamma_d}{d\omega d\xi_d d\cos\theta_d} = \mathcal{B}_d \frac{d\Gamma_{\rm SL}}{d\omega}\Big\{F_0^d(\omega,\xi_d) + F_1^d(\omega,\xi_d)\cos\theta_d + F_2^d(\omega,\xi_d)P_2(\cos\theta_d)\Big\},$$

where

$$F_0(\omega,\xi_d) = C_n(\omega,\xi_d) + C_{P_L}(\omega,\xi_d)\langle P_L^{\rm CM}\rangle(\omega)$$

$$F_1(\omega,\xi_d) = C_{A_{FB}}(\omega,\xi_d)A_{FB}(\omega) + C_{Z_L}(\omega,\xi_d)Z_L(\omega) + C_{P_T}(\omega,\xi_d)\langle P_T^{\rm CM}\rangle(\omega)$$

$$F_2(\omega,\xi_d) = C_{A_Q}(\omega,\xi_d)A_Q(\omega) + C_{Z_Q}(\omega,\xi_d)Z_Q(\omega) + C_{Z_\perp}(\omega,\xi_d)Z_\perp(\omega).$$

The C_i functions are kinematical factors that depend on the τ decay mode (π, ρ or $\mu\bar{\nu}_\mu$).

Observables

	Independent functions	Observables
Unpolarized τ^-	$\mathcal{A}, \mathcal{B}, \mathcal{C}$	$n = \frac{d\Gamma_{\rm SL}}{d\omega}, A_{FB}, A_Q$
Polarized τ^-	$\mathcal{A}_{\mathcal{H}}, \mathcal{B}_{\mathcal{H}}, \mathcal{C}_{\mathcal{H}}, \mathcal{D}_{\mathcal{H}}, \mathcal{E}_{\mathcal{H}}$	$\langle P_L^{CM}\rangle, \langle P_T^{CM}\rangle, Z_L, Z_Q, Z_\perp$
Complex WC's //CP violation	$\mathcal{F}_{\mathcal{H}}, \mathcal{G}_{\mathcal{H}}$	$\langle P_{TT}\rangle, Z_T$

Tau angular asymmetries

Tau spin asymmetries

Tau angular-spin asymmetries

τ angular, spin and angular-spin asymmetries

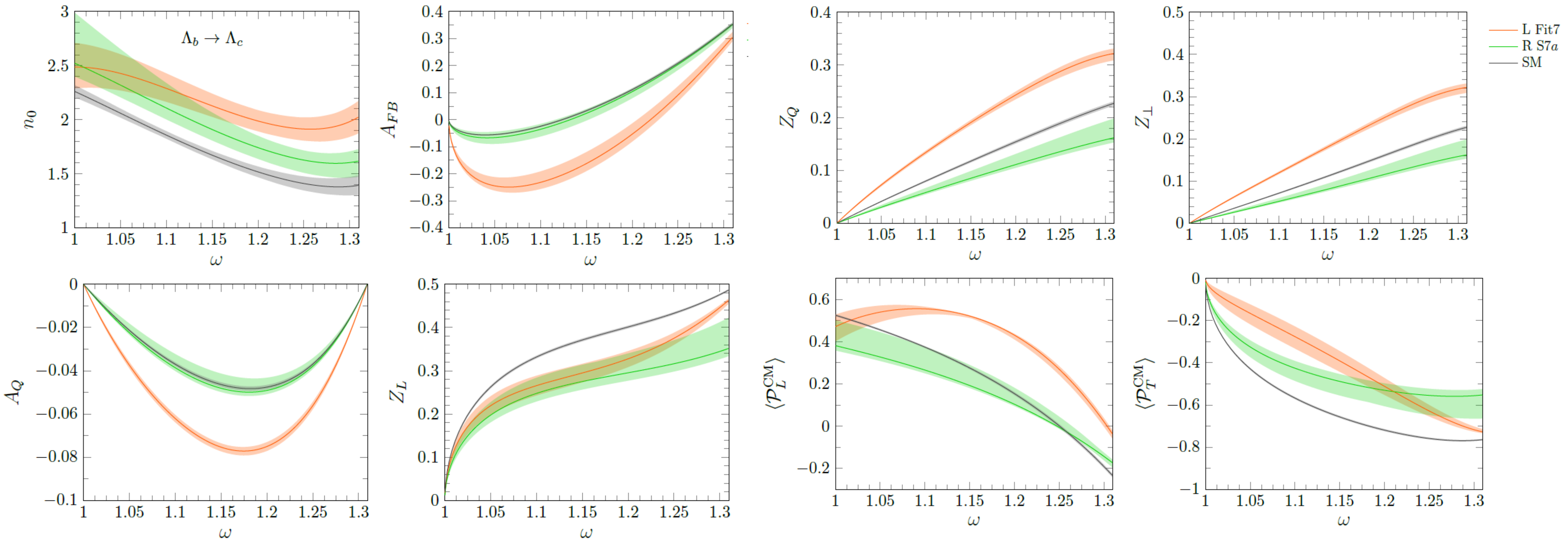


Figure: $n_0(\omega)$ and the full set of tau angular, spin and spin angular asymmetries introduced before, as a function of ω, for the $\Lambda_b \to \Lambda_c \tau^- \bar{\nu}_\tau$ decay.

Partially integrated visible distributions

We integrate some of the variables
to increase statistics

$$\frac{d^3\Gamma_d}{d\omega d\xi_d d\cos\theta_d}$$

$$\frac{d^2\Gamma_d}{d\omega d\cos\theta_d} \qquad \frac{d^2\Gamma_d}{d\omega d\xi_d}$$

$$\frac{d\Gamma_d}{d\cos\theta_d} \qquad \frac{d\Gamma_d}{d\omega} = \mathcal{B}_d \frac{d\Gamma_{SL}}{d\omega} \qquad \frac{d\Gamma_d}{dE_d}$$

The normalized $\mathrm{d}\Gamma/dE_d$ distribution

The only surviving term is:

$$\widehat{F}_0^d(E_d) = \frac{1}{\Gamma_{\mathrm{SL}}} \int_1^{\omega_{\mathrm{sup}}(E_d)} \frac{1}{\gamma} \frac{d\Gamma_{\mathrm{SL}}}{d\omega} \left\{ C_n^d(\omega, E_d) + C_{P_L}^d(\omega, E_d) \langle P_L^{\mathrm{CM}} \rangle(\omega) \right\} d\omega,$$

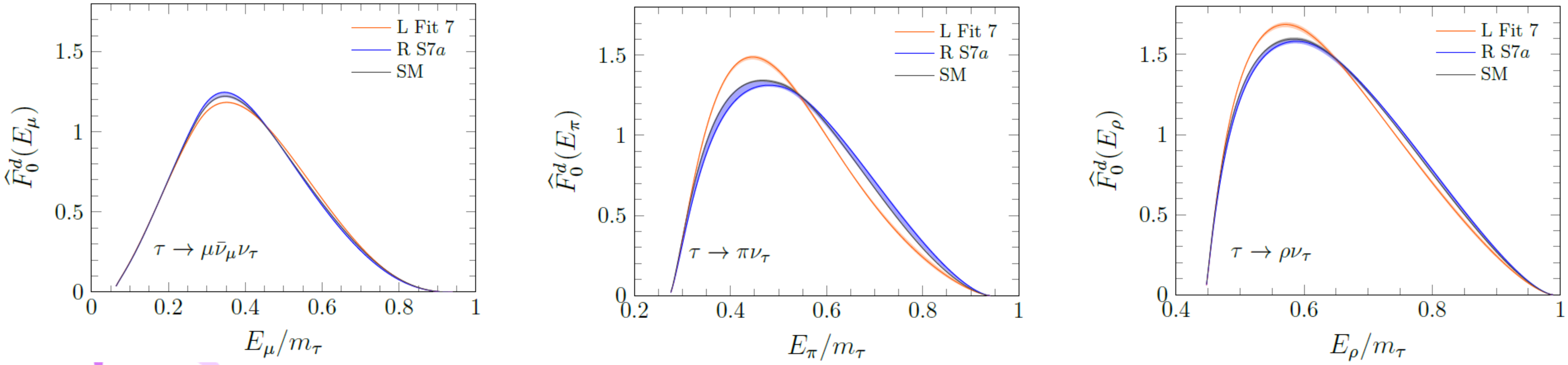


Figure: Energy $d\Gamma/dE_d$ distribution for the $\Lambda_b \to \Lambda_c\tau\bar{\nu}_\tau$ decay and each of the τ decay modes.

The $\mathrm{d}\Gamma/(d\cos\theta_d)$ distribution

We loose the information on $\langle \mathcal{P}_L^{\mathrm{CM}} \rangle$ but not in the remaining asymmetries.

$$\frac{d\Gamma_d}{d\cos\theta_d} = \mathcal{B}_d \Gamma_{\mathrm{SL}} \left[\frac{1}{2} + \widehat{F}_1^d \cos\theta_d + \widehat{F}_2^d \, P_2(\cos\theta_d) \right].$$

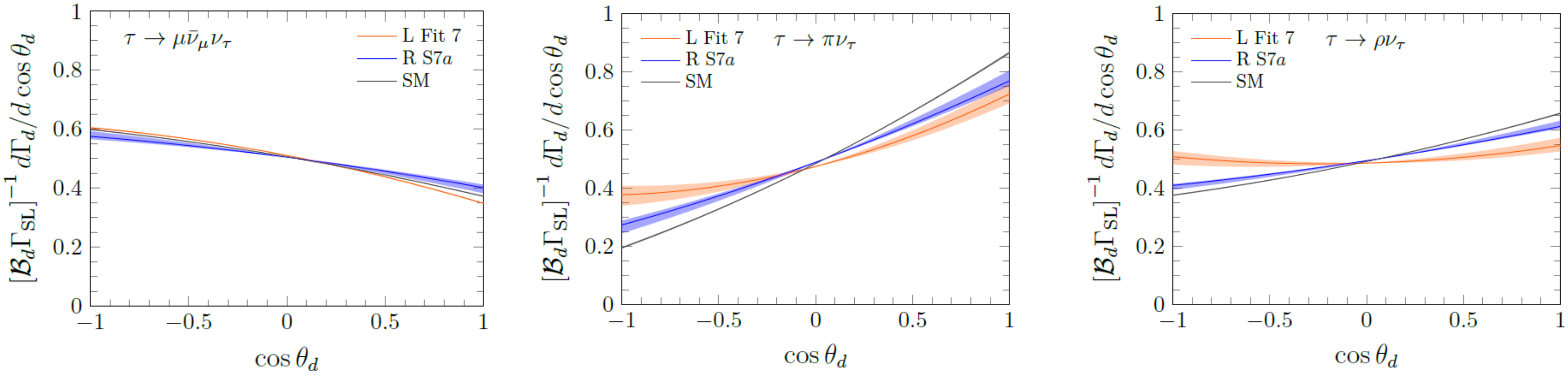


Figure: Angular $d\Gamma/d\cos\theta_d$ distribution for the $\Lambda_b \to \Lambda_c$ decay and each of the τ decay modes.

Final remarks

- We have proposed different observables that are sensitive to NP effects, which we have computed for different extensions of the SM.

- If the LFU anomalies observed in semileptonic B meson decays are confirmed, we still need to combine as many observables and decays as possible, in order to determine the preferred NP extension of the SM.

GRÀCIES!!