Junta de Andalucía

Functional Matching at Two Loop Order

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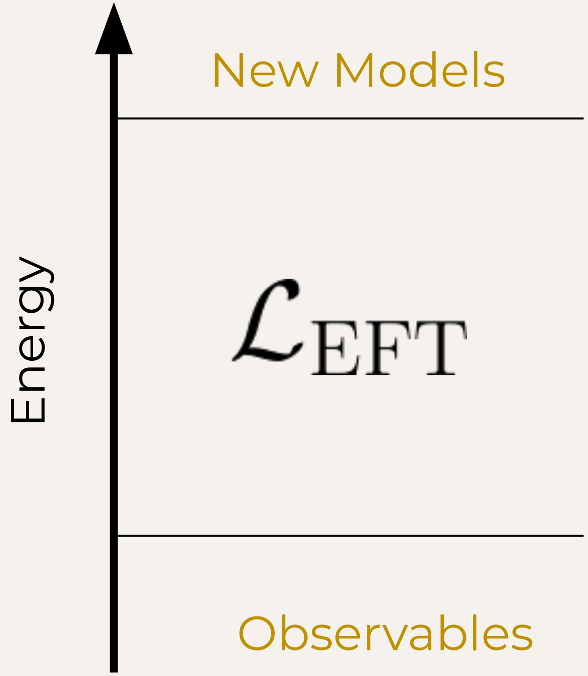
EFT-2024

Separation of Scales



BSM requires multiscale physics

- **Matching** of UV theories to low energy observables
- Process of matching automatized up to one loop

[Carmona et al-2112.10787]

[Fuentes-Martín et al-2212.04510]

- Running of the theory via RG evolution

Amplitude Matching

$$\mathcal{L}_{\mathrm{UV}}(z_h, z_l) \xrightarrow{q_i << \Lambda} \{\mathcal{A}_{\mathrm{UV}}(q_i)\}$$

Matching:
Determining Wilson Coefficients

$$\mathcal{L}_{\mathrm{EFT}}(z_l) \longrightarrow \{\mathcal{A}_{\mathrm{EFT}}(q_i)\}$$

Feynman Diagrams

- Well-established
- Ansatz: Redundancies, redefinitions…
- Explicit break of Gauge Symmetry in intermediate steps
- Normally uses off-shell computations

Functional Matching

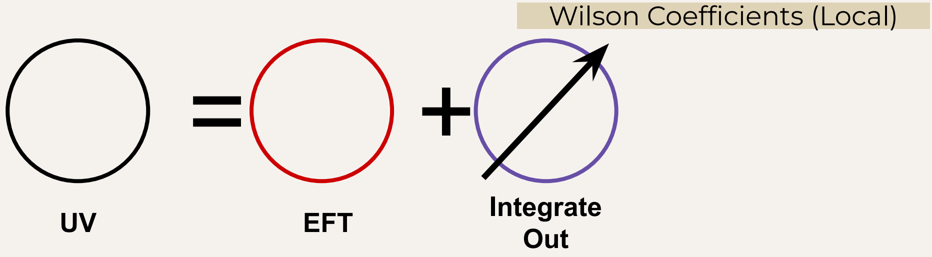
Quantum Effective Action: We are going to **"Integrate Out"** the heavy fields

$$e^{i\Gamma_{\mathrm{UV}}[\hat{\phi}]} = \int [D\phi] \exp\left(\int \mathrm{d}^d x \mathcal{L}_{\mathrm{UV}}(\bar{\phi} + \hat{\phi}) \right)$$

Matching Condition

[Fuentes-Martín, Palavrić , Eller Thomsen-2311.13630]

$$S_{\mathrm{EFT}} = \Gamma_{\mathrm{UV}}[\bar{\phi}_H[\hat{\phi}_L], \hat{\phi}_L]\Big|_{\mathrm{hard}}$$

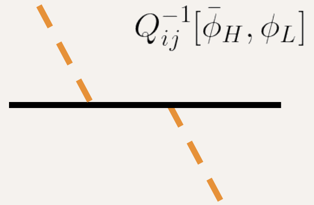


Background Field Method

$$\phi = \bar{\phi} + \hat{\phi}$$

$\bar{\phi}$ **Classical Configuration:** Tree Level

$\hat{\phi}$ **Quantum Fluctuation:** Loops



Expanding Lagrangian

$$\mathcal{L}_{\mathrm{UV}}(\bar{\phi} + \hat{\phi}) = \mathcal{L}_{\mathrm{UV}}(\bar{\phi}) + \frac{1}{2}\phi_i Q_{ij} \phi_j + \ldots$$

- **One-loop:**

$$\exp(i\Gamma^{(1)}_{\mathrm{UV}}[\hat{\phi}]) = \int [D\phi] \exp \frac{1}{2} \left(\int \mathrm{d}^d x \phi_i Q_{ij} \phi_j \right)$$

↓ Gaussian Integration

$$\Gamma^{(1)}_{\mathrm{UV}} = \frac{i}{2} \mathrm{STr} \ln Q$$

Evaluation of Traces

[Fuentes-Martín, Palavrić , Eller Thomsen, AM]

Differential operators under a Gauge Symmetry

$$\delta_c{}^b(x,y) = \delta(x-y)U_c{}^b(x,y)$$

$$Q^{ab}_{ij}(x,y) = \mathcal{Q}^{ac}_{ij}(x,P_x)\delta_c{}^b(x,y)$$

Locality and gauge invariance of the action:

Functional Traces are dressed loop integrals

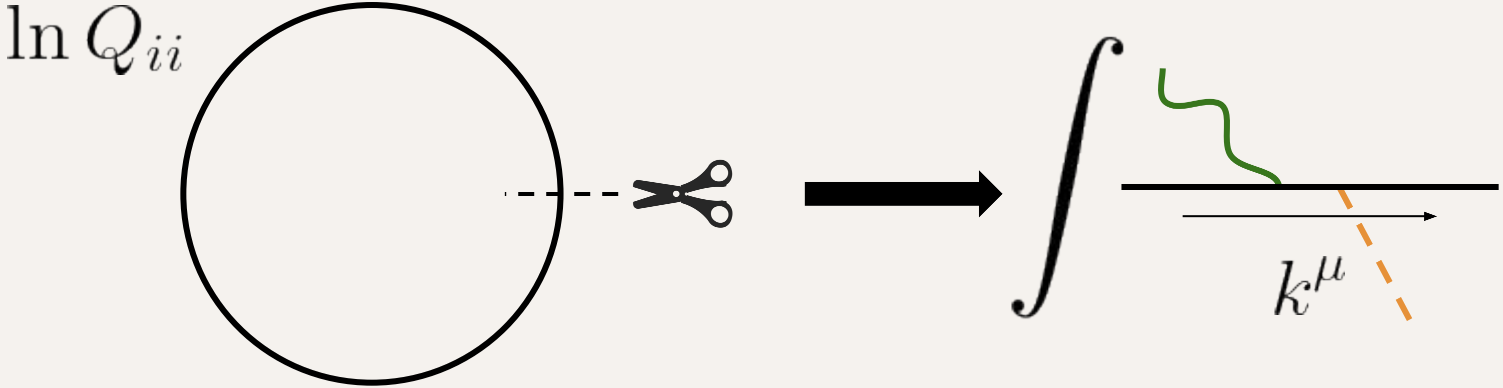
$$\begin{aligned}\mathrm{Tr}\ln Q\big|_{\mathrm{hard}} &= \int_{x,y} \delta_b{}^a(x,y))\ln \mathcal{Q}^{bc}_{ii}(x,P_x)\delta_c{}^a(x,y) \\ &= \int_{x,k} \ln \mathcal{Q}^{ab}_{ii}(x,P_x+k)U_c{}^a(x,y)\Big|_{x=y}\end{aligned}$$

OPE around k~Λ

=

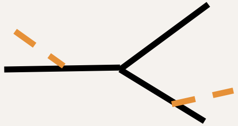
Explicit Gauge Invariance

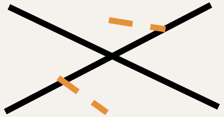
Diagrammatically,



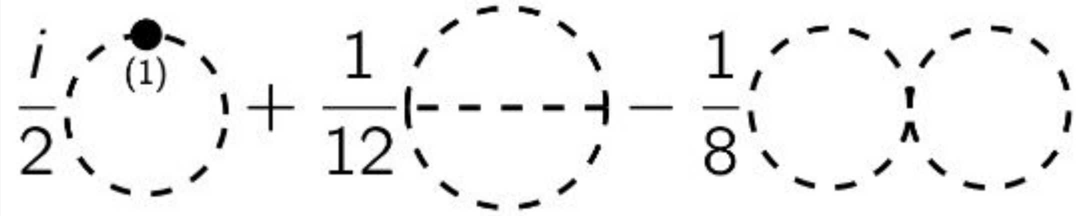
Operators traced in different points of spacetime remain local by a **momentum shift** operation

- **Two Loops:** More Topologies involved

C_{ijk} 

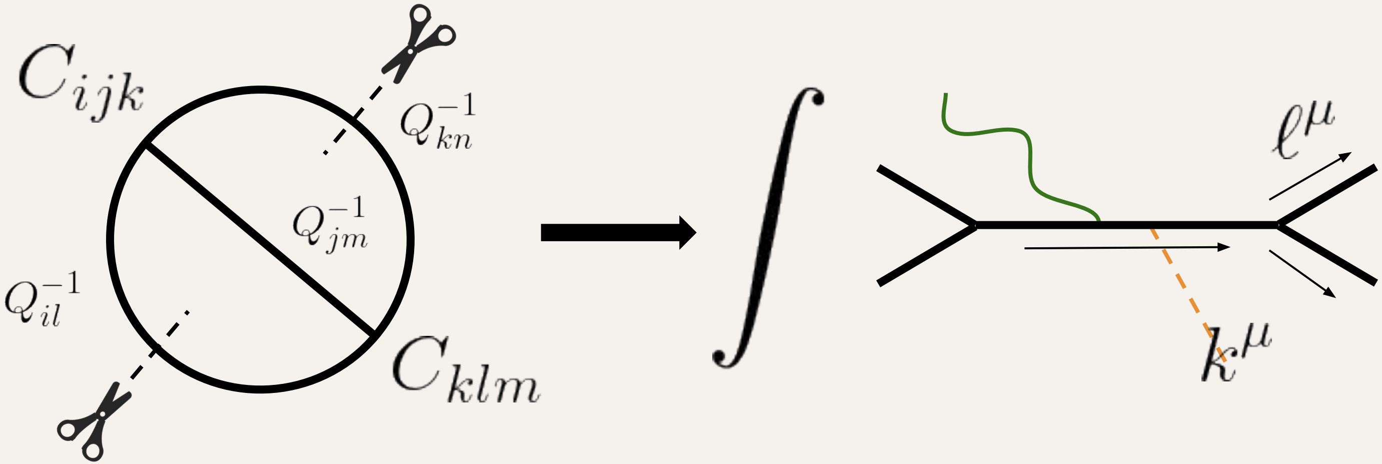
D_{ijkl} 

$$\Gamma^{(2)}_{UV}[\bar{\phi}] = \frac{i}{2} Q^{-1}_{ij} B_{ij} - \frac{1}{8} Q^{-1}_{ij} D_{ijkl} Q^{-1}_{kl} + \frac{1}{12} C_{ijk} Q^{-1}_{il} Q^{-1}_{jm} Q^{-1}_{kn} C_{lmn}$$



Two Loops Traces

$$G_{\rm ss}|_{\rm hard} = \sum_{n,m,n'm'} (-1)^{n+m} \int_x \int_{k,\ell} C^{(n,m)}_{abc}\, \mathcal{Q}^{-1}_{aa'}(y, P_y - k - \ell)\, C^{(n',m')}_{a'b'c'}(y)$$
$$\times [(P_x+k)^m \mathcal{Q}^{-1}_{be}(x, P_x+k)(P_x+k)^{m'} U_{b'}{}^{e}(x,y)]$$
$$\times [(P_x+\ell)^n \mathcal{Q}^{-1}_{cf}(x, P_x+\ell)(P_x+\ell)^{n'} U_{c'}{}^{f}(x,y)]|_{x=y}$$



Outlook

- EFT is given by the formalism (not by an ansatz)
- Basis reduction is still needed
- Better for Automation: Matchete
- Future study of applications of interest to the community

Functional methods in EFTs

Two-loop counterterms for the bosonic SMEFT

Lukas Born

University of Bern

July 15, 2024

b UNIVERSITÄT BERN

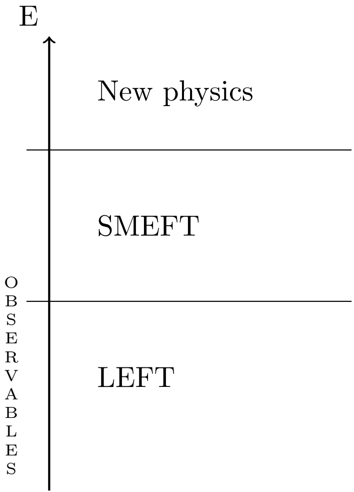
AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

with J. Fuentes-Martín, S. Kvedaraitė and A. E. Thomsen

Swiss National Science Foundation

Introduction

- Top-down approach to EFTs



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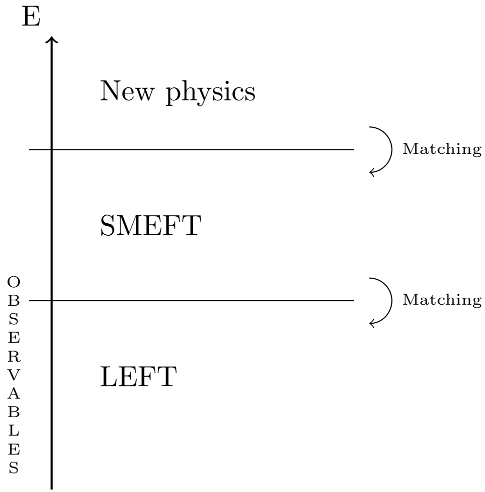

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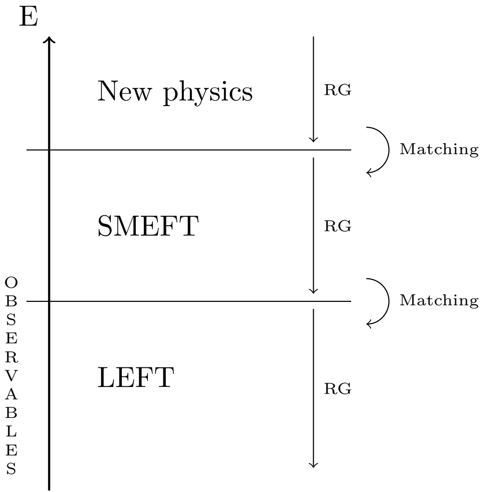
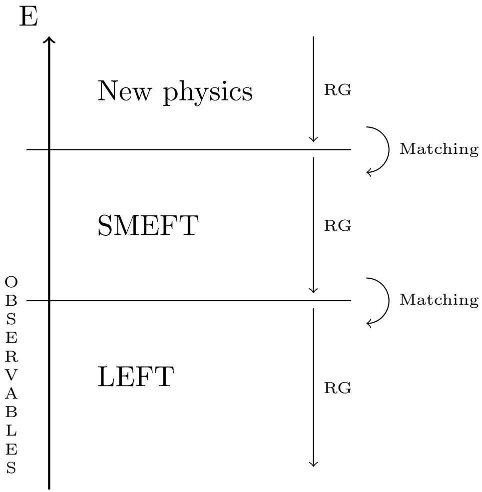


Introduction

- Top-down approach to EFTs
- Goal: Fully automated two-loop RG calculations
  1. Theoretical precision required
  2. Scheme independence of one-loop matching



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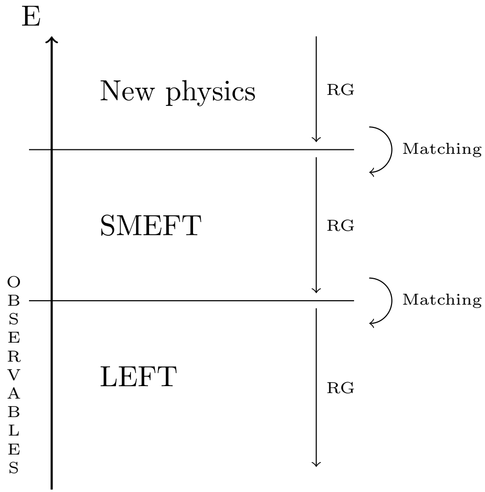
- Build on `Matchete`

Fuentes-Martín et al [2212.04510]



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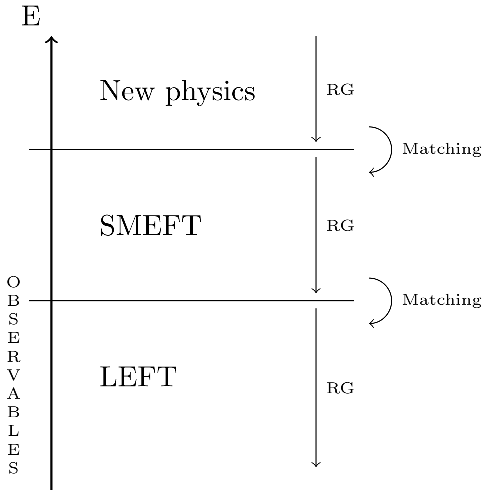
MATCHETE

Fuentes-Martín et al [2212.04510]

- Counterterms give β-functions



Functional methods

Effective action

$$\Gamma[\eta] = \; S^{(0)} \; + \; \hbar S^{(1)} + \tfrac{i\hbar}{2}\mathrm{STr}\left\{\log Q\right\} \; + \; \hbar^2 S^{(2)} + \tfrac{i\hbar^2}{2} Q^{-1}_{ij} B^{(1)}_{ji}$$

$$-\tfrac{\hbar^2}{8} D^{(0)}_{ijkl} Q^{-1}_{ij} Q^{-1}_{kl} + \tfrac{\hbar^2}{12} C^{(0)}_{ijk} Q^{-1}_{il} Q^{-1}_{jm} Q^{-1}_{kn} C^{(0)}_{lmn} \; + \mathcal{O}(\hbar^3)$$

Fuentes-Martín, Palavrić, Thomsen [2311.13630]

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- Tree-level

Fuentes-Martín, Palavrić, Thomsen [2311.13630]

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- One-loop

Fuentes-Martín, Palavrić, Thomsen [2311.13630]

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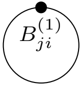
- Tree-level
- One-loop
- Two-loop

Fuentes-Martín, Palavrić, Thomsen [2311.13630]

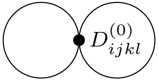
Two-loop counterterms

$$\Gamma^{(2)}[\eta] = \hbar^2 S^{(2)} + \frac{i\hbar^2}{2} Q^{-1}_{ij} B^{(1)}_{ji} - \frac{\hbar^2}{8} D^{(0)}_{ijkl} Q^{-1}_{ij} Q^{-1}_{kl} + \frac{\hbar^2}{12} C^{(0)}_{ijk} Q^{-1}_{il} Q^{-1}_{jm} Q^{-1}_{kn} C^{(0)}_{lmn}$$

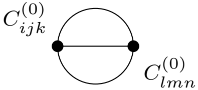
$$= \hbar^2 S^{(2)} + \frac{i\hbar^2}{2}$$



$$- \frac{\hbar^2}{8}$$



$$+ \frac{\hbar^2}{12}$$



Two-loop counterterms: Simple example

B C D Q

$$\mathcal{L} = \partial_\mu \phi^\dagger \, \partial^\mu \phi - m^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2 - \frac{\zeta}{36} (\phi^\dagger \phi)^3$$

$B^{(1)}_{ji}$: One-loop counterterm insertions

Two-loop counterterms: Simple example

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$B^{(1)}_{ji}$: One-loop counterterm insertions

$$C^{(0)}_{\phi^\dagger \phi \phi^\dagger}(x) = \frac{\delta^3 S^{(0)}[\phi]}{\delta\phi^\dagger \, \delta\phi \, \delta\phi^\dagger} = -\lambda\phi - \frac{\zeta}{2}\phi\phi^\dagger\phi \quad , \quad C^{(0)}_{\phi\phi\phi} = -\frac{\zeta}{6}\phi^\dagger\phi^\dagger\phi^\dagger \quad , \quad \text{etc.}$$

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$$D^{(0)}_{\phi^\dagger \phi \, \phi^\dagger \phi}(x) = \frac{\delta^4 S^{(0)}[\phi]}{\delta \phi^\dagger \, \delta \phi \, \delta \phi^\dagger \delta \phi} = -\lambda - \zeta \phi^\dagger \phi \quad , \quad D^{(0)}_{\phi \phi \phi \phi} = 0 \quad , \quad \text{etc.}$$

Two-loop counterterms: Simple example

B C D Q

$$\mathcal{L} = \partial_\mu \phi^\dagger \, \partial^\mu \phi - m^2 \phi^\dagger \phi - \frac{\lambda}{4}(\phi^\dagger \phi)^2 - \frac{\zeta}{36}(\phi^\dagger \phi)^3$$

$$Q^{-1}(x,k) = \frac{1}{(k + i\partial)^2 - X(x)} = \frac{1}{k^2} \sum_{n=0}^{\infty} \left(\frac{X(x) + \partial^2 - 2ik_\mu \partial^\mu}{k^2} \right)^n$$

Two-loop counterterms: Simple example

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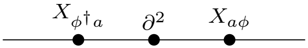
$Q^{-1}_{\phi^\dagger \phi}(x,k) =$ 

$= \frac{X_{\phi^\dagger a}\partial^2 X_{a\phi}}{k^6}$

Two-loop counterterms: Simple example

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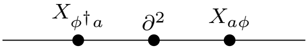

$= \dfrac{X_{\phi^\dagger a} \partial^2 X_{a\phi}}{k^6}$

$$X_{\phi^\dagger \phi}(x) = m^2 + \lambda \phi^\dagger \phi + \frac{\zeta}{4}(\phi^\dagger \phi)^2$$

Two-loop counterterms: Simple example

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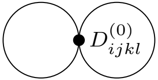
$$\mathcal{L} = \partial_\mu \phi^\dagger \, \partial^\mu \phi - m^2 \phi^\dagger \phi - \frac{\lambda}{4}(\phi^\dagger \phi)^2 - \frac{\zeta}{36}(\phi^\dagger \phi)^3$$

$G_{\mathrm{f8}} =$



$$= \int_x \int_{kl} D^{(0)}_{ijkl}(x) \, Q^{-1}_{ij}(x,k) \, Q^{-1}_{kl}(x,l)$$

Two-loop counterterms: Simple example

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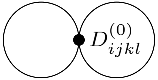
$G_{f8} =$ 

$$= \int_x \int_{kl} D^{(0)}_{ijkl}(x) \, Q^{-1}_{ij}(x,k) \, Q^{-1}_{kl}(x,l)$$

$$G_{f8} \supset \int_x \int_{kl} D^{(0)}_{\phi^\dagger \phi \phi^\dagger \phi}(x) \left(\frac{1}{(k + i\partial)^2 - X(x)} \right)_{\phi^\dagger \phi} \left(\frac{1}{(l + i\partial)^2 - X(x)} \right)_{\phi^\dagger \phi}$$

Two-loop counterterms: Integrals

$$\int_x \int_{kl} D^{(0)}_{\phi^\dagger\phi\phi^\dagger\phi}(x) \left(\frac{1}{(k+i\partial)^2 - X(x)} \right)_{\phi^\dagger\phi} \left(\frac{1}{(l+i\partial)^2 - X(x)} \right)_{\phi^\dagger\phi}$$

- Subdivergences
- Spurious IR divergences

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- Subdivergences
- Spurious IR divergences

Solution: R^*-method

Two-loop counterterms: Integrals

R^*-method:

$$\Delta_{\mathrm{UV}}(G) = -K\bar{R}^*\mathcal{T}^{(n)}G$$

$$\bar{R}^* = \sum \Delta_{\mathrm{IR}}(S') * \Delta_{\mathrm{UV}}(S) * G/S \setminus S'$$

Herzog, Ruijl [1703.03776]

Two-loop counterterms: Integrals

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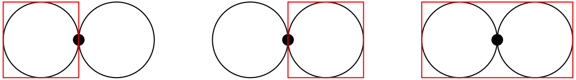

Herzog, Ruijl [1703.03776]

Two-loop counterterms: Integrals

$$\Delta_{\text{UV}}(G_{\text{f8}}) \supset -K\bar{R}^*\mathcal{T}^{(4)} \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(x) \left(\frac{1}{(k+i\partial)^2 - X(x)}\right)_{\phi^\dagger\phi} \left(\frac{1}{(l+i\partial)^2 - X(x)}\right)_{\phi^\dagger\phi}$$

$$\supset -K\bar{R}^* \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(x) \frac{X_{\phi^\dagger\phi}(x)}{k^4} \frac{X_{\phi^\dagger\phi}(x)}{l^4}$$

Two-loop counterterms: Integrals

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$$\supset -K\bar{R}^{*} \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(x) \frac{X_{\phi^\dagger\phi}(x)}{k^4} \frac{X_{\phi^\dagger\phi}(x)}{l^4} \propto \int_{kl} \frac{1}{(k^2 - a)^2} \frac{1}{(l^2 - a)^2}$$

- IR rearrangement removes spurious IR divergences

Two-loop counterterms: Integrals

$$\Delta_{\mathrm{UV}}(G_{\mathrm{f8}}) \supset -K\bar{R}^*\mathcal{T}^{(4)} \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(x) \left(\frac{1}{(k+i\partial)^2 - X(x)}\right)_{\phi^\dagger\phi} \left(\frac{1}{(l+i\partial)^2 - X(x)}\right)_{\phi^\dagger\phi}$$

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- IR rearrangement removes spurious IR divergences
- UV subdivergences: $\int_k \frac{1}{(k^2-a)^2}$ and $\int_l \frac{1}{(l^2-a)^2}$

Two-loop counterterms: Integrals

$$\Delta_{\mathrm{UV}}(G_{\mathrm{f8}}) \supset -K\bar{R}^{*}\mathcal{T}^{(4)} \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(x) \left(\frac{1}{(k+i\partial)^2 - X(x)} \right)_{\phi^\dagger\phi} \left(\frac{1}{(l+i\partial)^2 - X(x)} \right)_{\phi^\dagger\phi}$$

$$\supset -K\bar{R}^{*} \int_x \int_{kl} D_{\phi^\dagger\phi\phi^\dagger\phi}(x) \, \frac{X_{\phi^\dagger\phi}(x)}{k^4} \, \frac{X_{\phi^\dagger\phi}(x)}{l^4} \propto \int_{kl} \frac{1}{(k^2-a)^2} \frac{1}{(l^2-a)^2}$$

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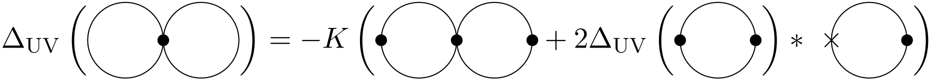


Two-loop counterterms: Bosonic SMEFT

15 effective operators in the bosonic SMEFT:

	X^2H^2
C_{HB}	$H^\dagger H\, B^{\mu\nu}\, B_{\mu\nu}$
C_{HW}	$H^\dagger H\, W^{I\,\mu\nu}\, W^I_{\mu\nu}$
C_{HG}	$H^\dagger H\, G^{A\,\mu\nu}\, G^A_{\mu\nu}$
$C_{H\widetilde{B}}$	$\frac{1}{2} H^\dagger H\, B^{\mu\nu}\, \widetilde{B}_{\mu\nu}$
$C_{H\widetilde{W}}$	$\frac{1}{2} H^\dagger H\, W^{I\,\mu\nu}\, \widetilde{W}^I_{\mu\nu}$
$C_{H\widetilde{G}}$	$\frac{1}{2} H^\dagger H\, G^{A\,\mu\nu}\, \widetilde{G}^A_{\mu\nu}$
C_{HWB}	$2H^\dagger H\, \tau^I\, B^{\mu\nu}\, W^I_{\mu\nu}$
$C_{H\widetilde{W}B}$	$H^\dagger H\, \tau^I\, B^{\mu\nu}\, \widetilde{W}^I_{\mu\nu}$

	X^3
C_W	$f^{IJK}\, W^{I\,\nu}_\mu\, W^{J\,\rho}_\nu\, W^{K\,\mu}_\rho$
C_G	$f^{ABC}\, G^{A\,\nu}_\mu\, G^{B\,\rho}_\nu\, G^{C\,\mu}_\rho$
$C_{\widetilde{W}}$	$-\frac{1}{2} f^{IJK}\, \widetilde{W}^{I\,\nu}_\mu\, W^{J\,\rho}_\nu\, W^{K\,\mu}_\rho$
$C_{\widetilde{G}}$	$-\frac{1}{2} f^{ABC}\, \widetilde{G}^{A\,\nu}_\mu\, G^{B\,\rho}_\nu\, G^{C\,\mu}_\rho$
	H^4D^2 and H^6
C_H	$\left(H^\dagger H\right)^3$
$C_{H\Box}$	$H^\dagger H \Box \left(H^\dagger H\right)$
C_{HD}	$\left(H^\dagger D_\mu H\right)^\dagger \left(H^\dagger D^\mu H\right)$

Two-loop counterterms: Bosonic SMEFT

```
Lag // NiceForm
```

$$-\frac{1}{4}B^{\mu\nu 2}-\frac{1}{4}G^{\mu\nu A2}-\frac{1}{4}W^{\mu\nu I2}+D_\mu\bar{H}_i D_\mu H^i+\mu^2\bar{H}_i H^i-\frac{1}{2}\lambda\bar{H}_i\bar{H}_j H^i H^j+cH\,\bar{H}_i\bar{H}_j\bar{H}_k H^i H^j H^k$$

```
CountertermLagrangian[Lag, EFTOrder → 6] // NiceForm
```

$$\left(\frac{1}{24}\hbar\frac{1}{\epsilon}gY^2+\frac{1}{16}\hbar^2\frac{1}{\epsilon}\left(3\,gL^2 gY^2+gY^4\right)\right)B^{\mu\nu 2}+\left(-\frac{7}{2}\hbar\frac{1}{\epsilon}gs^2-\frac{183}{16}\hbar^2\frac{1}{\epsilon}gs^4\right)G^{\mu\nu A2}+\left(-\frac{55}{24}\hbar\frac{1}{\epsilon}gL^2+\frac{1}{16}\hbar^2\frac{1}{\epsilon}\left(-77\,gL^4+gL^2 gY^2\right)\right)W^{\mu\nu I2}-$$
$$\left(\frac{1}{2}\hbar\frac{1}{\epsilon}\left(-3\,gL^2-gY^2\right)+\hbar^2\left(\frac{3}{16}\frac{1}{\epsilon^2}\left(29\,gL^4-4\,gL^2 gY^2-gY^4\right)+\frac{1}{192}\frac{1}{\epsilon}\left(-1179\,gL^4+54\,gL^2 gY^2+31\,gY^4+144\,\lambda^2\right)\right)\right)D_\mu\bar{H}_i D_\mu H^i+$$
$$\left(-\frac{1}{4}\hbar\frac{1}{\epsilon}\mu^2\left(3\,gL^2+gY^2-12\,\lambda\right)+\hbar^2\left(\frac{3}{32}\frac{1}{\epsilon}\mu^2\left(5\,gL^4+7\,gY^4+32\,\lambda\,gY^2-48\,\lambda^2+2\,gL^2\left(gY^2+48\,\lambda\right)\right)+\frac{1}{32}\frac{1}{\epsilon^2}\mu^2\left(105\,gL^4+19\,gY^4+6\,gL^2\left(7\,gY^2-48\,\lambda\right)-96\,\lambda\,gY^2+144\left(3\,\lambda^2+8\,cH\,\mu^2\right)\right)\right)\right)\bar{H}_i H^i+$$
$$\left(\frac{1}{16}\hbar\frac{1}{\epsilon}\left(-9\,gL^4-3\,gY^4-6\,gL^2\left(gY^2-2\,\lambda\right)+4\,\lambda\,gY^2-48\left(\lambda^2+4\,cH\,\mu^2\right)\right)+\hbar^2\left(\frac{1}{64}\frac{1}{\epsilon^2}\left(225\,gL^6-5\,gY^6+gL^4\left(57\,gY^2-336\,\lambda\right)-76\,\lambda\,gY^4+240\,gY^2\left(\lambda^2+4\,cH\,\mu^2\right)-1152\left(\lambda^3+14\,\lambda\,cH\,\mu^2\right)+gL^2\left(-17\,gY^4-168\,\lambda\,gY^2+720\left(\lambda^2+4\,cH\,\mu^2\right)\right)\right)+\right.\right.$$
$$\left.\left.\frac{1}{192}\frac{1}{\epsilon}\left(-1449\,gL^6+59\,gY^6+3\,gL^4\left(37\,gY^2-102\,\lambda\right)-198\,\lambda\,gY^4-144\,gY^2\left(3\,\lambda^2+16\,cH\,\mu^2\right)+288\left(7\,\lambda^3+48\,\lambda\,cH\,\mu^2\right)+gL^2\left(239\,gY^4-180\,\lambda\,gY^2-432\left(3\,\lambda^2+16\,cH\,\mu^2\right)\right)\right)\right)\right)\bar{H}_i\bar{H}_j H^i H^j+$$
$$\left(-\frac{3}{4}\hbar\frac{1}{\epsilon}cH\left(3\,gL^2+gY^2-36\,\lambda\right)+\hbar^2\left(\frac{9}{32}\frac{1}{\epsilon^2}cH\left(77\,gL^4+19\,gY^4+6\,gL^2\left(7\,gY^2-72\,\lambda\right)-144\,\lambda\,gY^2+1584\,\lambda^2\right)+\frac{9}{32}\frac{1}{\epsilon}cH\left(29\,gL^4+15\,gY^4+64\,\lambda\,gY^2-816\,\lambda^2+6\,gL^2\left(3\,gY^2+32\,\lambda\right)\right)\right)\right)\bar{H}_i\bar{H}_j\bar{H}_k H^i H^j H^k-$$
$$6\,\hbar^2\frac{1}{\epsilon}\lambda\,cH\left(\bar{H}_i D^2\bar{H}_j H^i H^j+\bar{H}_i\bar{H}_j D^2 H^i H^j\right)$$

*So far without ghosts

Conclusion & outlook

- Functional formalism works well for computers
- Also suitable for matching calculations
- Next: Fermions

Universidad de Granada

High Energy Theory

On-Shell matching in effective field theories

Fuensanta Vilches Bravo (she/her)

with M. Chala, J. López-Miras and J. Santiago [24xx.xxxxx]

EFT|2024

Junta de Andalucía

Why do we need effective field theories?

EFT's are perturbative (Taylor) expansions of a full theory

In QFT's :

Operators of mass dimension $d > 4$ → $\mathcal{O}_i^{(d)}$

EFT Lagrangian : $$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{d>4} \sum_i \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$

Green's basis and redundant operators

Valid operators
- Local operators
- Preserve the symmetries of the Lagrangian

Finite number of operators → Integration by parts → **Green's basis**

$$\partial_\mu(\phi\partial^\mu\phi) = \partial_\mu\phi\partial^\mu\phi + \phi\partial^2\phi$$

X^3		X^2H^2		H^2D^4	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{HG}$	$G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	H^4D^2	
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
X^2D^2		$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$	$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$	$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	H^6	
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$	$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	$\mathcal{O}_H$	$(H^\dagger H)^3$
		H^2XD^2			
		$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$		
		$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$		

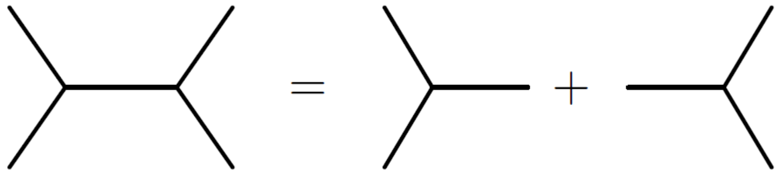
V. Gherardi, D. Marzocca y E. Venturini (2021) [2003.12525v5]

Matching: Off-Shell vs On-shell

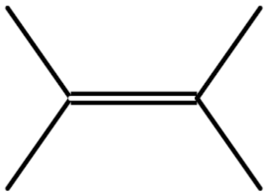
Construction of EFTs with the *Top- down* approach

Off-Shell matching

- Small number of diagrams (1 lPI)



- Local contribution of heavy bridges



$$\sim \frac{1}{p^2 - M^2} \approx -\frac{1}{M^2}\left(1 + \frac{p^2}{M^2} + \frac{p^4}{M^4} + \dots\right)$$

$$p^2 \ll M^2$$

- Gives effective Lagrangians efectivos containing **redundant operators**

Reduction to the physical basis

Identification of redundant operators

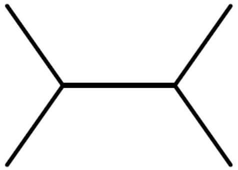
- Field redefinitions $\phi \to f(\phi)$
- EOMs (only valid up to linear order)

Non-trivial process

Hard to program it in a systematic way

On-Shell matching

- Huge number of diagrams



- There is delicate cancellation of non-local contributions between between UV and EFT

$$\left.\frac{1}{p^2 - m^2}\right|_{\text{UV}} - \left.\frac{1}{p^2 - m^2}\right|_{\text{EFT}} = Polynomial(p^2)$$

Reduction to the physical basis

Identification of redundant operators

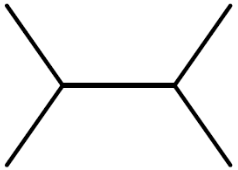
- Field redefinitions
- EOMs (only valid up to linear order)

Non-trivial process

Hard to program it in a systematic way

On-Shell matching

- Huge number of diagrams



- There is delicate cancellation of non-local contributions between between UV and EFT

Substitution of randomly generated physical momenta

M. Accettulli [2304.01589]

$$\left.\frac{1}{p^2 - m^2}\right|_{\text{UV}} - \left.\frac{1}{p^2 - m^2}\right|_{\text{EFT}} = Polynomial(p^2)$$

On-Shell matching approach

- Find the Green's basis up to dimension d → $\mathcal{L}_{Green}$

- Find the physical basis → $\mathcal{L}_{phys}$

  R. Fonseca [1907.12584]
  J.C. Criado [1901.03501]

- Compute n-points amplitudes with n $\leq d$ **on-shell** → **By the substitution of randomly generated physical momenta**

- Solve the system $\mathcal{M}_{i,Green} = \mathcal{M}_{i,phys}$

Some results in the SMEFT

HIGG'S SECTOR

$$\mathcal{L}_{Green} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{Green} \qquad \mathcal{L}_{phys} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}_{phys} + \mathcal{L}^{(8)}_{phys}$$

$$\mathcal{L}^{(4)} = \frac{1}{2}(D_\mu H)^\dagger (D^\mu H) - m_0^2 H^\dagger H - \lambda (H^\dagger H)^2$$

$$\mathcal{L}^{(6)}_{phys}$$

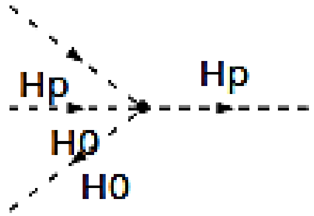
$$\mathcal{L}^{(6)}_{Green} = c_H (H^\dagger H)^3 + c_{H\Box} (H^\dagger H)\Box (H^\dagger H) + c_{HD} (H^\dagger D^\mu H)^\dagger (H^\dagger D_\mu H) + + r_{DH} (D^2 H)^\dagger (D^2 H)$$

$$\mathcal{L}^{(8)}_{phys} = c_{H^8} (H^\dagger H)^4 + c^{(1)}_{H^6 D^2} (H^\dagger H)^2 (D_\mu H^\dagger D^\mu H) + c^{(2)}_{H^6 D^2} (H^\dagger H)(H^\dagger \sigma^I H)(D_\mu H^\dagger \sigma^I D^\mu H) + c^{(1)}_{H^4 D^4} (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H) + c^{(2)}_{H^4 D^4} (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H) + c^{(3)}_{H^4 D^4} (D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$$

Some results in the SMEFT

1. Compute the n-point amplitudes

HpH0H0 → Hp



$$\mathcal{M}_{i,red} = -\text{lmbd} - 2\,\text{aHDD}\left(2\,\text{mH}^2 - \text{Pair[k[1], k[4]]} + \text{Pair[k[2], k[3]]}\right) - \text{aHD}\,(\text{Pair[k[1], k[3]]} - \text{Pair[k[2], k[4]]})$$

$$\mathcal{M}_{i,phys} = -\text{lmbd} - \text{aHD}\,(\text{Pair[k[1], k[3]]} - \text{Pair[k[2], k[4]]}) - 2\,\big(\text{aH43 Pair[k[1], k[4]]} \times \text{Pair[k[2], k[3]]} + \text{aHDD}\left(2\,\text{mH}^2 - \text{Pair[k[1], k[4]]} + \text{Pair[k[2], k[3]]}\right) + \text{aH41 Pair[k[1], k[3]]} \times \text{Pair[k[2], k[4]]} + \text{aH42 Pair[k[1], k[2]]} \times \text{Pair[k[3], k[4]]}\big)$$

Some results in the SMEFT

2 Replace random generated momenta

$$\mathcal{M}_{i,red} = -\text{lmbd} - \frac{(5\,110\,271\,456\,608\,372\,418\,158\,791\,262\,659\,188\,226\,358\,\text{aHD} + 5\,667\,343\,519\,855\,001\,567\,834\,436\,843\,671\,307\,395\,297\,\text{aHDD})\,\text{mH}^2}{289\,386\,244\,266\,659\,475\,135\,585\,493\,547\,103\,196\,896}$$

$$\mathcal{M}_{i,phys} = -\text{lmbd} - \frac{(5\,110\,271\,456\,608\,372\,418\,158\,791\,262\,659\,188\,226\,358\,\text{aHD} + 5\,667\,343\,519\,855\,001\,567\,834\,436\,843\,671\,307\,395\,297\,\text{aHDD})\,\text{mH}^2}{289\,386\,244\,266\,659\,475\,135\,585\,493\,547\,103\,196\,896} + \left(\left(104\,459\,497\,440\,905\,025\,370\,557\,004\,516\,941\,086\,685\,779\,240\,167\,863\,979\,569\,761\,544\,341\,444\,023\,383\,776\,656\,\text{aH41} + 252\,424\,935\,310\,185\,260\,931\,291\,954\,350\,757\,582\,753\,759\,781\,162\,189\,652\,241\,672\,962\,750\,585\,818\,884\,048\,169\,\text{aH42} + 20\,338\,282\,896\,536\,048\,399\,260\,842\,332\,945\,517\,645\,527\,660\,495\,092\,523\,562\,736\,657\,451\,352\,292\,159\,090\,369\,\text{aH43}\right)\text{mH}^4\right) / 669\,955\,186\,966\,101\,631\,552\,833\,832\,932\,406\,404\,259\,264\,606\,549\,036\,334\,762\,898\,940\,078\,586\,752\,278\,528\}$$

$$\mathcal{M}_{i,red} = \mathcal{M}_{i,phys}$$

Some results in the SMEFT

3 Solve the system

$$\text{lmbd}\left(-1+4\,\text{m0}^2\,\text{rDH}\left(1-4\,\text{m0}^2\,\text{rDH}\right)\right)+\frac{\text{m0}^2\left(1\,302\,951\,136\,961\,359\,158\,193\,395\,783\,076\,458\,713\;\text{aHD}\left(-1+5\,\text{m0}^2\,\text{rDH}\right)-5\,460\,929\,492\,434\,672\,006\,242\,912\,459\,802\,600\,180\left(-\text{aHDD}+\text{cHDD}+\left(5\,\text{aHDD}-\text{cHDD}\right)\text{m0}^2\,\text{rDH}\right)\right)}{58\,147\,936\,542\,012\,895\,949\,308\,641\,578\,673\,120}+\lambda==$$

$$\left(\left(1\,697\,681\,665\,308\,898\,511\,468\,388\,917\,556\,477\,488\,316\,567\,860\,814\,643\,982\,341\,460\,492\,793\,616\,369\;\text{cH41}+2\,037\,795\,104\,611\,025\,740\,512\,901\,972\,773\,488\,115\,167\,800\,811\,930\,872\,605\,116\,111\,581\,211\,256\,129\;\text{cH42}+8\,104\,046\,023\,605\,072\,941\,359\,410\,114\,418\,394\,024\,240\,778\,682\,484\,440\,344\,738\,753\,972\,022\,468\,900\;\text{cH43}\right)\text{m0}^4\right)\Big/\left(6\,762\,365\,048\,187\,917\,326\,896\,191\,289\,766\,646\,271\,391\,026\,071\,420\,935\,191\,444\,839\,621\,068\,800+\frac{1\,302\,951\,136\,961\,359\,158\,193\,395\,783\,076\,458\,713\;\text{cHD}\,\text{m0}^2\left(-1+\text{m0}^2\,\text{rDH}\right)}{58\,147\,936\,542\,012\,895\,949\,308\,641\,578\,673\,120}\right)$$

Sistema algebraico de ecuaciones lineales!!

$$\lambda\to\text{lmbd}-4\,\text{lmbd}\,\text{m0}^2\,\text{rDH}+16\,\text{lmbd}\,\text{m0}^4\,\text{rDH}^2$$

$$\text{cHD}\to\text{aHD}-4\,\text{aHD}\,\text{m0}^2\,\text{rDH},\quad \text{cHDD}\to\text{aHDD}-4\,\text{aHDD}\,\text{m0}^2\,\text{rDH}$$

$$\text{cH41}\to 0,\quad \text{cH42}\to 0,\quad \text{cH43}\to 0$$

J. Aebischer, M. Fael and J. Fuentes-Martín|2023
[2307.08745v1]

V. Gherardi, D. Marzocca and E. Venturini |2021
[2003.12525v5]

Future work

BOSONIC SECTOR

X^3		X^2H^2		H^2D^4	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{HG}$	$G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	H^4D^2	
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
X^2D^2		$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$	$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$	$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	H^6	
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$	$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	$\mathcal{O}_H$	$(H^\dagger H)^3$
		H^2XD^2			
		$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$		
		$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$		

$$m_0^2 \to m_0^2$$
$$\lambda \to \lambda - 2m_0^2 r'_{HD}$$
$$c^{(1)}_{H^4D^4} \to 2r^2_{BDH}$$
$$c^{(2)}_{H^4D^4} \to -2r^2_{BDH}$$
$$c^{(3)}_{H^4D^4} \to 0$$
$$c_{H\Box} \to c_{H\Box} + \frac{1}{2}g' r_{BDH} + \frac{1}{2}r'_{HD}$$
$$c_{HD} \to c_{HD} + 2g' r_{BDH}$$

$$D_\mu = \partial_\mu - ig' B_\mu$$
$$g' \to g'$$
$$c_{HB} \to c_{HB}$$

Future work

BOSONIC SECTOR

X^3		X^2H^2		H^2D^4	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{HG}$	$G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	H^4D^2	
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
X^2D^2		$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$	$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$	$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	H^6	
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$	$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	$\mathcal{O}_H$	$(H^\dagger H)^3$
		H^2XD^2			
		$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$		
		$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$		

$$m_0^2 \to m_0^2$$
$$\lambda \to \lambda - 2m_0^2 r'_{HD}$$
$$c^{(1)}_{H^4D^4} \to 2r^2_{BDH}$$
$$c^{(2)}_{H^4D^4} \to -2r^2_{BDH}$$
$$c^{(3)}_{H^4D^4} \to 0$$
$$c_{H\Box} \to c_{H\Box} + \frac{1}{2}g' r_{BDH} + \frac{1}{2}r'_{HD}$$
$$c_{HD} \to c_{HD} + 2g' r_{BDH}$$

$$g' \to g'$$
$$c_{HB} \to c_{HB}$$

$$D_\mu = \partial_\mu - ig' B_\mu$$

Fermions ✓

Evanescent operators

Notice that …

Reduction of the Green's basis

- Field redefinitions and EOMs
  - Non-systematic
  - Highly non-trivial
- On-Shell matching approach
  - Systematic
  - Algebraic system of linear equations

The reduction of **ANY** theory to **ANY** physical basis will be completely **AUTOMATIC**

Universidad de Granada

High Energy Theory

THANKS FOR YOUR ATTENTION !

Generation of random momenta

$$SL(2,\mathbb{C}) \cong SU(2)_L \times SU(2)_R \quad \left\{ \begin{array}{c} \lambda \in SU(2)_L \\ \tilde{\lambda} \in SU(2)_R \end{array} \right\} \quad \begin{array}{l} \lambda^\alpha = \varepsilon^{\alpha\beta}\lambda_\beta \\ \tilde{\lambda}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}^{\dot{\beta}} \end{array}$$

Massless momenta : $P_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ $\Rightarrow$ $P = p_\mu \sigma^\mu = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$

Massive momenta : $P^\mu := q^\mu + \frac{m^2}{2q \cdot k} k^\mu$

$$\begin{array}{rcl} q^2, k^2 & = & 0 \\ q_{\alpha\dot{\alpha}} & = & \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \\ k_{\alpha\dot{\alpha}} & = & \mu_\alpha \tilde{\mu}_{\dot{\alpha}} \end{array}$$

Evanescent operators

$\mathcal{R}=\alpha\,\mathcal{O}$ $\xrightarrow{d=4-2\epsilon}$ $\mathcal{R}=\alpha\mathcal{O}+\mathcal{E}$

$\mathcal{O}(\epsilon)$

Additional finite local contributions in loop amplitudes

$$IR^{(0)}+IR^{(1)}=UV^{(0)}+UV^{(1)}$$

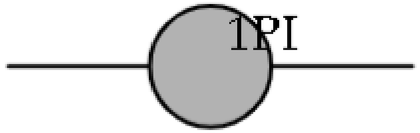
$$IR^{(0)}+IR^{(1)}_{soft}=UV^{(0)}+UV^{(1)}_{hard}+UV^{(1)}_{soft}$$

We take the hard region

$$\int\mathcal{O}=\frac{1}{\epsilon}(a+b_{\mathcal{O}}\epsilon)$$

$$\int\mathcal{R}=\frac{1}{\epsilon}(a+b_{\mathcal{R}}\epsilon)$$

$$\int\mathcal{R}-\mathcal{O}=\frac{1}{\epsilon}(b_{\mathcal{R}}\epsilon-b_{\mathcal{O}}\epsilon)=b$$



$$= \frac{i}{p^2 - m^2 - \Pi(p^2)} = \frac{iZ}{p^2 - m^2_{phys}} + \ldots \,,$$

$$p^2 - m^2 - \Pi(p^2)\Big|_{p^2-m^2_{phys}} = 0$$

$$\Pi(p^2) = \Pi(m^2_{phys}) + \Pi'(m^2_{phys})(p^2 - m^2_{phys}) + \ldots$$

$$\frac{i}{p^2 - m^2 - \Pi(p^2)} = \frac{i}{p^2 - m^2 - \Big(\Pi(m^2_{phys}) + \Pi'(m^2_{phys})(p^2 - m^2_{phys}) + \ldots\Big)}$$

$$= \frac{i}{\Big(p^2 - m^2_{phys}\Big)\Big(1 - \Pi'(m^2_{phys}) + \ldots\Big)} \backsim \frac{i\Big(1 - \Pi'(m^2_{phys})\Big)^{-1}}{\Big(p^2 - m^2_{phys}\Big)}$$

The gradient flow formalism: nEDM and operator renormalization

Òscar Lara Crosas

UZH and PSI

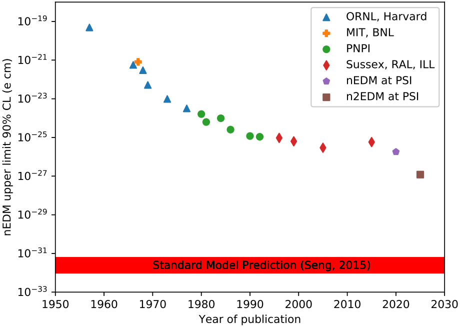
July 15th, 2024

Peter Stoffer's group

CP Violation Beyond the Standard Model

- CP violation present in the Standard Model (CKM phase and a possible theta term) is not enough to explain matter antimatter asymmetry.
- Electric Dipole Moments violate T $\implies$ CP violation (CPT theorem).

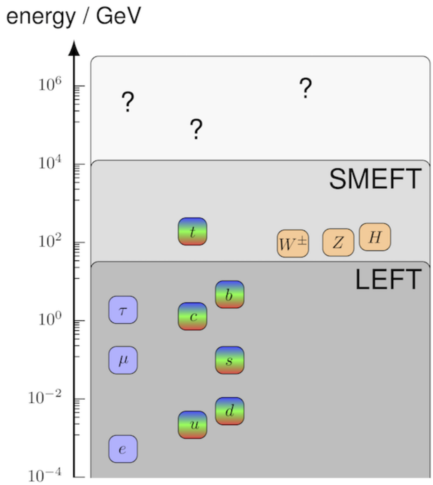
Neutron Electric Dipole Moment



Two possibilities:

- Detecting a signal in the unexplored region $\implies$ CP violating New Physics.
- Not detecting a signal, lowering the bound. Still very interesting! Constrains the shape of New Physics Models.

Effective Field Theories



- Effective Field Theories contain only the relevant degrees of freedom at a certain energy scale.
- Model independent way to encode the effects of all heavy particles (both BSM and SM).

$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}_{\text{QCD+QED}} + \sum_{d\geq 5} \sum_{i=1}^{n_d} L_i^{(d)} \mathcal{O}_i^{(d)}$$

Effective Field Theories

$$d_n \sim \sum_i L_i \langle N|\mathcal{O}_i|N\gamma\rangle \tag{1}$$

$$\begin{aligned} d_n \;=\;& -(1.5 \pm 0.7)\cdot 10^{-3}\,\bar{\theta}\,e\,\mathrm{fm} - (0.20 \pm 0.01) d_u \\ & + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.0016) d_s - (0.55 \pm 0.28) e\tilde{d}_u \\ & - (1.1 \pm 0.55) e\tilde{d}_d + (50 \pm 40)\,\mathrm{MeV}\,e\,\tilde{d}_G\,. \end{aligned} \tag{2}$$

where d_q denotes the EDM of a quark q, $\tilde{d}_q$ denotes its chromo EDM, and $\tilde{d}_G$ denotes the gluon-chromo EDM. Lessons learned:

- We need to measure multiple EDMs.
- Uncertainties in matrix elements are too big, we should aim for at most 25% uncertainty.
- Lattice is key!

Lattice Field Theory and scheme translations

$$d_N \sim \sum_i L_i^{\overline{\text{MS}}} \langle N|\mathcal{O}_i^{\overline{\text{MS}}}|N\gamma\rangle \implies D = 4 - 2\epsilon \tag{3}$$

However, lattice is tied to integer dimensions! We require a scheme translation:

$$\langle N|\mathcal{O}_i^{\overline{\text{MS}}}|N\gamma\rangle = \sum_j C_{ij} \langle N|\mathcal{O}_j^{\text{GF}}|N\gamma\rangle \tag{4}$$

Why the gradient flow?

- No need for renormalization, the gradient flow renders all Green's Functions finite. (!)
- It is widely used for scale setting.
- It smears statistical noise in the lattice.

The gradient flow

We extend D-dimensional Euclidean QCD by introducing an extra artificial dimension called flow-time t. The flowed field satisfies the flow equation

$$\partial_t B_\mu(x,t) = D_\nu B_{\nu\mu}(x,t) \tag{5}$$

together with the boundary condition that implies agreement with QCD at $t = 0$

$$B_\mu\,(x, t=0) = G_\mu(x) \tag{6}$$

The flow equation is turned into an integral equation and solved perturbatively, which we express in terms of Feynman diagrams.

Short flow-time expansion

- Goal: express renormalized flowed operators in terms of renormalized MS operators through a Short Flow-Time Expansion (SFTE):
$$\mathcal{O}_i^R(x,t) = \sum_j C_{ij}(t,\mu)\mathcal{O}_j^{MS}(x,\mu) + \mathcal{O}(t) \tag{7}$$
with the hard scale being $\Lambda = t^{-1/2}$.
- To extract the matching coefficients C_{ij} we consider insertions of the flowed operators $O_i^R(t)$ in suitable Green's functions.

LEFT Renormalization

The Short-Flow Time expansion reads:

$$\mathcal{O}_i^R(t) = \sum_j C_{ij}(t)\mathcal{O}_j^{MS} \tag{8}$$

But finite = finite is boring, let's switch to bare quantities

$$Z_{\text{external}}\mathcal{O}_i^{(0)}(t) = \sum_{j,k} C_{ij}(t) Z_{jk}\mathcal{O}_k^{(0)} \tag{9}$$

What we compute for the matching are Green's functions of the UV side (gradient flow) $< \mathcal{O}_i^R(t) >$, allowing us to extract the LEFT renormalization matrix Z!

Generalized Loop Integrals

The integrals that we have to compute are

$$
\begin{aligned}
&\int_k e^{-\beta t k^2}\left(k^2\right)^{\alpha} \\
&\int_0^t dt_1 \int_k e^{-\beta t k^2-\gamma t_1 k^2}\left(k^2\right)^{\alpha} \\
&\int_0^t dt_2 \int_0^t dt_1 \int_k e^{-\beta t k^2-\gamma t_1 k^2-\tau t_2 k^2}\left(k^2\right)^{\alpha} \\
&\int_0^t dt_2 \int_0^{t_2} dt_1 \int_k e^{-\beta t k^2-\gamma t_1 k^2-\tau t_2 k^2}\left(k^2\right)^{\alpha}
\end{aligned}
\tag{10}
$$

We can make use of normal IBPs and flowed-IBPs:

$$
\int_k \frac{\partial}{\partial k_\mu} f_\mu = 0\,, \quad \int_0^t dt_1 \partial_{t_1} f\left(t_1, \ldots\right) = f\left(t, \ldots\right) - f\left(0, \ldots\right) \tag{11}
$$

Summary

- Electric Dipole Moments are excellent places to look for CP violating New Physics.
- Effective Field Theories parametrize New Physics in a model independent way.
- We require matrix elements from Lattice Field Theory, and a corresponding translation to Minimal Subtraction.
- At one-loop, the SFTE of all operators contributing to the nEDM is now known: (2111.11449), (2304.00985), (2308.16221)
- Precision is key if we want to disentangle the different sources of CP violation $\implies$ we need to compute higher orders.
- The gradient flow allows us to extract the LEFT renormalization!

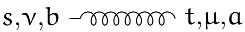
Back up slides

Perturbative solution of the flow equation

Full solution:

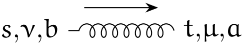
$$B_\mu^a(t,x) = \int_y K_{\mu\nu}(t, x-y) G_\nu^a(y) + \int_y \int_0^t ds\, K_{\mu\nu}(t-s, x-y) R_\nu^a(s,y) \tag{12}$$

Gluon two point function at LO: You just get an extra exponential due to the heat kernel:



$$= g_0^2 \delta^{ab} \frac{1}{p^2} \delta_{\mu\nu} e^{-(s+t)p^2}$$

At higher orders, you can take the R_ν part, what will give additional vertices. We view the heat kernel that brings you to the vertex as a generalized propagator (flow line):



$$= g_0^2 \delta^{ab} \theta(t-s) \delta_{\mu\nu} e^{-(t-s)p^2}$$

which will always connect to a flow vertex.

Perturbative solution of the flow equation

$$\partial_t B_\mu(t) = D_\nu B_{\nu\mu} \implies \partial_t B^a_\mu(t) = \partial_\nu \partial_\nu B^a_\mu + \underbrace{R^a_\mu}_{\text{non-linear}} \tag{13}$$

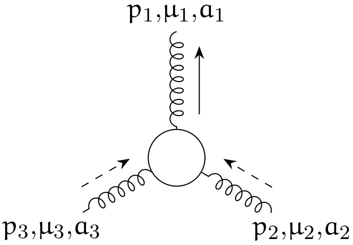
Solution to the linear part:

$$B^a_\mu(t,x) = \int_y K_{\mu\nu}(t, x-y) G^a_\nu(y)\,, \quad \widetilde{K}_{\mu\nu}(t,p) = e^{-tp^2} \tag{14}$$

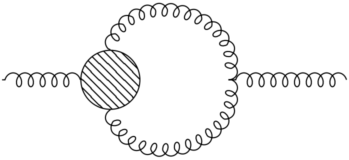
Full solution:

$$B^a_\mu(t,x) = \int_y K_{\mu\nu}(t, x-y) G^a_\nu(y) + \int_y \int_0^t ds\, K_{\mu\nu}(t-s, x-y) R^a_\nu(s,y) \tag{15}$$

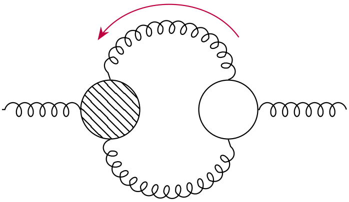
Flow vertices and Feynman diagrams



$$= -if^{a_1 a_2 a_3} \int_0^{+\infty} dt \Big(\delta_{\mu_2 \mu_3} (p_2 - p_3)_{\mu_1} + 2\delta_{\mu_1 \mu_3} p_{3,\mu_1} - 2\delta_{\mu_1 \mu_2} p_{2,\mu_3} \Big)$$

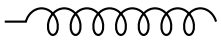


(a) Linear part: ″QCD″

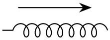


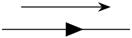
(b) Non-linear part: R_ν

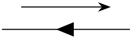
Flow lines

s,ν,b  t,μ,a $= g_0^2 \delta^{ab} \frac{1}{p^2} \delta_{\mu\nu} e^{-(s+t)p^2}$

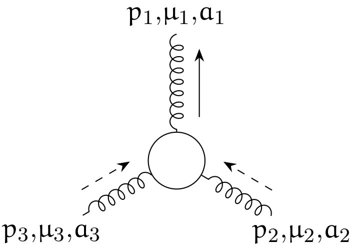
s,β  t,α $= \delta^{\alpha\beta} \frac{-i\not{p} + m}{p^2 + m^2} e^{-(s+t)p^2}$

s,ν,b  t,μ,a $= g_0^2 \delta^{ab} \theta(t-s) \delta_{\mu\nu} e^{-(t-s)p^2}$

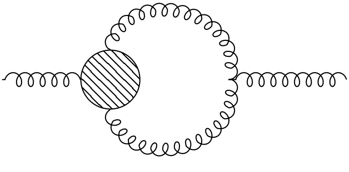
s,β  t,α $= \delta^{\alpha\beta} \theta(t-s) e^{-(t-s)p^2}$

s,β  t,α $= \delta^{\alpha\beta} \theta(t-s) e^{-(t-s)p^2}$

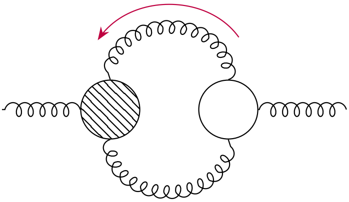
Flow vertices and Feynman diagrams



$$= -if^{a_1 a_2 a_3} \int_0^{+\infty} dt \Big(\delta_{\mu_2 \mu_3} (p_2 - p_3)_{\mu_1} + 2\delta_{\mu_1 \mu_3} p_{3,\mu_1} - 2\delta_{\mu_1 \mu_2} p_{2,\mu_3} \Big)$$



(a) Linear part: ″QCD″



(b) Non-linear part: R_ν

CP-odd Three-gluon operator

Short Flow-Time Expansion of the CP-odd three-gluon operator: (PLB **847** (2023) 138301)

$$\mathcal{O}_{\widetilde{G}}(x,t) = \frac{1}{g^2} Tr[G_{\mu\nu}(x,t) G_{\nu\lambda}(x,t) \widetilde{G}_{\lambda\mu}(x,t)] \tag{16}$$

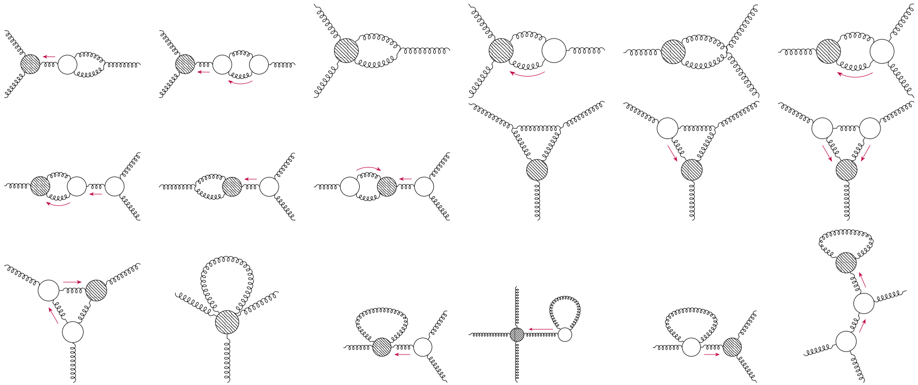
which reads

$$\mathcal{O}^R_{\widetilde{G}}(x,t) = \sum_i C_i(t,\mu) \mathcal{O}_i^{\overline{\text{MS}}}(x,\mu) + \sum_i C_{\mathcal{N}_i}(t,\mu) \mathcal{N}_i^{\overline{\text{MS}}}(x,\mu) + \sum_i C_{\mathcal{E}_i}(t,\mu) \mathcal{E}_i^{\overline{\text{MS}}}(x,\mu) \,. \tag{17}$$

The physical operators are

$$\mathcal{O}_\theta \sim Tr\left[G_{\mu\nu}\widetilde{G}_{\mu\nu}\right], \quad \mathcal{O}_{\widetilde{G}}, \quad \mathcal{O}_{CE} = m\left(\bar{q}\tilde{\sigma}_{\mu\nu}T^a q\right) G^a_{\mu\nu}, \\ \mathcal{O}_{\partial G} \sim \partial_\nu Tr\left[\left(D_\mu G_{\mu\lambda}\right)\widetilde{G}_{\nu\lambda}\right], \quad \mathcal{O}_{\Box\theta} \sim \Box Tr\left[G_{\mu\nu}\widetilde{G}_{\mu\nu}\right] \tag{18}$$

Sample diagrams to be computed



Results

$$
\begin{aligned}
C_\theta &= -\frac{9C_A\alpha_s}{16\pi t} \\
C_{\widetilde{G}} &= \frac{3C_A\alpha_s \log\left(8\pi\mu^2 t\right)}{2\pi} + (1-\delta)\,\frac{C_A\alpha_s}{12\pi} \\
C_{CE} &= \frac{3iC_A\alpha_s \log\left(8\pi\mu^2 t\right)}{32\pi} + \frac{31iC_A\alpha_s}{192\pi} + \delta\frac{iC_A\alpha_s}{96\pi} \\
C_{\partial G} &= -\frac{179C_A\alpha_s}{96\pi} - \delta\frac{C_A\alpha_s}{24\pi} \\
C_{\Box\theta} &= 0
\end{aligned}
\tag{19}
$$

With a perturbative uncertainty of $\sim$40%!

Evanescent operators

$$\mathcal{O}_{\widetilde{G}}(x,t) = \frac{1}{g^2} Tr[G_{\overline{\mu}\nu} G_{\nu\overline{\lambda}} \widetilde{G}_{\overline{\lambda}\overline{\mu}}] = C\overline{\mathcal{O}}^{MS} + C_{\mathcal{E}}\mathcal{E}^{MS}$$

vs.

$$\overline{\mathcal{O}}_{\widetilde{G}}(x,t) = \frac{1}{g^2} Tr[G_{\overline{\mu}\,\overline{\nu}} G_{\overline{\nu}\,\overline{\lambda}} \widetilde{G}_{\overline{\lambda}\overline{\mu}}] = C'\overline{\mathcal{O}}^{MS} + C'_{\mathcal{E}}\mathcal{E}^{MS}$$

In the D-dimensional scheme, we get a tree level contribution to

$$\mathcal{E}_{\widetilde{G}}(x,t) = \frac{1}{g^2} Tr[G_{\overline{\mu}\hat{\nu}} G_{\hat{\nu}\overline{\lambda}} \widetilde{G}_{\overline{\lambda}\overline{\mu}}] \tag{20}$$

Renormalized by imposing

$$\langle \mathcal{E}_{\widetilde{G}} \rangle_{phys} = 0 \implies \text{counterterm from } \mathcal{O}^{MS}!! \implies C' = C \tag{21}$$

THE FLAVOUR STRUCTURE OF THE LEFT

...and how to simplify running from W scale to b scale

Ben Smith

(based on WIP w/ S. Renner, D. Sutherland)

15th July 2024, **EFT 2024, Zurich**

University of Glasgow

Effective field theory valid below the electroweak scale.

$$\mathcal{L}_{LEFT} = \mathcal{L}_{\mathrm{QCD+QED}} + \sum_{k,D>4} c_k^{(D)} \mathcal{O}_k^{(D)},$$

where $c_k^{(D)}$ have an implicit suppression of $\frac{1}{\Lambda_{EW}^{D-4}}$.

Theory contains $n_u = 2$, $n_d = 3$, $n_e = 3$ and $n_{\nu_L} = 3$.

RUNNING AT ONE LOOP IN THE LEFT

Running first calculated at one-loop in (Jenkins, Manohar, and Stoffer 2018)

Focus on vectorial operators

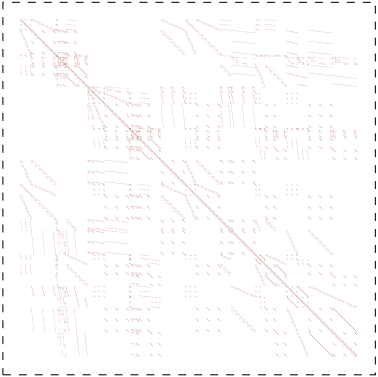
$$\left(\overline{\psi}\,\gamma_\mu P_{L/R}\,\psi\right)\left(\overline{\chi}\,\gamma^\mu P_{L/R}\,\chi\right) \qquad \psi,\chi \in \{d,e,u\}$$

Consider effects of dipole operators

$$\left(\overline{\psi}_L \sigma^{\mu\nu} \psi_R\right) F_{\mu\nu} \qquad \psi \in \{d,e,u\}$$

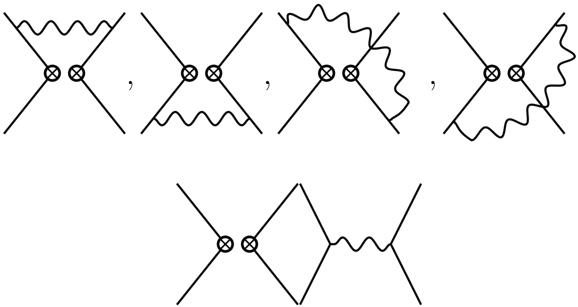
$$\left(\overline{\psi}_L \sigma^{\mu\nu} T^A \psi_R\right) G^A_{\mu\nu} \qquad \psi \in \{d,u\}$$

$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \boxed{\gamma(t)} c_V(t) + s_V(t) \qquad (t \equiv \ln \mu)$$



DSixTools basis (Fuentes-Martin, Ruiz-Femenia, Vicente, and Virto 2021)

$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \boxed{\gamma(t)}\, c_V(t) + s_V(t) \qquad (t \equiv \ln \mu)$$



(+identical diagrams with gluons)

THE LEFT HAS A LARGE (BROKEN) FLAVOUR SYMMETRY

$$(U(3)_{d_L} \times U(3)_{d_R} \rtimes \mathbb{Z}_{2,d}) \times (U(3)_{e_L} \times U(3)_{e_R} \rtimes \mathbb{Z}_{2,e}) \\ \times (U(2)_{u_L} \times U(2)_{u_R} \rtimes \mathbb{Z}_{2,u}) \times U(3)_{\nu_L}$$

Kinetic terms invariant under $d_L^i \to U_{d_L}^{ij} d_L^j$, $d_R^i \to U_{d_R}^{ij} d_R^j$, $d_L \leftrightarrow d_R$, ...

Masses and other operators break this — their components are *charged* under the flavour group.

$$\mathcal{L} = i\overline{d}_L^i D d_L^i + i\overline{d}_R^i D d_R^i + [\text{sim. for } u_L, u_R, e_L, e_R, \nu_L] \\ - [M^d]_{ij}\overline{d}_L^i d_R^j + \text{h.c.} + [\text{sim. for } M^u, M^e] \\ + c_{ijkl}\left(\overline{d}_L^i \gamma d_L^j\right)\left(\overline{e}_L^k \gamma e_L^l\right) + [\text{other ops}]$$

(Neglecting purely gluonic operators)

WE USE A SMALLER LARGE (BROKEN) FLAVOUR SYMMETRY

$$SU(3)_d \times SU(3)_e \times SU(2)_u \times \mathbb{Z}_2$$

Kinetic terms invariant under $d_L^i \to U_d^{ij} d_L^j$, $d_R^i \to U_d^{ij} d_R^j$, ..., $(d_L, e_L, u_L) \leftrightarrow (d_R, e_R, u_R)$.

Masses and other operators break this — their components are *charged* under the flavour group.

$$\begin{aligned}\mathcal{L} = & \, i\overline{d}_L^i D d_L^i + i\overline{d}_R^i D d_R^i + [\text{sim. for } u_L, u_R, e_L, e_R, \nu_L] \\ & - [M^d]_{ij} \overline{d}_L^i d_R^j + \text{h.c.} + [\text{sim. for } M^u, M^e] \\ & + c_{ijkl} \left(\overline{d}_L^i \gamma d_L^j \right) \left(\overline{e}_L^k \gamma e_L^l \right) + [\text{other ops}]\end{aligned}$$

(Neglecting purely gluonic operators)

FLAVOUR DECOMPOSITION

$$SU(3)_d \times SU(3)_e \times SU(2)_u$$

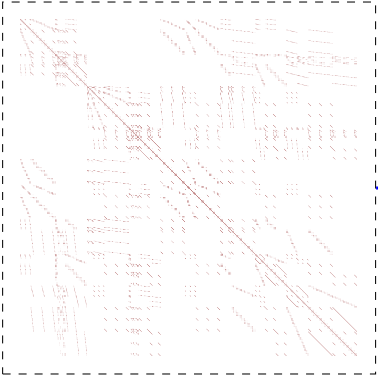
Following (Machado, Renner, and Sutherland 2023) we Clebsch-Gordan decompose under this flavour symmetry.

PARITY DECOMPOSITION (FOR VECTORIAL OPERATORS)

$$(\overline{\psi}\,\gamma_\mu P_{L/R}\,\psi)\,(\overline{\chi}\,\gamma^\mu P_{L/R}\,\chi) \qquad \psi,\chi \in \{d,e,u\}$$

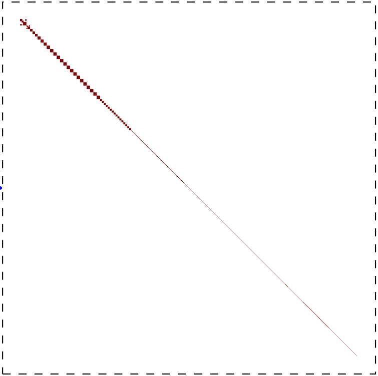
	$+$	$-$
'A' type	$LL+RR$	$LL-RR$
'B' type	$LR+RL$	$LR-RL$

$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \boxed{\gamma(t)}\, c_V(t) + s_V(t) \qquad (t \equiv \ln \mu)$$



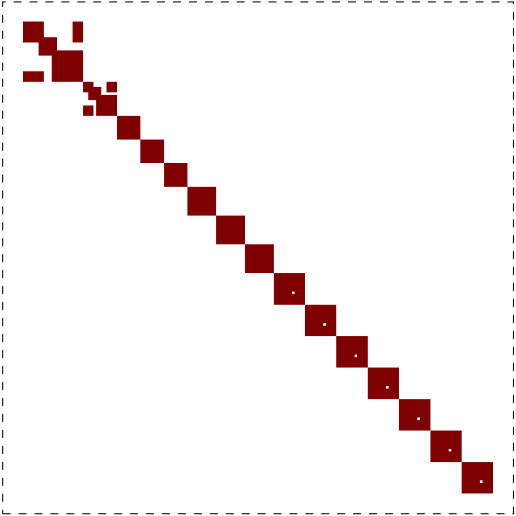
DSixTools/San Diego basis

→



Flavour & parity basis

FEW ZEROES WITHIN BLOCKS



SOLVING RUNNING

$$(4\pi)^2 \frac{\mathrm{d}}{\mathrm{d}t} c_V(t) = \gamma(t) c_V(t) + s_V(t) \qquad (t \equiv \ln \mu)$$

Solve running with integrating factor U

$$c_V(t_b) = U(t_b, t_W) c_V(t_W) + U(t_b, t_W) \int_{t_W}^{t_b} \mathrm{d}t\, U(t_W, t) s_V(t)\,.$$

U is given by exponentiating γ with SM couplings taking average values between M_W and m_b,

$$U(t_b, t_W) = e^{\frac{\langle\gamma\rangle}{16\pi^2}(t_b - t_W)}\,.$$

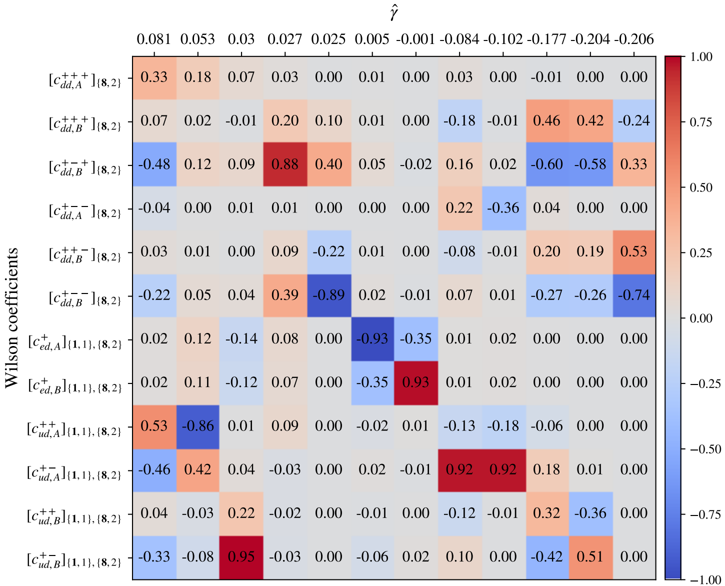
SOLVING RUNNING

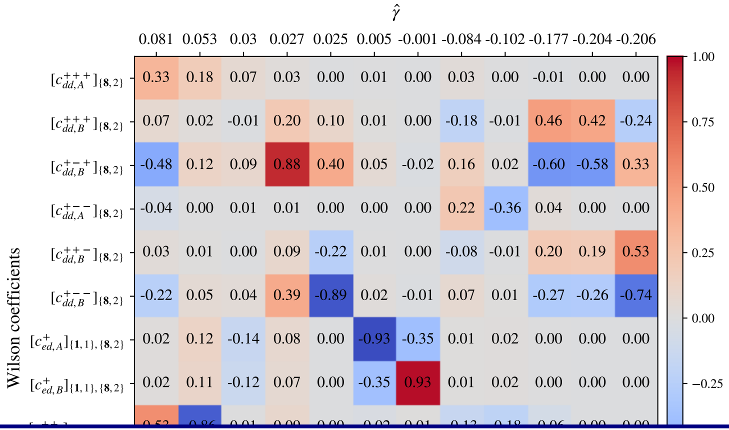
Diagonalise U to understand RG flow basis-independently

$$(S^{-1}US)_{ij} = \left(\frac{m_b}{m_W}\right)^{\frac{\hat{\gamma}_i}{(4\pi)^2}} \delta_{ij}$$

No mixing, directions with +ve $\hat{\gamma}$ shrink, -ve $\hat{\gamma}$ grow.

LEPTON UNIVERSAL OPERATORS MEDIATING $b \to s$



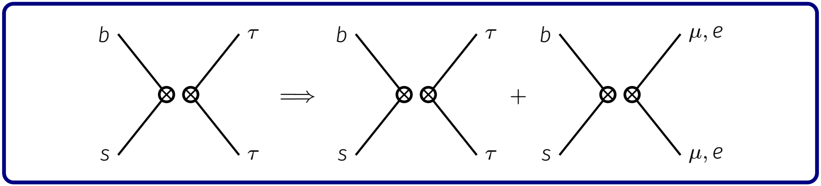


$$\delta E_n = \left\langle n \middle| \hat{H}_1 \middle| n \right\rangle$$

$$\delta \left| n \right\rangle = \sum_{k \neq n} \frac{\left\langle k \middle| \hat{H}_1 \middle| n \right\rangle}{E_n - E_k} \left| k \right\rangle \sim \sum_{k \neq n} \frac{O(10^{-4})}{\hat{\gamma}_n - \hat{\gamma}_k} \left| k \right\rangle \implies$$

higher loop corrections only a large effect for nearly degenerate eigenvectors

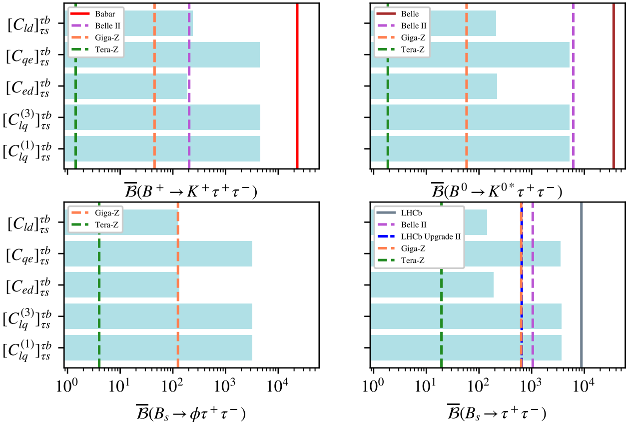
PHENO EXAMPLE



τ only at M_W scale $\Longrightarrow$ τ, and some e and μ, at m_b scale

PHENO EXAMPLE

$bs\mu\mu$ (teal bars) can be better than current/projected $bs\tau\tau$ (solid/dashed lines)



Also (Cornella, Faroughy, Fuentes-Martin, Isidori, and Neubert 2021)

SUMMARY

- Flavour and parity simplify running in the LEFT
- γ can be block diagonalised to all orders
- The map $M_W \to m_b$ is fully understandable in terms of eigenvalues and eigenvectors
- Many possible pheno applications!

BACKUP SLIDES

Schematically (Jenkins, Manohar, and Stoffer 2018)

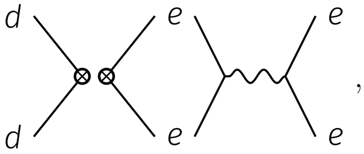
$$
\begin{aligned}
\text{masses} \to (4\pi)^2\,\dot{M} &= (e^2+g^2)M + (e+g)dM^2 + c_S M^3 + c_V M^3 + d^2M^3\,,\\
\text{QED} \to (4\pi)^2\,\dot{e} &= e^3 + e^2 dM + d^2M^2\,,\\
\text{QCD} \to (4\pi)^2\,\dot{g} &= g^3 + g^2 dM + d^2M^2\,,\\
\text{dipoles} \to (4\pi)^2\,\dot{d} &= (e^2+g^2+eg)d + eM(c_S+c_T) + (e+g)d^2M\,,\\
\text{4f scalar} \to (4\pi)^2\,\dot{c}_S &= (e^2+g^2)(c_S+c_T) + (e^2+g^2+eg)d^2\,,\\
\text{4f tensor} \to (4\pi)^2\,\dot{c}_T &= (e^2+g^2)(c_S+c_T)\,,\\
\text{4f vector} \to (4\pi)^2\,\dot{c}_V &= (e^2+g^2)c_V + (e^2+g^2+eg)d^2\,,
\end{aligned}
$$

Neglect operators in grey at $O(0.1\%)$ accuracy.

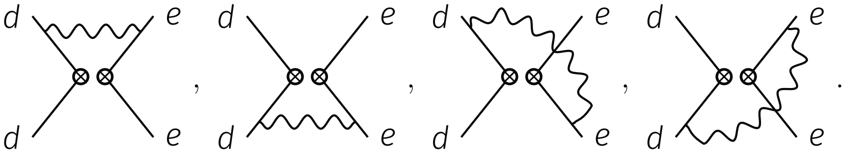
$\{c_S, c_T\}$ and c_V do not mix due to helicity selection rules. (Cheung and Shen 2015)

ACCIDENTAL ZERO

For each parity structure, zero due to contributions from



cancelling with contributions from



(Only occurs for this combination of charges.)

SOLVING RUNNING

$$\gamma(t) = e^2(t)\hat{\gamma}_e + g^2(t)\hat{\gamma}_g.$$

$$\begin{aligned}\ln U(t_b, t_W) = & \frac{1}{(4\pi)^2}\int \gamma(t_1) + \frac{1}{2\,(4\pi)^4}\int_{t_1>t_2}[\gamma(t_1), \gamma(t_2)] \\ & + \frac{1}{6\,(4\pi)^6}\int_{t_1>t_2>t_3}([\gamma(t_1), [\gamma(t_2), \gamma(t_3)]] + [\gamma(t_3), [\gamma(t_2), \gamma(t_1)]]) + \ldots, \\ = & -\hat{\gamma}_e \times 1.803\times 10^{-3} - \hat{\gamma}_g \times 3.783 \times 10^{-2} - \frac{1}{2}[\hat{\gamma}_e, \hat{\gamma}_g] \times 7.379 \times 10^{-6} + \ldots\end{aligned}$$

Neglecting higher order terms matches fully numerical solution to $\mathcal{O}(0.0001\%)$ accuracy for lepton universal $b \to s$ block.

TWO-LOOP ESTIMATION

$$\delta |n\rangle = \sum_{k\neq n} \frac{\left\langle k \middle| \hat{H}_1 \middle| n \right\rangle}{E_n - E_k} |k\rangle \sim \sum_{k\neq n} \frac{\left(\frac{g}{4\pi}\right)^4 \times 10}{E_n - E_k} |k\rangle$$

$$\sim \sum_{k\neq n} \frac{\left(\frac{1}{4\pi}\right)^4 \times 10}{E_n - E_k} |k\rangle \sim \sum_{k\neq n} \frac{1 \times 10^{-4}}{E_n - E_k} |k\rangle$$

Cheung, Clifford and Chia-Hsien Shen (2015). "Nonrenormalization Theorems without Supersymmetry". In: *Phys. Rev. Lett.* 115.7, p. 071601. DOI: 10.1103/PhysRevLett.115.071601. arXiv: 1505.01844 [hep-ph].

Cornella, Claudia et al. (2021). "Reading the footprints of the B-meson flavor anomalies". In: *JHEP* 08, p. 050. DOI: 10.1007/JHEP08(2021)050. arXiv: 2103.16558 [hep-ph].

Fuentes-Martin, Javier et al. (2021). "DsixTools 2.0: The Effective Field Theory Toolkit". In: *Eur. Phys. J. C* 81.2, p. 167. DOI: 10.1140/epjc/s10052-020-08778-y. arXiv: 2010.16341 [hep-ph].

Jenkins, Elizabeth E., Aneesh V. Manohar, and Peter Stoffer (2018). "Low-Energy Effective Field Theory below the Electroweak Scale: Anomalous Dimensions". In: *JHEP* 01. [Erratum: JHEP 12, 042 (2023)], p. 084. DOI: 10.1007/JHEP01(2018)084. arXiv: 1711.05270 [hep-ph].

Machado, Camila S., Sophie Renner, and Dave Sutherland (2023). "Building blocks of the flavourful SMEFT RG". In: *JHEP* 03, p. 226. DOI: 10.1007/JHEP03(2023)226. arXiv: 2210.09316 [hep-ph].

Cutting off SMEFT and HEFT

Konstantin Schmid

EFT School 2024 in Zürich

based on arXiv: 24xx.xxxxx with Ilaria Brivio and Ramona Gröber

Istituto Nazionale di Fisica Nucleare

The obligatory SMEFT slide

Lorentz invariance

Gauge invariance
$SU(3)_c \times SU(2)_L \times U(1)_Y$

physical Higgs part of doublet under $SU(2)_L$

removing redundancies
[Henning et al. 2015, 2017]

power counting in canonical dimension

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i^{(d=6)}}{\Lambda^2}\,\mathcal{O}_i^{(d=6)} + \sum_j \frac{c_j^{(d=8)}}{\Lambda^4}\,\mathcal{O}_j^{(d=8)} + \dots$$

LO | NLO | NNLO

[Grzadkowski et al. 2010] [Murphy 2020]

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But is SMEFT really the full story?

physical Higgs part of doublet under $SU(2)_L$ → (assumption can be relaxed; resolves correlations $(v+h)^n$) → treat physical Higgs and GBs separately

LO Higgs Effective Field Theory (HEFT) Lagrangian

3 GBs → $U(\boldsymbol{\pi}) = \exp\left(i\,\frac{\pi^a T^a}{v}\right)$

Higgs singlet → $\mathcal{F}_i(h) = \sum_n c_{in} \left(\frac{h}{v}\right)^n$

↑ tunable to SM

$$\mathcal{L}_2 = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{v^2}{4}\mathrm{Tr}\left(\mathbf{V}_\mu \mathbf{V}^\mu\right)\mathcal{F}_C(h) - V(h) + i\bar{Q}_L \not{D} Q_L + i\bar{Q}_R \not{D} Q_R + i\bar{L}_L \not{D} L_L + i\bar{L}_R \not{D} L_R - \frac{v}{\sqrt{2}}\left(\bar{Q}_L \mathbf{U}\mathcal{Y}_Q(h) Q_R + \text{h.c.}\right) - \frac{v}{\sqrt{2}}\left(\bar{L}_L \mathbf{U}\mathcal{Y}_L(h) L_R + \text{h.c.}\right)$$

$\mathbf{V}_\mu = (D_\mu \mathbf{U})\mathbf{U}^\dagger$

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How to construct HEFT at NLO?

- defining NLO is related to power counting
- HEFT: mixture of χPT (scalar sector) and SMEFT (gauge and fermion sector)

derivatives + canonical dimension = ?

- common approaches: Naive Dimensional Analysis (NDA) and chiral dimensions d_χ

[Manohar, Georgi 1984] [Gavela et al. 2016] [Buchalla, Catà 2012] [Buchalla, Catà, Krause 2014]

General idea: count loop orders **within the EFT**, $d_\chi = 2L + 2$

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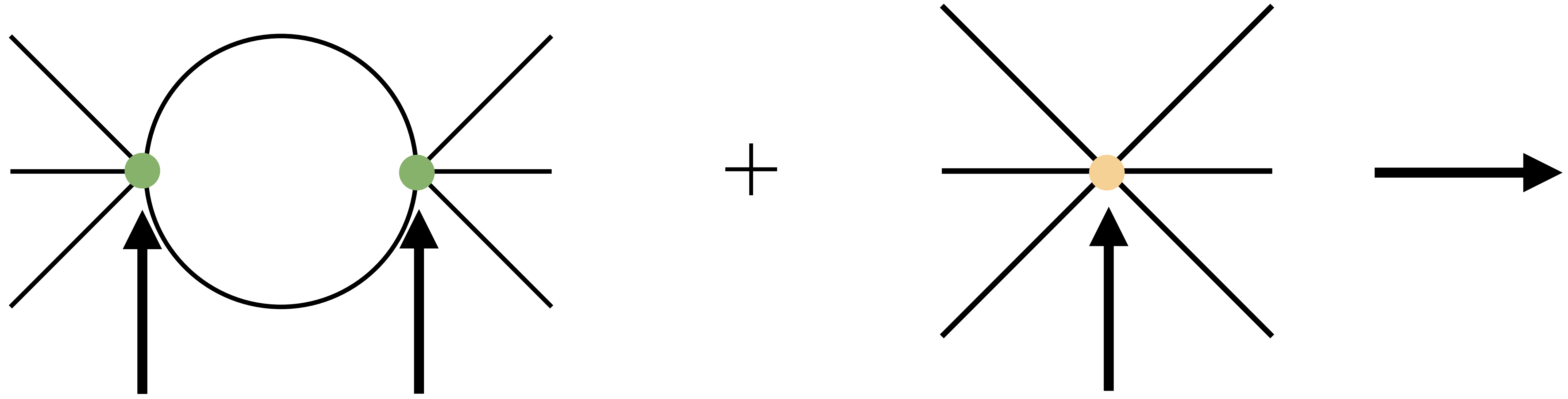
NLO in HEFT



vertices from LO ($d_\chi = 2$) Lagrangian

counterterm ($d_\chi = 4$)

required counterterms define NLO ($d_\chi = 4$)

operators carry loop order

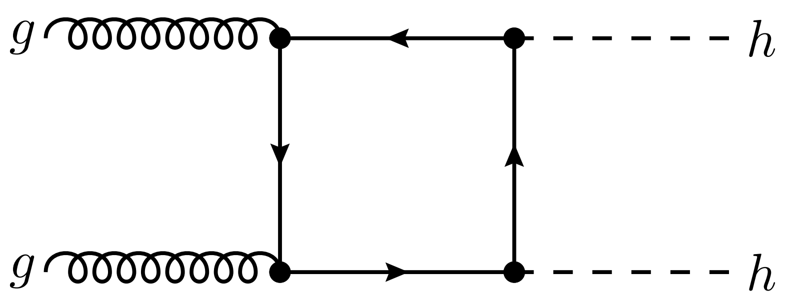
$$\mathcal{L}_{\text{HEFT}} = \mathcal{L}_2 + \left(\frac{1}{16\pi^2}\right)\sum_i \mathcal{O}_i^{d_\chi=4} + \left(\frac{1}{16\pi^2}\right)^2 \sum_j \mathcal{O}_j^{d_\chi=6} + \dots$$

LO | NLO | NNLO

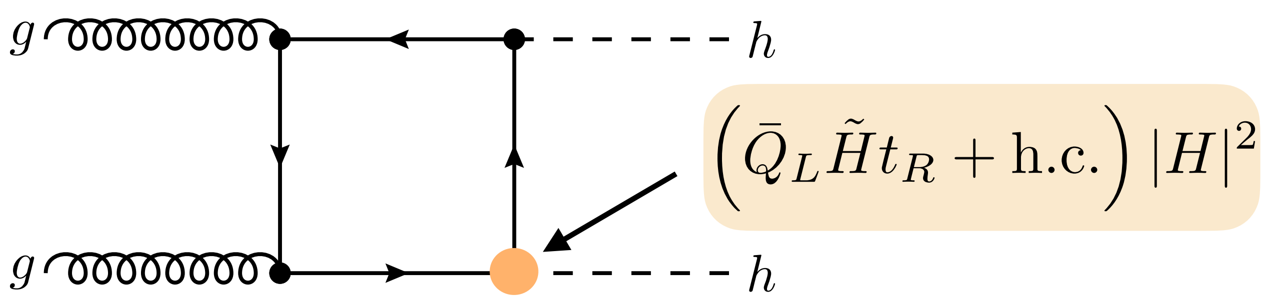
[Sun, Xiao, Yu 2022] [Sun, Xiao, Yu 2022]

Power counting on amplitude level

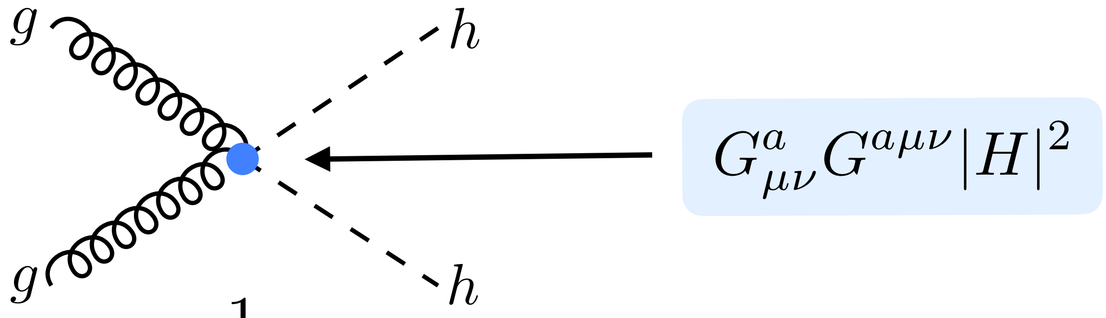
$$\mathcal{M}_{\text{SMEFT}} = \sum_{n,m} \frac{1}{\Lambda^n} \left(\frac{1}{16\pi^2}\right)^m \mathcal{M}_n^m$$



$\frac{1}{16\pi^2}$



$\frac{1}{\Lambda^2} \frac{1}{16\pi^2}$



$\frac{1}{\Lambda^2}$

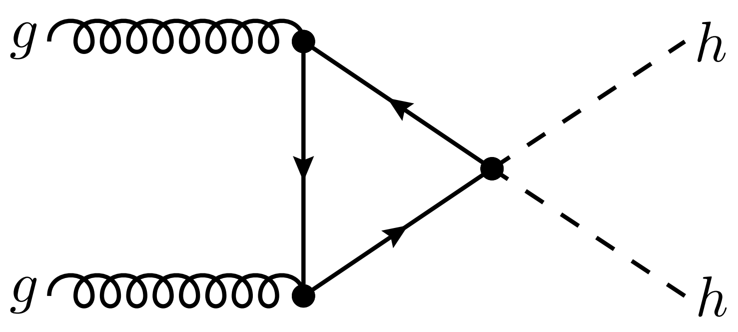
UV assumption:

$$\frac{[\text{scale}]^2}{\Lambda^2} \lesssim \frac{1}{16\pi^2}$$ and additional loop factors

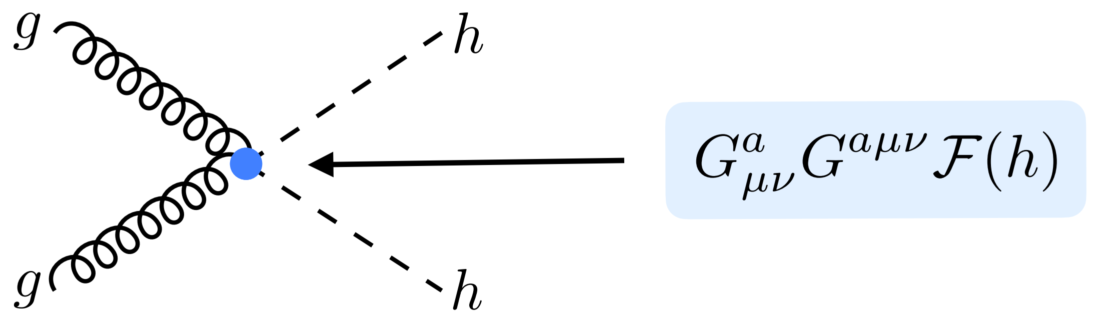
[Arzt, Einhorn, Wudka 1994] [Einhorn, Wudka 2013]

$$\mathcal{M}_{\text{HEFT}} = \sum_{n \in 2\mathbb{N}_{>0}} \left(\frac{1}{16\pi^2}\right)^{n/2-1} \mathcal{M}^n$$

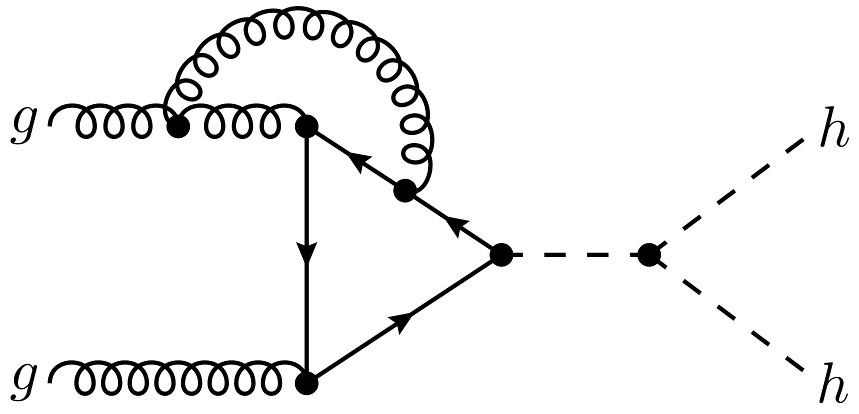
$d_\chi = 2L + 2$



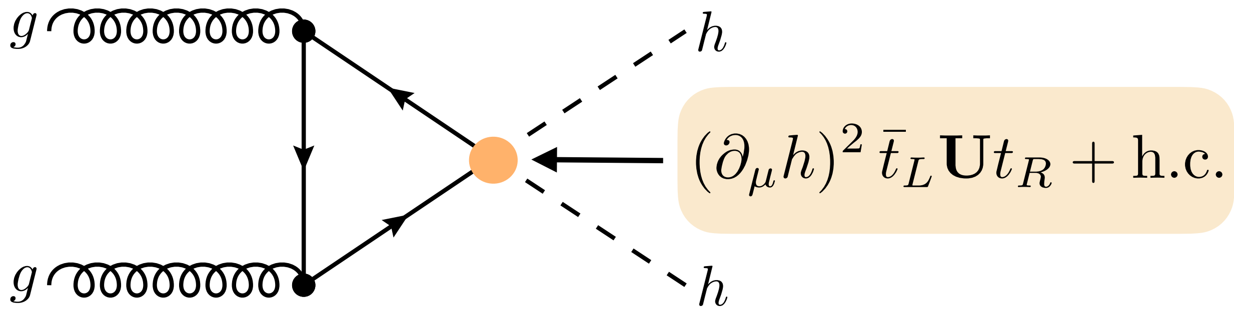
$\frac{1}{16\pi^2}$



$\frac{1}{16\pi^2}$



$\left(\frac{1}{16\pi^2}\right)^2$



$\left(\frac{1}{16\pi^2}\right)^2$

Combine loop order of topology and operators

Truncation in the squared amplitude

- squared amplitude enters observables (e.g. cross-sections)

SMEFT: max. order in loops and scale suppressions

$$|\mathcal{M}_{\text{SMEFT}}|^2 \simeq \sum_{\substack{n,n',m,m' \\ n+n' \leq N_\Lambda^{\max}, \\ m+m' \leq N_L^{\max}}} \frac{1}{\Lambda^{n+n'}} \left(\frac{1}{16\pi^2}\right)^{m+m'} \mathcal{M}_n^m \left(\mathcal{M}_{n'}^{m'}\right)^*$$

HEFT: max. chiral dimension (loop order)

$$|\mathcal{M}_{\text{HEFT}}|^2 \simeq \sum_{\substack{n,n' \in 2\mathbb{N}_{>0} \\ n+n'-2 \leq N_{d_\chi}^{\max}}} \left(\frac{1}{16\pi^2}\right)^{(n+n')/2-2} \mathcal{M}^n \left(\mathcal{M}^{n'}\right)^*$$

$$d_\chi^1 + d_\chi^2 = 2(L_1 + L_2) + 4$$

- truncation error can be estimated by next higher order

- our application of these power counting insights: **Higgs pair production**

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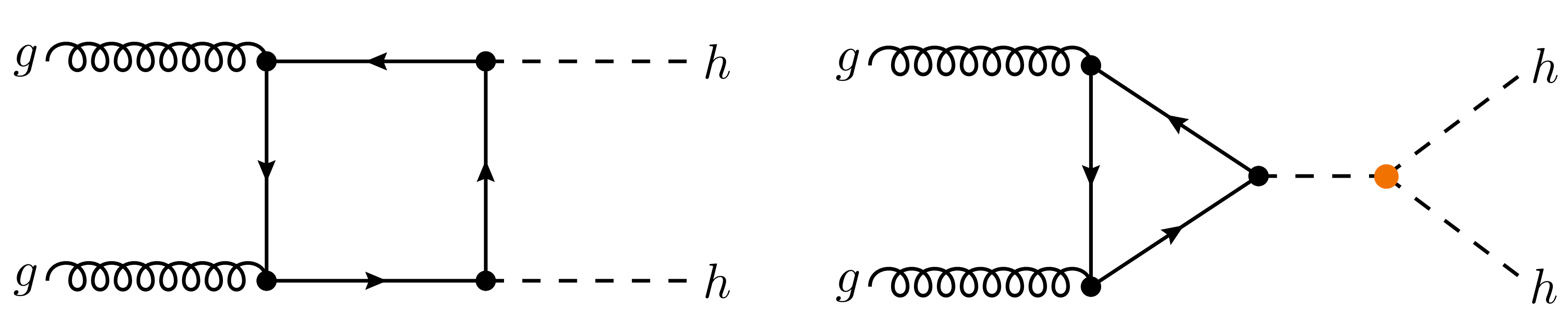
Application: Di-Higgs Production

[Plehn et al. 1998] [Gröber, Mühlleitner 2010] [Gillioz et al. 2012] [Hespel et al. 2014] [Goertz et al. 2015]

1988
LO in SM



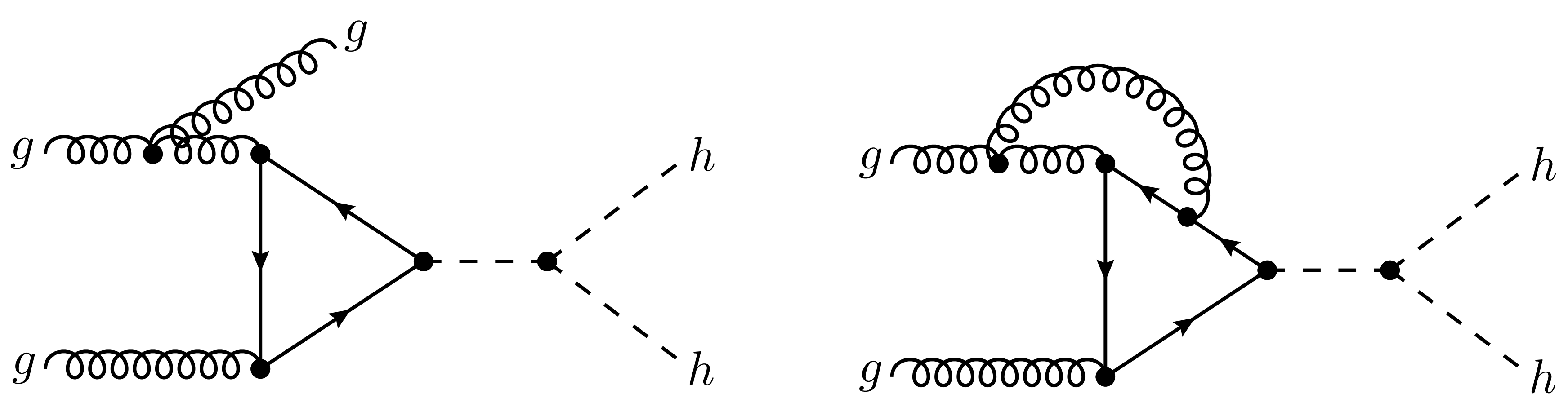
[Glover, van der Bij 1988]

BSM

- MSSM/2HDM
- Composite Higgs
- SMEFT
- Higgs Portals

2016
NLO QCD in SM

[Haisch, Koole 2022]
[Heinrich, Lang 2023]
[De Curtis et al. 2023]
[Haisch et al. 2024]



[Borowka et al. 2016]

From an experimental point of view:

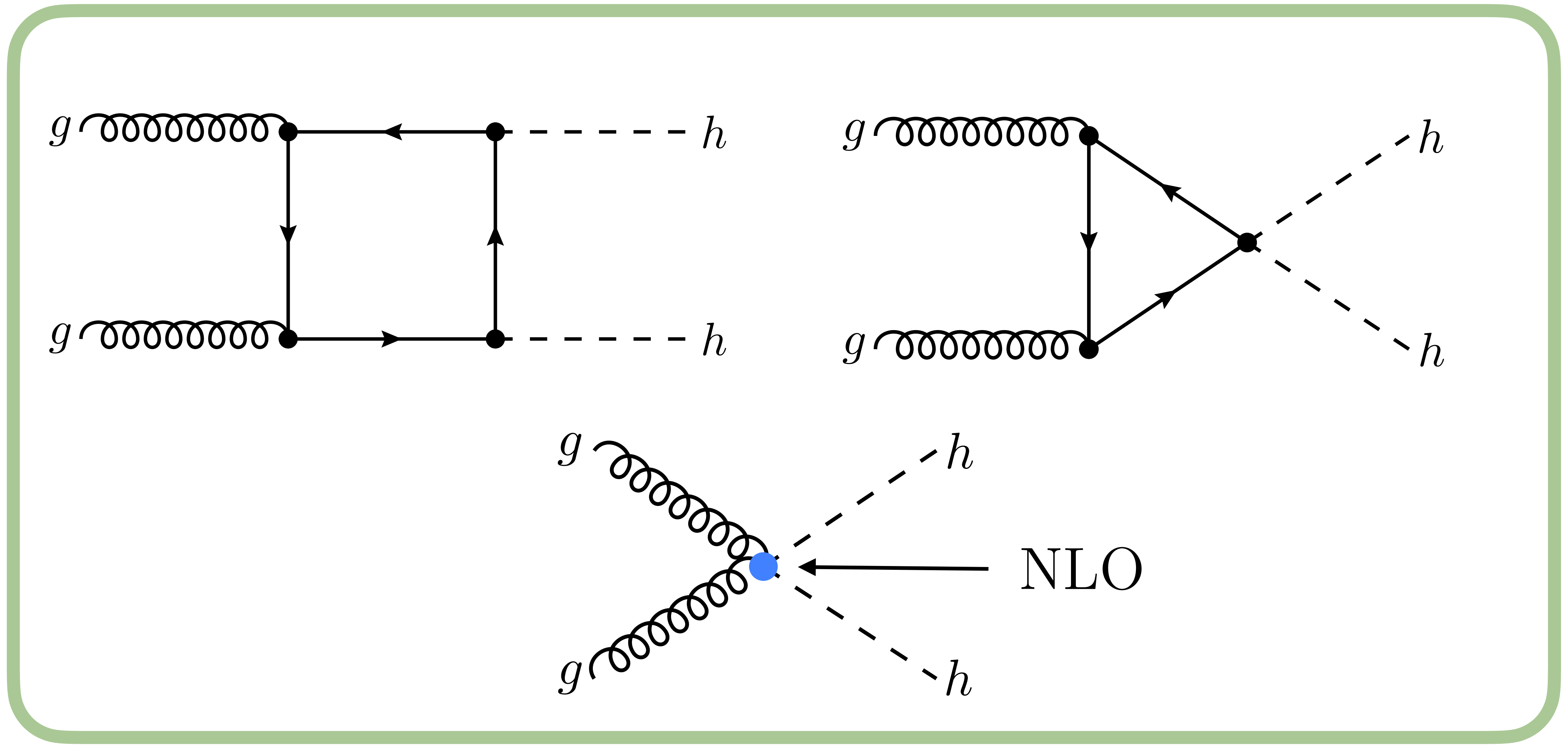
- advances in reconstruction of final states
- increase in luminosity at HL-LHC

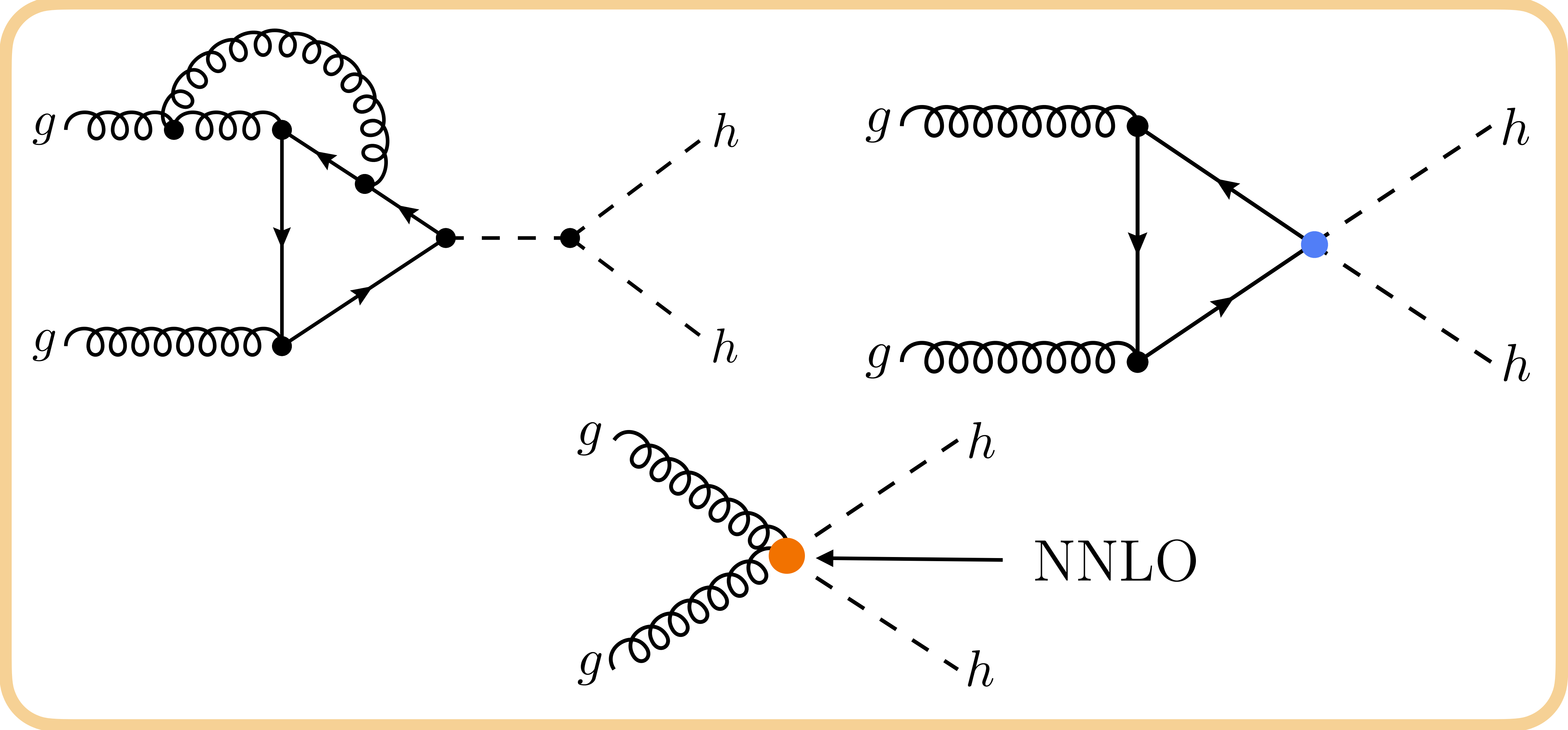
Di-Higgs Production in HEFT

- bottom-up EFT: manageable amount of states to include $\{g, t, h\}$

→ discussion up to NNLO ($d_\chi = 6$ on amplitude level)

$$|\mathcal{M}_{\text{HEFT}}|^2 \simeq \left(\frac{1}{16\pi^2}\right)^2 |\mathcal{M}^4|^2 + \left(\frac{1}{16\pi^2}\right)^3 2\text{Re}\{\mathcal{M}^4 \left(\mathcal{M}^6\right)^*\}$$





Conclusions and Outlook

- heavy BSM Physics can be treated with bottom-up EFTs: SMEFT and HEFT

- SMEFT bases: canonical dimension, HEFT bases: loop orders within EFT

- application of consistent power counting: Di-Higgs production

⟶ theoretically/phenomenologically well-stablished, experimentally viable

- outlook: comprehensive review and comparison of NDA and d_χ, further consistency discussion of loop counting, Di-Higgs production (SMEFT vs. HEFT)

Backup

NDA formula and chiral dimension assignments

NDA: $f^2\Lambda^2 \left(\frac{y}{4\pi}\right)^{N_y} \left(\frac{g}{4\pi}\right)^{N_g} \left(\frac{\lambda}{16\pi^2}\right)^{N_\lambda} \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^{N_\psi} \left(\frac{X}{f}\right)^{N_X} \left(\frac{\partial}{\Lambda}\right)^{N_p} \left(\frac{\phi}{f}\right)^{N_\phi}$ with $\Lambda \lesssim 4\pi f$

chiral dimension assignments:

$$[\partial_\mu]_\chi = 1, \quad [h]_\chi = [\pi]_\chi = 0, \quad [X_{\mu\nu}]_\chi = 1, \quad [\psi]_\chi = 1/2, \quad [g]_\chi = [y]_\chi = 1, \quad [\lambda]_\chi = 2$$

Di-Higgs Production in the SM

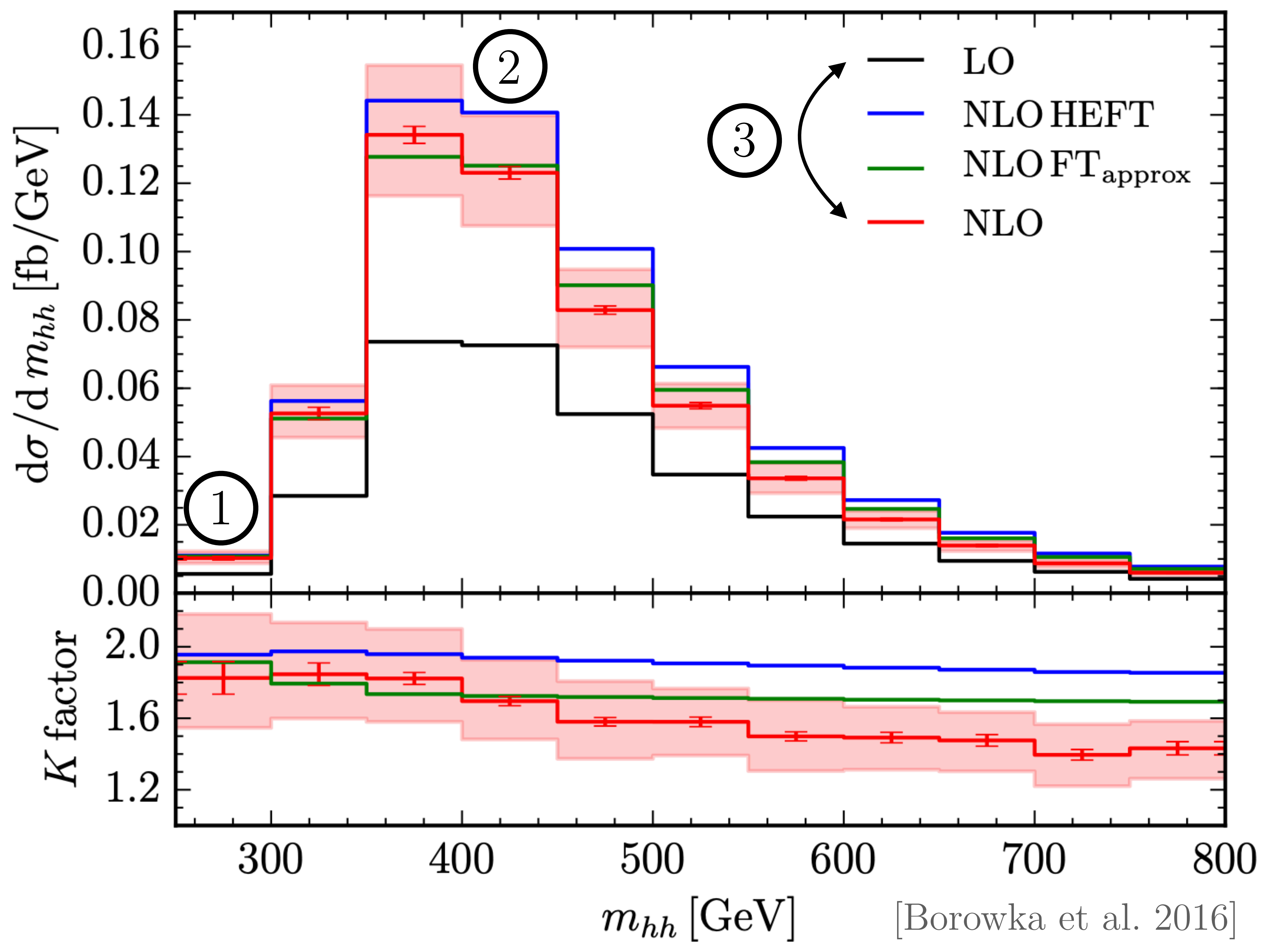
- general amplitude decomposition: $\mathcal{M}(g_a g_b \to hh) = \delta^{ab}\left(\mathcal{M}_1 A_1^{\mu\nu} + \mathcal{M}_2 A_2^{\mu\nu}\right)\epsilon_\mu \epsilon_\nu$

<u>Observations</u>

① approx. cancellation at threshold (exact in HTL)

② peak at $m_{hh} \simeq 2m_t$

③ NLO QCD important $\sigma_{\text{NLO}} \sim 2\sigma_{\text{LO}} \sim \mathcal{O}(30)\,\text{fb}$



[Borowka et al. 2016]

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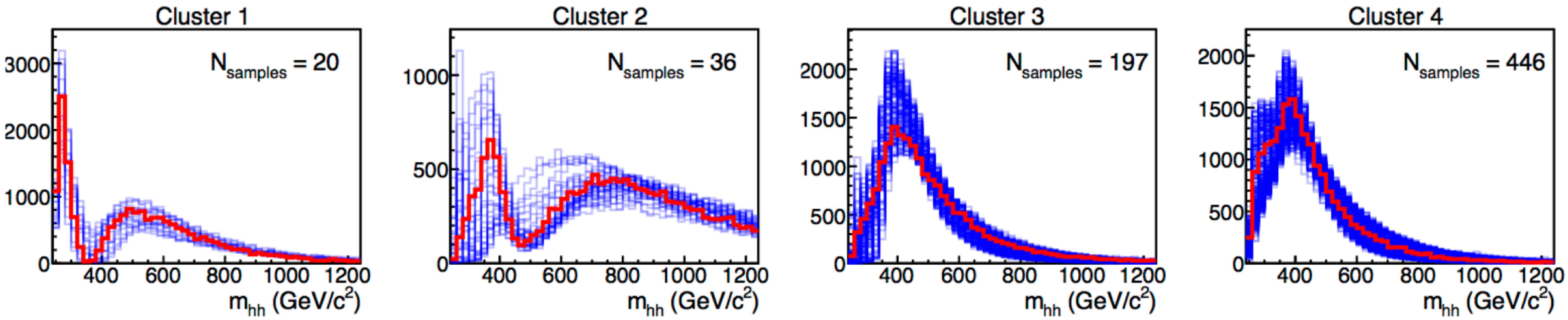
Di-Higgs Production in EFTs: Cluster Analysis

- classification of deviations from the SM: kinematic clusters (here: for inv. mass)



[Carvalho et al. 2016] [Buchalla et al. 2019]

Benchmark scenarios/clusters in HEFT vs. SMEFT?