

Strong cosmic censorship in de Sitter spacetime

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Based on:

OD, Felicity Eperon, Harvey Reall, Jorge Santos

1801.09694 (PRD) & 1808.02895 (JHEP) & 1808.04832 (CQG)

The vacuum of the Universe IV,

Barcelona

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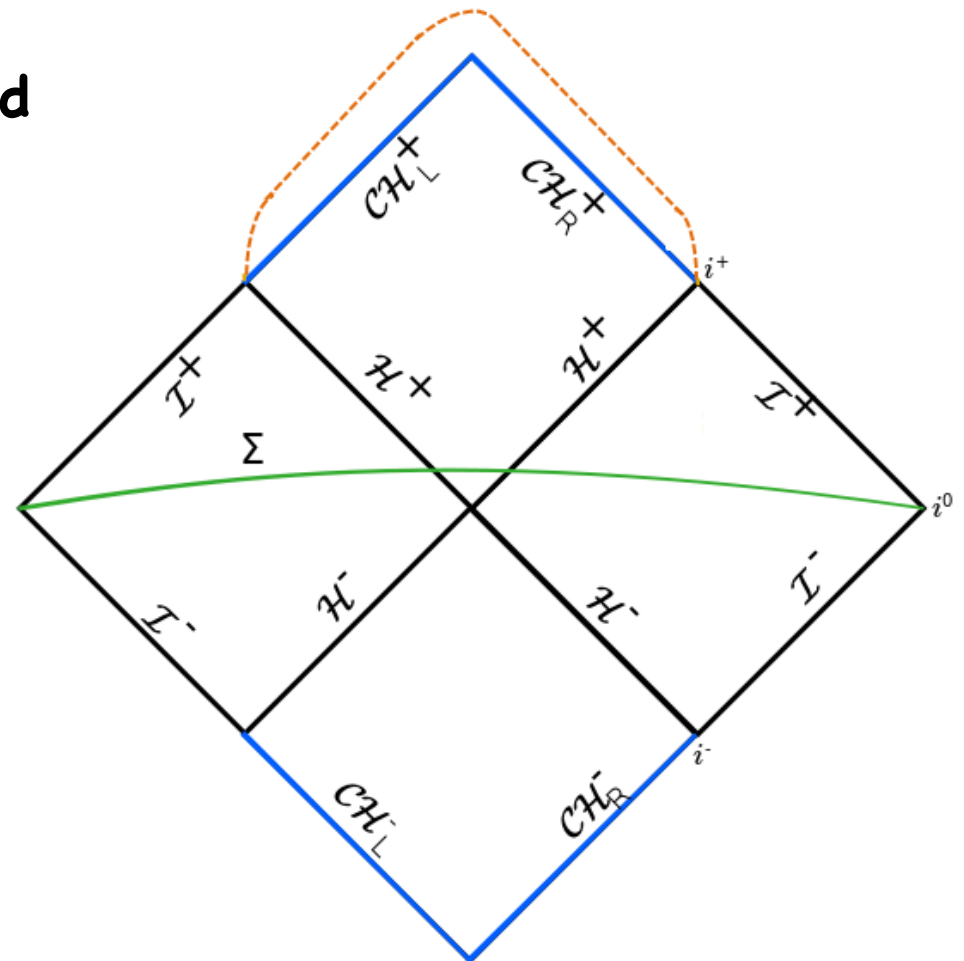
Outline:

1. Strong Cosmic censorship with $\Lambda = 0$
2. Strong Cosmic censorship with $\Lambda > 0$
3. Quantum effects

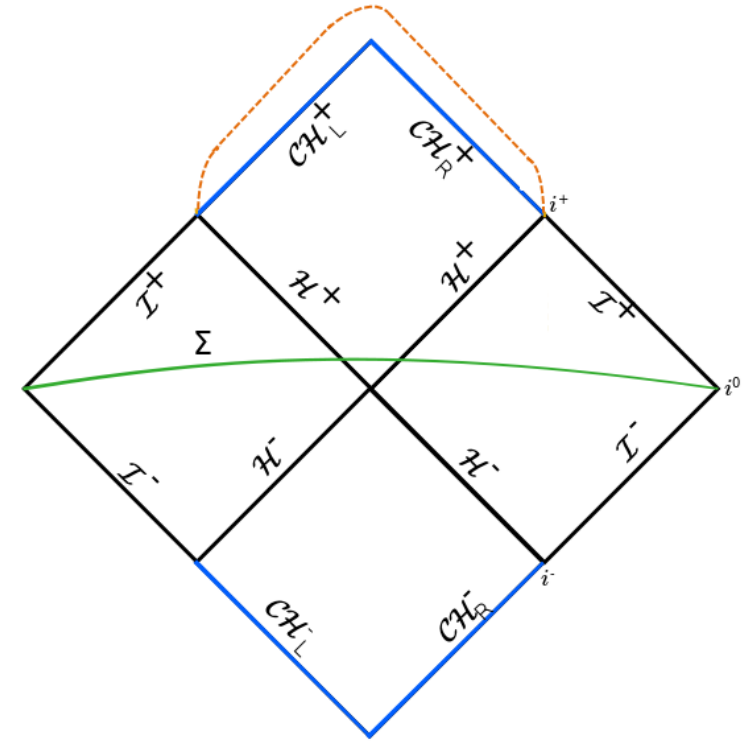
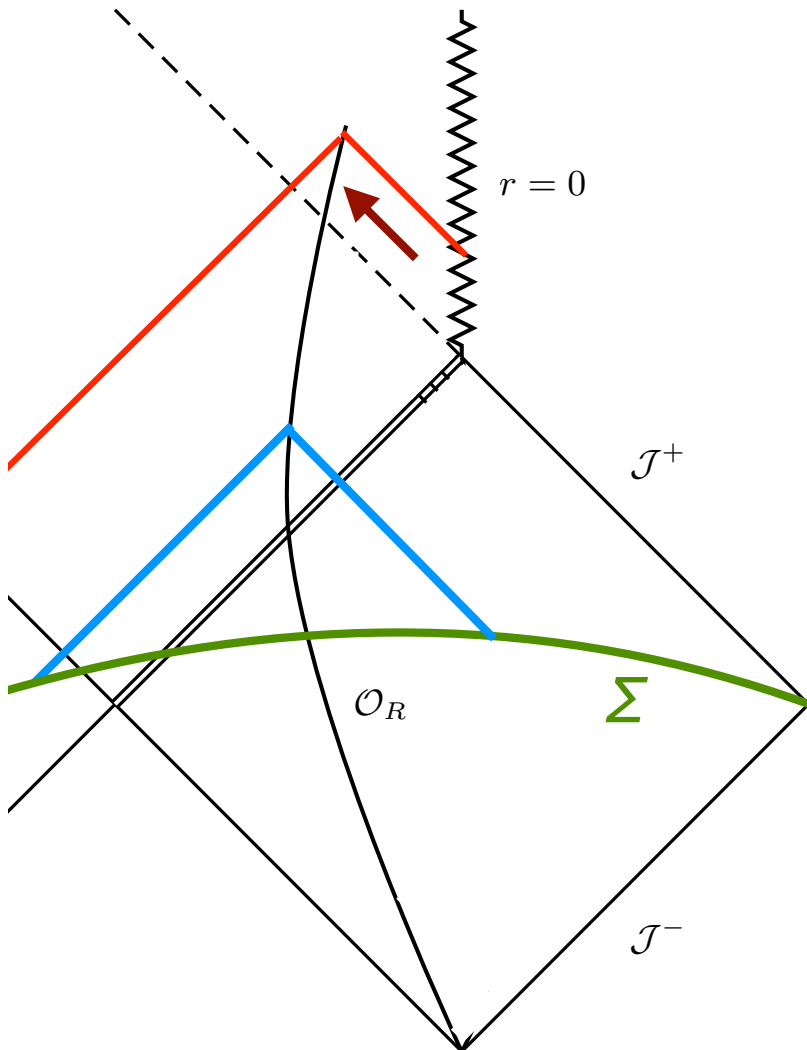
1. Strong Cosmic censorship with $\Lambda = 0$

→ Black holes with $\Lambda = 0$

- Consider **Einstein-Maxwell theory with $\Lambda = 0$** .
- **Reissner-Nordström** black hole (charged non-rotating BH).
Kerr BH (rotating, uncharged BH)
- Both solutions can be smoothly extended
across a horizon H^+ inside the BH.



- Inner horizon is a **Cauchy horizon** $CH_{L,R}^+$:
a **boundary** to the region of spacetime in which
physics can be predicted from
initial data prescribed on a surface Σ .

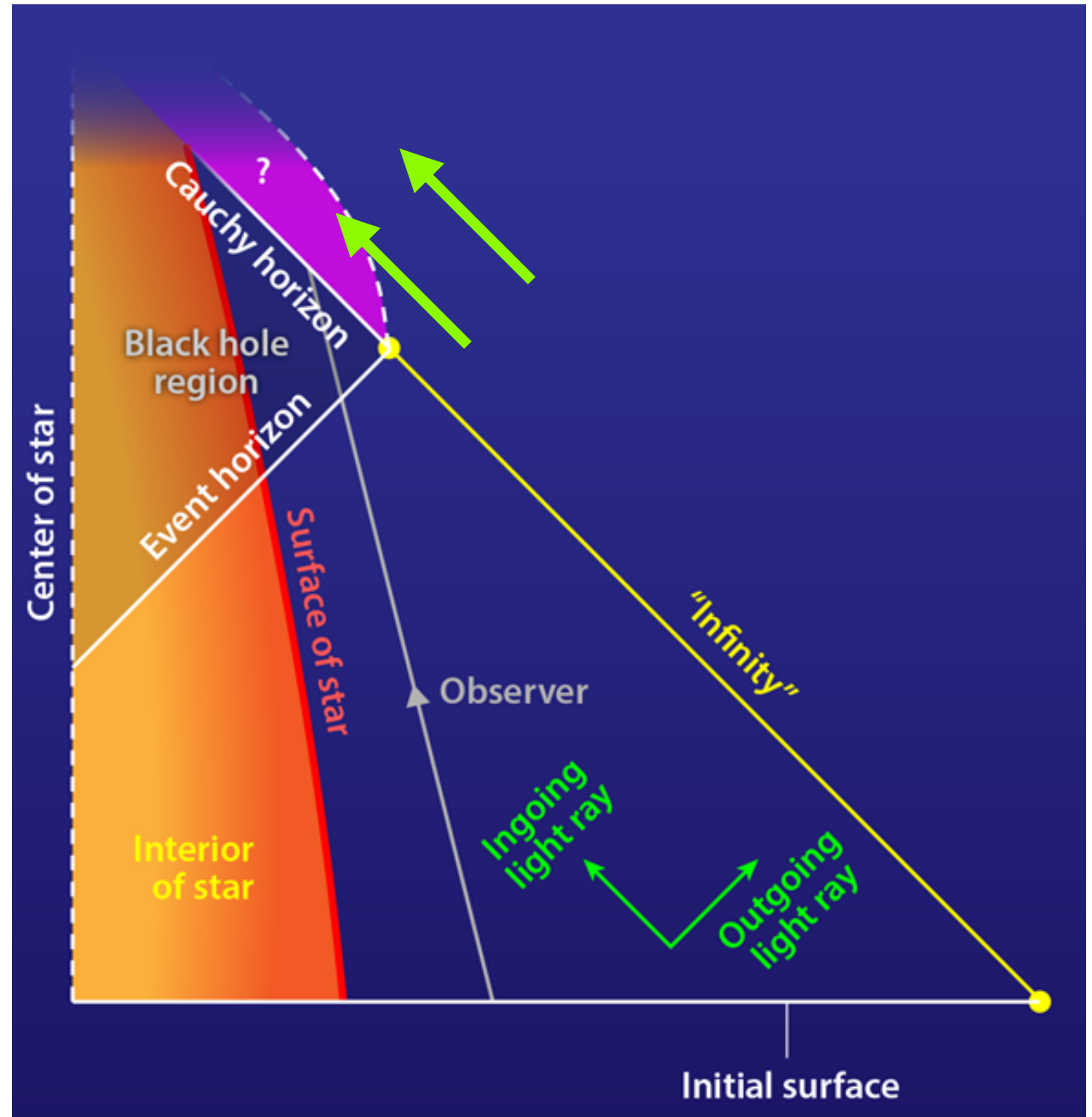
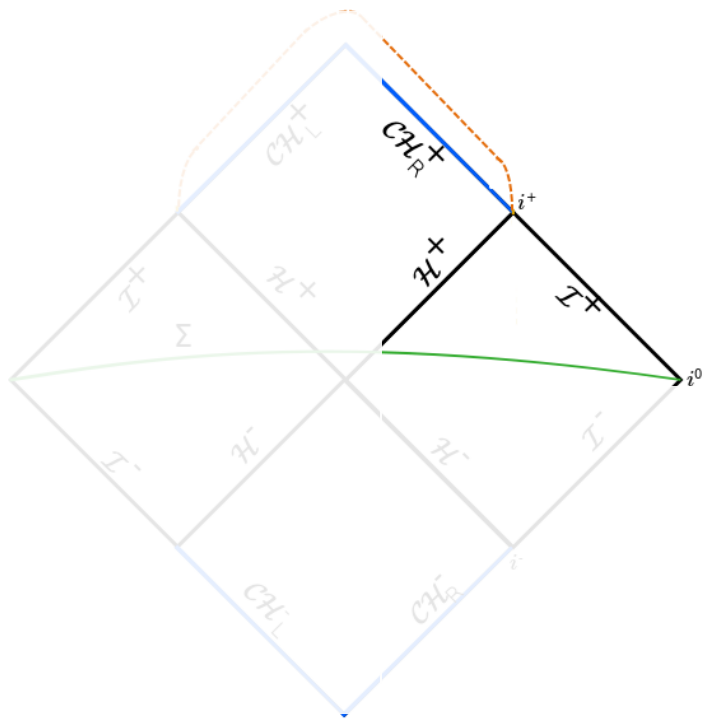


- Solution **beyond** the Cauchy horizon
is **NOT** determined by initial data on Σ .

There are **infinitely** many ways of smoothly
extending the solution **across** the **Cauchy** horizon.

- Interested in the ``*right*`` Cauchy horizon CH_R^+

because **early time section** of this is expected to be present in a **BH formed from collapse**.



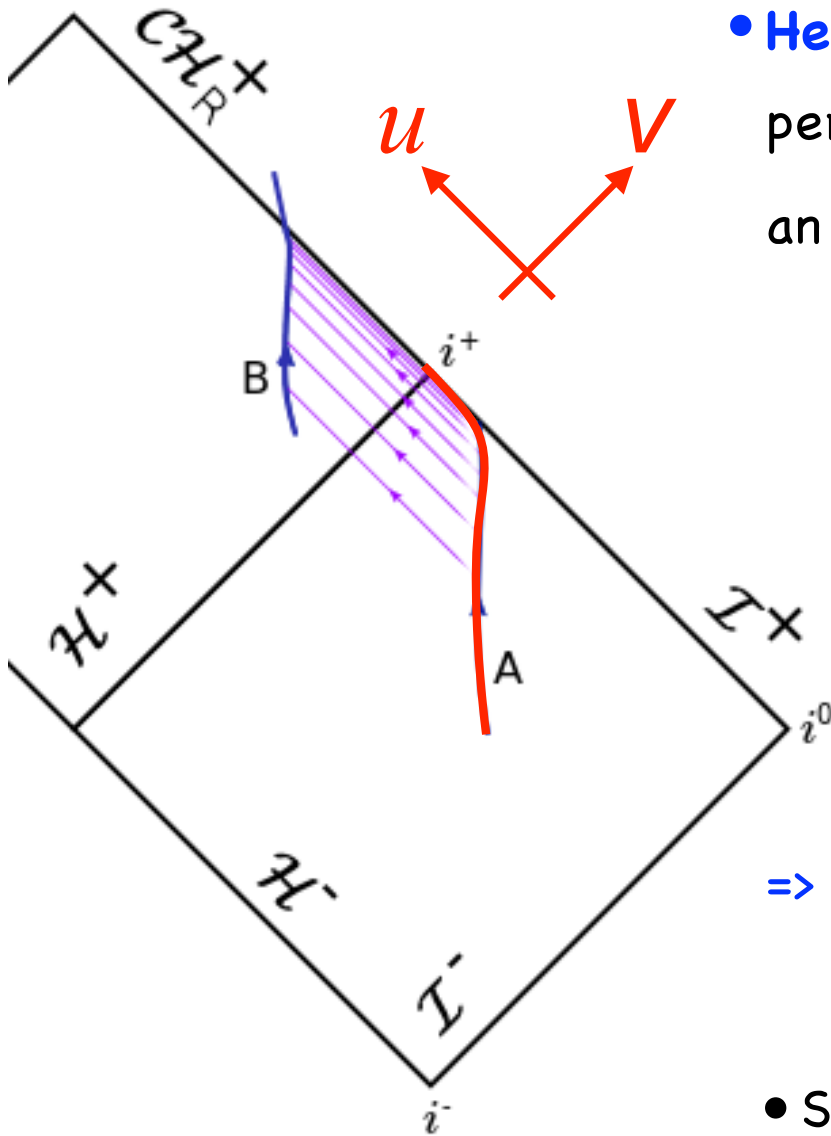
→ we have a problem ...

- **Newtonian physics, Maxwell theory, Yang-Mills, Schrödinger equation:**
solution **determined globally from initial data:** can predict future

- **BUT in GR we can smoothly extend a solution**
into a region of spacetime which canNOT be predicted from initial data !

This is a **worrying failure of determinism** in physics.

- Penrose: Cauchy horizon should be unstable! This would restore predictability!



- Heuristics:

perturbations entering from outside the BH experience an **infinite blue-shift** at the right Cauchy horizon.

$$\frac{\nu_B}{\nu_A} = \frac{\Delta\tau_A}{\Delta\tau_B} \sim e^{\kappa_- v} \quad \text{as } \begin{array}{l} r \rightarrow r_- \\ v \rightarrow \infty \end{array}$$

$v = t + r_*$: ingoing Eddington-Finkelstein time

=> **infinite energy densities** observed at the CH_R^+

- Suggests there will be a

large backreaction at the Cauchy horizon,

perhaps **causing** it to be replaced by a **singularity**

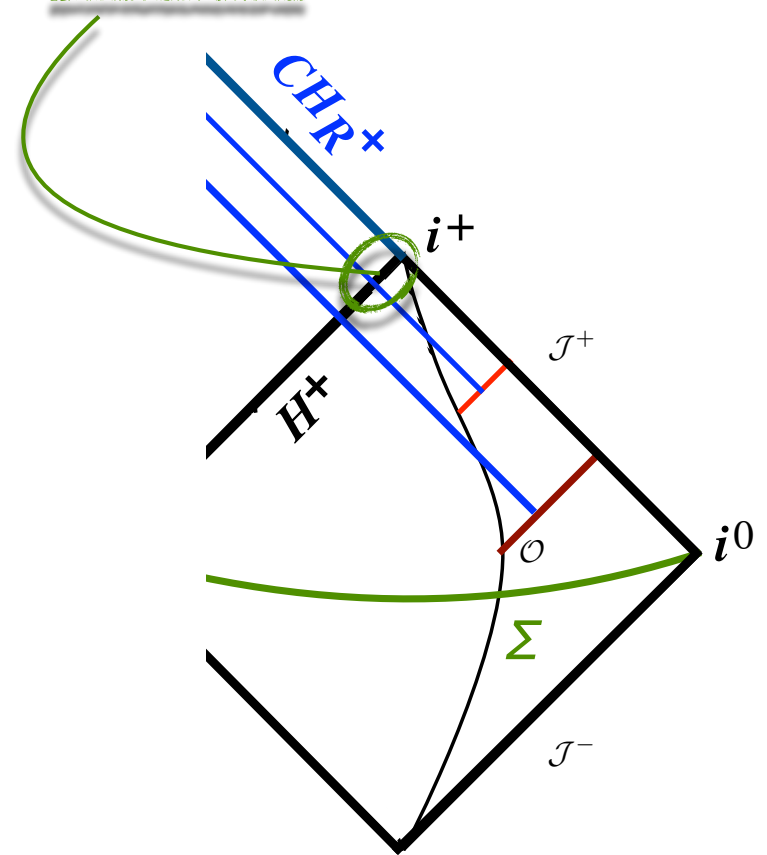
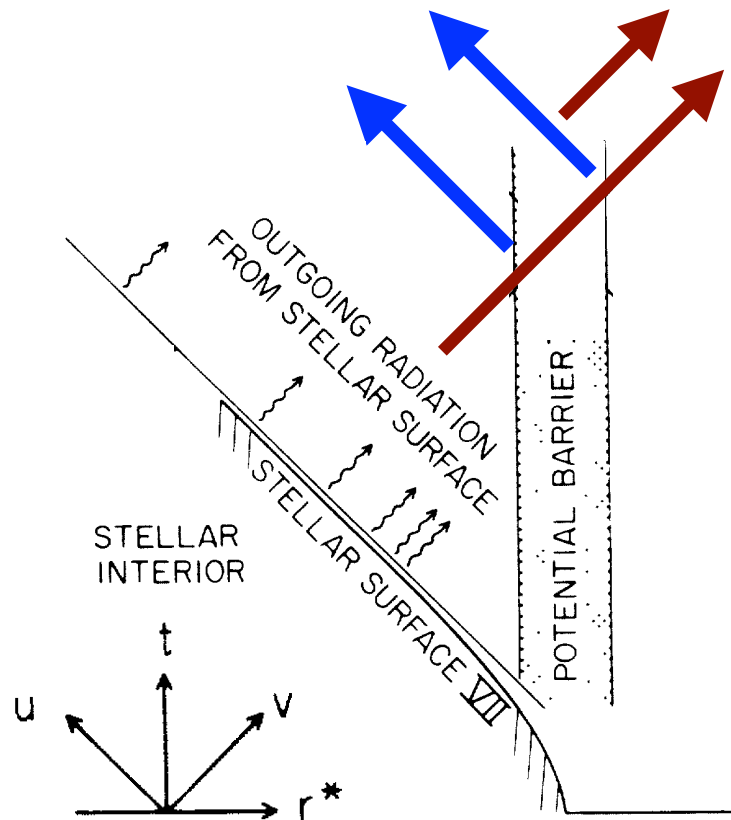
→ Strong cosmic censorship conjecture (Penrose 70's)

[aka Very SCC or C^0 version]

- Consider complete, asymptotically flat initial data for the Einstein–Maxwell eqns.
Then **generically** (generic initial data) the resulting **solution cannot be extended (continuously) across a Cauchy horizon**
(the maximal Cauchy development of a two-ended Σ is inextendible)
- If correct, this conjecture **restores predictability**
without invoking poorly understood physics
(e.g. backreaction of quantum effects).
- NOT related to Penrose's **weak cosmic censorship** conjecture:
“naked singularities don't form from collapse”

→ Evidence for the conjecture: linear

- Consider **linear** perturbations of RN BHs:
massless scalar (or linearized gravito-electromagnetic perturbations).
- Take **initial data compactly supported** on Σ .
- Perturbations outside BH exhibit **power-law decay** (“late time tails”) [Price 72]
- This is **slow enough** to **trigger the blue-shift instability** at the Cauchy horizon
- Inverse-power law tail of grav. collapse provides **initial data** for internal problem



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- **Result:**

- **gradient of scalar diverges** at Cauchy horizon.
- **Energy density** measured by **observer crossing CH** is **divergent**:

$$\rho = T_{\mu\nu} u^\mu u^\nu$$

=> Expect large backreaction on metric.

[McNamara (1978) Chandrasekhar & Hartle (1982)]

→ Evidence for the conjecture: nonlinear

[Poisson-Israel (1990), Ori (1991)]

“Null dust” model (charged Vaidya with infalling null dust):

- Backreaction causes the invariant “Hawking mass” to **diverge** at the Cauchy horizon (“**mass inflation**”).
 - ⇒ Energy density $\rho = T_{ab} u^a u^b$ measured by free-falling observer also **diverges**.
 - ⇒ Cauchy horizon becomes **singular**, in agreement with conjecture.
- **However:** the metric can be continuously extended across Cauchy horizon (so the Cauchy horizon can still be defined).
 - ⇒ the singularity at CH_R^+ is **null** at least at “early time”.

(Maybe)
NOT what Penrose
had in mind !

→ Rigorous results: Kerr

Kerr: [Dafermos & Luk (2017)]

if (nonlinear) gravitational perturbations decay along the H^+

at the expected inverse power-law rate

then the **perturbed solution can be continuously extended**

across a Cauchy horizon at early time.

→ Crossing the Cauchy horizon

- Summary so far:

a perturbed RN BH solution can be continuously extended across a CH

(i.e. Penrose's C^0 version of SCC is false)

but the extension is not in C^2 , i.e., the CH is a curvature singularity.

=> Great! **Predictability is restored!**

- Not so fast! Ori (1991) ... **a twist in the story:**

– Consider an (extended) observer approaching the CH.

The **total tidal distortion** felt by this observer can **remain bounded!**

– So **what happens** to such an observer?

– To answer this question, one must **specify:**

what is the **matter field content** of the observer & their **EOM**.

If these EOM still “**make sense**” at the CH then we still have a **problem with predictability!** (i.e. CH has singularity but **observer survives its crossing**)

→ Weak solutions

- KEY question: What is the minimal regularity required to make sense of EOM?
- Can make sense of solutions **less regular than C^2** as weak solutions.

Example: **shocks** in compressible perfect fluid.

- Scalar field Φ : $\square\Phi = 0$.
- Treat this as a **1st order** perturbation, sourcing a **2nd order** metric pert $h_{\mu\nu}^{(2)}$. One has:

$$\mathcal{L} h_{\mu\nu}^{(2)} = 8\pi T_{\mu\nu}[\Phi] \quad \sim \quad \nabla^2 h_{\mu\nu}^{(2)} = 8\pi T_{\mu\nu}[\Phi]. \quad (1)$$

Well defined if $h_{\mu\nu}^{(2)}$ **twice continuously differentiable** (of class C^2) and Φ of class C^1 .

- Multiply (1) by a smooth, compactly supported, symmetric tensor, $\psi^{\mu\nu}$ & integrate by parts:

$$\int d^4x \sqrt{-g} \left(-\nabla h_{\mu\nu}^{(2)} \nabla \psi^{\mu\nu} - 8\pi \psi^{\mu\nu} T_{\mu\nu} \right) = 0 \quad (2)$$

Keypoint: Can have Φ , $h_{\mu\nu}^{(2)}$ obeying (2) (if C^1) but, **if $h_{\mu\nu}^{(2)}$ not C^2 , (1) is not obeyed**

- If (2) is obeyed for *any* $\psi^{\mu\nu} \implies$ we have a weak solution of **original** Einstein eqn (1)

→ Weak solutions

- KEY question: What is the minimal regularity required to make sense of EOM?
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Keypoint: Can have $\Phi, h_{\mu\nu}^{(2)}$ obeying (2) (if C^1) but, if $h_{\mu\nu}^{(2)}$ not C^2 , (1) is not obeyed

- If (2) is obeyed for *any* $\psi^{\mu\nu} \implies$ we have a weak solution of original Einstein eqn (1)
- EOM (2) “makes sense”, if terms involving Φ are **finite**

\implies **require that Φ belongs to Sobolev space H_{loc}^1** :

space of functions Φ s.t. for any smooth compactly supported function ψ ,


$(\hat{\Phi}^2 + \partial_\mu \hat{\Phi} \partial_\mu \hat{\Phi})$ is integrable (where $\hat{\Phi} \equiv \psi\Phi$).

locally square integrable:
 $\psi\Phi$ is square integrable

→ Christodoulou's formulation of SCC

- Criterion for weak solutions of the full nonlinear **vacuum** Einstein equation:

A **weak solution** of the Einstein equation


$$[T[\Phi] \rightarrow T[(h^{(1)})^2, (\partial h^{(1)})^2]$$

must have **locally square integrable** (H^1_{loc}) **Christoffel symbols** in some chart.

- **Christodoulou's version of the SCC conjecture (2009):**



generically, it is **not** possible to **extend** the maximal Cauchy development
across the Cauchy Horizon

as a **weak metric solution of EOM** (\Rightarrow also not C^2)

- If correct then generically there is **no way of extending** beyond the CH
in such a way that **(at least the weak version of) EOM are satisfied** there
 \Rightarrow predictability restored!

- This version of the conjecture is **believed** to be **true** with $\Lambda = 0$.

→ Summary so far ($\Lambda = 0$)

- Different SCC versions distinguished by **smoothness level** required at Cauchy CH_R
- In order of **decreasing strength**:
 - **C^0 version (Very SCC)**: generically no continuous extension exists across **CH**
=> singularity exists before Cauchy horizon forms.

 - **Christodoulou version**: generically no weak extension (with H^1_{loc} Christoffel symb.)
=> Cauchy surface is a weak null singularity.

 - **C^2 version**: generically no C^2 extension.
- For **Einstein-Maxwell(-massless scalar) theory**, with $\Lambda = 0$:
 - **C^0 version** is **false** [proof by Christodoulou; Dafermos-Luk]
 - **Christodoulou version** **believed** to be **true** [proof for Einstein-scalar: Dafermos-Rothman; Luk-Oh]
 - Strong evidence that **C^2 version is true**.

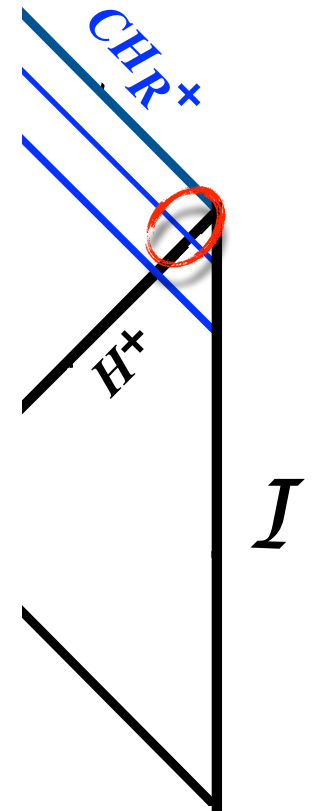
2. Strong Cosmic censorship with $\Lambda > 0$

→ The cosmological constant: $\Lambda < 0$ (AdS)

- $\Lambda < 0$: perturbations outside an AdS_4 BH decay very slowly ($\sim 1/(\log t)^\#$),
i.e. more slowly than power-law decay of $\Lambda=0$ [Holzegel-Smulevici (2013)].

- This is likely to **strengthen** the **instability**
of the **Cauchy horizon**.

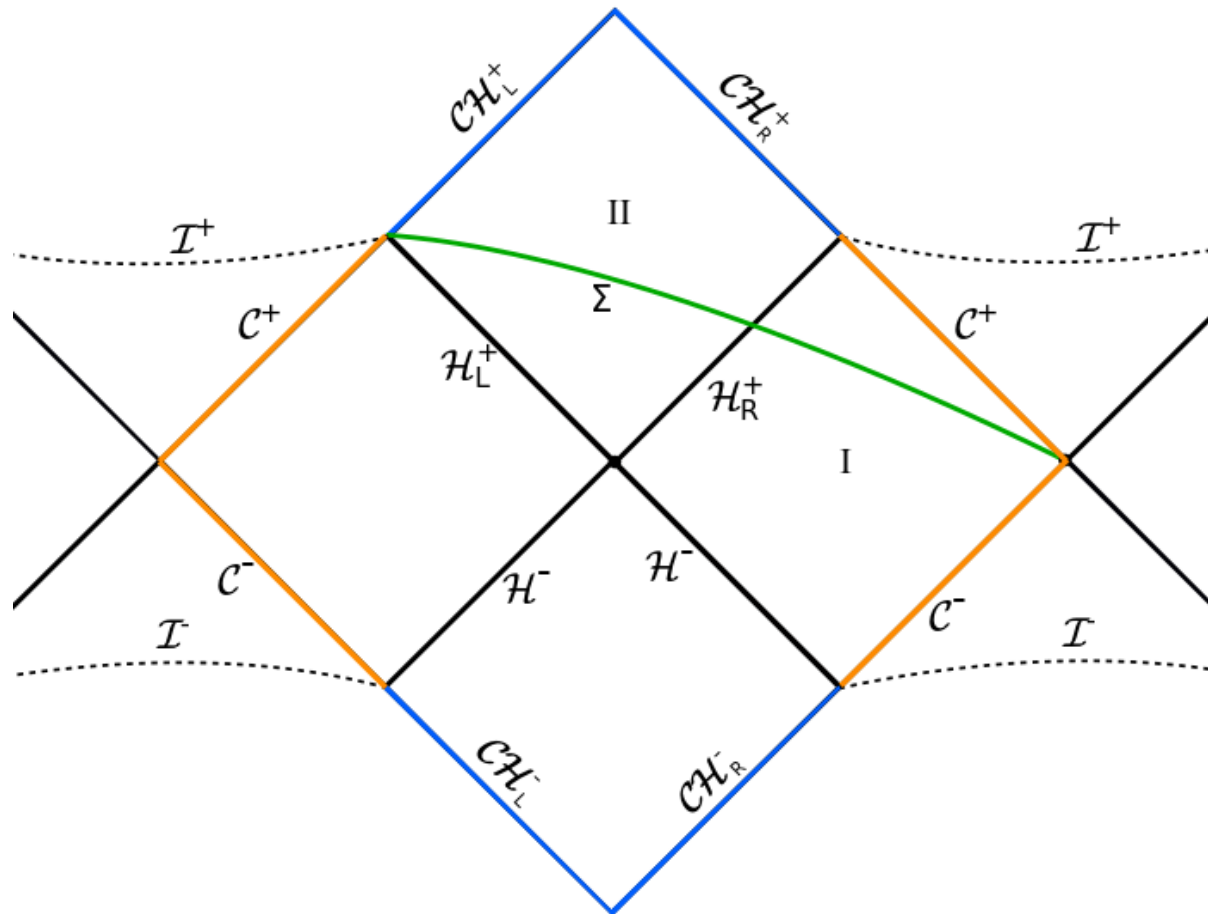
=> **Christodoulou version of SCC** expected to be **true**.



- But (surprisingly) **C^0 version** still **false** [Kehle (2018)]

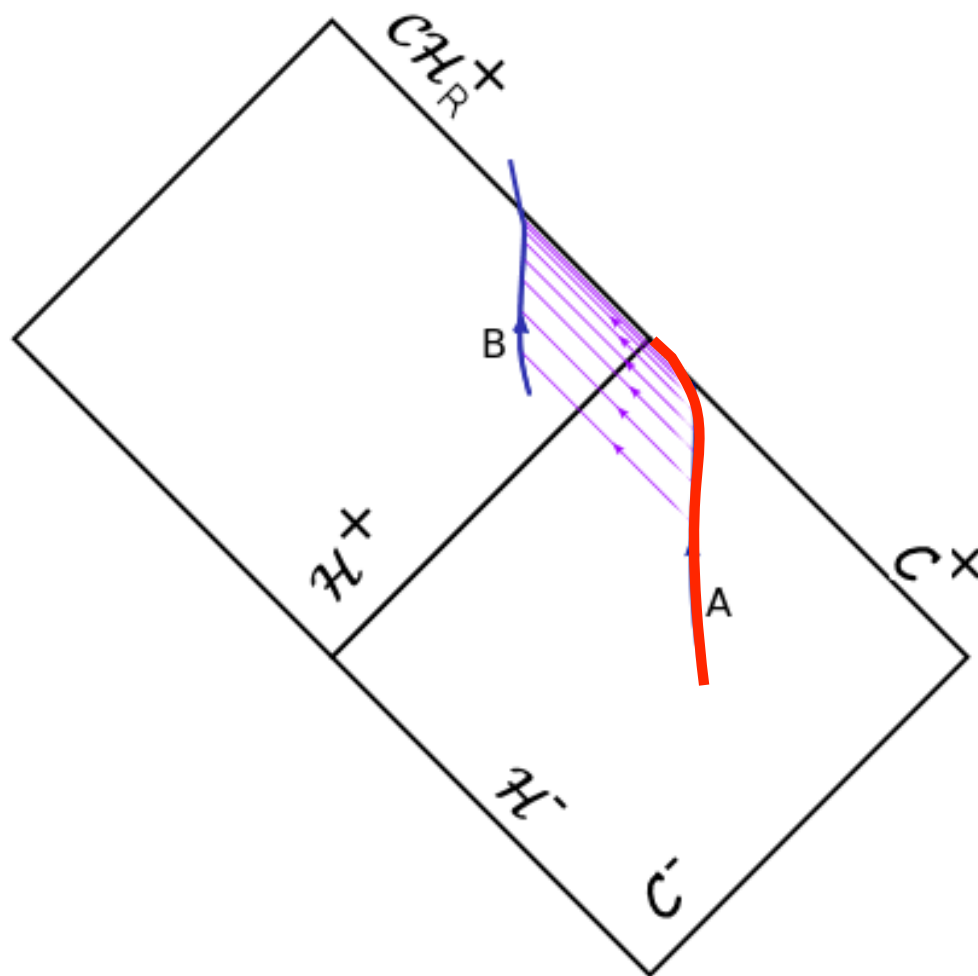
→ The cosmological constant $\Lambda > 0$ (de Sitter)

Onwards, assume $\Lambda > 0$: we now have a cosmological horizon $C^{+,-}$.



→ Blue vs red shift competition

- Perturbations entering H^+ at late time have to **climb out of the potential well** associated with the **cosmological horizon**, suffering a **red-shift**.
- This competes with **blue-shift at Cauchy horizon**: **which effect wins ?**

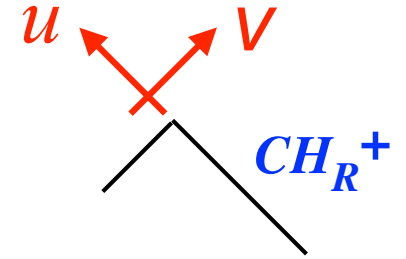


→ Mellor-Moss (1990)

- Perturbations of Reissner-Nordstrom-de Sitter (RNdS) **decay very fastly (exp)!**
- They argued that **late time decay** of linear perturbations outside the BH is **determined** by the **slowest decaying quasinormal mode** (damped oscillation).
- **Quasinormal mode**: solution with time dependence $e^{-i\omega t}$ smooth on future event horizon \mathbf{H}^+ and future cosmological horizon \mathbf{C}^+ . ω is complex with $\text{Im}(\omega) < 0$.
- Define **spectral gap α** to be the **minimum value of $-\text{Im}(\omega)$**
So **generic** perturbations **decay exponentially** in time as $e^{-\alpha t}$,

(in contrast with power-law decay for $\Lambda = 0$)
- **Does this decay trigger the blue-shift instability** of Cauchy horizon CH_R^+ ?

- Introduce **double null coordinates** (U, V) so that **Cauchy horizon** is at $V = 0$ with **BH interior** in $V < 0$.



- MM showed that **near Cauchy horizon**,

generic linear perturbations are proportional to $(-V)^\beta$ with

$$\beta = \frac{\alpha}{\kappa_-}$$

κ_- is the (positive) **surface gravity** of the **Cauchy horizon**

- So **gradient** of perturbation $\partial\Phi \sim (-V)^{\beta-1}$ **blows up** at Cauchy horizon **if $\beta < 1$**

- MM showed (numerically) that **most RNdS BHs** have $\beta < 1$.

=> **backreaction** causes curvature to **blow up** at the **Cauchy horizon**

=> good, **C^2 version of SCC** is obeyed.

- But MM also claimed that **near-extremal** RN holes have $\beta > 1$.

If correct, this would entail a **violation** of the **C^2 version of SCC** !!!

→ Confusion

- [Brady, Moss & Myers "Cosmic censorship: as strong as ever" (1998)]:

argued that the MM analysis overlooks

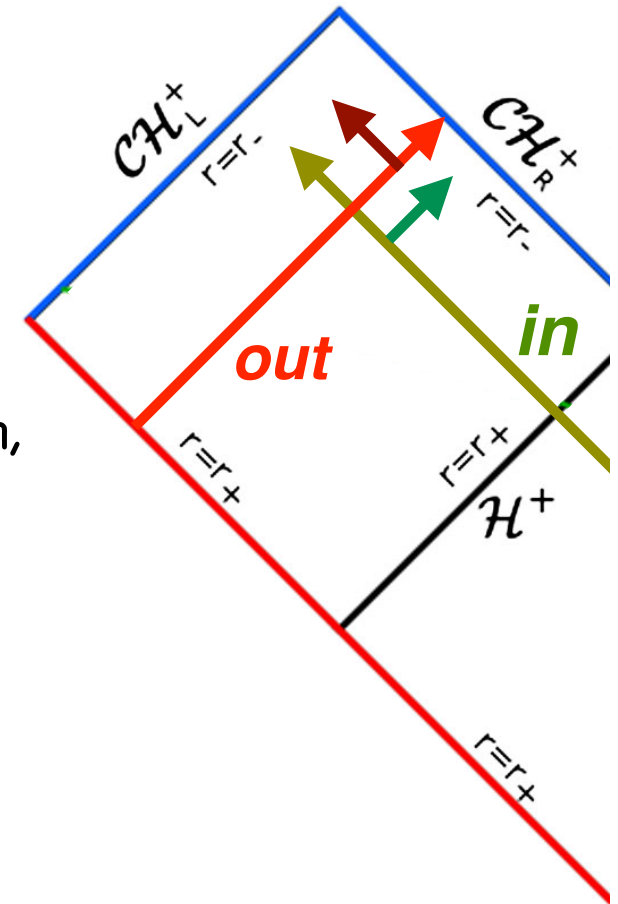
the effect of **outgoing radiation** inside the hole.

- BMM claimed that **backscattering** of such radiation would always have a **large effect** at the **Cauchy** horizon, restoring the **C²** version of SCC for $\Lambda > 0$.

- It turns out that the **BMM argument has a problem:**

we showed that their **initial** perturbation

is **not** smooth (not even **C¹**) at the **event horizon** !



[OD, Reall, Santos 1808.02895]

However, main conclusions correct!!! → see **rough** Dafermos & Shlapentokh-Rothman (2018) **later**

→ Recent developments

- [Hintz & Vasy (2015)]

For smooth initial perturbations,

HV have (mathematically) proved that the **behaviour of linear perturbations**
at the Cauchy horizon

is **indeed determined** by **quasinormal modes**,
as originally claimed by Mellor–Moss.

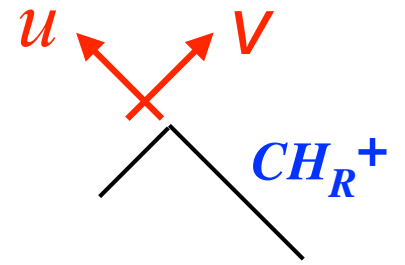
- Therefore:

- **C² version respected iff $\beta < 1$**

- If nonlinearities behave as expected then

- Christodoulou version of SCC is respected iff $\beta < 1/2$**

is $\beta < 1/2$? [later slide]



- Origin of critical $\beta \equiv -\frac{\text{Im}(\omega)}{\kappa_-} = \frac{1}{2}$?

– Near \mathcal{CH}_R^+ , QNM is sum of two independent solutions (u : outgoing EF time):

$$\Phi|_{\mathcal{CH}_R^+} = A\Phi^{(1)} + B\Phi^{(2)} \quad \text{with} \quad \begin{cases} \Phi^{(1)} = e^{-i\omega u} Y_\ell(\theta) \hat{R}_{\omega\ell}^{(1)}(r) \\ \Phi^{(2)} = e^{-i\omega u} Y_\ell(\theta) (r - r_-)^{i\omega/\kappa_-} \hat{R}_{\omega\ell}^{(2)}(r) \end{cases}$$

$$[\hat{R}_{\omega\ell}^{(1,2)}(r_-) \neq 0 \text{ and smooth }]$$

$\text{Im}(\omega) < 0 \Rightarrow \Phi^{(2)}(r_-) = 0$ **BUT** $\Phi^{(2)}$ is **not** smooth at $r = r_-$.

NO reason for $B = 0 \Rightarrow$ **regularity of QNM** is determined by the **non-smooth** $\Phi^{(2)}$.

- *What is the condition for $\Phi^{(2)}$ to be locally square integrable?*

$$\Phi^{(2)} \sim (r - r_-)^p \quad \text{with } p = i\omega/\kappa_- \quad \Rightarrow \quad \partial_r \Phi^{(2)} \sim (r - r_-)^{p-1}$$

which is **square integrable** ($\int (\partial_r \Phi^{(2)})^2$ finite) **iff** $2(\beta - 1) > -1$ where $\beta = \text{Re}(p)$.

\Rightarrow **QNM** $\in H_{\text{loc}}^1$ **at** \mathcal{CH}_R^+ **iff** $\beta \equiv -\frac{\text{Im}(\omega)}{\kappa_-} > \frac{1}{2} \longrightarrow$ Christodoulou's SCC may be violated

- Therefore:

- if \exists **a QNM** with $\beta < 1/2 \Rightarrow \Phi$ cannot be extended across CH_R^+ in H^1_{loc} \Rightarrow **SCC ok**
- if **all QNM** have $\beta > 1/2 \Rightarrow$ **SCC may be violated**

- **[Cardoso, Costa, Destounis, Hintz & Jansen (2017)]:**

careful numerical study of massless scalar field QNMs of RNdS.

- Calculate slowest-decaying QNMs to determine β
- Found that **near-extremal RNdS holes** have $1/2 < \beta < 1$
 \Rightarrow so massless scalar perturbations

violate the Christodoulou version but not the C^2 version of SCC.

- Very recently: numerical confirmation that this is true with **backreaction**, in **spherical** symmetry. **[Cardoso et al (2018)]**

→ Our work: RNdS

[OD, Reall, Santos 1808.02895]

- We studied linearized (coupled) **gravitational-electromagnetic** perturb of RNdS

- We found that **near-extremal** RNdS BHs **always** have $\beta > 2$

- Hence, in **pure** Einstein-Maxwell theory, not only is the

Christodoulou's version of SCC violated but so is the **C² version** !!!

- **In fact**, generic perturbations are C^r at the Cauchy horizon,

where r can be made arbitrarily large

by making the **BH sufficiently near-extremal & sufficiently large** (in units of Λ)

- So **SCC** is very badly violated in **Einstein-Maxwell** theory with $\Lambda > 0$!!!

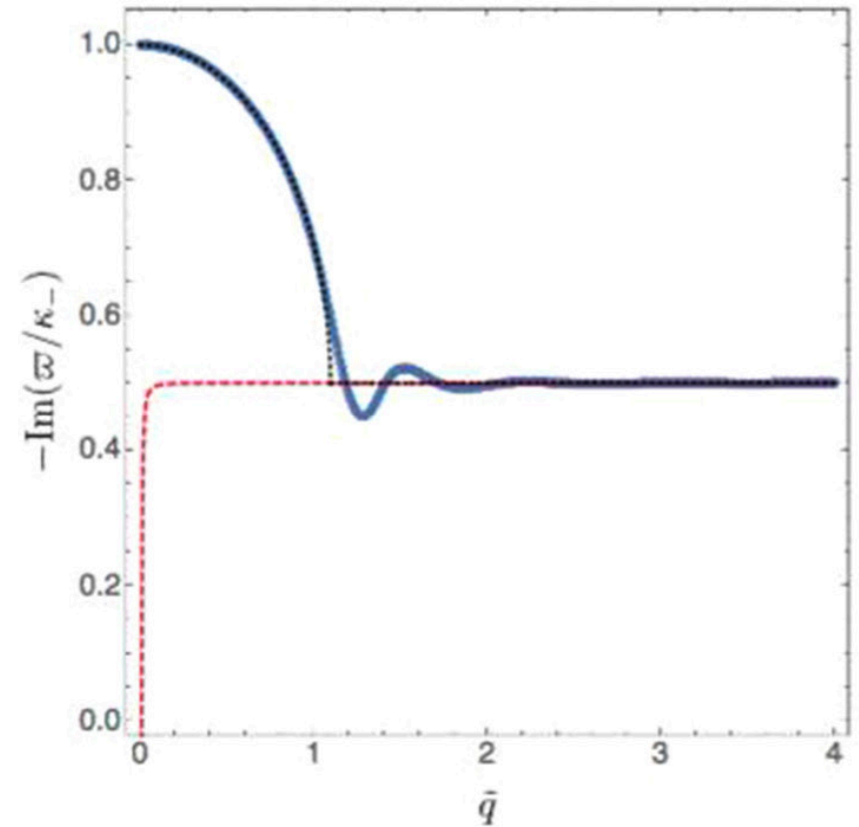
→ Our work: Adding charged matter to RNdS

- In Einstein-Maxwell(-uncharged scalar) theory,
it is **impossible** to **form** a **RNdS** black hole: **no charged matter**.

- So **add** a **charged** scalar field.

- Our results:

[OD, Reall, Santos, 1808.04832]



for “physical” (large) values of charge,

β is very close to $1/2$ **BUT** oscillates around $1/2$ as BH approaches extremality

=> **Christodoulou’s** version of **SCC is violated** ($\beta > 1/2$).

- [see also Hod (2018), Cardoso et al (2018)]: $\beta > 1/2$ not seen
because not close enough to extremality

→ Our work: Kerr dS

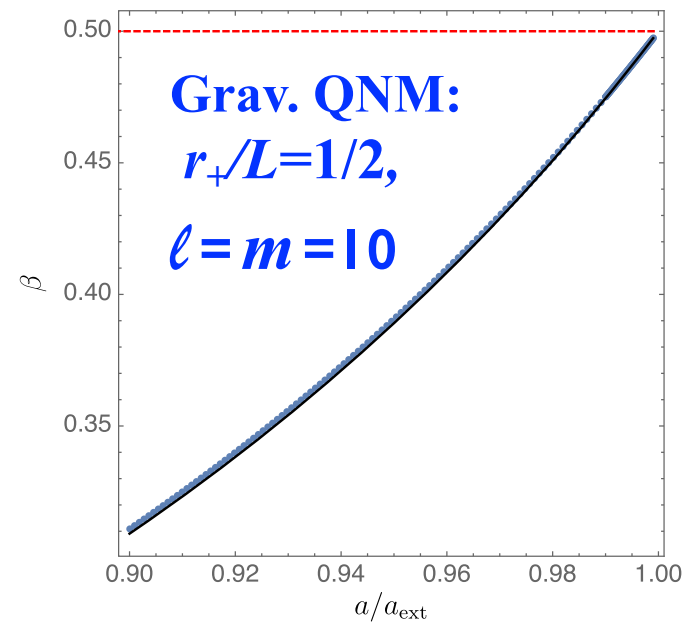
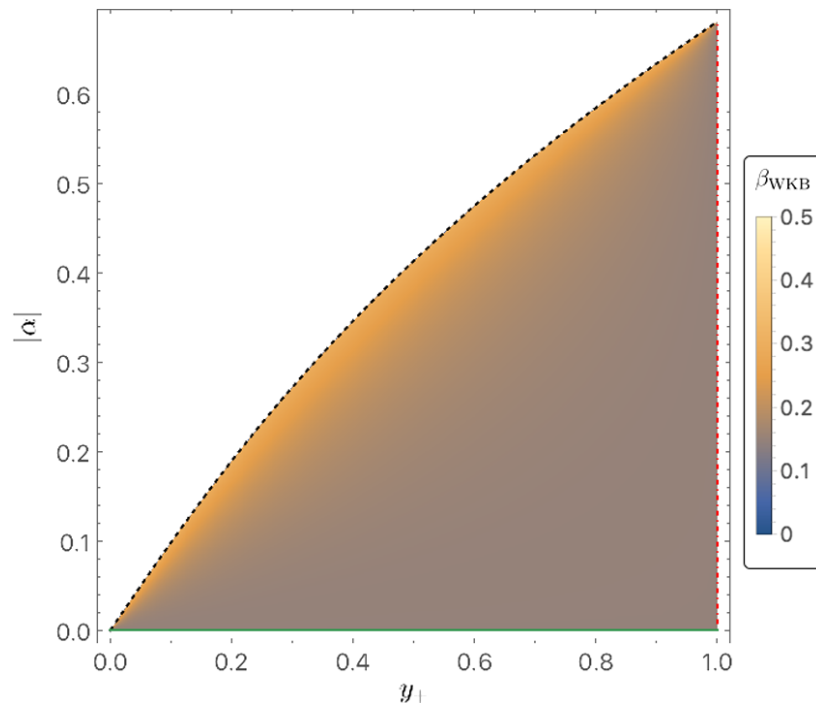
- $\Lambda > 0$ is our **Universe**. But near-extremal RNdS black holes are unphysical:
highly charged BHs don't exist in Nature!
- RN is often regarded as a **toy model** for Kerr within spherical symmetry.
- **So is there a violation of SCC for near-extremal Kerr-dS ?**
- We studied QNMs with large angular momentum, i.e., $\Phi \propto e^{im\varphi}$, $|m| \gg 1$,
both for **scalar field** and **gravitational** perturbations.
- In this limit, the modes can be calculated using **geometric optics**,
i.e., looking at **null geodesics**.
- **Ingoing/outgoing boundary conditions** at the **event/cosmological horizons**
↔ **null geodesics** which do **not cross** any horizons to the past.

→ Our work: Kerr dS

[OD, Eperon, Reall, Santos, 1801.09694]

- The **photon sphere** consists of null geodesics which **remain forever** at **fixed r** .
- These are **unstable** trajectories:
the **decay rate** when perturbed fixes the **imaginary part** of associated QNM.
- From such trajectories we can place an **upper bound on β** : **$\beta < 1/2$**
- Thus **Christodoulou's version of SCC** is **respected** by **Kerr-dS !!!**
- So when $\Lambda > 0$, Christodoulou's SCC is **respected** in **pure Einstein** gravity

BUT violated in **Einstein-Maxwell** theory.



→ Taking the rough with the smooth

- How do we rescue SCC in Einstein-Maxwell theory with $\Lambda > 0$?
- So far we've considered perturbations arising from smooth initial data.
- [Dafermos & Shlapentokh-Rothman (2018)]: consider rough initial data.
=> Late time behaviour is no longer dictated by quasinormal modes.

- The **smoothness** of the **solution** (in the sense of **Sobolev** spaces)
generically **gets worse** at the **Cauchy horizon**.

(In fact this is precisely what happens in the earlier argument of Brady, Moss, Myers!)
[OD, Reall, Santos 1808.02895]

- A **generic** perturbation arising from
initial data with the **minimum acceptable** level of **smoothness** **will not have**
this **minimum acceptable level** of **smoothness** at the **Cauchy horizon**.

3. Quantum effects

→ Quantum effects: theory with charged particles

- $\Lambda \leq 0$: Christodoulou's version of SCC appears to be true
so we do **not** need to invoke quantum effects to restore predictability.
- $\Lambda > 0$: do quantum effects help?
- **If** theory contains **charged particles**: RNdS BH can **radiate** them & **lose** its **charge**
- This does not depend on the mass of the particles:
redshifted away at cosmological horizon!
- So **Hawking radiation** will **drive BH away from extremality** \Rightarrow **SCC is saved**.

→ Quantum effects: theory with NO charged particles

- Now consider RNdS in pure Einstein–Maxwell theory (no charged matter).
- We have **Hawking radiation** of **photons** and **gravitons**
from both the BH horizon and the cosmological horizon.
- This will **drive a RNdS BH away from extremality** towards a “**lukewarm**” solution
(for which the two horizons have **equal temperatures**)
- **BUT** such BHs are **still close enough to extremality**
to **violate Christodoulou’s version of SCC** (but not C^2 version).

→ Quantum effects: theory with NO charged particles

- Expect the quantum state of the fields to approach the Hartle-Hawking (thermal) state at late time.
 - It would be interesting to calculate $\langle T_{\mu\nu} \rangle$ in this HH state near the Cauchy horizon of a lukewarm BH.
 - Does $\langle T_{\mu\nu} \rangle$ diverge at the Cauchy horizon sufficiently rapidly to ensure that $G_{\mu\nu} = 8\pi\langle T_{\mu\nu} \rangle$ cannot be satisfied there, even weakly?
 - If so then predictability would be enforced by vacuum polarization.
 - Calculations in a 2d toy model show that $\langle T_{\mu\nu} \rangle$ does diverge at a Cauchy horizon. [Birrell & Davies 1977]
- The 4d case is much harder!
- $\langle T_{\mu\nu} \rangle$ has only been calculated outside a lukewarm RNdS black hole.
- [Winstanley & Young (2007), Breen & Ottewill (2010)]

→ Summary

- **Strong cosmic censorship:** physics should be predictable from initial data!
- Very likely to be **true** for $\Lambda \leq 0$.
True for **pure Einstein** theory with $\Lambda > 0$.
- **Badly** (very! not even **c²**) violated in **pure Einstein-Maxwell** theory with $\Lambda > 0$
unless we allow **rough** initial data.
- Opportunity for quantum effects to save the day?
- Further reading:
 - **OD, Felicity Eperon, Harvey Reall, Jorge Santos**
1801.09694 (PRD) & 1808.02895 (JHEP) & 1808.04832 (CQG)
 - **Mihalis Dafermos' website**