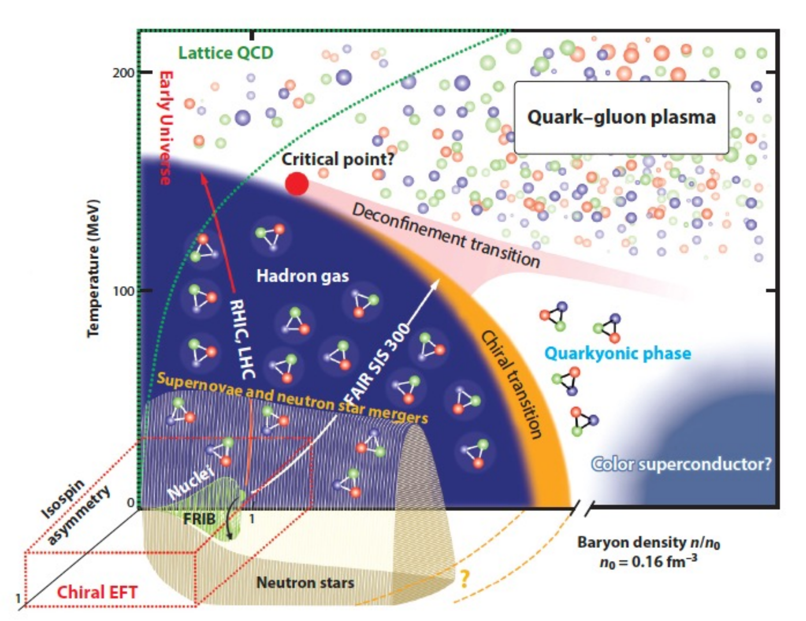
The equation of state of neutron-star matter

Anthea F. Fantina

Nuclei in the Cosmos XVIII,
16 – 20 June 2025, Girona (Spain)

Probing extreme conditions in NSs



Drischler et al., Ann. Rev. Nucl. Part. Sci. 71, 403 (2021)

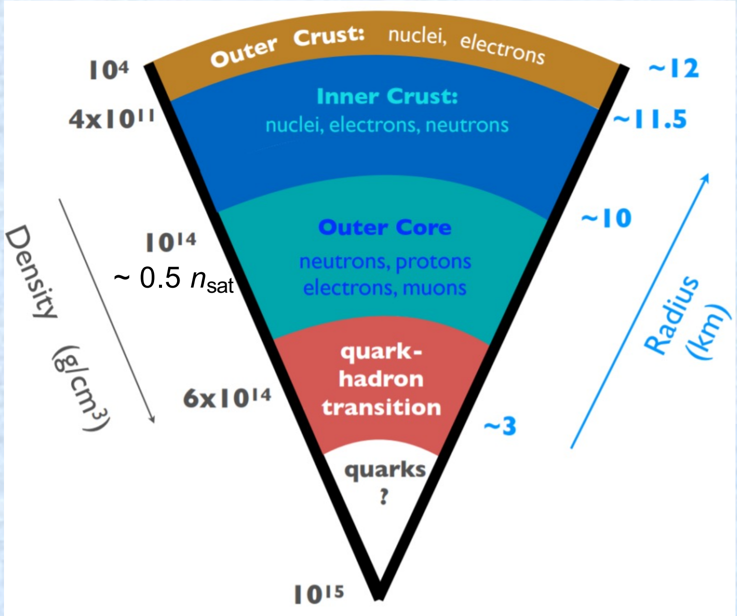
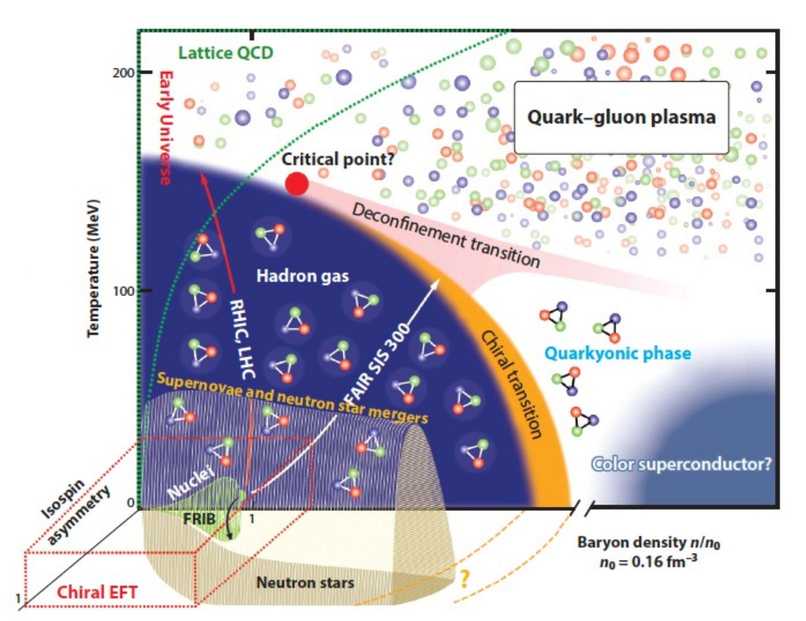


Image Credit: 3G Science White Paper

different states of matter spanned in NSs
→ inhomogeneous (crust), "pasta" phase, homogeneous (core), "exotic" particles (?)
+ superfluidity, (strong) magnetic field, etc.

N.B.: In this talk: beta-equilibrated T=0 matter, NS static properties
→ OK for cold NS and pre-merger binary NSs

Probing extreme conditions in NSs



Drischler et al., Ann. Rev. Nucl. Part. Sci. 71, 403 (2021)

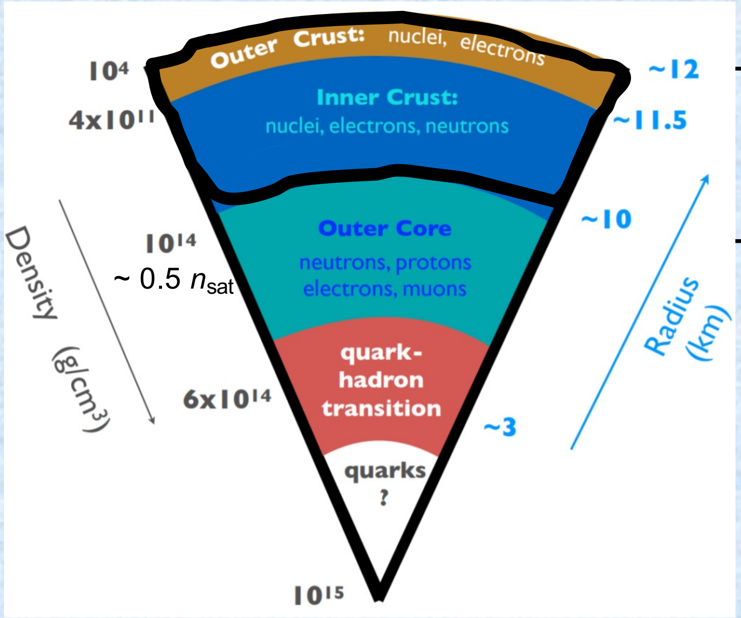


Image Credit: 3G Science White Paper

CRUST:
~ 1% of M_{NS}
~ 10% R_{NS}
$\rho < \rho_{sat}$

different states of matter spanned in NSs
→ inhomogeneous (crust), “pasta” phase, homogeneous (core), “exotic” particles (?)
+ superfluidity, (strong) magnetic field, etc.

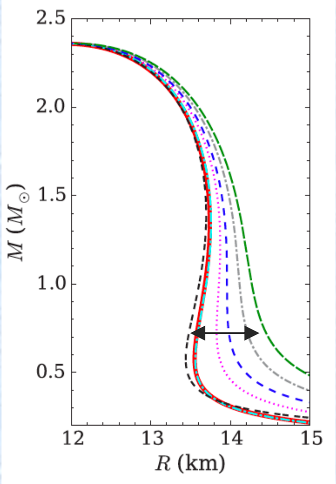
→ Not all conditions can be probed in terrestrial labs → theoretical models !
→ Consistent description very challenging

N.B.: In this talk: beta-equilibrated T=0 matter, NS static properties
→ OK for cold NS and pre-merger binary NSs

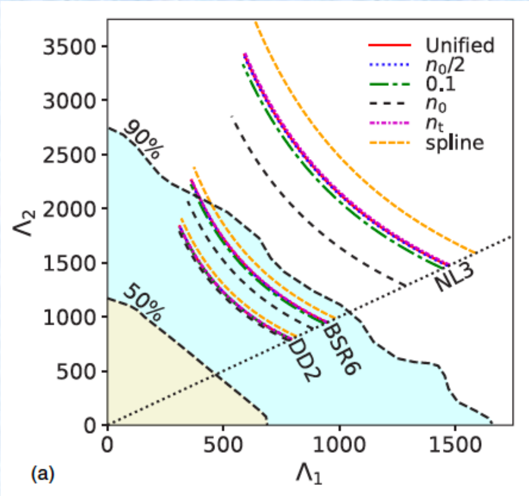
Why a consistent and unified treatment ?

Unified treatment of inhomogeneous & homogeneous matter
→ same nuclear model employed in different regions of star

- Challenging because of wide range of thermodynamic conditions and different states of matter
- But: essential to avoid spurious non-physical effects in numerical modelling



Fortin et al., PRC 94, 035804 (2016)

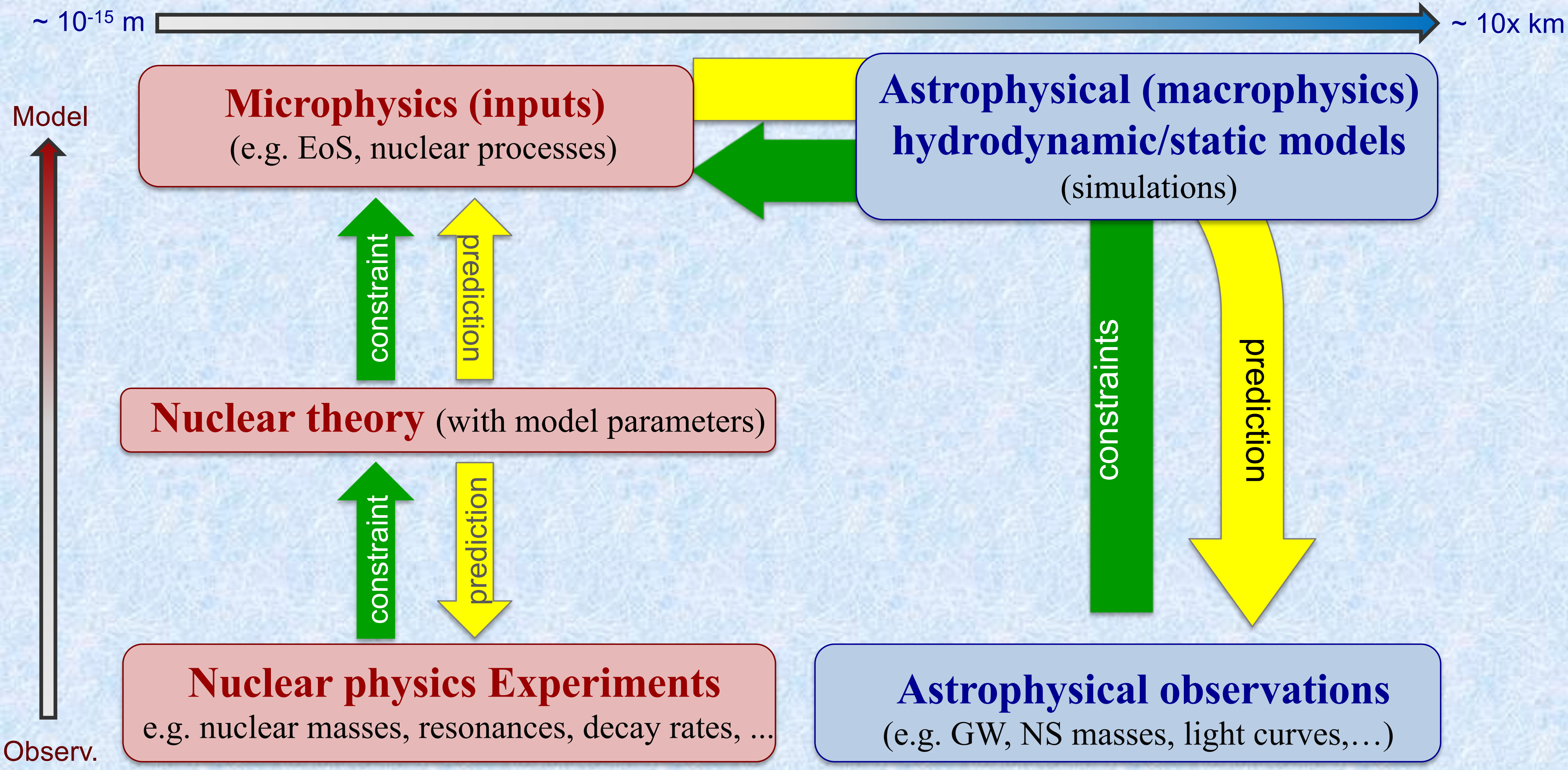


Suleiman et al., PRC 104, 015801 (2021)

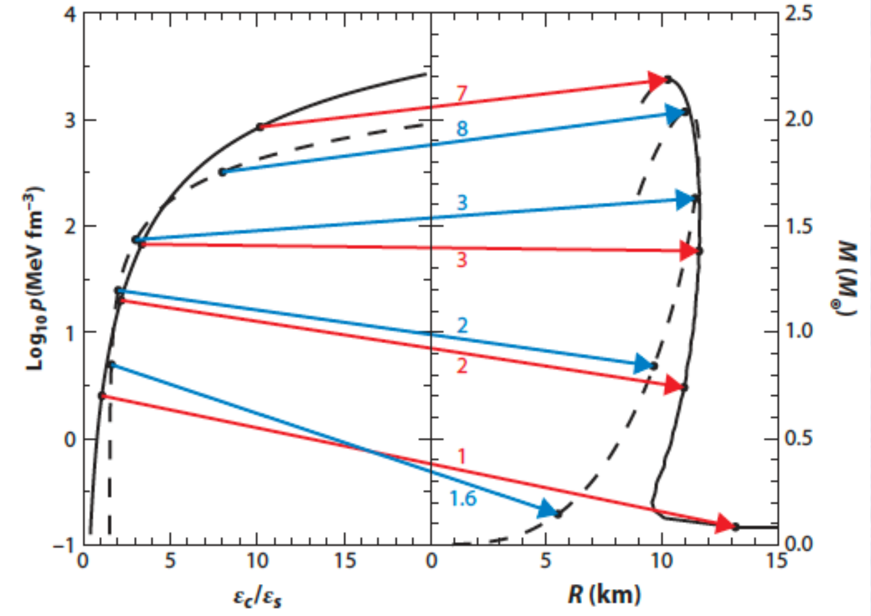
see also Ferreira&Providençia 2020, Davis et al. 2024

Thermodynamically consistent and unified EoSs for astro modelling & inference analyses
(but not many available, e.g. Douchin&Haensel 2001; Fantina et al. 2013; Raduta&Gulminelli 2015; Viñas et al. 2021; Pearson et al. 2018; Grams et al. 2022; Xia et al. 2022; Scurto et al. 2024; see CompOSE database)

Micro to macro through modelling and scales



EoS ←→ NS (static) observables (1)



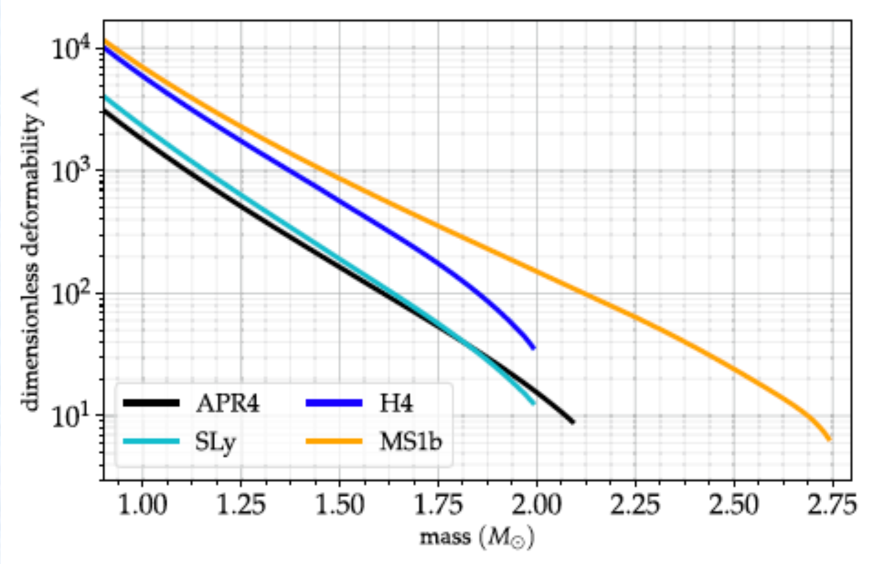
Lattimer, Annu. Rev. Part. Nucl. Sci. 62, 485 (2012)

- **TOV → *M*(*R*)** (Tolmann 1939; Oppenheimer&Volkoff 1939; see also Haensel et al. Springer 2007)

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2}\left[1+\frac{P(r)}{c^2\rho(r)}\right]\left[1+\frac{4\pi P(r)r^3}{c^2\mathcal{M}(r)}\right]\left[1-\frac{2G\mathcal{M}(r)}{c^2 r}\right]^{-1}$$

$$\mathcal{M}(r) = 4\pi\int_0^r \rho(r')r'^2\mathrm{d}r'$$

➡ $P(\mathcal{E})$ EoS needed!



Dietrich et al., Gen. Rel. Gravit. 53, 27 (2021)

➡ GR → direct correspondence
EoS ←→ NS static properties

?
→ trace back to EoS and composition?

N.B.: GR in slow rotation limit w/o magnetic field !

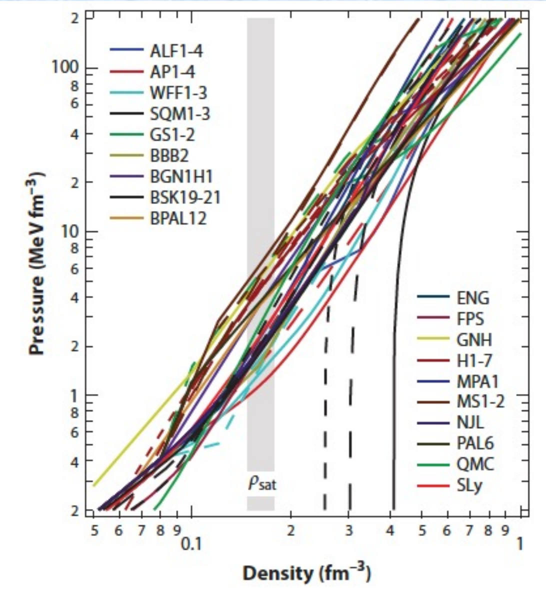
EoS ←→ NS (static) observables (2)

but:

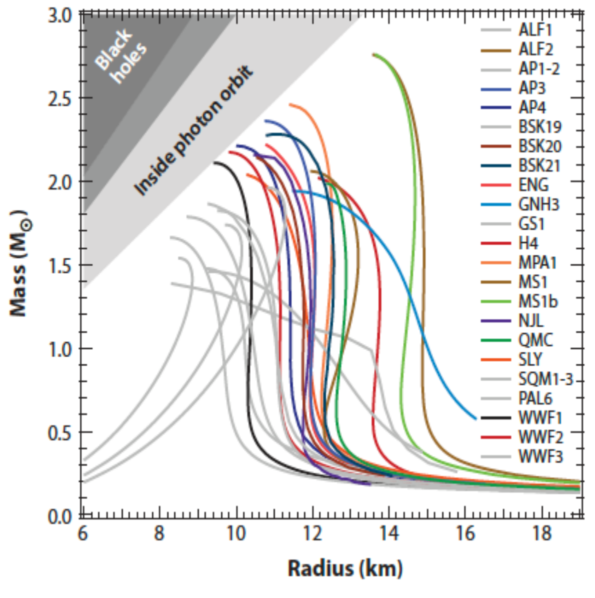
✗ EoS model dependent ! (higher uncertainties at higher ρ)

✗ no ab-initio dense-matter calculations
in all regimes → phenomenological models

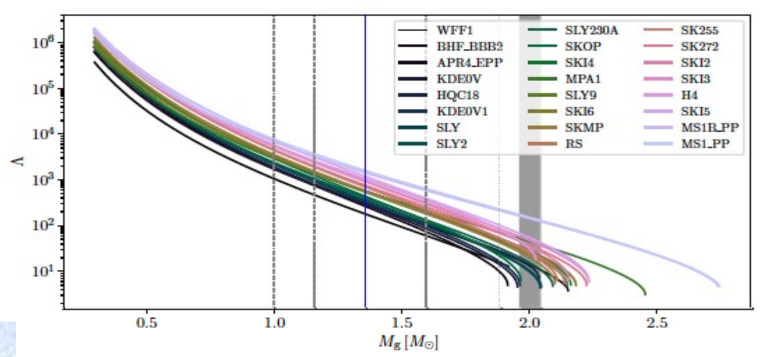
✗ composition ←→ EoS → $M(R)$?



Ozel & Freire, ARAA 54, 401 (2016)



Ozel & Freire, ARAA 54, 401 (2016)

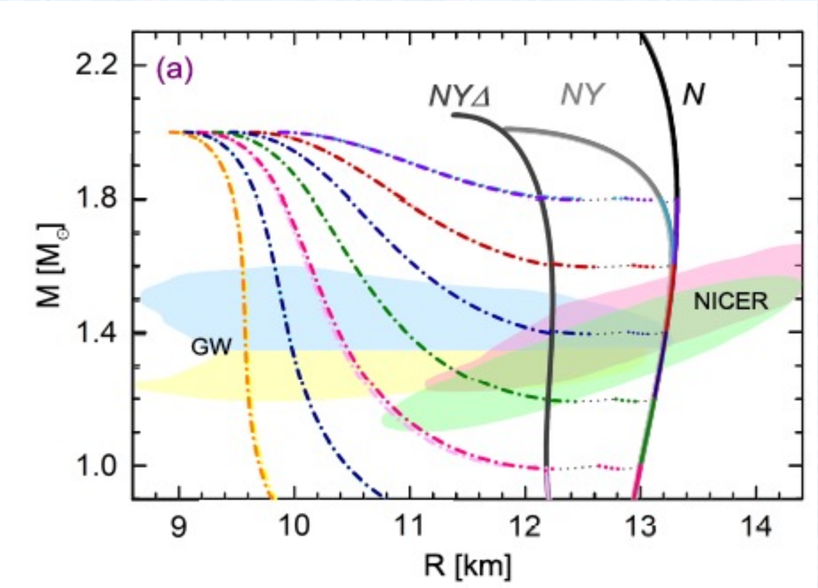


Abbott et al., Class. Quantum Grav, 37, 045006 (2020)

(see e.g. Haensel et al., "Neutron Star Structure", Springer 2007; Oertel et al., Rev. Mod. Phys. 2017; Burgio & Fantina, ASSL Springer 2018; Blaschke & Chamel, ASSL Springer 2018 for a review)

High-density EoS → additional d.o.f.?

- Role of "exotic" degrees of freedom?
  Hyperons → softer EoS → lower M_{max} (+ reduction of R and Λ for intermediate-mass) but large uncertainties on Y !
  Quarks → not clear

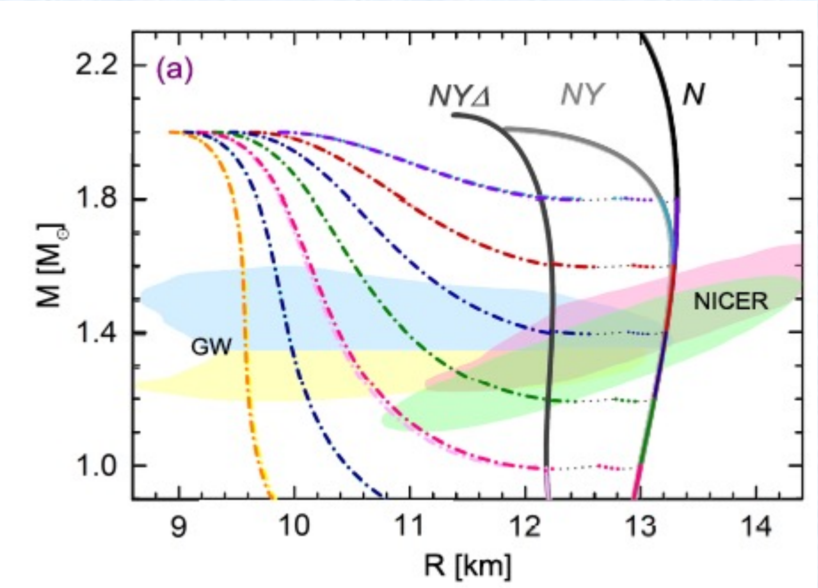


Li et al., PRD 101, 063022 (2020)

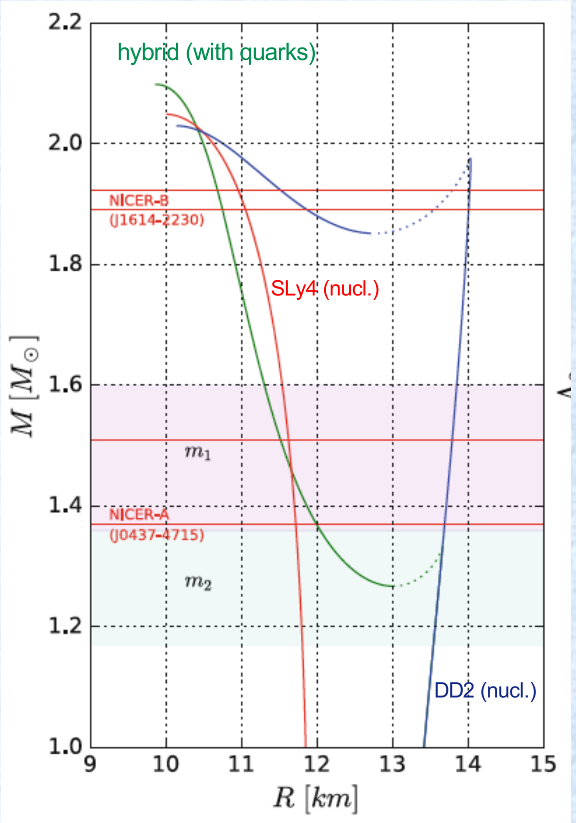
High-density EoS → additional d.o.f.?

- Role of "exotic" degrees of freedom?
  Hyperons → softer EoS → lower M_{max} (+ reduction of R and Λ for intermediate-mass) but large uncertainties on Y !
  Quarks → not clear

- "Masquerade" effect



Li et al., PRD 101, 063022 (2020)



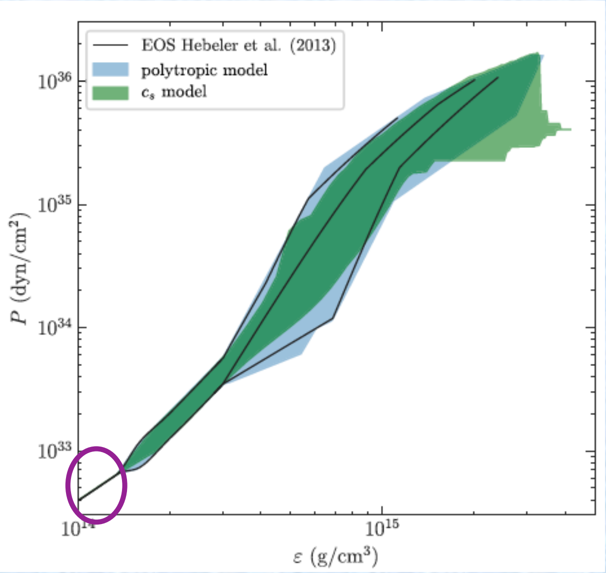
Blaschke & Chamel, ASSL 457, 337 (2018);

High-density EoS → additional d.o.f.?

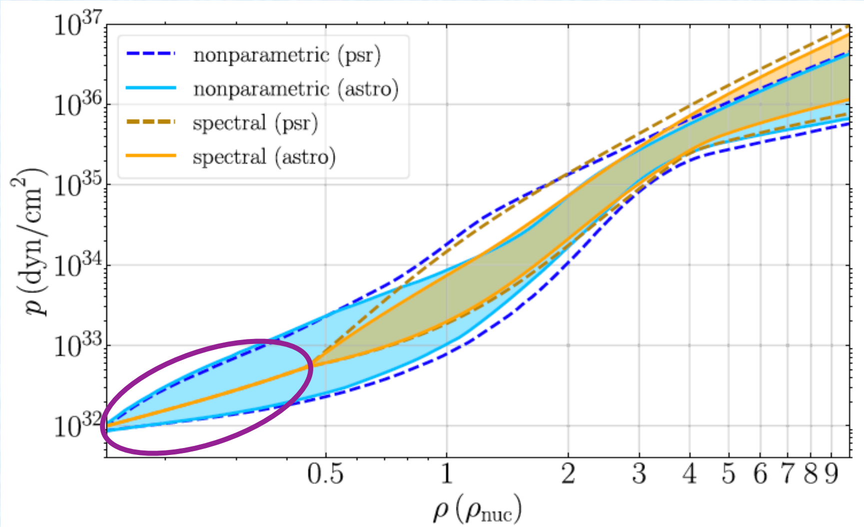
- Role of "exotic" degrees of freedom?
  Hyperons → softer EoS → lower M_{max} (+ reduction of R and Λ for intermediate-mass) but large uncertainties on Y !
  Quarks → not clear

- "Masquerade" effect

➢ *Agnostic* ("non-nuclear") approaches for NS core (e.g. piecewise polytropes, c_s models, Gaussian process, etc…) (conditioned by astro)



Hebeler, Phys. Rep. 980, 1 (2021)



Legred et al. PRD 105, 043016 (2022)

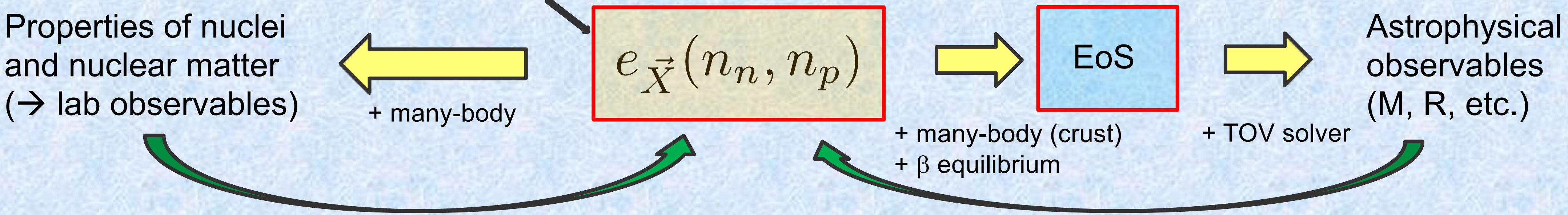
✓ poweful → no underlying hypotheses
✗ no info on composition
✗ often unique (non-consistent) low-density EoS
→ uncertainties underestimated

A semi-agnostic approach: nucleonic meta-model

- **Nucleonic Meta-model (MM)** (Margueron et al., PRC 97, 025805 (2018), Char et al., PRD 108 (2023))
  see also e.g. Lim&Holt 2019, Tsang et al. 2020

→ nucleonic → no additional particles
→ flexible functional, based on Taylor expansion around saturation density.
Model parameters $\{X_i\}$ related to properties of nuclear matter (nuclear empirical param.)

$$\vec{X} = \{E_{\mathrm{sat,sym}}, L_{\mathrm{sym}}, K_{\mathrm{sat,sym}}, \ldots\}$$

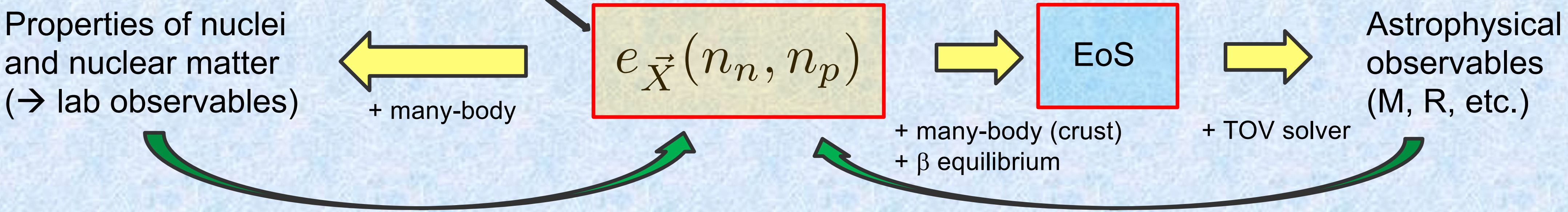


A semi-agnostic approach: nucleonic meta-model

- **Nucleonic Meta-model (MM)** (Margueron et al., PRC 97, 025805 (2018), Char et al., PRD 108 (2023))
  see also e.g. Lim&Holt 2019, Tsang et al. 2020

→ nucleonic → no additional particles
→ flexible functional, based on Taylor expansion around saturation density.
Model parameters $\{X_i\}$ related to properties of nuclear matter (nuclear empirical param.)

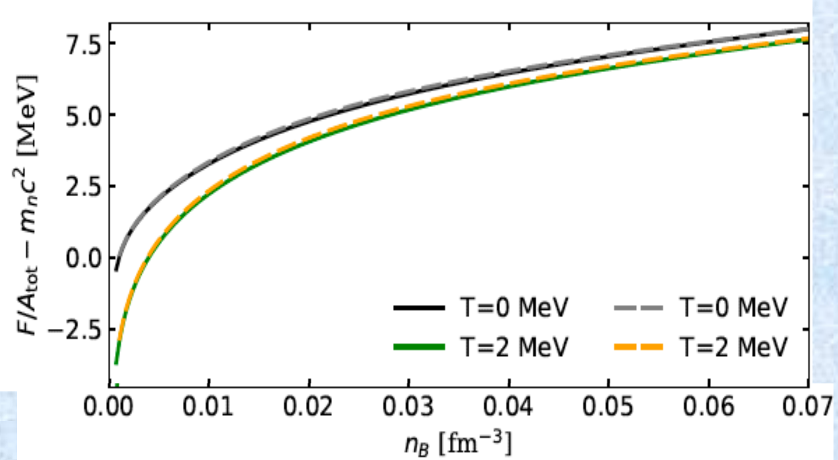
$$\vec{X} = \{E_{\mathrm{sat,sym}}, L_{\mathrm{sym}}, K_{\mathrm{sat,sym}}, \ldots\}$$

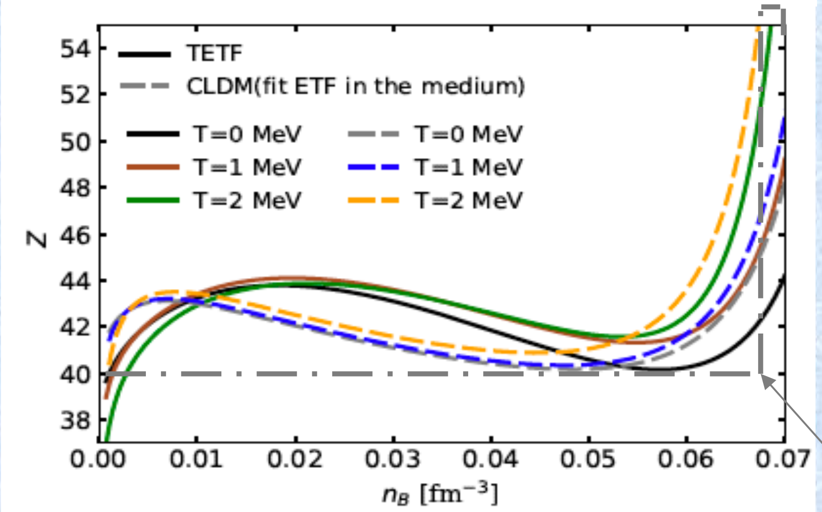


- ✓ Different $\{X\}$ → reproduce different effective models (EDF / RMF)
- ✓ Vary $\{X\}$ → large parameter space → statistical (Bayesian) analysis → possible correlation with nuclear param.
- ✓ Possible to couple MM approach for crust to agnostic EoS (e.g. polytropes, etc.) at high density

- N.B. : For application to NS crust → many-body method (e.g. CLDM or ETF)
  e.g. Carreau et al., EPJA 2019; Dinh Thi et al., A&A 2021; Grams et al., EPJA 2022; Mondal et al., MNRAS 2023; Davis et al., A&A 2024; Montefusco et al., A&A 2025, Klausner et al. arXiv2505.16929

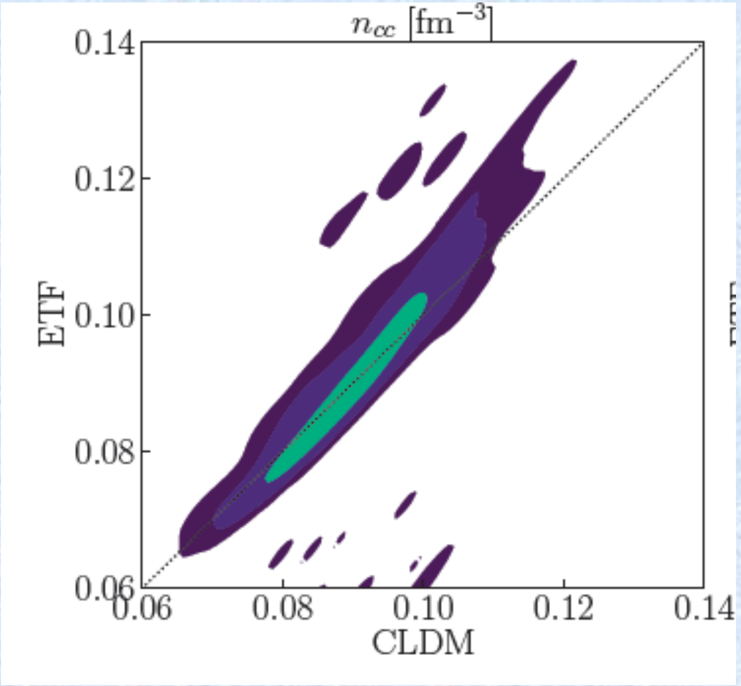
Crustal properties: CLDM vs ETF





Pearson et al., MNRAS 2018 (ETFSI)

Grams, Shchechilin, Diverrès, et al., Universe 11, 172 (2025)
(see also Carreau et al., A&A 2021; Grams et al., J. Phys. 2022)



Klausner et al., arXiv:2505.16929v1

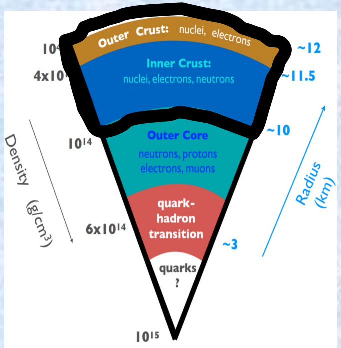
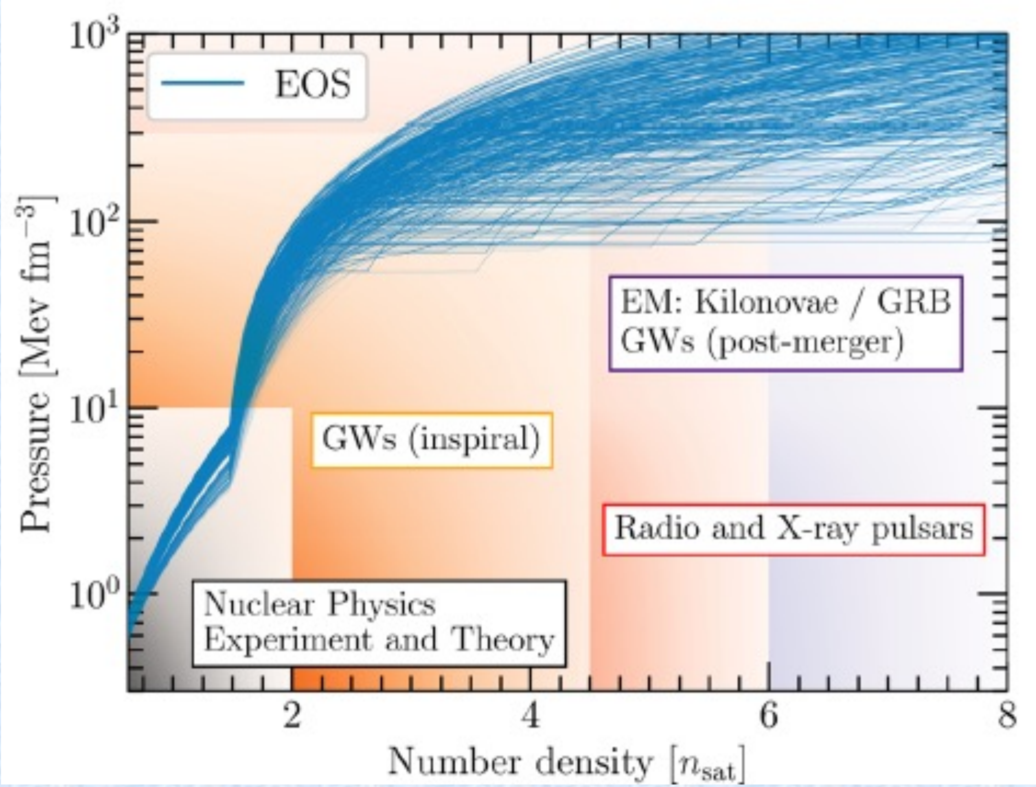


Image Credit: 3G Science White Paper

→ CLDM in good agreement with (T)ETF calculations

Model exploration: Bayesian inference

$$p_{\text{post}}(\vec{X}|\vec{c}) \propto p(\vec{X}) \prod_i p(c_i|\vec{X})$$



Pang et al., Nat. Comm. 14, 8352 (2023)

$p(\vec{X})$ prior → large parameter space (non-informative / nuc.-phys. informed)

$p(c_{\text{nuc}}|\vec{X})$ Nuclear data (e.g. nuclear masses)
Ab-initio calculations

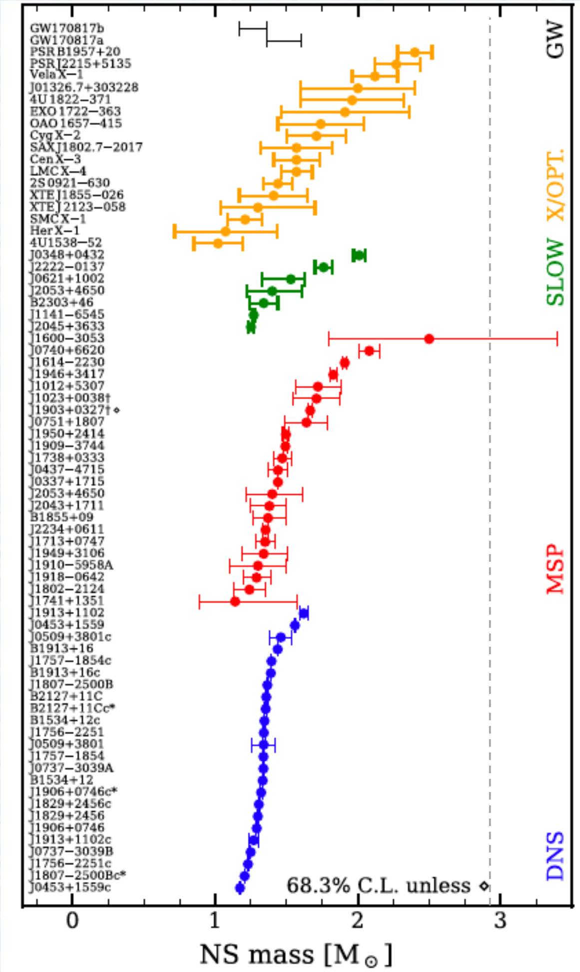
→ Low-density constraints

$p(c_{\text{astro}}|\vec{X})$ Causality + thermo stability
Astro data: M_{max}, GW, NICER

→ High-density constraints

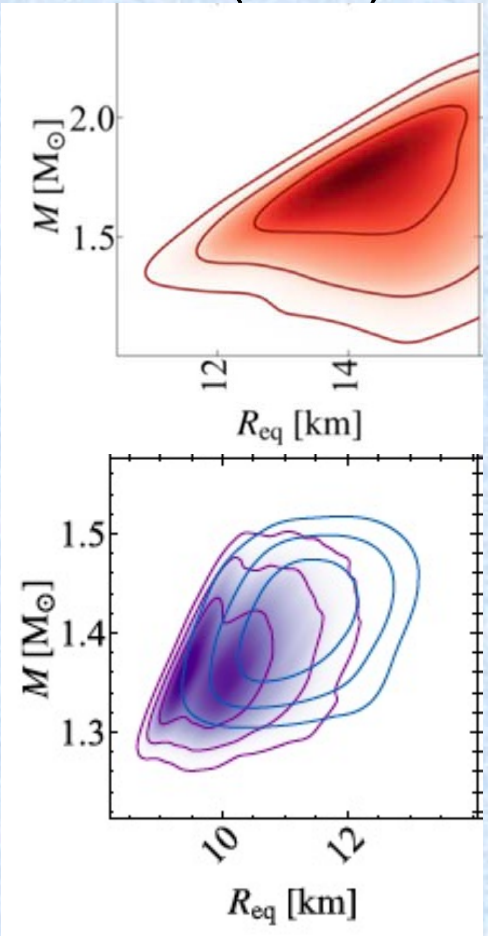
Constraints from astrophysics

MASSES



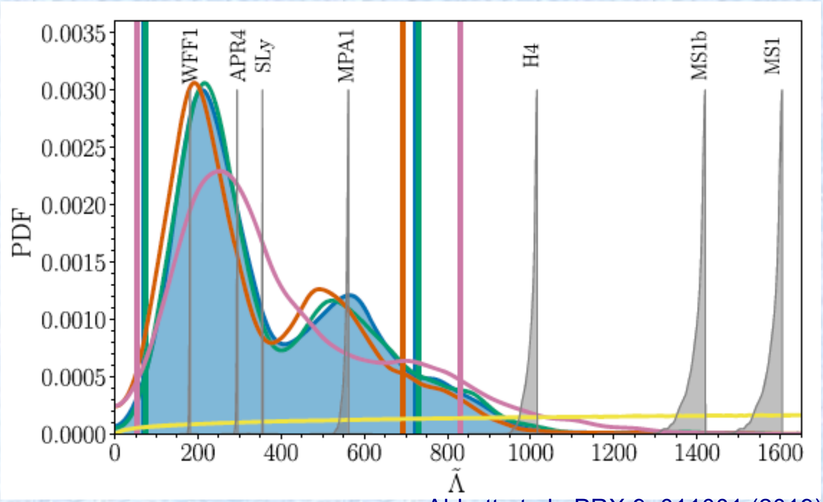
Suleiman et al., PRC 104, 015801 (2021)

RADII (NICER)



Choudhury et al., ApJL 971, L20 (2024);
Vinciguerra et al., ApJ 961, 62 (2024)
see also Miller et al. , Riley et al., ApJL 2019, 2021

TIDAL DEFORMABILITY



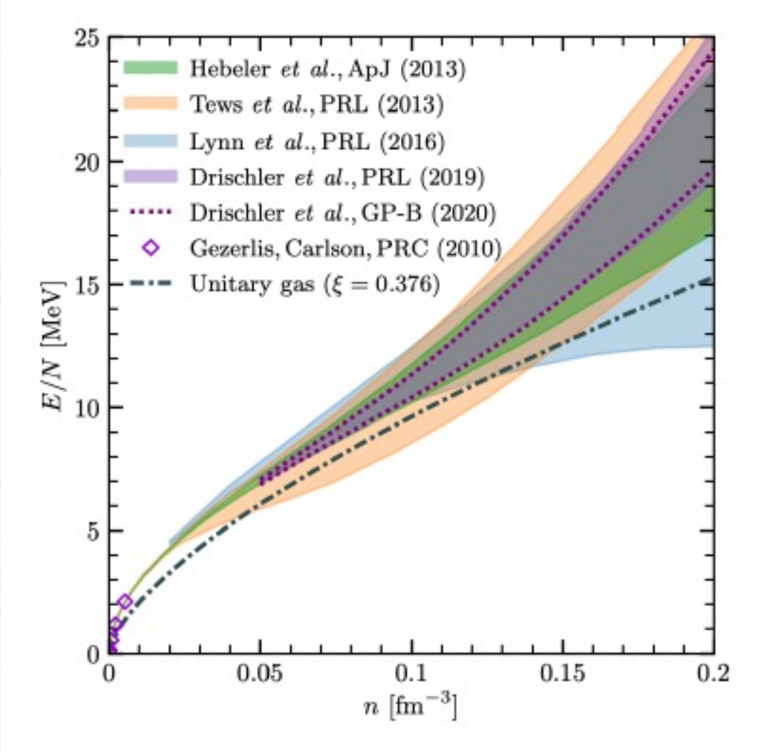
Abbott et al., PRX 9, 011001 (2019)

see talk by L. Suleiman, A. Bauswein

see also Koehn et al., arXiv:2402.04172 for a compilation

Constraints from nuclear physics: theory

PURE NEUTRON MATTER



Huth et al., Nature 606, 276 (2022)

- different many-body methods
- (relatively) low densities

→ "uncertainty band"

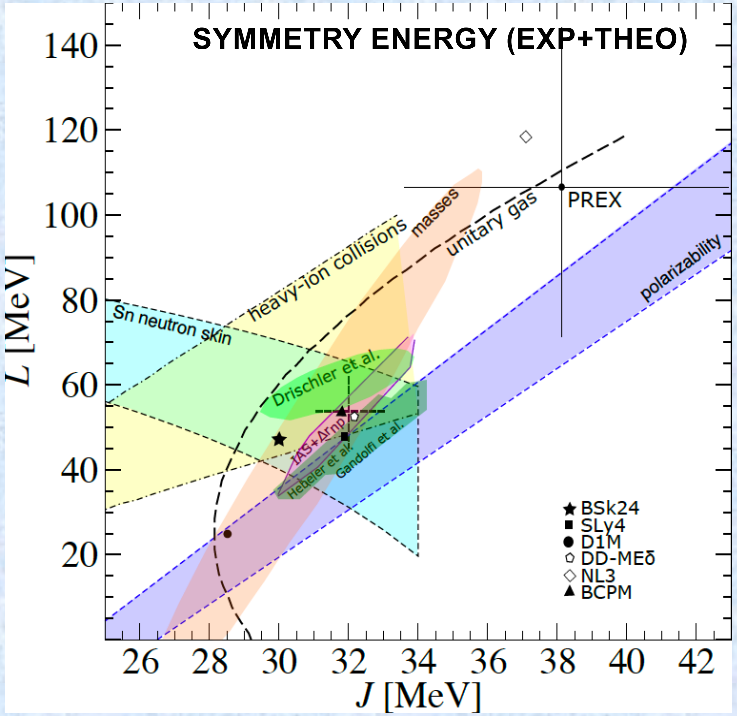
N.B.: ab-initio for symmetric matter much less constraining

→ Reasonable agreement of ab-initio (PNM) up to ~ saturation density
→ benchmark (for phenomenological models) & constraints

see also Oertel et al., Rev. Mod. Phys. 89, 015007 (2017), Burgio&Fantina, ASSL Springer 2018, Burgio et al., PPNP 120, 103879 (2021)

Constraints from nuclear physics: experiments

(1)



Gulminelli&Fantina, Nucl. Phys. News 31, 9 (2021);
Fantina&Gulminelli, J.Phys.Conf.Ser. 2586, 012112 (2023)

(2)

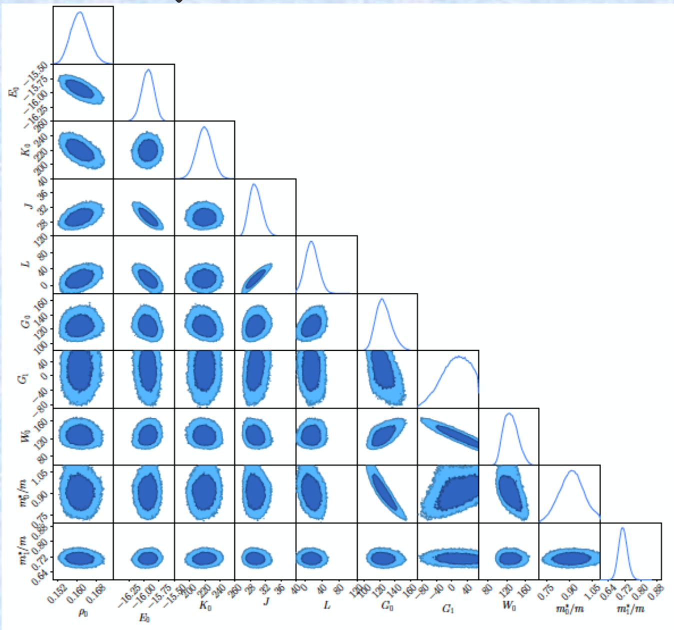
Ground-state properties			
	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]
^{208}Pb	1636.4 ± 2.0*	5.50 ± 0.05*	2.02 ± 0.50*
^{48}Ca	416.0 ± 2.0*	3.48 ± 0.05*	1.72 ± 0.50*
^{40}Ca	342.1 ± 2.0*	3.48 ± 0.05*	-
^{56}Ni	484.0 ± 2.0*	-	-
^{68}Ni	590.4 ± 2.0*	-	-
^{100}Sn	825.2 ± 2.0*	-	-
^{132}Sn	1102.8 ± 2.0*	4.71 ± 0.05*	-
^{90}Zr	783.9 ± 2.0*	4.27 ± 0.05*	-

Isoscalar resonances		
	E^{IS}_{GMR} [MeV]	E^{IS}_{GQR} [MeV]
^{208}Pb	13.5 ± 0.5*	10.9 ± 0.5*
^{90}Zr	17.7 ± 0.5*	-

Isovector properties			
	α_D [fm^3]	$m(1)$ [MeV fm^2]	A_{PV} (ppb)
^{208}Pb	19.60 ± 0.60	961 ± 22	550 ± 18
^{48}Ca	2.07 ± 0.22	-	2668 ± 113

Klausner et al., PRC 111, 014311 (2025)

parameter fit
through emulator



“compilation”
of results (”bands”)

observables directly included in model fit
→ nuc.-phy. informed prior

→ Constraints at “low” densities → low-order parameters
N.B.: model dependence of constrains

Crustal properties

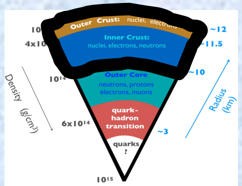
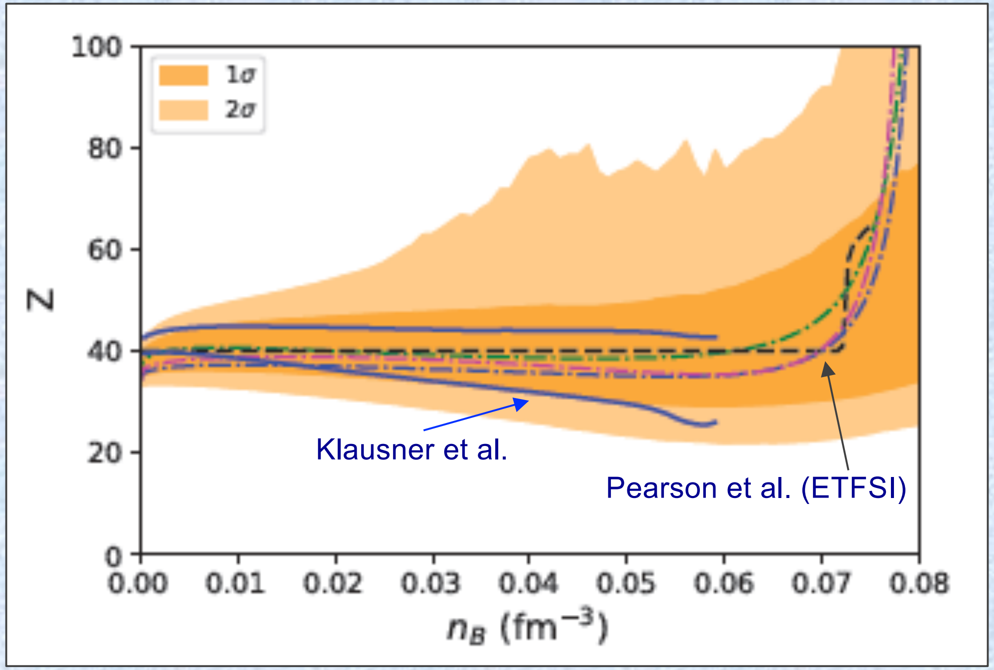


Image Credit: 3G Science White Paper

→ *important e.g. for pulsar glitches, NS cooling / transport properties*

COMPOSITION



Diverrès et al., in prep. (2025)

→ inclusion of nuclear phys. exp. observables constrains NS-crust properties

→ constraints on transport/elastic properties

see also Carreau et al., PRC 100, 055803 (2019), Balliet et al., ApJ 918, 79 (2021)

Crustal properties

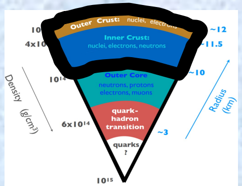
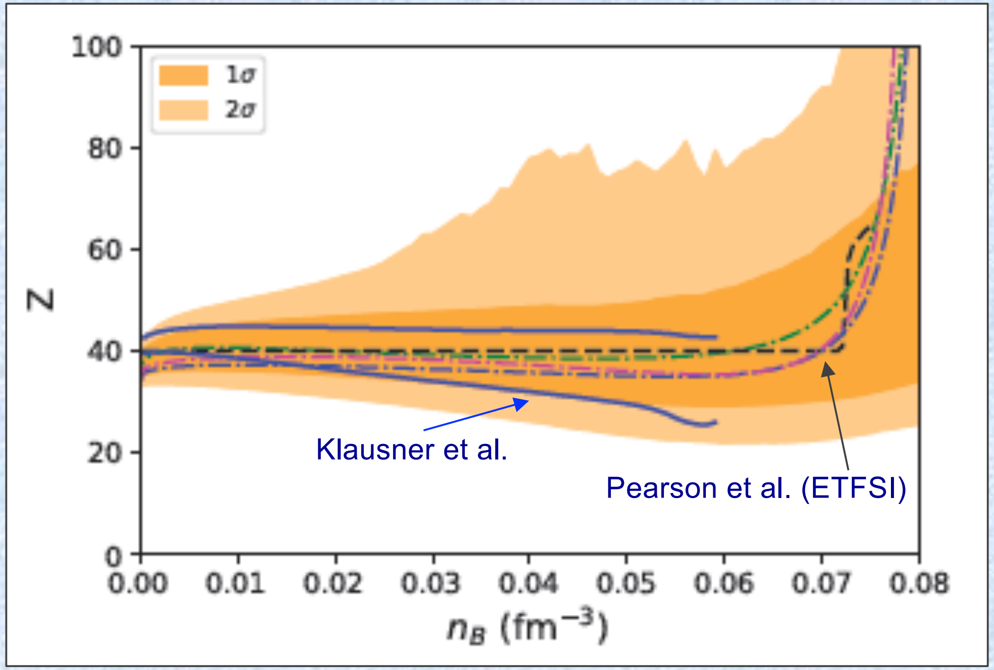


Image Credit: 3G Science White Paper

→ *important e.g. for pulsar glitches, NS cooling / transport properties*

COMPOSITION

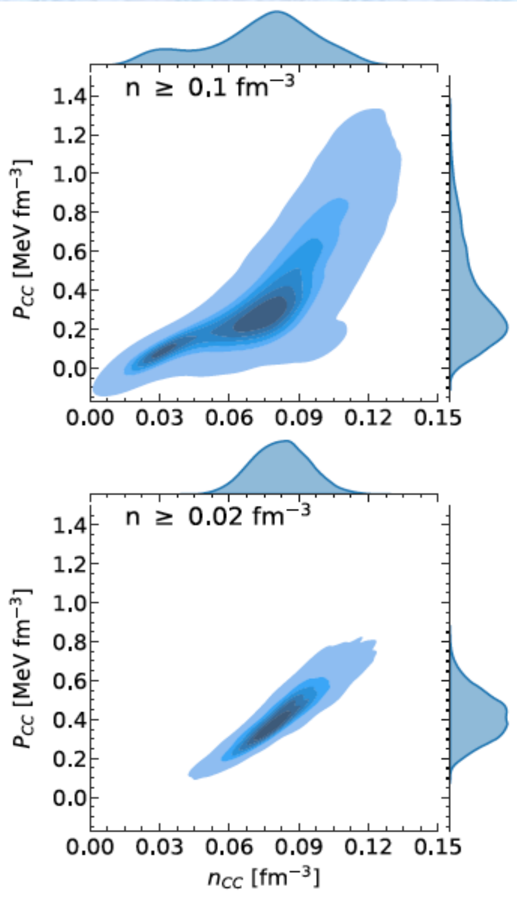


Diverrès et al., in prep. (2025)

→ inclusion of nuclear phys. exp. observables constrains NS-crust properties

→ constraints on transport/elastic properties

CC TRANSITION

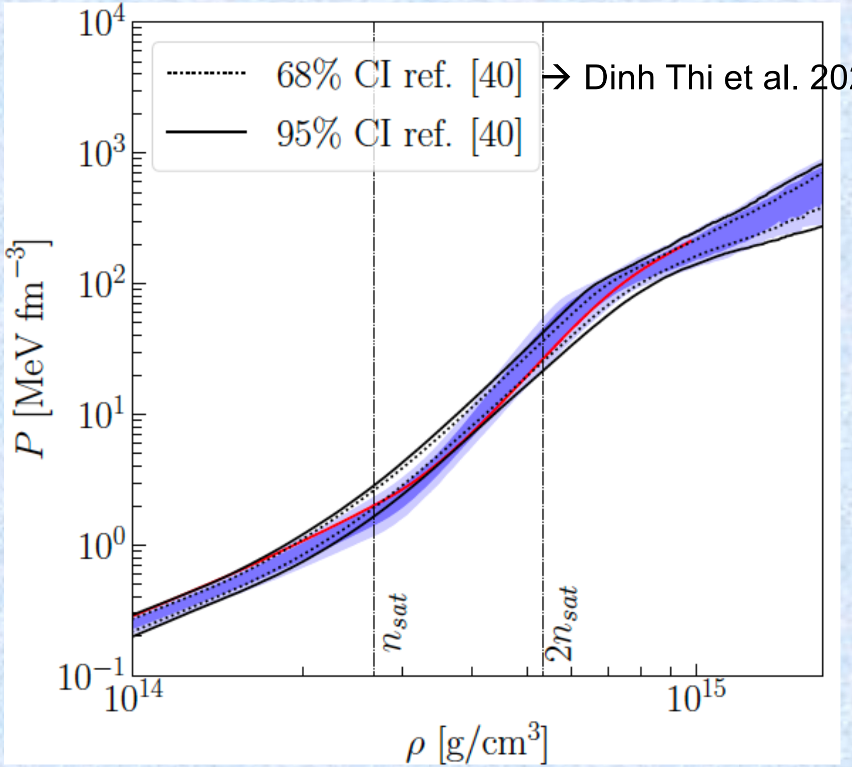


Dinh Thi et al., A&A 654, A114 (2021); EPJA 57, 296 (2021)

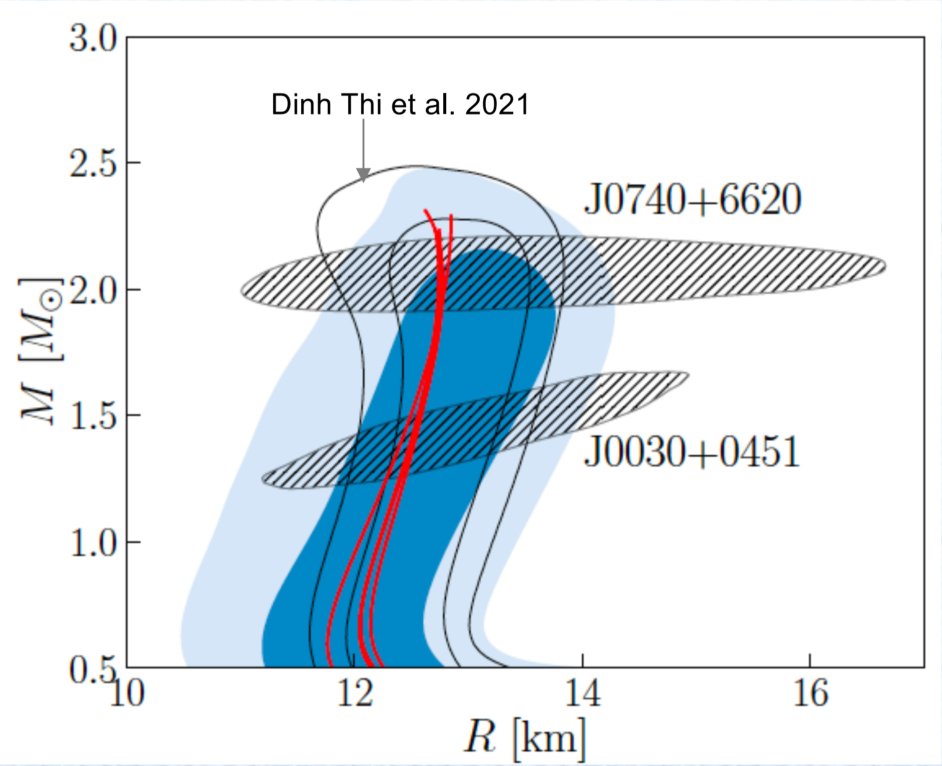
→ importance of low-density EoS

N.B.: improvement of meta-model at very low density: Burrello et al., arXiv:2506.05603

EoS and observables



→ Dinh Thi et al. 2021



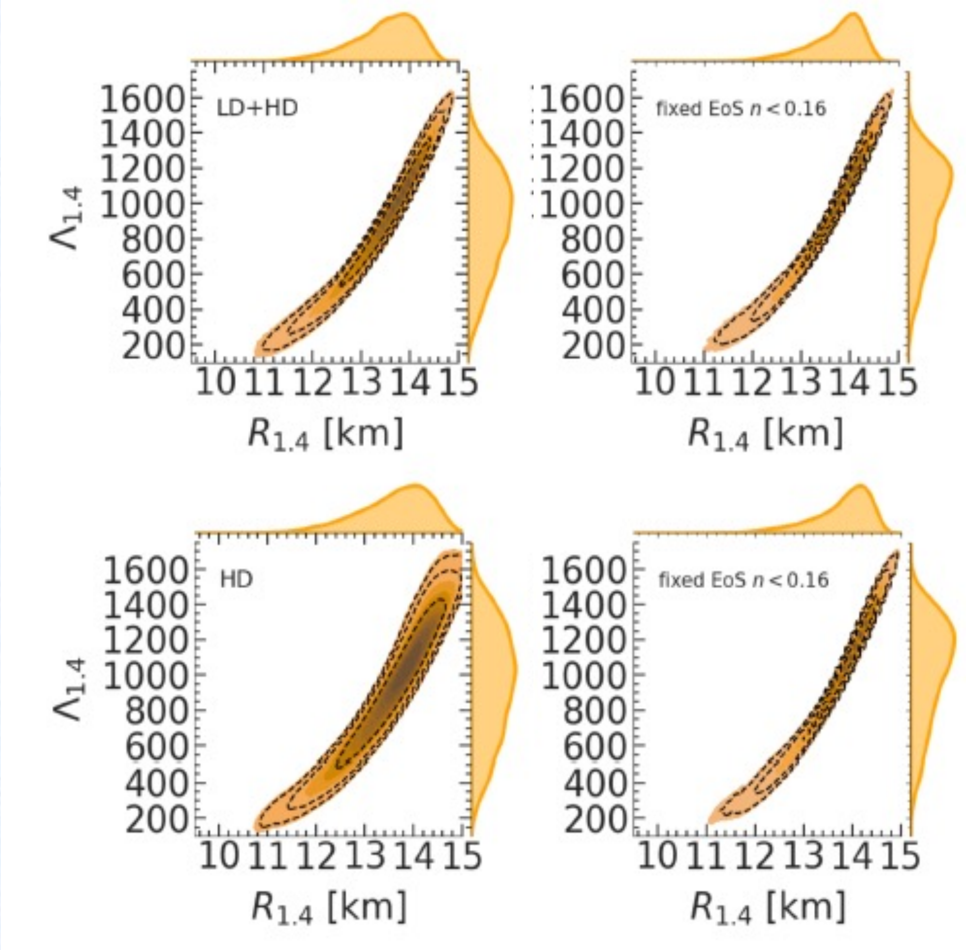
Klausner et al., arXiv2505.16929 (see also Dinh Thi et al., Universe 2021, Fantina&Gulminelli, J. Phys. 2023)

→ inclusion of nuc.-phys. exp. observables effect on EoS and M-R: lower R (softer EoS) and thinner 1σ predictions for low-mass NSs

→ nucleonic hp compatible with observations → observations not yet constraining enough!

similar conclusions in Lim&Holt, EPJA 2019; Malik et al., ApJ 2022

Effect of the (non-unified) crust



Davis et al., A&A 687, A44 (2024); Davis et al., EPJA 61, 120 (2025); Fantina & Gulminelli, PoS QNP2024, 153 (2025)

→ use of unique low-density EoS does not change much averages (~ few %)

→ ok for current GW detectors, but next generation ?

(see also Gamba et al., Class. Quant. Grav. 37, 025008 (2020) → ~ 3%)

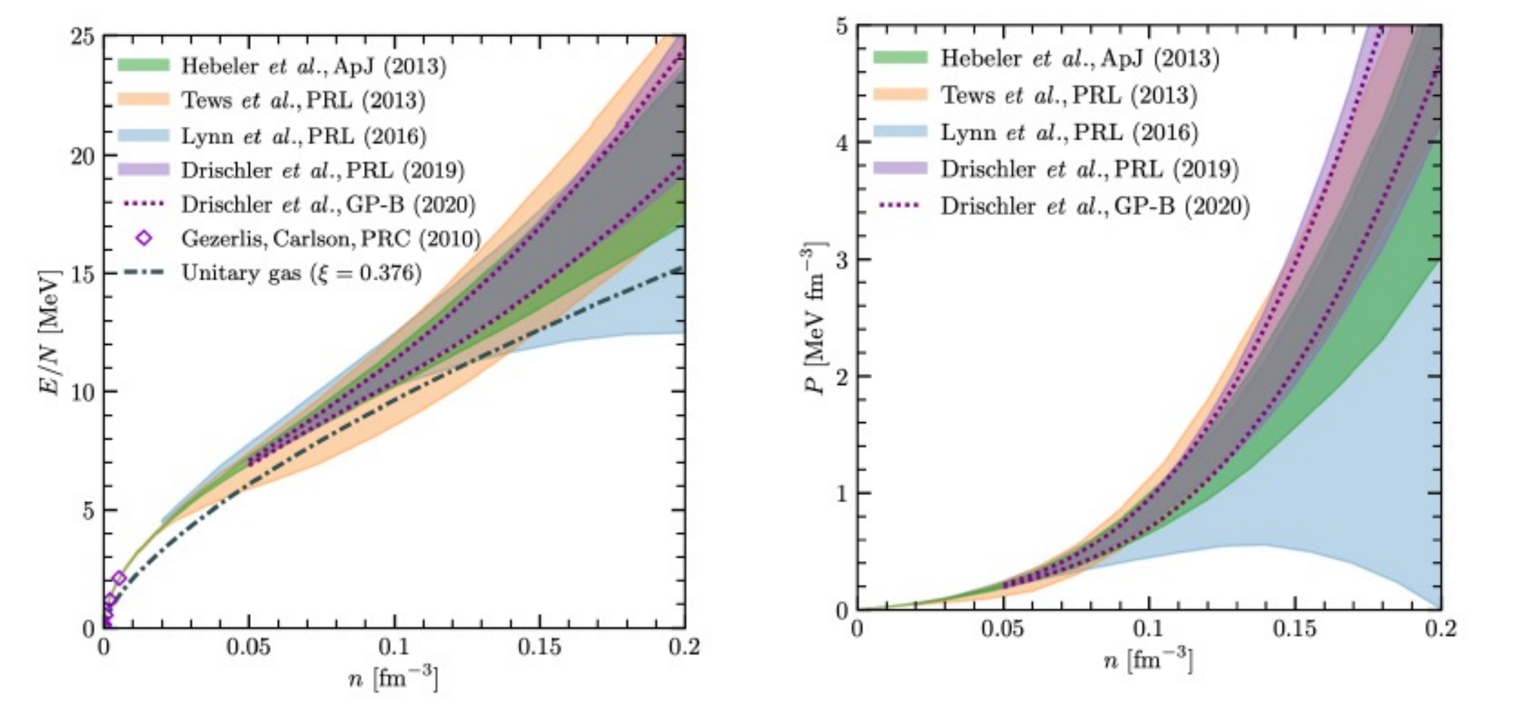
→ use of unique crust underestimates uncertainties

→ **CUTER code** to reconstruct a *thermodynamically consistent and unified* low-density EoS from a given (high-density) β-equilibrium EoS (non necessarily based on a nuclear model) (v1 and v2 publicly available on Zenodo)

CUTER = Crust Unified Tool for Equation-of-state Reconstruction

How to discriminate models? (theory)

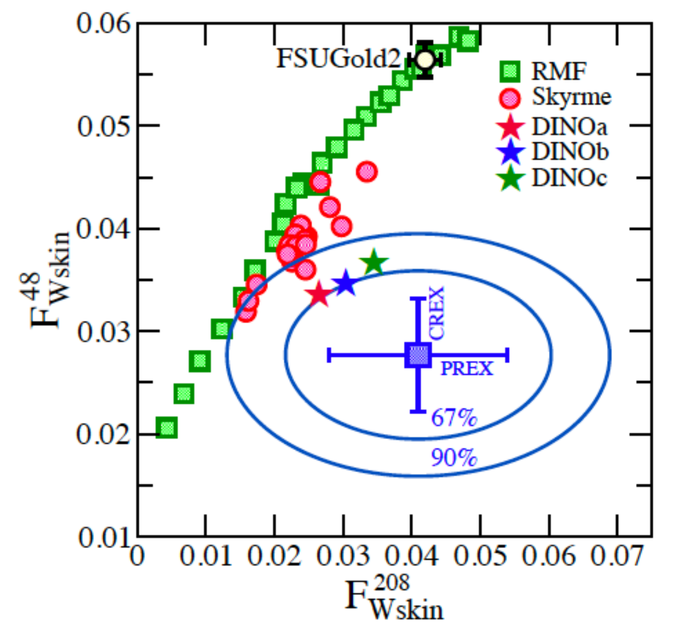
- ***Nuclear physics (theory + experiments)*** → information up to ~ 1.5 – 2 n_{sat}
  - *Ab-initio prediction of derived quantities (not only E/A, but also P, etc.)*
    → propagation of uncertainties
    → constraints on more phenomenological functionals



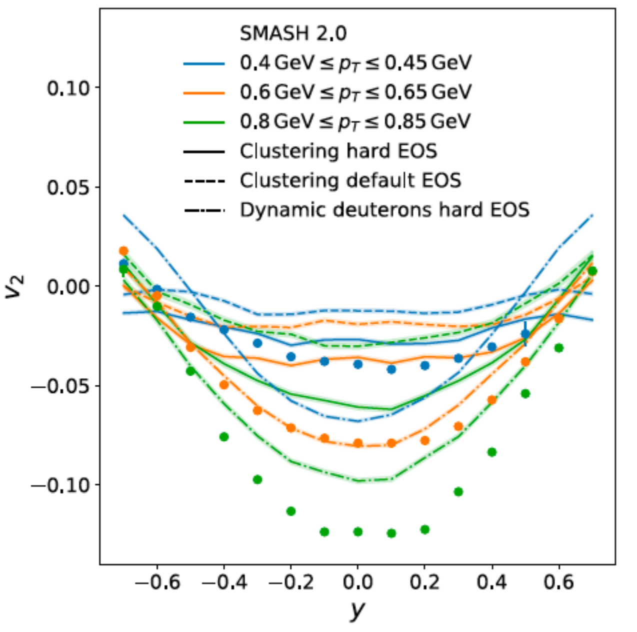
Hebeler, talk @MICRA2023; Huth et al., Nature 606, 276 (2022)

How to discriminate models? (exp)

- ***Nuclear physics (theory + experiments)*** → information up to ~ 1.5 – 2 n_{sat}
  - *reduced error bar in neutron skin measurements (e.g. PREX/CREX)*
    → constraints on low-order parameters in isospin sector
  - *constraints at high density (e.g. HADES collaboration, transport model vs data)*
    → constraints on higher order parameters in isoscalar sector



Reed et al., PRC 109, 035803 (2024)

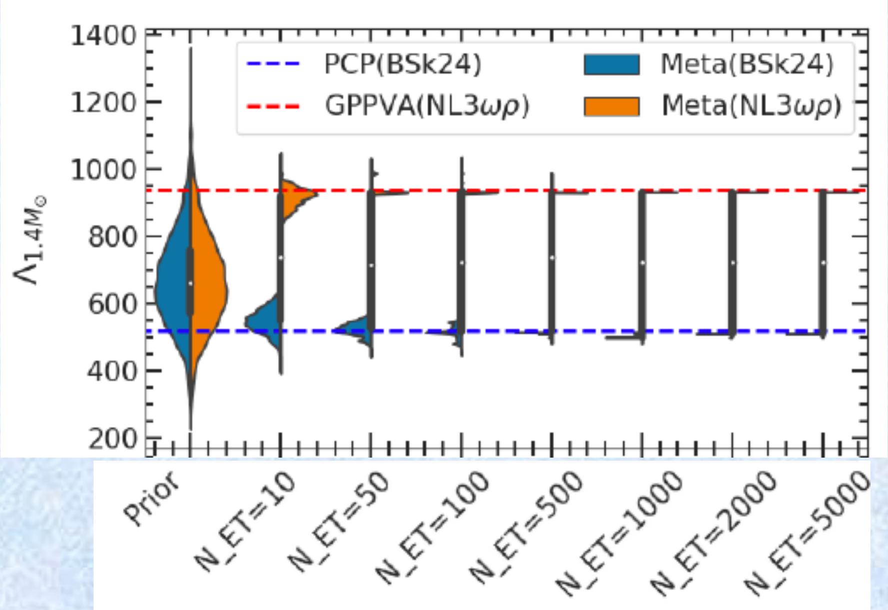


Mohs et al., PRC 105, 034906 (2022)

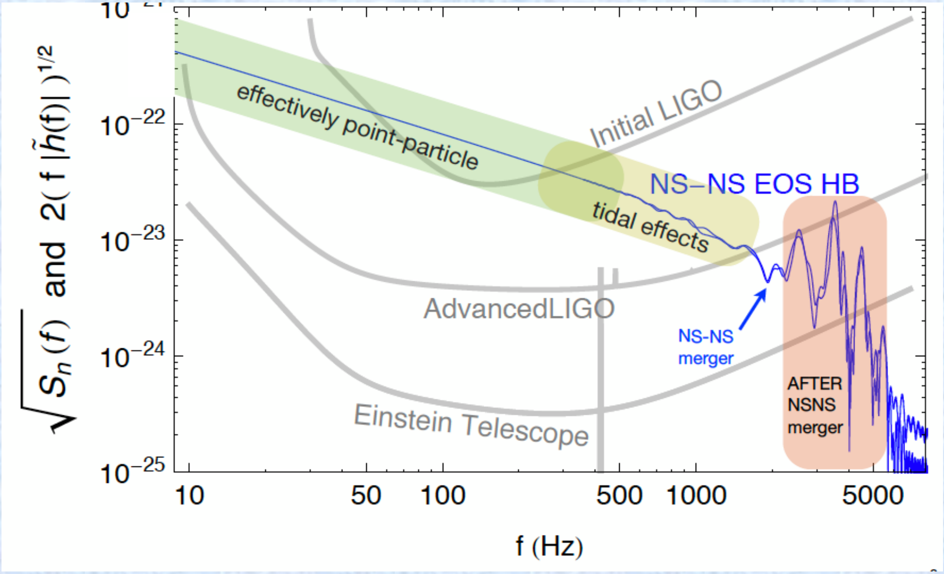
→ Better extrapolation of models

How to discriminate models? (astro 1)

- more and more precise data (e.g. M, R, Λ, ...) – or even a "smoking gun" observation!
- more sensitive detectors → new generation (ET, CE) → post-merger



Iacovelli et al., PRD 2023 (also Mondal et al., MNRAS 2023)



Read, CGWAS lecture (2015)

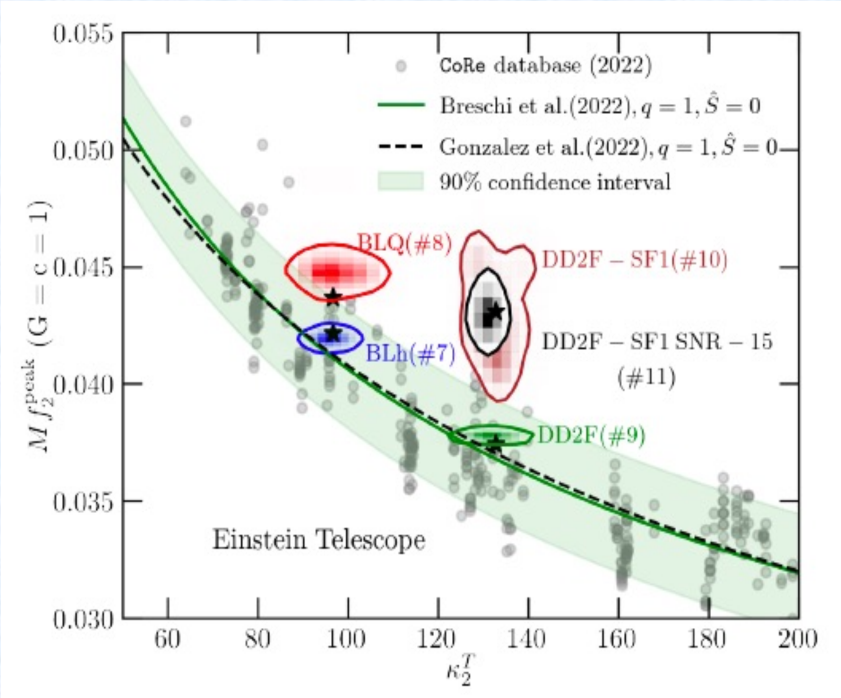
- More reliable prediction / interpretation of astrophysical observations
- Better knowledge of dense matter in compact stars
  → *Phase transition to deconfined matter (quarks, …) ?*

see talk by L. Suleiman

How to discriminate models? (astro 2)

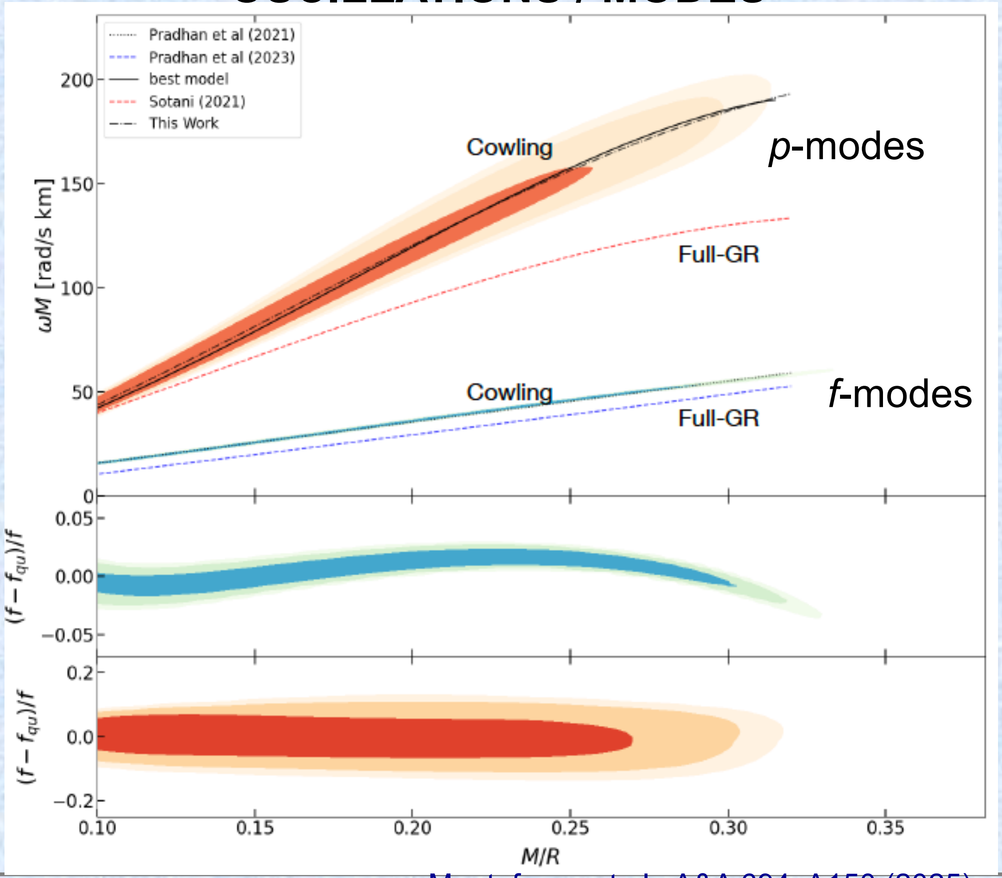
- more sensitive detectors → new generation (ET, CE)
  → possible identification of properties from (quasi-)universal relations (or discrepancy from them)

PHASE TRANSITION → POST-MERGER



Prakash et al., PRD 109, 103008 (2024); see also e.g. Bauswein et al. 2019, Kochankovksi et al. 2025

OSCILLATIONS / MODES



Montefusco et al., A&A 694, A150 (2025)

see talk by A. Bauswein

Conclusions & open questions

- ❖ Nuclear physics + astrophysics → constraints on EoS but still hard to discriminate
  → nucleonic hp compatible with observations, no (dis)proof of exotic matter to now

  - ✓ need of (microscopic) reliable theoretical model + experimental data to calibrate models
  - ✓ need of (more precise / numerous) astrophysical observations + post-merger?
    → phase transition, modes (?)

- ❖ **`CUTER`** tool for consistent crust EoS reconstruction available

---

- ➢ Extrapolation from raw data → **model dependence of the constraints**

- ➢ Lab. exper. mostly "low" density (~ saturation density), low *T* probed
  → **extrapolation to astro conditions (high *T* and density, asymmetry, charge neutral) ?**

- ➢ Uncertainties in high-density EoS → **blurring of different effects ?**

- ➢ Astro simulations vs microphysics inputs → **effects of microphysics inputs in astro modelling** ?
  → consistent implementation
  → systematic studies / bayesian analysis → uncertainties, implementation of consistent probab. distributions
  → static vs dynamical properties

Thank you