Asymptotically Safe Quantum Gravity
(and black holes)

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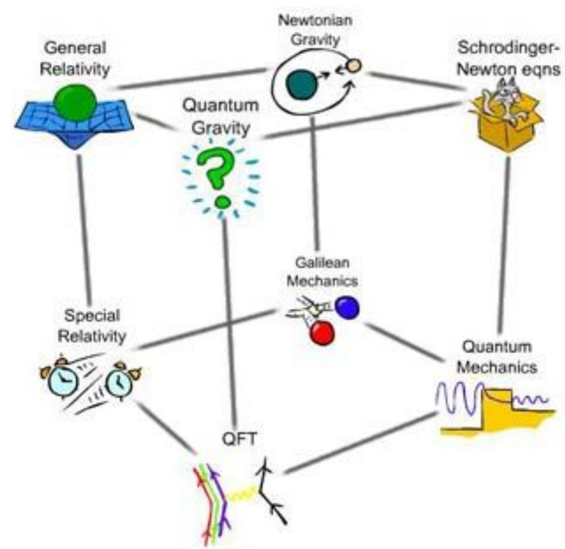
General Relativity

- Based on Einstein field equations
- Describes the gravitational interaction

How do quantum effects modify gravity at short distances?

Standard Model of particle physics

- Based on Quantum Field Theory
- Describes electromagnetic, strong and weak interactions



Einstein Gravity and Renormalization

Einstein-Hilbert action

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R-2\Lambda)$$

Power counting: the Newton's constant has negative mass dimension ⇒ Ultraviolet divergences

General Relativity is not (perturbatively) renormalizable

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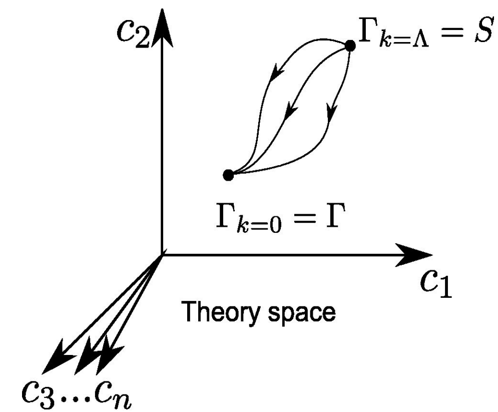
Asymptotically Safe Gravity (Weinberg, 1976)

- General Relativity might be renormalizable from a *non-perturbative* point of view
- Key idea: generalized notion of renormalizability based on the Wilsonian Renormalization Group

S. Weinberg, Erice Subnucl. Phys.1976:1

Wilsonian Renormalization Group



Consider a theory described by the fundamental action

$$S[\Phi] = \int d^4x\, \mathcal{L}(\Phi, \partial_\mu \Phi, \ldots)$$

How does the theory look like at a different *resolution scale k*?

- **RG flow**: evolution of the action in the theory space

$$k\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left\{ \left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\partial_k \mathcal{R}_k \right\}$$

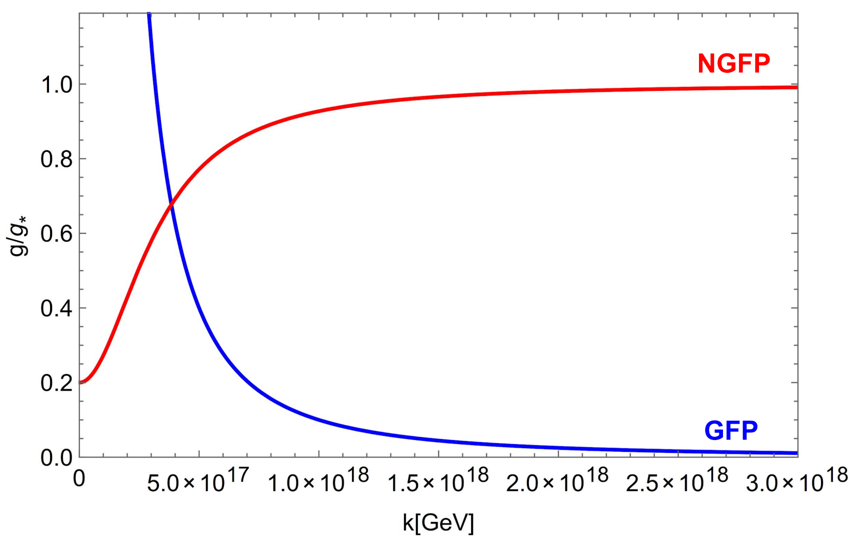
C. Wetterich. PLB 301:90 (1993)
M. Reuter. PRD **57** (2): 971 (1998)

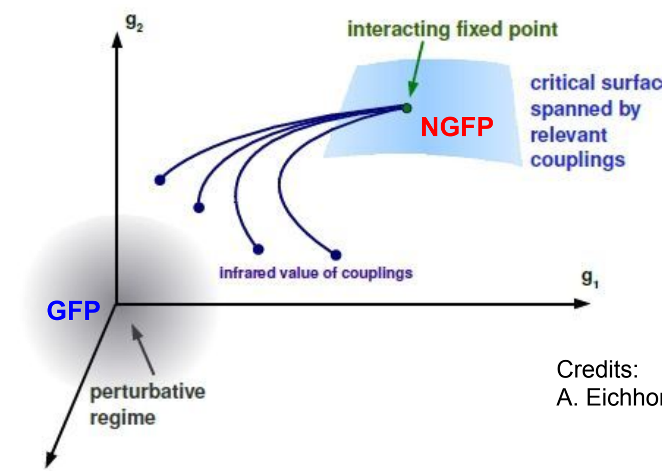
- **RG fixed points**: endpoints of the RG flow (⇔ scale invariant regimes)

RG fixed points and renormalizability

Two types of well-definite **ultraviolet completion** (microscopic/fundamental theory)

- **Gaussian Fixed Point (GFP)**: free theory ⇒ *Asymptotic Freedom*
- **Non-gaussian Fixed Point (NGFP)**: interacting theory ⇒ *Asymptotic Safety*



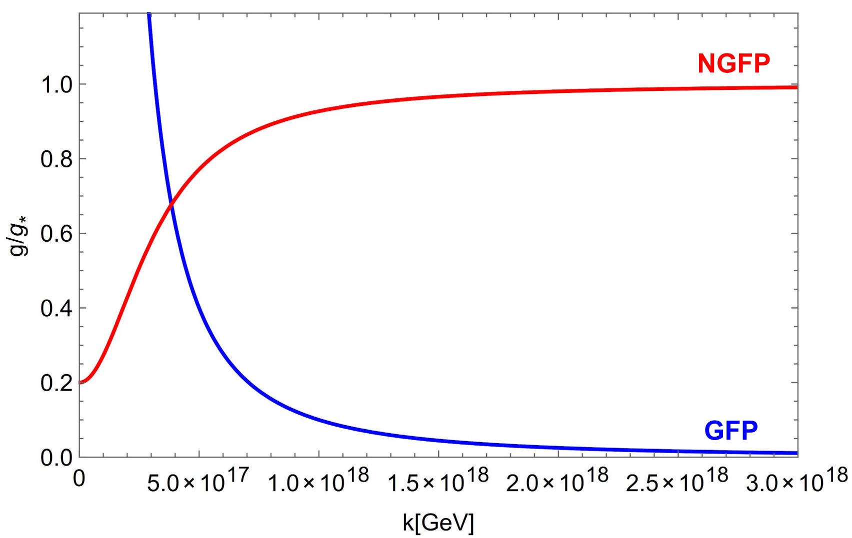


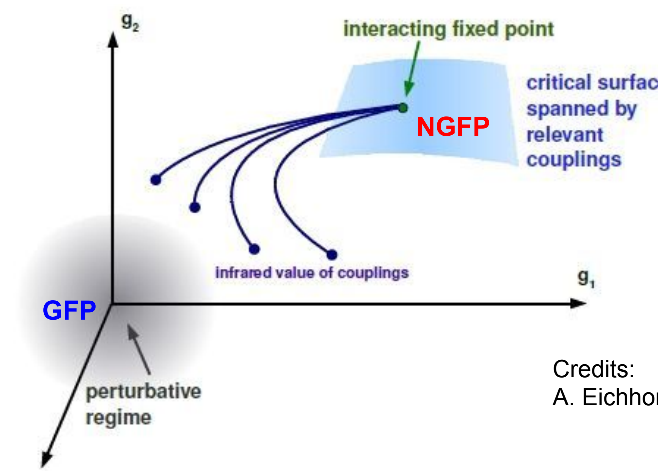
Credits:
A. Eichhorn

RG fixed points and renormalizability

Generalized (non-perturbative) renormalizability

- **Ultraviolet completion** ⇔ UV-attractive **fixed point** (microscopic theory)
  - **Gaussian Fixed Point (GFP)**: free theory ⇒ *Asymptotic Freedom*
  - **Non-gaussian Fixed Point (NGFP)**: interacting theory ⇒ *Asymptotic Safety*
- **Predictivity** ⇔ finite number of relevant directions (finite-dimensional UV critical manifold)





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Asymptotic Safety in Quantum Gravity

Einstein-Hilbert truncation

$$S_k = \frac{1}{16\pi G_k} \int d^4x \sqrt{-g}\, (R - 2\Lambda_k)$$

$$\boxed{G_k = k^{-2} g_k \qquad \Lambda_k = k^2 \lambda_k}$$

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Fixed points of the RG flow:

- **GFP** $\rightarrow$ saddle point
- **NGFP** $\rightarrow$ UV-attractor

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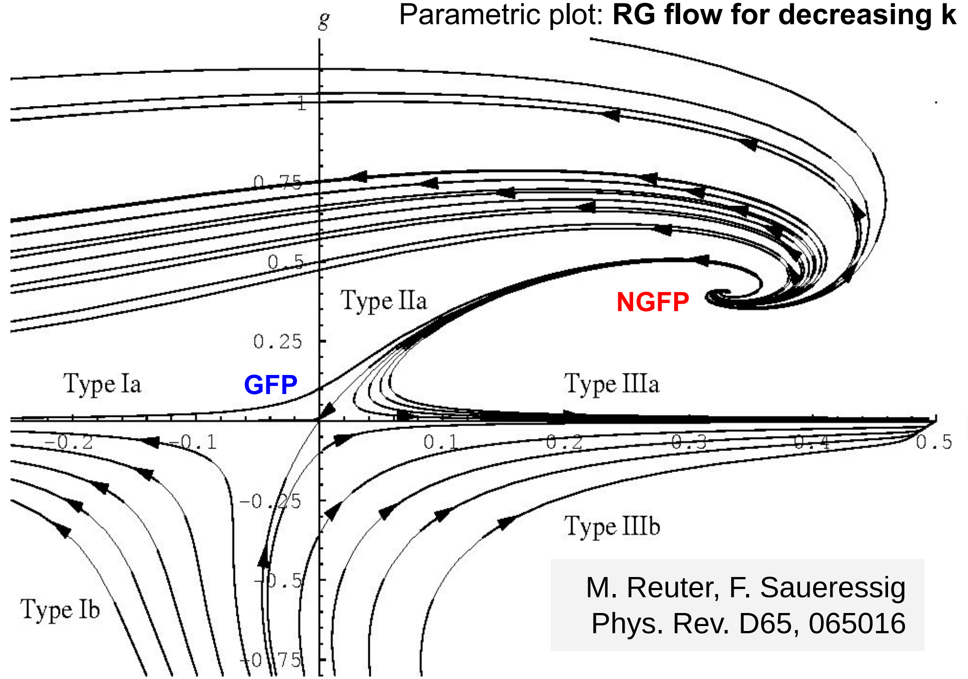
Parametric plot: **RG flow for decreasing k**



M. Reuter, F. Saueressig
Phys. Rev. D65, 065016

- UV-attractive NGFP ⇒ the theory is well-defined and UV-complete
- NGFP stable against the addition of higher derivatives terms (3 relevant directions)

Looking for Asymptotic Safety: a (long) history

Functional RG equations for different ansatz allows to check the existence of a NGFP

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right\}$$

$$f(R) \simeq \sum_{n=0}^{N} a_n R^n$$

Polynomial up to N=71:
Reuter, Lauscher, '02; Codello, Percacci, Rahmede '09; Benedetti, Caravelli, '12; Dietz, Morris, '12; Falls, Litim, Nikolakopoulos, Rahmede, '13, '14 Demmel, Saueressig, Zanusso, '15; Falls, Litim, Schoeder, '18
Beyond polynomial:
Benedetti, Caravelli, '12; Demmel, Saueressig, Zanusso, '12; Dietz, Morris, '13

$$R^2 + R_{\mu\nu}R^{\mu\nu}$$

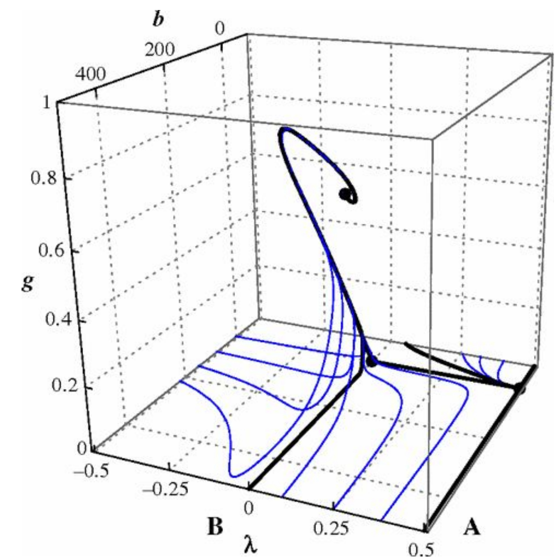
Benedetti, Machado, Saueressig, '09. Christiansen, '16, Oda, Yamada '17

$$C_{\mu\nu}^{\kappa\lambda} C_{\kappa\lambda}^{\rho\sigma} C_{\rho\sigma}^{\mu\nu}$$

Gies, Knorr, Lippoldt, Saueressig '16

→ **NGFP + 3 relevant directions**
→ Canonical power counting is still a good guideline

$$\mathcal{L}_k = \frac{k^2}{16\pi g(k)}[R - 2k^2\lambda(k)] - \beta(k)R^2$$



S. Rechenberger, F. Saueressig Phys. Rev. D **86**, 024018

RG-flow:
Gravity-matter systems

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

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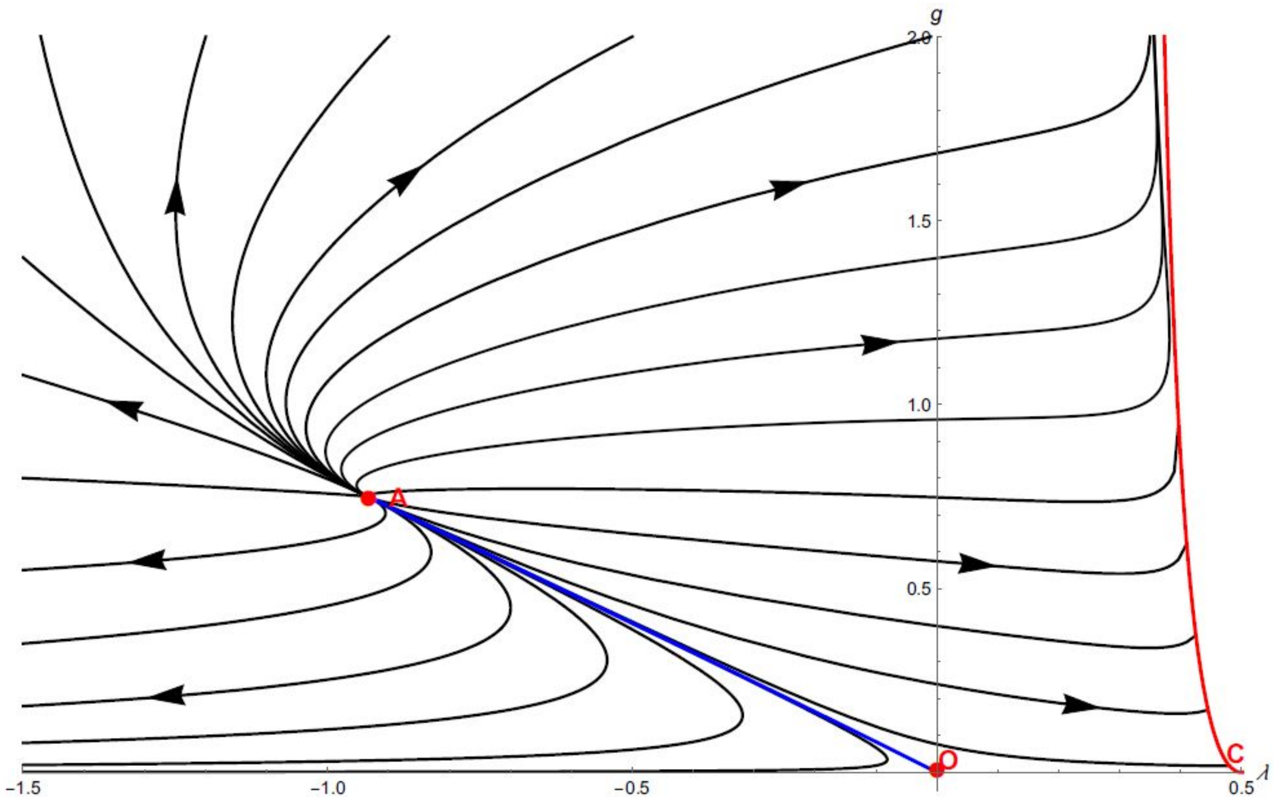
$$S_k = S_k^{\text{grav}} + S_k^{\text{matter}}$$

Is the gravitational RG flow influenced by the presence of matter fields?

RG flow: gravity coupled to SM matter

J. Biemans, AP, F. Saueressig
JHEP 05 (2017) 093 (2017)



Important properties:

- Positive and real critical exponents
- Negative ultraviolet cosmological constant

$$\lambda_* < 0$$

Astrophysical and cosmological implications

Black Holes in Asymptotically Safe Gravity

Spacetime singularities

- **Spacetime singularities** are a general feature of General Relativity
    - Cosmological singularity at the “origin of time”
    - Black hole singularities
- Implications
    - Divergence of physical quantities (curvature, energy density, etc..)
    - Impossible to determine the evolution of the spacetime beyond the singularity

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Can Asymptotically Safe Gravity solve this problem?

Quantum spacetime in Asymptotically Safe Gravity

The Wilsonian action tells us how the theory looks like at different energy scales.

➔ When quantum-relativistic effects are taken into account, the classical Einstein field equations are modified

$$\frac{\delta\Gamma_k}{\delta g_{\mu\nu}}[\langle g\rangle_k] = 0 \longrightarrow \textit{Effective average geometry}$$

Goal: understand how *quantum fluctuations* modify the classical solutions of GR in the high-energy regime

Antiscreening of the gravitational interaction at high energies

- Renormalization group equations ⇒ **running Newton's coupling**

$$G_k = \frac{G_0}{1 + G_0\, g_*^{-1}\, k^2}$$

A. Bonanno, M. Reuter
Phys. Rev. D 62, 043008 (2000)

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⇒ "**anti-screening**" effects of the gravitational interaction

- The anti-screening behavior of the Newton's constant implies a **weakening of the gravitational interaction at high energies**

⇒ weakening of the *singularities* typically appearing in the classical theory

How to take into account this antiscreening effect?
How "quantum-corrected BHs" look like?

A. Bonanno, M. Reuter. PRD 62 (2000) 043008
A. Bonanno, B. Koch, AP CQG 34 (2017) 095012

The case of QED: screening of the electric charge

The classical Coulomb potential is modified by the **vacuum-polarization effects**

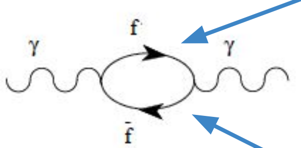
Coulomb potential

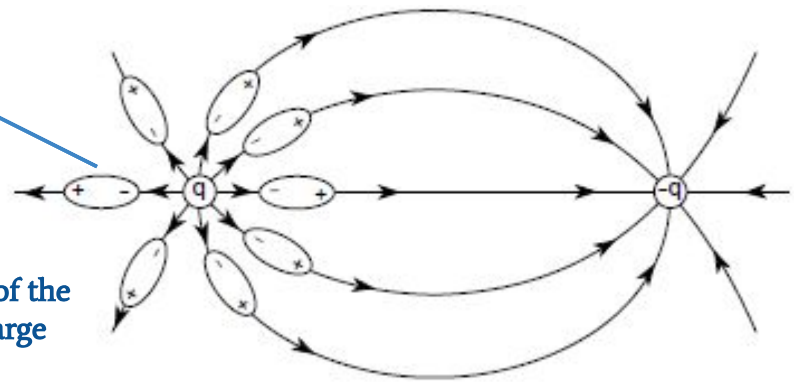
$$V(k) = -\frac{e^2}{k^2} \longrightarrow V(r) = -\frac{\alpha}{r}$$

Uehling potential

$$V(k) = -\frac{e^2}{k^2\,(1-\Pi(k^2))} \longrightarrow V(r) = -e^2 \int \frac{e^{iqx}}{k^2\,(1-\underbrace{\Pi(k^2)}_{\text{Photon self-energy}})} \frac{d^3q}{(2\pi)^3}$$

Photon self-energy





Screening of the electric charge

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Coulomb potential $V(k) = -\frac{e^2}{k^2} \longrightarrow V(r) = -\frac{\alpha}{r}$

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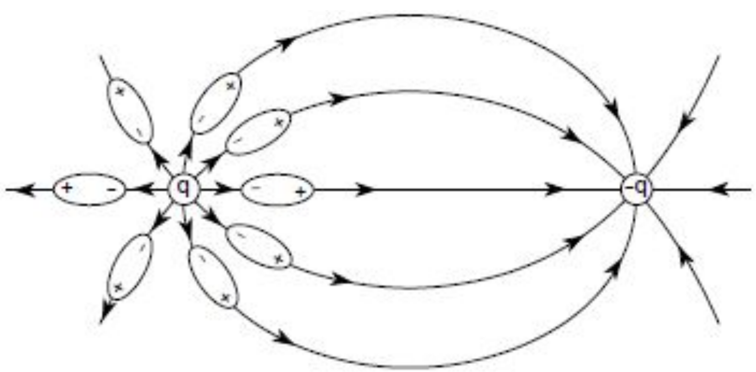
Uehling potential $V(k) = -\frac{e^2}{k^2(1-\Pi(k^2))} \longrightarrow V(r) = -e^2 \int \frac{e^{iqx}}{k^2\,(1-\underbrace{\Pi(k^2)}_{\text{Photon self-energy}})} \frac{d^3q}{(2\pi)^3}$



Electric potential + 1-loop radiative corrections

$$V_1(r) \sim -\frac{e^2(r_0^{-1})}{4\pi r}(1 + b\log(r_0/r))$$

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One way to write the potential with radiative corrections is by introducing the running electric charge

$$V(k) = -\frac{e_k^2}{k^2}$$

$$e_k^2 \simeq e^2(k_0)(1 + b\log(k^2/k_0^2))$$

and Fourier-transform to coordinate space.

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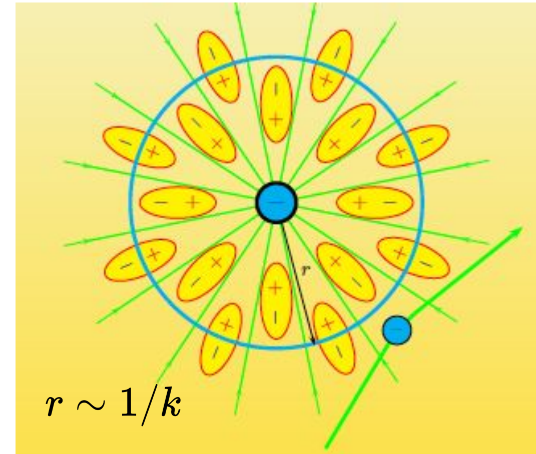
The Renormalization Group improvement

In the context of QFT, the **RG-improvement** is widely used to study the *effect of leading order quantum corrections*

- ➢ Start from a classical system
- ➢ Replace coupling constants with the corresponding running couplings
- ➢ Close the system by identifying the RG-scale with a characteristic energy scale of the system



Example: RG-improvement of the **electric potential** in ***Quantum Electrodynamics***

$$V(r) = -\frac{e^2}{r} \quad \rightarrow \quad V_k(r) = -\frac{e_k^2}{r} \quad \xrightarrow[k \sim 1/r]{} \quad V_1(r) \sim -\frac{e^2(r_0^{-1})}{4\pi r}(1 + b\log(r_0/r))$$

1-loop Uehling potential

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2$$

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 0 \\ f(r) = 1 - \dfrac{2\, m\, G_0}{r} \end{cases}$$

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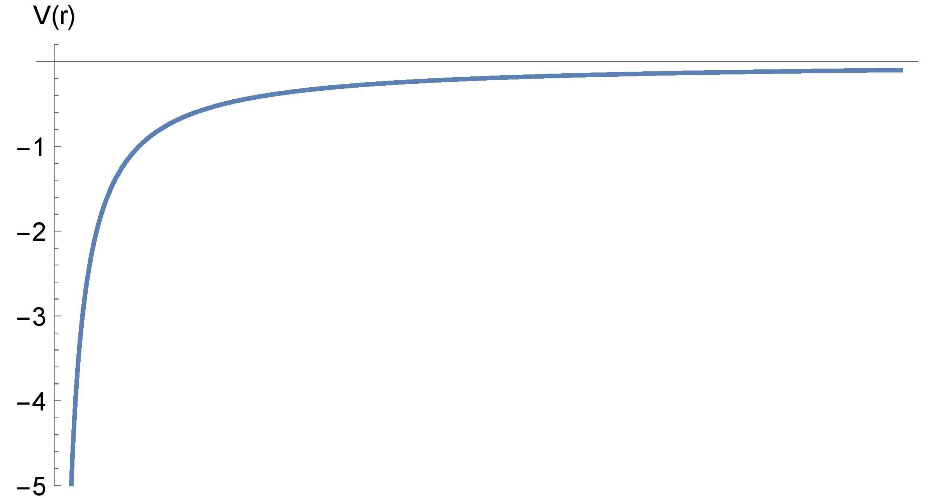

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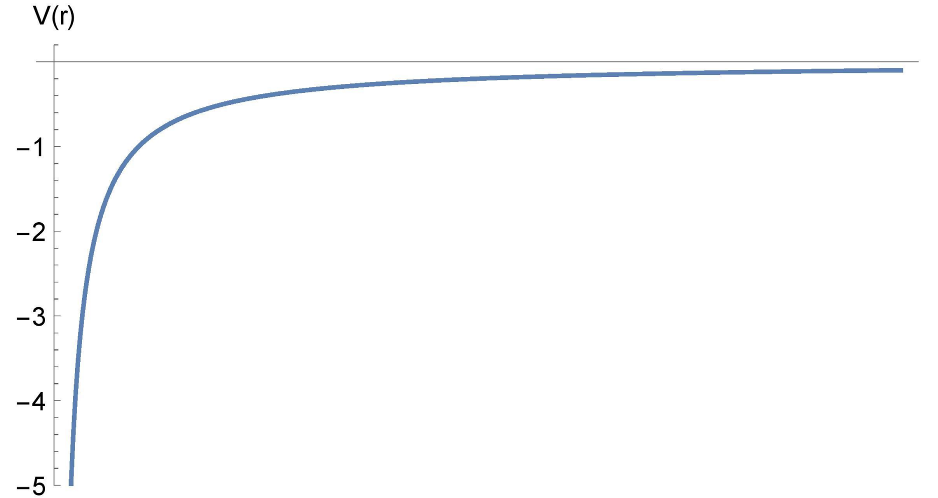

$k \gg M_{\rm Pl}$
$G_k \sim g_* k^{-2}$ $\longrightarrow$ $k \sim k_{\rm obs}$
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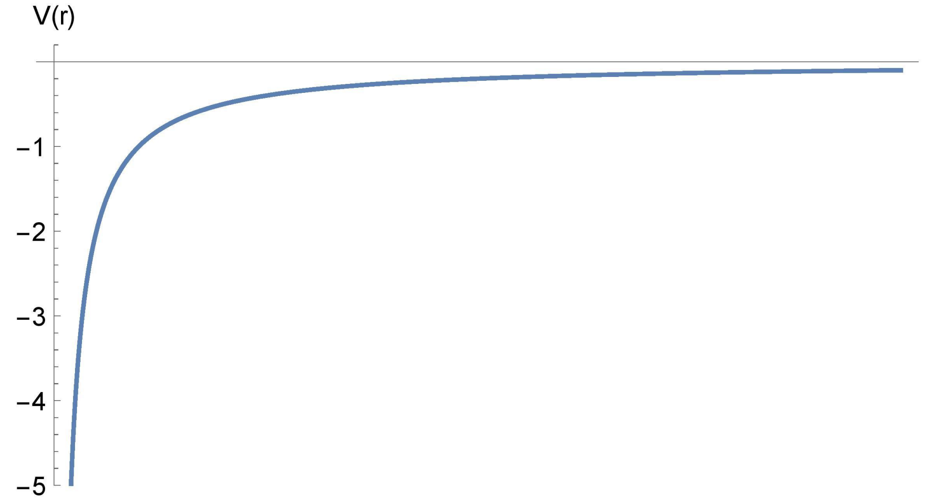

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[obtained by solving the beta functions from RG equations]

$$G_k = \frac{G_0}{1 + \omega\, G_0 k^2} \qquad \omega = g_*^{-1}$$

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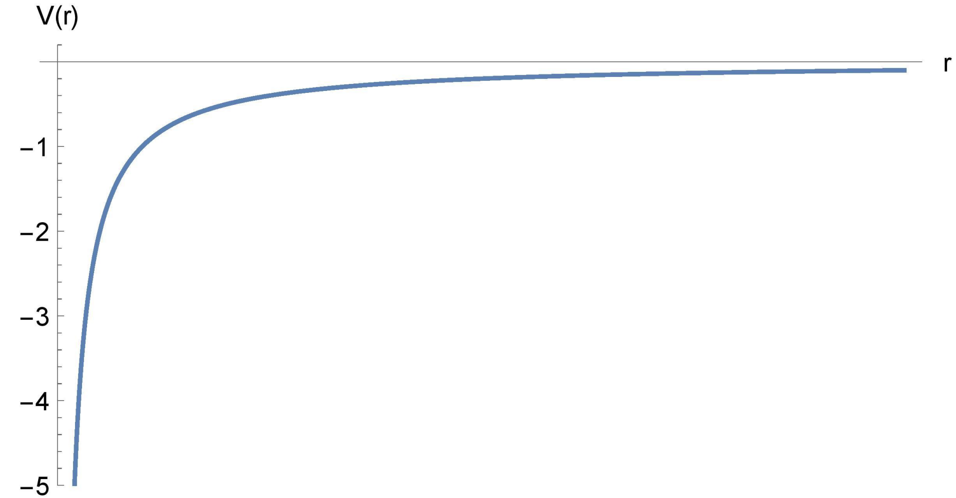

$r = 0$ $\qquad$ $r \to \infty$

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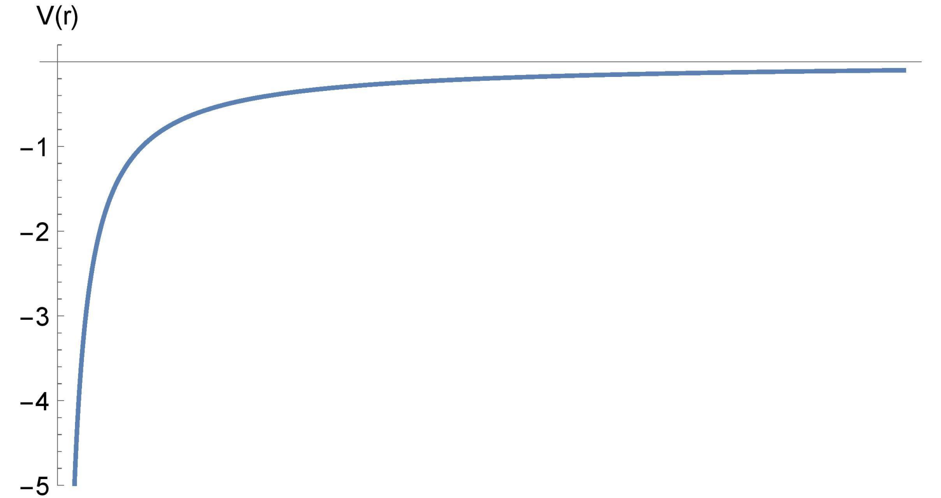

$r = 0$ → $r \to \infty$

By analogy with the case of QED:

$$G_{(0)} \longrightarrow G_{(1)} = \frac{G_0}{1 + \omega\, G_0\, k_{(1)}^2}$$

$k_{(1)} \sim 1/d_0(r)$ Radial coordinate r → proper distance

$$G_{(1)}(r) = \frac{G_0 r^3}{r^3 + \omega\, G_0 (r + \gamma\, G_0 m)}$$

A. Bonanno, M. Reuter.
Phys.Rev. D62 (2000) 043008

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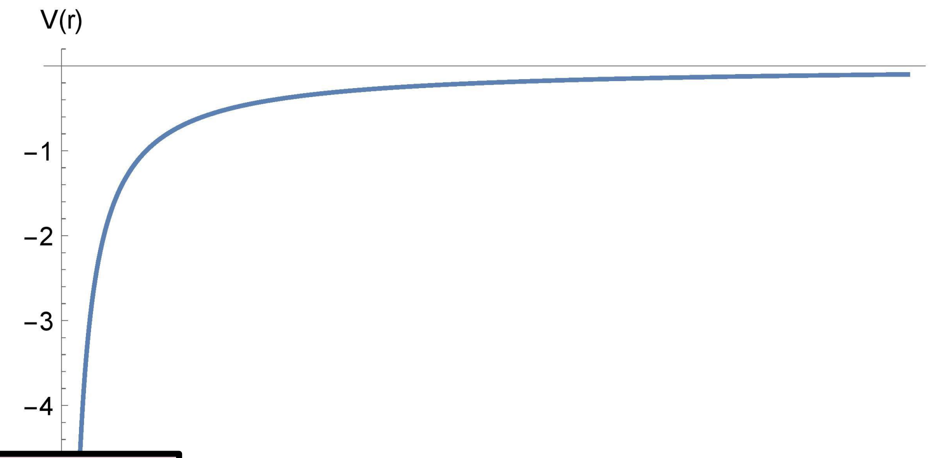

...several interrelated issues:

- The RG scale k(r) depends on the classical background geometry
- In the strong-curvature regime, this metric is no longer valid
- The metric depends explicitly on the Newton coupling
- Quantum effects modify the Einstein equations

⇒ backreaction effects must be taken into account

$r \to \infty$

$k \sim k_{\rm obs}$

$G_k \equiv G_0$

$^{-2}$

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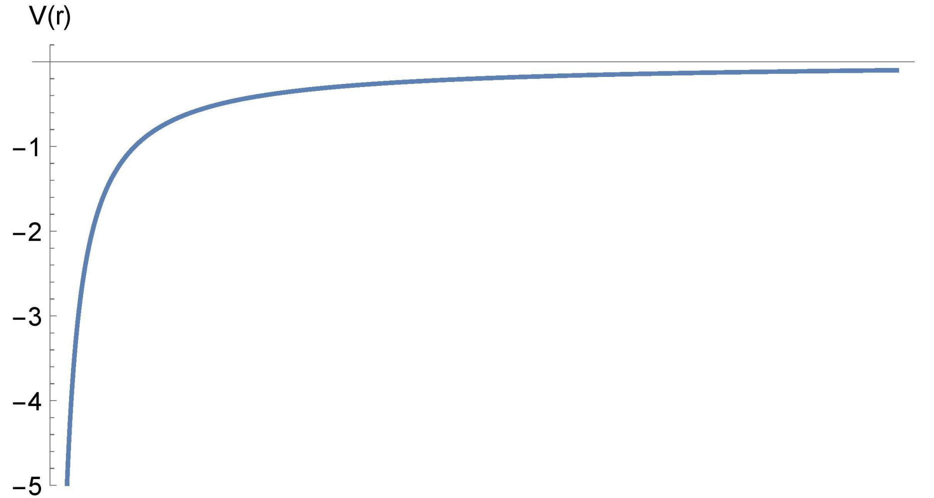

$r = 0$ $\qquad$ $r \to \infty$

Goal: modify the system (metric + Einstein eqs) to find a black hole solution encoding the running of G in a self-consistent way

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$$G_k = \frac{G_0}{1 + \omega\, G_0 k^2} \qquad \omega = g_*^{-1}$$

Construction of self-consistent BH solutions

1. Start from the classical metric and introduce the running of the Newton coupling as a perturbation

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 0 \\ f_{(0)}(r) = 1 - \dfrac{2\, m\, G_{(0)}}{r} \end{cases} \quad \Longrightarrow \quad G_{(0)} \longrightarrow G_{(1)} = \frac{G_0}{1 + \omega\, G_0\, k_{(1)}^2}$$

$$k_{(1)} \propto r^{-\alpha}$$

$$\omega = g_*^{-1}$$

AP. EPJC 79 (2019) 470

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2. The running of the Newton's coupling produces an **effective energy-momentum tensor**:

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}^{(1)} \\ f_{(1)}(r) = 1 - \dfrac{2\, m\, G_{(1)}(r)}{r} \end{cases} \longrightarrow \quad \rho_{(1)} = \frac{m}{4\pi r^2} \frac{G'_{(1)}(r)}{G_{(1)}(r)}$$

Effective quantum gravitational self-energy

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Effective quantum gravitational self-energy

3. The effective energy density gives a measure of the **strength of QG effects**. We can use it to construct the next steps of the iteration and look for **fixed points**

$$k_{(n+1)}^2 = K[\rho_{(n)}] \Longrightarrow \begin{cases} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_{(n+1)} T_{\mu\nu}^{(n+1)} \\ f_{(n+1)}(r) = 1 - \dfrac{2mG_{(n+1)}(r)}{r} \end{cases} \Longrightarrow G_{(n+1)} = \frac{G_0}{1 + \omega\, G_0\, K[\rho_n]}$$

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Construction of self-consistent BH solutions

1. Start from the classical metric and introduce the running of the Newton coupling as a perturbation

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 0 \\ f_{(0)}(r) = 1 - \dfrac{2\, m\, G_{(0)}}{r} \end{cases} \quad \Longrightarrow \quad G_{(0)} \longrightarrow G_{(1)} = \frac{G_0}{1 + \omega\, G_0\, k_{(1)}^2}$$

$$k_{(1)} \propto r^{-\alpha}$$

$$\omega = g_*^{-1}$$

2. The running of the Newton's coupling produces an **effective energy-momentum tensor**:

$$\begin{cases} R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}^{(1)} \\ f_{(1)}(r) = 1 - \dfrac{2\, m\, G_{(1)}(r)}{r} \end{cases} \longrightarrow \quad \rho_{(1)} = \frac{m}{4\pi r^2} \frac{G'_{(1)}(r)}{G_{(1)}(r)}$$

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$$G_{(n+1)} = \frac{G_0}{1 + g_*^{-1} G_0\, k_{(n+1)}^2} \quad \xrightarrow[n \to \infty]{k_{(n+1)}^2 = K[\rho_{(n)}]} \quad K[G'(r)] = g_* \frac{G_0 - G(r)}{G_0\, G(r)}$$

RG-scale in dependence of the effective energy density

$$G_{(n+1)} = \frac{G_0}{1 + g_*^{-1} G_0 \, k^2_{(n+1)}} \quad \xrightarrow[n \to \infty]{k^2_{(n+1)} = K[\rho_{(n)}]} \quad K[G'(r)] = g_* \frac{G_0 - G(r)}{G_0 \, G(r)}$$

- **Case with a (running) cosmological constant**

  The relation we are looking for is determined by a *consistency relation* arising from the Bianchi identity.
  In the proximity of the UV-fixed point

  M. Reuter, H. Weyer. Phys.Rev. D69 (2004) 104022
  Babic, Guberina, et al. Phys.Rev. D71 (2005) 124041
  Bonanno, Esposito, et al. Class. Quant. Grav. 23 (2006) 3103

$$\nabla_\mu G_{\mu\nu} = 0 \qquad \Rightarrow \qquad k^2 \sim \frac{R}{4\lambda_*}$$

- **Case at hand**

  The contracted Bianchi identity is not enough to constrain the scaling relation k(r)

  ⇒ Physical arguments needed

Let us analyse the **quantum-corrected Ricci and Kretschmann scalars**

$$R_{(n)} = \frac{G_{(n)}}{G_0}\left\{\bar{R}_{cl} + c(r)\left(\rho_{(n)} G_0\right)\right\}$$

$$K_{(n)} = \frac{G^2_{(n)}}{G^2_0}\left\{\bar{K}_{cl} + a(r)\sqrt{\bar{K}_{cl}}\left(\rho_{(n)} G_0\right) + b(r)\left(\rho_{(n)} G_0\right)^2\right\}$$

$$R_{\rm cl} = 0$$

$$K_{\rm cl} = \frac{48 m G_0}{r^6}$$

- The coefficients a, b, c are dimensionless functions of r and they can also be negative!

- The strength of the *classical tidal forces* is counterbalanced by additional terms that depend on a single mass-scale. This scale acts as a *scale-dependent regulator* for the bare curvature invariants

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Strength of the classical gravitational field

Measure of the strength of quantum effects

- The coefficients a, b, c are dimensionless functions of r and they can also be negative!
- The strength of the *classical tidal forces* is counterbalanced by additional terms that depend on a single mass-scale. This scale acts as a *scale-dependent regulator* for the bare curvature invariants

$\longrightarrow$ $$k^2 \equiv \xi G_0 \rho$$

Iteration and self-consistent solution

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Fixed point of the RG-improvement procedure

$$G_\infty(r) = G_0 \left\{ 1 - \exp\left(-\frac{r^3}{r_s\, l_*^2} \right) \right\}$$

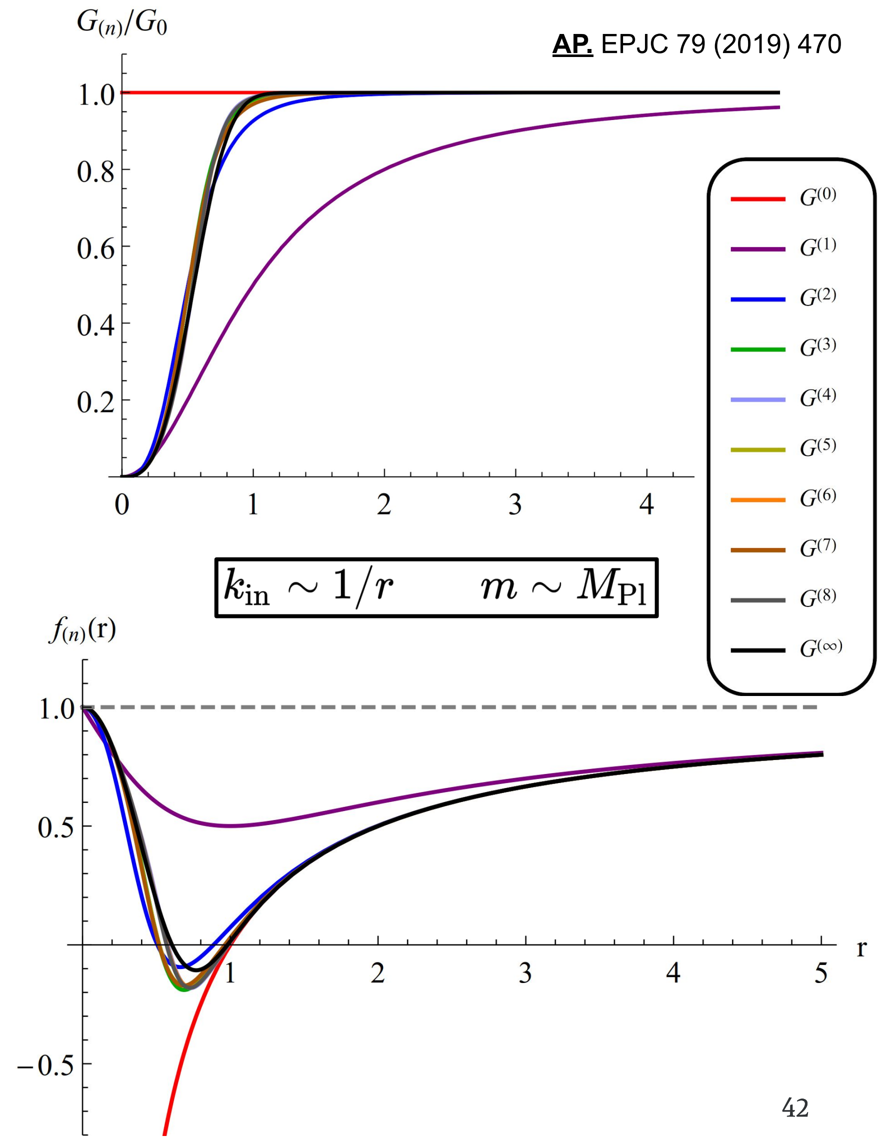
Dymnikova (1996)

$$l_* = \sqrt{\frac{3\xi}{8\pi g_*}}\, l_{\rm Pl} \qquad r_s = 2mG_0$$

Three possibilities:

1) **No RG fixed-point** ⇒ the Schwarzschild metric is recovered
2) **Asymptotic Freedom** ⇒ the Newton constant vanishes everywhere
3) **Asymptotic Safety** ⇒ Dymnikova BHs

Non-perturbative renormalizability ⇒ Effective "renormalization" of the spacetime geometry (singularity-resolution)



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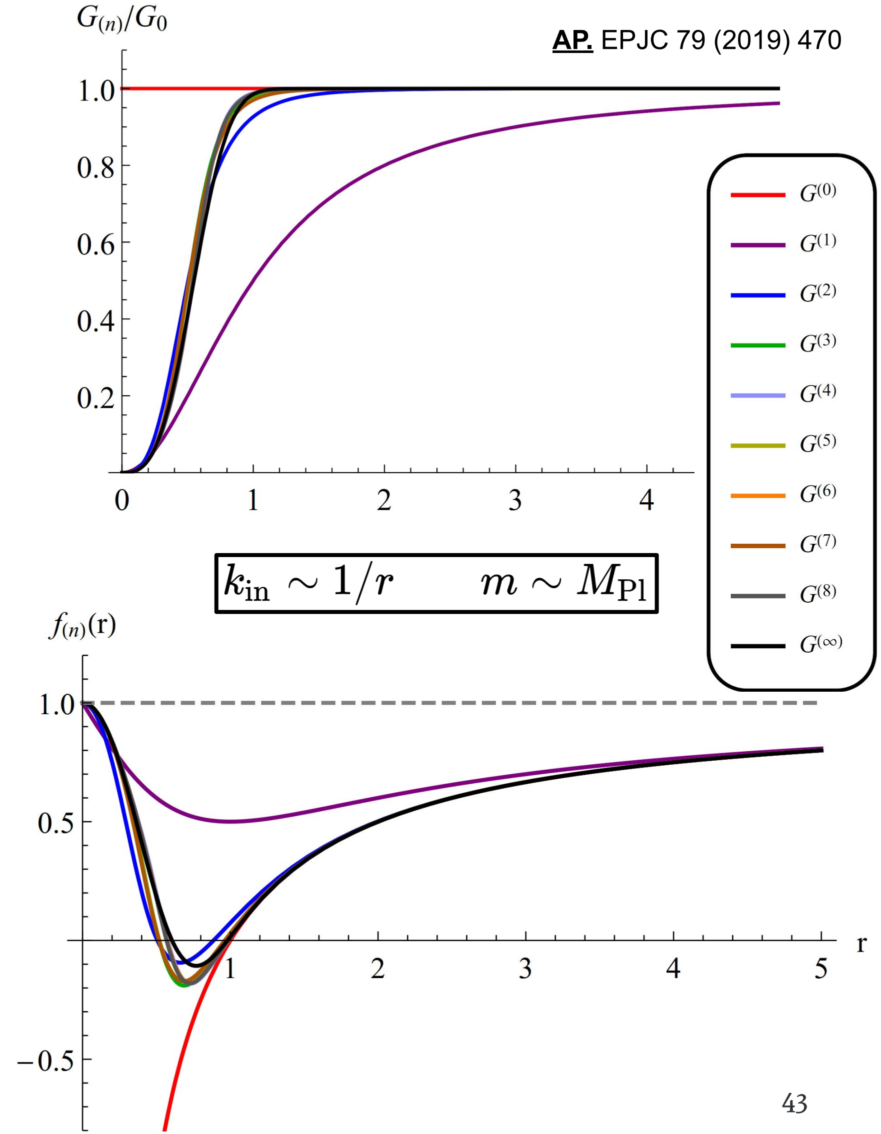
Properties of the solution

➔ Singularity replaced by a de-Sitter core

➔ Number of horizons determined by a *critical mass* of the order of the Planck mass

➔ The Hawking temperature drops to zero when the mass approaches the critical value



Summary

- AS-gravity: Mechanism for constructing a consistent theory of Quantum Gravity

- NGFP providing a well defined UV-completion for the gravitational interaction
⇒ *anti-screening effects* at high energies

- Self-consistent BH solutions can be constructed by relating the RG-scale with the effective self-energy generated by the running of the Newton's coupling

- The procedure converges rapidly to a *self-consistent solution*, which is regular if a UV-attractive fixed point exists:
Asymptotic Freedom → the Newton constant is everywhere zero
Asymptotic Safety → Dymnikova black-hole, regular de-Sitter core
⇒ The appearance spacetime singularity might be related to the perturbative non-renormalizability of General Relativity

- Important generalizations:
    - Cosmological constant (internal consistency)
    - Cosmological solutions (similar mechanism? Bouncing cosmologies?)