Probing cosmic inflation via gravitational waves

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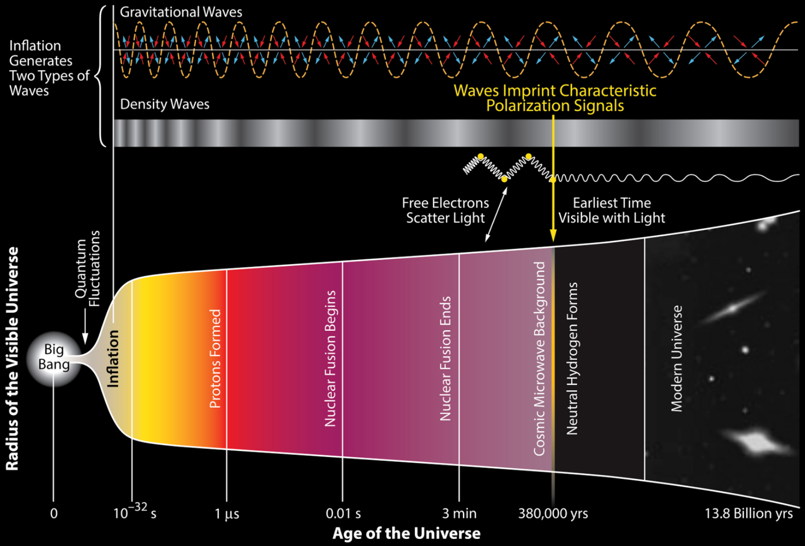
History of the Universe



Flatness problem

From cosmological observations like Type Ia supernova and cosmic microwave background (CMB) radiation:

$$|\Omega_k|_{t=t_0} = |\Omega_{\rm tot} - 1|_{t=t_0} < 1$$

Why our Universe is flat?

Flatness problem

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Why our Universe is flat?

- Big bang nucleosynthesis: $|\Omega_{\rm tot} - 1|_{t=1s} < 10^{-18}$
- Electroweak SB scale: $|\Omega_{\rm tot} - 1|_{t=10^{-12}s} < 10^{-30}$

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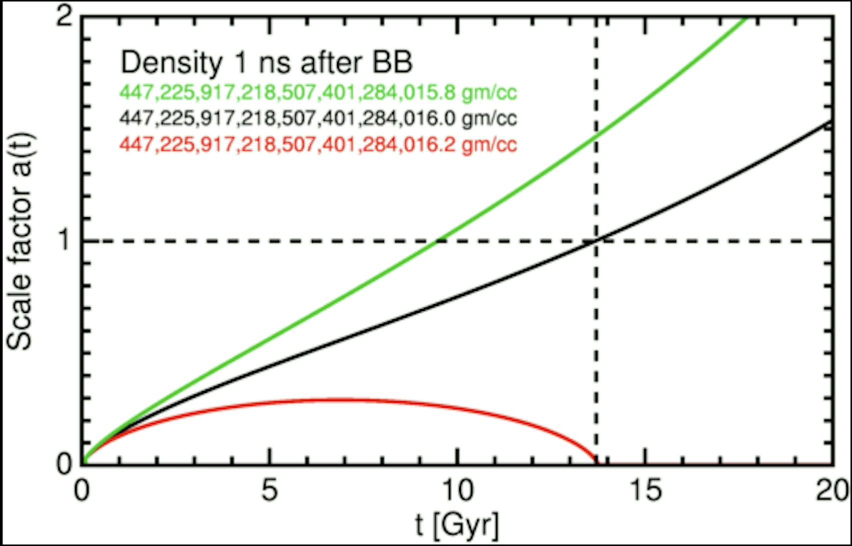
At the Big bang nucleosynthesis we need a fine tuning:

$$0.999999999999999999 \leq |\Omega_{\text{tot}}|_{t=1s} \leq 1.000000000000000001$$

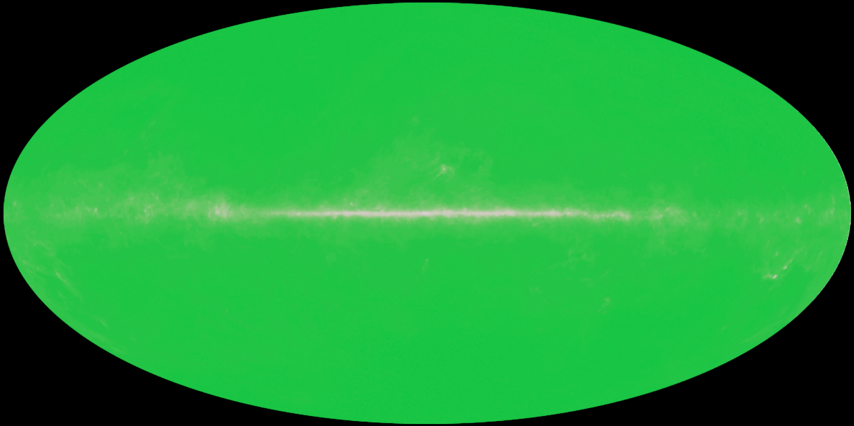
Flatness problem



Ned Wright

Horizon problem

Why the opposite sides of the universe have the same temperature of $T \simeq 2.7$ K?



NASA/WMAP

Solution to the Big Bang problems

A short rapid accelerated expansion at early times can solve the horizon and flatness problems

- **Flatness (problem):**
  Inflation sets

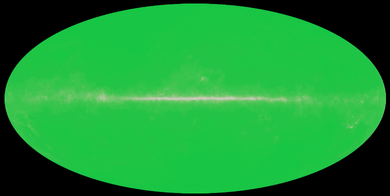
  $$|\Omega_{\rm tot}| \to 1.000000000000\ldots$$

  at early times.
- **Horizon (problem):**
  The whole observable universe are created from one causally connected region through the rapid exponential inflationary expansion.

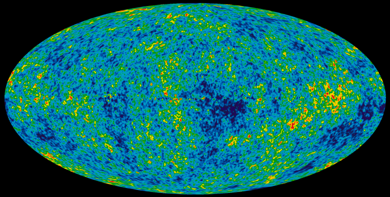
Inflation

Inflation also explains the origin of the structures in the Universe

$T \simeq 2.7$ **K**



$T \simeq 2.7(1 \pm 10^{-5})$ **K**

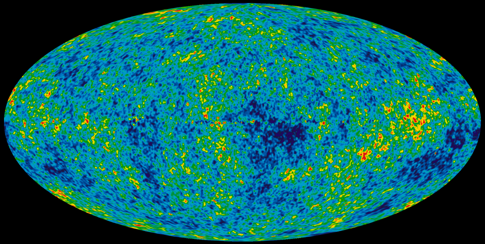


We need small inhomogeneities as a seed for the observed structures in the universe like stars, galaxies, clusters, $\cdots$

Cosmic microwave background (CMB)

The superhorizon curvature perturbation generates temperature fluctuations on CMB

$$\langle \mathcal{R}\mathcal{R} \rangle \propto \left\langle \frac{\delta T}{T} \frac{\delta T}{T} \right\rangle$$



NASA/WMAP

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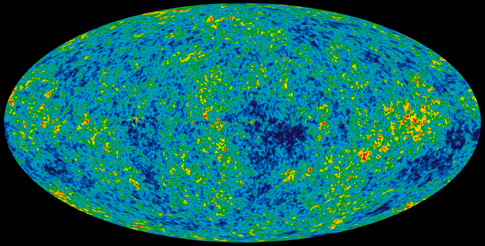


NASA/WMAP

Curvature perturbations:

$\langle \mathcal{R}\mathcal{R} \rangle \propto \mathcal{P}_{\mathcal{R}} = A_S \left(k/k_* \right)^{n_S - 1}$

$A_S = \mathcal{O}\left(10^{-9}\right)$, $n_S - 1 = -\mathcal{O}\left(10^{-2}\right)$

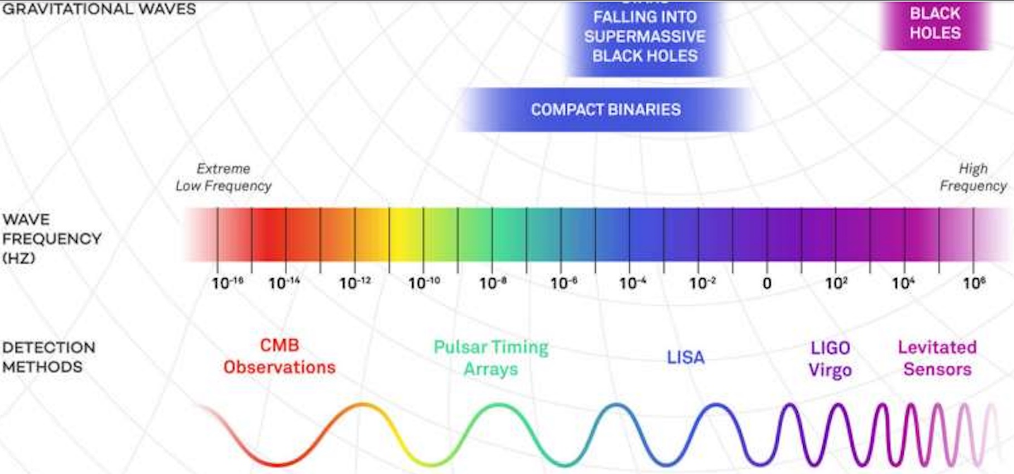
Metric tensor perturbations (GWs):

$\mathcal{P}_h = r\, A_S \left(k/k_* \right)^{-r/8}$

$r < \mathcal{O}\left(10^{-2}\right)$

Inflation generates
almost Gaussian,
almost scale-invariant, and
almost adiabatic perturbations

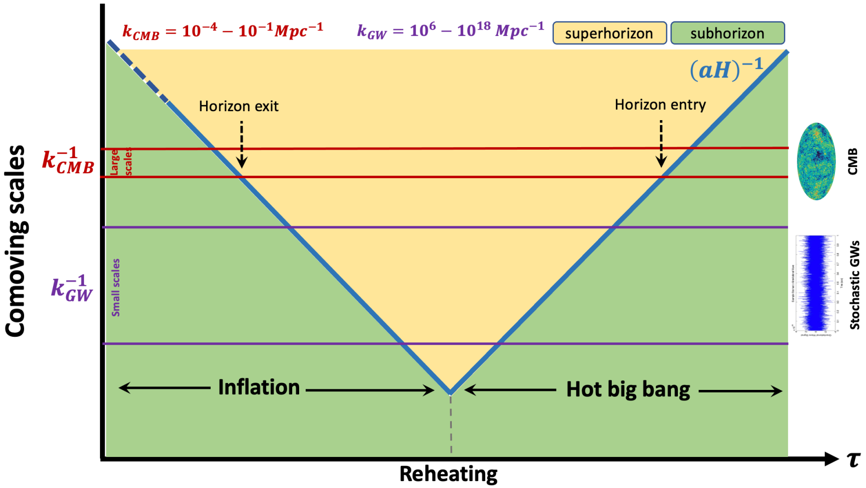
Gravitational wave detectors frequency bounds



Credit:https://phys.org

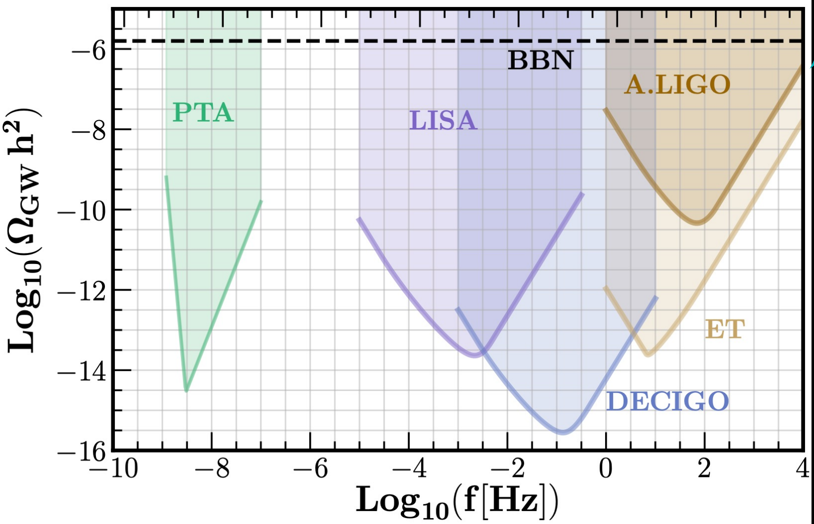
CMB scales: $10^{-4} \lesssim k_{CMB} \lesssim 10^{-1}\mathrm{Mpc}^{-1} / 10^{-19} \lesssim f \lesssim 10^{-16}\mathrm{Hz}$

GWs scales: $10^{6} \lesssim k_{GW} \lesssim 10^{18}\,\mathrm{Mpc}^{-1} / 10^{-8} \lesssim f \lesssim 10^{3}\ \mathrm{Hz}$



We can probe the late stage of inflation (that is not accessible by the CMB) via GW observations.

Gravitational wave detectors



The future GW detectors can detect $\Omega_{GW} > 10^{-16}$ in the frequency bounds $10^{-9} \leq f[Hz] \leq 10^{4}$

Credit:G. Domenech, arXiv:2109.01398

Stochastic GWs with primordial origin

Spectral density of GWs:

$$\Omega_{\rm GW} = \frac{1}{12}\Omega_{0,r}\mathcal{P}_h = \mathcal{O}(10^{-5})\mathcal{P}_h$$

- At CMB scales $k \sim k_{CMB}$, $\mathcal{P}_h \sim r\,\mathcal{P}_{\mathcal{R}} \lesssim 10^{-11}$. Since the power spectra are almost scale-invariant and red-tilted, $\mathcal{P}_h < 10^{-11}$ at $k \sim k_{GW}$ giving

$$\Omega_{\rm GW}^{\rm pri} < 10^{-16}$$

- Scalar perturbations contribute since $\mathcal{P}_h^{\rm induced} \sim \mathcal{P}_{\mathcal{R}}^2$ (nonlinear interaction) and similarly $\mathcal{P}_{\mathcal{R}} < 10^{-9}$ at $k \sim k_{GW}$ giving

$$\Omega_{\rm GW}^{\rm induced} < 10^{-23}$$

The sensitivity of future GW detectors might reach $\Omega_{\rm GW} = \mathcal{O}(10^{-16})$ (very optimistic). So, $\Omega_{\rm GW}^{\rm pri}$ and $\Omega_{\rm GW}^{\rm induced}$ are too small to be detected by GW detectors.

Contribution of primordial tensor and scalar perturbations to stochastic GWs in too small to be detected by GW detectors.

“Secondary scalar-induced” GWs

Spectral density of GWs:

$$\Omega_{\rm GW}^{\rm induced} = \mathcal{O}(10^{-5})\mathcal{P}_{\mathcal{R}}^2$$

At CMB scales $k \sim k_{CMB}$, $\mathcal{P}_{\mathcal{R}} \sim 10^{-9}$. If power spectrum be scale-dependent such that $\mathcal{P}_{\mathcal{R}} \gg \mathcal{P}_{\mathcal{R}}$ where $\mathcal{P}_{\mathcal{R}}$ is the power spectrum at GW scale $k \sim k_{GW}$, one may achieve

$$\Omega_{\rm GW}^{\rm induced} \gtrsim \mathcal{O}(10^{-16})$$

which is large enough to be detected by the GW detectors. Thus, $\mathcal{P}_{\mathcal{R}} \gtrsim 10^{-5}$ is needed which means the initial power spectrum $\mathcal{P}_{\mathcal{R}} \sim 10^{-9}$ should be enhanced at least by “four orders of magnitude” at small scales.

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Why $\Omega_{\rm GW}^{\rm induced}$ is proportional to $\mathcal{P}_{\mathcal{R}}^{2}$?

Because $\mathcal{R}$ gives a source to h which starts at the quadratic order

$$h_{ij}'' + 2\frac{a'}{a}h_{ij}' - \partial^2 h_{ij} \propto \mathcal{T}_{ij}^{mn}[\partial_m\mathcal{R}\partial_n\mathcal{R}]$$

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Why $\Omega_{\rm GW}^{\rm induced}$ is proportional to $\mathcal{P}_{\mathcal{R}}^2$?

Because $\mathcal{R}$ gives a source to h which starts at the quadratic order

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \partial^2 h_{ij} \propto \mathcal{T}_{ij}^{mn}[\partial_m\mathcal{R}\partial_n\mathcal{R}]$$

Is it NOT possible to have a “linear” source by $\mathcal{R}$.

SVT decomposition theorem

Curvature perturbation $\mathcal{R}$: Perturbation of the (effective) field which is dominant at the background.

$$h_{ij}'' + 2\frac{a'}{a}h_{ij}' - \partial^2 h_{ij} \propto \mathcal{T}_{ij}^{mn}[\partial_m \mathcal{R} \partial_n \mathcal{R}]; \quad \Omega_{\mathrm{GW},\mathcal{R}}^{\mathrm{induced}} \propto \mathcal{P}_{\mathcal{R}}^2$$

Spectator fields (s, v, t): Extra fields which do not significantly contribute to the background. These fields can provide scalar, vector, and even tensor perturbations:

$$h_{ij}'' + 2\frac{a'}{a}h_{ij}' - \partial^2 h_{ij} \propto \mathcal{T}_{ij}^{mn}[\partial_m s \partial_n s]; \quad \Omega_{\mathrm{GW},s}^{\mathrm{induced}} \propto \mathcal{P}_s^2$$

$$h_{ij}'' + 2\frac{a'}{a}h_{ij}' - \partial^2 h_{ij} \propto \mathcal{T}_{ij}^{mn}[v_m v_n]; \quad \Omega_{\mathrm{GW},v}^{\mathrm{induced}} \propto \mathcal{P}_v^2$$

$$h_{ij}'' + 2\frac{a'}{a}h_{ij}' - \partial^2 h_{ij} \propto \mathcal{T}_{ij}^{mn}[t_{mn}]; \quad \Omega_{\mathrm{GW},t}^{\mathrm{induced}} \propto \mathcal{P}_t$$

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Typical situation is $\mathcal{P}_{s,v,t} \ll \mathcal{P}_{\mathcal{R}}$. Even if $\mathcal{P}_t \ll \mathcal{P}_{\mathcal{R}}$, $\Omega_{\mathrm{GW},t}^{\mathrm{induced}}$ and $\Omega_{\mathrm{GW},\mathcal{R}}^{\mathrm{induced}}$ may be at the same order since tensor modes can appear at the "linear" level.

"Primary tensor-induced" GWs [MAG and M. Sasaki, PLB (2023)]

$$\Omega_{\mathrm{GW},\mathcal{R}}^{\mathrm{induced}} = \mathcal{O}(10^{-5})\mathcal{P}_{\mathcal{R}}^2 \qquad \Omega_{\mathrm{GW},t}^{\mathrm{induced}} = \mathcal{O}(10^{-5})\mathcal{O}(10^{-?})\mathcal{P}_t$$

What is the order of the unknown factor $\mathcal{O}(10^{-?})$?

“Primary tensor-induced” GWs [MAG and M. Sasaki, PLB (2023)]

$$\Omega^{\text{induced}}_{\text{GW},\mathcal{R}} = \mathcal{O}(10^{-5})\mathcal{P}^2_{\mathcal{R}} \qquad \Omega^{\text{induced}}_{\text{GW},t} = \mathcal{O}(10^{-5})\mathcal{O}(10^{-?})\mathcal{P}_t$$

What is the order of the unknown factor $\mathcal{O}(10^{-?})$?

It depends on the model.

Models that can provide extra tensor modes:

- A spin-two (or higher spin) spectator field.
  [L. Bordin, P. Creminelli, A. Khmelnitsky, L. Senatore, JCAP (2018)]
  [L. Iacconi, M. Fasiello, H. Assadullahi, E. Dimastrogiovanni, D. Wands, JCAP (2020)]
- Yang-Mills theories with homogeneous and isotropic vev.
  [A. Maleknejad and M.M. Sheikh-Jabbari, PLB (2013)]
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  [LISA Cosmology Working Group; E. Belgacem *et al*, JCAP (2019)]
  [J. B. Jiménez, J. M. Ezquiaga, L. Heisenberg, JCAP (2020)]
- Modified gravity theories with dynamical torsion.
  [K. Aoki, S. Mukohyama, JCAP (2020)]
  [K. Aoki, S. Bahamonde, J. Gigante Valcarcel, MAG, arXiv:2310.16007 [gr-qc]]
- . . .

"Primary tensor-induced" GWs [MAG and M. Sasaki, PLB (2023)]

$$\Omega^{\rm induced}_{{\rm GW},\mathcal{R}} = \mathcal{O}(10^{-5})\mathcal{P}^2_{\mathcal{R}} \qquad \Omega^{\rm induced}_{{\rm GW},t} = \mathcal{O}(10^{-5})\mathcal{O}(10^{-?})\mathcal{P}_t$$

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Implementing EFT method, we found $\mathcal{O}(1)$ is possible. So, we should not neglect contribution of the extra tensor perturbations even if they are spectator.

Tensor-induced origin for the PTA signal: No PBH production

MAG, M. Sasaki and T. Suyama, PLB (2023)

The recent PTA signal reported by NANOGrav, EPTA/InPTA, PPTA, and CPTA:

$$10^{-9} \lesssim \Omega_{\rm GW}^{\rm PTA} \lesssim 10^{-7} \qquad (10^{-9} \lesssim f^{\rm PTA}/{\rm Hz} \lesssim 10^{-7})$$

- **Secondary scalar-induced GWs** (see, i.e., D. G. Figueroa, *et al*, arXiv:2307.02399 and references therein):

$$\Omega_{{\rm GW},\mathcal{R}}^{\rm induced} = \mathcal{O}(10^{-5})\mathcal{P}_{\mathcal{R}}^2 \Rightarrow 10^{-2} \lesssim \mathcal{P}_{\mathcal{R}}^{\rm PTA} \lesssim 10^{-1}$$

Large values of $\mathcal{P}_{\mathcal{R}}^{\rm PTA}$ may lead to the overproduction of PBH!

- **Primary tensor-induced GWs**:

$$\Omega_{{\rm GW},t}^{\rm induced} = \mathcal{O}(10^{-5})\mathcal{O}(1)\mathcal{P}_t \Rightarrow 10^{-4} \lesssim \mathcal{P}_t^{\rm PTA} \lesssim 10^{-2}$$

t_{ij} is a spectator field and since it's energy density is subdominant, it will not lead to any PBH formation.

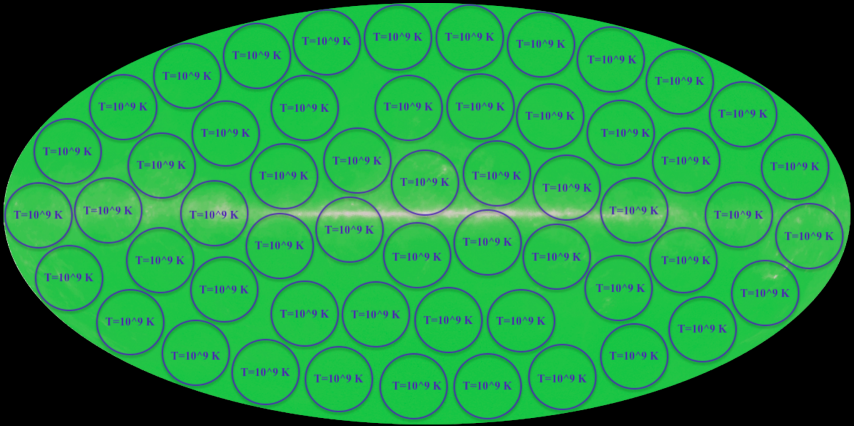
Summary

- GW observations can give us valuable information about the late stage of inflation which is not accessible by the CMB.
- The contribution of the spectator fields to stochastic GWs is expected to be very small compared with the one from curvature perturbation or the field which dominates the background.
- If a spectator field provides extra tensor perturbation (on top of the metric tensor perturbation), its contribution to the stochastic GWs can be comparable to the one from curvature perturbation.
- The scenario with extra tensor perturbation (primary tensor-induced GWs) is in principle distinguishable from the curvature perturbation case (secondary scalar-induced GWs) (e.g. the recent PTA signal): PBH will hardly form in the first case.

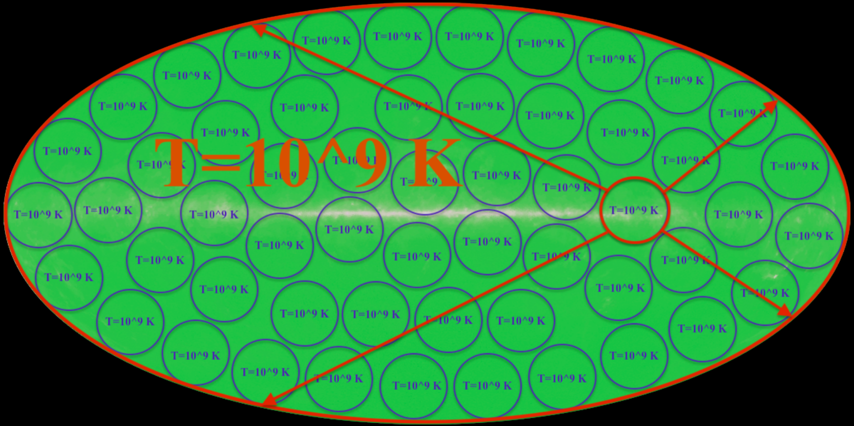
Backup slides

Horizon problem

There are many($\sim 10^8$) casually disconnected regions with the same temperatures $T \simeq 10^9$ K at Big Bang nucleosynthesis!



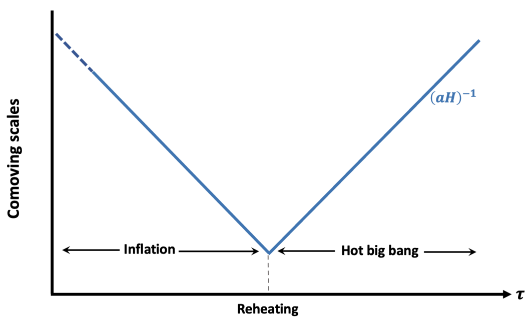
Inflation creates our observable universe from one causally disconnected region through the rapid exponential expansion



Inflation is a short accelerated expansion $\ddot{a} > 0$ at early times, say before the Big Bang nucleosynthesis

The comoving Hubble horizon $(aH)^{-1} = \dot{a}^{-1}$ is

- decreasing in accelerating universe $\downarrow$: $(aH)^{-1} = -\tau$ with $\tau \in [-\infty, 0]$ ($\tau \equiv \int dt/a(t)$ is conformal time)
- increasing in decelerating universe $\uparrow$: $(aH)^{-1} = \tau$ with $\tau \in [0, \infty]$

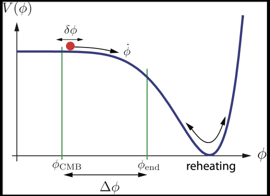


Slow-roll inflation

Scalar field with slow-roll potential

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{M_{\rm Pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1$$

$$\eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \simeq -\epsilon + M_{\rm Pl}^2 \frac{V_{,\phi\phi}}{V} \ll 1$$



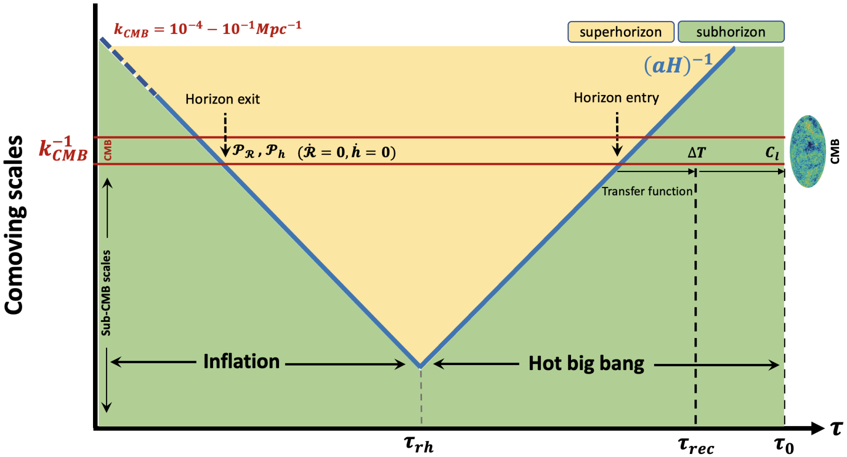
Danial Baumann (2009)

can drive inflation.

Its quantum fluctuations $\delta\phi(t, \mathbf{x}) = \phi(t, \mathbf{x}) - \langle\phi(t)\rangle$ characterized by curvature perturbations $\mathcal{R} = \frac{H}{\dot{\phi}}\delta\phi$ (in spatially flat gauge) satisfy

$$(a\mathcal{R}_k)'' + \left[(k\tau)^2 - 2 + \beta_{\mathcal{R}}\right]\frac{a\mathcal{R}_k}{\tau^2} = 0\,; \qquad \beta_{\mathcal{R}} \equiv \frac{m_{\mathcal{R}}^2}{H_{inf}^2} \simeq 6\epsilon - 3\eta \ll 1$$

Superhorizon curvature perturbations $-k\tau < \sqrt{2}$ or $k < \sqrt{2}aH_{inf}$ are produced through interaction with gravity.



Slow-roll inflation generates almost Gaussian, almost scale-invariant, and almost adiabatic perturbations

Spectator fields

Curvature perturbation $\mathcal{R}$: Perturbation of the (effective) field which is dominant at the background.

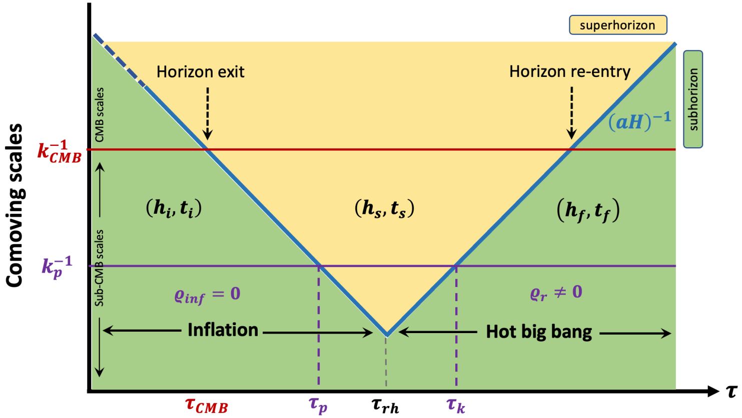
Spectator fields: Extra fields which do not significantly contribute to the background. These fields can provide scalar, vector, and even tensor perturbations:

$$S_{ij}^{\mathrm{TT}} = \mathcal{O}(\epsilon_t) + \mathcal{O}(\epsilon_s^2) + \mathcal{O}(\epsilon_s^2) + \mathcal{O}(\epsilon_v^2) + \mathcal{O}(\epsilon_t^2) + \cdots ,$$

- **Due to the hierarchy $\mathcal{P}_{s,v,t} \ll \mathcal{P}_{\mathcal{R}}$, contributions of spectator fields seem to be very small.**
- **Even if $\mathcal{P}_t \ll \mathcal{P}_{\mathcal{R}}$, $\mathcal{O}(\epsilon_t)$ and $\mathcal{O}(\epsilon_s^2)$ may be at the same order since tensor modes can appear at the LINEAR level.**

A simple subset

A simple model: i) assuming $\varrho_{\rm inf} = 0$ while $\varrho_{\rm r} \neq 0$,
ii) t_i enhances at $\tau \sim \tau_p$, i.e., by a dip in f and/or c_t s.t. $|t_{ij}(\tau_k)| \gg |h_{ij}(\tau_k)|$.



"Primary tensor-induced" stochastic GWs

MAG and M. Sasaki, arxiv:2302.14080

$$\Omega^{\rm induced}_{{\rm GW},t}(k) = \mathcal{O}(10^{-6}) K_h(k,\tau)\varrho_{\rm r}^2 \frac{c_t^2 \mathcal{P}_{t,k}(k)}{3M_{\rm Pl}^2} = \mathcal{O}(10^{-6}) K_h(k,\tau)\varrho_{\rm r}^2\, \Omega_{t,{\rm r}}(k)$$

where

$$K_h(k,\tau) \approx \begin{cases} \dfrac{1}{(1-c_t^2)^2} & c_t < 1\,, \\ \dfrac{1}{\varrho_{\rm r}^2}\sin^2(\varrho_{\rm r}\Delta\mathcal{N}/2) & c_t = 1\,, \end{cases}$$

where $\Delta\mathcal{N} = \ln(a/a_k)$ is the number of e-folds after the horizon crossing.

Constraints from BBN

$$\Omega_{t,{\rm r}}(k) = \frac{7}{16}\left(\frac{4}{11}\right)^{4/3} \Delta N_{\rm eff} < 0.034$$

The model

The effective action for extra traceless-transverse tensor modes t_{ij}, coupled minimally to gravity, in FLRW background is [L. Bordin, P. Creminelli, A. Khmelnitsky, L. Senatore, JCAP (2018)]

$$S = \frac{1}{8}\int \mathrm{d}^3x\,\mathrm{d}\tau\, a^2 \left\{ M_{\rm Pl}^2 \left[\left(h'_{ij}\right)^2 - (\partial_i h_{jk})^2 \right] + f^2 \left[\left(t'_{ij}\right)^2 - c_t^2\,(\partial_i t_{jk})^2 \right] \right\}$$
$$+ \frac{M_{\rm Pl}}{4}\varrho \int \mathrm{d}^3x\,\mathrm{d}\tau\, a a' \left[t^{ij} h'_{ij} \right]$$

where c_t, f, ϱ are functions of time. EoMs are given by

$$M_{\rm Pl}\left(h''_{\mathbf{k}} + 2\frac{a'}{a}h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} \right) = -\varrho\frac{a'}{a}\left[t'_{\mathbf{k}} + \frac{(aa')'}{aa'} t_{\mathbf{k}} \right]$$
$$t''_{\mathbf{k}} + 2\frac{(af)'}{af} t'_{\mathbf{k}} + c_t^2 k^2 t_{\mathbf{k}} = \frac{\varrho}{f^2}\frac{a'}{a}\left(M_{\rm Pl} h'_{\mathbf{k}} \right)$$

Thinking in the context of EFT, $\varrho \ll 1$ is needed to have weakly coupled (consistent) setup.

Oscillatory features

The system can be diagonalized by substituting

$$h_{\mathbf{k}} = \frac{1}{a}\left(\alpha X_{\mathbf{k}} + \beta Y_{\mathbf{k}}\right), \qquad t_{\mathbf{k}} = \frac{1}{af}\left(-\beta X_{\mathbf{k}} + \alpha Y_{\mathbf{k}}\right),$$

such that

$$X_{\mathbf{k}}'' + \omega_X^2 X_{\mathbf{k}} = 0\,, \qquad Y_{\mathbf{k}}'' + \omega_Y^2 Y_{\mathbf{k}} = 0\,,$$

For the modes deep inside the horizon $k \gg \mathcal{H}$, the positive frequency WKB solutions of are

$$X_{\mathbf{k}} \propto e^{-i\int^{\tau}\omega_X(\tilde{\tau})d\tilde{\tau}}\,, \qquad Y_{\mathbf{k}} \propto e^{-i\int^{\tau}\omega_Y(\tilde{\tau})d\tilde{\tau}}\,,$$

Even if there is no oscillation in the power spectra of $X_{\mathbf{k}}$ and $Y_{\mathbf{k}}$

$$P_X \propto |X_{\mathbf{k}}|^2\,, \qquad P_Y \propto |Y_{\mathbf{k}}|^2\,,$$

there will be oscillatory features in the power spectra of $h_{\mathbf{k}}$ and $t_{\mathbf{k}}$ for $\varrho \neq 0$

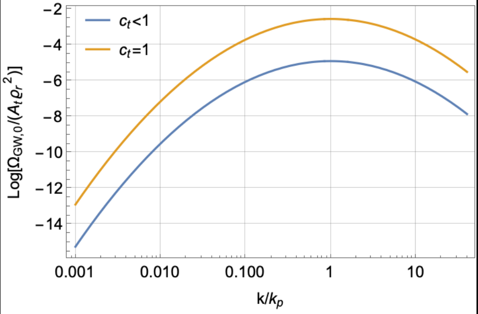
$$P_h \propto |\alpha X_{\mathbf{k}} + \beta Y_{\mathbf{k}}|^2\,, \qquad P_t \propto |-\beta X_{\mathbf{k}} + \alpha Y_{\mathbf{k}}|^2\,,$$

The observable quantity in GW detectors is the power spectrum of $h_{\mathbf{k}}$ and not diagonalized eigenstates $X_{\mathbf{k}}$ and $Y_{\mathbf{k}}$.

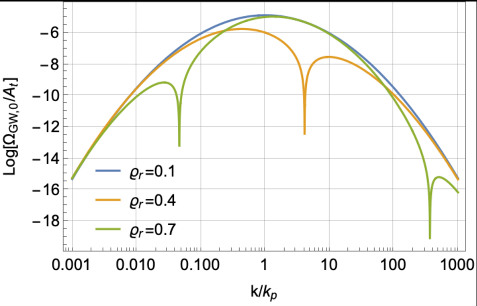
A simple model

$$\Omega^{\text{induced}}_{\text{GW},t}(k) = \begin{cases} \mathcal{O}(10^{-5})\, \varrho_{\rm r}^2\, \Omega_{t,\rm r}(k) & c_t < 1 \\ \mathcal{O}(10^{-5})\, \varrho_{\rm r}^2 \Delta\mathcal{N}^2\, \Omega_{t,\rm r}(k) & c_t = 1 \;\;\&\;\; \varrho_{\rm r} \lesssim \Delta\mathcal{N}^{-1} \\ \mathcal{O}(10^{-5})\, \sin^2\left[\varrho_{\rm r}\Delta\mathcal{N}/2\right] \Omega_{t,\rm r}(k) & c_t = 1 \;\;\&\;\; \varrho_{\rm r} \gtrsim \Delta\mathcal{N}^{-1} \end{cases}$$

where, i.e., $\Delta\mathcal{N} \sim 30$ corresponds to LISA scale with $k = 10^{-3}$Hz. Assuming lognormal power spectrum $\Omega_{t,\rm r}(k) = \frac{A_t}{\sqrt{2\pi}\Delta_t} \exp\left[-\frac{\ln^2(k/k_p)}{2\Delta_t^2}\right]$,



When $\varrho_{\rm r} \lesssim \Delta\mathcal{N}^{-1} \ll 1$ ($\varrho_{\rm r} \sim 10^{-2}$) & $c_t = 1$, there is an enhancement due to the resonance.



For $\varrho_{\rm r} \gtrsim \Delta\mathcal{N}^{-1} \sim 1$ & $c_t = 1$ multiple peaks show up. $\Delta\mathcal{N} = 30$ and $\Delta_t = 1$ are considered.

A simple model

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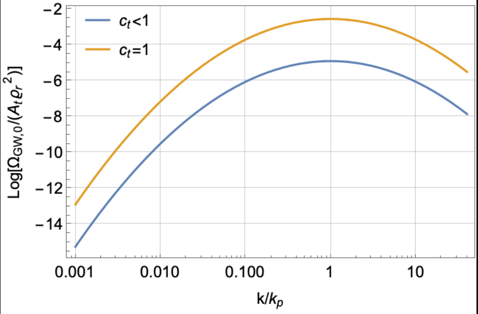

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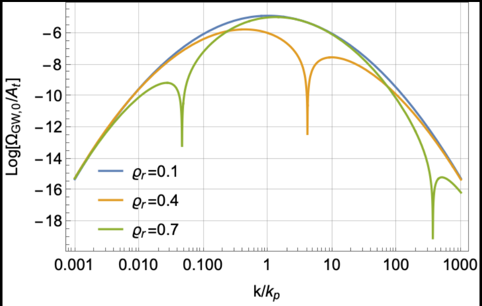

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$$\mathcal{P}_{h_s} = \frac{2H_{\rm inf}^2}{\pi^2 M_{\rm Pl}^2}\left[1 - \varrho_{\rm inf}\,(N - N_k)\right]^2 ,$$

$$\mathcal{P}_{t_s} = \frac{2H_{\rm inf}^2}{\pi^2 M_{\rm Pl}^2}\frac{1}{c_t}\left[1 - \frac{2}{3}\varrho_{\rm inf}^2\,(N - N_k)\right] ,$$

$$\mathcal{P}_{\mathcal{R}} = \frac{H_{\rm inf}^2}{8\pi^2 M_{\rm Pl}^2}\frac{1}{\epsilon}\left[1 + \frac{\varrho_{\rm inf}^2}{6\epsilon c_s}\frac{2 + c_s}{(1 + c_s)^2}\right] ,$$

$$\varrho_{\rm inf} < \mathrm{Min}\left[(N - N_k)^{-1}, 2\sqrt{2\epsilon c_s}\right] ,$$

$\varrho_{\rm inf} \lesssim 10^{-2}$ for $N - N_k \approx \mathcal{O}(50)$, $\epsilon = \mathcal{O}(10^{-3} - 10^{-2})$, $c_s = 1$