QFT and Effective Field Theories

Drop me a line if you need anything!

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High Energy Theory

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What is (not) this course?

- Not an introduction to QFT (basic knowledge is assumed)
- No formal, rigorous proofs but plausibility arguments.
- Emphasis is on EFTs: we'll discuss the QFT that we need for EFTs
- Not a complete EFT course: focus on particle physics (mostly BSM)
- We will sacrifice completeness for detailed specific examples (emphasis not only on concepts but mostly on calculability)
- Most of the calculations will be done in tutorials (can be done by hand but we will also use computer tools)
- If there is ANYTHING you don't understand please stop me and ask.
- Use some of the (very useful) resources: Skiba (TASI 2009, arXiv 1006.2142), Manohar (Les Houches 2017, arXiv 1804.05863), Cohen (TASI 2018, arXiv 1903.03622), Pich (ph/9806303) … and many others

Why EFT?

- Because nature decouples! Observations always have a finite precision. Given that precision we only need to care about certain degrees of freedom, symmetries and dynamics.
- Because it's easier: EFTs split complicated multi-scale calculations into series of simpler single-scale calculations.
- Because we have to (I): Sometimes we do not know (or can't compute) the dynamics at high energies. EFTs allow us to parametrize the low energy effects of such unknown UV dynamics.
- Because we have to (II): In multi-scale problems large logs can ruin perturbation theory (even in renormalizable models) these large logs can only be resummed by using EFTs and RGE.

What is EFT?

- It’s the one thing that we constantly do in physics: dimensional analysis + (Taylor) perturbative expansion … with a few subtleties from QFT.
  - At least in some cases we can prove that the result is analytic (and therefore can genuinely been expanded).
  - Locality and renormalization: we have to perform the expansion carefully
- Like any perturbative expansion:
  - It is useful because experimental measurements have a finite precision.
  - It’s usefulness (range of validity) depends on the size of the expansion parameter (and the nature of the expansion itself).

Observables in QFT

- The relevant observables in particle physics are given by S-matrix elements, which can be computed from correlators via the LSZ reduction formula

$$\lim_{\substack{q_i^2\to m^2\\ q_i^0>0}}\ \lim_{\substack{p_j^2\to m^2\\ p_j^0>0}}\ \prod_{i=1}^{m}\left(q_i^2-m^2\right)\prod_{j=1}^{n}\left(p_j^2-m^2\right)G(q_1,\ldots,q_m;p_1,\ldots,p_n)$$
$$=\prod_{i=1}^{m}\left(i\sqrt{\mathcal{R}_i}\right)\prod_{j=1}^{n}\left(i\sqrt{\mathcal{R}_j}\right)\quad {}_{\mathrm{out}}\langle q_1,\ldots,q_m|p_1,\ldots,p_n\rangle_{\mathrm{in}}\,,$$

where the correlator is defined as

$$G(q_1,\ldots,q_m;p_1,\ldots,p_n)=\prod_{i=1}^{m}\int \mathrm{d}^4y_i\ e^{iq_i\cdot y_i}\prod_{j=1}^{n}\int \mathrm{d}^4x_j\ e^{-ip_j\cdot x_j}\ \langle 0|T\left\{\phi(y_1)\ldots\phi(y_m)\phi(x_1)\ldots\phi(x_n)\right\}|0\rangle$$

- This is valid for any interpolating field

$$_{\mathrm{in}}\langle k|\phi(x)|\Omega\rangle=\sqrt{\mathcal{R}}\mathrm{e}^{\mathrm{i}k\cdot x}\Leftrightarrow D_F(p)=\frac{\mathrm{i}\mathcal{R}}{p^2-m^2}+\ldots$$

Observables in QFT

- Correlators can be computed in perturbation theory via

$$\langle\Omega|T\{\phi(x_1)\ldots\phi(x_n)\}|\Omega\rangle=\frac{\langle 0|T\{\phi_0(x_1)\ldots\phi_0(x_n)\,e^{i\int d^4x\,\mathcal{L}_{int}[\phi_0(x)]}\}|0\rangle}{\langle 0|T\{e^{i\int d^4x\,\mathcal{L}_{int}[\phi_0(x)]}\}|0\rangle}$$

$$=\langle 0|T\{\phi_0(x_1)\ldots\phi_0(x_n)\,e^{i\int d^4x\,\mathcal{L}_{int}[\phi_0(x)]}\}|0\rangle_{\text{no vac. bubbles.}}$$

and Wick's theorem

$$T\{\phi_0(x_1)\ldots\phi_0(x_n)\}=:\phi_0(x_1)\ldots\phi_0(x_n):+\text{ all possible contractions}$$

$$\text{contraction }\ \overbrace{\phi_0(x_1)\phi_0(x_2)}\equiv D_F(x_1-x_2)=\text{Feynman propagator.}$$

Which Lagrangian?

- Our QFT will be defined by the Lagrangian, a sum of local, invariant (gauge, Lorenzt, …) operators built with a finite number of fields and their (covariant) derivatives.
- Which operators? In principle all local invariant ones, each with an arbitrary coefficient called Wilson coefficient. (We will see that some operators are more relevant than others.)
- Quadratic operators are special: they fix the global scale (kinetic term) via canonical normalization or fix the on-shell condition (mass term), plus we know how to solve them (free theory = harmonic oscillator).

$$\mathcal{L}_{kin} = \frac{1}{2}\kappa_S(\partial_\mu S)^2 + \kappa_\varphi|\partial_\mu\varphi|^2 + \bar\Psi\kappa_\Psi i\not\partial\Psi - \frac{1}{4}\kappa_A F_{\mu\nu}^2 \longrightarrow \frac{1}{2}(\partial_\mu S)^2 + |\partial_\mu\varphi|^2 + \bar\Psi i\not\partial\Psi - \frac{1}{4}F_{\mu\nu}^2$$

$S \to (\kappa_S)^{-\frac{1}{2}} S$ $\quad \Psi \to \kappa_\Psi^{-\frac{1}{2}}\Psi$ $\quad A_\mu \to \kappa_A^{-\frac{1}{2}} A_\mu$

$\varphi \to (\kappa_\varphi)^{-\frac{1}{2}}$ $\quad \bar\Psi \to \bar\Psi\kappa_\Psi^{-1/2}$

These field redefinitions have to be applied to all terms in the Lagrangian.

$$\mathcal{L}_{int} = -\lambda_\Psi \bar\Psi S\Psi \longrightarrow -\frac{\lambda_\Psi}{\kappa_\Psi\sqrt{\kappa_S}}\bar\Psi S\Psi$$

Which Lagrangian?

$[\hbar = c = 1]$

- Quadratic operators also fix the (mass) dimension of the fields

Let's assume we are in D space-time dimensions (D=4 usually, but will be D=4-2ε in dim. reg. that we'll use for loop calculations).

The mass dimensions of the fields are obtained from the kinetic terms and $[S]=0$, $[x]=-1$, $[\partial_\mu]=[p_\mu]=[m]=1$

$$\Rightarrow [S] = \left[\int d^Dx\, \mathcal{L}\right] = -D + [\mathcal{L}] \Rightarrow [\mathcal{L}] = D$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + |\partial_\mu \varphi|^2 - m^2|\varphi|^2$$
$$+ \bar{\Psi}(i\not{\partial} - m)\Psi - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m^2A_\mu^2 + \cdots$$

Which Lagrangian?

$[\hbar = c = 1]$

- Quadratic operators also fix the (mass) dimension of the fields

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 + |\partial_\mu \varphi|^2 - m^2 |\varphi|^2$$

$$+ \bar{\Psi}(i\not{\partial} - m)\Psi - \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} m^2 A_\mu^2 + \dots$$

$$D = [\partial_\mu \phi]^2 = 2 + 2[\phi] \Rightarrow [\phi] = \frac{D-2}{2} = [\varphi] = [A_\mu] = 1 - \epsilon$$

$$D = [\bar{\Psi} i \not{\partial} \Psi] = 1 + 2[\Psi] \Rightarrow [\Psi] = \frac{D-1}{2} = \frac{3}{2} - \epsilon$$

- For any field f (boson or fermion) $[f]_{D=4-2\epsilon} = [f]_{D=4} - \epsilon$

Which Lagrangian?

$[\hbar = c = 1]$

- Quadratic operators also fix the (mass) dimension of the fields

Interactions

$$1 = [D_\mu] = [\partial_\mu - i g A_\mu] \Rightarrow 1 = [g A_\mu] = [g] + [A_\mu] = [g] + \frac{D-2}{2}$$

$$\Rightarrow [g] = 1 - \frac{D-2}{2} = \frac{4-D}{2} = \epsilon$$

$$D = [y \bar{\psi} \psi \varphi] = [y] + (D-1) + \frac{D-2}{2} \Rightarrow [y] = 1 - \frac{D-2}{2} = \frac{4-D}{2} = \epsilon$$

$$D = [\lambda \varphi^4] = [\lambda] + 4 \frac{D-2}{2} = [\lambda] + 2D - 4 \Rightarrow [\lambda] = 4 - D = 2\epsilon$$

Higher dimensional operators

$$D = [C_6 \varphi^6] = [C_6] + 6 \frac{D-2}{2} = [C_6] + 3D - 6 \Rightarrow [C_6] = 6 - 2D$$

$$= -2 + 4\epsilon$$

In $D = 4$ $[C_6] = -2$, $[\varphi^6] = 6$

Which Lagrangian?

$[\hbar = c = 1]$

- Quadratic operators also fix the (mass) dimension of the fields

We can ensure that couplings have the same mass dimension in $D=4$ as in $D=4-2\epsilon$ by compensating with powers of a dimensionful scale μ.

$[\mu^{-\epsilon} g] = 0$ in $D=4-2\epsilon$

in practice, when working in D dimensions we write $g \to \mu^{\epsilon} g$ to have $[g]=0$.

Similarly $c_6 \to \mu^{4\epsilon} c_6$ to have $[c_6] = -2$.

In general: $c_i f_1 \ldots f_{e_i} \Rightarrow c_i \to \mu^{n_i \epsilon} c_i \Rightarrow n_i = (e_i - 2)$

Regularization and renormalization

- Feynman rules instruct us to integrate over loop momenta, resulting sometimes in divergent expressions. These require regularization and renormalization in order to make sensible quantitative predictions for physical observables.
- There is a whole machinery for loop calculations that is worth mastering but here we will either use computer tools to do the loop calculations or use special techniques useful in EFT calculations.
- We will use dimensional regularization:
  - Analytic continuation from $D=4$ to $D=4-2\varepsilon$.
  - Divergences appear as poles at $\varepsilon=0$.

Regularization and renormalization

- Some useful properties in dimensional regularization:
  - Scaleless integrals vanish

$$\int \frac{d^Dk}{(2\pi)^D}\frac{1}{k^4} = \frac{\mathrm{i}}{16\pi^2}\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right)$$

all others identically 0

  - Tadpole (and higher)

$$\mu^{2\epsilon}\int \frac{d^Dk}{(2\pi)^D}\frac{1}{k^2 - M^2} = \frac{\mathrm{i}M^2}{16\pi^2}\left[\frac{1}{\bar{\epsilon}} + 1 + \log\left(\frac{\mu^2}{M^2}\right) + \mathcal{O}(\epsilon)\right]$$

$$\int \frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2 - M^2)^{n+1}} = \frac{D - 2n}{2n}\frac{1}{M^2}\int \frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2 - M^2)^n}$$

$$\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} - \gamma_E + \log(4\pi)$$

Regularization and renormalization

- Some other properties we will use:
  - Partial fractioning

$$\frac{1}{(k^2-M_1^2)(k^2-M_2^2)} = \frac{1}{M_1^2-M_2^2}\left[\frac{1}{k^2-M_1^2} - \frac{1}{k^2-M_2^2}\right], \quad M_1 \neq M_2$$

  - Propagator expansions

$$\frac{1}{(k+p)^2 - M^2} = \frac{1}{k^2-M^2}\left[1 - \frac{p^2+2k\cdot p}{(k+p)^2-M^2}\right]$$

useful when $k^2 \sim M^2 \gg p^2$

$$\frac{1}{(k+p)^2 - m^2} = \frac{1}{k^2}\left[1 - \frac{p^2+2k\cdot p-m^2}{(k+p)^2-m^2}\right]$$

useful when $k^2 \gg m^2 \sim p^2$

Regularization and renormalization

- Before renormalizing let's discuss what UV divergences look like.
  - Up to sub-divergences, UV divergences coming from loop integrals are proportional to polynomials in external momenta.

A loop integral that can have a potentially non-polynomial dependence on external momenta is generically of the form

$$I(p) = \int_0^\infty dk \, \frac{k}{k+p}$$

This is a linearly divergent integral (it's superficial degree of divergence sdod is +1).

Regularization and renormalization

Every time we take a derivative wrt an external momentum we reduce the degree of divergence by one unit. Taking 2 derivatives results in a finite integral

$$I''(p) = \int_0^\infty dk \, \frac{2k}{(k+p)^3} = \frac{1}{p}$$

We can now integrate over p with (divergent) unknown integration constants

$$\int I''(p)\, dp = \ln p + c_1$$

$$\int\int I''(p)\, dp = \int (\ln p + c_1)\, dp = p \log p - p + c_1 p + c_2$$

$$\Rightarrow \quad I(p) = p \log p + p(c_1 - 1) + c_2$$

divergences can only live here.

Regularization and renormalization

This result is general, we just need to take $n+1$ derivatives (n=sd of div.) and we get a finite integral, which is not necessarily a polynomial in p. Divergences come from the integration constants upon integrating this finite result $n+1$ times and therefore always a polynomial in external momenta.

But polynomials in external momenta is what local operators produce ⇒ all UV divergences (after subdivergences have been subtracted)
⇒ all UV divergences can be absorbed in the WC of local operators.

Regularization and renormalization

- The practical idea behind renormalization is that terms in the Lagrangian are not observable (and could therefore be anything, even infinity). Each Wilson coeff. has to be fixed by computing a physical observable that depends on it.
- We will use $\overline{\text{MS}}$ renormalization, that eliminates only the $1/\bar{\epsilon}$ UV divergences (renormalized WC still have to be fixed by experiment).
- The original terms in the Lagrangian are called “bare” terms (fields and WCs):

$$\mathcal{L} = \sum_i C_i^{(0)} O_i^{(0)}$$

- Bare objects are written in terms of renormalized ones times renormalization constants:

$$C_i^{(0)} = \mu^{n_i \epsilon} Z_i C_i \qquad\qquad \phi^{(0)}(x) = \sqrt{Z_\phi}\phi(x)$$

Regularization and renormalization

- Focusing on 1-loop renormalization we can write $Z_i = 1 + \delta_i$
- The bare Lagrangian is then written as

$$\mathcal{L} = \sum_i C_i^{(0)} O_i^{(0)} = \sum_i \mu^{n_i \epsilon} \Big[\underbrace{C_i O_i}_{\text{Renormalized Lagrangian}} + \underbrace{(\delta_i + \overset{\text{Wave function renormalization}}{\delta_i^F}) C_i O_i}_{\text{Counterterm Lagrangian}} \Big]$$

- The counterterms $\delta_i, \ \delta_i^F$,are fixed by cancelling the $1/\bar{\epsilon}$ UV divergences (which are local operators and we have written all of them).
- Sometimes it's useful to write the mass dimension of the WCs explicitly

$$C_i = \frac{c_i}{\Lambda^{d_i - 4}}, \qquad d_i = [O_i] \ (\text{in D=4})$$

Power counting and renormalizability

- Let's consider the contribution of a single insertion of an operator of dimension *d* in an low energy amplitude normalized to be dimensionless

$$\mathcal{M} \sim \left(\frac{p}{\Lambda}\right)^{d-4}, \qquad [p \text{ any low energy scale or mass}]$$

- The higher the dimension of the operator the smaller its contribution at low energies.
- The general power counting equation is $\mathcal{M} \sim \left(\frac{p}{\Lambda}\right)^{n}, \quad n = \sum_i (d_i - 4)$
- Also true at loop level for mass-independent regularization schemes.
- Operators of dimension higher than 4 have suppressed effects at low energies. Given a finite experimental precision we only need operators up to certain dimension (and there are a finite number of these).

Power counting and renormalizability

- Let's sketch renormalizability of renormalizable theories (operators of dimension 4 or less)

Let's include one non-renormalizable operator O_5 with $d=5$

Then two insertions of O_5 in a loop calculation will lead to a UV divergence $\sim \left(\frac{p}{\Lambda}\right)^2$, which needs a counterterm corresponding to a local operator of dimension 6, O_6.

Including more insertions we need higher dim operators ⇒ we need an infinite # of operators to absorb all divergencies.

If we only include renormalizable operators with $d \leq 4$, more insertions give divergencies of the same or smaller dimension, but the number of operators of dimension ≤ 4 is finite ⇒ renormalizable theories (can be renormalized with a finite # of operators).

Power counting and renormalizability

- What about non-renormalizable theories (those with operators of dimension larger than 4)?
  - Formally they are non-renormalizable: more insertions require higher-dimensional operators which themselves induce even higher-dimensional divergencies so that an infinite number of counterterms are required to renormalize the theory.
  - In practice, given the finite precision of experimental data, we only need to consider operators up to certain dimension.
  - An EFT is the set of all allowed local operators with mass dimension less than some maximum one. This theory will generate divergences of higher dimension but the corresponding operators produce such a small phenonenological effect that they are irrelevant and therefore we don't need to consider them.

In practice, non-renormalizable theories are as good for loop calculations as renormalizable theories (only a finite number of counterterms are needed to renormalize them).

Redundancies and EoM

[Arzt, ph/9304230; Criado, Pérez-Victoria, 1811.09413]

- Let’s consider the following Lagrangian

$$\mathcal{L}_\varphi = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\eta}{4!}\varphi^4 - c_1\varphi^6 + c_2\varphi^3\partial^2\varphi,$$

- And perform the field redefinition $\varphi \to \varphi' = \varphi + c_2\varphi^3$ Still an interpolating field

- The resulting Lagrangian is

$$\begin{aligned}\mathcal{L}_\varphi \;&\to\; \frac{(\partial_\mu\varphi')^2}{2} - c_2\varphi'^3\partial^2\varphi' - \frac{m^2}{2}\varphi'^2 - c_2 m^2\varphi'^4 - \frac{\eta}{4!}\varphi'^4 - \frac{\eta}{3!}c_2\varphi'^6 - c_1\varphi'^6 + c_2\varphi'^3\partial^2\varphi' + \\ &= \frac{(\partial_\mu\varphi')^2}{2} - \frac{m^2}{2}\varphi'^2 - (\frac{\eta}{4!} + c_2 m^2)\varphi'^4 - (c_1 + \frac{\eta c_2}{3!})\varphi'^6 + \dots,\end{aligned}$$

- The last operator has disappeared! But the physics is the same.

- This field redefinition is equivalent, at the linear level, to using EoM of L_4 into L_6

$$c_2 O_2 = c_2\varphi^3\partial^2\varphi \to c_2\varphi^3[-m^2\varphi - \eta/3!\varphi^3]$$

- Operators that can be eliminated via EoM are called redundant and are not necessary to compute physical observables (but are to compute off-shell quantities). For details see [Criado, Pérez-Victoria, 1811.09413]

Redundancies in 4D: evanescent operators

- Some properties are only valid in D=4. Corrections of order ϵ can hit a pole and give a finite (possibly ambiguous – scheme dependence –) “rational” contribution [Dekens, Stoffer 1908.05295]

$$
\begin{aligned}
P_L\gamma^\mu\gamma^\nu P_L \otimes P_L\gamma_\mu\gamma_\nu P_L &= (4-2\epsilon)P_L \otimes P_L - P_L\sigma^{\mu\nu}P_L \otimes P_L\sigma_{\mu\nu}P_L\,, \\
P_L\gamma^\mu\gamma^\nu P_L \otimes P_R\gamma_\mu\gamma_\nu P_R &= 4(1+a_{\rm ev}\epsilon)P_L \otimes P_R + E^{(2)}_{LR}\,, \\
P_R\gamma^\mu\gamma^\nu\gamma^\lambda P_L \otimes P_R\gamma_\mu\gamma_\nu\gamma_\lambda P_L &= 4(4-b_{\rm ev}\epsilon)P_R\gamma^\mu P_L \otimes P_R\gamma_\mu P_L + E^{(3)}_{LL}\,, \\
P_R\gamma^\mu\gamma^\nu\gamma^\lambda P_L \otimes P_L\gamma_\mu\gamma_\nu\gamma_\lambda P_R &= 4(1+c_{\rm ev}\epsilon)P_R\gamma^\mu P_L \otimes P_L\gamma_\mu P_R + E^{(3)}_{LR}\,, \\
P_L\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma P_L \otimes P_L\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\sigma P_L &= 32(2-3d_{\rm ev}\epsilon)P_L \otimes P_L \\
&\quad - 8(2-e_{\rm ev}\epsilon)P_L\sigma^{\mu\nu}P_L \otimes P_L\sigma_{\mu\nu}P_L + E^{(4)}_{LL}\,, \\
P_L\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma P_L \otimes P_R\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\sigma P_R &= 16(1+8f_{\rm ev}\epsilon)P_L \otimes P_R + E^{(4)}_{LR}\,,
\end{aligned}
$$

[Herrlich, Nierste ph-9412375]

- The one-loop matching, and the RGEs from two loops depend on the coefficients of the evanescent operators

Redundancies in 4D: evanescent operators

- There is some freedom in defining the evanescent structures but some terms have to be included by consistency. [Fuentes-Martin, et al 2211.09144]
- Example Fierz identities (valid in 4D) for tree-level generated operators.

$$\mathcal{L} \supset \mathcal{L}_{\rm SM} + D_\mu\Phi^\dagger D^\mu\Phi - M_\Phi^2\,\Phi^\dagger\Phi - \left(y_{\Phi e}^{pr}\,\bar\ell_p\Phi e_r + \text{h.c.}\right)$$

Tree level matching

$$[R_{\ell e}]^{prst} = (\bar\ell_p e_r)(\bar e_s \ell_t)\,, \qquad [c_{\ell e}]^{prst} = \frac{y^{pr}_{\Phi e}\, y^{*ts}_{\Phi e}}{M_\Phi^2}$$

Going to the physical basis

$$[R_{\ell e}]^{prst} = \left(-\frac{1}{2}(\bar\ell_p\gamma_\mu\ell_t)(\bar e_s\gamma^\mu e_r) \equiv -\frac{1}{2}[Q_{\ell e}]^{ptsr}\right) + [E_{le}]^{prst}$$

Does not generate dipoles

Does generate dipoles

Can be fixed with a finite renormalization

$$\Delta[C_{eB}]^{pr} = \frac{3g_Y y_e^{ts}}{128\pi^2}[c_{\ell e}]^{prst} \qquad \Delta[C_{eW}]^{pr} = -\frac{g_L y_e^{ts}}{128\pi^2}[c_{\ell e}]^{prst}\,.$$

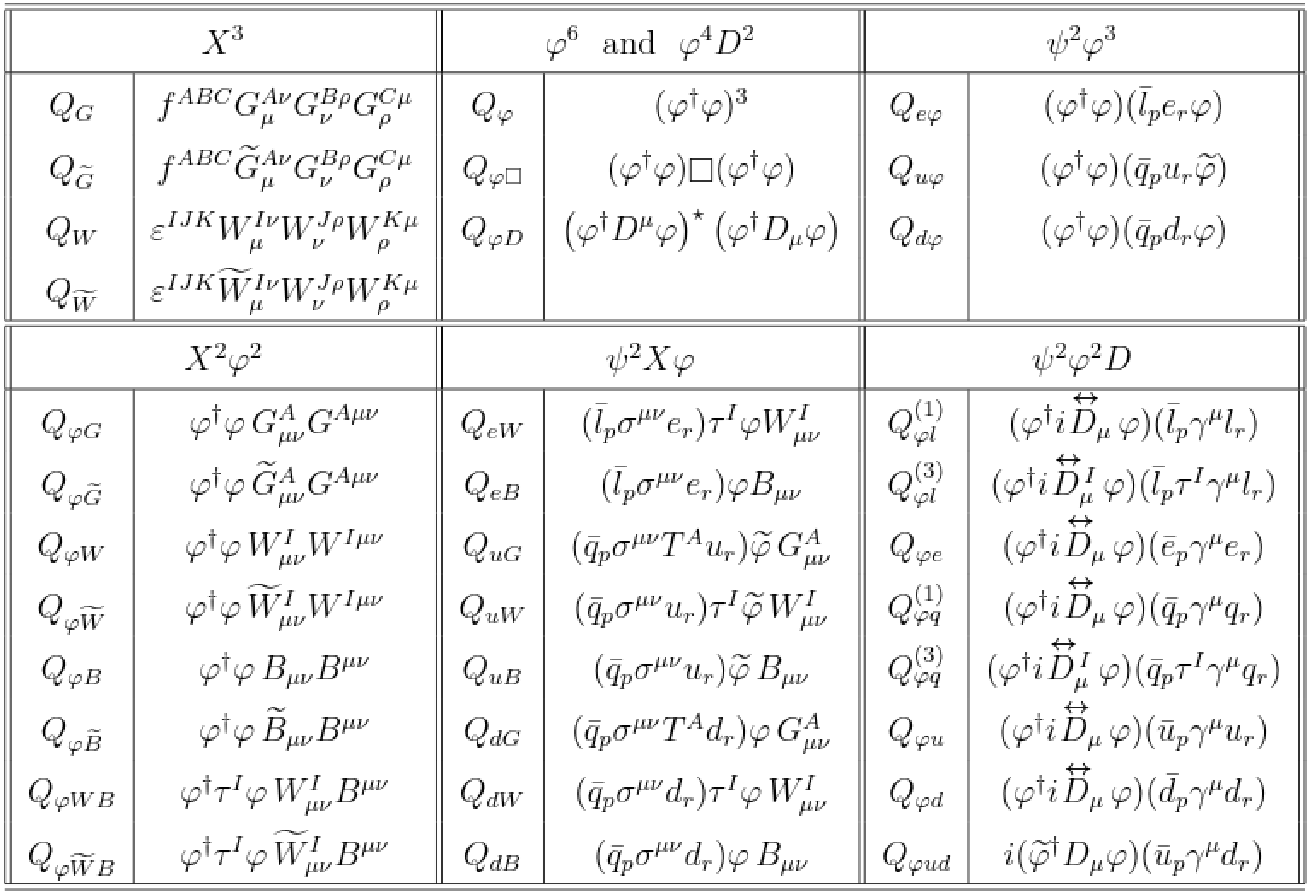
Bases in EFTs

- Which basis should we use?
  - We can always use integration by parts (momentum conservation).
  - We can use 4D properties for tree-level calculations (no evanescent) or one-loop RGEs (only interested in divergent terms).
  - We can use EoM (field redefinitions) when computing on-shell quantities (minimal basis).
  - We have to include redundant operators when computing off-shell quantities (Green basis).
- It is non-trivial to build minimal or Green bases but they have to be built only once for each EFT (and not always).

Bases in EFTs

- Minimal (Warsaw) basis (SM EFT dim 6)

[Grzadkowski et al 1008.4884]

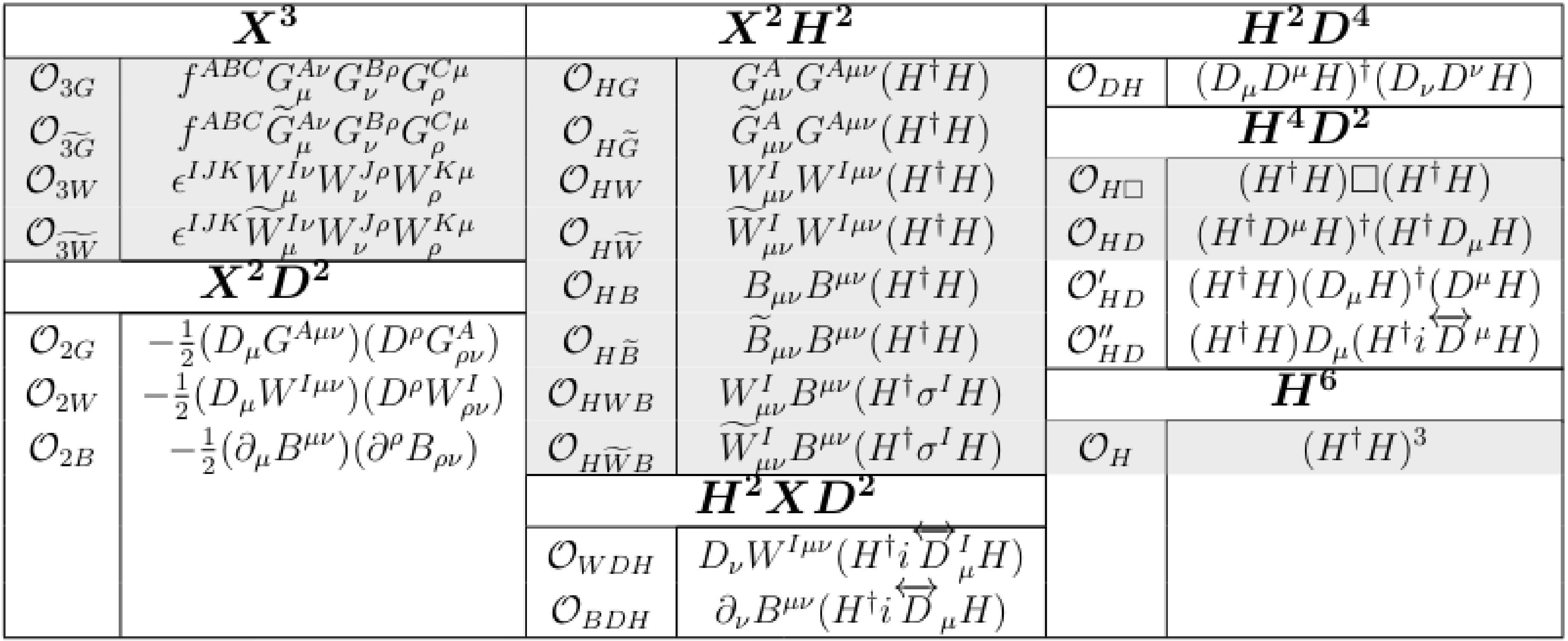


X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Bases in EFTs

- Green basis (SM EFT dim 6)

[Gherardi, Marzocca, Venturini 2003.12525]



X^3	
$\mathcal{O}_{3G}$	$f^{ABC}G^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho}$
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}^{I\nu}_{\mu}W^{J\rho}_{\nu}W^{K\mu}_{\rho}$

X^2D^2	
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_{\mu}G^{A\mu\nu})(D^{\rho}G^{A}_{\rho\nu})$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_{\mu}W^{I\mu\nu})(D^{\rho}W^{I}_{\rho\nu})$
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_{\mu}B^{\mu\nu})(\partial^{\rho}B_{\rho\nu})$

X^2H^2	
$\mathcal{O}_{HG}$	$G^{A}_{\mu\nu}G^{A\mu\nu}(H^{\dagger}H)$
$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^{A}_{\mu\nu}G^{A\mu\nu}(H^{\dagger}H)$
$\mathcal{O}_{HW}$	$W^{I}_{\mu\nu}W^{I\mu\nu}(H^{\dagger}H)$
$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^{I}_{\mu\nu}W^{I\mu\nu}(H^{\dagger}H)$
$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^{\dagger}H)$
$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^{\dagger}H)$
$\mathcal{O}_{HWB}$	$W^{I}_{\mu\nu}B^{\mu\nu}(H^{\dagger}\sigma^{I}H)$
$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^{I}_{\mu\nu}B^{\mu\nu}(H^{\dagger}\sigma^{I}H)$

H^2XD^2	
$\mathcal{O}_{WDH}$	$D_{\nu}W^{I\mu\nu}(H^{\dagger}i\overleftrightarrow{D}^{I}_{\mu}H)$
$\mathcal{O}_{BDH}$	$\partial_{\nu}B^{\mu\nu}(H^{\dagger}i\overleftrightarrow{D}_{\mu}H)$

H^2D^4	
$\mathcal{O}_{DH}$	$(D_{\mu}D^{\mu}H)^{\dagger}(D_{\nu}D^{\nu}H)$

H^4D^2	
$\mathcal{O}_{H\Box}$	$(H^{\dagger}H)\Box(H^{\dagger}H)$
$\mathcal{O}_{HD}$	$(H^{\dagger}D^{\mu}H)^{\dagger}(H^{\dagger}D_{\mu}H)$
$\mathcal{O}'_{HD}$	$(H^{\dagger}H)(D_{\mu}H)^{\dagger}(D^{\mu}H)$
$\mathcal{O}''_{HD}$	$(H^{\dagger}H)D_{\mu}(H^{\dagger}i\overleftrightarrow{D}^{\mu}H)$

H^6	
$\mathcal{O}_{H}$	$(H^{\dagger}H)^{3}$

On gauge invariance: Background field method

[Abbott, NPB185 (1981)]

- When computing in gauge theories we have to fix the gauge, the quantum theory is no longer gauge invariant but just BRST invariant.
- This is enough to get gauge invariant results for physical quantities but not for non-physical ones (off-shell Green functions, counterterms, …).
- We split the fields into classical background fields and quantum fluctuations and fix the gauge for the latter (leaving the theory invariant under gauge transformations of the background fields).
- Background fields never appear in loops, quantum fields can only be in loops.
  - Off-shell Green functions, UV divergences, are explicitly gauge invariant.
  - The covariant derivative does not renormalize (divergence must be proportional to $(F^a_{\mu\nu})^2$ but

$$(F^a_{\mu\nu})_0 = \sqrt{Z_A}[\partial_\mu A_\nu - \partial_\nu A_\mu + Z_g \sqrt{Z_A} g f^{abc} A^b_\mu A^c_\nu] \propto F^a_{\mu\nu} \Leftrightarrow Z_g \sqrt{Z_A} = 1$$

RGE for general theories (at 1 loop)

- Let's consider our EFT Lagrangian $\mathcal{L} = \sum_i C_i^{(0)} O_i^{(0)}$
- UV divergences generated from it can be parameterized in terms of local operators (after canonical normalization and reduction to physical basis)

$$\mathscr{L}^{\rm div}_{1-\rm loop} = \sum_j \frac{1}{16\pi^2\epsilon} c'_j(c)\mathcal{O}_j,$$

- These divergences can be cancelled by counterterms

$$\mathscr{L}^{(0)}_{\rm EFT} = \sum_i c_i^{(0)} \mathcal{O}_i^{(0)} = \mu^{n_i\epsilon} Z_i c_i \mathcal{O}_i, \qquad Z_i = 1 - \frac{1}{16\pi^2\epsilon}\frac{c'_i(c)}{c_i}$$

- Using that the bare WC are independent of μ we get

$$\dot{c}_i \equiv 16\pi^2 \mu \frac{\mathrm{d}c_i}{\mathrm{d}\mu} = n_i c'_i - \sum_j n_j c_j \frac{\partial c'_i}{\partial c_j} = -2c'_i$$

RGE for general theories (at 1 loop)

- Using that the bare WC are independent of μ we get

$$Z_i = 1 - \frac{1}{16\pi^2\epsilon}\frac{c_i'(c)}{c_i}$$

$$0 = \dot{c}_i^{(0)} = \mu^{n_i\epsilon}[16\pi^2 n_i\epsilon Z_i c_i + \dot{Z}_i c_i + Z_i\dot{c}_i]$$

$$\dot{c}_i = -16\pi^2 n_i\epsilon c_i - \frac{\dot{Z}_i}{Z_i}c_i = -16\pi^2 n_i\epsilon c_i + \frac{c_i}{16\pi^2\epsilon}\dot{\left(\frac{c_i'}{c_i}\right)}$$

$$\dot{c}_i^{\text{tree}} = -16\pi^2 n_i\epsilon c_i$$

$$\dot{c}_i^{1\ \text{loop}} = \frac{c_i}{16\pi^2\epsilon}\dot{\left(\frac{c_i'}{c_i}\right)} = -\frac{c_i}{16\pi^2\epsilon}\frac{c_i'\dot{c}_i}{c_i^2} + \frac{1}{16\pi^2\epsilon}\frac{\partial c_i'}{\partial c_j}\dot{c}_j$$
$$= n_i c_i' - n_j c_j\frac{\partial c_i'}{\partial c_j}$$

EFTs: bottom-up vs top-down

- In the bottom-up approach to EFTs we only care about the EFT: it parameterizes the low energy effects of any UV dynamics.
  - It helps us parameterize experimental data in a model-independent way in the form of global fits.
  - Examples: Chiral Lagrangian (low energy QCD), SMEFT ("any" BSM)
- In the top-down approach we consider specific UV models and match them to the EFT (compute the WCs of the EFT in terms of the parameters of the UV theory).
  - We lose model independence in favor of model discrimination.
  - Smaller number of parameters (easier to handle in fits).
  - Only way to compare direct and indirect limits, range of validity of EFT, ...
  - Can be used to completely classify new physics models: IR/UV dictionaries.

EFTs from the top-down: matching and running

- The idea behind matching is to Taylor expand in the heavy mass limit

$$\frac{1}{p^2 - M^2} = -\frac{1}{M^2} \sum_n \left(\frac{p^2}{M^2} \right)^n , \quad p^2 \ll M^2$$

- What about loops? Because of divergences loop integration and heavy mass expansion do not commute … but the difference is local!

[Witten, NPB104 (1976), NPB122 (1977)]

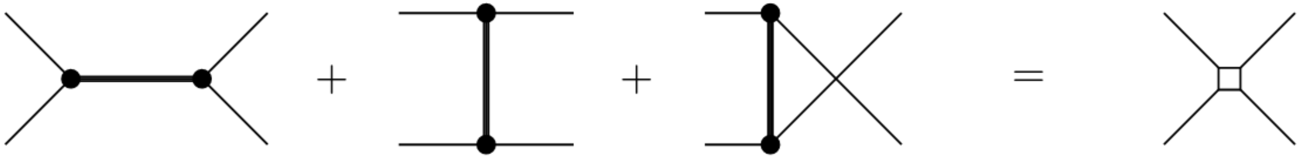
The difference corresponds to the hard region contribution of the full theory (soft region contributions cancel between full and EFT theories), which is local.

EFTs from the top-down: matching and running

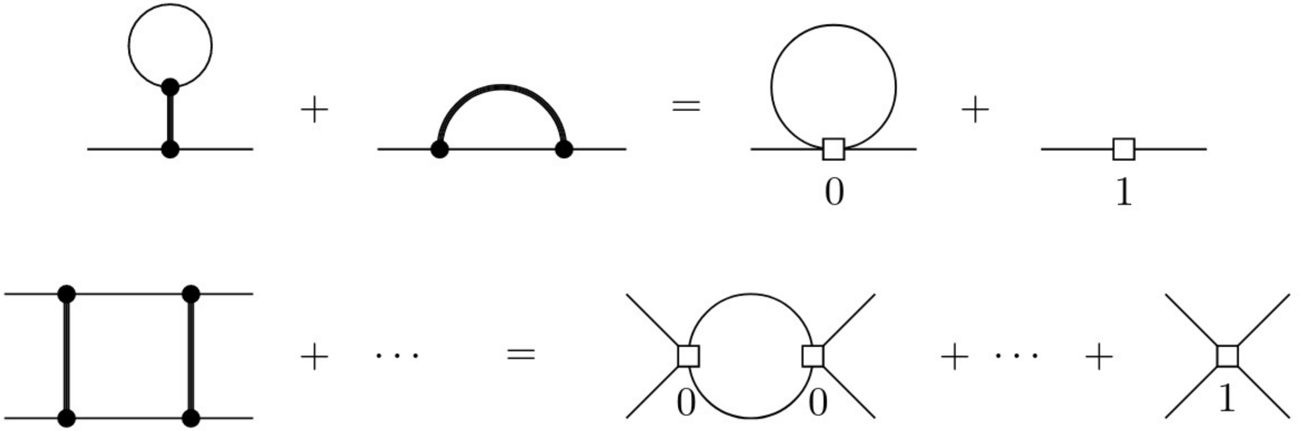
- The difference between the UV and the EFT is local!

Taken from Pich's lectures ph/9806303

Tree level



One loop: Compare renormalized amplitudes (we have to be consistent!)



EFTs from the top-down: matching and running

- Which amplitudes do we compute?
  - On-shell matching: all connected amplitudes with external, on-shell, light particles up to the dimension we need (number of fields and derivatives).
    - PROs: We don't need redundant operators.
    - CONs: The number of diagrams is in general very large; we lose cross-checks; delicate non-local cancellations between UV and EFT.
  - Off-shell matching: 1lPI (one-light-particle-irreducible) off-shell Green functions with external light particles up to the needed dimension.
    - PROs: Fewer diagrams; large redundancy, more cross-checks.
    - CONs: Redundant operators needed.

Why 1lPI?

Because contributions with light (off-shell) bridges are accounted for by adding operators in the EFT at tree-level. In on-shell matching light bridges account for the redundancies in the off-shell one and have to be included in the matching.

Efficient matching: expansion by regions

Taken from Manohar’s lectures 1804.05863

- Before studying a specific model let’s consider the following integral:

$$I_F = \mu^{2\epsilon} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2-M^2} \frac{1}{k^2-m^2} \,, \quad m^2 << M^2$$

$$= \frac{i}{16\pi^2} \left\{ \frac{1}{\epsilon} + 1 + \log\frac{\mu^2}{M^2} + \frac{m^2}{M^2-m^2} \log\frac{m^2}{M^2} \right\}$$

$$= \frac{i}{16\pi^2} \left\{ \frac{1}{\epsilon} + 1 + \log\frac{\mu^2}{M^2} + \log\frac{m^2}{M^2} \left(\frac{m^2}{M^2} + \frac{m^4}{M^4} + \dots \right) \right\}$$

- Let’s now expand the integrand first

$$I_{EFT} = \mu^{2\epsilon} \int \frac{1}{k^2-m^2} \left(-\frac{1}{M^2} \right) \left(1 + \frac{k^2}{M^2} + \frac{k^4}{M^4} + \dots \right)$$

$$= \left(\frac{1}{\epsilon} + 1 + \log\frac{\mu^2}{m^2} \right) \frac{-i}{16\pi^2} \left[\frac{m^2}{M^2} + \frac{m^4}{M^4} + \dots \right]$$

$$= -\frac{i}{16\pi^2} \left(\frac{1}{\epsilon} + 1 + \log\frac{\mu^2}{m^2} \right) \frac{m^2}{M^2-m^2}$$

Efficient matching: expansion by regions

Taken from Manohar’s lectures 1804.05863

- We got

$$I_F = \frac{i}{16\pi^2}\left[\frac{1}{\epsilon} + 1 + \log\frac{\mu^2}{M^2} + \log\frac{m^2}{M^2}\left(\frac{m^2}{M^2} + \frac{m^4}{M^4} + \dots\right)\right]$$

$$I_{EFT} = \frac{i}{16\pi^2}\left[-\frac{1}{\epsilon} - 1 - \log\frac{\mu^2}{m^2}\right]\left(\frac{m^2}{M^2} + \frac{m^4}{M^4} + \dots\right)$$

- We learn a few interesting lessons:
  - $I_F \neq I_{EFT}$, the expansion vs integration order matters when there are divergences.
  - UV poles are different in both integrals.
  - Non-analytic dependence on light scales is the same in I_F and I_{EFT}.
  - Dependence on M can be non-analytic in I_F but it is analytic in I_{EFT}.
  - I_F has a large log that can (sometimes has to) be resummed via RGE.
  - The difference between the two integrals, after renormalization, gives the matching condition.

$$I_M = \left[I_F + I_F^{ct}\right] - \left[I_{EFT} + I_{EFT}^{ct}\right] = \frac{i}{16\pi^2}\left(1 + \log\frac{\mu^2}{M^2}\right)\left[1 + \frac{m^2}{M^2} + \dots\right]$$

Analityc in m!

Efficient matching: expansion by regions

Taken from Manohar's lectures 1804.05863

- There is a better way thanks to the expansion by regions technique. The integrand in I$_F$ is singular for $k^2 \sim m^2$ (soft region) and for $k^2 \sim M^2$ (hard region). Let's compute the integral expanding in both regions.

$$I_F = \mu^{2\epsilon}\int \frac{d^Dk}{(2\pi)^D}\frac{1}{k^2-M^2}\frac{1}{k^2-m^2} = \frac{i}{16\pi^2}\left\{\frac{1}{\epsilon}+1+\log\frac{\mu^2}{M^2}+\frac{m^2}{M^2-m^2}\log\frac{m^2}{M^2}\right\}$$

$$I_F^{(s)} = \mu^{2\epsilon}\int \frac{1}{k^2-m^2}\frac{-1}{M^2}\left(1+\frac{k^2}{M^2}+\dots\right) = \frac{-i}{16\pi^2}\left(\frac{1}{\epsilon}+1+\log\frac{\mu^2}{m^2}\right)\frac{m^2}{M^2-m^2}$$

$$I_F^{(h)} = \mu^{2\epsilon}\int \frac{1}{k^2-M^2}\frac{1}{k^2}\left(1+\frac{m^2}{k^2}+\dots\right) = \frac{i}{16\pi^2}\left(\frac{1}{\epsilon}+1+\log\frac{\mu^2}{M^2}\right)\frac{M^2}{M^2-m^2}$$

$$\boxed{I_F = I_F^{(s)} + I_F^{(h)}}$$

$I_{EFT}^{(s)} = I_{EFT} = I_F^{(s)}$ ← IR is the same in both

$I_{EFT}^{(h)} = 0$ ← always scaleless

$$\boxed{I_M = I_F^{\text{renorm.}} - I_{EFT}^{\text{renorm}} = I_F^{(h)r} + I_F^{(s)r} - I_{EFT}^{(s)r} = I_F^{(h)r}}$$

Efficient matching: expansion by regions

Taken from Manohar's lectures 1804.05863

- There is a better way thanks to the expansion by regions technique. The integrand in I_{F} is singular for $k^2 \sim m^2$ (soft region) and for $k^2 \sim M^2$ (hard region). Let's compute the integral expanding in both regions.
- The matching comes from the hard region contribution of the UV theory
  - No need to compute in the EFT.
  - No need to do the full UV calculation.
  - Only the tadpole integral is needed for the calculation.
- New matching procedure (1-loop):
  - Compute the hard region contribution in the UV theory.
  - Forget about $\frac{1}{\epsilon}$ terms (UV MS-barred away, IR cancel in the difference)
- Match the result to the tree level contribution of the EFT.

But we have to be consistent between UV and EFT

EFTs from the top-down: tree level matching

- Let's consider the following UV theory $[m^2 \ll M^2]$

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\not{\partial} - m)\psi + \frac{1}{2}[(\partial_\mu \phi)^2 - m^2\phi^2] - \eta\bar{\psi}\psi\phi + \frac{1}{2}[(\partial_\mu \Phi)^2 - M^2\Phi^2] - \lambda\bar{\psi}\psi\Phi$$

- We want to find the EFT that reproduces its low-energy effects.

$$\mathcal{L}_{\mathrm{EFT}} = c_\psi \bar{\psi}\mathrm{i}\not{\partial}\psi - c_{m_\psi}\bar{\psi}\psi + \frac{1}{2}c_\phi(\partial_\mu\phi)^2 - \frac{1}{2}c_{m_\phi^2}\phi^2$$
$$+c_{\phi^3}\phi^3 + c_{\phi^4}\phi^4 + c_{\phi^5}\phi^5 + c_{\phi^3 d^2}\phi^2\partial^2\phi + \ldots$$
$$+c_{\psi^2\phi^2}\bar{\psi}\psi\phi^2 + c_{\psi^4}(\bar{\psi}\psi)^2 + d_{\psi^4}\bar{\psi}\psi(\partial_\mu\bar{\psi})(\partial^\mu\psi) + \ldots$$

EFTs from the top-down: tree level matching

- How do we match (in a systematic way) off-shell?

  1) Build a Green basis (only once per EFT).

  2) Compute, in the full theory, the hard region of the 1lPI contribution to all the amplitudes needed from the (tree-level) EFT side.

  3) Match all kinematic invariants to the tree-level EFT (imposing momentum conservation = ibp in the EFT Lagrangian).

  4) Solve for the Wilson Coefficients and check that all off-shell kinematic invariants are matched (non-trivial cross-check!).

  5) A further cross-check (sometimes necessary) is gauge invariance (compute amplitudes with momentum replaced with gauge bosons), when using the background field method.

EFTs from the top-down: tree level matching

- How do we match (in a systematic way) off-shell?

1) Build a Green basis (only once per EFT).

  - Let’s focus on four-fermion interactions up to dim 8 (only with derivatives)

$$\mathcal{O}_{\psi^4} = (\bar\psi\psi)(\bar\psi\psi)$$

$$\mathcal{O}^{(1)}_{d^2\psi^4} = (\bar\psi\psi)(\bar\psi\partial^2\psi)$$

$$\mathcal{O}^{(2)}_{d^2\psi^4} = (\bar\psi\psi)(\partial_\mu\bar\psi\partial^\mu\psi)$$

$$\mathcal{O}^{(3)}_{d^2\psi^4} = (\bar\psi\partial^\mu\psi)(\partial_\mu\bar\psi\psi)$$

$$\mathcal{L}_{\rm EFT} = \frac{C_{\psi^4}}{2}\mathcal{O}_{\psi^4} + \left[C^{(1)}_{d^2\psi^4}\mathcal{O}^{(1)}_{d^2\psi^4} + \text{h.c.}\right] + \sum_{i=2}^{3} C^{(i)}_{d^2\psi^4}\mathcal{O}^{(i)}_{d^2\psi^4}$$

2) Compute, in the full theory, the hard region of the 1lPI contribution to all the amplitudes needed from the (tree-level) EFT side.

  - Since we only want these operators it’s enough to compute $\psi\psi \to \psi\psi$ to order p^2

EFTs from the top-down: tree level matching

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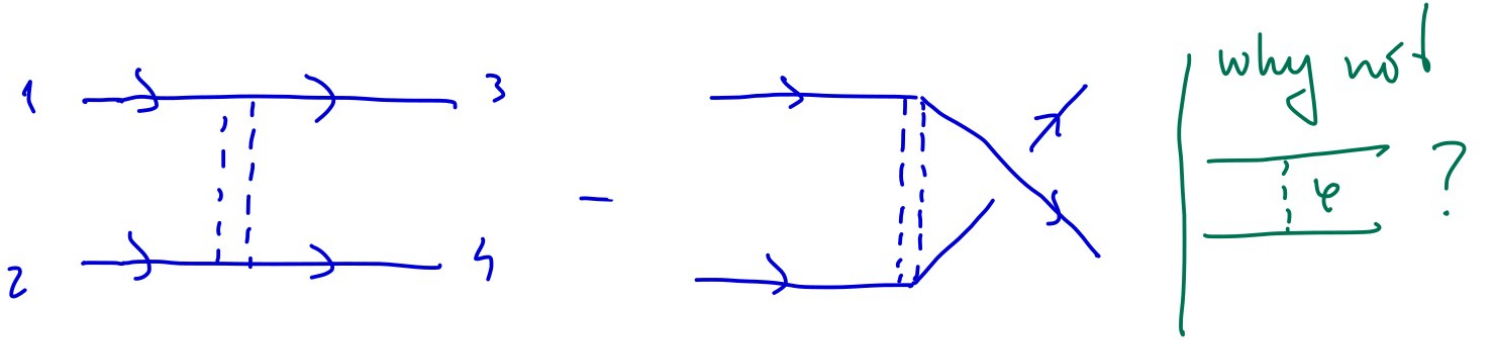


$$i\mathcal{M}_F = \bar{u}_3 u_1 \bar{u}_4 u_2 (-i\lambda)^2 \frac{i}{(p_3-p_1)^2 - M^2} - (3 \leftrightarrow 4)$$

$$= \bar{u}_4 u_1 \bar{u}_3 u_2 \frac{i\lambda^2}{M^2} \left(1 + \frac{p_1^2 + p_3^2 - 2p_1 \cdot p_3}{M^2} + \ldots\right) - (3 \leftrightarrow 4)$$

EFTs from the top-down: tree level matching

- How do we match (in a systematic way) off-shell?

  3) Match all kinematic invariants to the tree-level EFT (imposing momentum conservation = ibp in the EFT Lagrangian). $p_4 \to p_1 + p_2 - p_3$

$$\begin{aligned} \mathrm{i}\mathcal{M}_E = \bar{u}_4 u_1 \bar{u}_3 u_2 \mathrm{i}\Big\{ & C_{\psi^4} + \big[C^{(3)}_{d^2\psi^4} - C^{(1)}_{d^2\psi^4} - \big(C^{(1)}_{d^2\psi^4}\big)^*\big]p_1^2 \\ & + \big[C^{(2)}_{d^2\psi^4} - C^{(1)}_{d^2\psi^4} - (C^{(1)}_{d^2\psi^4})^*\big]p_2^2 - 2\big(C^{(1)}_{d^2\psi^4}\big)^* p_3^2 \\ & + \big[C^{(2)}_{d^2\psi^4} + C^{(3)}_{d^2\psi^4} - 2\big(C^{(1)}_{d^2\psi^4}\big)^*\big]p_1 \cdot p_2 \\ & + \big[C^{(2)}_{d^2\psi^4} - C^{(3)}_{d^2\psi^4} + 2\big(C^{(1)}_{d^2\psi^4}\big)^*\big]p_1 \cdot p_3 \\ & + \big[-C^{(2)}_{d^2\psi^4} + C^{(3)}_{d^2\psi^4} + 2\big(C^{(1)}_{d^2\psi^4}\big)^*\big]p_2 \cdot p_3 \Big\} - (3 \leftrightarrow 4) \end{aligned}$$

$$\mathrm{i}\mathcal{M}_F = \bar{u}_4 u_1 \bar{u}_3 u_2 \frac{\mathrm{i}\lambda^2}{M^2}\left(1 + \frac{p_1^2 + p_3^2 - 2p_1 \cdot p_3}{M^2} + \ldots\right) - (3 \leftrightarrow 4)$$

EFTs from the top-down: tree level matching

- How do we match (in a systematic way) off-shell?

4) Solve for the Wilson Coefficients and check that all off-shell kinematic invariants are matched (non-trivial cross-check!).

$$C_{\psi^4} = \frac{\lambda^2}{M^2} \qquad C^{(2)}_{d^2\psi^4} = -\frac{\lambda^2}{M^4}$$

$$C^{(1)}_{d^2\psi^4} = -\frac{\lambda^2}{2M^4} \qquad C^{(3)}_{d^2\psi^4} = 0$$

$$\mathrm{i}\mathcal{M}_E = \mathrm{i}\mathcal{M}_F + \mathcal{O}\left(\frac{p^4}{M^6}\right)$$ ✓

The same procedure is used for matching at arbitrary loops!!

EFTs from the top-down: tree level matching

- Matching can be done also functionally. At tree level it corresponds to just solving the classical EoM for the heavy fields, introducing the solutions back in the Lagrangian and expanding.

$$\mathcal{L}_{\ell h} = -\frac{1}{2}\Phi(\partial^2+M^2)\Phi - \Phi\lambda\bar{\psi}\psi \Rightarrow (\partial^2+M^2)\Phi = -\lambda\bar{\psi}\psi$$

$$\Rightarrow \Phi = \Pi(-\lambda\bar{\psi}\psi) \qquad \text{where } \Pi = (\partial^2+M^2)^{-1} = \frac{1}{M^2}\left(1-\frac{\partial^2}{M^2}+\ldots\right)$$

$$\mathcal{L}_{\ell h} \to -\frac{1}{2}\Pi(-\lambda\bar{\psi}\psi)(\partial^2+M^2)\Pi(-\lambda\bar{\psi}\psi) - \lambda\bar{\psi}\psi\,\Pi(-\lambda\bar{\psi}\psi)$$

non-local lagrangian

$$= -\frac{\lambda^2}{2}\Pi(\bar{\psi}\psi)\bar{\psi}\psi + \lambda^2\bar{\psi}\psi\,\Pi(\bar{\psi}\psi) = \frac{\lambda^2}{2}\bar{\psi}\psi\,\Pi(\bar{\psi}\psi)$$

can be expanded in local operators

$$= \frac{\lambda^2}{2}\left\{\frac{(\bar{\psi}\psi)^2}{M^2} - \frac{\bar{\psi}\psi\,\partial^2(\bar{\psi}\psi)}{M^4} + \ldots\right\}$$

$$= \frac{\lambda^2}{2M^2}(\bar{\psi}\psi)^2 - \frac{\lambda^2}{2M^4}\left[\bar{\psi}\psi(\partial^2\bar{\psi})\psi + \bar{\psi}\psi\,\bar{\psi}\partial^2\psi + 2\bar{\psi}\psi\,\partial_\mu\bar{\psi}\partial^\mu\psi\right] + \ldots$$

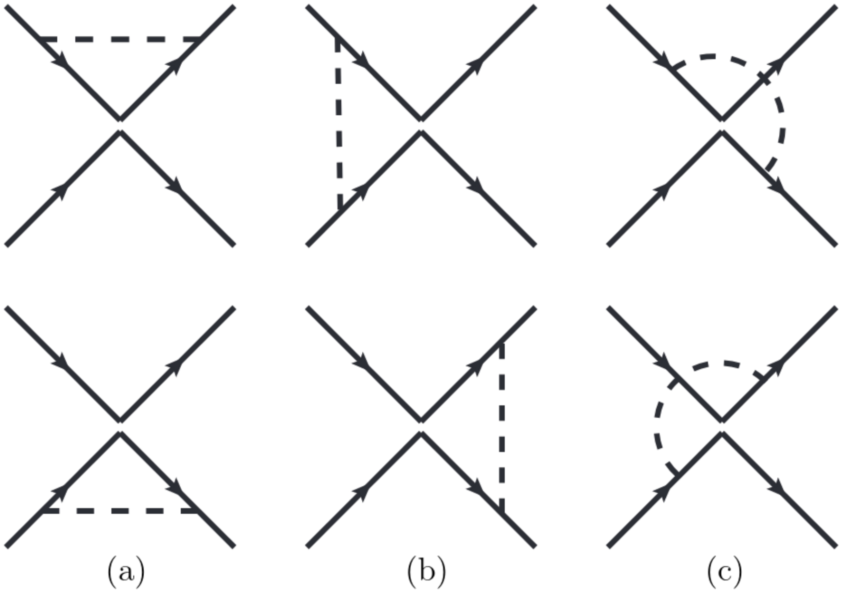
which agrees with the results from the diagrammatic calculation.

EFTs from the top-down: RGE running

- Let's see an explicit example of how to compute the RGEs for an EFT. We start with the following EFT

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{d\leq 4} + \frac{c}{2}(\bar{\psi}\psi)^2$$

- We want to compute the UV divergences up to dim 6: 1 insertion of c



$$-(3 \leftrightarrow 4)$$

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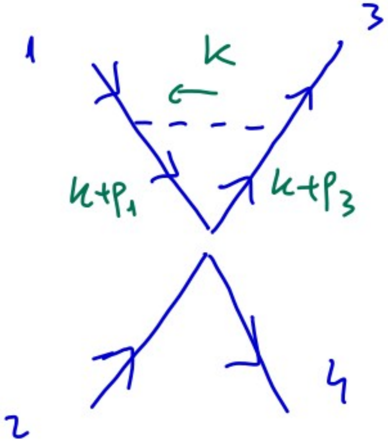

$$= \left(-i y \mu^{\epsilon}\right)^2 (ic) \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - M^2} \bar{u}_4 u_2 \bar{u}_3 \frac{i(\not{k} + \not{p}_3 + m)}{(k+p_3)^2 - m^2} \frac{i(\not{k} + \not{p}_1 + m)}{(k+p_1)^2 - m^2} u_1$$

$$\sim -y^2 c \mu^{2\epsilon} \int \bar{u}_4 u_2 \; \bar{u}_3 \frac{\not{k}\not{k}}{k^6} u_1 = -y^2 c \frac{i}{16\pi^2} \frac{1}{\epsilon} \bar{u}_4 u_2 \bar{u}_3 u_1 + \ldots$$

↳ UV div. $k \gg p, m, M$

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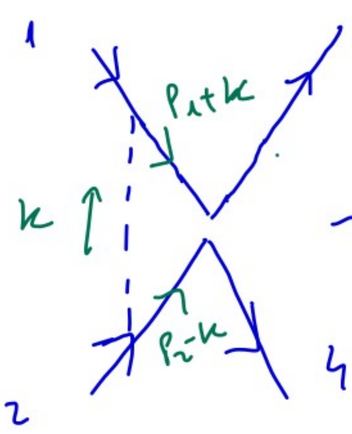


$$\sim (-iy\mu^{\epsilon})^2 (ic) \int \frac{i}{k^2} \bar{u}_3 \frac{i\not{k}}{k^2} u_1 \bar{u}_4 \frac{(-i)\not{k}}{k^2} u_2 \quad \Big| \quad k^\mu k^\nu \to \frac{g^{\mu\nu}}{D} k^2$$

$$= \frac{y^2 c}{4} \frac{i}{16\pi^2} \frac{1}{\epsilon} \bar{u}_3 \gamma^\mu u_1 \bar{u}_4 \gamma_\mu u_2 + \ldots$$

EFTs from the top-down: RGE running

- Let's see an explicit example of how to compute the RGEs for an EFT. We start with the following EFT

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{d\leq 4} + \frac{c}{2}(\bar\psi\psi)^2$$

- We want to compute the UV divergences up to dim 6: 1 insertion of c

$$i\mathcal{M}_a = \frac{i}{16\pi^2\epsilon}\left(-2y^2c\right)\bar u_3 u_1 \bar u_4 u_2 + \dots$$

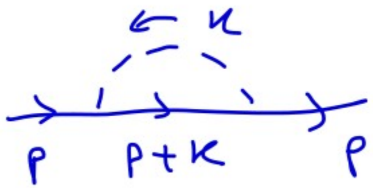
$$i\mathcal{M}_b = \frac{i}{16\pi^2\epsilon}\left(\frac{y^2c}{2}\right)\bar u_3 \gamma^\mu u_1 \bar u_4 \gamma_\mu u_2 + \dots$$

$$i\mathcal{M}_c = \frac{i}{16\pi^2\epsilon}\left(-\frac{y^2c}{2}\right)\bar u_3 \gamma^\mu u_1 \bar u_4 \gamma_\mu u_2 + \dots$$

$$i\mathcal{M} = i\mathcal{M}_a = \frac{i}{16\pi^2\epsilon}\left(-2y^2c\right)\bar u_3 u_1 \bar u_4 u_2 + \text{finite}$$

EFTs from the top-down: RGE running

- We also need the UV divergence for the kinetic term



$$= (-iy)^2 \int \frac{d^D k}{(2\pi)^D} \frac{i(\not k + \not p)}{(k+p)^2} \frac{i}{k^2}$$

Let's deal with the integrand

$$\frac{\not k + \not p}{(k+p)^2} \frac{1}{k^2} = \left\{ \begin{aligned} &\frac{1}{(k+p)^2} = \frac{1}{k^2}\left[1 - \frac{p^2 + 2k\cdot p}{(k+p)^2}\right] = \frac{1}{k^2}\left[1 - \frac{p^2+2k\cdot p}{k^2}\left(1 - \frac{p^2+2k\cdot p}{(k+p)^2}\right)\right] \\ &= \frac{1}{k^2} - \frac{2k\cdot p}{k^4} + O(p^2) \leftarrow \text{we are only interested in the } \not p \text{ term and UV div.} \Rightarrow k^2 >> p^2. \end{aligned} \right.$$

$$= (\not k + \not p)\left[\frac{1}{k^4} - \frac{2k\cdot p}{k^6} + \dots\right] = \overbrace{\frac{\not k}{k^4}}^{\text{0 upon int.}} + \frac{\not p}{k^4} - \frac{2k\cdot p \not k}{k^6} + O(p^2)$$

$$\underset{\text{upon int.}}{\sim} \frac{\not p}{k^4} - \frac{2 \not p k^2}{D k^6} + \dots \sim \not p \left(1 - \frac{2}{4-2\epsilon}\right)\frac{1}{k^4} \sim \frac{\not p}{2}\frac{i}{16\pi^2\epsilon} + \text{finite}.$$

Thus  $\sim \frac{y^2}{2} \frac{i}{16\pi^2} \not p \frac{1}{\epsilon} + \text{finite}.$

EFTs from the top-down: RGE running

- The original plus divergent Lagrangian fixes the counterterms and therefore the RGE

$$\mathcal{L} = \left(1+\frac{y^2}{2}\frac{1}{16\pi^2\epsilon}\right)\bar\psi i\partial\!\!\!/\psi + \frac{c}{2}\left(1-\frac{2y^2}{16\pi^2\epsilon}\right)(\bar\psi\psi)^2+\dots$$

$$\longrightarrow \bar\psi i\partial\!\!\!/\psi + \frac{c}{2}\left(1-\frac{3y^2}{16\pi^2\epsilon}\right)(\bar\psi\psi)^2+\dots \quad \left\{ \psi\to\left(1-\frac{y^2}{4}\frac{1}{16\pi^2\epsilon}\right)\psi\right.$$

$$\Longrightarrow \quad c' = -3c\eta^2 \Rightarrow \dot c = -2c' = 6c\eta^2$$

- At one loop level we can consider the couplings on the RHS not to run. This is the leading log (LL) approximation:

$$C(\mu) = C(M)\left[1-\frac{3}{16\pi^2}y^2\ln\frac{M^2}{\mu^2}\right] = \frac{d^2}{M^2}\left[1-\frac{3}{16\pi^2}y^2\ln\frac{M^2}{\mu^2}\right]$$

EFTs from the top-down: RGE running

- RGE can be used to resum all loop order contributions of the form $(\alpha \log)^n$

The RGE for y is $\boxed{\dot{y} = \frac{5}{16\pi^2} y^3}$ $\Rightarrow y^2(\mu) = \frac{y^2(\Lambda)}{1 - \frac{10}{16\pi^2} y^2(\Lambda) \log\frac{\mu}{\Lambda}}$

$$\frac{d\ln C}{d\ln y} = \frac{d\ln C}{d\ln\mu}\left(\frac{d\ln y}{d\ln\mu}\right)^{-1} = \frac{6y^2}{16\pi^2}\,\frac{16\pi^2}{5y^2} = \frac{6}{5}$$

$$\Rightarrow C(\mu) = C(\Lambda)\left(\frac{y^2(\mu)}{y^2(\Lambda)}\right)^{3/5} = \frac{d^2}{M^2}\left(1 + \frac{3}{5} y^2(\Lambda) \frac{10}{16\pi^2} \ln\frac{\mu}{\Lambda} + \dots\right)$$

$$= \frac{d^2}{M^2}\left(1 + \frac{6}{16\pi^2} y^2 \ln\frac{\mu}{\Lambda} + \dots\right)$$

- Which leads to RG-improved perturbation theory:
  - LO $(\alpha \log)^n$
  - NLO $\alpha(\alpha \log)^n$
  - …

Important when $\alpha \ll 1$ but $(\alpha \log) \sim 1$

EFTs from the top-down: RGE running

- In general counterterms depend on new operators (operator mixing)

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi + \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{m^2}{2}\varphi^2 - y\bar{\psi}\psi\varphi + \frac{c_V}{2}\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi + \dots$$

$$\sim \bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi$$

but

$+$

$$\sim \bar{\psi}\sigma^{\mu\nu}\psi\bar{\psi}\sigma_{\mu\nu}\psi \qquad \left\{ \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \right.$$

but that means that we need a different operator to cancel the divergence

$$\mathcal{L}' = \mathcal{L} + \frac{C_T}{2}(\bar{\psi}\sigma^{\mu\nu}\psi)^2$$

The tree-level matching gives $C_T(M) = 0$, $C_V(M) \neq 0$.

but the fact that we need O_T to renormalize insertions of O_V means that at low energies we generate a non-zero C_T

EFTs from the top-down: RGE running

- In general counterterms depend on new operators (operator mixing)

$$\kappa_V = \frac{y^2}{16\pi^2}\, 6\, \frac{C_T}{C_V}$$

$$\kappa_T = \frac{y^2}{16\pi^2}\left(1+\frac{C_V}{C_T}\right)$$

$$\dot{C}_V = \frac{12\, y^2}{16\pi^2}\, C_T$$

$$\dot{C}_T = \frac{2y^2}{16\pi^2}\left[C_T + C_V\right]$$

or in matrix notation:

$$\boxed{16\pi^2 \frac{d}{dt}\begin{pmatrix} C_V \\ C_T \end{pmatrix} = 2y^2 \begin{pmatrix} 0 & 6 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} C_V \\ C_T \end{pmatrix}}$$

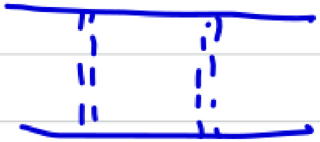
$C_V(M)$ induces C_T at lower energies.

$C_T(M)$ induces both C_V and C_T at lower energies.

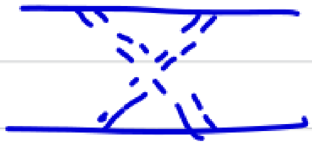
EFTs from the top-down: 1-loop matching

- Let's now go on to compute the 1-loop matching. We will consider the λ^4 contribution to fermion-fermion scattering.

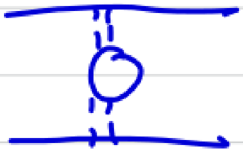
The relevant diagrams in the full theory (up to $3 \leftrightarrow 4$) are



(a)



(b)



(c)

(d) 4×2 ()

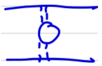
EFTs from the top-down: 1-loop matching

- Let’s now go on to compute the 1-loop matching. We will consider the λ^4 contribution to fermion-fermion scattering.

$$u_V \equiv \bar{u}_3 \gamma^\mu u_1 \bar{u}_4 \gamma_\mu u_2 , \quad u_S \equiv \bar{u}_3 u_1 \bar{u}_4 u_2$$

$$(a)_F = \frac{i\lambda^4}{16\pi^2 M^2}\left[U_V\left(\frac{1}{4}+\frac{1}{4}\frac{m^2}{M^2}\left(3-2\log\frac{M^2}{m^2}\right)\right)+U_S\frac{m^2}{M^2}\left(\log\frac{M^2}{m^2}-2\right)\right]+\ldots$$

$$(b)_F = \frac{i\lambda^4}{16\pi^2 M^2}\left[-U_V\left(\frac{1}{4}+\frac{1}{4}\frac{m^2}{M^2}\left(3-2\log\frac{M^2}{m^2}\right)\right)+U_S\frac{m^2}{M^2}\left(\log\frac{M^2}{m^2}-2\right)\right]+\ldots$$



$$(c)_F = -\frac{i\lambda^4}{16\pi^2 M^2}\, 4\frac{m^2}{M^2} U_S\left(\frac{3}{\bar\epsilon}+3\log\frac{\mu^2}{m^2}+1\right)+\ldots$$

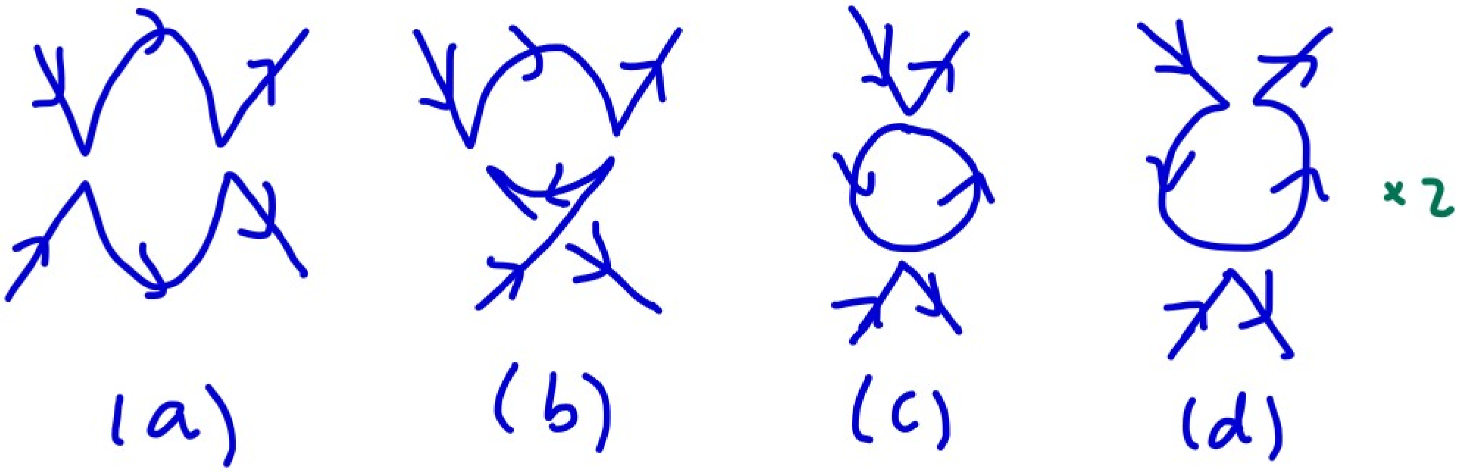
$$(d)_F = -\frac{i\lambda^4}{16\pi^2 M^2}\, 2 U_S\left[\frac{1}{\bar\epsilon}+1+\log\frac{\mu^2}{M^2}+\frac{m^2}{M^2}\left(2-3\log\frac{M^2}{m^2}\right)\right]+\ldots$$

$$(a+\ldots+d)_F = \frac{2i\lambda^4}{16\pi^2}\frac{U_S}{M^2}\left[-\frac{1}{\bar\epsilon}-1-\log\frac{\mu^2}{M^2}+\frac{m^2}{M^2}\left(-\frac{6}{\bar\epsilon}-6\log\frac{\mu^2}{m^2}-6+4\log\frac{M^2}{m^2}\right)\right]+\ldots$$

$$= \frac{i\not{p}}{2}\frac{\lambda^2}{16\pi^2}\left(\frac{1}{\bar\epsilon}+\log\frac{\mu^2}{M^2}+\frac{1}{2}+\ldots\right)$$

EFTs from the top-down: 1-loop matching

- Let’s now go on to compute the 1-loop matching. We will consider the λ^4 contribution to fermion-fermion scattering.



EFTs from the top-down: 1-loop matching

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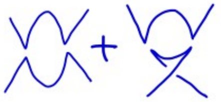


$$(a+b)_\epsilon = \frac{2ic^2m^2}{16\pi^2} U_s \left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{m^2}\right)\right] + \ldots$$

$$(c)_\epsilon = -\frac{4ic^2m^2}{16\pi^2} U_s \left[\frac{3}{\epsilon} + 3\log\left(\frac{\mu^2}{m^2}\right) + 1\right] + \ldots$$

$$(d)_\epsilon = 2\,\frac{ic^2m^2}{16\pi^2} U_s \left[\frac{3}{\epsilon} + 3\log\left(\frac{\mu^2}{m^2}\right) + 1\right] + \ldots$$

Thus

$$(a+\ldots+d)_\epsilon = \frac{-2ic^2m^2}{16\pi^2} U_s \left[\frac{2}{\epsilon} + 2\log\left(\frac{\mu^2}{m^2}\right) + 1\right] + \ldots$$

$$= -\frac{2i\lambda^4}{16\pi^2}\frac{m^2}{M^4} U_s \left[\frac{2}{\epsilon} + 2\log\left(\frac{\mu^2}{m^2}\right) + 1\right] + \ldots$$

No EFT contribution to kinetic term.

EFTs from the top-down: 1-loop matching

- Let’s now go on to compute the 1-loop matching. We will consider the λ^4 contribution to fermion-fermion scattering.

$$(a+\dots+d)_F = \frac{2i\lambda^4}{16\pi^2}\frac{U_s}{M^2}\left[-\frac{1}{\bar{\epsilon}}-1-\log\frac{\mu^2}{M^2}+\frac{m^2}{M^2}\left(-\frac{6}{\bar{\epsilon}}-6\log\frac{\mu^2}{m^2}-6+4\log\frac{M^2}{m^2}\right)\right]+\dots$$

$$(a+\dots+d)_E = -\frac{2i\lambda^4}{16\pi^2}\frac{m^2}{M^4}U_s\left[\frac{2}{\bar{\epsilon}}+2\log\left(\frac{\mu^2}{m^2}\right)+1\right]+\dots$$

$$(\times)^{\text{ren}}_F-(\times)^{\text{ren}}_E = \frac{i}{16\pi^2}U_s\frac{2\lambda^4}{M^2}\left[-1-\log\frac{\mu^2}{M^2}+\frac{m^2}{M^2}\left(-5-4\log\frac{\mu^2}{M^2}\right)\right]$$

$$(-\bullet-)^{\text{ren}}_F-(-\bullet-)^{\text{ren}}_E = \frac{i\not{p}}{2}\frac{\lambda^2}{16\pi^2}\left(\log\frac{\mu^2}{M^2}+\frac{1}{2}\right)$$

EFTs from the top-down: 1-loop matching

- Let's now go on to compute the 1-loop matching. We will consider the λ^4 contribution to fermion-fermion scattering.

$$\mathcal{L}_{EFT} = Z_\psi \bar\psi i\not\partial\psi + \frac{C}{2}(\bar\psi\psi)^2 + \dots$$

$$Z_\psi = 1 + \frac{\lambda^2}{16\pi^2}\left(\frac{1}{4} + \frac{1}{2}\log\frac{\mu^2}{M^2}\right)$$

$$C = \frac{\lambda^2}{M^2}\left[1 - \frac{2\lambda^2}{16\pi^2}\left(1 + \log\frac{\mu^2}{M^2}\right)\right]$$

Normalizing canonically

$$\boxed{C^{\text{canonical}}(\mu) = Z_\psi^{-2}\, C(\mu) = \frac{\lambda^2}{M^2}\left[1 - \frac{\lambda^2}{16\pi^2}\left(\frac{5}{2} + 3\log\frac{\mu^2}{M^2}\right)\right]}$$

Consistent with RGE?

$$\frac{d}{d\ln\mu} C(\mu) = \frac{6}{16\pi^2}\, y^2 C = \frac{6}{16\pi^2}\frac{y^2\lambda^2}{M^2} + \dots$$

$$(\dot C^{\text{can.}}) = \left(\dot{\frac{\lambda^2}{M^2}}\right) - \frac{6\lambda^4}{16\pi^2 M^2} = \frac{2\lambda\dot\lambda}{M^2} - \frac{\lambda^2(\dot{M^2})}{M^4} - \frac{6\lambda^4}{16\pi^2 M^2}$$

we need to renormalize λ and M in the UV theory.

EFTs from the top-down: 1-loop matching

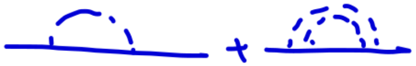
- Let’s now go on to compute the 1-loop matching. We will consider the λ^4 contribution to fermion-fermion scattering.

$$(\dot{C}^{\text{can.}}) = \left(\frac{\overset{\bullet}{\lambda^2}}{M^2}\right) - \frac{6\lambda^4}{16\pi^2 M^2} = \frac{2\lambda\dot{\lambda}}{M^2} - \frac{\lambda^2(\overset{\bullet}{M^2})}{M^4} - \frac{6\lambda^4}{16\pi^2 M^2}$$

we need to renormalize λ and M in the UV theory.

$$= \frac{2i\lambda^2}{16\pi^2\epsilon}\left(p^2 - 6m^2\right) + \text{finite}$$

$$= \frac{i\lambda}{16\pi^2\epsilon}\left(y^2+\lambda^2\right) + \text{finite}$$



$$= \frac{i\not{p}}{16\pi^2\epsilon}\left(\frac{y^2+\lambda^2}{2}\right) + \text{finite} + \ldots$$

EFTs from the top-down: 1-loop matching

- Let's now go on to compute the 1-loop matching. We will consider the λ^4 contribution to fermion-fermion scattering.

$$\mathcal{L} = \left(1+\frac{1}{16\pi^2\epsilon}\frac{y^2+\lambda^2}{2}\right)\bar{\Psi} i\partial\!\!\!/\Psi + \left(1+\frac{2\lambda^2}{16\pi^2\epsilon}\right)\frac{1}{2}(\partial_\mu\Phi)^2 - \frac{1}{2}M^2\Phi^2$$

$$- \lambda\left(1-\frac{y^2+\lambda^2}{16\pi^2\epsilon}\right)\bar{\Psi}\Psi\Phi + \ldots$$

$$\longrightarrow \bar{\Psi} i\partial\!\!\!/\Psi + \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{1}{2}M^2\left(1-\frac{2\lambda^2}{16\pi^2\epsilon}\right)\Phi^2$$

$$-\lambda\left[1-\frac{1}{16\pi^2\epsilon}\frac{3y^2+5\lambda^2}{2}\right]\bar{\Psi}\Psi\Phi$$

$$k_{M^2} = \frac{2\lambda^2}{16\pi^2} \qquad \Rightarrow \qquad (\dot{M^2}) = \frac{4\lambda^2}{16\pi^2}M^2$$

$$k_\lambda = \frac{1}{16\pi^2}\frac{3y^2+5\lambda^2}{2} \qquad \dot{\lambda} = \frac{(3y^2+5\lambda^2)\lambda}{16\pi^2}$$

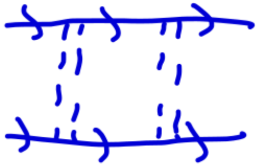
$$(\dot{C}^{\text{can.}}) = \frac{2\lambda\dot{\lambda}}{M^2} - \frac{\lambda^2(\dot{M^2})}{M^4} - \frac{6\lambda^4}{16\pi^2M^2} = \frac{\lambda^2}{16\pi^2M^2}\left[2(3y^2+5\lambda^2) - 4\lambda^2 - 6\lambda^2\right] = \frac{6\lambda^2y^2}{16\pi^2M^2}$$

$$\frac{d}{d\ln\mu}C(\mu) = \frac{6}{16\pi^2}y^2C = \frac{6}{16\pi^2}\frac{y^2\lambda^2}{M^2} + \ldots$$

EFTs from the top-down: 1-loop matching

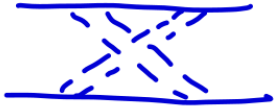
- Let’s do the matching the more efficient way (neglecting m^2 terms)

At dim 6 we can set $p_i = 0 = m^2$

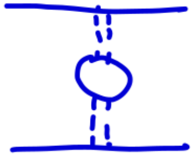


$$= (-i\lambda)^4 \int \bar{u}_3 \frac{i\not{k}}{k^2} u_1 \bar{u}_4 \frac{-i\not{k}}{k^2} u_2 \frac{i^2}{(k^2-M^2)^2} + \dots$$

$$= -\lambda^4 \bar{u}_3 \gamma^\alpha u_1 \bar{u}_4 \gamma^\beta u_2 \int \frac{k^\alpha k^\beta}{k^4 (k^2-M^2)^2} + \dots$$



$$= \lambda^4 \bar{u}_3 \gamma^\alpha u_1 \bar{u}_4 \gamma^\beta u_2 \int \frac{k^\alpha k^\beta}{k^4 (k^2-M^2)^2} + \dots$$



$= 0$ A purely light loop always results in scaleless integrals in the hard region.

EFTs from the top-down: 1-loop matching

- Let’s do the matching the more efficient way (neglecting m^2 terms)

$2\times$

$$= 2\times(-i\lambda)^4\,\frac{i}{-M^2}\int\frac{i}{k^2-M^2}\,\bar u_3\,\frac{i\not k}{k^2}\,\frac{i\not k}{k^2}\,u_1\,\bar u_4 u_2$$

scaleless ↙

$$= -\frac{2\lambda^4}{M^2}\,u_s\int\frac{1}{(k^2-M^2)k^2} = -\frac{2\lambda^4}{M^2}\,u_s\int\frac{1}{M^2}\left[\frac{1}{k^2-M^2}-\frac{1}{k^2}\right]$$

$$= -\frac{i}{16\pi^2}\,\frac{2\lambda^4}{M^2}\,u_s\left[\frac{1}{\epsilon}+1+\log\frac{\mu^2}{M^2}\right]+\dots$$

Thus

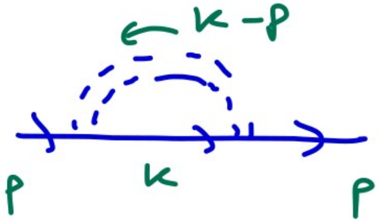
$$\Bigg|^{\text{ren}}_{\text{hard}} = -\frac{2i\lambda^4}{16\pi^2 M^2}\,u_s\left(1+\log\frac{\mu^2}{M^2}\right)$$

which agrees with our previous calculation.

EFTs from the top-down: 1-loop matching

- Let’s do the matching the more efficient way (neglecting m^2 terms)

Let's do the two point function



$$= (-i\lambda)^2 \int \frac{i}{(k-p)^2-M^2} \, \frac{i(\slashed{k}+m)}{k^2-m^2}$$

$$= \lambda^2 \int \frac{1}{k^2-M^2}\left[1-\frac{p^2-2p\cdot k}{(k-p)^2-M^2}\right] \frac{(\slashed{k}+m)}{k^2}\left[1+\frac{m^2}{k^2}+\frac{m^4}{k^4}+\dots\right]$$

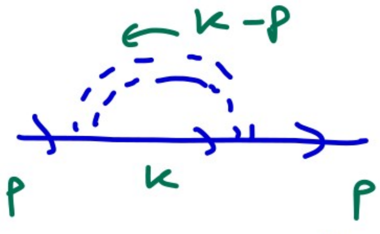
$$= \lambda^2 \int \frac{\slashed{k}+m}{k^2(k^2-M^2)}\left[1+\frac{2k\cdot p}{k^2-M^2}\right]+O(p^2,m^2)$$

$$= \lambda^2 \int \frac{m}{k^2(k^2-M^2)} + \frac{2k\cdot p\,\slashed{k}}{k^2(k^2-M^2)^2}+\dots$$

$$\rightarrow \lambda^2 \int \frac{m}{k^2(k^2-M^2)}+\frac{2}{D}\frac{\slashed{p}}{(k^2-M^2)^2}+\dots$$

EFTs from the top-down: 1-loop matching

- Let’s do the matching the more efficient way (neglecting m^2 terms)



$$\to \lambda^2 \int \frac{m}{k^2(k^2-M^2)} + \frac{2}{D}\frac{\not{k}}{(k^2-M^2)^2} + \dots$$

$$= \lambda^2 \int \frac{m}{M^2}\left[\frac{1}{k^2-M^2} - \overbrace{\frac{1}{k^2}}^{\text{scaleless}}\right] + \frac{2}{D}\,\frac{D-2}{2M^2}\,\frac{\not{p}}{k^2-M^2} + \dots$$

$$= \frac{i\lambda^2}{16\pi^2}\left(\frac{1}{\epsilon}+1+\log\frac{\mu^2}{M^2}\right)\left[\overbrace{\frac{2-2\epsilon}{4-2\epsilon}}^{=\frac{1}{2}(1-\epsilon/2)}\not{p} + m\right] + \dots$$

$$= \frac{i\lambda^2}{16\pi^2}\left\{\frac{\not{p}}{2}\left(\frac{1}{\epsilon}+\frac{1}{2}+\log\frac{\mu^2}{M^2}\right) + m\left(\frac{1}{\epsilon}+1+\log\frac{\mu^2}{M^2}\right)\right\} + \dots$$

$$\boxed{Z_\psi = 1 + \frac{\lambda^2}{16\pi^2}\left(\frac{1}{4}+\frac{1}{2}\log\frac{\mu^2}{M^2}\right)}$$

EFTs and mass-(in)dependent renorm. schemes

- Why did we use a mass-independent renormalization scheme?

  The EFT expansion is a double expansion in $1/\Lambda$ and loops. Using a mass independent renormalization scheme is crucial to keep these two expansions meaningful.

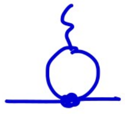
Taken from Manohar’s lectures ph/9606222

- Let’s see an example

$$\mathcal{L}_{EFT} = -\frac{4G_F}{\sqrt{2}} V_{ui} V_{ui}^{*} (\bar{u}\gamma^\mu P_L q_i)(\bar{q}_i \gamma_\mu P_L u)$$

This term induces a 1-loop correction to $Z_{u_L \bar{u}_L}$

→ Momentum cutoff $\Lambda \sim M_W$



$$\sim I_6 \sim \frac{1}{M_W^2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2} \sim \frac{\Lambda^2}{M_W^2} \sim O(1)$$

A similar operator with 2 extra derivatives (dim 8) contributes

$$I_8 \sim \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\frac{k^2}{M_W^4} \sim \frac{\Lambda^4}{M_W^4} \sim O(1)$$

Higher dim ops are not suppressed!

With mass-indep. schemes the expansions don't mix

$$I_6 \sim \frac{1}{M_W^2}\int \frac{d^Dk}{(2\pi)^D}\frac{1}{k^2} \sim \frac{m^2}{M_W^2}\left(a+b\log\left(\frac{\mu^2}{M_W^2}\right)\right) << 1$$

$$I_8 \sim \frac{1}{M_W^4}\int \frac{d^Dk}{(2\pi)^D} \sim \frac{m^4}{M_W^4}\left(a'+b'\log\left(\frac{\mu^2}{M_W^2}\right)\right) << I_6 << 1 .$$

EFTs and mass-(in)dependent renorm. schemes

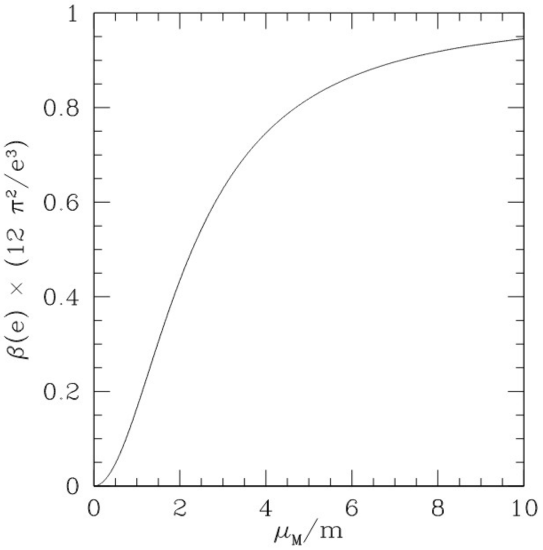
- The problem with mass independent schemes is that they don't decouple!

In QED

$$\beta^{\overline{MS}}(e) = \frac{e^3}{12\pi^2}$$

mom. subs.

$$\beta(e) = \frac{e^3}{2\pi^2}\int_0^1 dx\, x(1-x)\frac{\mu^2 x(1-x)}{M^2+\mu^2 x(1-x)} \sim \begin{cases} \frac{e^3}{12\pi^2}, & M \ll \mu \\ \frac{e^3}{60\pi^2}\frac{\mu^2}{M^2}, & \mu \ll M \end{cases}$$



The solution is to consider a new EFT without the heavy particle and match, then run to the next mass threshold and repeat the process until you reach the energies you are interested in.

When EFT is the only way

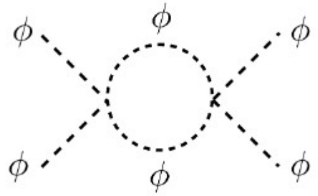
Taken from Cohen's lectures 1903.03622

- Let's consider the following renormalizable Lagrangian

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu\phi)^2 - m^2\phi^2] + \frac{1}{2}[(\partial_\mu\Phi)^2 - M^2\Phi^2] - \frac{\eta}{4!}\phi^4 - \frac{\kappa}{4}\phi^2\Phi^2$$

- We want to compute $\phi\phi \to \phi\phi$ scattering at threshold

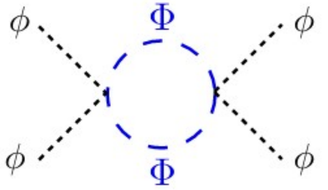
$3 \times$



$$= \frac{3i}{32\pi^2}\,\mu^{2\epsilon}\,\eta^2\left(\frac{1}{\epsilon} + \log\frac{\tilde{\mu}^2}{m^2} + \frac{2}{3}\right)$$

$$K_\eta = \frac{3/2}{16\pi^2}\left[\eta + \frac{\kappa^2}{\eta}\right]$$

$3 \times$



$$= \frac{3i}{32\pi^2}\,\mu^{2\epsilon}\,\kappa^2\left(\frac{1}{\epsilon} + \log\frac{\tilde{\mu}^2}{M^2}\right)$$

$$\mathrm{i}\mathcal{M}_F^{\text{tree+1L}} = -\mathrm{i}\eta + \frac{\mathrm{i}}{16\pi^2}\left[\eta^2\left(1 + \frac{3}{2}\log\frac{\tilde{\mu}^2}{m^2}\right) + \kappa^2\frac{3}{2}\log\frac{\tilde{\mu}^2}{M^2}\right]$$

There is no choice of $\tilde{\mu}$ that makes both logs small so perturbation theory breaks down for $m \ll M$

When EFT is the only way

Taken from Cohen's lectures 1903.03622

- Let's do it the EFT way

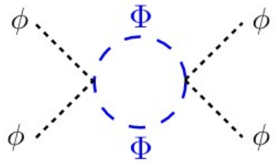
$$\mathcal{L}_{\text{EFT}} = \frac{1}{2}[(\partial_\mu\phi)^2 - m^2\phi^2] - \frac{C_4}{4!}\phi^4 + \ldots$$

- The matching, up to 1 loop gives (no correction to kinetic term)

$$C_4(\tilde{\mu}_M) = \eta(\tilde{\mu}_M) - \frac{3}{2}\frac{1}{16\pi^2}\big(\kappa(\tilde{\mu}_M)\big)^2 \log\frac{\tilde{\mu}_M^2}{M^2}$$

The log is small if we choose $\tilde{\mu}_M \sim M$

$3\times$



$$= \frac{3i}{32\pi^2}\mu^{2\epsilon}\kappa^2\left(\frac{1}{\epsilon} + \log\frac{\tilde{\mu}^2}{M^2}\right)$$

- We can now use the RGE to run from $\tilde{\mu}_M$ to $\tilde{\mu}_L$

$$\frac{\mathrm{d}C_4}{\mathrm{d}\log\mu^2} = \frac{3}{2}\frac{C_4^2}{16\pi^2} \Rightarrow C_4(\tilde{\mu}_L) = \frac{C_4(\tilde{\mu}_M)}{1 - \frac{3}{32\pi^2}C_4(\tilde{\mu}_M)\log\frac{\tilde{\mu}_L^2}{\tilde{\mu}_M^2}}$$

The large log is resummed to all loop order

- And now compute the amplitude in the EFT at $\tilde{\mu}_L$

$$\mathrm{i}\mathcal{M}_E^{\text{NLO}} = -\mathrm{i}C_4(\tilde{\mu}_L) + \frac{\mathrm{i}}{16\pi^2}C_4(\tilde{\mu}_L)^2\left(1 + \frac{3}{2}\log\frac{\tilde{\mu}_L^2}{m^2}\right)$$

This log is also small if we choose $\tilde{\mu}_L \sim m$

When EFT is the only way

Taken from Cohen's lectures 1903.03622

- What happened? The RGE in the EFT allowed us to resum (to all loop orders) the large log.
- Indeed, if we expand to leading log our NLO solution we get the original amplitude

$$C_4(\tilde{\mu}_L) = \frac{C_4(\tilde{\mu}_M)}{1 - \frac{3}{32\pi^2} C_4(\tilde{\mu}_M) \log \frac{\tilde{\mu}_L^2}{\tilde{\mu}_M^2}} = C_4(\tilde{\mu}_M) + \frac{3}{32\pi^2} C_4^2(\tilde{\mu}_M) \log \frac{\tilde{\mu}_L^2}{\tilde{\mu}_M^2} + \dots$$

$$= \eta(\tilde{\mu}_M) + \frac{3}{2}\frac{1}{16\pi^2}\left[\eta^2(\tilde{\mu}_M) \log \frac{\tilde{\mu}_L^2}{\tilde{\mu}_M^2} - \kappa^2(\tilde{\mu}_M) \log \frac{\tilde{\mu}_M^2}{M^2}\right] + \dots$$

$$\begin{aligned} \mathrm{i}\mathcal{M}_E^{\mathrm{LL}} &= -\mathrm{i}C_4(\tilde{\mu}_L) + \frac{\mathrm{i}}{16\pi^2} C_4(\tilde{\mu}_L)^2 \left(1 + \frac{3}{2}\log\frac{\tilde{\mu}_L}{m^2}\right) \\ &= -\mathrm{i}\eta(\tilde{\mu}_M) - \mathrm{i}\frac{3}{2}\frac{1}{16\pi^2}\left[\eta^2(\tilde{\mu}_M) \log \frac{\tilde{\mu}_L^2}{\tilde{\mu}_M^2} - \kappa^2(\tilde{\mu}_M) \log \frac{\tilde{\mu}_M^2}{M^2}\right] \\ &\quad + \frac{\mathrm{i}}{16\pi^2}\eta(\tilde{\mu}_M)^2 \left(1 + \frac{3}{2}\log\frac{\tilde{\mu}_L}{m^2}\right) \end{aligned}$$

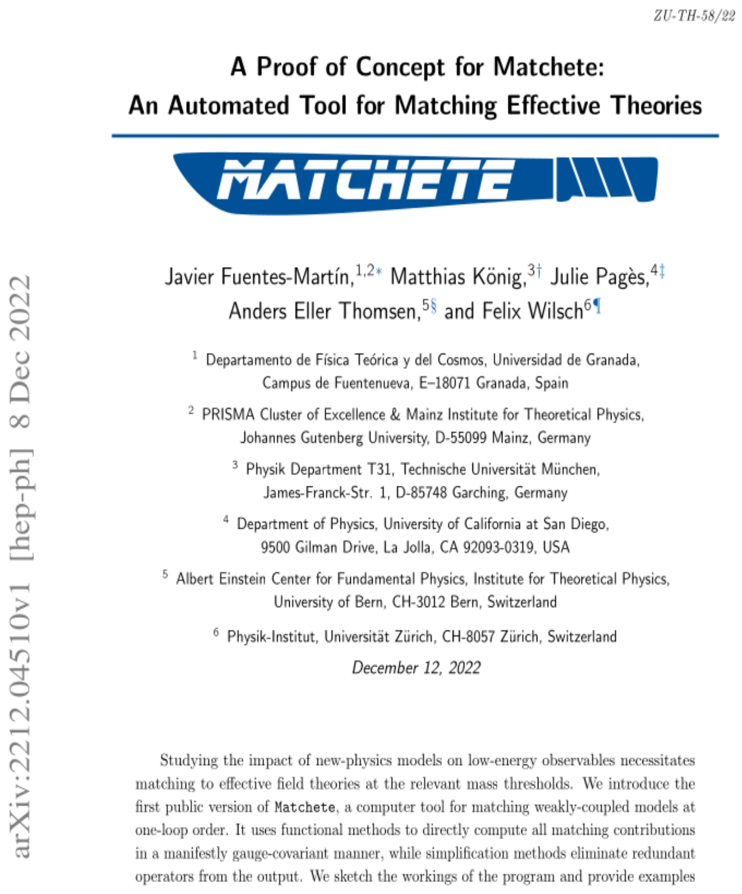
$$\mathrm{i}\mathcal{M}_F^{\mathrm{tree+1L}} = -\mathrm{i}\eta + \frac{\mathrm{i}}{16\pi^2}\left[\eta^2\left(1 + \frac{3}{2}\log\frac{\tilde{\mu}^2}{m^2}\right) + \kappa^2 \frac{3}{2}\log\frac{\tilde{\mu}^2}{M^2}\right]$$

What's new now in EFTs?

- Bottom-up:
  - Global fits with increasing number of experimental observables (EW, Higgs, top, flavor, LHC tails, …).
  - One-loop dim-6 in the EFT slowly being incorporated.
  - Explicit construction of bases up to dim 9 (with and without neutrinos).
  - Dim 8 effects starting to be taken into account.
- Bottom-up/top-down:
  - RGEs: known for SMEFT and LEFT up to dim 6, partial results for dim 8
  - Matching: Matching from SMEFT to LEFT up to 1-loop known
  - Both implemented in computer codes
  - RGEs for beyond the SMEFT (ALPs, neutrinos, …)
  - Positivity bounds

What’s new now in EFTs?

- Top-down:
  - Impressive progress in functional matching up to one-loop.
    - Codes available to make the matching easier (but no fully automated yet).



ZU-TH-58/22

A Proof of Concept for Matchete:
An Automated Tool for Matching Effective Theories

MATCHETE

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December 12, 2022

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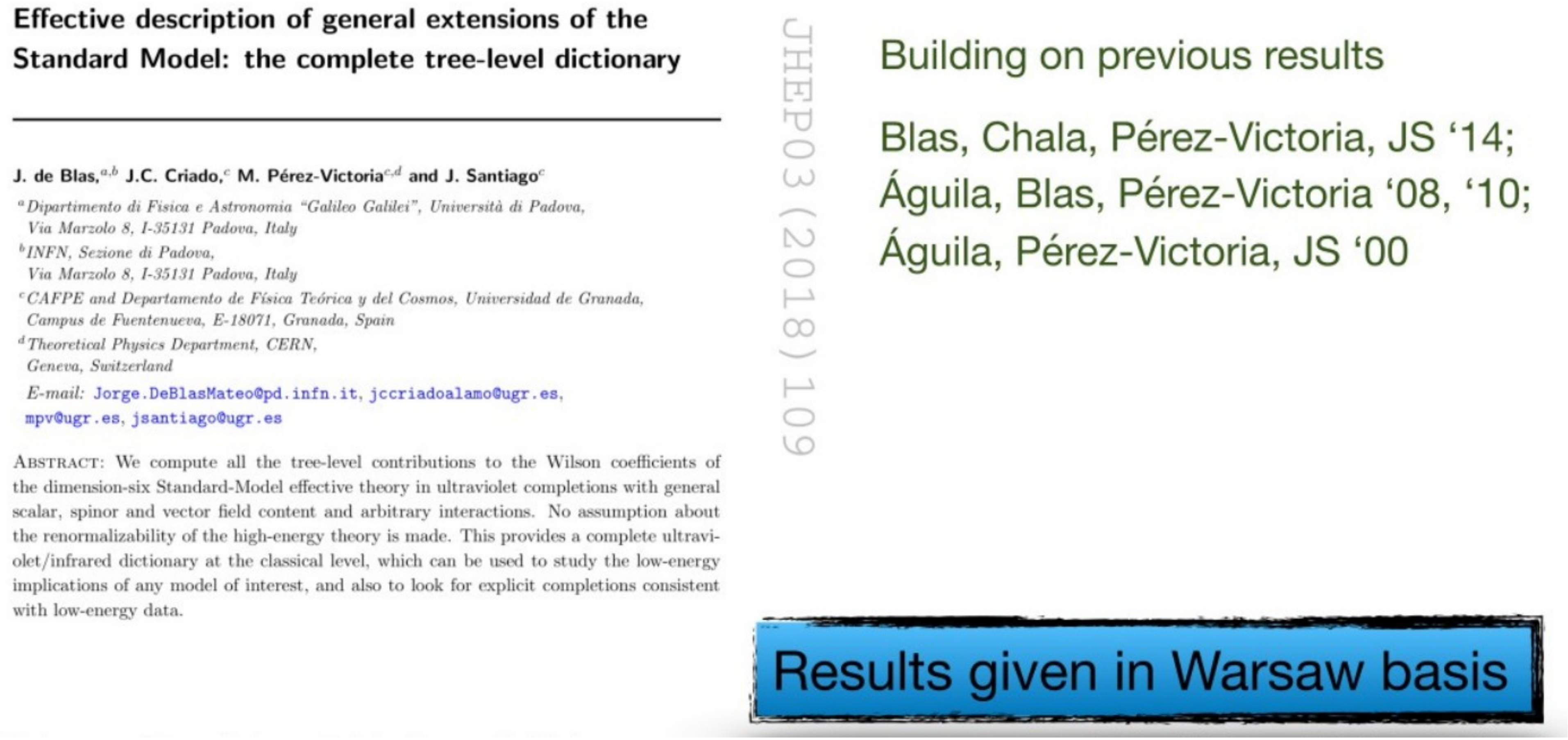
Studying the impact of new-physics models on low-energy observables necessitates matching to effective field theories at the relevant mass thresholds. We introduce the first public version of `Matchete`, a computer tool for matching weakly-coupled models at one-loop order. It uses functional methods to directly compute all matching contributions in a manifestly gauge-covariant manner, while simplification methods eliminate redundant operators from the output. We sketch the workings of the program and provide examples

What's new now in EFTs?

- Top-down:
  - Impressive progress in functional matching up to one-loop.
    - Codes available to make the matching easier (but no fully automated yet).
  - Fully automated matching up to one loop via Feynman diagrams now available. [Matchmakereft, A. Carmona, A. Lazopoulos, P. Olgoso, J. Santiago, 2112.10787]
  - IR/UV dictionaries being developed:
    - Complete classification of all models that contribute to the EFT at certain order and matching to the EFT. [Blas, Criado, Pérez-Victoria, Santiago, 1711.10391]
    - Leading contribution (tree-level, dimension 6) already finished, next ones in progress.

IR/UV dictionaries

- The leading IR/UV dictionary (tree-level, dimension 6 SMEFT) was computed a few years ago. [Blas, Criado, Pérez-Victoria, Santiago ‘18]



Effective description of general extensions of the Standard Model: the complete tree-level dictionary

JHEP03(2018)109

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ABSTRACT: We compute all the tree-level contributions to the Wilson coefficients of the dimension-six Standard-Model effective theory in ultraviolet completions with general scalar, spinor and vector field content and arbitrary interactions. No assumption about the renormalizability of the high-energy theory is made. This provides a complete ultraviolet/infrared dictionary at the classical level, which can be used to study the low-energy implications of any model of interest, and also to look for explicit completions consistent with low-energy data.

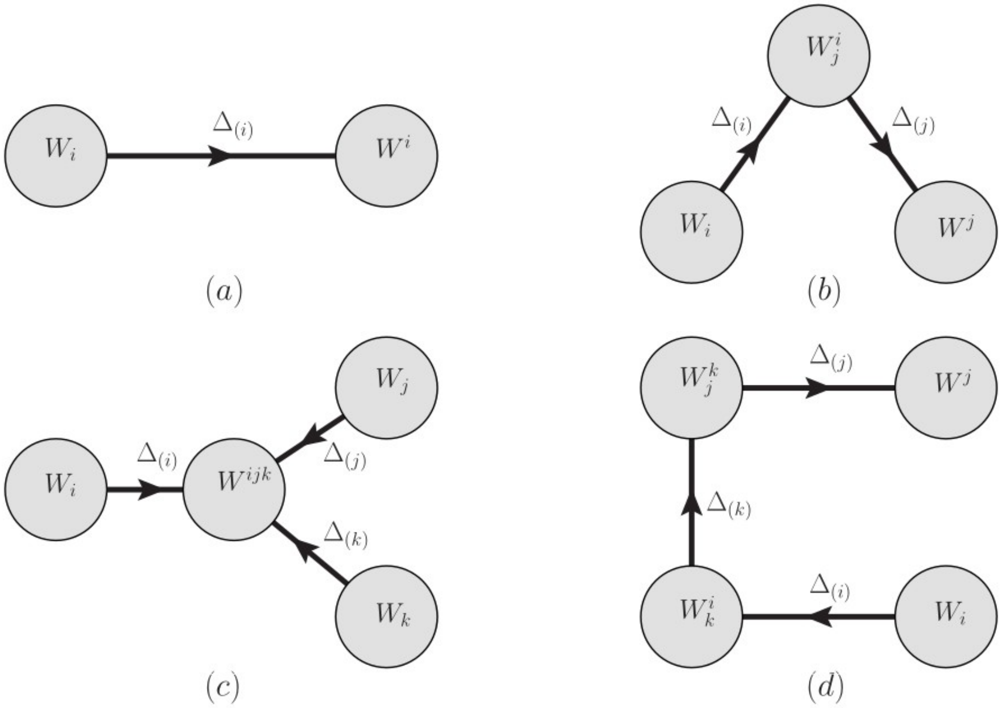
Building on previous results

Blas, Chala, Pérez-Victoria, JS ‘14;
Águila, Blas, Pérez-Victoria ‘08, ‘10;
Águila, Pérez-Victoria, JS ‘00

Results given in Warsaw basis

IR/UV dictionaries

- The leading IR/UV dictionary (tree-level, dimension 6 SMEFT) was computed a few years ago. [Blas, Criado, Pérez-Victoria, Santiago ‘18]
- Complete list of all possible models that contribute to experiment at tree-level and dim 6 (and their contributions).



IR/UV dictionaries

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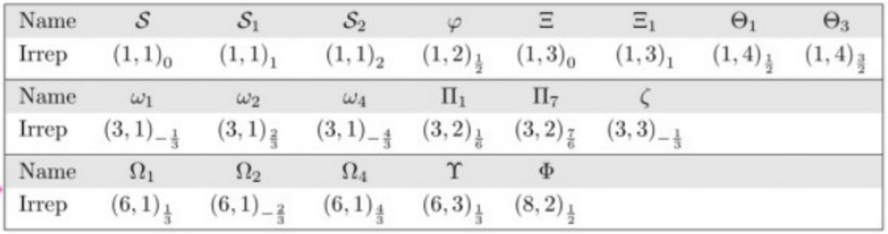
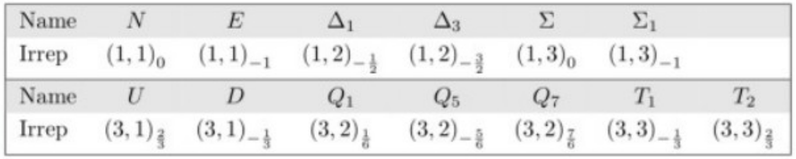
19 scalars



Name	$\mathcal{S}$	$\mathcal{S}_1$	$\mathcal{S}_2$	φ	Ξ	Ξ_1	Θ_1	Θ_3
Irrep	$(1,1)_0$	$(1,1)_1$	$(1,1)_2$	$(1,2)_{\frac{1}{2}}$	$(1,3)_0$	$(1,3)_1$	$(1,4)_{\frac{1}{2}}$	$(1,4)_{\frac{3}{2}}$
Name	ω_1	ω_2	ω_4	Π_1	Π_7	ζ		
Irrep	$(3,1)_{-\frac{1}{3}}$	$(3,1)_{\frac{2}{3}}$	$(3,1)_{-\frac{4}{3}}$	$(3,2)_{\frac{1}{6}}$	$(3,2)_{\frac{7}{6}}$	$(3,3)_{-\frac{1}{3}}$		
Name	Ω_1	Ω_2	Ω_4	Υ	Φ			
Irrep	$(6,1)_{\frac{1}{3}}$	$(6,1)_{-\frac{2}{3}}$	$(6,1)_{\frac{4}{3}}$	$(6,3)_{\frac{1}{3}}$	$(8,2)_{\frac{1}{2}}$			

Table 1. New scalar bosons contributing to the dimension-six SMEFT at tree level.

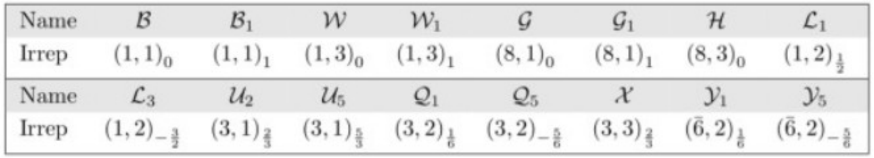
13 fermions



Name	N	E	Δ_1	Δ_3	Σ	Σ_1	
Irrep	$(1,1)_0$	$(1,1)_{-1}$	$(1,2)_{-\frac{1}{2}}$	$(1,2)_{-\frac{3}{2}}$	$(1,3)_0$	$(1,3)_{-1}$	
Name	U	D	Q_1	Q_5	Q_7	T_1	T_2
Irrep	$(3,1)_{\frac{2}{3}}$	$(3,1)_{-\frac{1}{3}}$	$(3,2)_{\frac{1}{6}}$	$(3,2)_{-\frac{5}{6}}$	$(3,2)_{\frac{7}{6}}$	$(3,3)_{-\frac{1}{3}}$	$(3,3)_{\frac{2}{3}}$

Table 2. New vector-like fermions contributing to the dimension-six SMEFT at tree level.

16 vectors



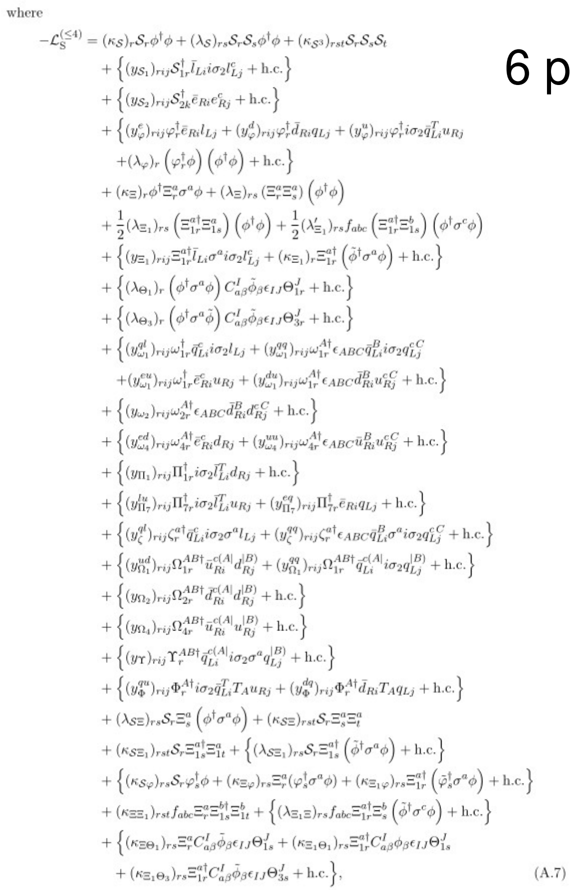
Name	$\mathcal{B}$	$\mathcal{B}_1$	$\mathcal{W}$	$\mathcal{W}_1$	$\mathcal{G}$	$\mathcal{G}_1$	$\mathcal{H}$	$\mathcal{L}_1$
Irrep	$(1,1)_0$	$(1,1)_1$	$(1,3)_0$	$(1,3)_1$	$(8,1)_0$	$(8,1)_1$	$(8,3)_0$	$(1,2)_{\frac{1}{2}}$
Name	$\mathcal{L}_3$	$\mathcal{U}_2$	$\mathcal{U}_5$	$\mathcal{Q}_1$	$\mathcal{Q}_5$	$\mathcal{X}$	$\mathcal{Y}_1$	$\mathcal{Y}_5$
Irrep	$(1,2)_{-\frac{3}{2}}$	$(3,1)_{\frac{2}{3}}$	$(3,1)_{\frac{5}{3}}$	$(3,2)_{\frac{1}{6}}$	$(3,2)_{-\frac{5}{6}}$	$(3,3)_{\frac{2}{3}}$	$(\bar{6},2)_{\frac{1}{6}}$	$(\bar{6},2)_{-\frac{5}{6}}$

Table 3. New vector bosons contributing to the dimension-six SMEFT at tree level.

48 new fields

IR/UV dictionaries

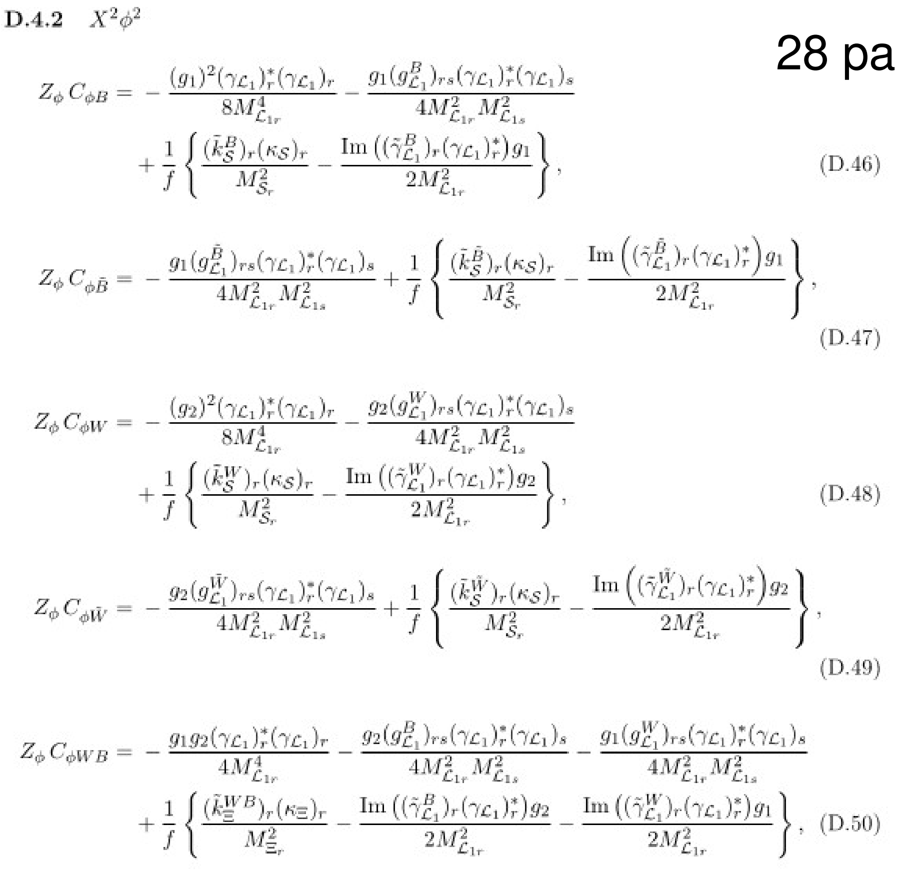
- The leading IR/UV dictionary (tree-level, dimension 6 SMEFT) was computed a few years ago. [Blas, Criado, Pérez-Victoria, Santiago '18]
- Complete list of all possible models that contribute to experiment at tree-level and dim 6 (and their contributions).



6 pages

where

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{S}}^{(\leq 4)} = {} & (\kappa_{\mathcal{S}})_r \mathcal{S}_r \phi^\dagger \phi + (\lambda_{\mathcal{S}})_{rs} \mathcal{S}_r \mathcal{S}_s \phi^\dagger \phi + (\kappa_{\mathcal{S}^3})_{rst} \mathcal{S}_r \mathcal{S}_s \mathcal{S}_t \\
& + \left\{ (y_{\mathcal{S}_1})_{rij} \mathcal{S}_{1r}^\dagger \bar{l}_{Li} i\sigma_2 l_{Lj}^c + \text{h.c.} \right\} \\
& + \left\{ (y_{\mathcal{S}_2})_{rij} \mathcal{S}_{2k}^\dagger \bar{e}_{Ri} e_{Rj}^c + \text{h.c.} \right\} \\
& + \left\{ (y_\varphi^e)_{rij} \varphi_r^\dagger \bar{e}_{Ri} l_{Lj} + (y_\varphi^d)_{rij} \varphi_r^\dagger \bar{d}_{Ri} q_{Lj} + (y_\varphi^u)_{rij} \varphi_r^\dagger i\sigma_2 \bar{q}_{Li}^T u_{Rj} \right. \\
& \quad \left. + (\lambda_\varphi)_r \left(\varphi_r^\dagger \phi \right) \left(\phi^\dagger \phi \right) + \text{h.c.} \right\} \\
& + (\kappa_\Xi)_r \phi^\dagger \Xi_r^a \sigma^a \phi + (\lambda_\Xi)_{rs} \left(\Xi_r^a \Xi_s^a \right) \left(\phi^\dagger \phi \right) \\
& + \frac{1}{2} (\lambda_{\Xi_1})_{rs} \left(\Xi_{1r}^{a\dagger} \Xi_{1s}^a \right) \left(\phi^\dagger \phi \right) + \frac{1}{2} (\lambda'_{\Xi_1})_{rs} f_{abc} \left(\Xi_{1r}^{a\dagger} \Xi_{1s}^b \right) \left(\phi^\dagger \sigma^c \phi \right) \\
& + \left\{ (y_{\Xi_1})_{rij} \Xi_{1r}^{a\dagger} \bar{l}_{Li} \sigma^a i\sigma_2 l_{Lj}^c + (\kappa_{\Xi_1})_r \Xi_{1r}^{a\dagger} \left(\tilde{\phi}^\dagger \sigma^a \phi \right) + \text{h.c.} \right\} \\
& + \left\{ (\lambda_{\Theta_1})_r \left(\phi^\dagger \sigma^a \phi \right) C_{a\beta}^I \tilde{\phi}_\beta \epsilon_{IJ} \Theta_{1r}^J + \text{h.c.} \right\} \\
& + \left\{ (\lambda_{\Theta_3})_r \left(\phi^\dagger \sigma^a \tilde{\phi} \right) C_{a\beta}^I \tilde{\phi}_\beta \epsilon_{IJ} \Theta_{3r}^J + \text{h.c.} \right\} \\
& + \left\{ (y_{\omega_1}^{ql})_{rij} \omega_{1r}^\dagger \bar{q}_{Li}^c i\sigma_2 l_{Lj} + (y_{\omega_1}^{qq})_{rij} \omega_{1r}^{A\dagger} \epsilon_{ABC} \bar{q}_{Li}^B i\sigma_2 q_{Lj}^{cC} \right. \\
& \quad \left. + (y_{\omega_1}^{eu})_{rij} \omega_{1r}^\dagger \bar{e}_{Ri}^c u_{Rj} + (y_{\omega_1}^{du})_{rij} \omega_{1r}^{A\dagger} \epsilon_{ABC} \bar{d}_{Ri}^B u_{Rj}^{cC} + \text{h.c.} \right\} \\
& + \left\{ (y_{\omega_2})_{rij} \omega_{2r}^{A\dagger} \epsilon_{ABC} \bar{d}_{Ri}^B d_{Rj}^{cC} + \text{h.c.} \right\} \\
& + \left\{ (y_{\omega_4}^{ed})_{rij} \omega_{4r}^{A\dagger} \bar{e}_{Ri}^c d_{Rj} + (y_{\omega_4}^{uu})_{rij} \omega_{4r}^{A\dagger} \epsilon_{ABC} \bar{u}_{Ri}^B u_{Rj}^{cC} + \text{h.c.} \right\} \\
& + \left\{ (y_{\Pi_1})_{rij} \Pi_{1r}^\dagger i\sigma_2 \bar{l}_{Li}^T d_{Rj} + \text{h.c.} \right\} \\
& + \left\{ (y_{\Pi_7}^{lu})_{rij} \Pi_{7r}^\dagger i\sigma_2 \bar{l}_{Li}^T u_{Rj} + (y_{\Pi_7}^{eq})_{rij} \Pi_{7r}^\dagger \bar{e}_{Ri} q_{Lj} + \text{h.c.} \right\} \\
& + \left\{ (y_\zeta^{ql})_{rij} \zeta_r^{a\dagger} \bar{q}_{Li}^c i\sigma_2 \sigma^a l_{Lj} + (y_\zeta^{qq})_{rij} \zeta_r^{a\dagger} \epsilon_{ABC} \bar{q}_{Li}^B \sigma^a i\sigma_2 q_{Lj}^{cC} + \text{h.c.} \right\} \\
& + \left\{ (y_{\Omega_1}^{ud})_{rij} \Omega_{1r}^{AB\dagger} \bar{u}_{Ri}^{c(A|} d_{Rj}^{|B)} + (y_{\Omega_1}^{qq})_{rij} \Omega_{1r}^{AB\dagger} \bar{q}_{Li}^{c(A|} i\sigma_2 q_{Lj}^{|B)} + \text{h.c.} \right\} \\
& + \left\{ (y_{\Omega_2})_{rij} \Omega_{2r}^{AB\dagger} \bar{d}_{Ri}^{c(A|} d_{Rj}^{|B)} + \text{h.c.} \right\} \\
& + \left\{ (y_{\Omega_4})_{rij} \Omega_{4r}^{AB\dagger} \bar{u}_{Ri}^{c(A|} u_{Rj}^{|B)} + \text{h.c.} \right\} \\
& + \left\{ (y_\Upsilon)_{rij} \Upsilon_r^{AB\dagger} \bar{q}_{Li}^{c(A|} i\sigma_2 \sigma^a q_{Lj}^{|B)} + \text{h.c.} \right\} \\
& + \left\{ (y_\Phi^{qu})_{rij} \Phi_r^{A\dagger} i\sigma_2 \bar{q}_{Li}^T T_A u_{Rj} + (y_\Phi^{dq})_{rij} \Phi_r^{A\dagger} \bar{d}_{Ri} T_A q_{Lj} + \text{h.c.} \right\} \\
& + (\lambda_{\mathcal{S}\Xi})_{rs} \mathcal{S}_r \Xi_s^a \left(\phi^\dagger \sigma^a \phi \right) + (\kappa_{\mathcal{S}\Xi})_{rst} \mathcal{S}_r \Xi_s^a \Xi_t^a \\
& + (\kappa_{\mathcal{S}\Xi_1})_{rst} \mathcal{S}_r \Xi_{1s}^{a\dagger} \Xi_{1t}^a + \left\{ (\lambda_{\mathcal{S}\Xi_1})_{rs} \mathcal{S}_r \Xi_{1s}^{a\dagger} \left(\tilde{\phi}^\dagger \sigma^a \phi \right) + \text{h.c.} \right\} \\
& + \left\{ (\kappa_{\mathcal{S}\varphi})_{rs} \mathcal{S}_r \varphi_s^\dagger \phi + (\kappa_{\Xi\varphi})_{rs} \Xi_r^a (\varphi_s^\dagger \sigma^a \phi) + (\kappa_{\Xi_1\varphi})_{rs} \Xi_{1r}^{a\dagger} \left(\tilde{\varphi}_s^\dagger \sigma^a \phi \right) + \text{h.c.} \right\} \\
& + (\kappa_{\Xi\Xi_1})_{rst} f_{abc} \Xi_r^a \Xi_{1s}^{b\dagger} \Xi_{1t}^b + \left\{ (\lambda_{\Xi_1\Xi})_{rs} f_{abc} \Xi_{1r}^{a\dagger} \Xi_s^b \left(\tilde{\phi}^\dagger \sigma^c \phi \right) + \text{h.c.} \right\} \\
& + \left\{ (\kappa_{\Xi\Theta_1})_{rs} \Xi_r^a C_{a\beta}^I \tilde{\phi}_\beta \epsilon_{IJ} \Theta_{1s}^J + (\kappa_{\Xi_1\Theta_1})_{rs} \Xi_{1r}^{a\dagger} C_{a\beta}^I \phi_\beta \epsilon_{IJ} \Theta_{1s}^J \right. \\
& \quad \left. + (\kappa_{\Xi_1\Theta_3})_{rs} \Xi_{1r}^{a\dagger} C_{a\beta}^I \tilde{\phi}_\beta \epsilon_{IJ} \Theta_{3s}^J + \text{h.c.} \right\},
\end{aligned}
\tag{A.7}
$$



28 pages

D.4.2 $X^2\phi^2$

$$
\begin{aligned}
Z_\phi\, C_{\phi B} = {} & -\frac{(g_1)^2 (\gamma_{\mathcal{L}_1})_r^* (\gamma_{\mathcal{L}_1})_r}{8 M_{\mathcal{L}_{1r}}^4} - \frac{g_1 (g_{\mathcal{L}_1}^B)_{rs} (\gamma_{\mathcal{L}_1})_r^* (\gamma_{\mathcal{L}_1})_s}{4 M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} \\
& + \frac{1}{f} \left\{ \frac{(\tilde{k}_{\mathcal{S}}^B)_r (\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2} - \frac{\operatorname{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^B)_r (\gamma_{\mathcal{L}_1})_r^* \right) g_1}{2 M_{\mathcal{L}_{1r}}^2} \right\},
\end{aligned}
\tag{D.46}
$$

$$
Z_\phi\, C_{\phi \tilde{B}} = -\frac{g_1 (g_{\mathcal{L}_1}^{\tilde{B}})_{rs} (\gamma_{\mathcal{L}_1})_r^* (\gamma_{\mathcal{L}_1})_s}{4 M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} + \frac{1}{f} \left\{ \frac{(\tilde{k}_{\mathcal{S}}^{\tilde{B}})_r (\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2} - \frac{\operatorname{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{\tilde{B}})_r (\gamma_{\mathcal{L}_1})_r^* \right) g_1}{2 M_{\mathcal{L}_{1r}}^2} \right\},
\tag{D.47}
$$

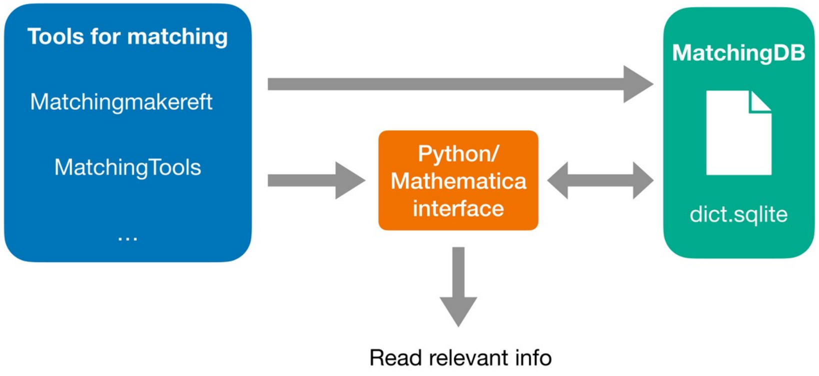
$$
\begin{aligned}
Z_\phi\, C_{\phi W} = {} & -\frac{(g_2)^2 (\gamma_{\mathcal{L}_1})_r^* (\gamma_{\mathcal{L}_1})_r}{8 M_{\mathcal{L}_{1r}}^4} - \frac{g_2 (g_{\mathcal{L}_1}^W)_{rs} (\gamma_{\mathcal{L}_1})_r^* (\gamma_{\mathcal{L}_1})_s}{4 M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} \\
& + \frac{1}{f} \left\{ \frac{(\tilde{k}_{\mathcal{S}}^W)_r (\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2} - \frac{\operatorname{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^W)_r (\gamma_{\mathcal{L}_1})_r^* \right) g_2}{2 M_{\mathcal{L}_{1r}}^2} \right\},
\end{aligned}
\tag{D.48}
$$

$$
Z_\phi\, C_{\phi \tilde{W}} = -\frac{g_2 (g_{\mathcal{L}_1}^{\tilde{W}})_{rs} (\gamma_{\mathcal{L}_1})_r^* (\gamma_{\mathcal{L}_1})_s}{4 M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} + \frac{1}{f} \left\{ \frac{(\tilde{k}_{\mathcal{S}}^{\tilde{W}})_r (\kappa_{\mathcal{S}})_r}{M_{\mathcal{S}_r}^2} - \frac{\operatorname{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^{\tilde{W}})_r (\gamma_{\mathcal{L}_1})_r^* \right) g_2}{2 M_{\mathcal{L}_{1r}}^2} \right\},
\tag{D.49}
$$

$$
\begin{aligned}
Z_\phi\, C_{\phi WB} = {} & -\frac{g_1 g_2 (\gamma_{\mathcal{L}_1})_r^* (\gamma_{\mathcal{L}_1})_r}{4 M_{\mathcal{L}_{1r}}^4} - \frac{g_2 (g_{\mathcal{L}_1}^B)_{rs} (\gamma_{\mathcal{L}_1})_r^* (\gamma_{\mathcal{L}_1})_s}{4 M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} - \frac{g_1 (g_{\mathcal{L}_1}^W)_{rs} (\gamma_{\mathcal{L}_1})_r^* (\gamma_{\mathcal{L}_1})_s}{4 M_{\mathcal{L}_{1r}}^2 M_{\mathcal{L}_{1s}}^2} \\
& + \frac{1}{f} \left\{ \frac{(\tilde{k}_{\Xi}^{WB})_r (\kappa_\Xi)_r}{M_{\Xi_r}^2} - \frac{\operatorname{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^B)_r (\gamma_{\mathcal{L}_1})_r^* \right) g_2}{2 M_{\mathcal{L}_{1r}}^2} - \frac{\operatorname{Im}\left((\tilde{\gamma}_{\mathcal{L}_1}^W)_r (\gamma_{\mathcal{L}_1})_r^* \right) g_1}{2 M_{\mathcal{L}_{1r}}^2} \right\},
\end{aligned}
\tag{D.50}
$$

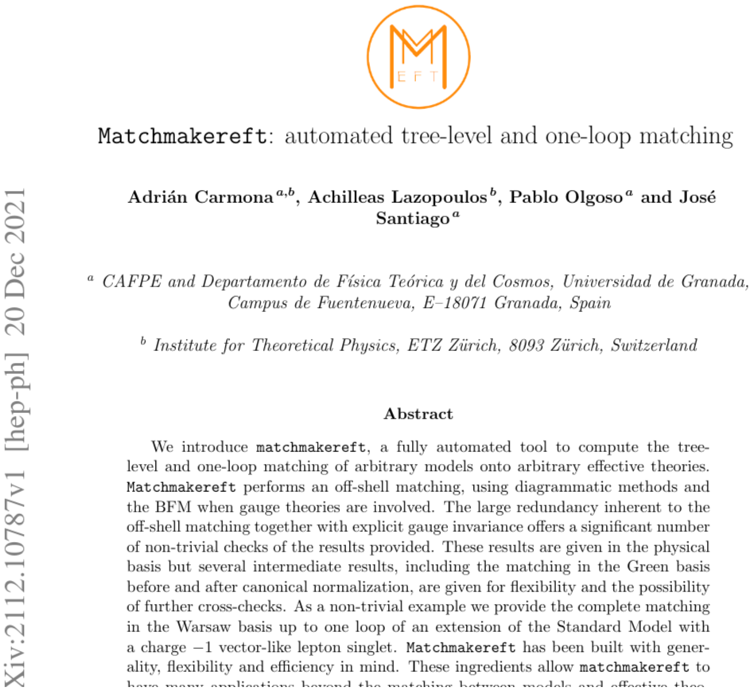
IR/UV dictionaries

- The leading IR/UV dictionary (tree-level, dimension 6 SMEFT) was computed a few years ago. [Blas, Criado, Pérez-Victoria, Santiago ‘18]
- Complete list of all possible models that contribute to experiment at tree-level and dim 6 (and their contributions).
- Tree-level and dimension 6 is not enough for current experimental precision. Going beyond requires automation.
- The next (tree-level dimension 8 or 1-loop dimension 6) dictionaries will need to be published in electronic form. We are working on a standard database format to store them [with J.C. Criado]



gitlab.com/jccriado/matchingdb

Automated matching with MME



`Matchmakereft`: automated tree-level and one-loop matching

Adrián Carmona[a,b], Achilleas Lazopoulos[b], Pablo Olgoso[a] and José Santiago[a]

[a] *CAFPE and Departamento de Física Teórica y del Cosmos, Universidad de Granada, Campus de Fuentenueva, E–18071 Granada, Spain*

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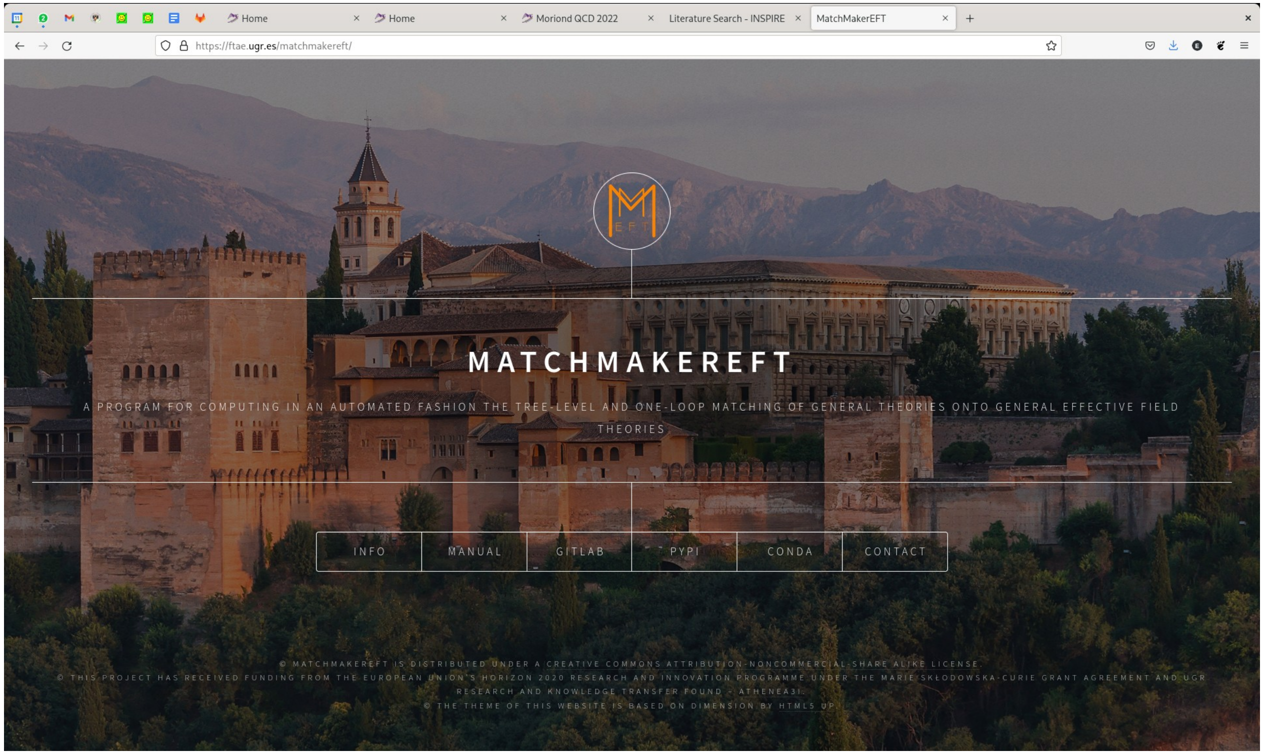
Abstract

We introduce `matchmakereft`, a fully automated tool to compute the tree-level and one-loop matching of arbitrary models onto arbitrary effective theories. `Matchmakereft` performs an off-shell matching, using diagrammatic methods and the BFM when gauge theories are involved. The large redundancy inherent to the off-shell matching together with explicit gauge invariance offers a significant number of non-trivial checks of the results provided. These results are given in the physical basis but several intermediate results, including the matching in the Green basis before and after canonical normalization, are given for flexibility and the possibility of further cross-checks. As a non-trivial example we provide the complete matching in the Warsaw basis up to one loop of an extension of the Standard Model with a charge −1 vector-like lepton singlet. `Matchmakereft` has been built with generality, flexibility and efficiency in mind. These ingredients allow `matchmakereft` to

https://ftae.ugr.es/matchmakereft/



MATCHMAKEREFT

A PROGRAM FOR COMPUTING IN AN AUTOMATED FASHION THE TREE-LEVEL AND ONE-LOOP MATCHING OF GENERAL THEORIES ONTO GENERAL EFFECTIVE FIELD THEORIES

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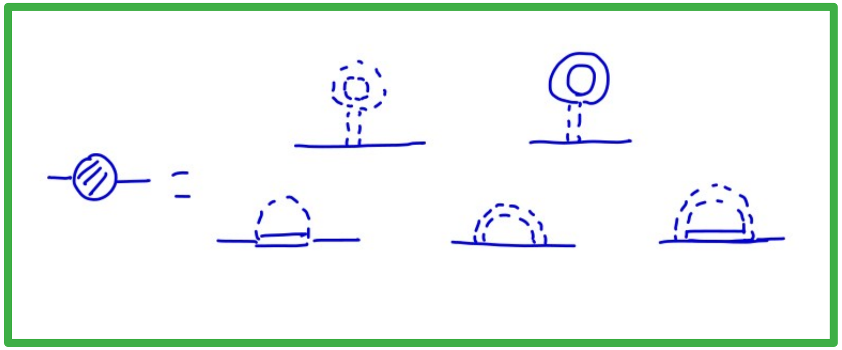
© THE THEME OF THIS WEBSITE IS BASED ON DIMENSION BY HTML5 UP.

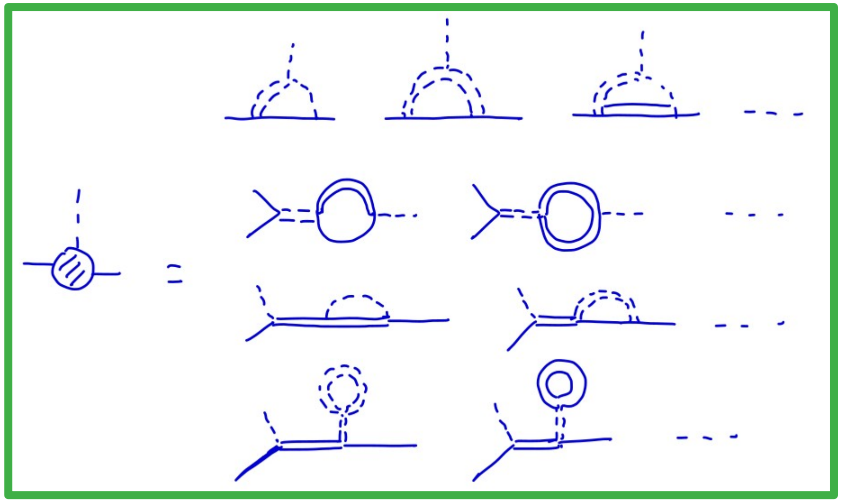
- Also RGEs, operator independence, …

Towards the next IR/UV dictionaries

Preliminary!

- We are working on the one-loop, dimension-6 IR/UV dictionary [with G. Guedes and P. Olgoso]. see also [Cepedello, Esser, Hirsch, Sanz 2207.13714]
  - We have started with operators that cannot be generated at tree level in weakly-coupled extensions $[X^3, X^2\phi^2, \psi^2 X\phi]$, with heavy scalars and fermions [heavy vectors currently under study with J. Fuentes-Martín, P. Olgoso, A.E. Thomsen] and renormalizable interactions.
    - Extend the SMEFT with heavy fields in arbitrary gauge configurations.
    - Just need 2 and 3 point functions (plus gauge boson insertions).





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    - Extend the SMEFT with heavy fields in arbitrary gauge configurations.
    - Just need 2 and 3 point functions (plus gauge boson insertions).
    - Perform the matching with MME using the kinematics but leave gauge directions general [MME is very well suited for this task: matching from EFT, gauge numerics replaced only at the end of the calculation].
    - Result for specific models can be obtained doing a simple group-theoretical calculation [we use GroupMath by R. Fonseca].

Towards the next IR/UV dictionaries

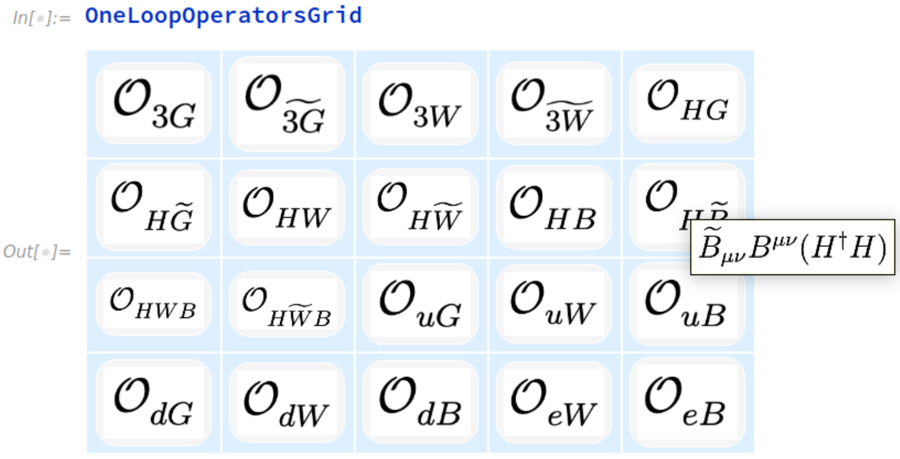
Preliminary!

- We are working on the one-loop, dimension-6 IR/UV dictionary [with G. Guedes and P. Olgoso].

```
In[∘]:= << OneLoopDictionary`

One Loop Dictionary loaded
Version: 0.0.1
Authors: Guilherme Guedes, Pablo Olgoso, José Santiago
Reference: _____

XXXXXXXXXXXXXXXXXXXXXXXXXXX GroupMath XXXXXXXXXXXXXXXXXXXXXXXXXXX
Version: 1.1.2 (6/May/2020)
Author: Renato Fonseca
Reference: 2011.01764 [hep-th]
Website: renatofonseca.net/groupmath
Built-in documentation: here
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

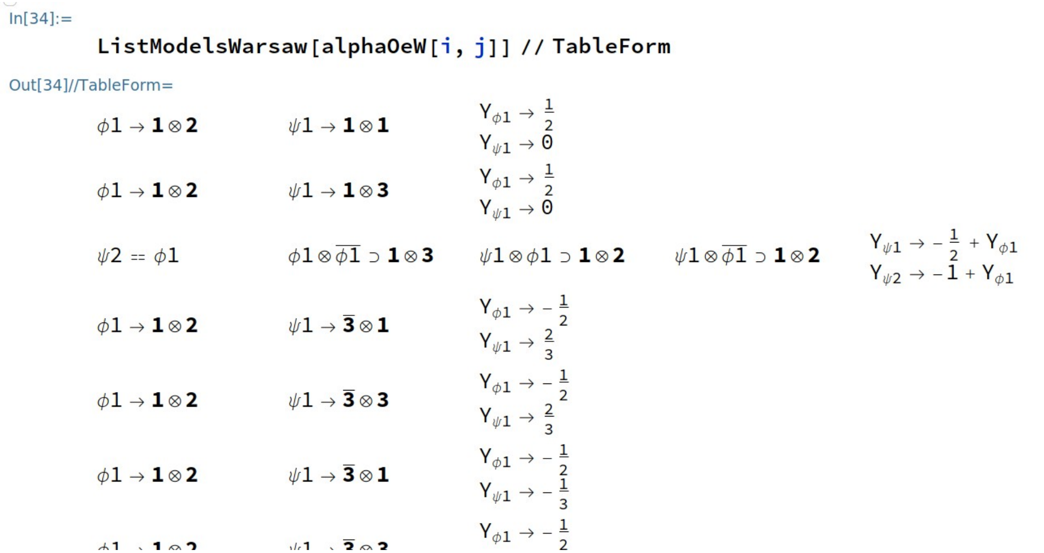


In[∘]:= **OneLoopOperatorsGrid**

Out[∘]=

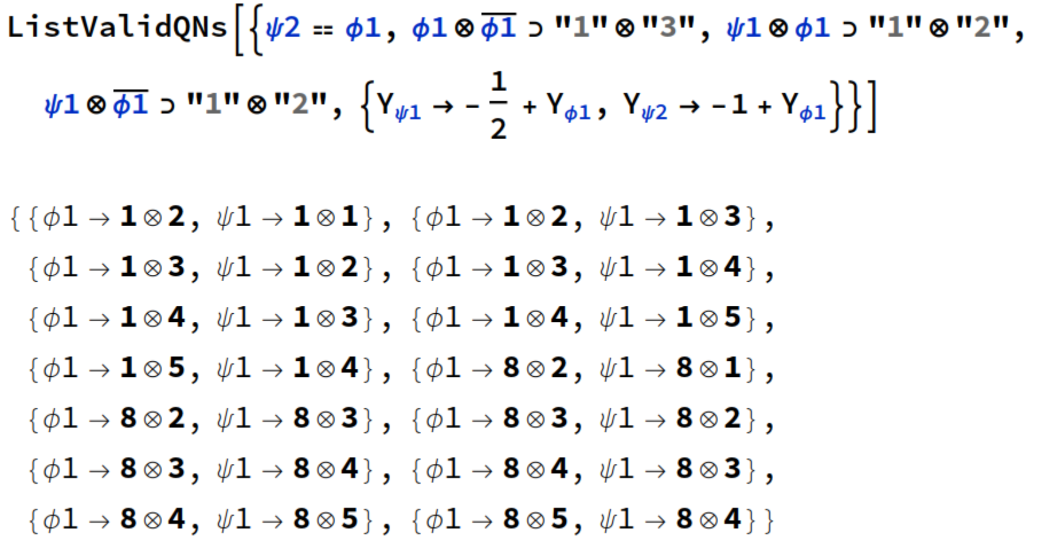
$\mathcal{O}_{3G}$	$\mathcal{O}_{3\widetilde{G}}$	$\mathcal{O}_{3W}$	$\mathcal{O}_{3\widetilde{W}}$	$\mathcal{O}_{HG}$
$\mathcal{O}_{H\widetilde{G}}$	$\mathcal{O}_{HW}$	$\mathcal{O}_{H\widetilde{W}}$	$\mathcal{O}_{HB}$	$\mathcal{O}_{H\widetilde{B}}$
$\mathcal{O}_{HWB}$	$\mathcal{O}_{H\widetilde{W}B}$	$\mathcal{O}_{uG}$	$\mathcal{O}_{uW}$	$\mathcal{O}_{uB}$
$\mathcal{O}_{dG}$	$\mathcal{O}_{dW}$	$\mathcal{O}_{dB}$	$\mathcal{O}_{eW}$	$\mathcal{O}_{eB}$

$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$



```
In[34]:=
        ListModelsWarsaw[alphaOeW[i, j]] // TableForm

Out[34]//TableForm=
        ϕ1 → 1⊗2        ψ1 → 1⊗1        Yϕ1 → 1/2
                                         Yψ1 → 0
        ϕ1 → 1⊗2        ψ1 → 1⊗3        Yϕ1 → 1/2
                                         Yψ1 → 0
        ψ2 == ϕ1        ϕ1⊗ϕ̄1 ⊃ 1⊗3     ψ1⊗ϕ1 ⊃ 1⊗2     ψ1⊗ϕ̄1 ⊃ 1⊗2     Yψ1 → -1/2 + Yϕ1
                                                                         Yψ2 → -1 + Yϕ1
        ϕ1 → 1⊗2        ψ1 → 3̄⊗1        Yϕ1 → -1/2
                                         Yψ1 → 2/3
        ϕ1 → 1⊗2        ψ1 → 3̄⊗3        Yϕ1 → -1/2
                                         Yψ1 → 2/3
        ϕ1 → 1⊗2        ψ1 → 3̄⊗1        Yϕ1 → -1/2
                                         Yψ1 → -1/3
                                         Yϕ1 → -1/2
```



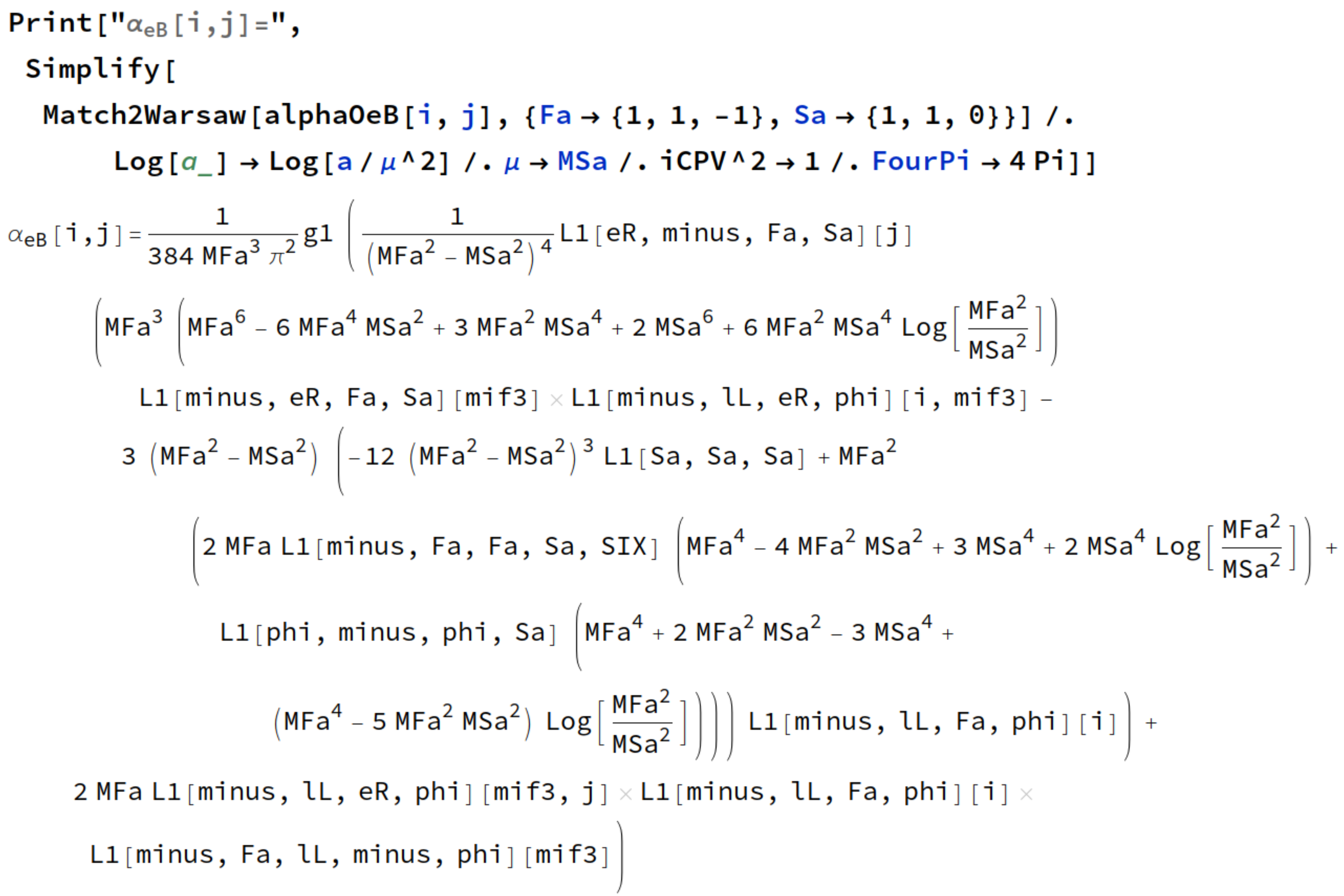
```
ListValidQNs[{ψ2 == ϕ1, ϕ1⊗ϕ̄1 ⊃ "1"⊗"3", ψ1⊗ϕ1 ⊃ "1"⊗"2",
  ψ1⊗ϕ̄1 ⊃ "1"⊗"2", {Yψ1 → -1/2 + Yϕ1, Yψ2 → -1 + Yϕ1}}]

{{ϕ1 → 1⊗2, ψ1 → 1⊗1}, {ϕ1 → 1⊗2, ψ1 → 1⊗3},
 {ϕ1 → 1⊗3, ψ1 → 1⊗2}, {ϕ1 → 1⊗3, ψ1 → 1⊗4},
 {ϕ1 → 1⊗4, ψ1 → 1⊗3}, {ϕ1 → 1⊗4, ψ1 → 1⊗5},
 {ϕ1 → 1⊗5, ψ1 → 1⊗4}, {ϕ1 → 8⊗2, ψ1 → 8⊗1},
 {ϕ1 → 8⊗2, ψ1 → 8⊗3}, {ϕ1 → 8⊗3, ψ1 → 8⊗2},
 {ϕ1 → 8⊗3, ψ1 → 8⊗4}, {ϕ1 → 8⊗4, ψ1 → 8⊗3},
 {ϕ1 → 8⊗4, ψ1 → 8⊗5}, {ϕ1 → 8⊗5, ψ1 → 8⊗4}}
```

Towards the next IR/UV dictionaries

Preliminary!

- We are working on the one-loop, dimension-6 IR/UV dictionary [with G. Guedes and P. Olgoso].



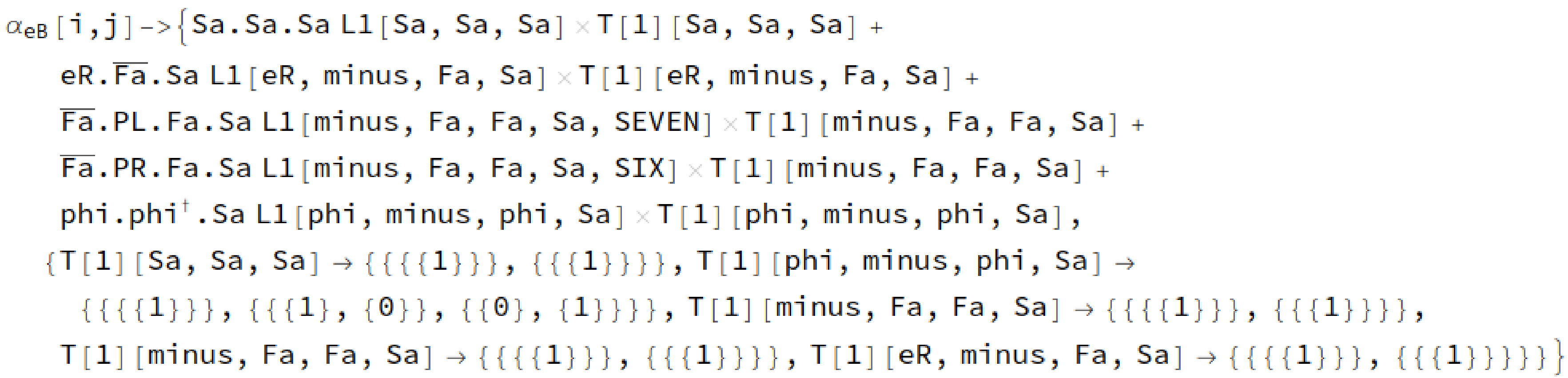
```
Print["αeB[i,j]=",
 Simplify[
  Match2Warsaw[alphaOeB[i, j], {Fa → {1, 1, -1}, Sa → {1, 1, 0}}] /.
      Log[a_] → Log[a / μ^2] /. μ → MSa /. iCPV^2 → 1 /. FourPi → 4 Pi]]
```

$$
\begin{aligned}
\alpha_{eB}[i,j] = {} & \frac{1}{384\,\mathrm{MFa}^3\,\pi^2}\,\mathrm{g1}\,\Bigg(\frac{1}{(\mathrm{MFa}^2-\mathrm{MSa}^2)^4}\,\mathrm{L1}[\mathrm{eR},\mathrm{minus},\mathrm{Fa},\mathrm{Sa}][\mathrm{j}] \\
& \Big(\mathrm{MFa}^3\,\Big(\mathrm{MFa}^6-6\,\mathrm{MFa}^4\,\mathrm{MSa}^2+3\,\mathrm{MFa}^2\,\mathrm{MSa}^4+2\,\mathrm{MSa}^6+6\,\mathrm{MFa}^2\,\mathrm{MSa}^4\,\mathrm{Log}\Big[\frac{\mathrm{MFa}^2}{\mathrm{MSa}^2}\Big]\Big) \\
& \quad \mathrm{L1}[\mathrm{minus},\mathrm{eR},\mathrm{Fa},\mathrm{Sa}][\mathrm{mif3}]\times\mathrm{L1}[\mathrm{minus},\mathrm{lL},\mathrm{eR},\mathrm{phi}][\mathrm{i},\mathrm{mif3}] - \\
& \quad 3\,(\mathrm{MFa}^2-\mathrm{MSa}^2)\,\Big(-12\,(\mathrm{MFa}^2-\mathrm{MSa}^2)^3\,\mathrm{L1}[\mathrm{Sa},\mathrm{Sa},\mathrm{Sa}]+\mathrm{MFa}^2 \\
& \qquad \Big(2\,\mathrm{MFa}\,\mathrm{L1}[\mathrm{minus},\mathrm{Fa},\mathrm{Fa},\mathrm{Sa},\mathrm{SIX}]\,\Big(\mathrm{MFa}^4-4\,\mathrm{MFa}^2\,\mathrm{MSa}^2+3\,\mathrm{MSa}^4+2\,\mathrm{MSa}^4\,\mathrm{Log}\Big[\frac{\mathrm{MFa}^2}{\mathrm{MSa}^2}\Big]\Big)+ \\
& \qquad \mathrm{L1}[\mathrm{phi},\mathrm{minus},\mathrm{phi},\mathrm{Sa}]\,\Big(\mathrm{MFa}^4+2\,\mathrm{MFa}^2\,\mathrm{MSa}^2-3\,\mathrm{MSa}^4+ \\
& \qquad (\mathrm{MFa}^4-5\,\mathrm{MFa}^2\,\mathrm{MSa}^2)\,\mathrm{Log}\Big[\frac{\mathrm{MFa}^2}{\mathrm{MSa}^2}\Big]\Big)\Big)\Big)\Big)\,\mathrm{L1}[\mathrm{minus},\mathrm{lL},\mathrm{Fa},\mathrm{phi}][\mathrm{i}]\Bigg)+ \\
& 2\,\mathrm{MFa}\,\mathrm{L1}[\mathrm{minus},\mathrm{lL},\mathrm{eR},\mathrm{phi}][\mathrm{mif3},\mathrm{j}]\times\mathrm{L1}[\mathrm{minus},\mathrm{lL},\mathrm{Fa},\mathrm{phi}][\mathrm{i}]\times \\
& \mathrm{L1}[\mathrm{minus},\mathrm{Fa},\mathrm{lL},\mathrm{minus},\mathrm{phi}][\mathrm{mif3}]\Bigg)
\end{aligned}
$$

Towards the next IR/UV dictionaries

Preliminary!

- We are working on the one-loop, dimension-6 IR/UV dictionary [with G. Guedes and P. Olgoso].



```
αeB[i,j]->{Sa.Sa.Sa L1[Sa, Sa, Sa] × T[1][Sa, Sa, Sa] +
    eR.F̅a̅.Sa L1[eR, minus, Fa, Sa] × T[1][eR, minus, Fa, Sa] +
    F̅a̅.PL.Fa.Sa L1[minus, Fa, Fa, Sa, SEVEN] × T[1][minus, Fa, Fa, Sa] +
    F̅a̅.PR.Fa.Sa L1[minus, Fa, Fa, Sa, SIX] × T[1][minus, Fa, Fa, Sa] +
    phi.phi†.Sa L1[phi, minus, phi, Sa] × T[1][phi, minus, phi, Sa],
   {T[1][Sa, Sa, Sa] → {{{{1}}}, {{{1}}}}, T[1][phi, minus, phi, Sa] →
     {{{{1}}}, {{{1}, {0}}, {{0}, {1}}}}, T[1][minus, Fa, Fa, Sa] → {{{{1}}}, {{{1}}}},
    T[1][minus, Fa, Fa, Sa] → {{{{1}}}, {{{1}}}}, T[1][eR, minus, Fa, Sa] → {{{{1}}}, {{{1}}}}}}
```

- We will also provide a function to automatically generate Matchmakereft models for specific choices of field quantum numbers to perform the complete one-loop matching.

On-shell matching

Preliminary!

- Off-shell matching is very efficient:
  - Small(ish) number of diagrams (1lPI).
  - Hard region contribution directly local, many cross-checks.
- But requires the construction and reduction of a Green basis.
- On-shell matching can be done in terms of a Physical basis but:
  - There are many diagrams contributing (light bridges have to be included).
  - There is a delicate cancellation of non-local contributions between UV and EFT that is non-trivial to follow analytically.
- Our solution [with M. Chala, J. López-Miras and F. Vilches]:
  - We rely on QGRAF (very efficient even for a large number of diagrams).
  - We do kinematics numerically (trivial cancellation of non-local terms).
  - We stick to tree level.

On-shell matching

Preliminary!

- Tree level on-shell matching of the Green basis to the physical basis provides a simple reduction (which has to be done only once, for the EFT at the end of the chain of EFTs across thresholds), including higher order terms.
- Simplest example: a real scalar to dimension 8 (Z2 symmetric)

$$\mathcal{L}_{IR} = -\frac{1}{2}s(\partial^2 + m^2)s - \lambda s^4 + \alpha_{61}s^6 + \alpha_{81}s^8 + \alpha_{82}s^2(\partial_\mu\partial_\nu s)^2$$

$$\mathcal{L}_{UV} = -\frac{1}{2}s(\partial^2 + m^2)s - \lambda s^4 + \alpha_{61}s^6 + \beta_{61}(\partial^2 s)^2 + \beta_{62}s^3\partial^2 s$$

$$+\alpha_{81}s^8 + \alpha_{82}s^2(\partial_\mu\partial_\nu s)^2 + \beta_{81}s\partial^2\partial^2\partial^2 s + \beta_{82}s^3\partial^2\partial^2 s$$

$$+\beta_{83}s^2(\partial^2 s)^2 + \beta_{84}s^5\partial^2 s$$

On-shell matching

Preliminary!

- Simplest example: a real scalar to dimension 8 (Z2 symmetric)
  - Corrections to the 2-point function have to be carefully included in the UV theory

$$m^2_{\text{phys}} = m^2 - 2\beta_{61}m^4 + 2(\beta_{81} + 4\beta_{61}^2)m^6 + \dots$$

$$\sqrt{Z} = 1 - 2\beta_{12}m^2 + (3\beta_{81} + 10\beta_{61}^2)m^4 + \dots$$

  - Connected, amputated amplitudes have to be computed with full propagators, $\sqrt{Z}$ factors and $p_i^2 = m^2_{\text{phys}}$

$$\alpha_{61} \to \alpha_{61} + 16\lambda^2\beta_{61} - 4\lambda\beta_{62} + m^2\Big[-\frac{304}{5}\lambda^4\beta_{81} + \frac{65}{5}\lambda\beta_{82} + 8\lambda\beta_{83} - \beta_{84} \\ -12\alpha_{61}\beta_{61} - \frac{1728}{5}\lambda^2\beta_{61}^2 - \frac{22}{5}\beta_{62}^2 + \frac{512}{5}\lambda\beta_{61}\beta_{62}\Big]$$

$$\alpha_{81} \to \alpha_{81} - \frac{576}{2}\lambda^3\beta_{81} + 6\alpha_{61}\beta_{62} + \dots$$

Automatic basis generation

Preliminary!

- Producing a Green basis is non-trivial.

  [Buchmuller, Wyller '86]
  [Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884]
  [Gherardi, Marzocca, Venturini 2003.12525]

  SMEFT at dim 6

- Tools can help us do that in an automated (and error-free) way.
- Why not do the calculation once and for all? [with R. Fonseca, P. Olgoso].
  - Write down a generic EFT up to dimension 6 [with Sym2Int].
  - Compute its RGEs [using Matchmakereft].
  - The result is valid for arbitrary EFTs (only the group theory remains to be done).

- The next step is to compute the finite matching [with R. Fonseca, G. Guedes and P. Olgoso].

RGEs of general EFTs

Preliminary!

- Build the most general EFT using Sym2Int.

$$\begin{aligned}\mathscr{L}_{d\leq 4} = &-\frac{1}{4}(a_{KF})_{AB}F^{A}_{\mu\nu}F^{B\,\mu\nu}+\frac{1}{2}(a_{K\phi})_{ab}D_{\mu}\phi_{a}D^{\mu}\phi_{b}+(a_{K\psi})_{ij}\bar{\psi}_{i}\mathrm{i}\not{D}\psi_{j}-\frac{1}{2}\Big[(m_{f})_{ij}\psi_{i}^{T}C\psi_{j}+\text{h.c.}\Big]\\ &-\frac{1}{2}(m^{2}_{\phi})_{ab}\phi_{a}\phi_{b}-\frac{1}{2}\Big[Y_{ija}\psi_{i}^{T}C\psi_{j}+\text{h.c.}\Big]\phi_{a}-\frac{\kappa_{abc}}{3!}\phi_{a}\phi_{b}\phi_{c}-\frac{\lambda_{abcd}}{4!}\phi_{a}\phi_{b}\phi_{c}\phi_{d},\end{aligned}$$

$$\begin{aligned}\mathscr{L}^{\text{phys}}_{5} &= \left[\frac{1}{2}\big(a^{(5)}_{\psi F}\big)_{Aij}\psi_{i}^{T}C\sigma^{\mu\nu}\psi_{j}F^{A}_{\mu\nu}+\frac{1}{4}\big(a^{(5)}_{\psi\phi^{2}}\big)_{ijab}\psi_{i}^{T}C\psi_{j}\phi_{a}\phi_{b}+\text{h.c.}\right]\\ &\quad+\frac{1}{2}\big(a^{(5)}_{\phi F}\big)_{ABa}F^{A\,\mu\nu}F^{B}_{\mu\nu}\phi_{a}+\frac{1}{2}\big(a^{(5)}_{\phi\widetilde{F}}\big)_{ABa}F^{A\,\mu\nu}\widetilde{F}^{B}_{\mu\nu}\phi_{a}+\frac{1}{5!}\big(a^{(5)}_{\phi}\big)_{abcde}\phi_{a}\phi_{b}\phi_{c}\phi_{d}\phi_{e},\\ \mathscr{L}^{\text{red}}_{5} &= \frac{1}{2}\big(r^{(5)}_{\phi\Box}\big)_{abc}(D_{\mu}D^{\mu}\phi_{a})\phi_{b}\phi_{c}+\left[\frac{1}{2}\big(r^{(5)}_{\psi}\big)_{ij}(D_{\mu}\psi_{i})^{T}CD^{\mu}\psi_{j}+\big(r^{(5)}_{\psi\phi}\big)_{ija}\bar{\psi}_{i}\mathrm{i}\not{D}\psi_{j}\phi_{a}+\text{h.c.}\right],\end{aligned}$$

- Compute its beta functions using MME.

$$\begin{aligned}\big(\dot{a}^{(5)}_{\phi\widetilde{F}}\big)_{ABa} = &-2g^{2}\theta^{C}_{ab}\theta^{C}_{bc}\big(a^{(5)}_{\phi\widetilde{F}}\big)_{ABc}-2g^{2}\left\{\left[\frac{11}{6}f^{CDB}f^{CDE}-\frac{1}{12}\theta^{B}_{bc}\theta^{E}_{cb}-\frac{1}{3}t^{B}_{ij}t^{E}_{ji}\right]\big(a^{(5)}_{\phi\widetilde{F}}\big)_{AEa}+(A\leftrightarrow B)\right\}\\ &+2\mathrm{i}g\Big[\big(a^{(5)}_{\psi F}\big)_{Aij}t^{B}_{jk}\bar{Y}^{a}_{ki}-[\big(a^{(5)}_{\psi F}\big)_{Aij}]^{*}t^{B}_{kj}Y^{a}_{ki}+(A\leftrightarrow B)\Big]+\frac{1}{2}\big(a^{(5)}_{\phi\widetilde{F}}\big)_{ABc}\mathrm{Tr}[Y^{c}\bar{Y}^{a}+Y^{a}\bar{Y}^{c}],\end{aligned}$$

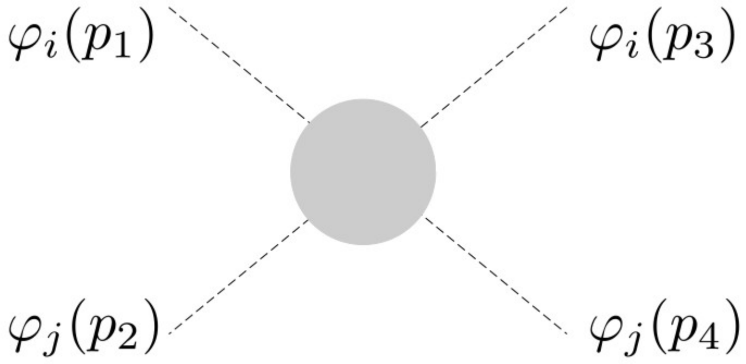
- Only a straight-forward group theory calculation remains for any specific model.

Positivity bounds

- Positivity bounds: restrictions on WC based on locality, unitarity and crossing symmetry

Taken from Chala @ SMEFT TOOLS 2022

Deriving positivity



$$s = (p_1 + p_2)^2$$

$$t = (p_1 + p_3)^2$$

$$u = (p_2 + p_3)^2$$

$$s + t + u = 0$$

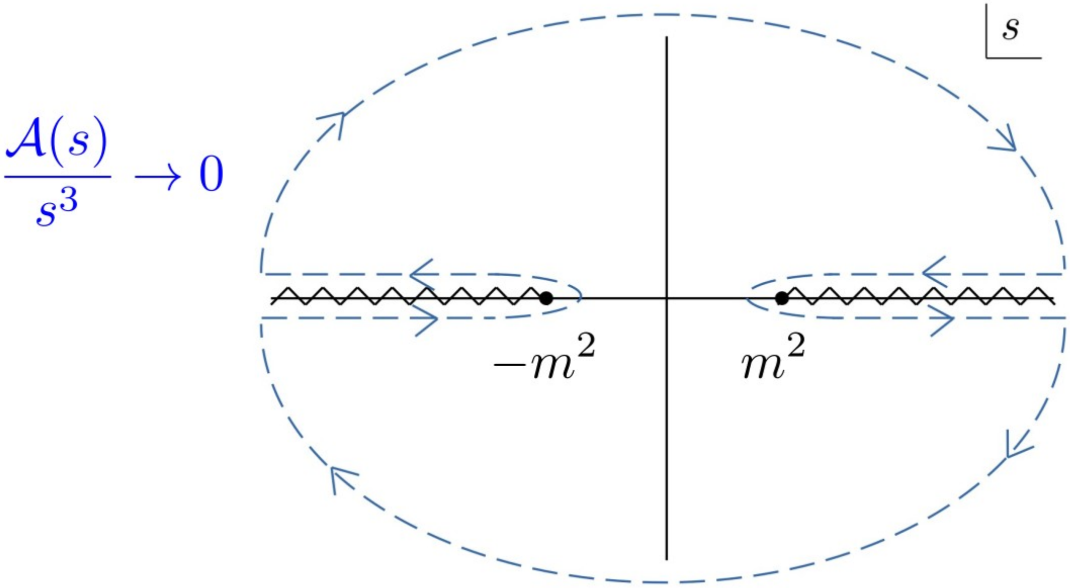
$$\mathcal{A}(s) = \mathcal{A}(-s)$$

$$\mathcal{A}(s) = a_0 + {\color{red}a_1} s + a_2 s^2 + \cdots$$

Positivity bounds

- Positivity bounds: restrictions on WC based on locality, unitarity and crossing symmetry

Taken from Chala @ SMEFT TOOLS 2022



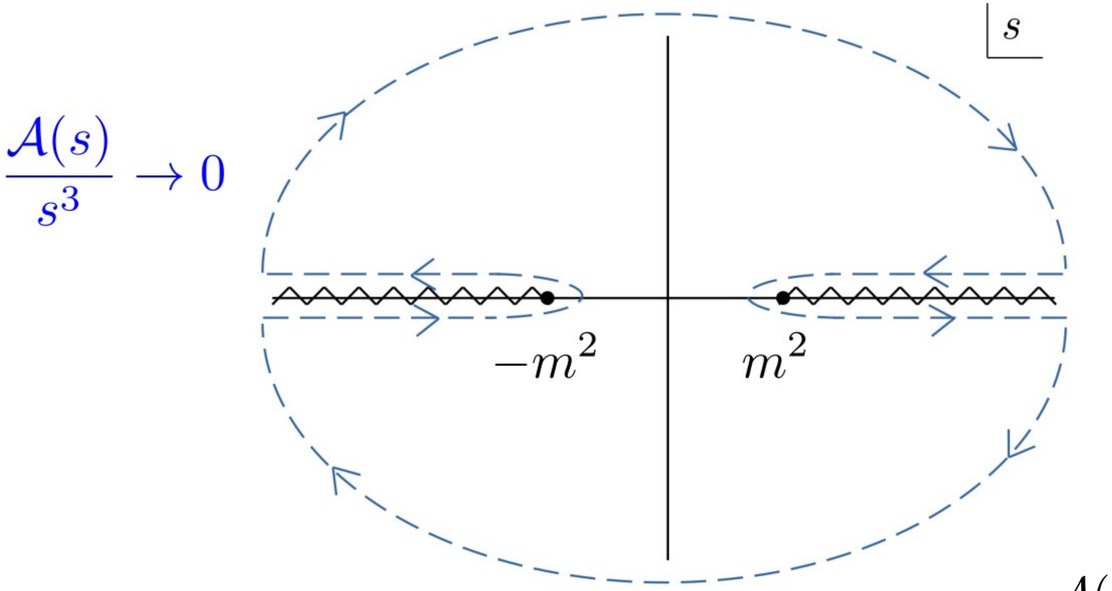
$$\int \frac{\mathcal{A}(s)}{s^3} = 2\int_{m^2}^{\infty} \frac{1}{s^3} \lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s-i\epsilon)]$$

$$= 2\int_{m^2}^{\infty} \frac{1}{s^3} \lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s+i\epsilon)^*] = 2i\int_{m^2}^{\infty} \frac{\sigma(s)}{s^2}$$

Positivity bounds

- Positivity bounds: restrictions on WC based on locality, unitarity and crossing symmetry

Taken from Chala @ SMEFT TOOLS 2022



$$\mathcal{A}(s) = a_0 + a_1 s + a_2 s^2 + \cdots$$

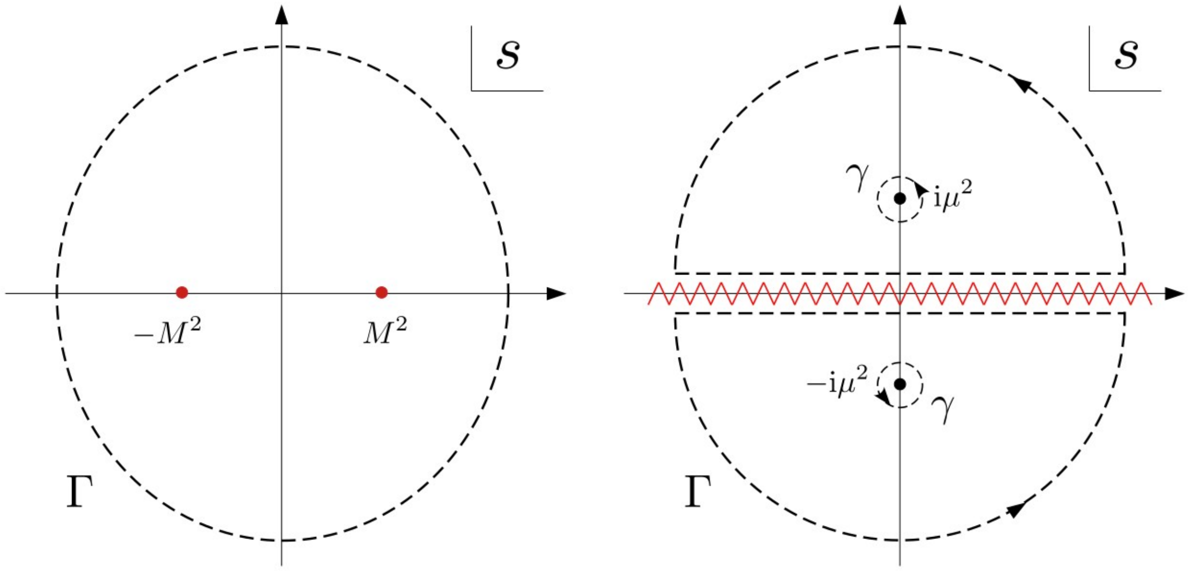
$$2i\int_{m^2}^{\infty}\frac{\sigma(s)}{s^2} = 2\pi i\,\mathrm{Res}[\frac{\mathcal{A}(s)}{s^3}, s=0] = 2\pi i a_2$$

$$\Rightarrow a_2 \geq 0$$

Adams et al ‘06

Positivity bounds

- Positivity bounds: restrictions on WC based on locality, unitarity and crossing symmetry
- These bounds can impose restrictions on theoretically allowed parameter space (global fits)
- Can also impose non-trivial conditions (signs, vanishing conditions) on anomalous dimensions (tricky when tree-level contributions vanish).



Chala 2301.09995

Figure 1: *Structure of the singularities of a two-to-two amplitude in the forward limit in the plane of the complex Mandelstam variable s at tree level (left) and at one loop (right).*

Positivity bounds

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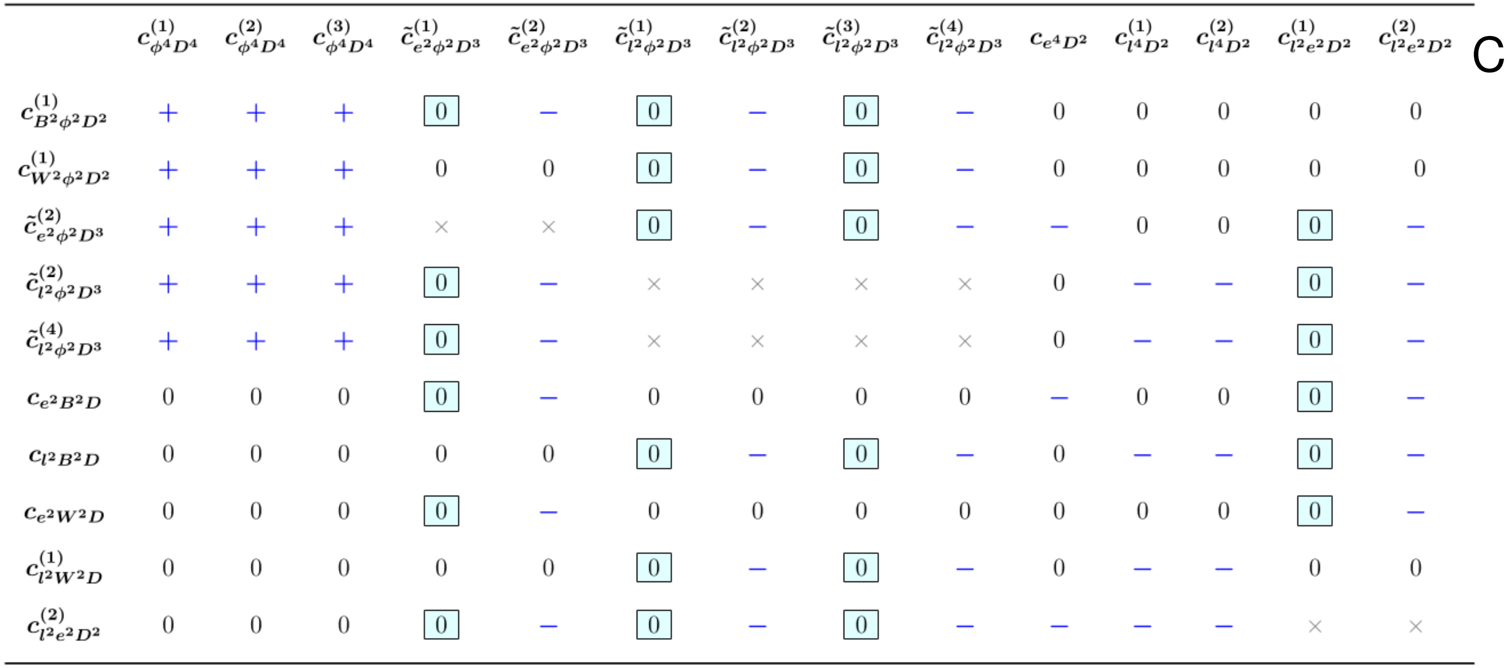

	$c^{(1)}_{\phi^4D^4}$	$c^{(2)}_{\phi^4D^4}$	$c^{(3)}_{\phi^4D^4}$	$\tilde{c}^{(1)}_{e^2\phi^2D^3}$	$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$\tilde{c}^{(1)}_{l^2\phi^2D^3}$	$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$\tilde{c}^{(3)}_{l^2\phi^2D^3}$	$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$c_{e^4D^2}$	$c^{(1)}_{l^4D^2}$	$c^{(2)}_{l^4D^2}$	$c^{(1)}_{l^2e^2D^2}$	$c^{(2)}_{l^2e^2D^2}$
$c^{(1)}_{B^2\phi^2D^2}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$c^{(1)}_{W^2\phi^2D^2}$	$+$	$+$	$+$	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	0	0	0	0
$\tilde{c}^{(2)}_{e^2\phi^2D^3}$	$+$	$+$	$+$	$\times$	$\times$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	0	0	$\boxed{0}$	$-$
$\tilde{c}^{(2)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$\tilde{c}^{(4)}_{l^2\phi^2D^3}$	$+$	$+$	$+$	$\boxed{0}$	$-$	$\times$	$\times$	$\times$	$\times$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2B^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	$-$	0	0	$\boxed{0}$	$-$
$c_{l^2B^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	$\boxed{0}$	$-$
$c_{e^2W^2D}$	0	0	0	$\boxed{0}$	$-$	0	0	0	0	0	0	0	$\boxed{0}$	$-$
$c^{(1)}_{l^2W^2D}$	0	0	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	0	$-$	$-$	0	0
$c^{(2)}_{l^2e^2D^2}$	0	0	0	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$\boxed{0}$	$-$	$-$	$-$	$-$	$\times$	$\times$

Chala 2301.09995

Thank you for your attention!

If you are interested in going deeper into state-of-the-art research in EFTs feel free to talk to me, there’s a lot to be done!