EMISSION OF GRAVITATIONAL WAVES FROM STRONGLY-COUPLED PLASMAS

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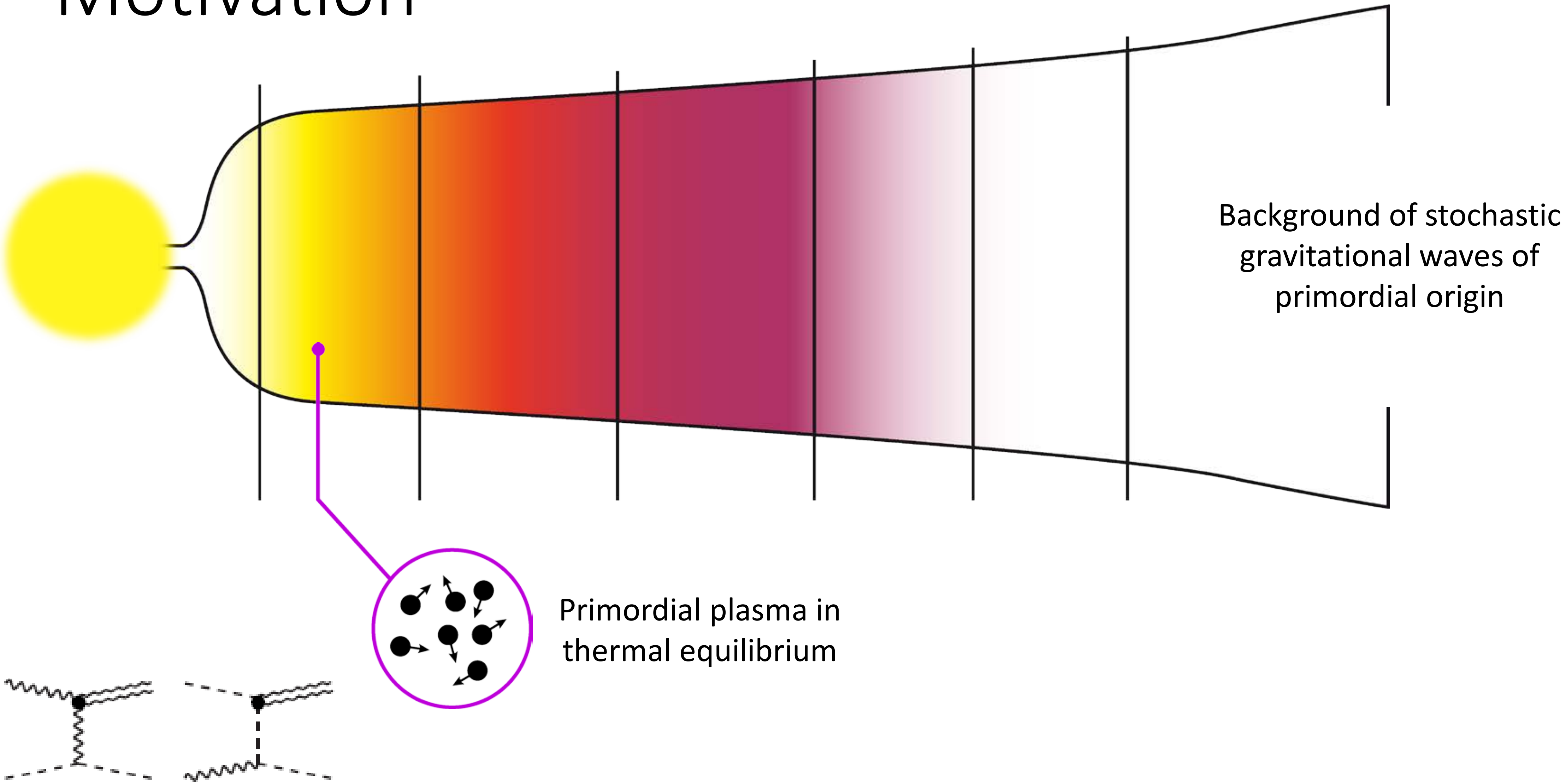
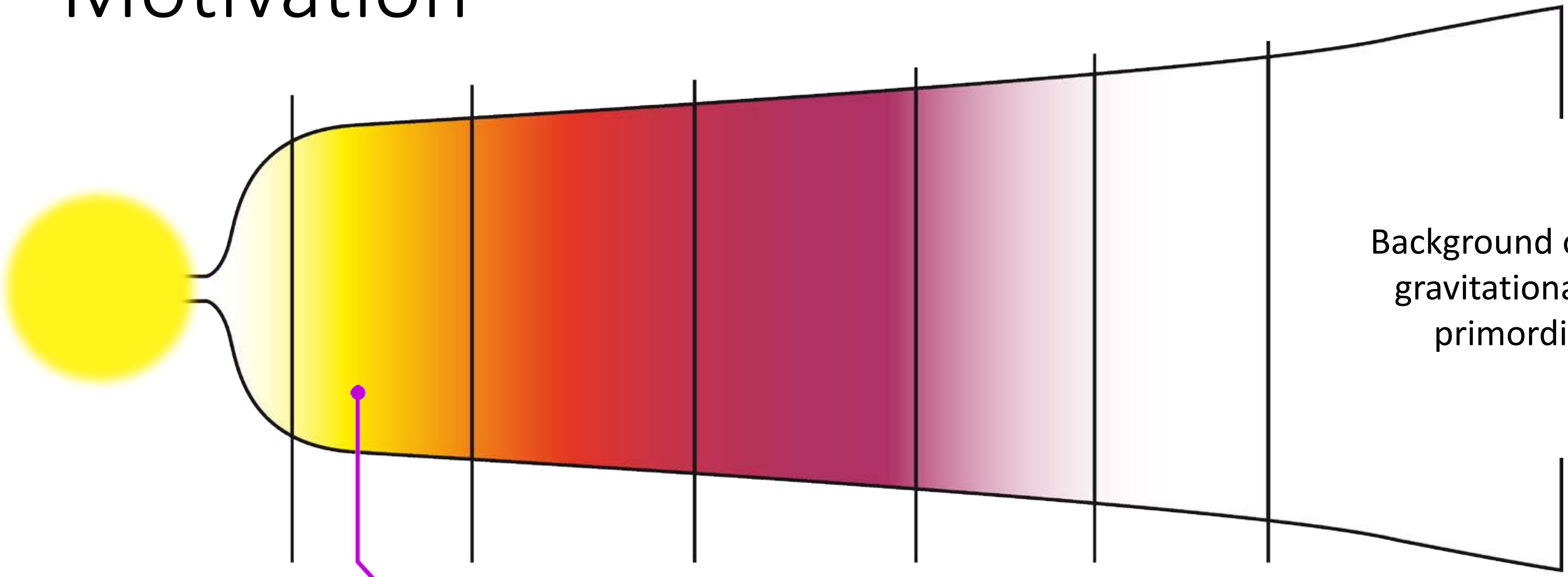
Outline

- We explore the strong coupling effects on the emission of gravitational waves generated in different mechanisms.
- We start by analyzing the emission from a primordial plasma in thermal equilibrium.
- I review the emission of gravitational waves resulting from first-order cosmological phase transitions via their two possible realizations: bubble nucleation and spinodal instability.

Motivation


Background of stochastic gravitational waves of primordial origin
Primordial plasma in thermal equilibrium

Motivation



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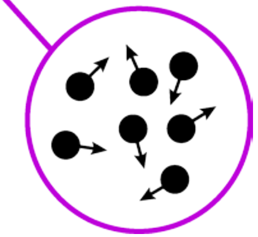
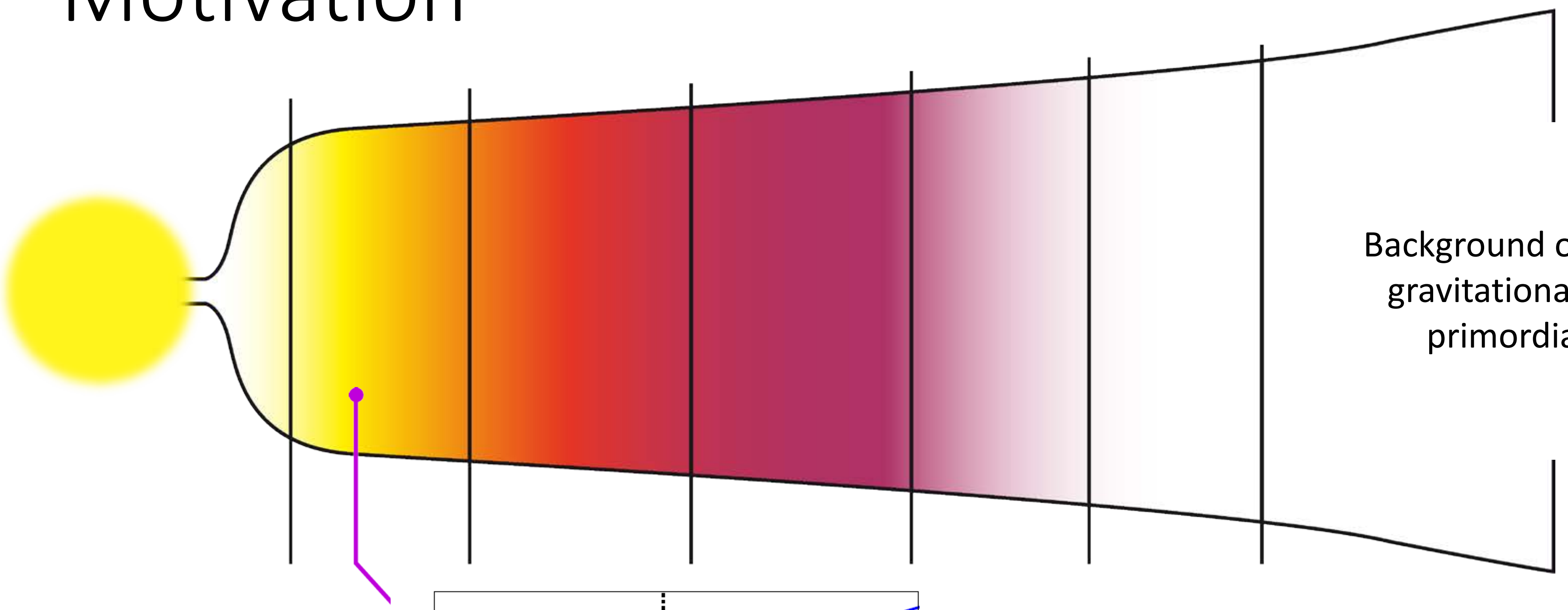


Primordial plasma in thermal equilibrium

Why do we care about this background?

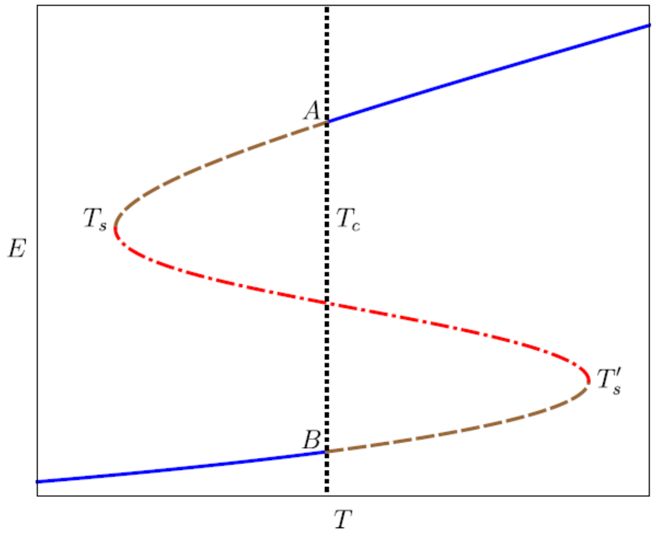
- Max. temperature of the hot Big Bang
- Probe high-energy physics
- Dark Matter?

Motivation



Background of stochastic gravitational waves of primordial origin

First-order cosmological phase transition



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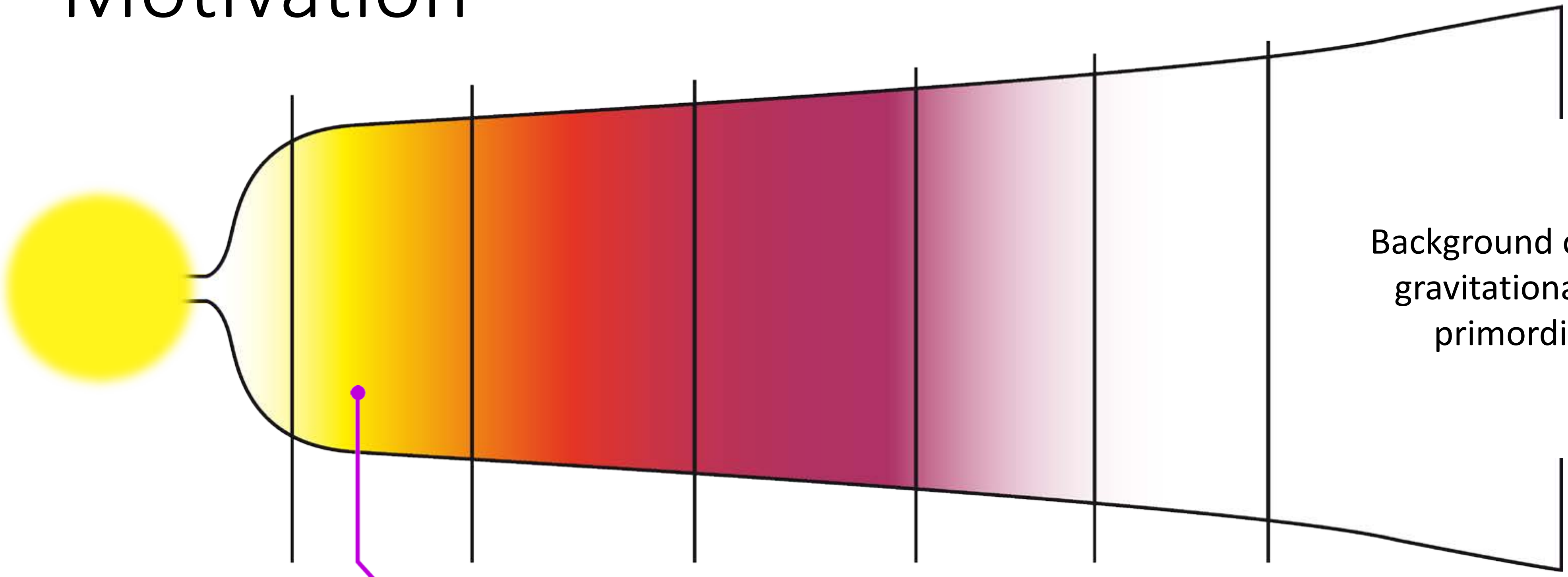

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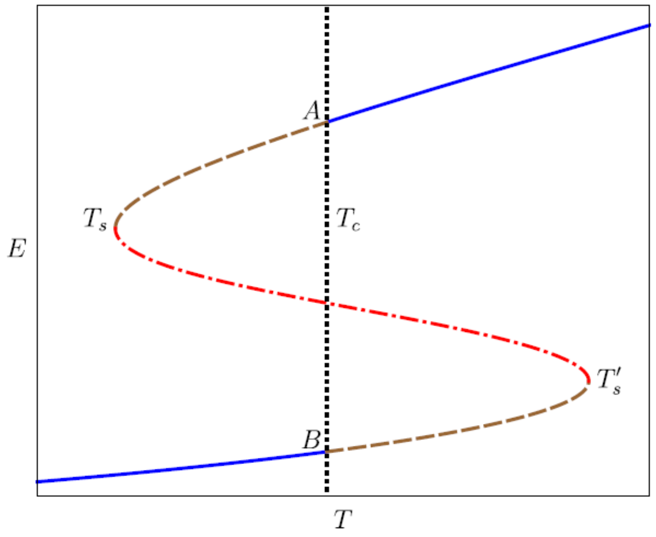
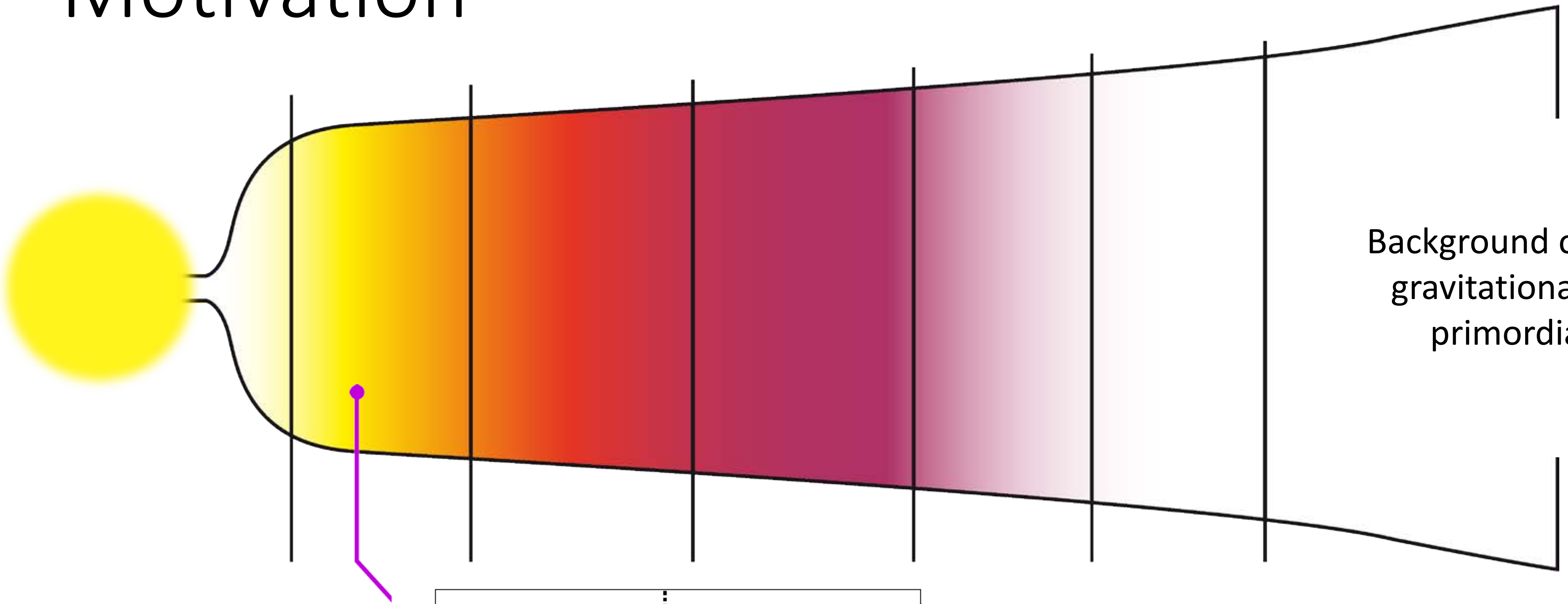


This background could lead to new physics:

- Extentions of the SM
- Hidden sectors gravitationally-coupled to the SM

Motivation



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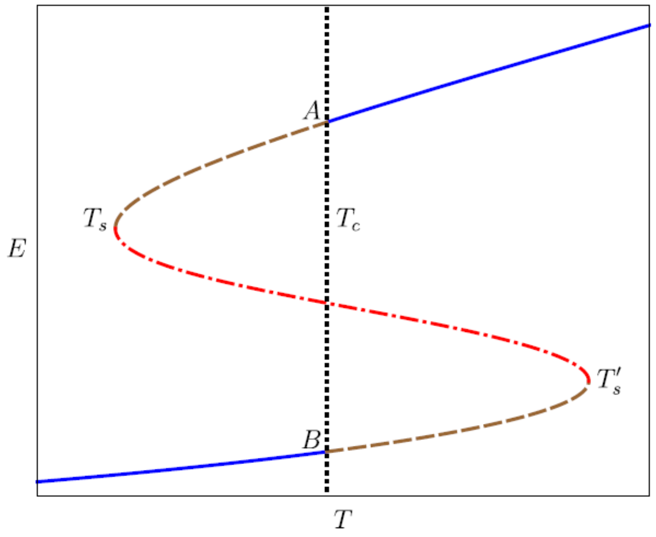


The phase transition could be undergone via:

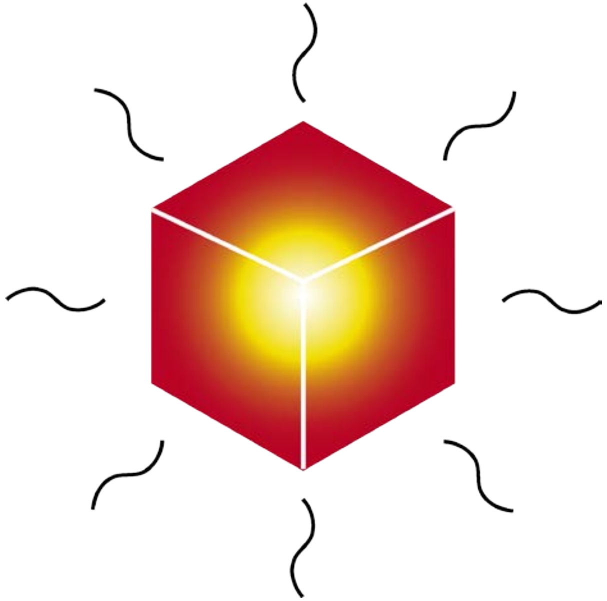
- Bubble nucleation
- Spinodal instability

EMISSION FROM A PLASMA IN THERMAL EQUILIBRIUM

Gravitational waves from a thermal source

Energy production rate of thermal gravitational waves:

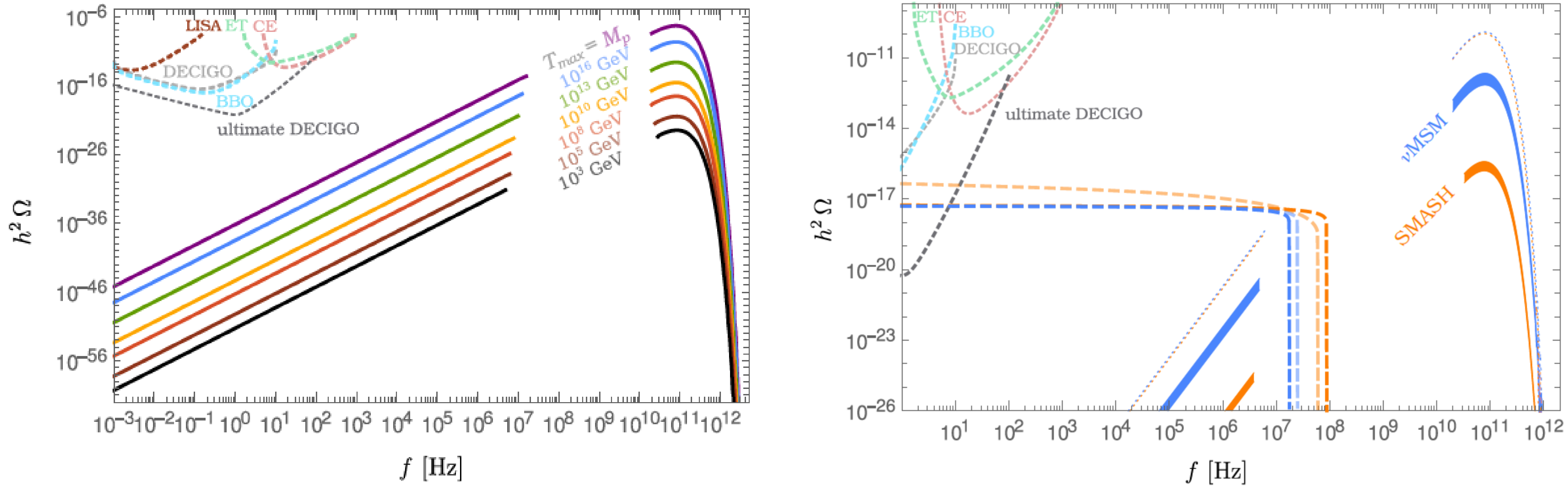
$$\frac{d\rho_{GW}}{dt\,d^3k} = \frac{4\pi G}{(2\pi)^3}\Lambda_{ijmn}\int \boldsymbol{d^4x\, e^{i(\omega t - \mathbf{kx})}\langle T_{ij}(0,0)T_{mn}(t,\mathbf{x})\rangle}$$



under light-like condition $\omega = k$ and Λ_{ijmn} the projector onto spin-2 modes.

The energy density carried by thermal gravitational waves depends on the equilibrium correlator of the energy-momentum tensor in field theory.

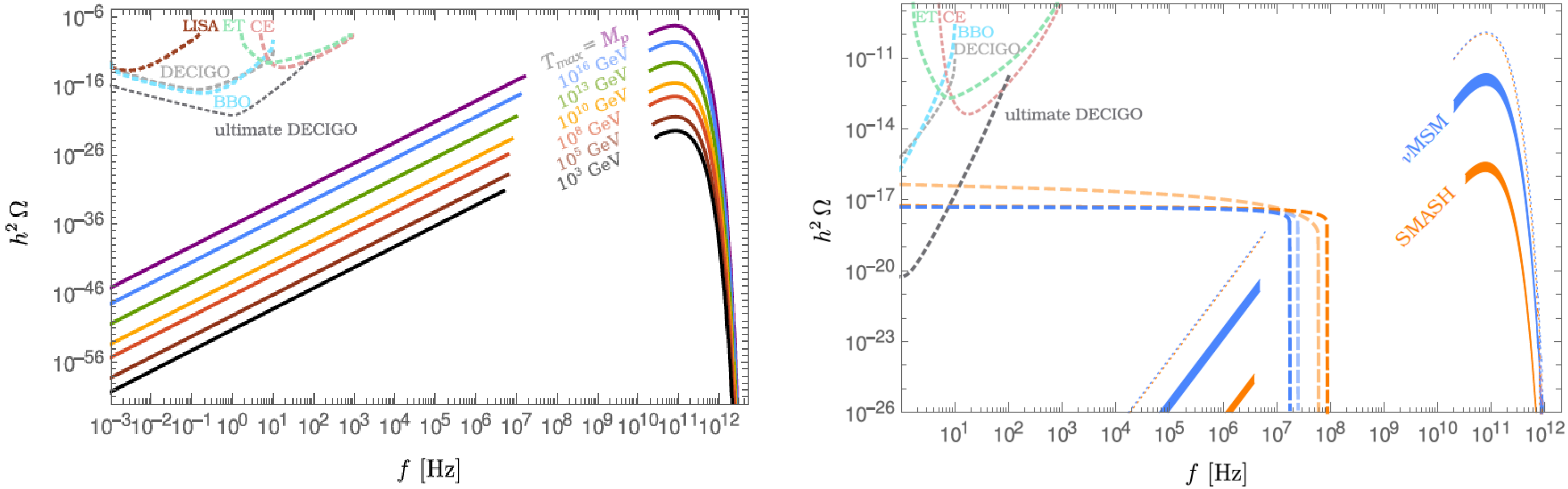
Energy production rate in SM theories



Energy density of gravitational waves from the primordial thermal plasma in the SM and two of its extensions: Neutrino Minimal SM (νMSM) and SM-Axion-Seesaw-Higgs (SMASH)

Figures from Ringwald, Schütte-Engel & Tamarit (2021)

Energy production rate in SM theories



What if physics beyond the Standard Model is strongly-coupled?

Figures from Ringwald, Schütte-Engel & Tamarit (2021)

Gravitational waves from a thermal source

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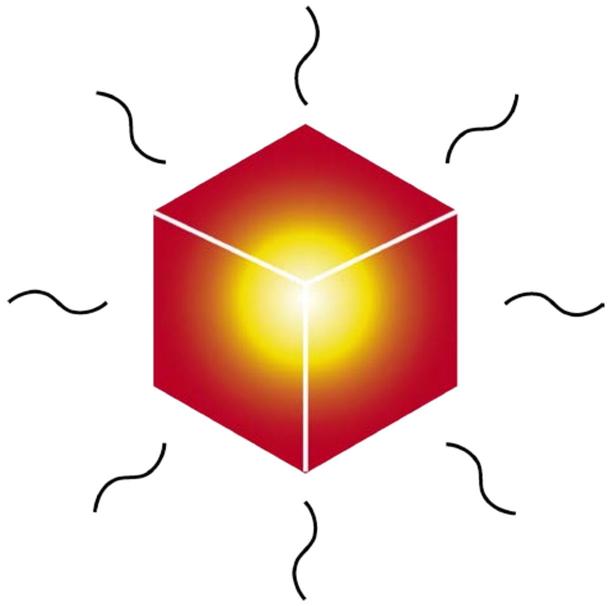

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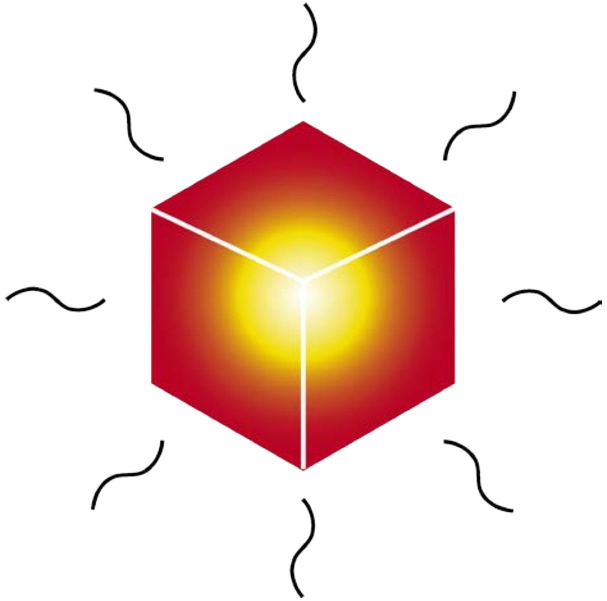


under light-like condition $\omega = k$ and Λ_{ijmn} the projector onto spin-2 modes.

Computation of the correlator:

- Weak coupling limit $\lambda \to 0$: Perturbation theory
- Strong coupling limit $\lambda \to \infty$: ~~Perturbation theory~~ **Holography**

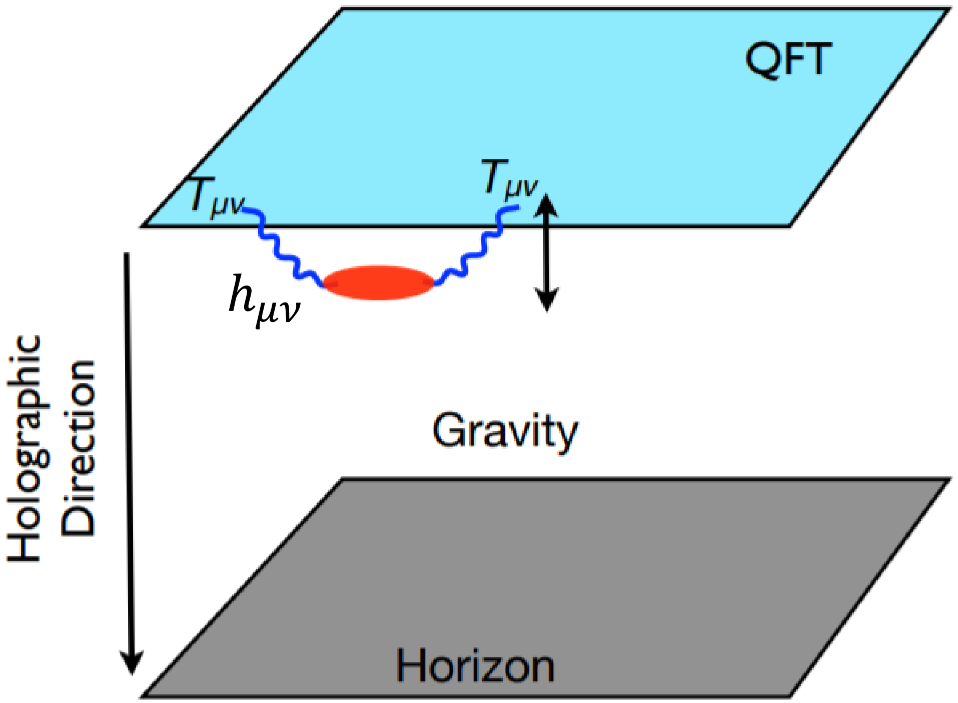
Gauge/gravity duality: AdS/CFT correspondence

Strongly-coupled, non-perturbative quantum field theories with conformal invariance

Holography ↕

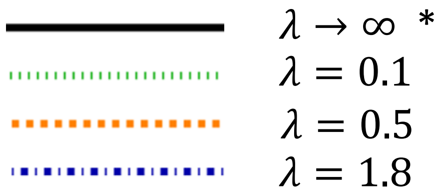
Weakly-coupled gravity theories with black hole horizons in asymptotically Anti de Sitter spacetime

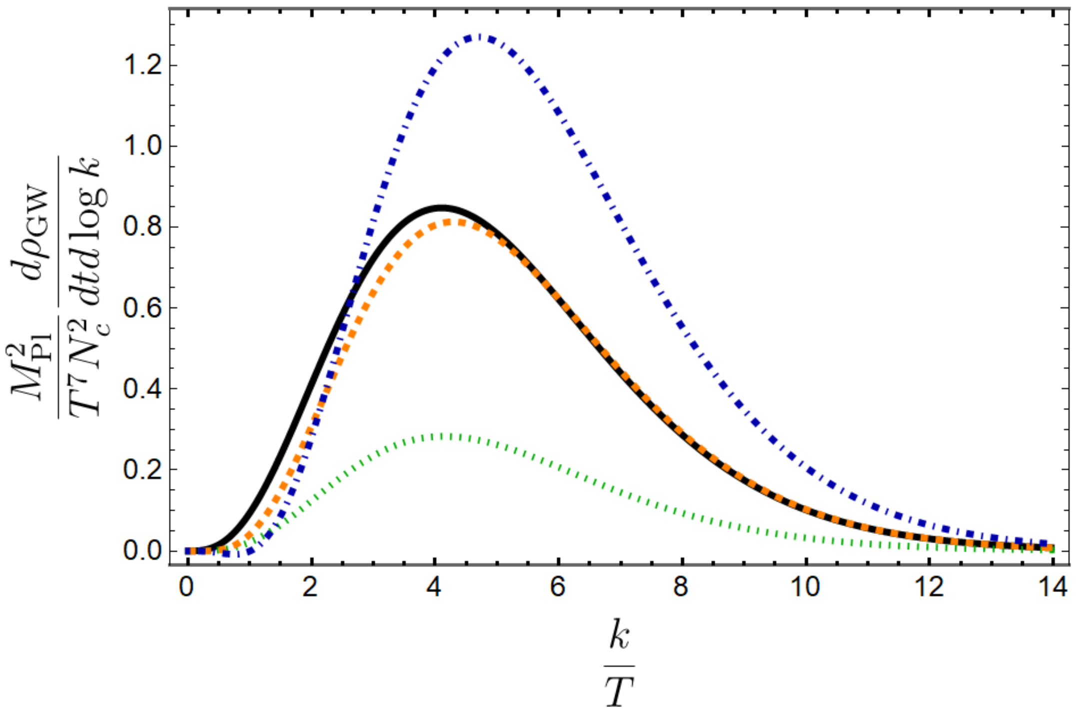
Gauge/gravity duality allows us to compute correlation functions in the field theory in terms of the gravity prescription.



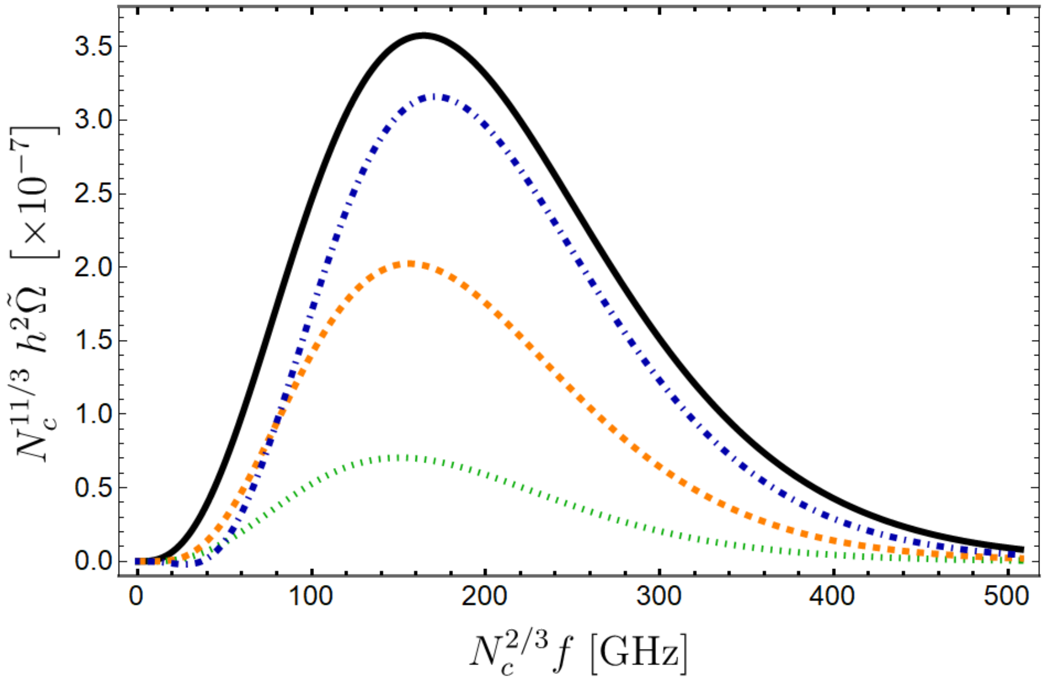
J.M. Maldacena, "Adv. Theor. Math. Phys." 2 (1998), pp. 231-252.

Energy production rate in $\mathcal{N} = 4$ Super Yang-Mills





Energy density from a static source



Energy density after convolution with the expansion of the Universe, with $\tilde{\Omega} \equiv \frac{M_{Pl}}{T_{max}} \frac{\Omega}{N_c^2}$

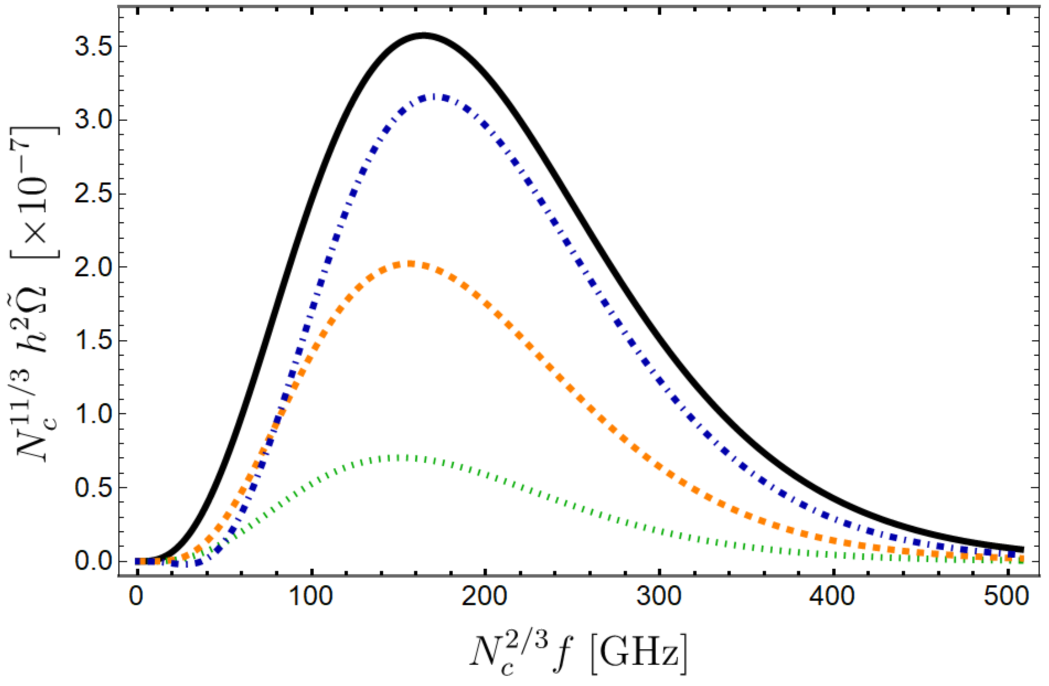
* With $\lambda = g^2 N$ the coupling of the theory

Castells-Tiestos & Casalderrey-Solana (2022)

Energy production rate in $\mathcal{N} = 4$ Super Yang-Mills

- The strong coupling computation exhibits a similar behavior as the one in the weakly-coupled regime.

- The spectrum from a thermal plasma gives no intuition about the coupling characterizing the theory.



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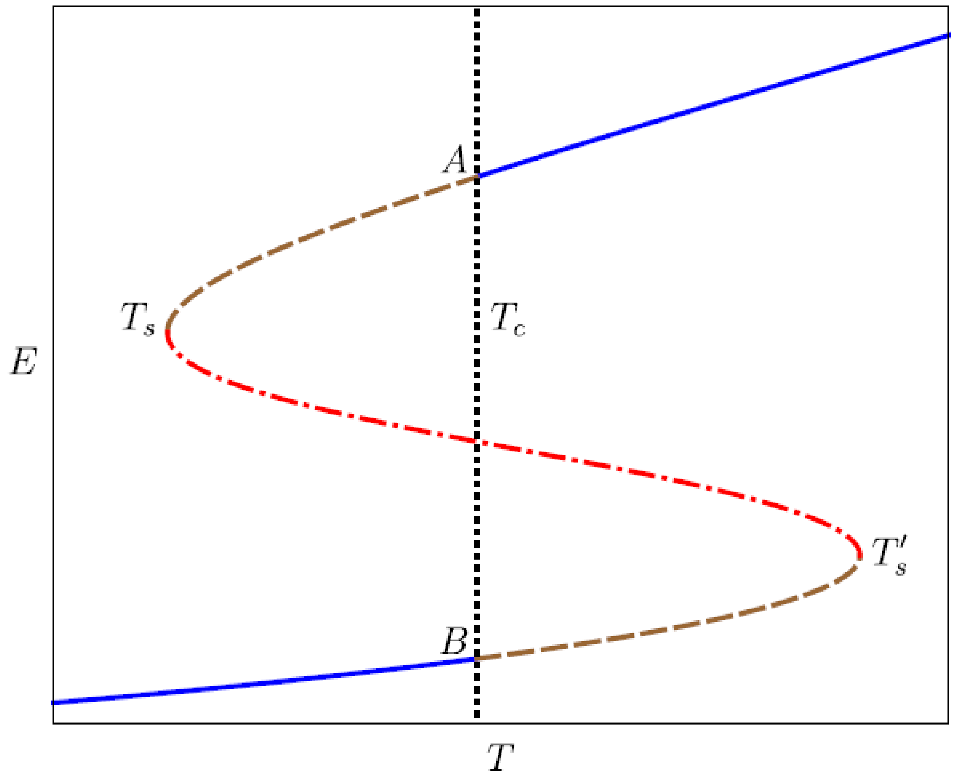
Castells-Tiestos & Casalderrey-Solana (2022)

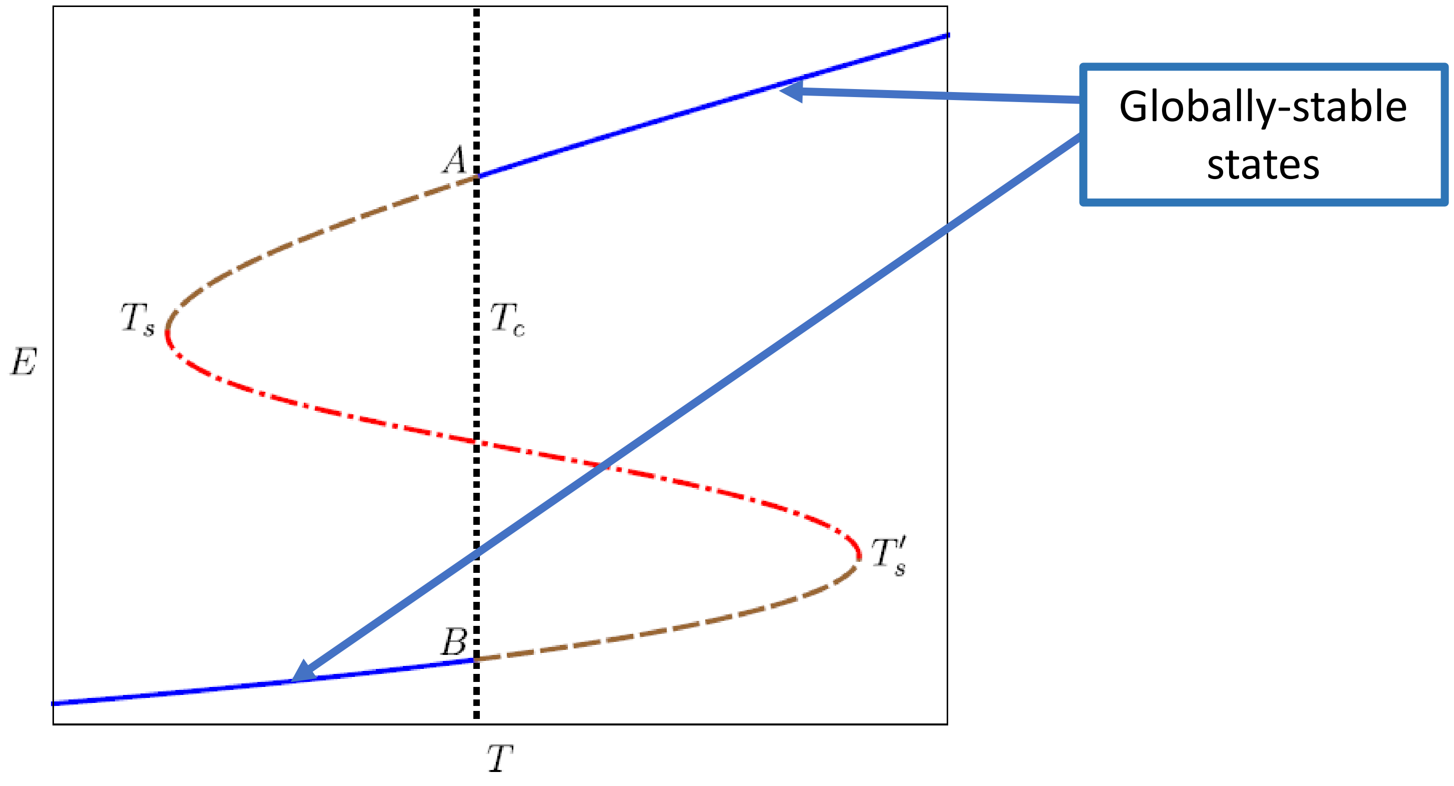
EMISSION FROM A FIRST-ORDER COSMOLOGICAL PHASE TRANSITION

First-order phase transition



Energy density is multivalued with respect to the temperature

First-order phase transition



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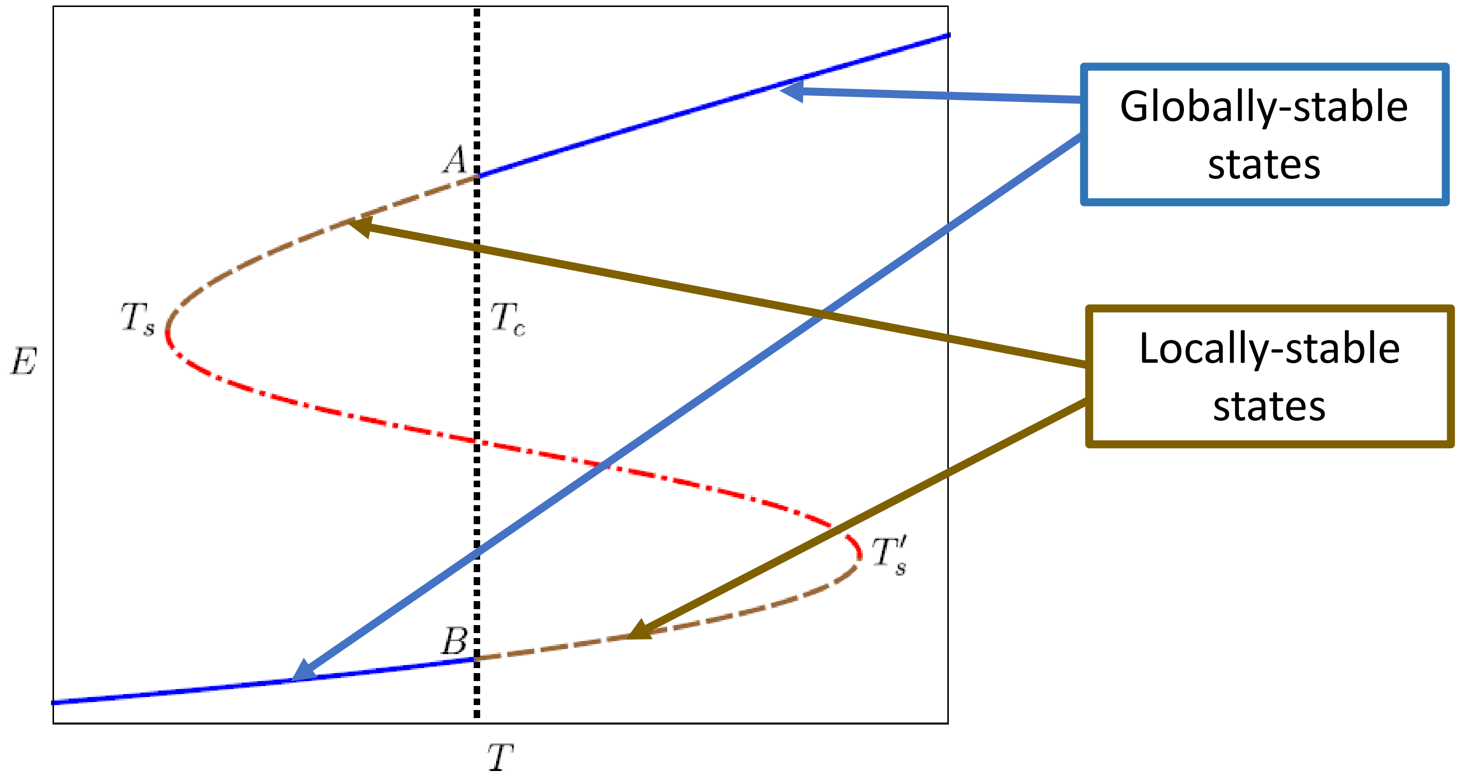

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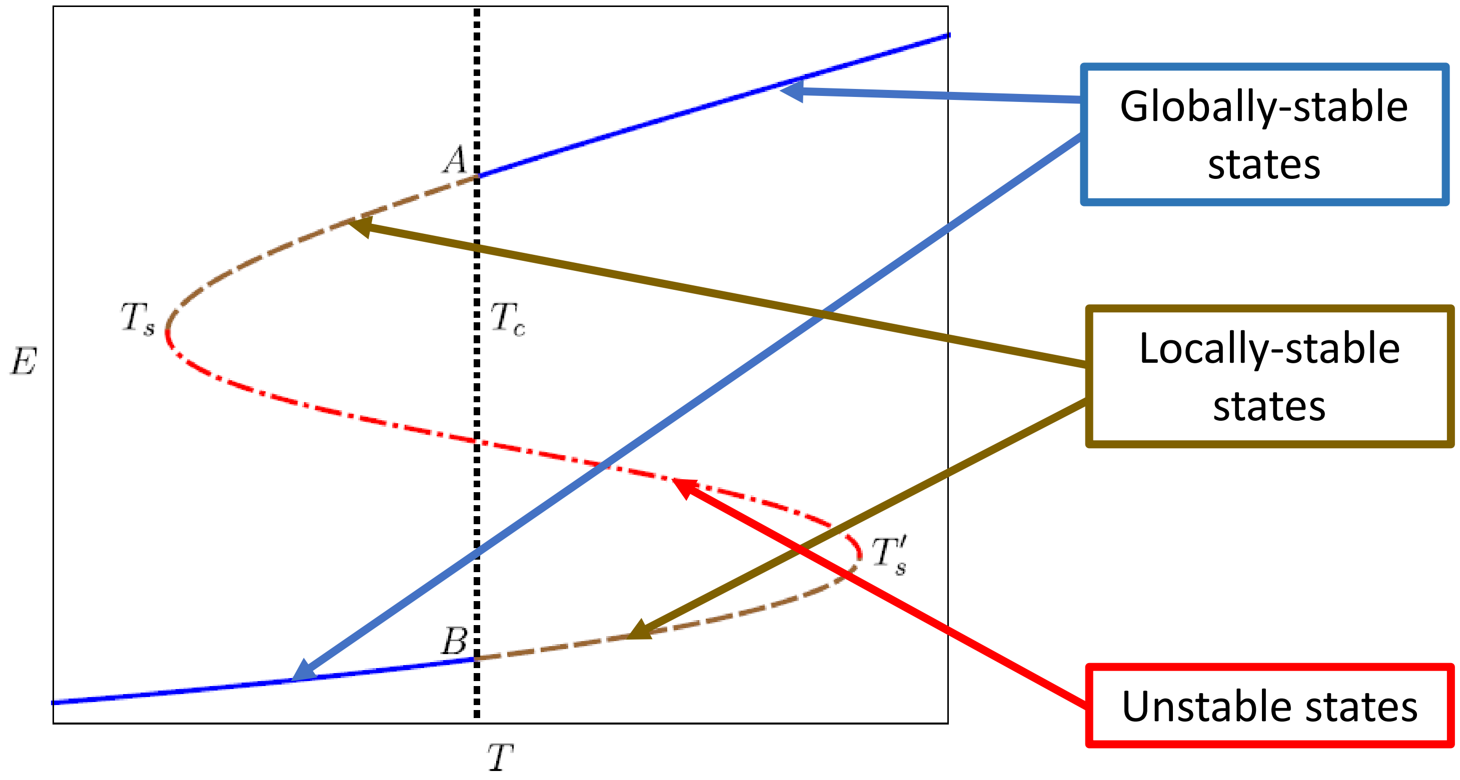

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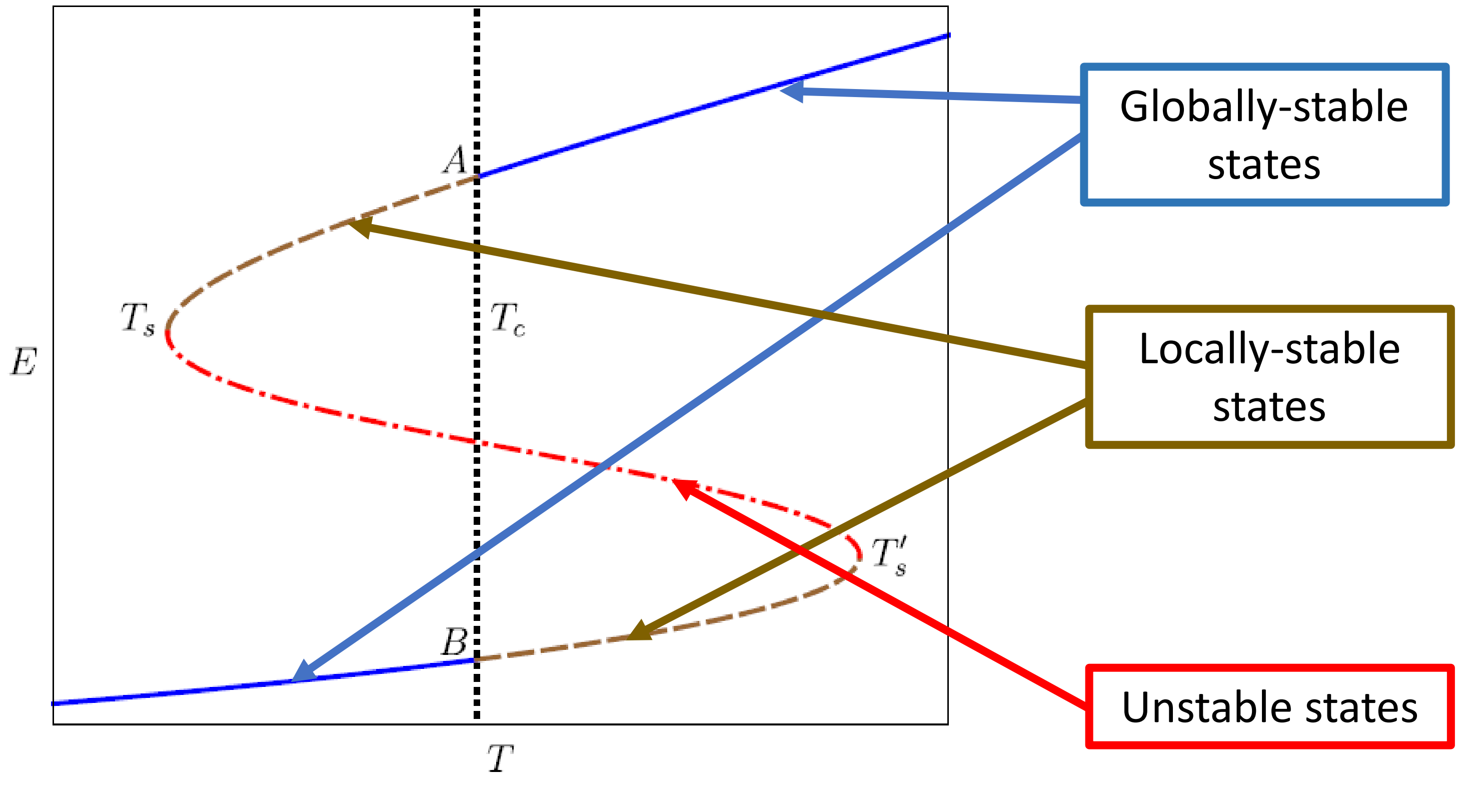

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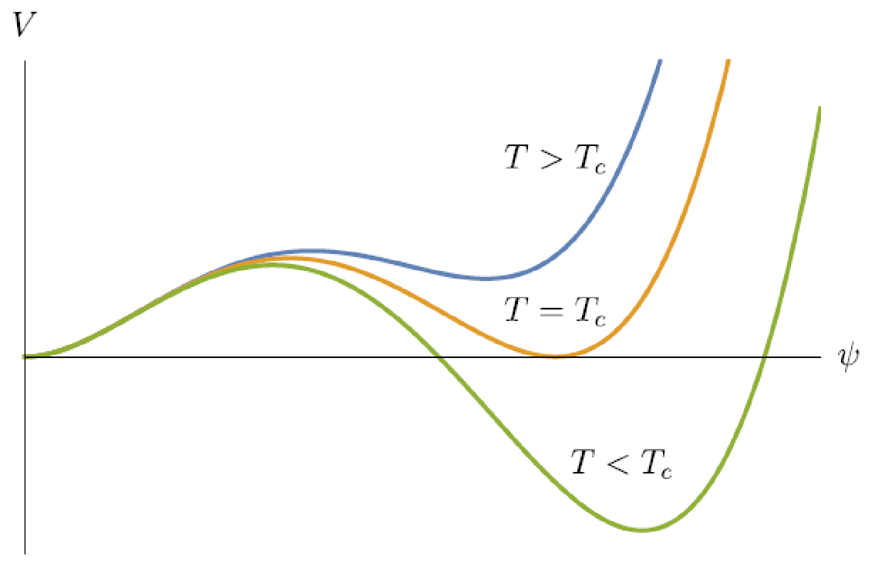
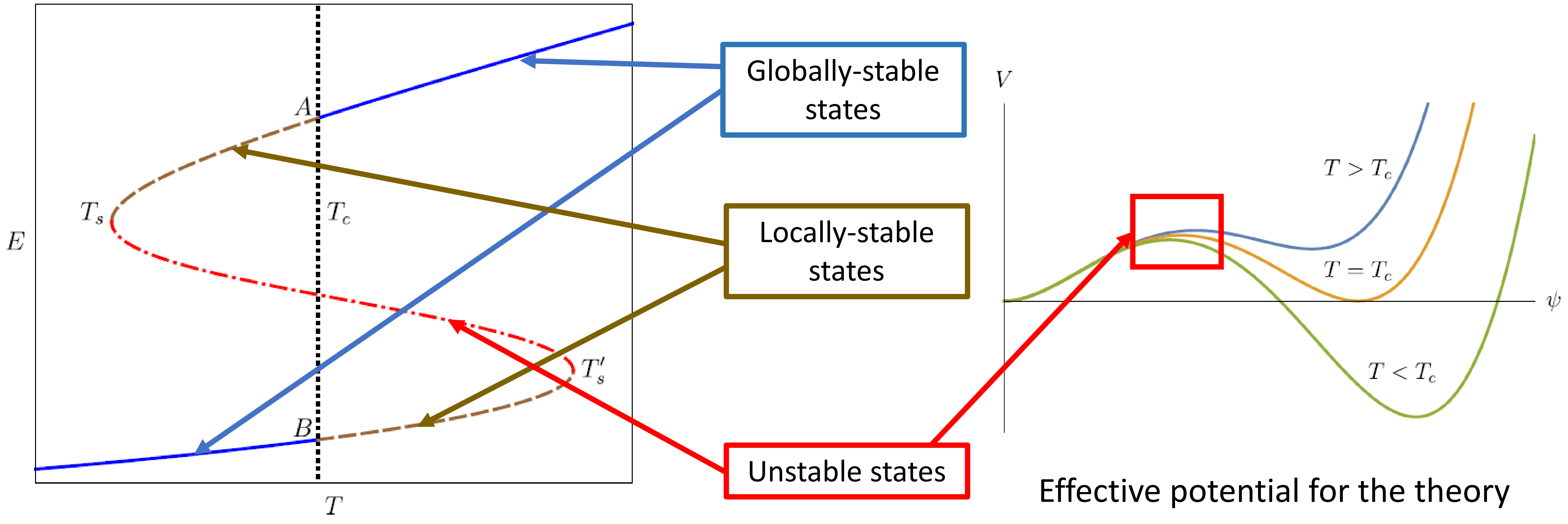


Effective potential for the theory

First-order phase transition



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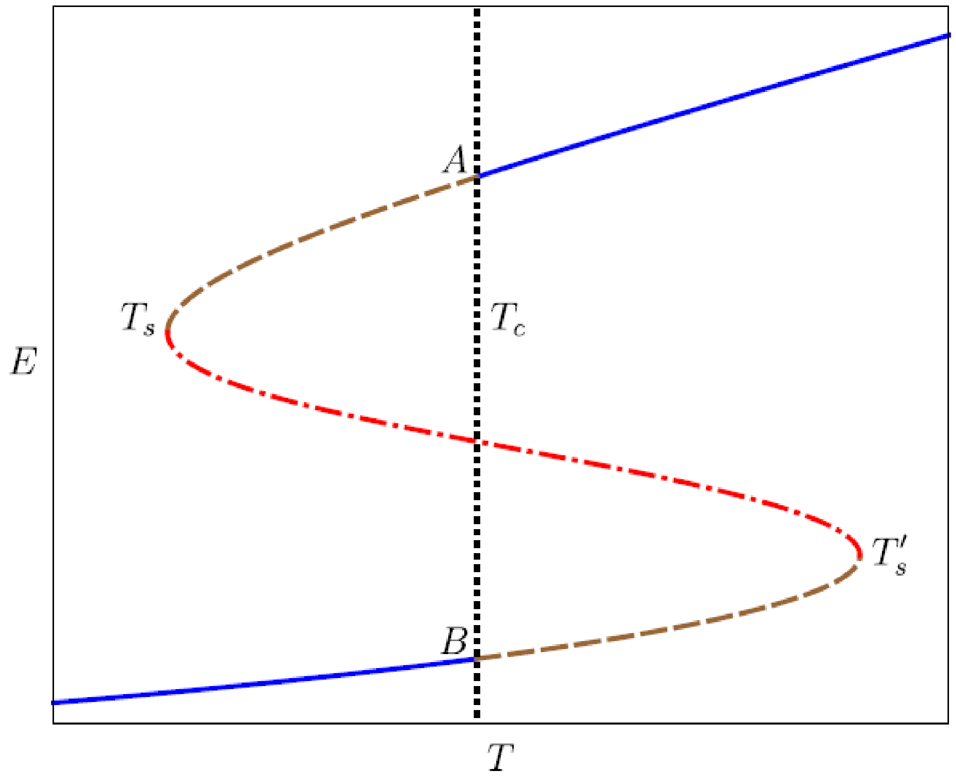
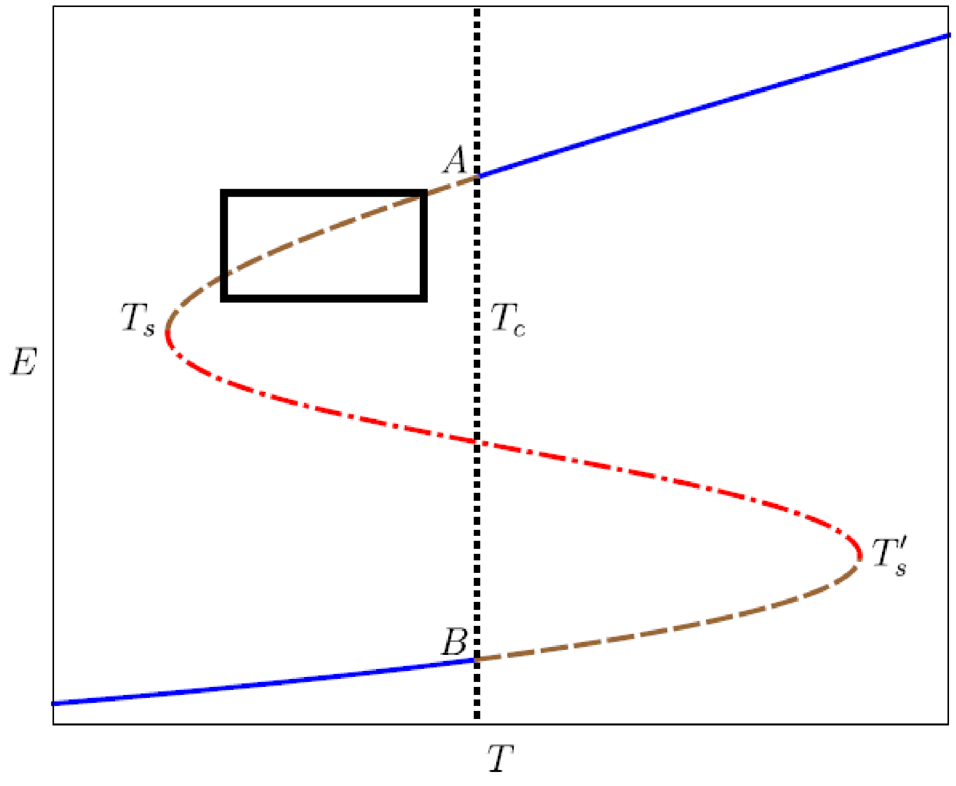


Energy density is multivalued with respect to the temperature

The phase transition can be realized in two ways:

- Bubble nucleation (commonly-assumed mechanism)
- Spinodal instability (alternative mechanism)

Bubble nucleation



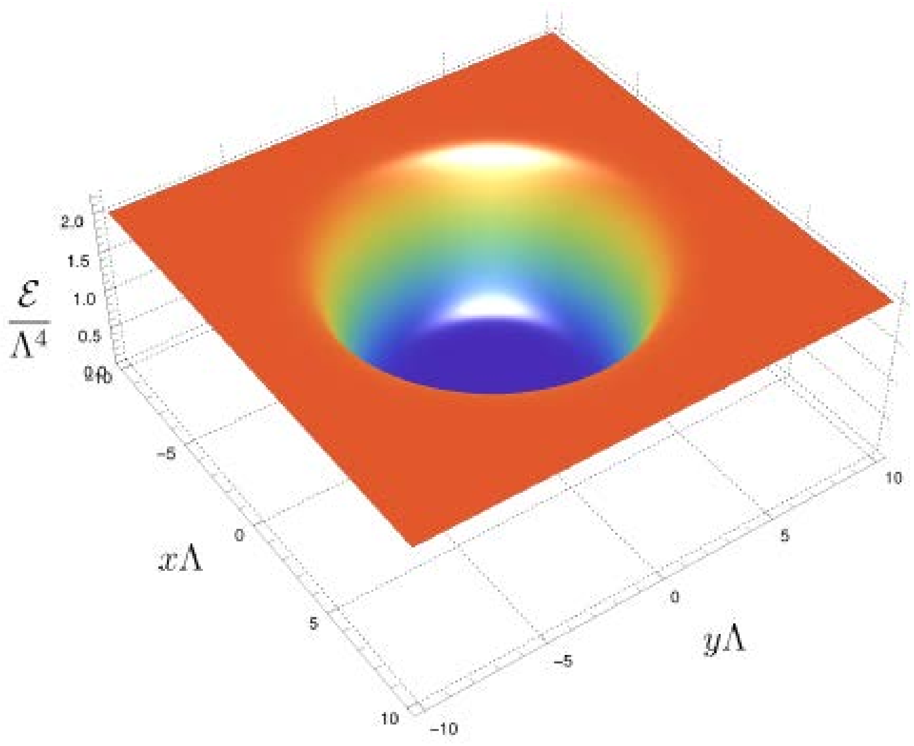
System in the locally-stable region

If the system is overcooled over T_c, it enters the metastable region.

This state is stable against small fluctuations, but not against large ones.

These fluctuations are **bubbles**, and the minimal fluctuation to induce the transition is the **critical bubble**.

Bubble nucleation



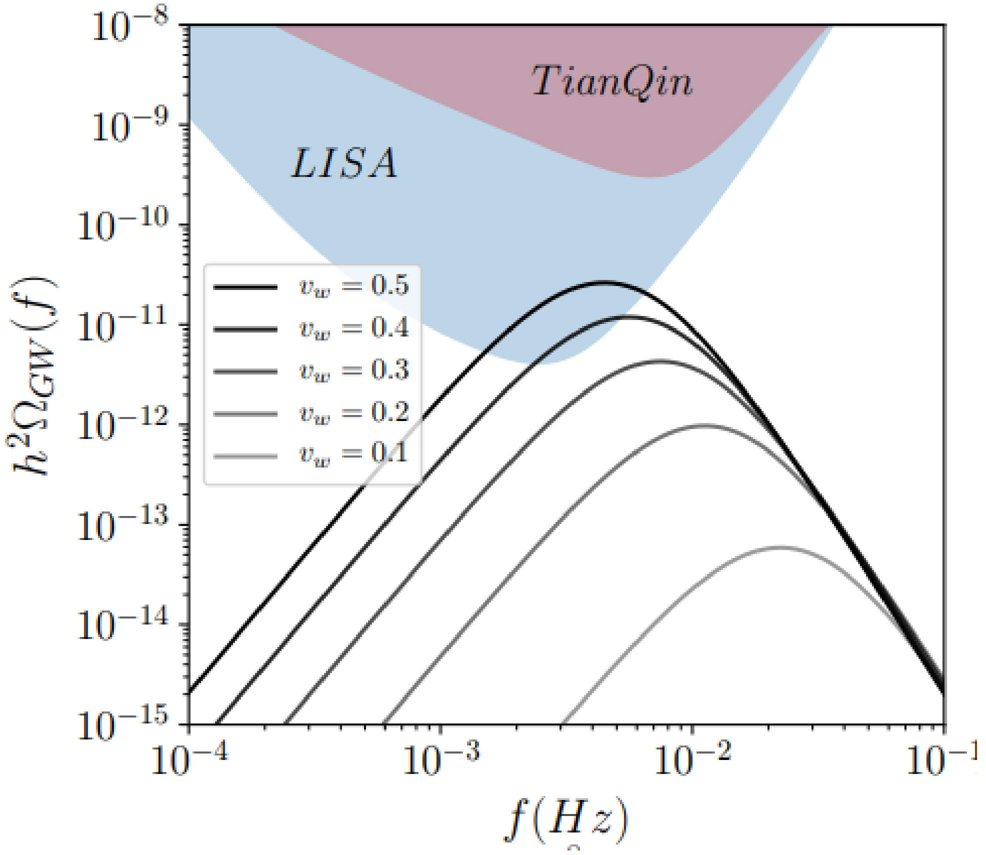
Bubbles are configurations for which the energy density within a certain region is reduced (i.e., in the stable branch).

Supercritical bubbles expand while undercritical bubbles collapse.

If multiple bubbles expand and collide, it leads to the **emission of gravitational waves**.

Figure from Bea, Casalderrey-Solana, Giannakopoulos, Jansen, Mateos, Sánchez-Garitaonandia & Zilhão (2022)

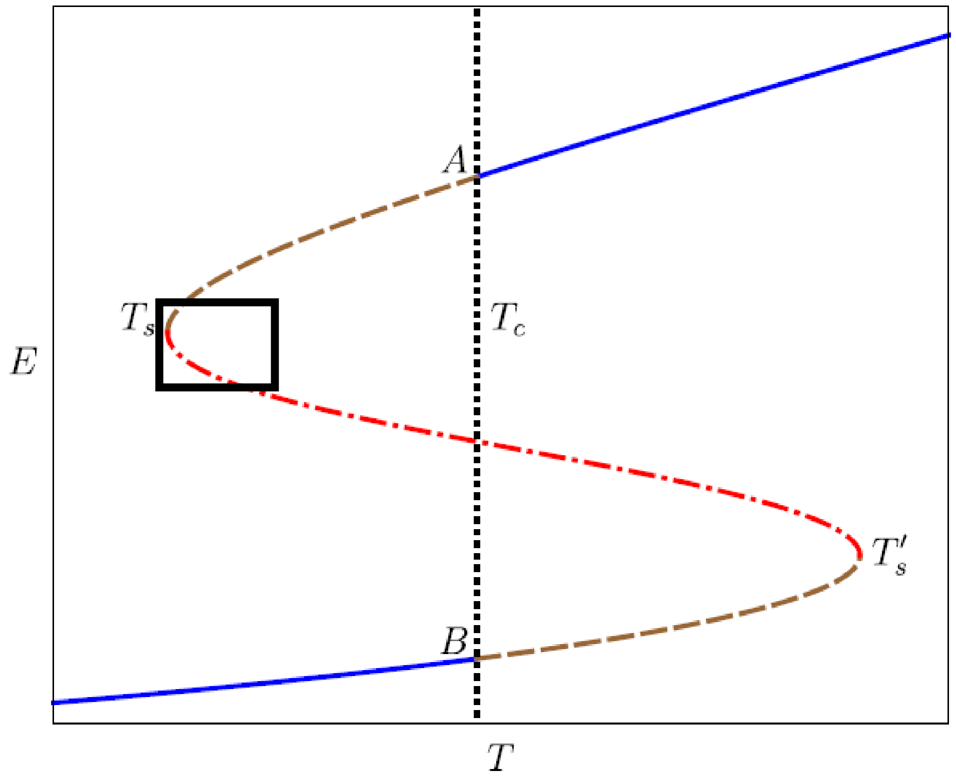
Gravitational waves from bubble nucleation



Spectrum of gravitational waves emitted from the collision of bubbles for different values of the bubble wall velocity

Figure from Ares, Hindmarsh, Hoyos & Jokela (2022)

Spinodal instability



System in the unstable region

If the bubble nucleation rate is sufficiently suppressed, the Universe can cool down to the spinodal branch.

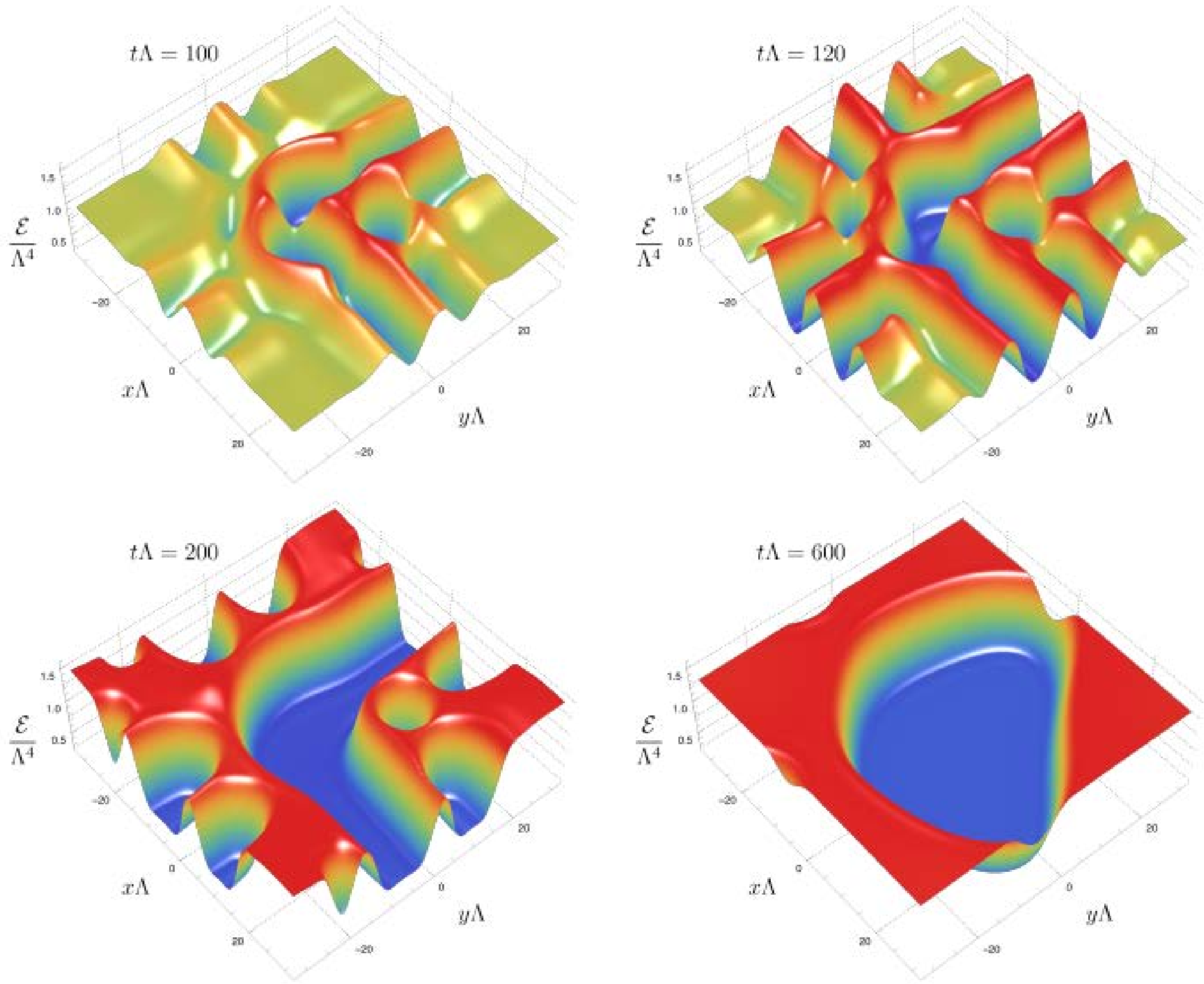
Fluctuations begin to grow, some in the form of **sound waves**.

Some modes grow exponentially as

$$\mathcal{A} \sim e^{\gamma(k)t} \,,$$

with $\gamma(k) > 0$ for long-wave length, small amplitude perturbatins, making the system very **unstable**.

Spinodal instability



Evolution of the phase transition via spinodal instability from an initial fluctuation

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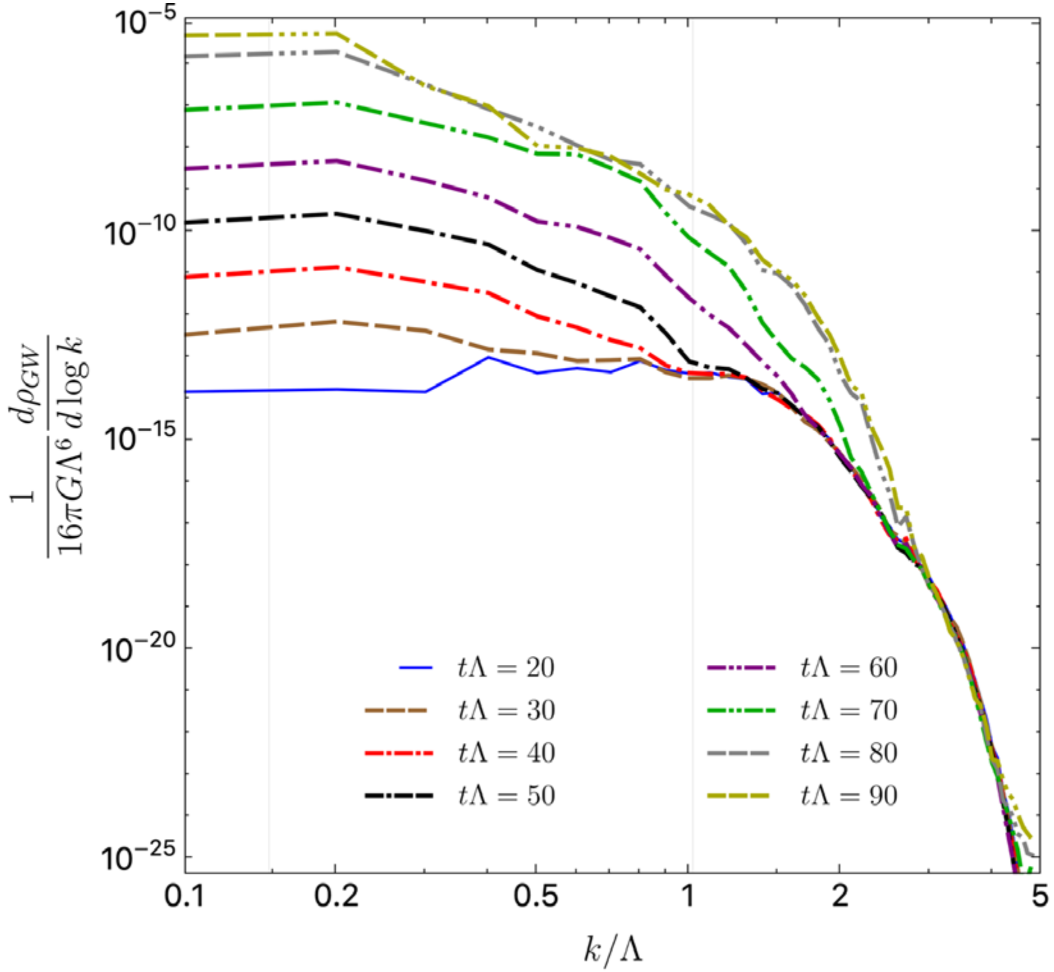
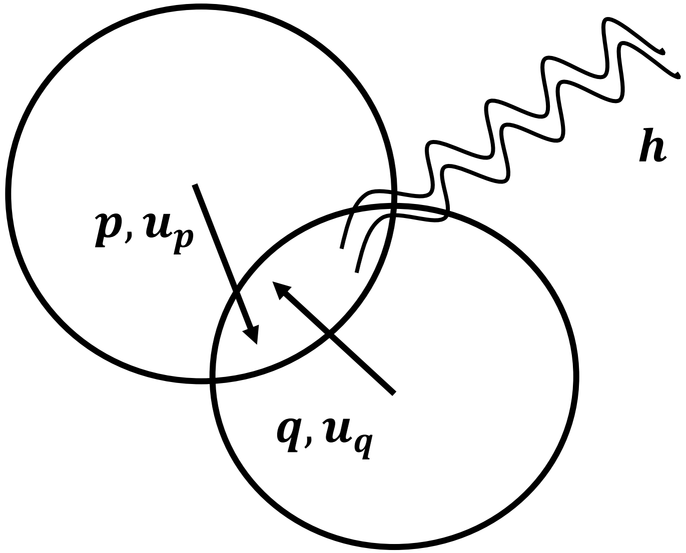
Gravitational waves from spinodal instability



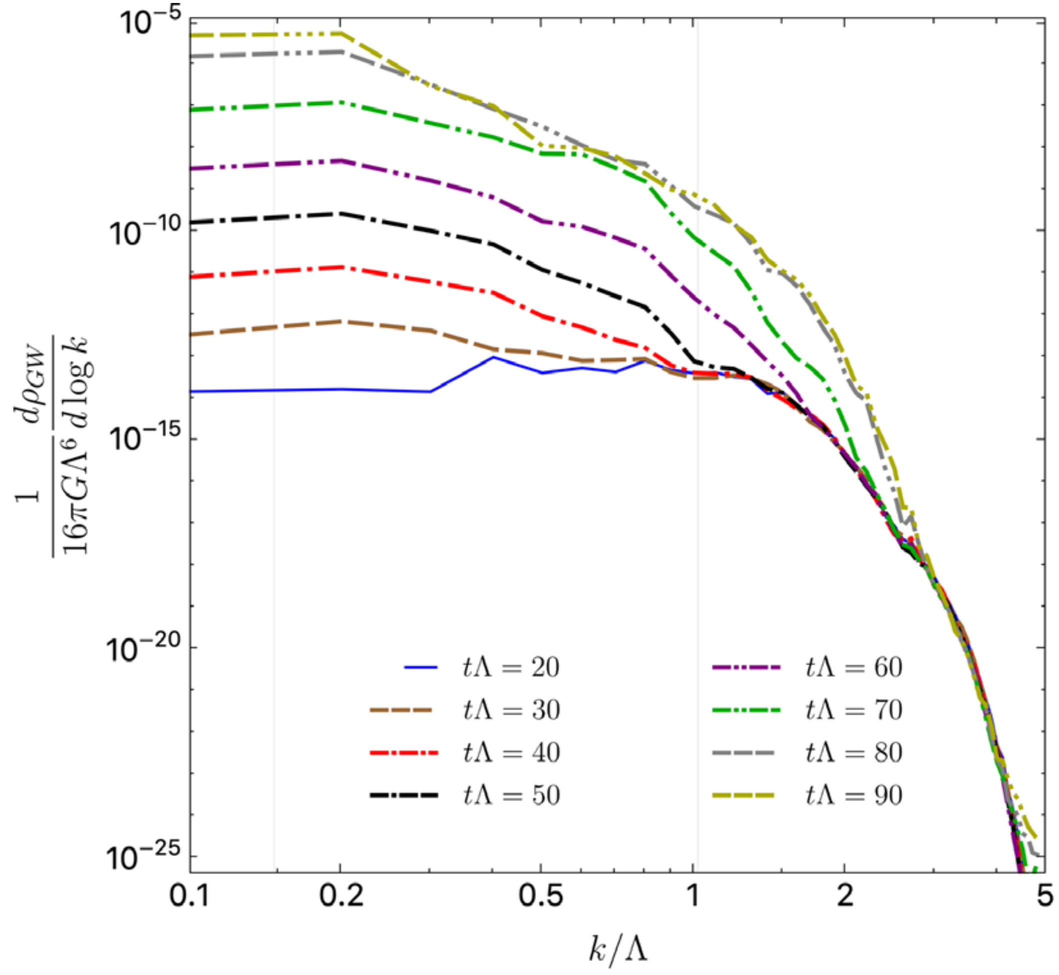
Gravitational waves energy density per unit logarithmic momentum



The **collision of sound waves** leads to the production of gravitational waves.

Figure from Bea, Casalderrey-Solana, Giannakopoulos, Jansen, Krippendorf, Mateos, Sánchez-Garitaonandia & Zilhão (2021)

Gravitational waves from spinodal instability



Gravitational waves energy density per unit logarithmic momentum

- The dynamics has been computed by setting by hand an initial fluctuation over the system.
- If we want to truly characterize the dynamics of the transition, we need to perform a full analysis of the fluctuations.

Figure from Bea, Casalderrey-Solana, Giannakopoulos, Jansen, Krippendorf, Mateos, Sánchez-Garitaonandia & Zilhão (2021)

Gravitational waves from spinodal instability

Energy production rate of gravitational waves:

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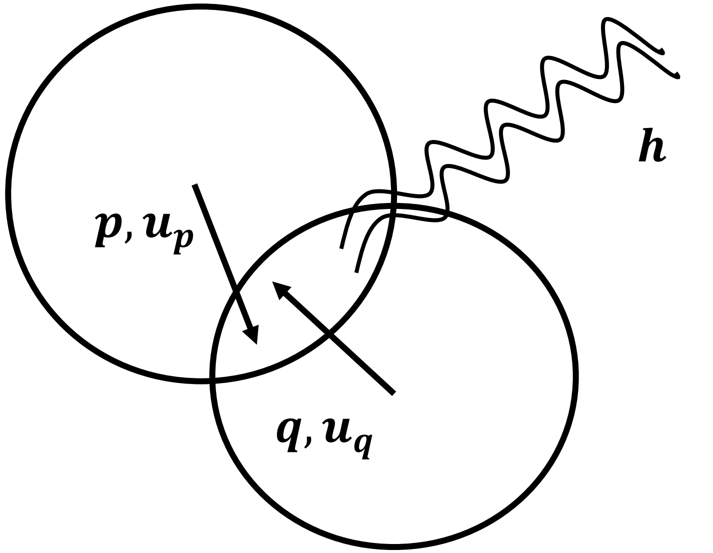

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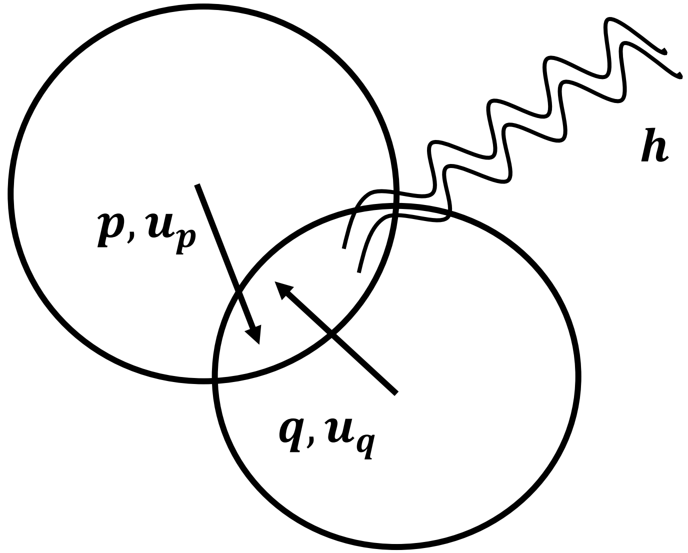

Sound waves propagate in the medium, leading to fluctuations $\boldsymbol{\delta T_{ij}}$ and $\boldsymbol{\delta v_i}$.

The piece of the fluctuation that contributes to the production of gravitational waves is

$$\delta T_{ij} = \omega_0 v_i v_j$$

The **collision of sound waves** leads to the production of gravitational waves.

Gravitational waves from spinodal instability

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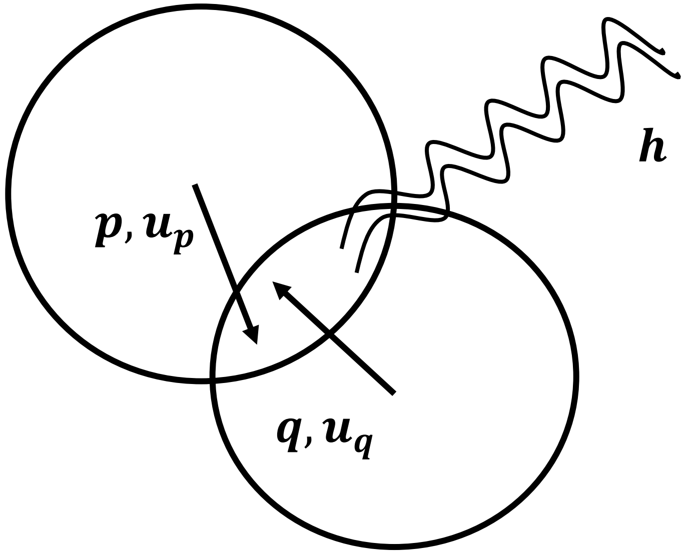

Fluctuations $\delta T_{ij} = \omega_0 v_i v_j$ with $v_i = \frac{T_{0i}}{\omega_0}$ and ω_0 the enthalpy.

The energy density of gravitational waves emitted by collision of sound modes depends on the correlator of the spin-0 components of the energy-momentum tensor:

$$\boldsymbol{\langle T_{0i}(0,0) T_{0j}(t,\mathbf{x}) \rangle}$$

Gravitational waves from spinodal instability: Holographic computation

Five-dimensional theory with Einstein gravity coupled to a scalar field ϕ

$$S = \frac{2}{\kappa_5^2} \int d^5x \sqrt{-g} \left[\frac{1}{4} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

$$\updownarrow$$

Non-conformal gauge theory obtained by addition of a source term to a conformal field theory

$$S \sim S_{CFT} + \int d^4x \, \Lambda \boldsymbol{\mathcal{O}}$$

For certain choice of the parameters in the gravity theory, the dual gauge theory exhibits a number of features such a first-order phase transition.

Gravitational waves from spinodal instability: Holographic computation

Ongoing work: Characterize the fluctuations $\langle \boldsymbol{T_{0i}(0,0)T_{0j}(t,x)} \rangle$

Duality toolbox:

Energy-momentum tensor $T_{\mu\nu}$ in the CFT **4D**

$\updownarrow$

Gravitational fluctuations $h_{\mu\nu}$ **5D**

Holography allows us to compute $\langle \boldsymbol{T_{0i}(0,0)T_{0j}(t,x)} \rangle$ by solving for the $\boldsymbol{h_{0i}}$ component.

Turn on spin-0 fluctuations

$$h_{tt}, h_{tr}, h_{rr}, h_{rx}, \varphi,$$

which can be expressed in terms of two independent scalars $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$ satisfying

$$\boldsymbol{\Box\Phi - V(r)\Phi = 0}$$

Conclusions

- With this analysis, we estimate the strong coupling effects on the gravitational wave spectrum emitted by different phenomena in the early Universe.

- In order to use this spectrum of gravitational waves in the search of new physics, we need to widen the range of possible signals.

- For the particular case of phase transitions, we will soon complete the analysis of spinodal-induced emissions and compare it with the spectrum of the standard bubble nucleation mechanism.