The fuzzball proposal for black holes

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The vacuum of the Universe: from cosmology to particle physics

Overview

Approaches to the information paradox

Proposed solutions to the black hole information paradox fall broadly in two classes

- Typical black hole microstates are described by geometries with a smooth horizon but there are non-local effects linking the region inside the horizon with a region arbitrary far from the black hole
  (ER=EPR, Papadodimas-Raju, ...)
- Effective field theory fails at distances of the order of the black hole horizon and the geometry of a typical microstate does not have a smooth horizon
  (Fuzzballs, firewalls, ...)

A holographic perspective

- In some situations, a black hole is dual to an ensemble in a 2D CFT

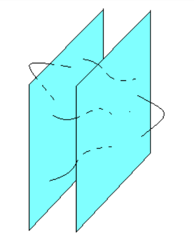
$$\text{Black hole} \xrightarrow{\text{decoupling}} \text{AdS}_3 \xleftrightarrow{\text{holography}} \text{2D CFT}$$

- A b.h. microstate is dual to a <u>"heavy"</u> operator O_H ($\Delta_H \sim c \gg 1$)
  What is the description of O_H when $g_s^2 c \gg 1$?

$g_s^2 c \ll 1$

$g_s^2 c \gg 1$

(EFT)



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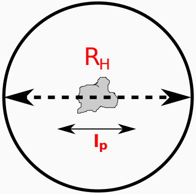

O_H

ds_H^2

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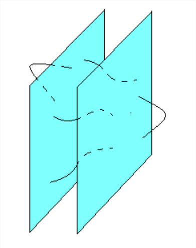
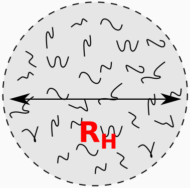
(Fuzzball)



O_H

$\longleftrightarrow$



ds_H^2

Hints from holography

- Microstates can be probed by "light" operators O_L ($\Delta_L \sim O(c^0)$)
- 3-point correlators

$$\langle \bar{O}_H(\infty) O_H(0) O_L(1) \rangle \longleftrightarrow \langle O_L(1) \rangle_{ds_H^2}$$

$\langle O_L(1) \rangle_{ds_H^2}$ are encoded in the asymptotic deviations of ds_H^2 from AdS_3
$\Rightarrow$ ds_H^2 differs "slightly" from the classical black hole already at large distances

- 4-point correlators

$$\langle \bar{O}_H(\infty) O_H(0) O_L(z) \bar{O}_L(1) \rangle \longleftrightarrow \langle O_L(z) \bar{O}_L(1) \rangle_{ds_H^2}$$

$\langle O_L(z) \bar{O}_L(1) \rangle_{ds_H^2}$ is extracted by solving a wave equation in ds_H^2 and it cannot vanish at large Lorentzian time
$\Rightarrow$ ds_H^2 cannot admit waves with imaginary frequencies

Plan of the talk

- The D1-D5-P black hole and the dual CFT
- Construction of the microstate geometries
- Holographic probes and consistency with unitarity
- Outlook and open problems

The D-brane system

Extremal near-horizon limits: 4D

- The extremal 4-charge black hole in type II on $\mathbb{R}^{3,1} \times T^6$

$$\begin{array}{lcl}
\mathrm{D6}_{123456}\,\mathrm{D2}_{12}\,\mathrm{D2}_{34}\,\mathrm{D2}_{56} & \xrightarrow{\text{decoupling}} & \mathrm{AdS}_2 \times S^2 \times T^6 \longleftrightarrow \mathrm{1D\,CFT} \\
\Big\downarrow \mathrm{U-duality} & & \\
\mathrm{KKM}_{12345(6)}\,\mathrm{D1}_5\,\mathrm{D5}_{12345}\,\mathrm{P}_5 & \xrightarrow{\text{decoupling}} & \mathrm{AdS}_3 \times S^2 \times T^5 \longleftrightarrow \mathrm{2D\,CFT}
\end{array}$$

- The 2D CFT is a $(4, 0)$ theory, not well-understood
- The system simplifies if KKM $\to 0$ and $R_6 \to \infty$

$\Rightarrow$

Extremal near-horizon limits: 5D (Strominger, Vafa)

- The extremal 3-charge black hole in type IIB on $\mathbb{R}^{4,1} \times S^1 \times T^4$

$$\mathrm{D1}_5\,\mathrm{D5}_{12345}\,\mathrm{P}_5 \xrightarrow{\text{decoupling}} \mathrm{AdS}_3 \times S^3 \times T^4 \longleftrightarrow \mathrm{2D\,CFT}$$

  with $\mathrm{vol}(T^4) \sim \ell_s^4$ and $R(S^1) \gg \ell_s$

- The 2D CFT is the $(4,4)$ D1D5 CFT with $c = 6n_1 n_5 \equiv 6N \gg 1$
- The CFT has a 20-dim moduli space:
  - free orbifold point $\longleftrightarrow$ $R_{AdS} \ll \ell_s$
  - strong coupling point $\longleftrightarrow$ $R_{AdS} \gg \ell_s$

The D1-D5 CFT

- Symmetries:
  (4,4) SUSY with $SU(2)_L \times SU(2)_R$ R-symmetry $\longleftrightarrow$ S^3 rotations
- The orbifold point: sigma-model on $(T^4)^N/S_N$
  The elementary fields are 4 bosons, 4 fermions and twist fields
- Chiral primary operators: $O_{(j,\bar{j})}$ with $h = j$, $\bar{h} = \bar{j}$ (and their descendants with respect to L_{-n}, J^-_{-n}, $G^-_{-n-1/2}$) are protected
- Spectral flow:
  - NS $\longrightarrow$ R
  - $j \longrightarrow j + \frac{N}{2} \quad , \quad h \longrightarrow h + j + \frac{N}{4}$
  - (anti)CPO $\longrightarrow$ RR ground states with $h = \bar{h} = \frac{N}{4}$

Microstate geometries

A simple D1D5 state

(Maldacena-Maoz et al.)

- States carrying D1-D5 charges are RR ground states ($h = \bar{h} = \frac{N}{4}$)
  Note: $h, \bar{h} \sim c \;\Rightarrow$ "heavy" states $\Rightarrow$ classical geometry
- A simple example:

$$\begin{array}{ccc}
|0\rangle_{\mathrm{NS}} & \xrightarrow{\text{spectral flow}} & |N/2, N/2\rangle_R \\
\downarrow & \downarrow & \downarrow \\
\mathrm{AdS}_3 \times S^3 & \phi \to \phi - \tau\,,\; \psi \to \psi - \sigma & \mathrm{AdS}_3 \times' S^3
\end{array}$$

with (ϕ, ψ) S^3 coordinates and (τ, σ) AdS_3 coordinates

- The geometry dual to the maximally rotating RR ground state $|N/2, N/2\rangle_R$ is $\mathrm{AdS}_3 \times' S^3$ with S^3 non-trivially fibered over AdS_3

The graviton gas

- If O_k is a (anti)CPO of dimension k one can consider its descendants

$$O_{k,m,n,q} \equiv (J_0^+)^m (L_{-1})^n (G^{+1}_{-\frac{1}{2}} G^{+2}_{-\frac{1}{2}})^q \, O_k$$

- Spectral flow maps $O_{k,m,n,q}$ to a D1-D5-P state with $h > \bar{h} = \frac{N}{4}$
- “Semi-classical” states are coherent states

$$|B_1, B_2, \ldots\rangle_{\mathrm{NS}} \equiv \sum_{p_1, p_2, \ldots} (B_1 \, O_{k_1,m_1,n_1,q_1})^{p_1} (B_2 \, O_{k_2,m_2,n_2,q_2})^{p_2} \ldots |0\rangle_{\mathrm{NS}}$$

- When $B_i^2 \sim N \gg 1$ the p_i-sum is peaked for $p_i \approx B_i^2/k$

What is the gravitational description of $|B_1, B_2, \ldots\rangle_{\mathrm{NS}}$?

Superstrata: construction

- Holography associates to O_k a sugra field ϕ_k : $O_k \longleftrightarrow \phi_k$
- At linear order in B_i $|B_1,\ldots\rangle_{\rm NS}$ is a perturbation of the vacuum

$$|0\rangle_{\rm NS} + B_i\, O_{k_i,m_i,n_i,q_i}\, |0\rangle_{\rm NS} \longleftrightarrow \mathrm{AdS}_3 \times S^3 + B_i\, \phi_{k_i,m_i,n_i,q_i}$$

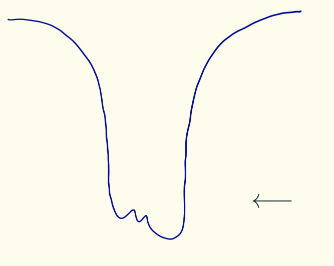
where ϕ_{k_i,m_i,n_i,q_i} solves the linearised sugra eqs. around $\mathrm{AdS}_3 \times S^3$

$$\phi_{k,m,n,0} = \frac{\rho^n}{(\rho^2+1)^{\frac{n+k}{2}}} \sin^{k-m}\theta\, \cos^m\theta\, e^{i\,[(k-m)\phi - m\psi + (k+n)\tau + n\sigma]}$$

- One can extend the linearised solution to an exact solution of the sugra eqs. valid for $B_i^2 \sim N$
- The non-linear extension is non-unique: ambiguities are fixed by imposing regularity

Superstrata: result

- The non-linear solutions are smooth and horizonless
- The solutions are asymptotically $\mathrm{AdS}_3 \times S^3$ but in the interior AdS_3 and S^3 are non-trivially mixed
- The R-sector solutions can be glued back to flat space $\to \mathbb{R}^{4,1} \times S^1$
- There is a continuous family of solutions, parametrised by B_i, for fixed values of the global D1, D5, P charges



$\mathbb{R}^{4,1} \times S^1$

$\mathrm{AdS}_3 \times S^3$

$\longleftarrow$ $r \sim R_{\mathrm{Hor}}$ no horizon!

Microstate geometries: the state of the art

- All D1-D5 geometries are known

  (Lunin, Mathur; Kanitscheider, Skenderis, Taylor)

- The D1-D5-P graviton gas geometries are known

  (Bena, Ceplak, Heidmann, SG, Martinec, Russo, Shigemori, Turton, Warner)

  Note that $S_{\text{graviton gas}} \ll S_{\text{D1}-\text{D5}-\text{P}}$

- Some BPS 4-charge geometries (with no clear CFT dual) are known

  (Bena, Berglund, Gimon, SG, Levi, Martinec, Peet, Saxena, Turton, Warner)

- Some non-BPS 3 and 4-charge geometries (with known and unknown CFT dual) are known

  (JMaRT; Bossard, Katmadas, Turton, et al.)

Holographic probes

HHL correlators (Kanitscheider, Skenderis, Taylor; SG, Moscato, Rawash, Russo, Turton)

- Consider

$$\langle O_L \rangle_H \equiv \langle \bar{O}_H(\infty) O_H(0) O_L(1) \rangle$$

  with

  - $O_H \xrightarrow{\text{spectral flow}} \sum_{p_1,\ldots} (B_1 O_{k_1,m_1,n_1,q_1})^{p_1} \ldots \xleftrightarrow{\text{holography}} ds^2_H$
  - $O_L = O_k \qquad \xleftrightarrow{\text{holography}} \phi_k$

- $\langle \bar{O}_H O_H O_L \rangle$ do not depend on the CFT moduli $\Rightarrow$ One can extract $\langle O_k \rangle_H$ from the geometry ds^2_H

$$\phi_k \overset{\rho\to\infty}{\longrightarrow} \rho^{-k} \langle O_k \rangle_H$$

  and compare with the value computed in the orbifold CFT

- What we learn:
  - Microstate geometries must have non-trivial multiple moments
  - Non-trivial checks of the sugra construction, including the non-linear completion

HHLL correlators

- How to compute holographically

$$\mathcal{C}_H(z,\bar z) \equiv \langle \bar O_H(\infty) O_H(0) O_L(z,\bar z) \bar O_L(1)\rangle$$

- $O_L(z,\bar z) \equiv O_k(z,\bar z) \longleftrightarrow \phi_k(\rho; z,\bar z)$
- Solve the linearised e.o.m. for ϕ_k in the background $ds^2_H \longleftrightarrow O_H$
- Pick the non-normalisable solution such that
  - at the boundary ($\rho \to \infty$)

$$\phi_k(\rho; z,\bar z) \overset{\rho\to\infty}{\longrightarrow} \underbrace{\delta(z-1)}_{\text{source for } \bar O_L(1)}\,\rho^{k-2} + \overbrace{b(z,\bar z)}^{\text{vev of } O_L(z,\bar z)}\,\rho^{-k}$$

  - in the interior ($\rho \to 0$) $\phi(\rho; z,\bar z)$ is regular
- The correlator is given by

$$\mathcal{C}_H(z,\bar z) = \langle O_H | O_L(z,\bar z) \bar O_L(1) | O_H\rangle = b(z,\bar z)$$

A simple example (Bombini, Galliani, SG, Moscato, Russo)

- We take

$$O_H = \sum_p (B\, O_1)^p \quad , \quad O_L = O_1$$

- O_H flows to a RR ground state $\Rightarrow$ $P = 0$
- The ensemble of RR ground states corresponds to a "small black hole" (massless limit of BTZ)

$$\frac{ds^2}{R^2_{\mathrm{AdS}}} = \frac{d\rho^2}{\rho^2} + \rho^2(-d\tau^2 + d\sigma^2) + d\Omega_3^2$$

- The geometry ds^2_H dual to O_H approximates the small black hole geometry in the limit $B^2 \to N$
- Computing $\mathcal{C}_H$ for heavy states with $P \neq 0$ and finite B is harder, but see also Bena, Heidmann, Monten, Warner

Result

Gravity

$$\mathcal{C}_H = \alpha\, e^{-i\tau} \sum_{l\in\mathbb{Z}} e^{il\sigma} \sum_{n=1}^{\infty} \frac{\exp\left[-i\alpha\sqrt{(|l|+2n)^2 + \frac{(1-\alpha^2)l^2}{\alpha^2}}\tau\right]}{\sqrt{1+\frac{1-\alpha^2}{\alpha^2}\frac{l^2}{(|l|+2n)^2}}} + N(1-\alpha^2)e^{-i\tau}$$

with $z = e^{i(\tau+\sigma)}, \bar{z} = e^{i(\tau-\sigma)}$, $\alpha = \left(1-\frac{B^2}{N}\right)^{1/2}$

Free CFT

$$\mathcal{C}_H = \frac{1}{|z||1-z|^2} + \frac{B^2}{2N}\frac{|z|^2+|1-z|^2-1}{|z||1-z|^2} + \frac{(N-B^2)B^2}{N}\left(1-\frac{1}{N}\right)\frac{1}{|z|}$$

The late time behaviour of the HHLL correlator

- We focus on the limit $B^2 \to N \Leftrightarrow \alpha \to 0$
  in which ds^2_H approximates the "small b.h."
- In this limit the series giving $\mathcal{C}_H$ is dominated by terms with $n \gg \frac{|l|}{2\alpha}$:

$$\mathcal{C}_H \sim e^{-i\tau} \left[\frac{1}{1 - e^{i(\sigma-\tau)}} + \frac{1}{1 - e^{-i(\sigma+\tau)}} - 1 \right] \frac{\alpha}{1 - e^{-2i\alpha\,\tau}}$$

- The time-dependence of the correlator is controlled by α:
  - for $\tau \ll \alpha^{-1}$ one has $\mathcal{C}_H \sim \tau^{-1}$;
    this is the same behaviour of the 2-point function in the "small b.h."
  - for $\tau \gtrsim \alpha^{-1}$ $\mathcal{C}_H$ stops decreasing with τ and oscillates
- Correlators in a pure or thermal state in a unitary theory with finite entropy do not vanish at late times

The late-time behaviour of $\mathcal{C}_H$ is consistent with unitarity already at large c

Summary and outlook

Results

- At strong coupling *some* heavy states in the black hole ensemble are described by smooth horizonless geometries
- HHL correlators can be used to construct and check the map between states and geometries
- Microstate geometries contain non-trivial informations on HHLL correlators
- If probed for a short time microstates are indistinguishable from the black hole, but for sufficiently long times microstates deviate from the black hole and produce correlators that are consistent with unitarity already at large c
- These results are solid for *susy* states: there is a string-motivated mechanism to have non-trivial structure at the horizon scale

Open problems

- Classical supergravity works well for atypical states in the black hole ensemble
- For some observables, deviations from a typical state and the classical black hole should be exponentially suppressed in the entropy
- How much of our analysis can be extended to typical states?
- And what about microstates of non-BPS black holes?
- Can one make (semi)quantitative predictions that could be tested experimentally (GW, EHT)?
  - At which scale the geometry of a typical microstate starts to deviate from the classical black hole?
  - What is the dynamics controlling the interaction between a typical non-BPS fuzzball and infalling particles? How absorptive is the fuzzball surface?

Outlook

- Even if the general fuzzball paradigm is correct, it is possible that classical supergravity probes cannot resolve the structure of typical states
- Do we have quantitative tools to describe microstates beyond supergravity?
- Does one need to resort to full string theory?

(Massai, Martinec, Turton)