GeV ALP from TeV Vector-like Leptons

Marta Fuentes Zamoro

Based on 2402.14059, in collaboration with Arturo de Giorgi and Luca Merlo

Instituto de Física Teórica
UAM-CSIC

Motivation

Objectives

- Active neutrino mass → Linear low scale seesaw [J. Kersten, A. Y. Smirnov, Phys. Rev. D 76 (2007) 073005] [A. Abada et al. JHEP 12 (2007) 061]
- Solve $(g-2)_\mu$ anomaly [T. Aoyama et. al., Phys. Rept. 887 (2020) 1–166]

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- Solve $(g-2)_\mu$ anomaly [T. Aoyama et. al., Phys. Rept. 887 (2020) 1–166]

How?

- Two RH lepton singlets N, S
- EW Vector-like doublet ψ

} Neutral part = HNLs [A. de Giorgi, L. Merlo, S. Pokorski, Fortsch. Phys. 71 (2023), no. 4-5 2300020]

- $\mathcal{U}(1)_{PQ}$ symmetry → ALP

Strong CP problem and ALPs

$$\mathcal{L}_{QCD} \supset \theta_{QCD} \frac{g_S^2}{16\pi^2} \mathrm{tr}\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right)$$

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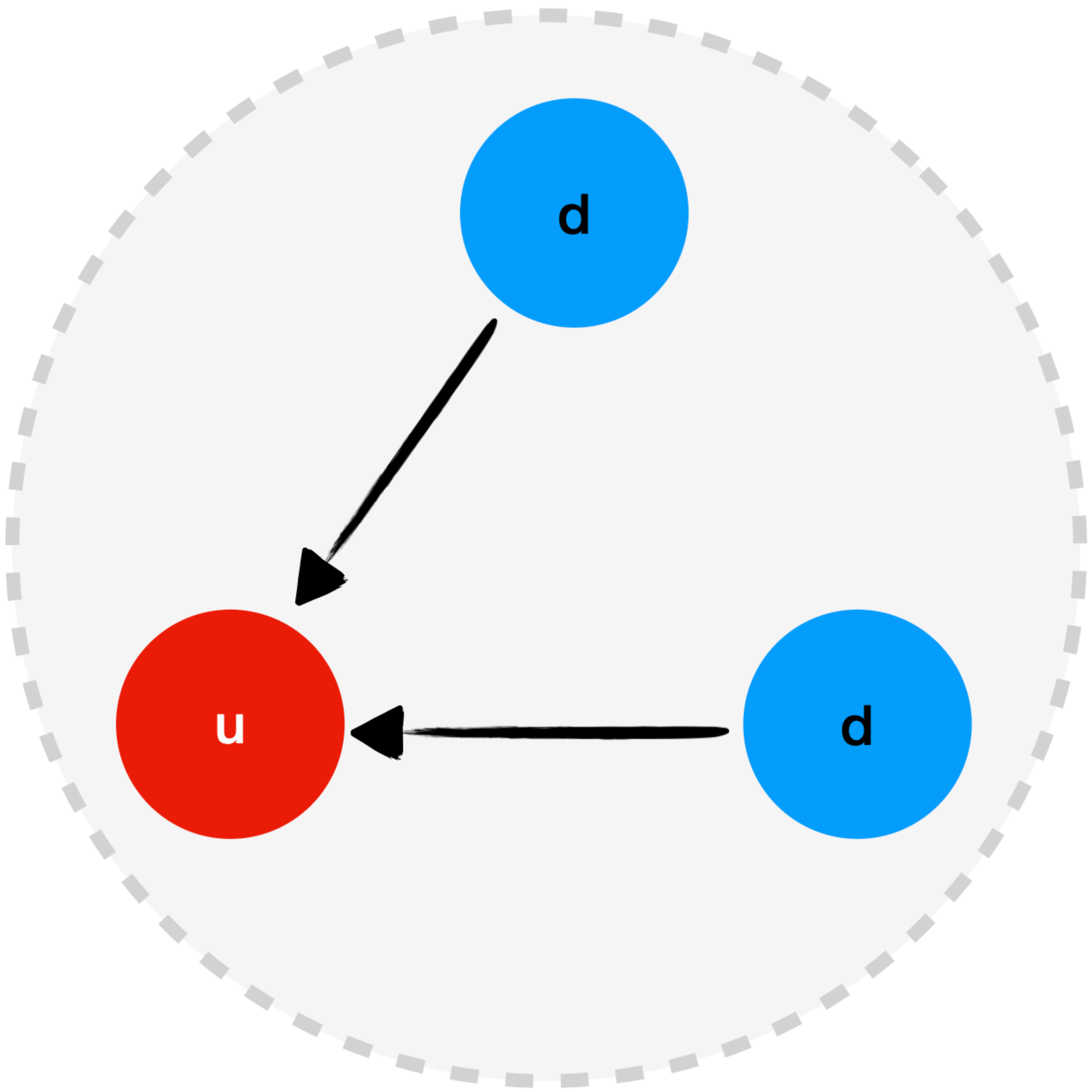
Neutron electric dipole moment



$$d_n \simeq 10^{-16}\bar{\theta}\,\mathrm{e}\cdot\mathrm{cm}$$

$$\bar{\theta} = \theta_{QCD} + \arg(\det(M))$$

$$d_n \lesssim \mathcal{O}(10^{-26})$$

$$\bar{\theta} \lesssim \mathcal{O}(10^{-10})$$

[C. Abel et al., Phys. Rev. Lett., vol. 124, no.8, p. 081803, 2020]

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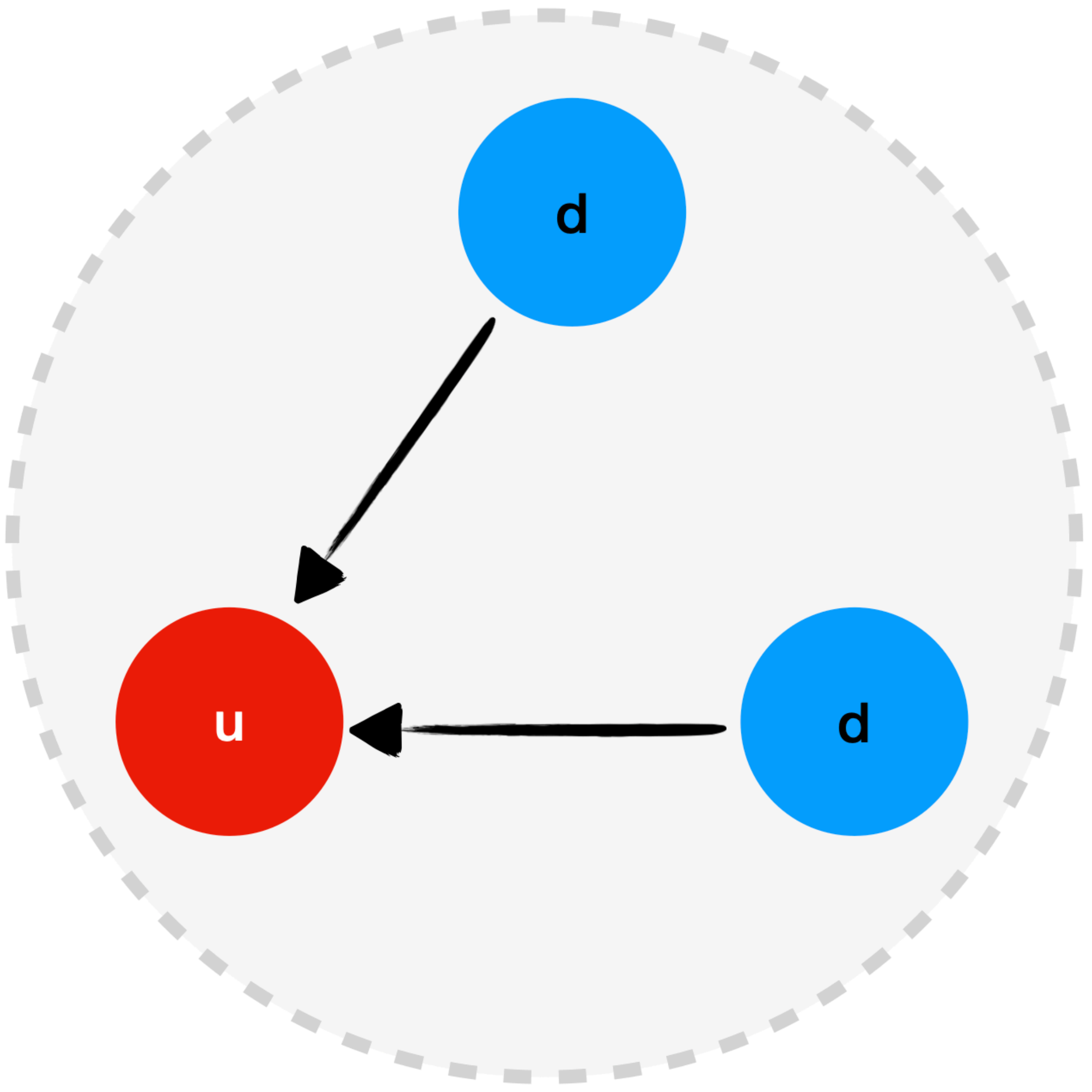

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Solution

$\mathcal{U}(1)_{PQ}$

SSB

Axion

[R. D. Peccei, H. R. Quinn, Phys. Rev. Lett., vol. 38, pp. 1440-1443, 1977]

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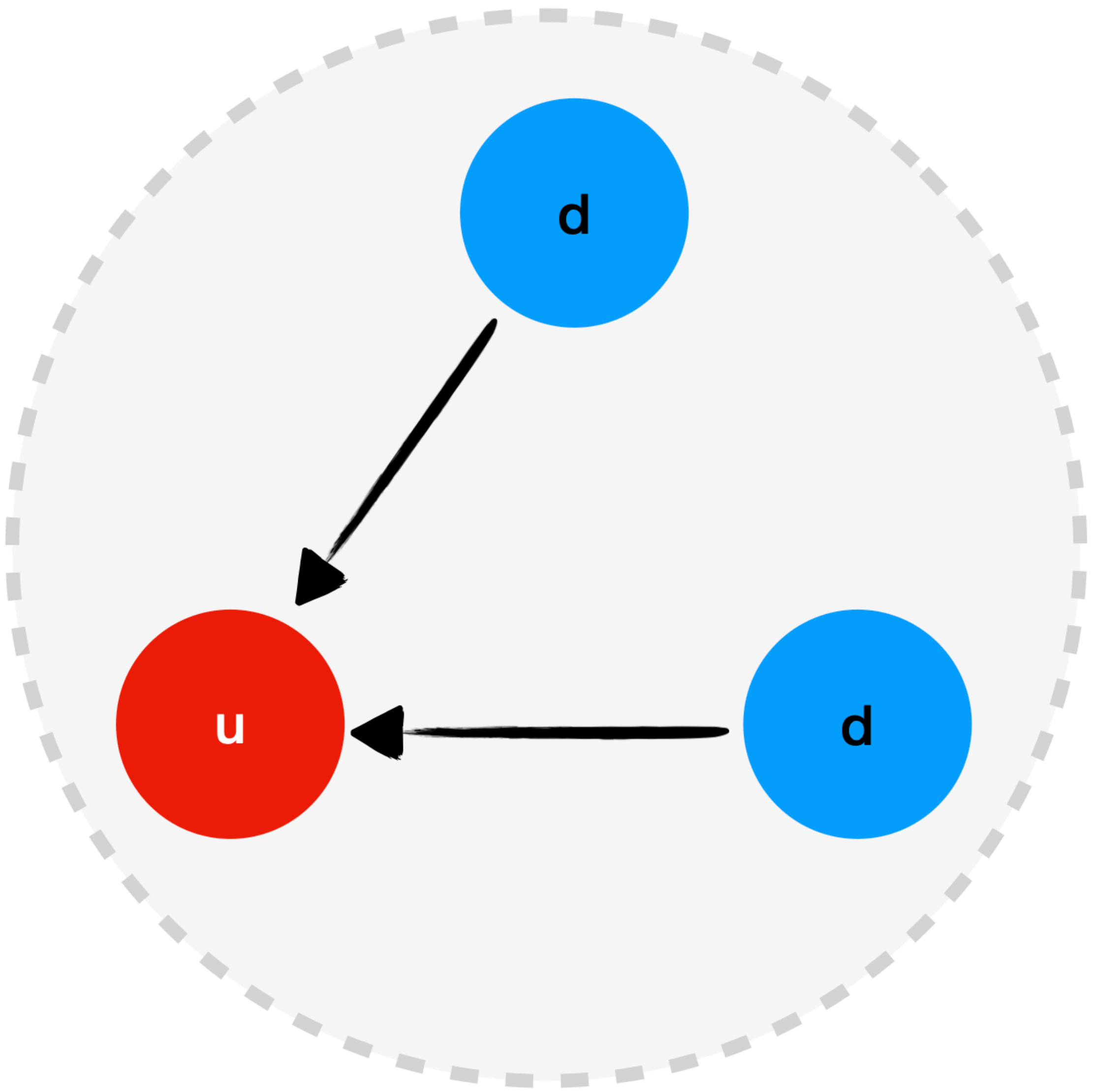

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Axion vs ALP

Axion $m_a \propto \frac{1}{f_a}$

ALP → Free parameters

Possible to work with **lower scales for ALPs** ⇒ Not solve Strong CP

ALP EFT theory

$$\mathcal{L}_{ALP} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{m_a^2}{2}a^2$$
$$+ C_{\tilde{B}}O_{\tilde{B}} + C_{\tilde{W}}O_{\tilde{W}} + C_{\tilde{G}}O_{\tilde{G}}$$
$$+ C_u O_u + C_d O_d + C_e O_e + C_Q O_Q + C_L O_L$$

$$O_{\tilde{B}} = \frac{a}{f_a}B_{\mu\nu}\tilde{B}^{\mu\nu},$$
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$$O_{f,ij} = \frac{\partial^\mu a}{f_a}(\overline{f_i}\gamma_\mu f_j)$$

[H. Georgi, D. Kaplan, L. Randall, PLB 169B(1986)73]

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Pure ALP

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Pure ALP

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Fermions

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Shift-invariant $a \to a + C$

[H. Georgi, D. Kaplan, L. Randall, PLB 169B(1986)73]

$(g-2)_\mu$: a long standing anomaly

Muon magnetic moment

$$\vec{\mu} = g\frac{q}{2m}\vec{S} \xrightarrow[\text{equation}]{\text{Dirac's}} g = 2$$

[T. Aoyama et. al., Phys. Rept. 887 (2020) 1–166]

Deviations $a_\mu = \frac{g-2}{2}$

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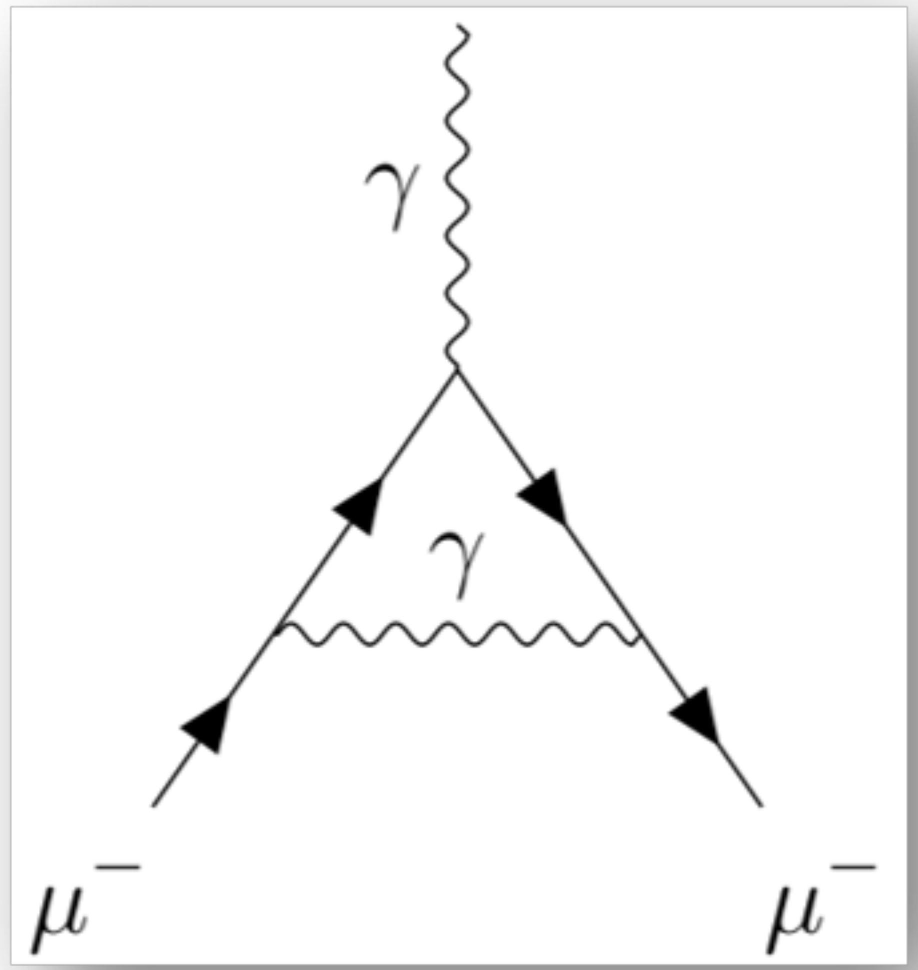

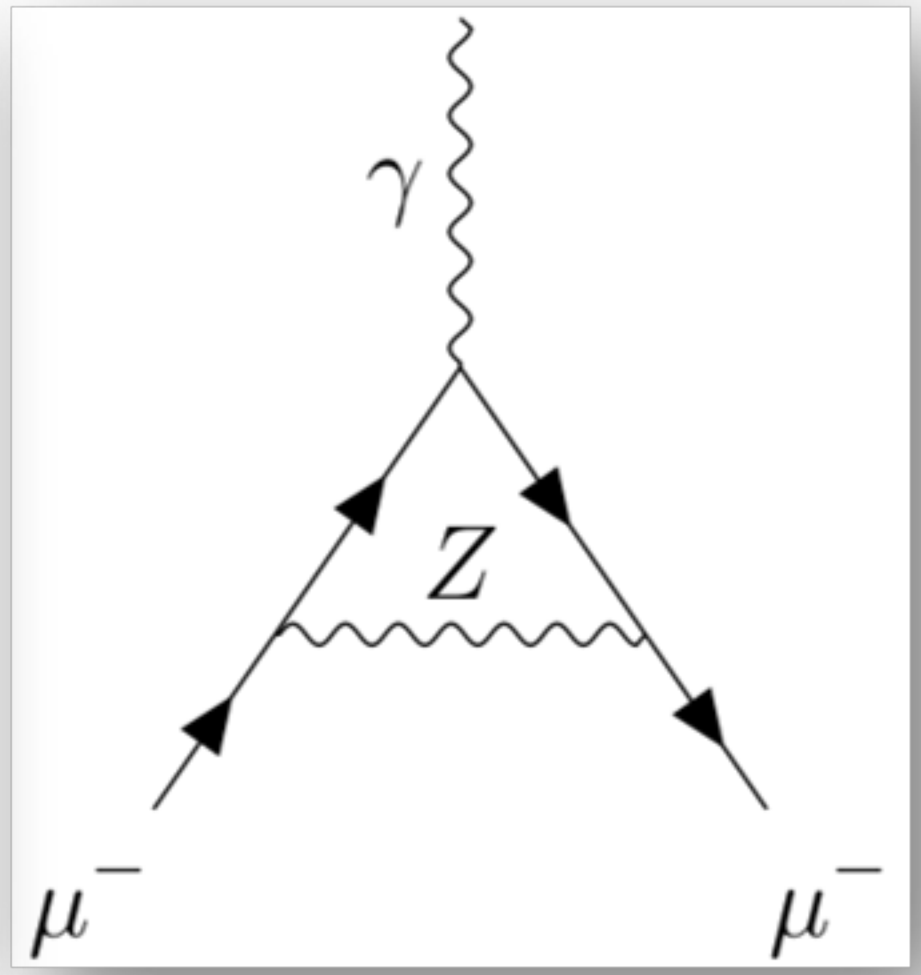

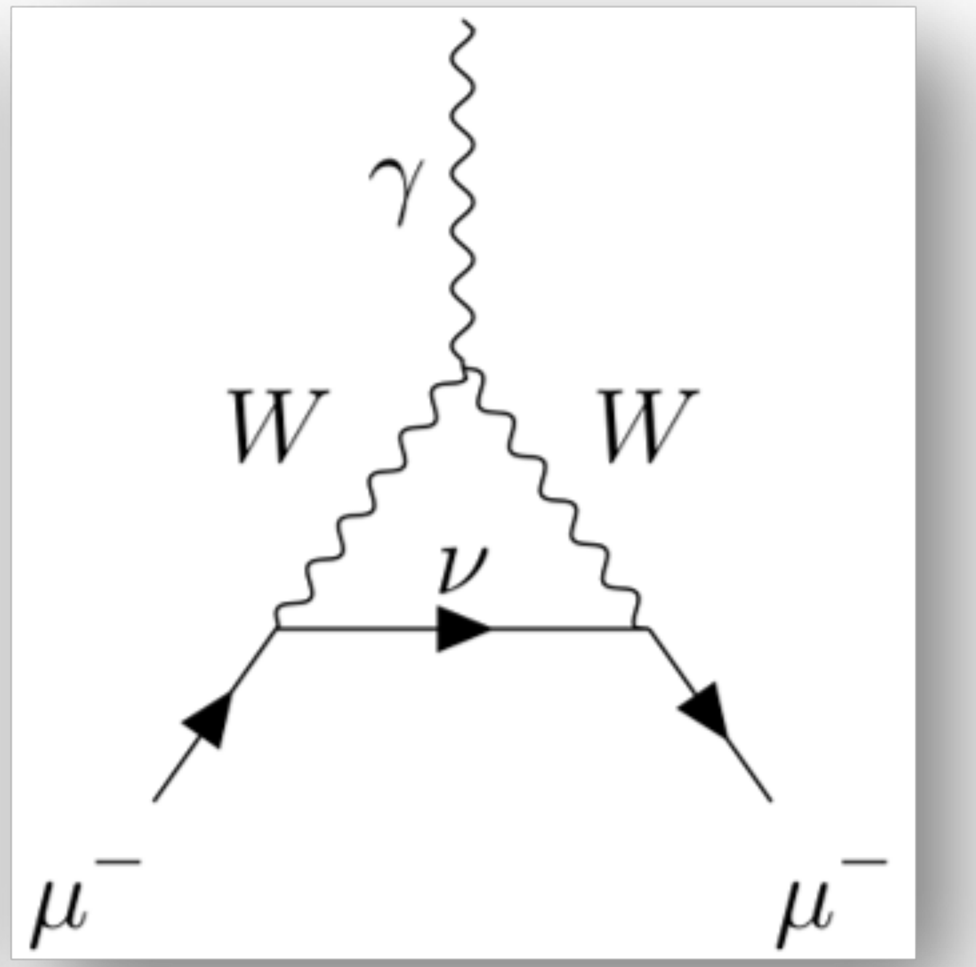

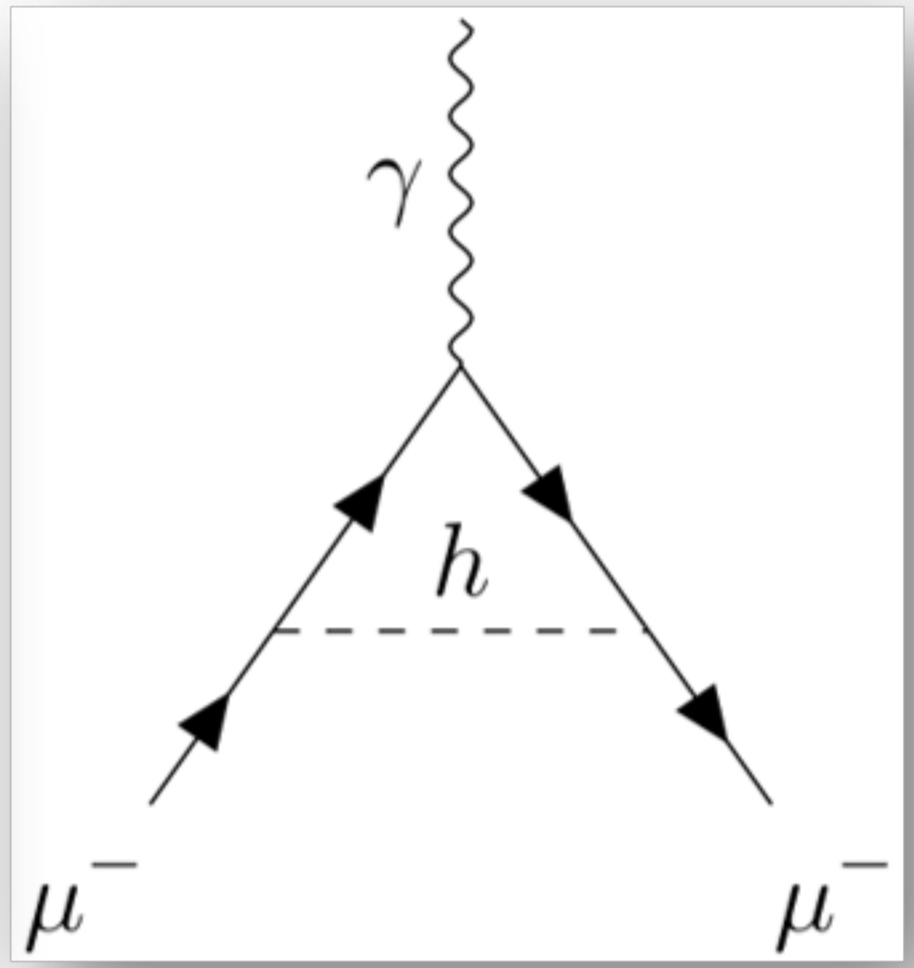

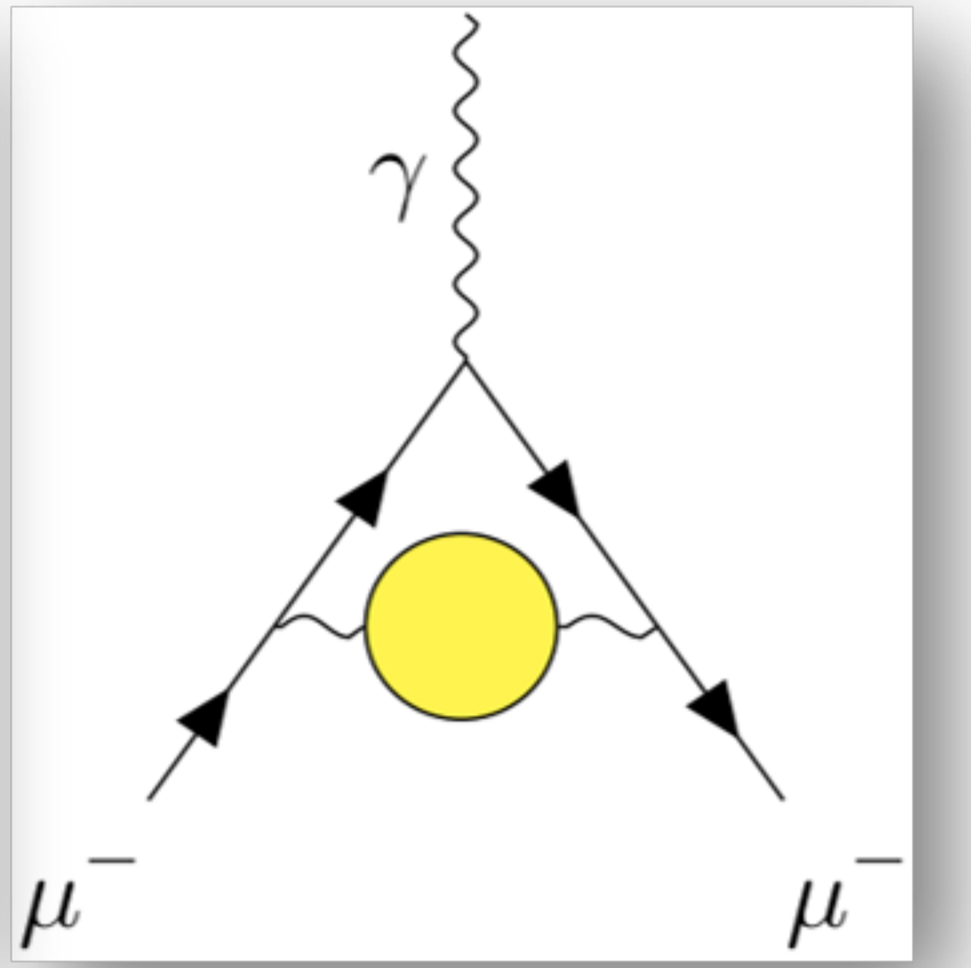

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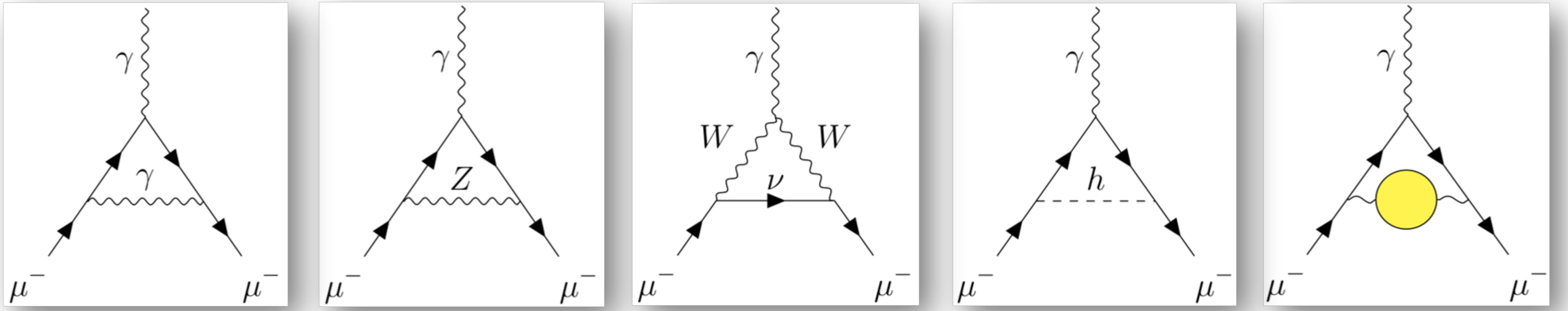

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 251(41)(43) \times 10^{-11} \Rightarrow 4.2\,\sigma$$

UV model

$$
\begin{aligned}
-\mathcal{L}_Y = & Y_N \overline{\ell_L} \widetilde{H} N_R + Y_R \overline{\psi_L} H \mu_R + \\
& + \delta_{x,0} \Lambda \overline{N_R^c} S_R + \delta_{|x|,1} \alpha_N \phi^{(*)} \overline{N_R^c} S_R + \delta_{y,0} M_\psi \overline{\psi_L} \psi_R + \delta_{|y|,1} \alpha_\psi \phi^{(*)} \overline{\psi_L} \psi_R + \\
& + Y_V \overline{S_R^c} \widetilde{H}^\dagger \psi_R + Y_{V'} \overline{\psi_L} \widetilde{H} N_R + \epsilon Y_S \overline{\ell_L} \widetilde{H} S_R + \text{h.c}
\end{aligned}
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Linear low scale seesaw

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Linear low scale seesaw

Yukawa terms

UV model

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Linear low scale seesaw

Yukawa terms

Terms with ϕ

UV model

Only 2nd generation of leptons

$$
\begin{aligned}
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Linear low scale seesaw

Yukawa terms

Terms with ϕ

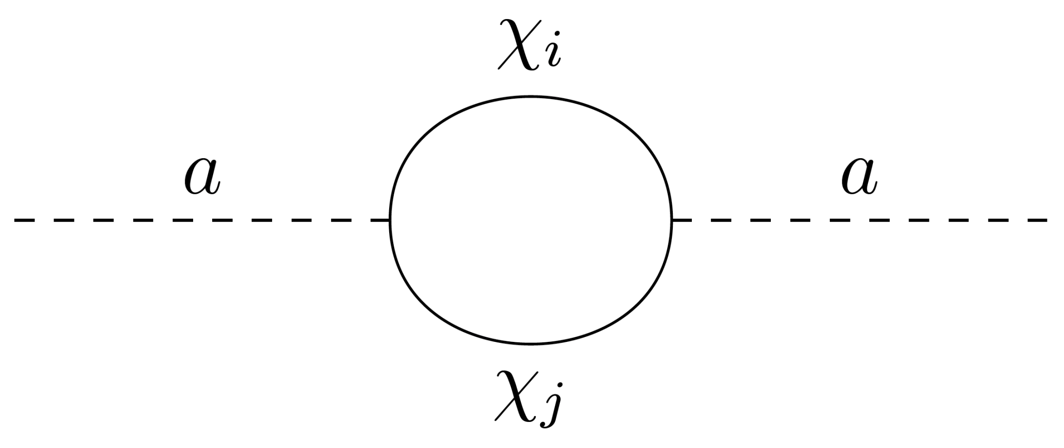
ALP phenomenology

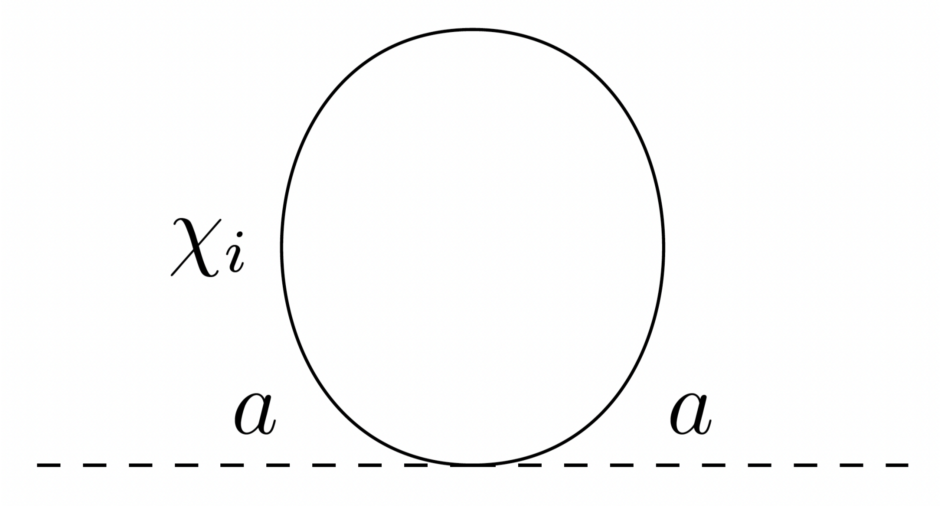
Specific model
"Model A"

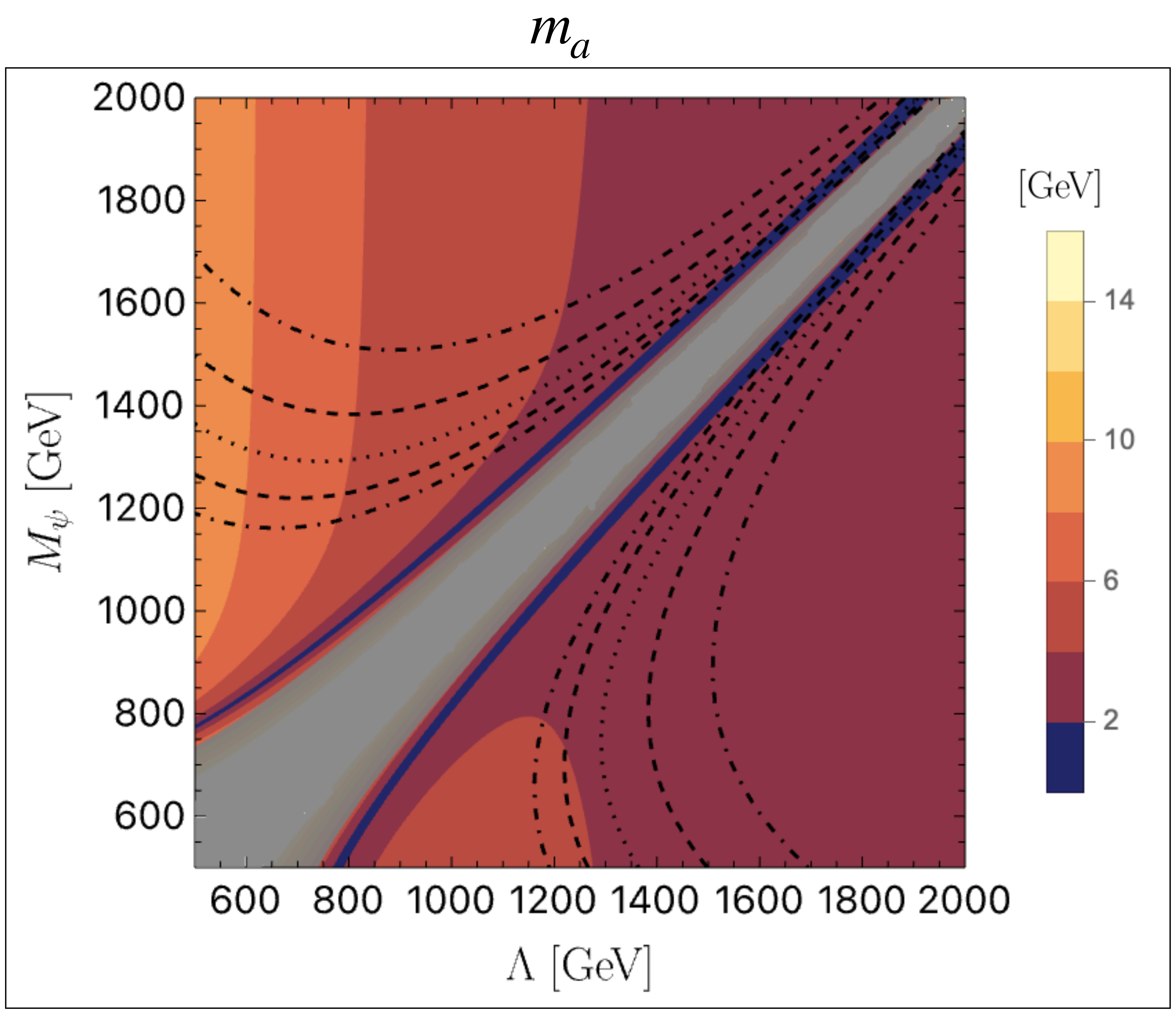
$$\phi^{(*)}\overline{N_R^c}S_R + M_\psi\overline{\psi_L}\psi_R$$

ALP mass

$$m_a^2 \propto Y_V Y_{V'} \Lambda M_\psi$$







$Y_V = 0.1$

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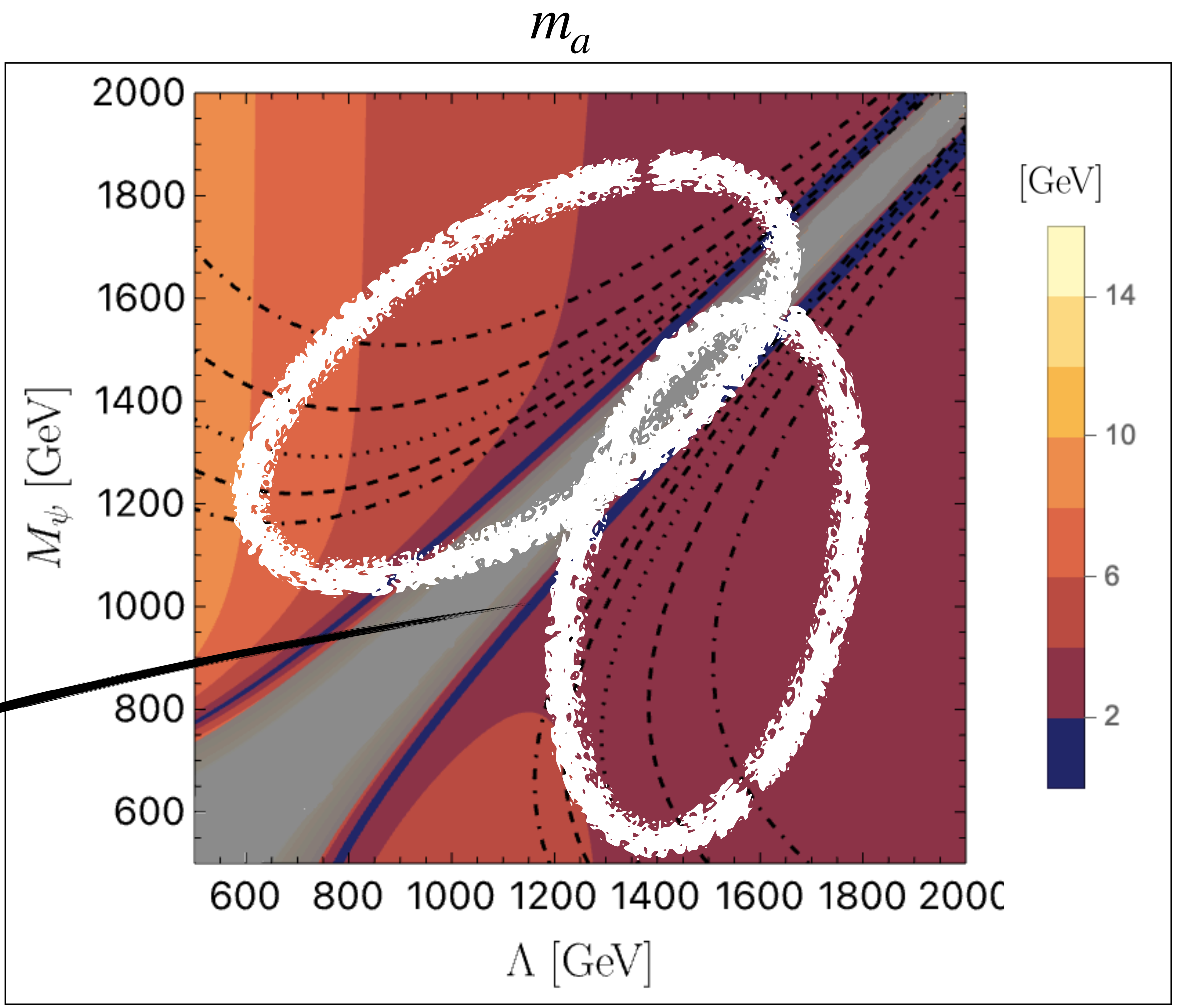
Solution to $(g-2)_\mu$!



$Y_V = 0.1$

Coupling to SM particles: μ coupling

W, Z, γ, charged particles →Calculated at 1-loop

Coupling to μ

$$\mathscr{L}_{\partial a} \supset \frac{\partial_\mu a}{f_a}\left(c_{\ell_L}\overline{\ell_L}\gamma^\mu \ell_L + c_{\mu_R}\overline{\mu_R}\gamma^\mu \mu_R\right)$$

Chirality-preserving basis

Coupling to SM particles: μ coupling

W, Z, γ, charged particles →Calculated at 1-loop

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$$\mu_R \to e^{i\,c_{\mu_R}\,a/f_a}\mu_R$$
$$\ell_L \to e^{i\,c_{\ell_L}\,a/f_a}\ell_L$$

ALP-dependent rotation

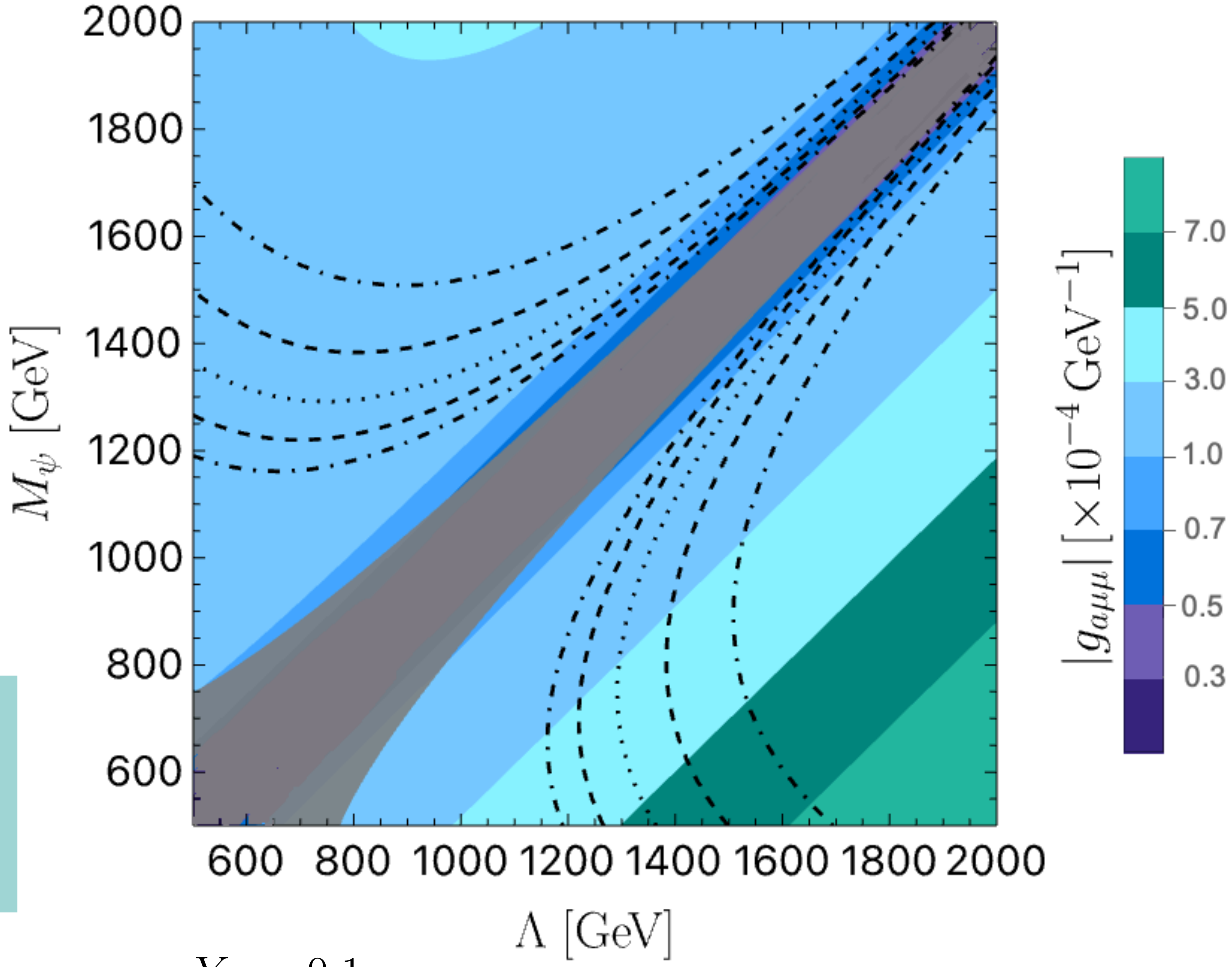
$$\mathscr{L}_a \supset \widehat{m}_\mu\, e^{i(c_{\mu_R}-c_{\ell_L})\,a/f_a}\overline{\mu_L}\mu_R + \mathrm{h.c.}$$ Chirality-flipping basis

Coupling to SM particles: μ coupling

Coupling to μ

$$g_{a\mu\mu} = \frac{(\bar{\delta}_{x,1} + \bar{\delta}_{y,1})}{f_a} \times \left(\frac{Y_V}{Y_V + \left(\frac{M_\psi}{\Lambda}\right) Y_{V'}} \right)$$

Spans over several orders of magnitude



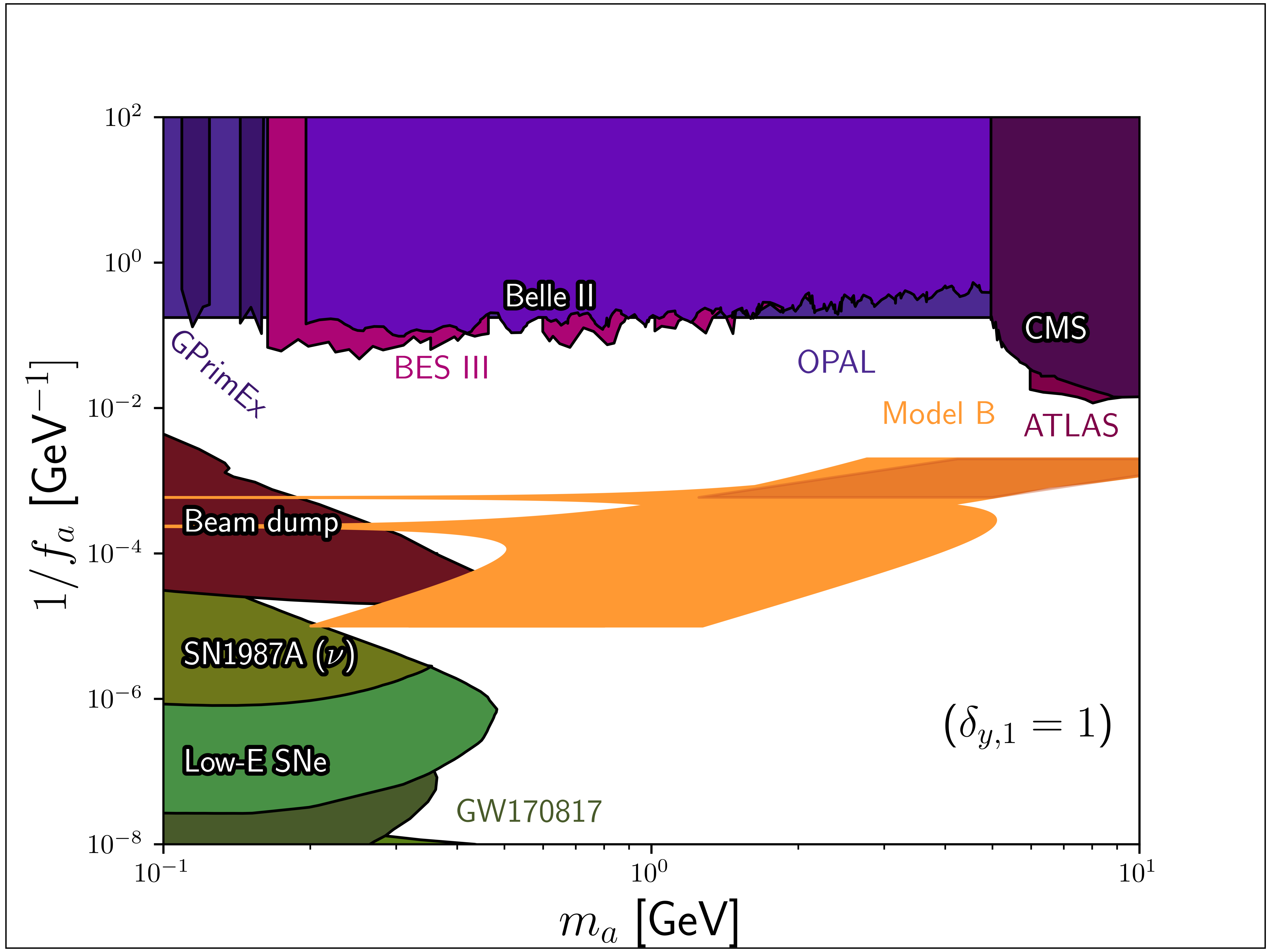
$Y_V = 0.1$

Coupling to SM particles: photon

Coupling to μ

$$g_{a\gamma\gamma} = \bar{\delta}_{y,1} \frac{\alpha_{em}}{\pi f_a}$$

Not excluded $\Rightarrow$ Testable!



Adapted from **Ciaran O'Hare,** https://cajohare.github.io/AxionLimits/

Conclusions

UV completion with **exotic lepton sector**

- Realistic **mass** for active **neutrinos**
- Viable solution to $(g-2)_\mu$
- **Coupling to** μ over several **orders of magnitude**
- TeV-scale HNLs and GeV-mass ALP with scale $\mathcal{O}(TeV)$

TESTABLE AT COLLIDERS

Thank you for your attention

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European Union
European Social Fund
AGENCIA ESTATAL DE INVESTIGACIÓN
EXCELENCIA SEVERO OCHOA
COSMIC WISPERS
CA21106

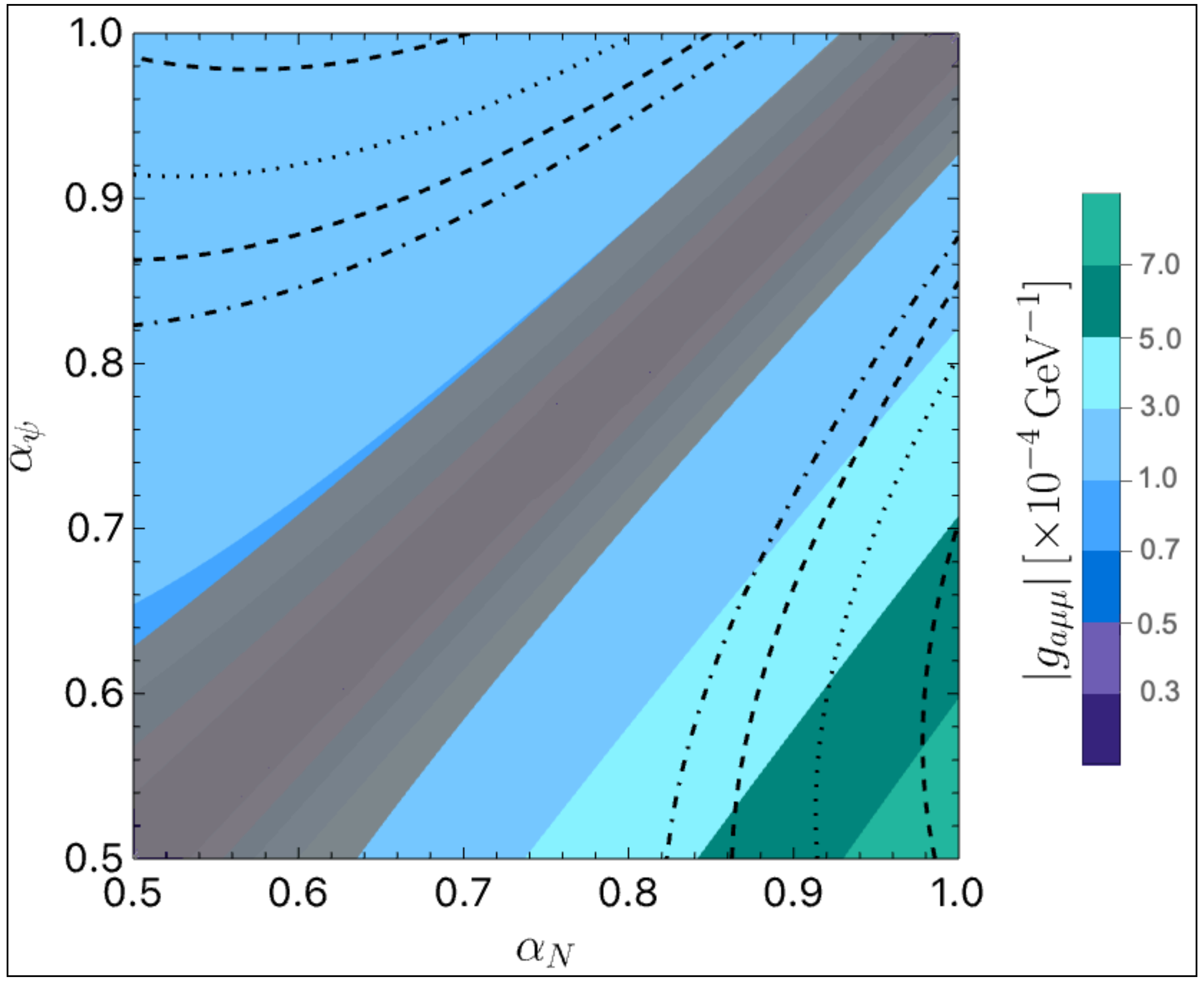
Couplings to SM

$$g_{aWW} = \bar{\delta}_{y,1} \frac{\alpha_{\text{em}}}{2\pi f_a s^2_{\theta_W}}$$

$$g_{aZZ} = \bar{\delta}_{y,1} \frac{\alpha_{\text{em}}}{6\pi f_a s^2_{2\theta_W}} \left(c_{4\theta_W} + 7\right)$$

$$f_a \sim \mathcal{O}(1)\ \text{GeV}$$

$$g_{a\mu\mu} = \frac{(\bar{\delta}_{x,1} + \bar{\delta}_{y,1})}{f_a} \times \left(\frac{Y_V}{Y_V + \left(\frac{M_\psi}{\Lambda}\right) Y_{V'}} \right)$$

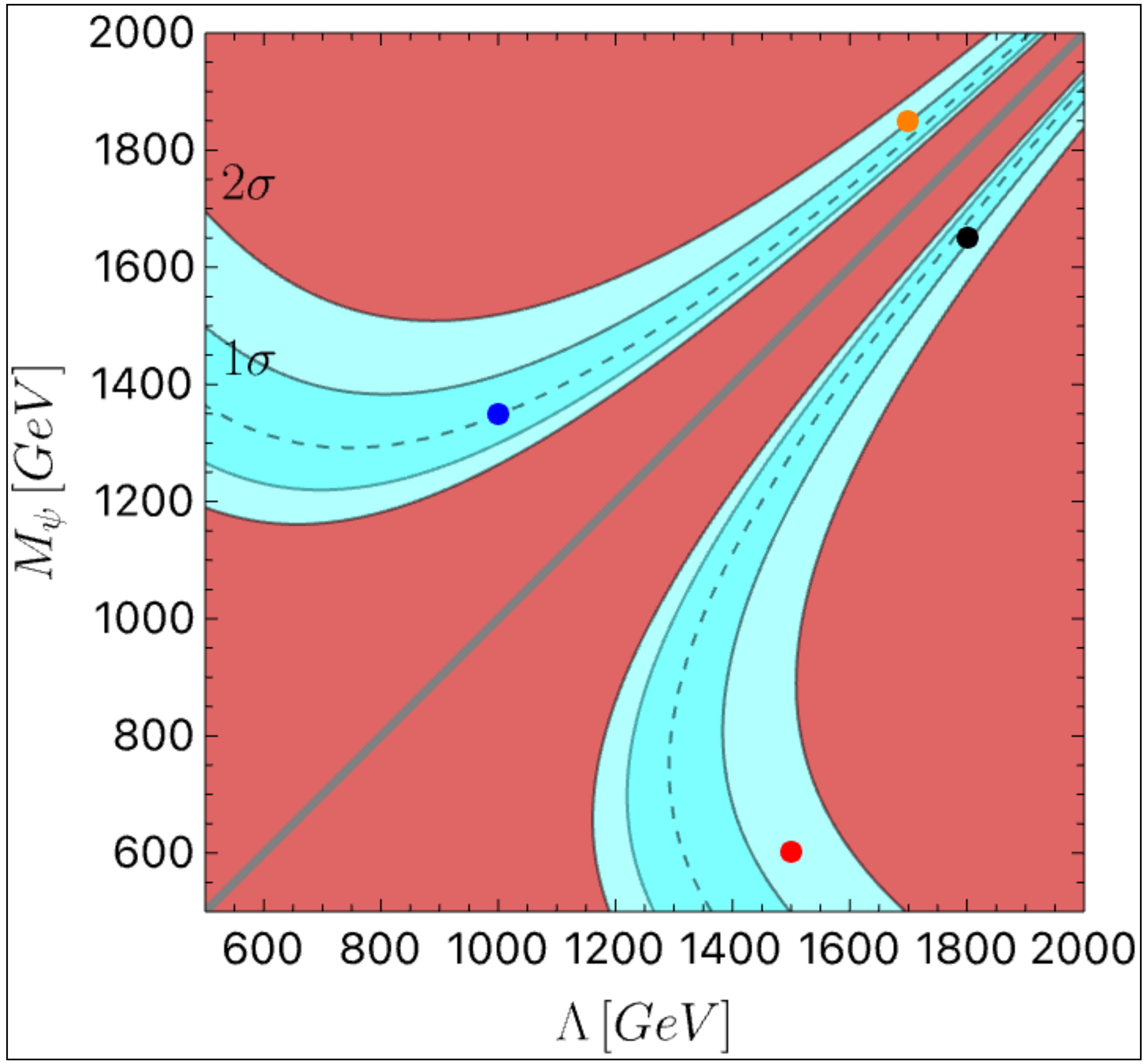


$$Y_V = 0.4$$

$(g-2)_\mu$ and μ mass

$$\delta a_\mu = \frac{3\, m_\mu^{\exp}}{4\,\pi^2\, v^2} \frac{M_W^2}{\Lambda M_\psi} \frac{m_N m_R}{M_\psi} \left(\frac{m_V}{M_\psi} + \frac{m_{V'}}{\Lambda} \right) F_0 \left(\frac{\Lambda^2}{M_W^2}, \frac{M_\psi^2}{M_W^2} \right)$$

$$F_0(x,y) \equiv \frac{3}{2} - \frac{x \log y - y \log x}{x - y}$$

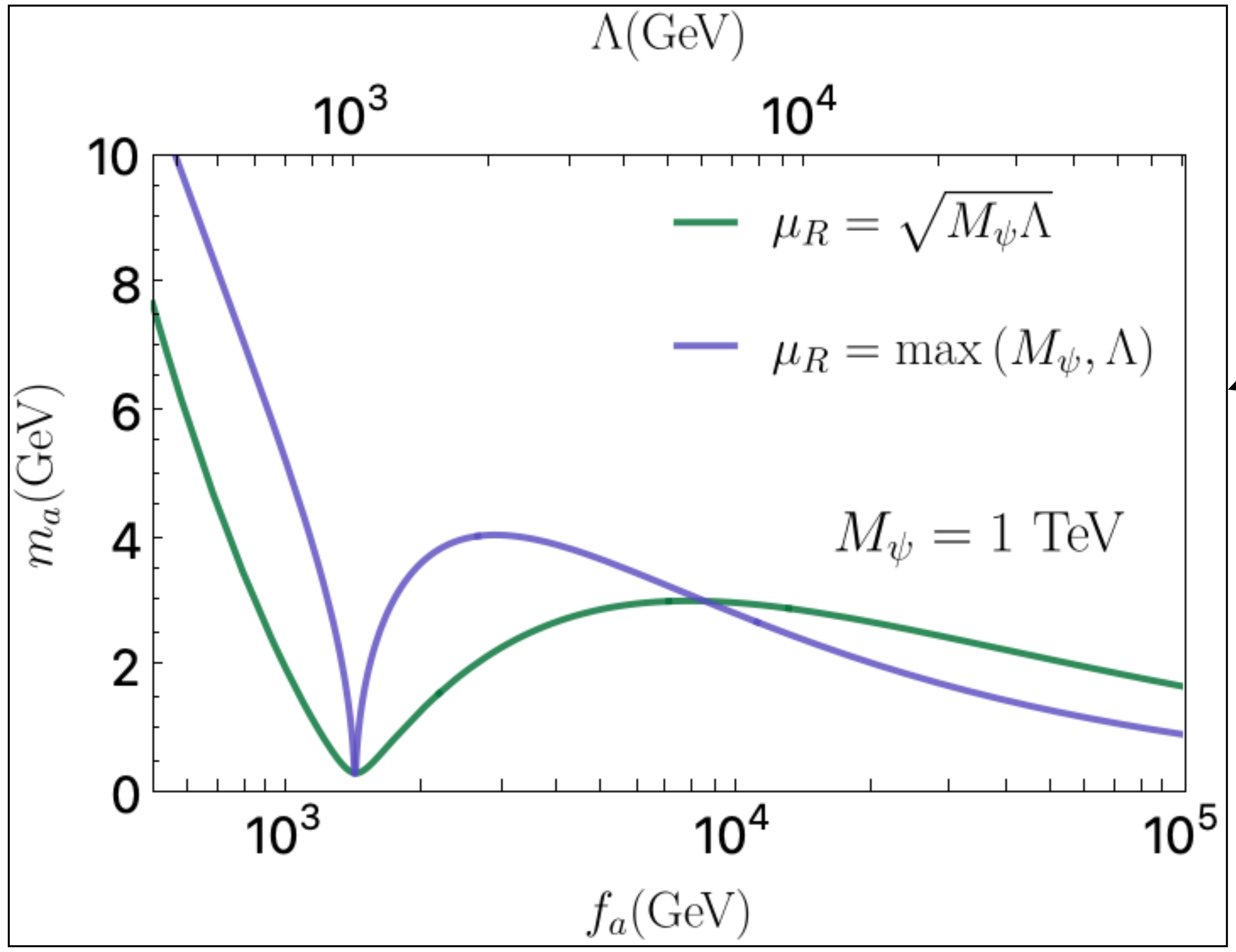


ALP mass

$$V_{\rm CW} = -\frac{1}{32\pi^2}\left\{ {\rm Tr}\left[(\mathcal{M}_\chi\mathcal{M}_\chi^\dagger)^2 \log\left(\frac{\mathcal{M}_\chi\mathcal{M}_\chi^\dagger}{\mu_R^2}\right)\right] - \frac{3}{2}{\rm Tr}\left[(\mathcal{M}_\chi\mathcal{M}_\chi^\dagger)^2\right]\right\}$$

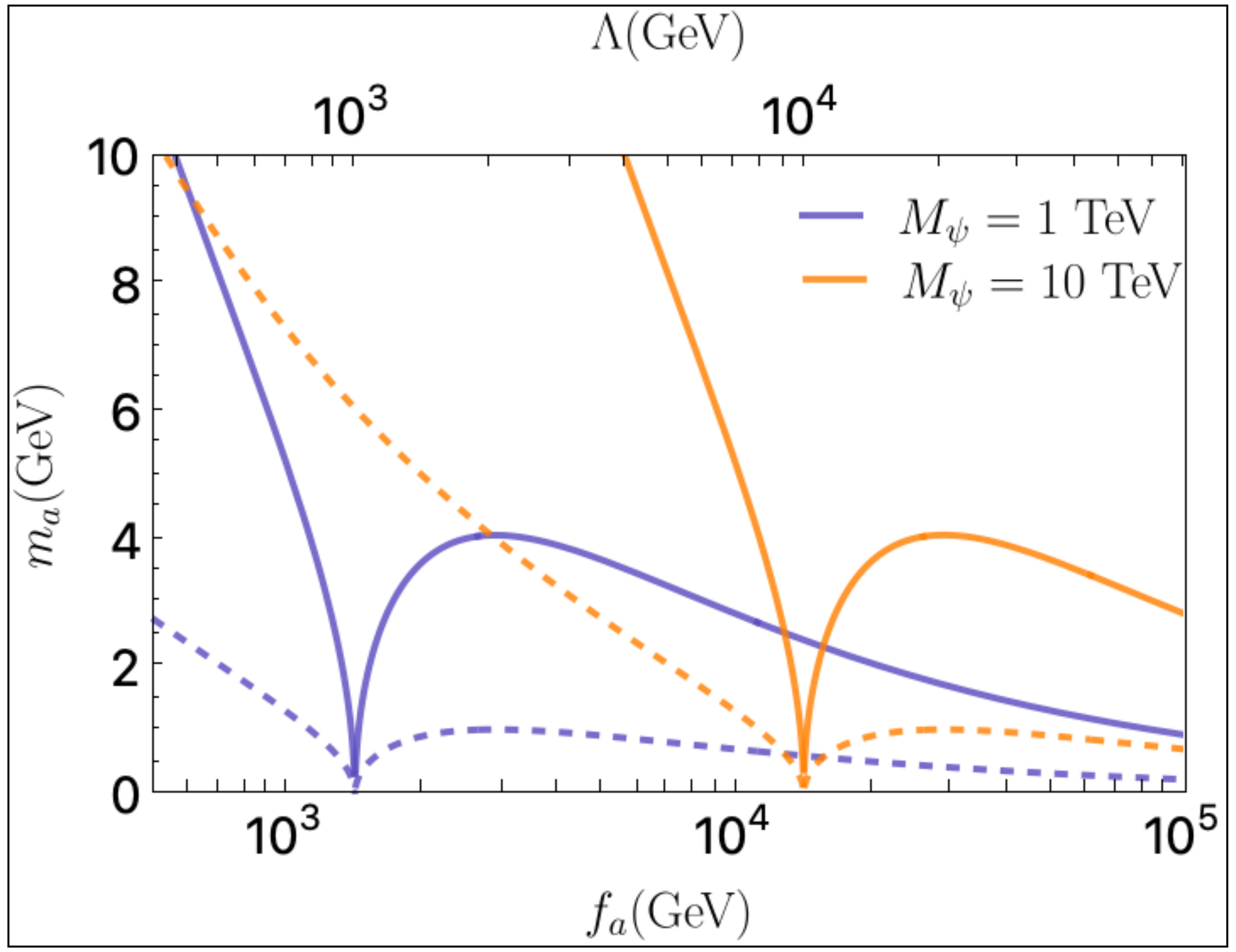
Coleman-Weinberg potential, from [A. de Giorgi, L. Merlo, X. Ponce Díaz, S. Rigolin, 2312.13417]

$$f_a^2 m_a^2 = \frac{(\bar\delta_{x,1}+\bar\delta_{y,1})^2}{4\pi^2}\left(\frac{m_V m_{V'}\Lambda M_\psi}{M_\psi^2-\Lambda^2}\right)\left[\frac{(M_\psi^2+\Lambda^2)}{2}\log\left(\frac{M_\psi^2}{\Lambda^2}\right) + (M_\psi^2-\Lambda^2)\left(\log\left(\frac{M_\psi\Lambda}{\mu_R^2}\right)-1\right)\right]$$



Renormalization scale dependence

Scale dependence



THE “DEFORMED”-TYPE II SEESAW MECHANISM

Wrishik Naskar

(based on U. Banerjee, C. Englert, **WN** 2024)

16th July 2024 — **EFT 2024, University of Zürich**

University of Glasgow

INTRODUCTION

TYPE-II SEESAW MECHANISM

Extends the SM scalar sector by a complex $SU(2)_L$ triplet (Δ) with $Y_\Delta = 1$.

(Chakrabortty et al. 2016; Primulando et al. 2019; Fuks et al. 2020; Antusch et al. 2019; Águila et al. 2014)

$$\mathcal{L}^{\text{type-II}} = \mathcal{L}_{\text{SM}} + \text{Tr}[D_\mu \Delta^\dagger D^\mu \Delta] - V(\Phi, \Delta) + \mathcal{L}^{\text{BSM}}_{\text{Yukawa}}$$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi^+ \\ (\phi + v_\Phi + i\eta) \end{pmatrix}, \quad \Delta = \frac{1}{\sqrt{2}} \begin{pmatrix} \delta^+ & \sqrt{2}\Delta^{++} \\ (\delta^0 + v_\Delta + i\chi) & -\delta^+ \end{pmatrix}$$

Physical scalars: $h, \Delta^0, A, \Delta^\pm, \Delta^{\pm\pm}$

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$$\mathcal{L}_{\text{Yukawa}}^{\text{BSM}} \supset -(Y_\Delta)_{ij} \bar{\psi}_{L_i}^c \Delta \psi_{L_j} + \text{h.c.}$$

Quintessential in generating non-zero neutrino masses!

TYPE-II SEESAW MECHANISM

$$\mathcal{L}^{\mathrm{BSM}}_{\mathrm{Yukawa}} \supset -(Y_\Delta)_{ij}\bar{\psi}^c_{L_i}\Delta\psi_{L_j} + \mathrm{h.c.} \supset \frac{v_\Delta}{\sqrt{2}}\left[(Y_\Delta + Y_\Delta^T)_{ij}\,\bar{\nu}^c_i \nu_j\right]$$

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(see Backup for details!) (NuFIT 2018)

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(see Backup for details!) (NuFIT 2018)

Y_Δ has 2 free-parameters: m_{ν_1}, v_Δ.

CONSTRAINTS

Production of $\Delta^{\pm\pm}$: smoking gun signal for the Type-II Seesaw Mechanism at colliders.

(CMS 2017; ATLAS 2018; ATLAS 2019; ATLAS 2023)

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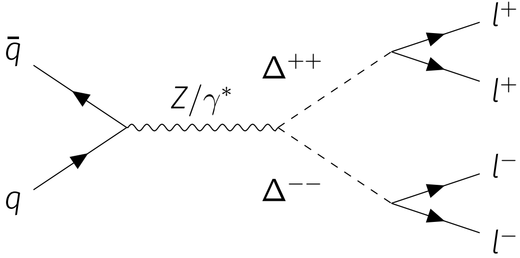
Strongest constraints: $pp \to \Delta^{++}\Delta^{--} \to l^+l^+l^-l^-$ $(l = e, \mu, \tau)$

(ATLAS 2018; ATLAS 2023)

100% branching for $v_\Delta \sim 1$ eV, $m_{\nu_1} \sim 0.05$ eV



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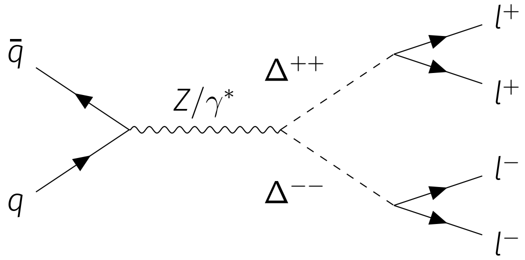
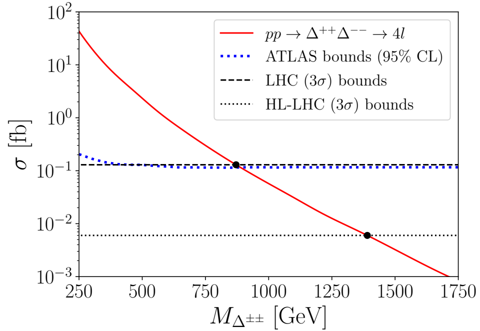


Cuts and methodology: (Anisha, Banerjee, et al. 2022)

Mass exclusions:

LHC: $\gtrsim$ 870 GeV

HL-LHC: $\gtrsim$ 1400 GeV



(ATLAS 2023)

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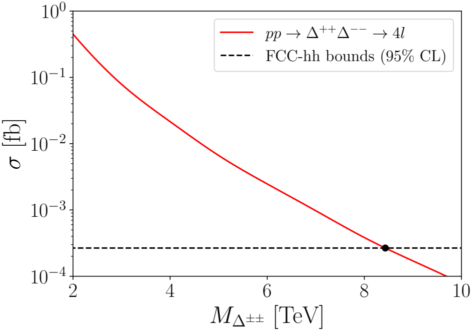
FCC-*hh*: $\gtrsim$ 8.5 TeV



THEORETICAL CONSTRAINTS

- Vacuum stability
- Perturbative Unitarity
- Higgs data
- Electroweak Precision

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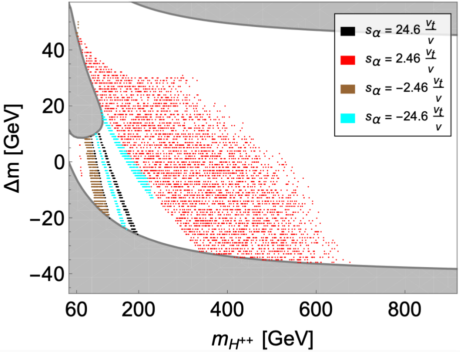
(Primulando et al. 2019)



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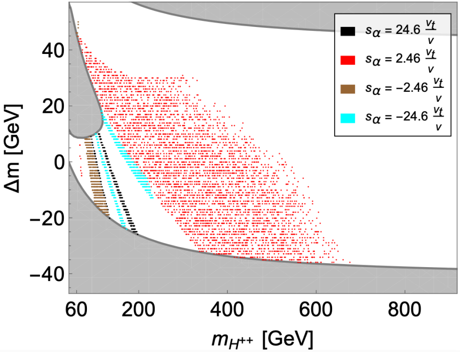


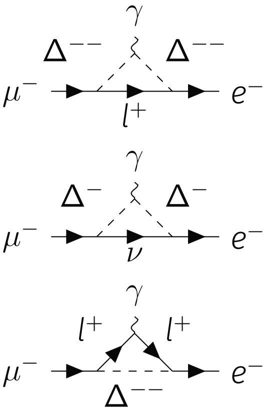
All constraints are within LHC-sensitivity!

EWP Constraints after FCC-*ee*: $M_{\Delta^{\pm\pm}} \gtrsim 105$ GeV

(Fan et al. 2015)

$$\mathrm{BR}(\mu \to e\gamma) = \frac{\alpha_{\mathrm{EM}} |Y_\Delta^\dagger Y_\Delta|^2_{\mu e}}{192\pi G_F^2} \left(\frac{1}{M^2_{\Delta^\pm}} + \frac{8}{M^2_{\Delta^{\pm\pm}}} \right)^2 \leq 3.1 \times 10^{-13}$$

(MEG 2016; MEG 2024)



LEPTON-FLAVOUR-VIOLATING (LFV) DECAYS

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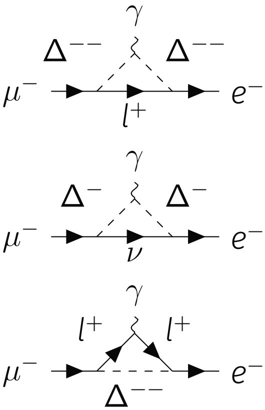
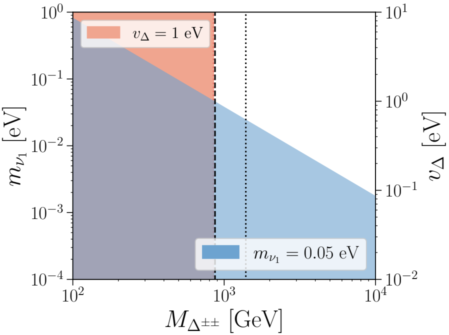
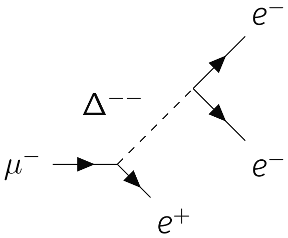




LEPTON-FLAVOUR-VIOLATING (LFV) DECAYS

$$\mathrm{BR}(\mu \to 3e) = \frac{|(Y_\Delta)_{ee}(Y_\Delta)^*_{\mu e}|^2}{4G_F^2 M^4_{\Delta^{\pm\pm}}} \leq 10^{-12}$$

(SINDRUM 1988)



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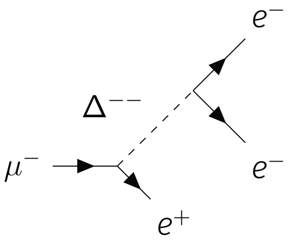

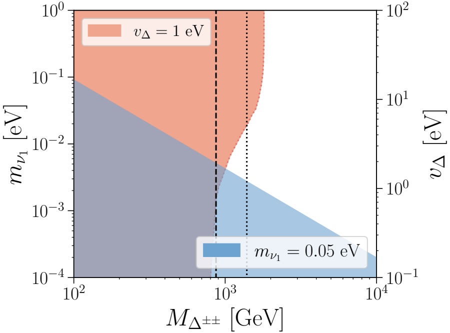

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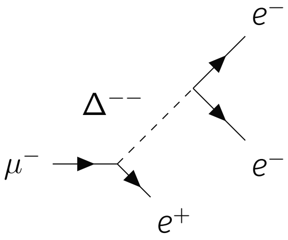

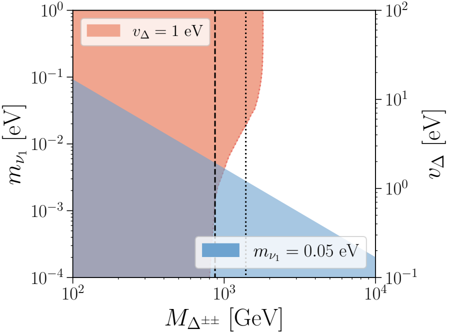

Much of the parameter space sensitive to the HL-LHC is already excluded!

THE “DEFORMED” MODEL

$$\mathcal{L}^{\text{BSM-EFT}} = \mathcal{L}^{\text{BSM}} + \frac{1}{\Lambda^2} \sum C_i \mathcal{O}_i^{(6)}$$

- **Motivation:** The lightest non-SM particle lies close to the EW scale.
- **Complex singlets:** (Cho et al. 2023; Oikonomou et al. 2024)
- **2HDM:** (Anisha, Biermann, et al. 2022; Anisha, Azevedo, et al. 2024; Ouazghour et al. 2023)
- **Triplet Extensions:** (Padhan et al. 2022; Das et al. 2023)
- **BSM-EFT basis:** (Banerjee et al. 2021)

IMPLICATIONS OF EFT-DEFORMATIONS ($\mu \to 3e$)

$$\mathcal{L}^{\text{BSM-EFT}} = \mathcal{L}^{\text{Type-II}} + \frac{1}{\Lambda^2} \sum C_i \mathcal{O}_i^{(6)}$$

$\mathcal{O}^{(1)}_{L\Phi\Delta,ij}$	$(\bar{\psi}^c_{L_i} \Delta \psi_{L_j})(\Phi^\dagger \Phi)$
$\mathcal{O}^{(2)}_{L\Phi\Delta,ij}$	$\bar{\psi}^c_{L_i,\alpha} \Delta \Phi^\alpha \Phi^\dagger_\beta \psi^\beta_{L_j}$
$\mathcal{O}^{(1)}_{L\Delta,ij}$	$(\bar{\psi}^c_{L_i} \Delta \psi_{L_j}) \text{Tr}[(\Delta^\dagger \Delta)]$
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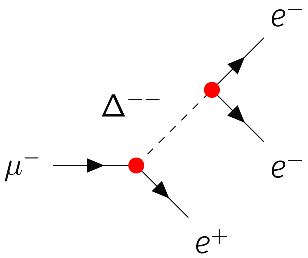

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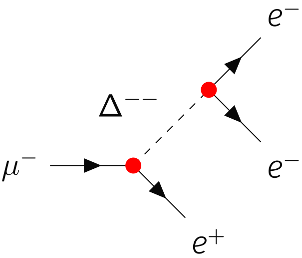
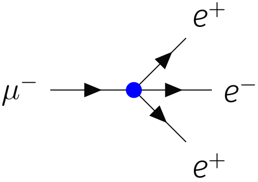
$\mathcal{O}^{ijkm}_{ll}$	$(\bar{\psi}_{L_i}\gamma_\mu\psi_{L_j})(\bar{\psi}_{L_k}\gamma^\mu\psi_{L_m})$
$\mathcal{O}^{ijkm}_{ee}$	$(\bar{e}_i\gamma_\mu e_j)(\bar{e}_k\gamma^\mu e_m)$
$\mathcal{O}^{ijkm}_{le}$	$(\bar{\psi}_{L_i}\gamma_\mu\psi_{L_j})(\bar{e}_k\gamma^\mu e_m)$





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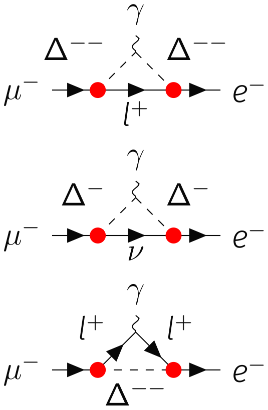
$$(Y^{\text{mod.}}_\Delta)_{ij} = (Y_\Delta)_{ij} - C^{\text{BSM}}_{ij} \frac{v^2}{2\Lambda^2}$$

$$\text{BR} \supset |(Y^{\text{mod.}}_\Delta)_{ee} (Y^{\text{mod.}}_\Delta)^*_{\mu e}|^2$$

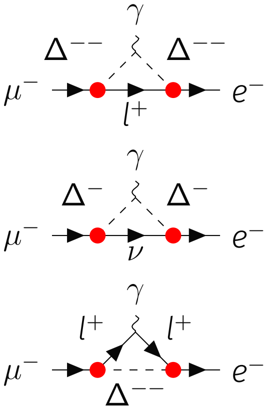
$$\text{BR} \supset \frac{(C^{\text{SMEFT}}_{ll,le,ee})^2}{\Lambda^4}$$

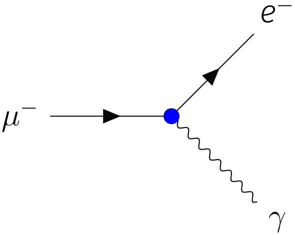
$\mathcal{O}^{(2)}_{L\Phi\Delta,ij}$	$\bar{\psi}^c_{L_i,\alpha}\Delta\Phi^\alpha\Phi^\dagger_\beta\psi^\beta_{L_j}$



$\mathcal{O}^{(2)}_{L\Phi\Delta,ij}$	$\bar{\psi}^{c}_{L_i,\alpha}\Delta\Phi^{\alpha}\Phi^{\dagger}_{\beta}\psi^{\beta}_{L_j}$

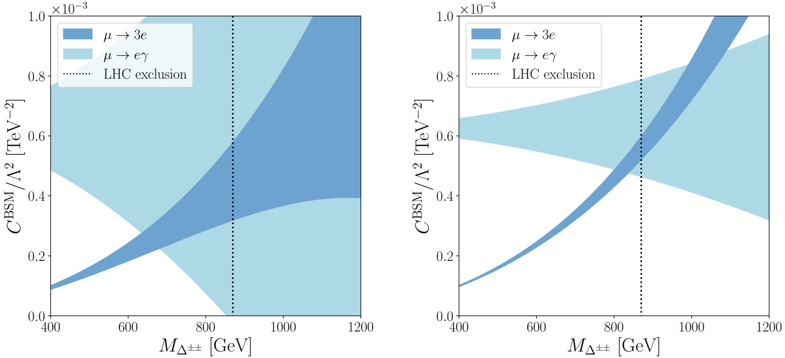
$\mathcal{O}_{eW}$	$(\bar{\psi}_{L_i}\sigma^{\mu\nu}e_j)\tau^{\alpha}\Phi W^{\alpha}_{\mu\nu}$
$\mathcal{O}_{eB}$	$(\bar{\psi}_{L_i}\sigma^{\mu\nu}e_j)\Phi B_{\mu\nu}$





IMPLICATIONS OF EFT-DEFORMATIONS

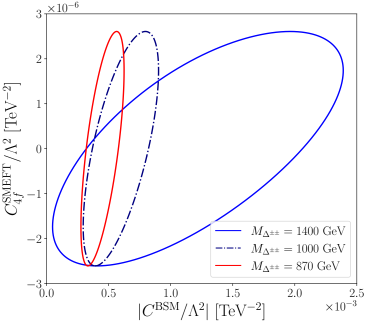
$C^{\mathrm{SMEFT}} = 0$ $\qquad\qquad$ $C^{\mathrm{SMEFT}} \neq 0$



$$C^{(2)}_{L\Phi\Delta,ee} = C^{(2)}_{L\Phi\Delta,\mu e} = C^{\mathrm{BSM}}$$

We can probe masses sensitive to the LHC through $\mu \to 3e/e\gamma$.

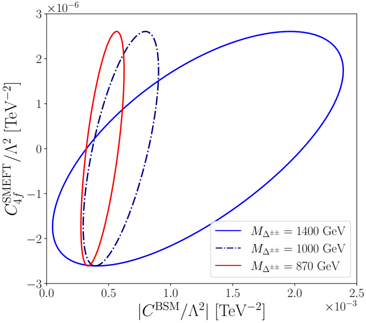
$$\mu \to 3e$$



$$C_{ll} = C_{le} = C_{ee} = C_{4f}^{\mathrm{SMEFT}}$$

IMPLICATIONS OF EFT-DEFORMATIONS

$\mu \to 3e$



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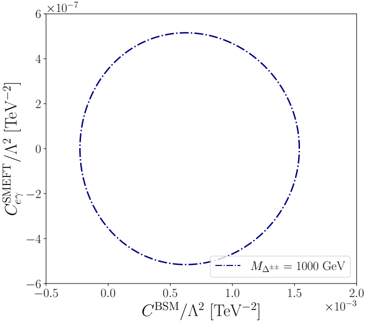
$\mu \to e\gamma$



$$C_{eB} = C_{eW} = C_{e\gamma}^{\mathrm{SMEFT}}$$

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Thank You!

BACKUP SLIDES

NUFIT CONSTRAINTS (NORMAL ORDERING)

Parameter	Best-fit
Δm^2_{21} [10^{-5} eV^2]	$7.55^{+0.20}_{-0.16}$
Δm^2_{31} [10^{-3} eV^2]	2.50 ± 0.03
$\sin\theta_{12}/0.1$	$3.20^{+0.20}_{-0.16}$
$\theta_{12}/^\circ$	$34.5^{+1.2}_{-1.0}$
$\sin\theta_{23}/0.1$	$5.47^{+0.20}_{-0.30}$
$\theta_{23}/^\circ$	$47.7^{+1.2}_{-1.7}$
$\sin\theta_{13}/0.1$	$2.160^{+0.083}_{-0.069}$
$\theta_{13}/^\circ$	$8.45^{+0.16}_{-0.14}$
δ/π	$1.21^{+0.21}_{-0.15}$
$\delta/^\circ$	218^{+38}_{-27}

Best-fit constraints from the global fit of neutrino oscillation data. (NuFIT 2018)

YUKAWA MATRIX

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$\boxed{c_{ij} = \cos\theta_{ij},\ s_{ij} = \sin\theta_{ij}}$$

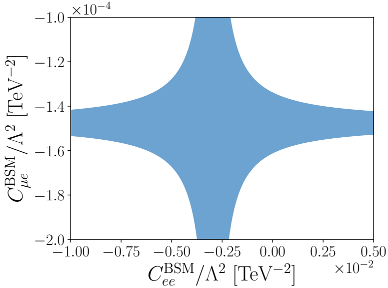
Plugging in the NuFIT constraints:

$$Y_\Delta = \begin{pmatrix} 0.0358 & 0.0018 & 0.0012 \\ 0.0018 & 0.0438 & 0.0069 \\ 0.0012 & 0.0069 & 0.0416 \end{pmatrix}$$

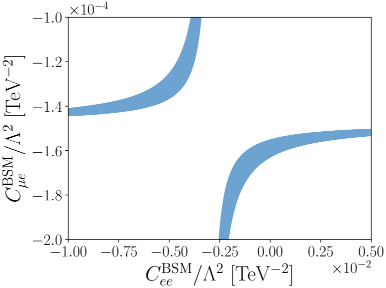
$$\boxed{v_\Delta = 1\text{ eV},\ m_{\nu_1} = 0.05\text{ eV}}$$

BSM-EFT CONSTRAINTS FROM $\mu \to 3e$

$C_{\text{SMEFT}} = 0$



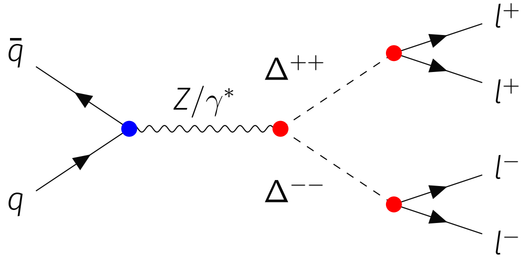
$C_{\text{SMEFT}} \neq 0$



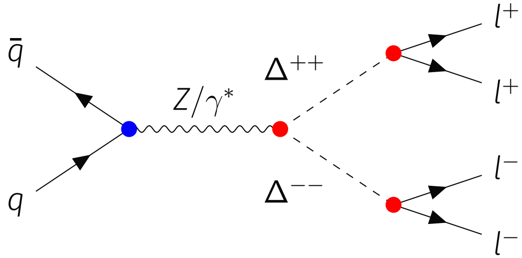
$$C_{L\Phi\Delta,ij} = C^{\text{BSM}}_{ij}$$

Since $(Y_\Delta)_{ee} >> (Y_\Delta)_{\mu e}$, we need bigger cancellations on the diagonal Yukawas compared to the off-diagonal ones.

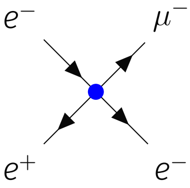
Hadron Colliders:



Hadron Colliders:



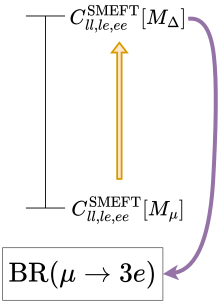
e^+e^--Colliders:

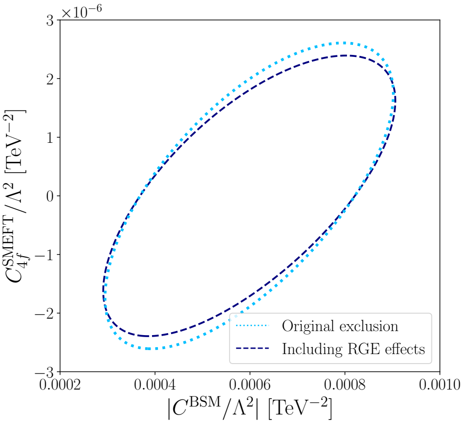


- **FCC**-*ee* (Z-pole, 192 ab^{-1}): $|C_{4f}^{\text{SMEFT}}| \lesssim 10^{-4}$ TeV^{-2}.
- **CLIC** (3 TeV, 5 ab^{-1}): $|C_{4f}^{\text{SMEFT}}| \lesssim 10^{-5}$ TeV^{-2}.

RGE effects computed for $\mu \to 3e$ using DSixTools.

(Celis et al. 2017; Fuentes-Martin et al. 2021)





RGE effects are small, and don't affect our results considerably.

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Leptonic invariant mass spectrum of the $B \to X_c l \bar{\nu}_l$

Mateusz Czaja

Institute of Theoretical Physics
University of Warsaw

16.07.2024

- The Standard Model depends on $\mathcal{O}(20)$ parameters that have to be extracted from comparisons of theoretical predictions with experiment. For BSM searches, one of the most important is the $|V_{cb}|$ CKM matrix element.

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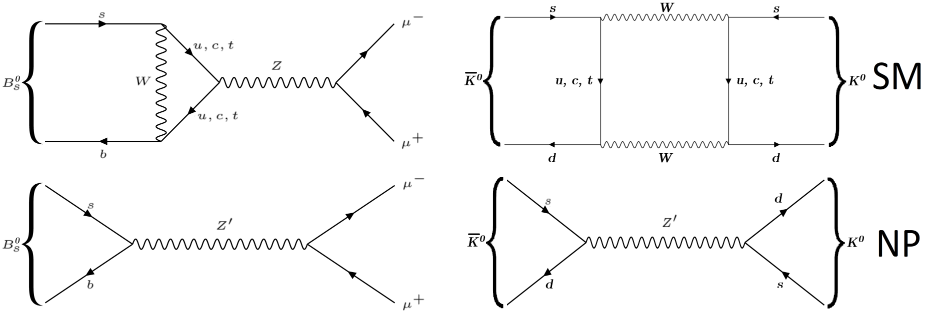
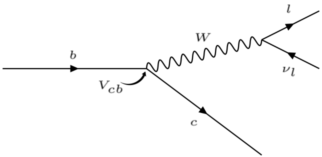
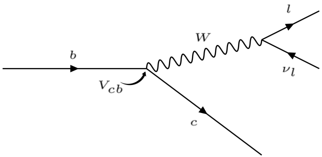


- $\delta\mathcal{B}(B_s^0 \to \mu^+\mu^-) = \sqrt{\underbrace{(2.3\%)^2}_{|V_{cb}|} + \underbrace{(2.2\%)^2}_{other}}$
  [arXiv: 2407.03810]

- Around 50% of the theoretical error of $|\epsilon_K|$ is due to $|V_{cb}|$
  [arXiv: 2401.08006]

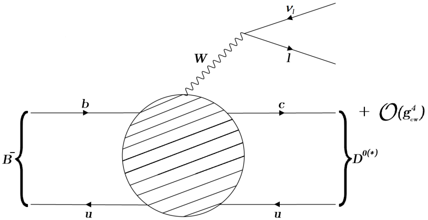


- The standard method of extracting the value of $|V_{cb}|$ is to study the semileptonic transition $b \to c l \bar{\nu}_l$, $l \in \{e, \mu\}$.

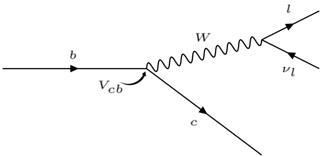


- The standard method of extracting the value of $|V_{cb}|$ is to study the semileptonic transition $b \to cl\bar{\nu}_l$, $l \in \{e, \mu\}$.

- On the hadronic level it can be realized in an exclusive or inclusive way:



- Exclusive $B^- \to D^{0(*)} l\bar{\nu}_l$ decay.
- $\mathcal{B}^{exp}(B^- \to D^* l\bar{\nu}_l) = (5.53 \pm 0.22)\%$



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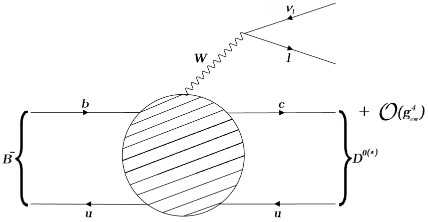
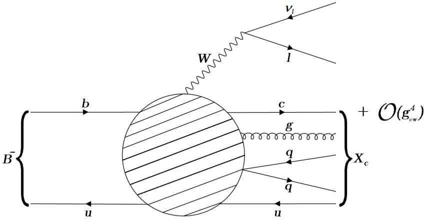
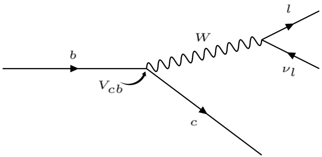
- On the hadronic level it can be realized in an exclusive or inclusive way:





- Exclusive $B^- \to D^{0(*)} l \bar{\nu}_l$ decay.
- $\mathcal{B}^{exp}(B^- \to D^* l \bar{\nu}_l) = (5.53 \pm 0.22)\%$

- Inclusive $B^- \to X_c l \bar{\nu}_l$ decay. All final states with $C = 1$ are summed over.
- $\mathcal{B}^{exp}(B^- \to X_c l \bar{\nu}_l) = (10.8 \pm 0.4)\%$



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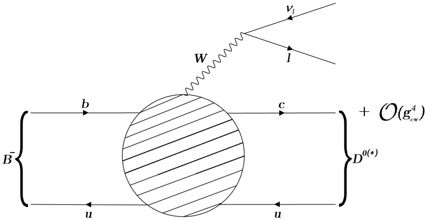
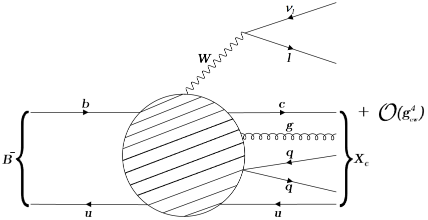
- On the hadronic level it can be realized in an exclusive or inclusive way:





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- $\mathcal{B}^{exp}(B^- \to X_c l \bar{\nu}_l) = (10.8 \pm 0.4)\%$

- The value of $|V_{cb}|$ can be extracted from fits of experimental data to spectra of the inclusive semileptonic decay width. In the leptonic invariant mass q^2 spectrum, some non-perturbative matrix elements drop out, which makes the fit more precise.

- At the leading order in EW interactions, the inclusive differential rate can be written as

$$\frac{d\Gamma}{dq^2 dr^2 dE_l} \propto G_F^2 |V_{cb}|^2 W^{\alpha\beta} L_{\alpha\beta},$$

where G_F is the Fermi constant, $q \equiv k_l + k_{\bar{\nu}}$, and $r \equiv p_B - q$.

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- The hadronic tensor $W^{\alpha\beta}$ is defined as

$$W^{\alpha\beta} \equiv \sum_{X_c} \langle B | J_H^\alpha | X_c \rangle \langle X_c | J_H^{\dagger\beta} | B \rangle , \qquad J_H^\alpha \equiv \bar{b}\gamma^\alpha P_L c,$$

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- The optical theorem and the Operator Product Expansion can be used to write $W^{\alpha\beta}$ as a series of matrix elements suppressed by powers of Λ_{QCD}/m_b:

$$W^{\alpha\beta} \propto \mathrm{Im} \sum_k \frac{C_k \langle B | O_k^{(n)\alpha\beta} | B \rangle}{m_b^{n(k)}}, \text{ where } \frac{\langle B | O_k^{(n)\alpha\beta} | B \rangle}{\langle B | B \rangle} \sim \Lambda_{QCD}^n.$$

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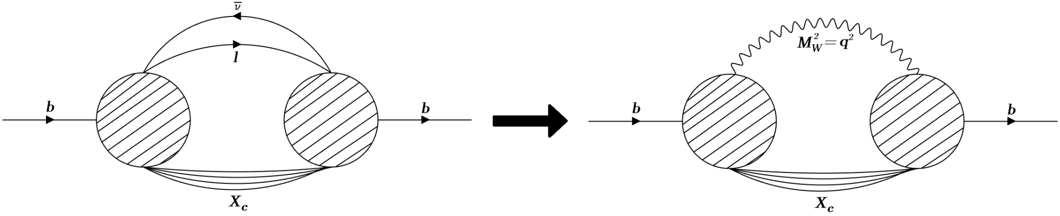
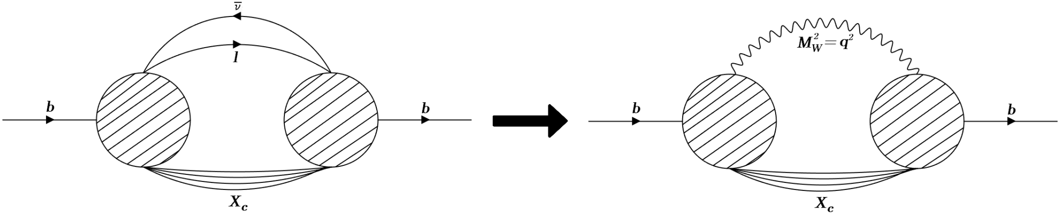
- At the LO and NLO:

$$\langle B | O_k^{(0)} | B \rangle = 2m_B \left(1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m_b^2} \right) \right), \; O_k^{(1)} = 0|_{EOM} + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m_b^2} \right)$$



$$L^{\alpha\beta} \equiv \frac{1}{2}\sum_{s_l s_{\bar{\nu}}} A_l^{\alpha} A_l^{\dagger\beta} = k_l^{\alpha} k_{\bar{\nu}}^{\beta} + k_l^{\beta} k_{\bar{\nu}}^{\alpha} - (k_l k_{\bar{\nu}}) g^{\alpha\beta} - i\epsilon^{\alpha\rho\beta\sigma} k_{l_\rho} k_{\bar{\nu}_\sigma}, \qquad A_l^{\alpha} \equiv \bar{u}_l^{(s_l)} \gamma^{\alpha} P_L v_{\bar{\nu}}^{(s_{\bar{\nu}})}.$$



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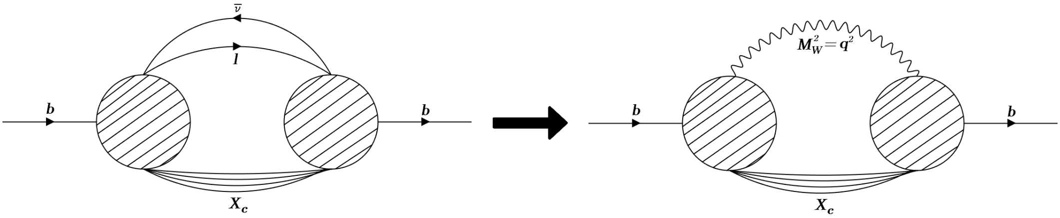
- Assuming massless leptons, one can integrate $L^{\alpha\beta}$ over E_l to obtain

$$\frac{d\Gamma}{dq^2 dr^2} \propto G_F^2 |V_{cb}|^2 W^{\alpha\beta} \int dE_l L_{\alpha\beta} \propto G_F^2 |V_{cb}|^2 \frac{|\vec{q}|}{3}(q_\alpha q_\beta - q^2 g_{\alpha\beta}) W^{\alpha\beta},$$



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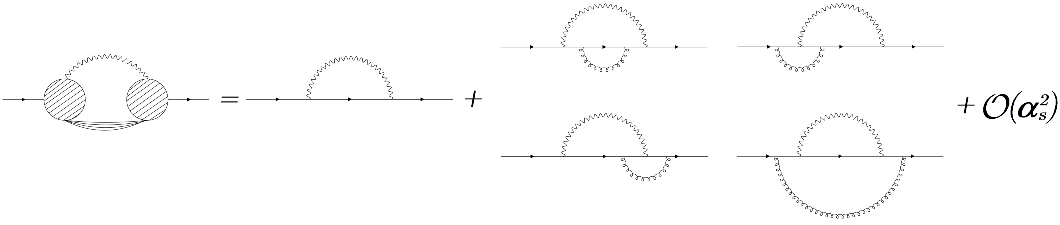
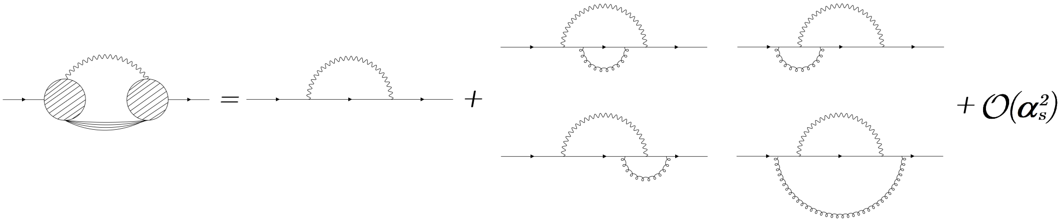
- The transverse structure can be reproduced by polarization vectors ε_μ of an auxiliary final state W-boson with $M_W^2 = q^2$:

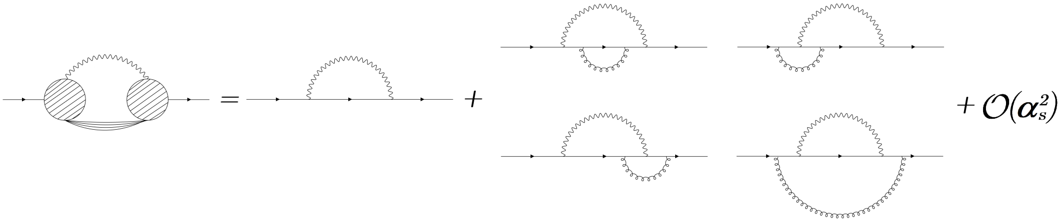
$$\sum_{polarizations} \varepsilon_\alpha \varepsilon_\beta = \frac{q_\alpha q_\beta - q^2 g_{\alpha\beta}}{q^2} \propto \int dE_l L_{\alpha\beta} \Longrightarrow \frac{d\Gamma}{dq^2} = \frac{1}{48\pi^2} \frac{q^2}{M_{W(SM)}^4} \Gamma_W.$$



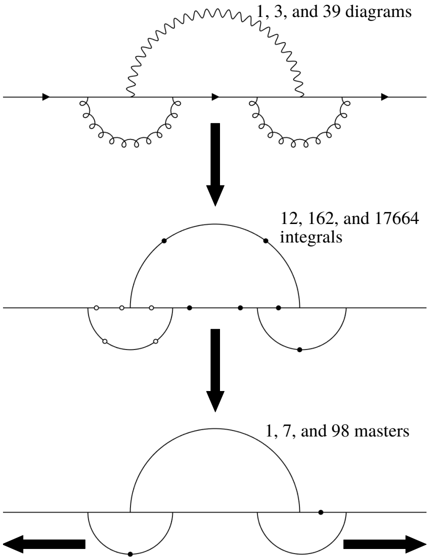
- The rate Γ_W is computed in perturbative QCD in powers of α_s.



- The rate Γ_W is computed in perturbative QCD in powers of α_s.
- The replacement $(B \to X_c l \bar{\nu}) \longrightarrow (B \to X_c W)$ allows one to retain the q^2 dependence of the process that would normally be integrated over when using the optical theorem.

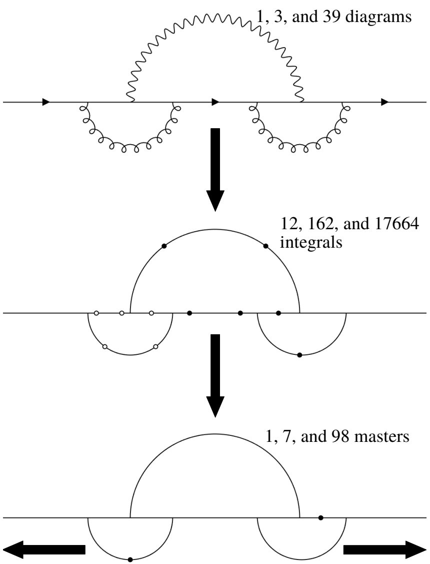


- The rate Γ_W is computed in perturbative QCD in powers of α_s.
- The replacement $(B \to X_c l \bar{\nu}) \longrightarrow (B \to X_c W)$ allows one to retain the q^2 dependence of the process that would normally be integrated over when using the optical theorem.
- Additionally, the lepton loop is integrated out for the price of an additional scale q^2.


1, 3, and 39 diagrams
12, 162, and 17664
integrals
1, 7, and 98 masters

Analytic solutions:
([arXiv:2403.03976])

- IBP relates derivatives of Masters back to the same Masters, resulting in first order partial DEs for Masters.
- The DEs for a large class of integrals can be solved using the canonical form.
- The boundary condition was found using AMFlow. [arXiv:2201.11669]
- Solution given in terms of Goncharov Polylogarithms.
- No analytic solution known for integrals with 3 cut charm quarks.



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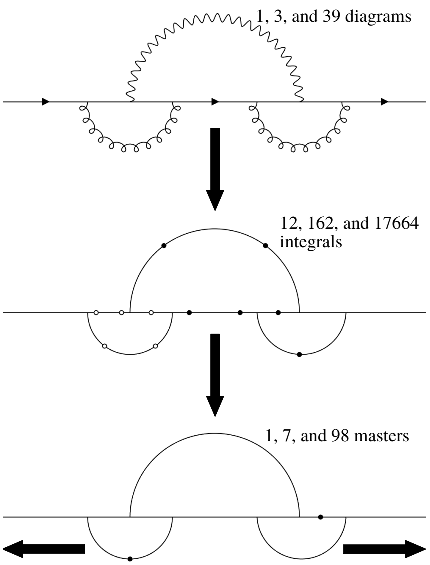
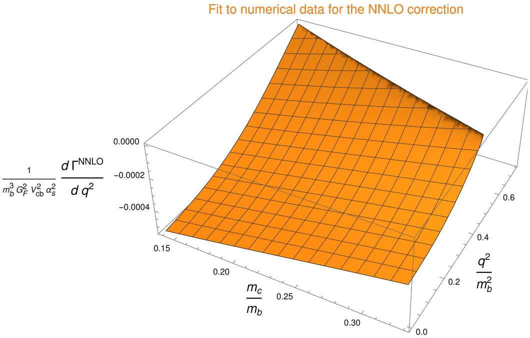
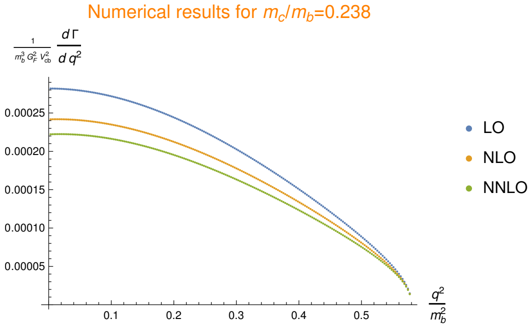


Fits to numerical solutions:
(in progress)

- Dense scans in the (m_c, q^2) space using AMFlow
- Requires a lot of resources. Computational cluster necessary.
- The result can be expressed using elementary functions.
- Accuracy of more than 4 significant digits when compared with exact results, far higher than experimental precision.
- Cuts through 3 charm quarks can be computed.


Fit to numerical data for the NNLO correction
$\frac{1}{m_b^3 G_F^2 V_{cb}^2 \alpha_s^2} \frac{d\Gamma^{NNLO}}{dq^2}$
0.0000
−0.0002
−0.0004
0.15
0.20
0.25
0.30
$\frac{m_c}{m_b}$
0.0
0.2
0.4
0.6
$\frac{q^2}{m_b^2}$


Numerical results for m_c/m_b=0.238
$\frac{1}{m_b^3 G_F^2 V_{cb}^2} \frac{d\Gamma}{dq^2}$
0.00025
0.00020
0.00015
0.00010
0.00005
0.1
0.2
0.3
0.4
0.5
$\frac{q^2}{m_b^2}$
LO
NLO
NNLO

Probing τ lepton dipole moments at future Lepton Colliders

ZeQiang Wang

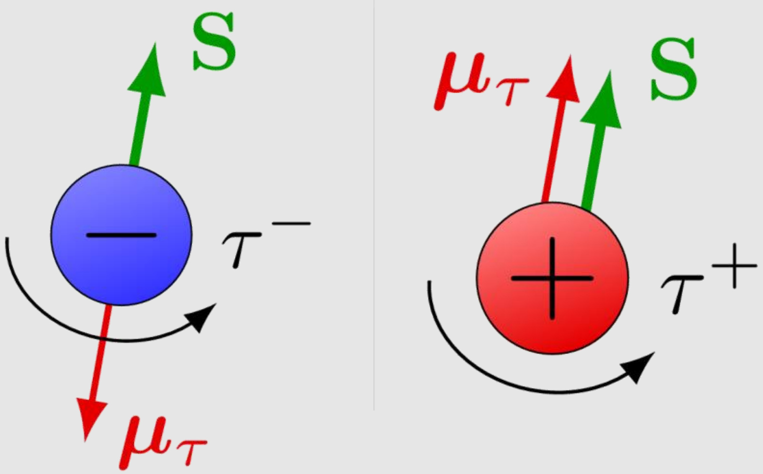
UCLouvain, CP3

With D. Buttazzo, G. Levati, F. Maltoni, P. Paradis

July 16th, 2024

Lepton magnetic moment

- **Introduction**



- **The magnetic moment of lepton related**

With Spin and factor g

$$\mu = g \frac{e}{2m} \mathbf{S}$$

Dirac equation predicts g=2

In QED, high order quantum effects modify
The value of g, an anomalous magnetic moment(tau):

$$a_l = (g-2)/2$$

The quantity Δa_l typically refers to the deviation of the measured anomalous magnetic moment of the lepton from its predicted value in the Standard Model. This can be expressed as:

$$\Delta a_l = a_l^{\text{exp}} - a_l^{\text{SM}}$$

Research measurements

a_l measurements

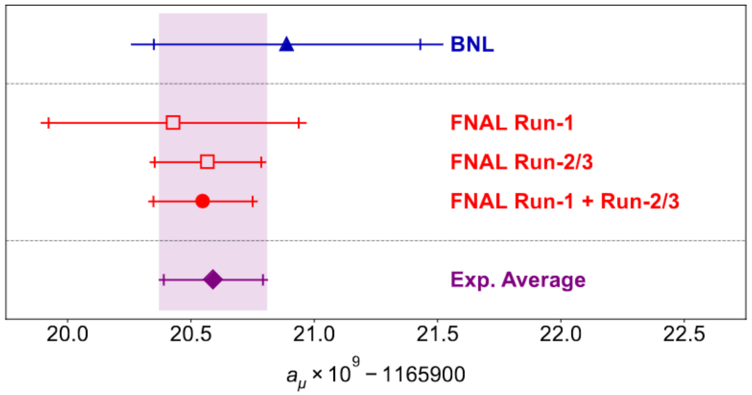
Electron

1. One of the most precisely measured quantities

2. Measurement aligns with QED prediction (12 decimal)

Muon

1. Discrepancy between experimental measurements and theoretical prediction.(9 decimal)
2. Latest measurements:



Tau

1. The short lifetime prevented precise measurements of its g-2.
2. Larger BSM effects: large than the value of $\mathcal{O}(10^{-6})$ predicted by naive scaling $\Delta a_\tau / \Delta a_\mu = m_\tau^2 / m_\mu^2$.

The Lagrangian of SMEFT

The relevant effective Lagrangian of leptonic g-2 up to one-loop order, at a scale Λ larger than the electroweak scale: $E \ll \Lambda$ by an effective Lagrangian containing non-renormalizable $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ invariant operators.

$$\mathcal{L} = \frac{C_{eB}^{\ell}}{\Lambda^2}\left(\bar{\ell}_L \sigma^{\mu\nu} e_R\right) H B_{\mu\nu} + \frac{C_{eW}^{\ell}}{\Lambda^2}\left(\bar{\ell}_L \sigma^{\mu\nu} e_R\right) \tau^I \, H W_{\mu\nu}^I$$

$$+ \frac{C_T^{\ell}}{\Lambda^2}(\bar{\ell}_L^a \sigma_{\mu\nu} e_R)\varepsilon_{ab}(\bar{Q}_L^b \sigma^{\mu\nu} u_R) + h.c$$

The resulting expression for Δa_τ at one-loop order is given by

,

$$\Delta a_\tau \simeq \frac{4 m_\tau v}{e\sqrt{2}\Lambda^2}\left(C_{e\gamma}^{\tau} - \frac{3\alpha}{2\pi}\frac{c_W^2 - s_W^2}{s_W c_W} C_{eZ}^{\ell} \log\frac{\Lambda}{m_Z}\right) - \frac{4 m_\tau m_t}{\pi^2}\frac{C_T^{\tau t}}{\Lambda^2} \log\frac{\Lambda}{m_t}$$

Replace the parameter, we will get that

$$\Delta a_\tau \approx 4 \times 10^{-5} \left(\frac{10\ \text{TeV}}{\Lambda}\right)^2 \left(C_{e\gamma}^{\tau} - 0.12\, C_T^{\tau t} - 0.02\, C_{eZ}^{\tau}\right)$$

*Δa_τ can reach values $\mathcal{O}(10^{-4})$ for $\Lambda \approx 10$ TeV and $C_{e\gamma}^{\tau} \sim 1$. This requires a strongly coupled NP sector where and a violation of the naive scaling $\Delta a_\tau \propto m_\tau^2$ by the chiral enhancement factor v/m_τ. In this case, a muon collider could still be able to directly produce NP particles. (*Journal of High Energy Physics*, *2010*(5), 1-48.)

*The NP responsible for $\Delta a_\tau \sim 10^{-4}$ can be also tested indirectly through the rare higgs decay $h \to \tau^+\tau^-\gamma$ and the high-energy processes $\mu^+\mu^- \to \tau^+\tau^-(h)$ and $\mu^+\mu^- \to \mu^+\mu^-\tau^+\tau^-(\bar{\nu}\nu\tau^+\tau^-)$ where the latter process enjoys a very large cross-section driven by vector-boson-fusion.

*The main advantage of a MC over other lepton colliders such as the FCC-ee in probing Δa_τ is the much larger c.o.m. energy with a corresponding higher luminosity and (sometimes) also larger cross-sections.

- **Leptonic g-2 from rare Higgs decays:**

The radiative Higgs decays $h \to \ell^+\ell^-\gamma$.

The dipole operator $O_{e\gamma}$ contributes to the rare decay $h \to \ell^+\ell^-\gamma$ as

$$\Gamma_{h\ell\ell\gamma} = \Gamma_{SM} + \frac{e y_\ell m_h^3}{64\pi^3}\frac{\mathrm{Re}\, C_{e\gamma}^\ell}{\Lambda^2} + \frac{m_h^5}{768\pi^3}\frac{\left|C_{e\gamma}^\ell\right|^2}{\Lambda^4}.$$

We obtain the following estimates

$$\frac{\mathcal{B}_{h\tau\tau\gamma}}{\mathcal{B}_{h\tau\tau\gamma}^{SM}} \approx 1 + 0.02\left(\frac{\Delta a_\tau}{10^{-4}}\right) + 2\times 10^{-4}\left(\frac{\Delta a_\tau}{10^{-4}}\right)^2$$

A sensitvity to Δa_τ of order $\Delta a_\tau \lesssim 10^{-4}$ could be attained through $h \to \tau^+\tau^-\gamma$. Notice that the leading NP effect in $\mathcal{B}_{h\tau\tau\gamma}$ is provided by the interference term with the SM contribution.

- **Leptonic g-2 from rare Higgs decays:**
- Since deviations in the Z(W) decays from the SM expectations are constrained to be below the few $\times 10^{-3(2)}$ level, we argue that electroweak precision tests prevent visible NP effects in the $h \to Z\tau\tau$ and $h \to W\tau\nu$ decays.
- In simpler terms, it means that any new theories or physics beyond what we currently understand from the Standard Model are unlikely to be observable in these particular decay processes of the Higgs boson as well as Z(W) dcays.
- A sensitvity to Δa_τ of order $\Delta a_\tau \lesssim 10^{-4}$ could be attained by measuring $h \to \tau^+\tau^-\gamma$ with percent precision.
- An important background could arise from the $Z + \gamma \to \tau^+\tau^-$ process (when the invariant mass $m_{\tau^+\tau^-\gamma} \approx m_h$).

- **Leptonic g-2 from Muon Collider**

It is worth pointing out that at a high-energy lepton collider Δa_τ can also be efficiently probed through the processes $\mu^+\mu^- \to \tau^+\tau^-(h)$, and especially $\mu^+\mu^- \to \mu^+\mu^-\tau^+\tau^-(\bar{\nu}\nu\,\tau^+\tau^-)$ which enjoys a very large cross-section driven by vector-boson-fusion. Furthermore, the process $\mu^+\mu^- \to \mu^+\mu^-\tau^+\tau^-h(\bar{\nu}\nu\,\tau^+\tau^-h)$ also need been consider.

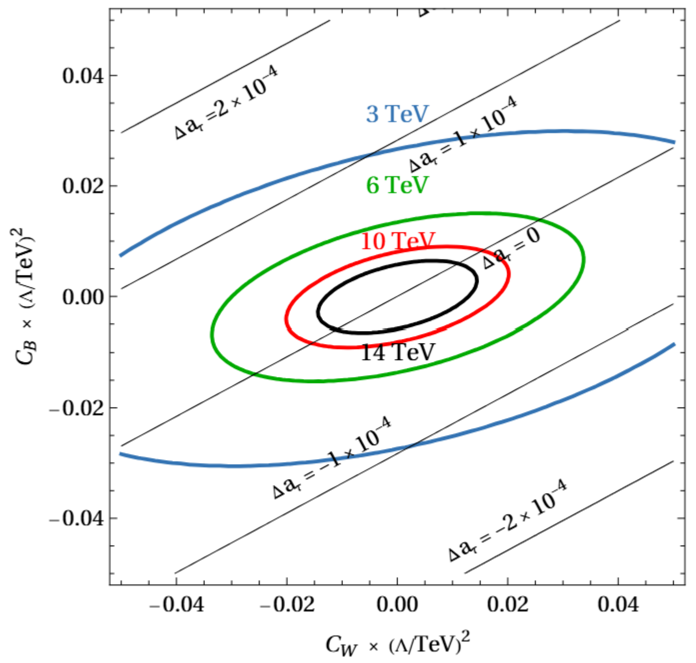
The dominant SM background is provided by the process $\mu^+\mu^- \to \tau^+\tau^-Z$ when the Z is misidentified for an Higgs.

We assume a mistag probability $\epsilon_{Z\to h} = 15\%$. In addition we include an 80% efficiency for tau identification, and a 50% efficiency for the reconstruction of a boosted Higgs decaying into $b\bar{b}$.

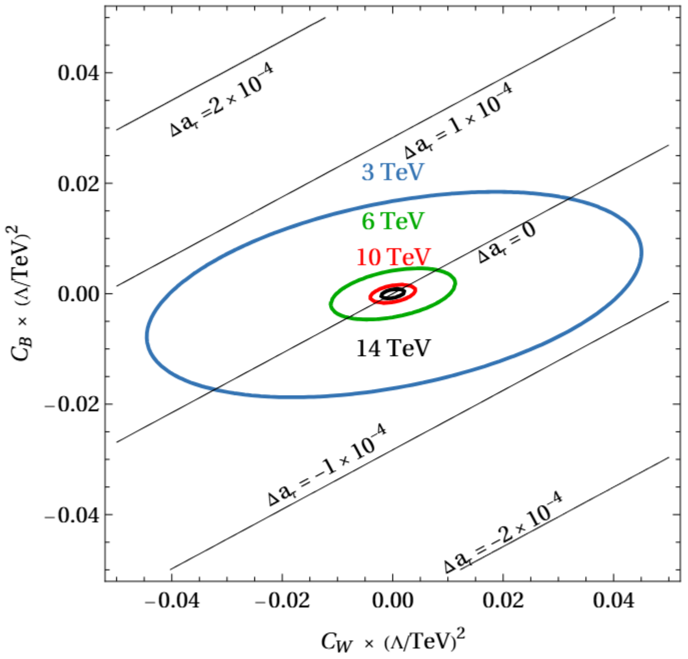
Leptonic g-2 from Muon Collider

The imposed cuts on madgraph calculation relevant to this case are as follows:

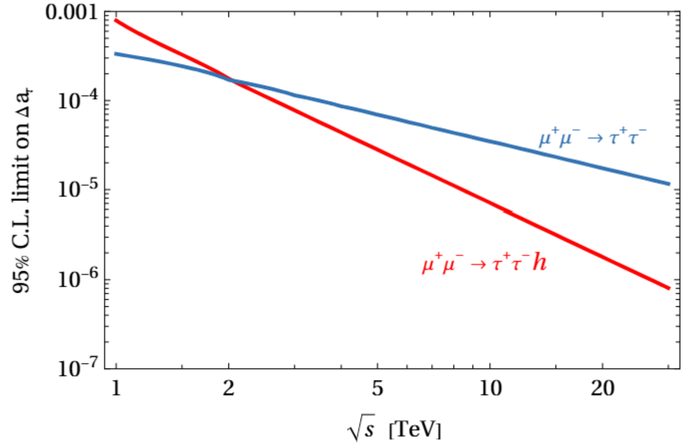
$p_{T,\tau} > E_{\text{cm}}/10,\ M_{\tau\tau} > E_{\text{cm}}/10,\ \Delta R_{\tau\tau} > 0.4,\ \eta < 3$



$\mu^+\mu^- \to \tau^+\tau^-$



$\mu^+\mu^- \to \tau^+\tau^-\ h$



The numerical example of $\mu^+\mu^- \to \tau^+\tau^- h$

After these cuts the number of signal and background events at 3 TeV is

$$N_{\rm sig}^{3\,\rm TeV} = (8.8C_{eB} - 3.1C_{eW})\left(\frac{\rm TeV}{\Lambda}\right)^2 + 10^2 \times \left(37.4C_{eW}^2 + 217C_{eB}^2 - 74.0C_{eW}C_{eB}\right)\left(\frac{\rm TeV}{\Lambda}\right)^4$$

$$N_{\rm bkg}^{3\,\rm TeV} = 0.014 + 3.5(\epsilon_{Z\to h}/15\%),$$

while at 10 TeV

$$N_{\rm sig}^{10\,\rm TeV} = (101C_{eB} - 35.7C_{eW})\left(\frac{\rm TeV}{\Lambda}\right)^2 + 10^4 \times \left(47.0C_{eW}^2 + 233C_{eB}^2 - 93.5C_{eW}C_{eB}\right)\left(\frac{\rm TeV}{\Lambda}\right)^4$$

$$N_{\rm bkg}^{10\,\rm TeV} = 0.005 + 4.3(\epsilon_{Z\to h}/15\%).$$

A 3 TeV muon collider can detect $\Delta a_\tau \approx 7 \times 10^{-4}$, while a 10 TeV collider can reach $\Delta a_\tau \approx 7 \times 10^{-5}$.

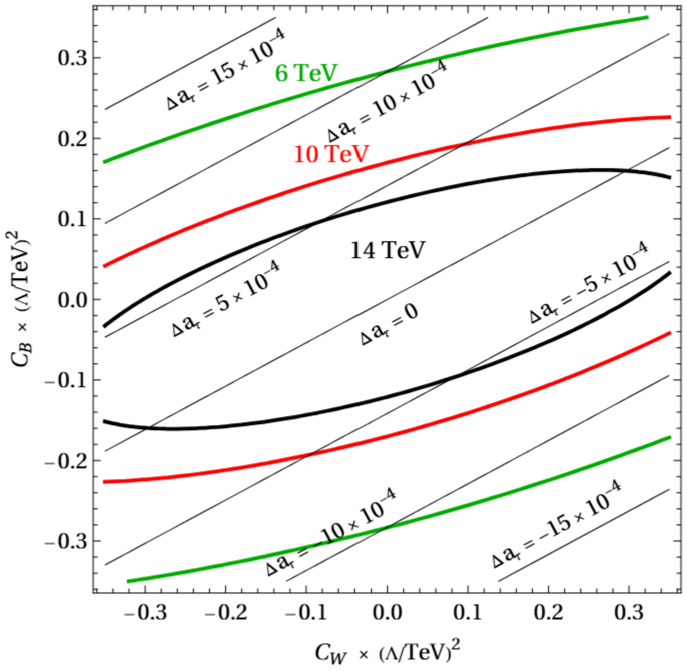
The sensitivity to Δa_τ as a function of center-of-mass energy ($E_{\rm cm}$) is shown, with the red line indicating constraints from $\mu^+\mu^- \to \tau^+\tau^- h$ and the blue line from $\mu^+\mu^- \to \tau^+\tau^-$ pair production.

The $2 \to 3$ process ($\mu^+\mu^- \to \tau^+\tau^- h$) dominates over the $2 \to 2$ process ($\mu^+\mu^- \to \tau^+\tau^-$) at energies above 2 TeV due to the additional Higgs vev involved.

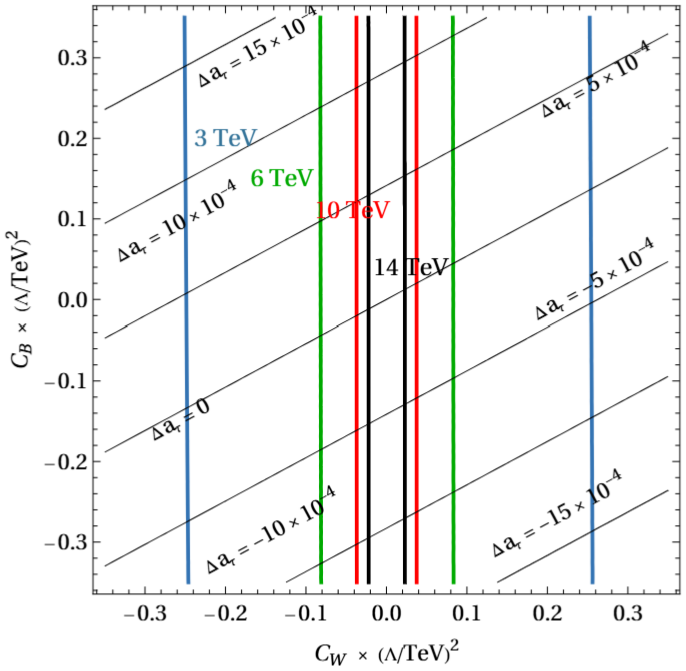
Leptonic g-2 from Muon Collider

The imposed cuts on madgraph calculation relevant to this case are as follows:

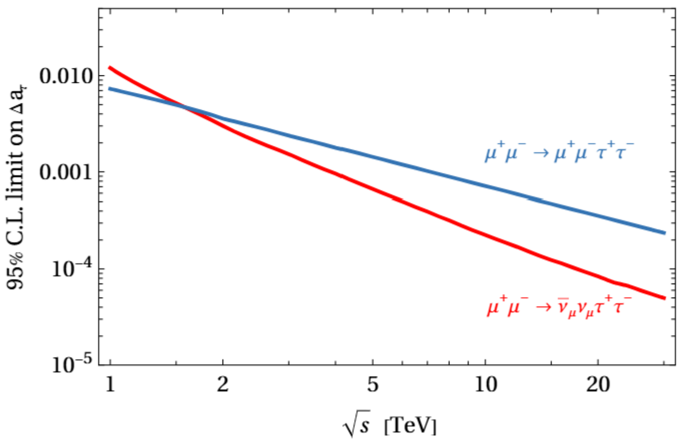
$$p_{T,\tau/\nu_\mu} > E_{\rm cm}/10,\ M_{\nu_\mu\bar{\nu}_\mu} > E_{\rm cm}/10,\ M_{\tau\tau} > E_{\rm cm}/10,\ \Delta R_{\tau\tau} > 0.4,\ \eta < 3$$



$\mu^+\mu^- \to \mu^+\mu^-\tau^+\tau^-$



$\mu^+\mu^- \to \bar{\nu}\nu\tau^+\tau^-$



Conclusion

- Examining tau g–2 sensitivity at future high-energy muon colliders, which have the advantage of higher c.o.m. energy, corresponding higher luminosity, and larger cross-sections.

- A 3 TeV muon collider would be sensitive to Δa_τ at the order around 10^{-4}, while a 10 TeV collider the sensitive would reach values 10^{-5}.

- The background analysis and the application of well-defined cuts are critical for the accurate interpretation of physics results at the Future Muon Collider.

- The NP responsible for $\Delta a_\tau < 10^{-4}$ can be also tested indirectly through the rare higgs decay $h \to \tau^+\tau^-\gamma$ and the high-energy processes $\mu^+\mu^- \to \tau^+\tau^-(h)$, $\mu^+\mu^- \to \mu^+\mu^-\tau^+\tau^-(\bar{\nu}\nu\tau^+\tau^-)$.

THANKS!

Fabian Esser
IFIC, Universidad de Valencia

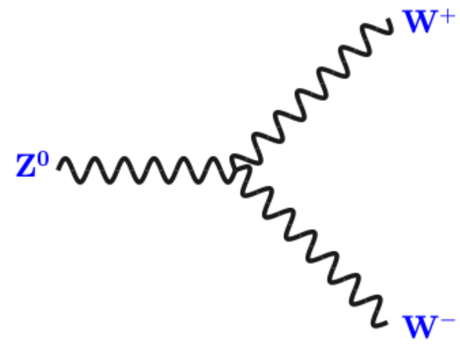
Neutral Triple Gauge Couplings in the SMEFT

with Ricardo Cepedello, Martin Hirsch & Veronica Sanz

2402.04306

1. Neutral Triple Gauge Couplings

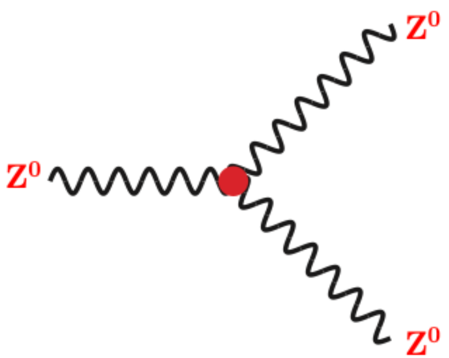
Triple gauge boson vertices



In the SM:

Triple gauge boson vertices from self-coupling in field strength tensor

$$W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu - g\epsilon^{IJK} W^J_\mu W^K_\nu$$



But:

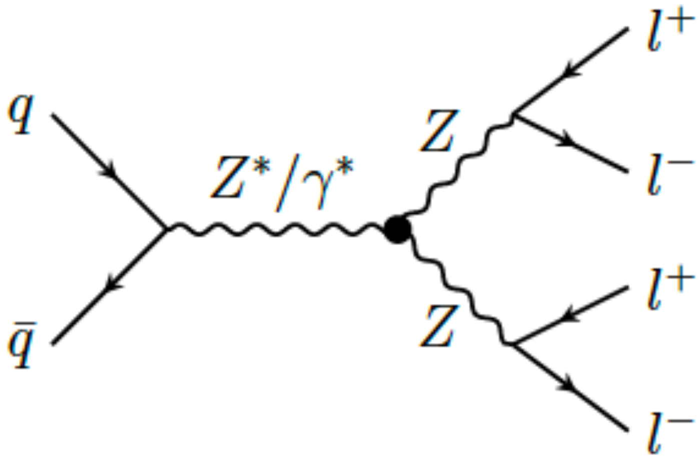
no Neutral Triple Gauge Couplings (NTGCs) due to ϵ^{IJK}

→ Anomalous NTGC (aNTGC)

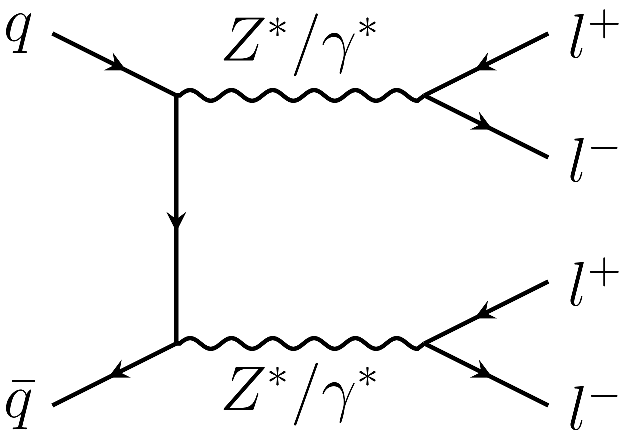
aNTGC provide important tests for the gauge structure of the SM

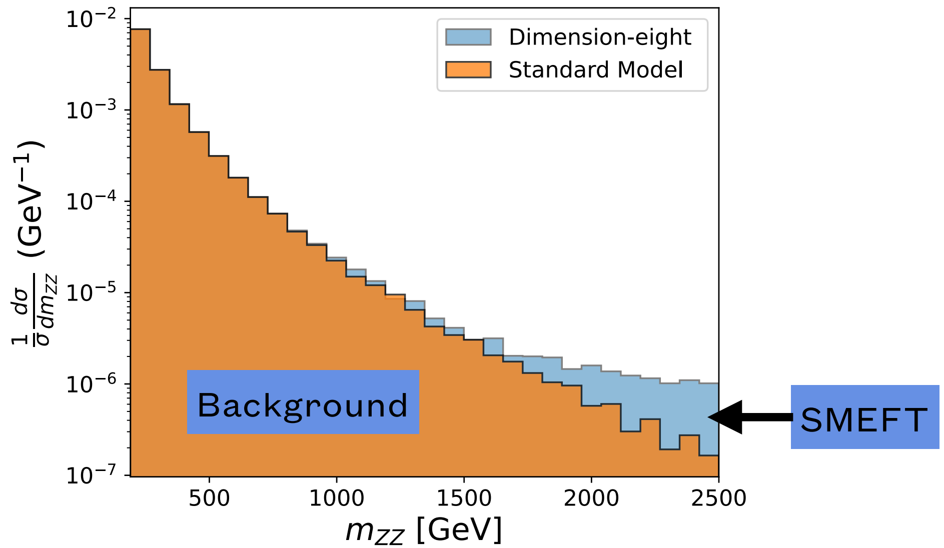
→ Searches for aNTGC at ATLAS and CMS

Searches for NTGCs



- Cleanest final state: $ZZ \to 4l$
- Rare channel, will increase its power with more data
- Not background limited
  $\Rightarrow$ Increase sensitivity with luminosity





Form factors for NTGCs

CP conserving (CPC) vertices after EWSB, taking into account Bose symmetry and gauge invariance

→ NTGC with 3 on-shell bosons vanish

→ $V = \gamma^*, Z^*$ has to be off-shell

[Gounaris et al. 1999]
[Gounaris et al. 2000]

$$
\begin{aligned}
ie\Gamma^{\alpha\beta\mu}_{ZZV}(q_1, q_2, q_3) &= e\frac{(q_3^2 - m_V^2)}{m_Z^2}\left[f_5^V \epsilon^{\mu\alpha\beta\rho}(q_1 - q_2)_\rho\right], \\
ie\Gamma^{\alpha\beta\mu}_{Z\gamma V}(q_1, q_2, q_3) &= e\frac{(q_3^2 - m_V^2)}{m_Z^2}\left[h_3^V \epsilon^{\mu\alpha\beta\rho} q_{2,\rho} + \frac{h_4^V}{m_Z^2} q_3^\alpha \epsilon^{\mu\beta\rho\sigma} q_{3,\rho} q_{2,\sigma}\right]
\end{aligned}
$$

- Form factors f_5^V, h_3^V and h_4^V are independent parameters, but h_4^V is not generated at 1-loop and dim-8
- CP-violating vertices or vertices with more than one boson off-shell are not discussed here
  → experimentally irrelevant

2. SMEFT operators for NTGCs

Gauge couplings in SMEFT

In Greens basis for SMEFT list all operators at $d = 6$ containing only bosons
MatchMakerEFT (1908.05295):

X^3		X^2H^2		H^2D^4	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{HG}$	$G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	$\mathcal{R}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$	H^4D^2	
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
X^2D^2		$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{R}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{R}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$	$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$	$\mathcal{R}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger \mathrm{i}\overleftrightarrow{D}^\mu H)$
$\mathcal{R}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$	$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	H^6	
$\mathcal{R}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$	$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$	$\mathcal{O}_H$	$(H^\dagger H)^3$
		H^2XD^2			
		$\mathcal{R}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger \mathrm{i}\overleftrightarrow{D}^I_\mu H)$		
		$\mathcal{R}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger \mathrm{i}\overleftrightarrow{D}_\mu H)$		

$\mathcal{O}_{HB}, \mathcal{O}_{HWB}, \mathcal{O}_{HW}, \mathcal{O}_{3W}$ contain TGC, but no NTGC
$\Rightarrow$ need to go to dimension-8

d=8 operators for NTGCs

Four d=8 operators that generate the effective Lagrangian, all in the class $X^2H^2D^2$

$$\mathcal{O}_{DB\tilde{B}} = i\frac{c_{DB\tilde{B}}}{\Lambda^4} H^\dagger \tilde{B}_{\mu\nu}(D^\rho B_{\nu\rho})D_\mu H + \text{h.c.}\,,$$

$$\mathcal{O}_{DW\tilde{W}} = i\frac{c_{DW\tilde{W}}}{\Lambda^4} H^\dagger \tilde{W}_{\mu\nu}(D^\rho W_{\nu\rho})D_\mu H + \text{h.c.}\,,$$

$$\mathcal{O}_{DW\tilde{B}} = i\frac{c_{DW\tilde{B}}}{\Lambda^4} H^\dagger \tilde{B}_{\mu\nu}(D^\rho W_{\nu\rho})D_\mu H + \text{h.c.}\,,$$

$$\mathcal{O}_{DB\tilde{W}} = i\frac{c_{DB\tilde{W}}}{\Lambda^4} H^\dagger \tilde{W}_{\mu\nu}(D^\rho B_{\nu\rho})D_\mu H + \text{h.c.}\,.$$

4 independent form factors $f_5^Z, f_5^\gamma, h_3^Z, h_3^\gamma$

⇒ these 4 operators are the maximal set

d=8 operators for NTGC

Relations to the form factors:

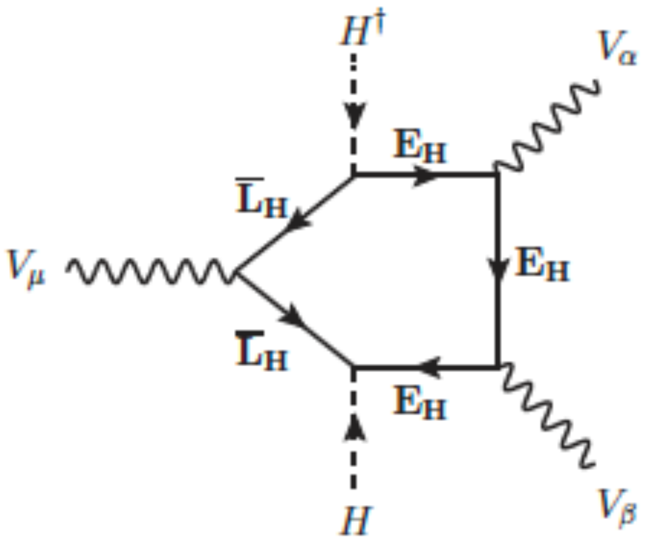
$$
\begin{aligned}
f_5^Z &= \frac{v^2 m_Z^2}{\Lambda^4} \frac{1}{c_W s_W} \left[s_W^2 c_{DB\tilde{B}} + c_W^2 c_{DW\tilde{W}} + \frac{1}{2} c_W s_W (c_{DW\tilde{B}} + c_{DB\tilde{W}}) \right], \\
f_5^\gamma &= \frac{v^2 m_Z^2}{\Lambda^4} \frac{1}{c_W s_W} \left[c_W s_W (-c_{DB\tilde{B}} + c_{DW\tilde{W}}) - \frac{1}{2} (s_W^2 c_{DW\tilde{B}} - c_W^2 c_{DB\tilde{W}}) \right], \\
h_3^Z &= \frac{v^2 m_Z^2}{\Lambda^4} \frac{1}{c_W s_W} \left[c_W s_W (-c_{DB\tilde{B}} + c_{DW\tilde{W}}) + \frac{1}{2} (c_W^2 c_{DW\tilde{B}} - s_W^2 c_{DB\tilde{W}}) \right], \\
h_3^\gamma &= \frac{v^2 m_Z^2}{\Lambda^4} \frac{1}{c_W s_W} \left[c_W^2 c_{DB\tilde{B}} + s_W^2 c_{DW\tilde{W}} - \frac{1}{2} c_W s_W (c_{DW\tilde{B}} + c_{DB\tilde{W}}) \right],
\end{aligned}
$$

For our models, we always find $c_{DW\tilde{B}} = c_{DB\tilde{W}}$

$\Rightarrow f_5^\gamma = h_3^Z$, only 3 independent form factors

3. Models for NTGCs

Prototype UV model for NTGCs



- We searched for models at d=8 using a diagrammatic approach
- contributions from *pentagon diagrams* with two fermions
- **prototype model**: "vector-like" leptons $L_H = F_{1,2,-1/2}$ and $E_H = F_{1,1,-1}$
- heavy-light Yukawa couplings are strongly constrained by dimension-6 operators at tree-level
  $\Rightarrow$ Both fermions must be heavy
- pentagon reduces to triangle diagram after EWSB and mass mixing

More fermionic models for NTGCs

scan different options for QNs: up to **hypercharge 4** and **SU(2) quintuplets**

couple to a Higgs boson: SU(2) products needs to contain a doublet & $\Delta Y = 1/2$

Singlets & Doublets

Model	Particles	$\tilde{c}_{DB\tilde{B}}$	$\tilde{c}_{DW\tilde{B}} = \tilde{c}_{DB\tilde{W}}$	$\tilde{c}_{DW\tilde{W}}$
MDS1	(L_H, E_H)	$\frac{23}{960}$	$-\frac{7}{480}$	$\frac{1}{320}$
MDS2	$(F_{1,2,-\frac{3}{2}}, F_{1,1,-1})$	$-\frac{21}{320}$	$-\frac{13}{480}$	$-\frac{1}{320}$
MDS3	$(F_{1,2,-\frac{3}{2}}, F_{1,1,-2})$	[illegible]	[illegible]	$\frac{1}{320}$
MDS4	$(F_{1,2,-\frac{5}{2}}, F_{1,1,-2})$	[illegible]	[illegible]	$-\frac{1}{320}$
MDS5	$(F_{1,2,-\frac{5}{2}}, F_{1,1,-3})$	[illegible]	[illegible]	$\frac{1}{320}$
MDS6	$(F_{1,2,-\frac{7}{2}}, F_{1,1,-3})$	$-\frac{141}{320}$	$-\frac{11}{160}$	$-\frac{1}{320}$
MDS7	$(F_{1,2,-\frac{7}{2}}, F_{1,1,-4})$	$\frac{563}{960}$	$-\frac{37}{480}$	$\frac{1}{320}$

Doublets & Triplets

Model	Particles	$\tilde{c}_{DB\tilde{B}}$	$\tilde{c}_{DW\tilde{B}} = \tilde{c}_{DB\tilde{W}}$	$\tilde{c}_{DW\tilde{W}}$
MTD1	$(F_{1,3,0}, F_{1,2,-\frac{1}{2}})$	[illegible]	[illegible]	$-\frac{49}{960\sqrt{3}}$
MTD2	$(F_{1,3,-1}, F_{1,2,-\frac{1}{2}})$	[illegible]	[illegible]	$\frac{49}{960\sqrt{3}}$
MTD3	$(F_{1,3,-1}, F_{1,2,-\frac{3}{2}})$	[illegible]	[illegible]	$-\frac{49}{960\sqrt{3}}$
MTD4	$(F_{1,3,-2}, F_{1,2,-\frac{3}{2}})$	$\frac{41\sqrt{3}}{320}$	$\frac{89}{480\sqrt{3}}$	$\frac{49}{960\sqrt{3}}$
MTD5	$(F_{1,3,-2}, F_{1,2,-\frac{5}{2}})$	$-\frac{203}{320\sqrt{3}}$	$\frac{37}{160\sqrt{3}}$	$-\frac{49}{960\sqrt{3}}$

Triplets & Quartuplets

Model	Particles	$\tilde{c}_{DB\tilde{B}}$	$\tilde{c}_{DW\tilde{B}} = \tilde{c}_{DB\tilde{W}}$	$\tilde{c}_{DW\tilde{W}}$
MQT1	$(F_{1,4,-\frac{1}{2}}, F_{1,3,0})$	$-\frac{\sqrt{\frac{3}{2}}}{160}$	$-\frac{19}{240\sqrt{6}}$	$-\frac{109}{480\sqrt{6}}$
MQT2	$(F_{1,4,-\frac{1}{2}}, F_{1,3,-1})$	[illegible]	[illegible]	$\frac{109}{480\sqrt{6}}$
MQT3	$(F_{1,4,-\frac{3}{2}}, F_{1,3,-1})$	[illegible]	[illegible]	$-\frac{109}{480\sqrt{6}}$
MQT4	$(F_{1,4,-\frac{3}{2}}, F_{1,3,-2})$	[illegible]	[illegible]	$\frac{109}{480\sqrt{6}}$
MQT5	$(F_{1,4,-\frac{5}{2}}, F_{1,3,-2})$	$-\frac{203}{160\sqrt{6}}$	$-\frac{53}{80\sqrt{6}}$	$-\frac{109}{480\sqrt{6}}$

Quartuplets & Quintuplets

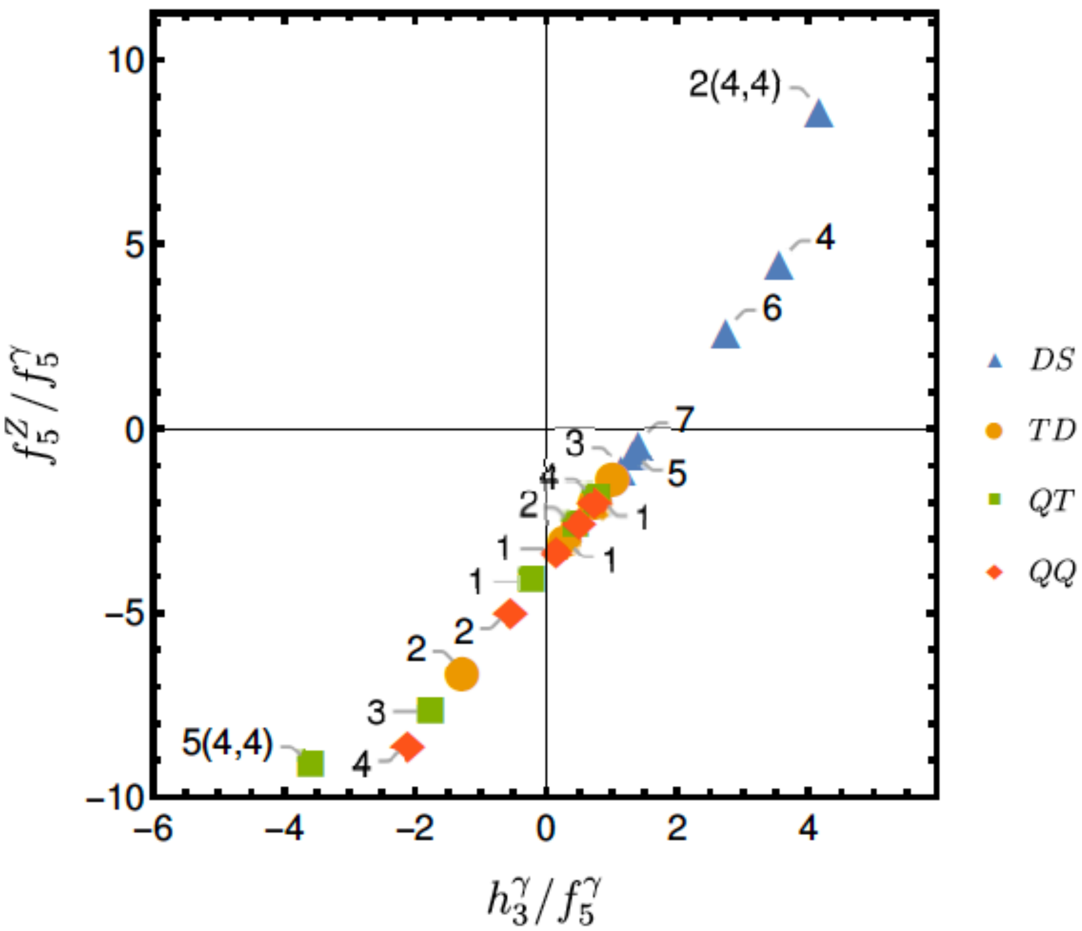
Model	Particles	$\tilde{c}_{DB\tilde{B}}$	$\tilde{c}_{DW\tilde{B}} = \tilde{c}_{DB\tilde{W}}$	$\tilde{c}_{DW\tilde{W}}$
MQQ1	$(F_{1,5,0}, F_{1,4,-\frac{1}{2}})$	$-\frac{1}{32\sqrt{10}}$	$\frac{7}{48\sqrt{10}}$	$-\frac{21}{32\sqrt{10}}$
MQQ2	$(F_{1,5,-1}, F_{1,4,-\frac{1}{2}})$	$\frac{23}{96\sqrt{10}}$	[illegible]	$\frac{21}{32\sqrt{10}}$
MQQ3	$(F_{1,5,-1}, F_{1,4,-\frac{3}{2}})$	[illegible]	[illegible]	$-\frac{21}{32\sqrt{10}}$
MQQ4	$(F_{1,5,-2}, F_{1,4,-\frac{3}{2}})$	$\frac{41}{32\sqrt{10}}$	$\frac{53}{48\sqrt{10}}$	$\frac{21}{32\sqrt{10}}$
MQQ5	$(F_{1,5,-2}, F_{1,4,-\frac{5}{2}})$	$-\frac{203}{96\sqrt{10}}$	$\frac{67}{48\sqrt{10}}$	$-\frac{21}{32\sqrt{10}}$

Matching done with ***Matchete***

$$c_{DAB} = \frac{1}{16\pi^2} g_A g_B |Y|^2 \tilde{c}_{DAB},$$

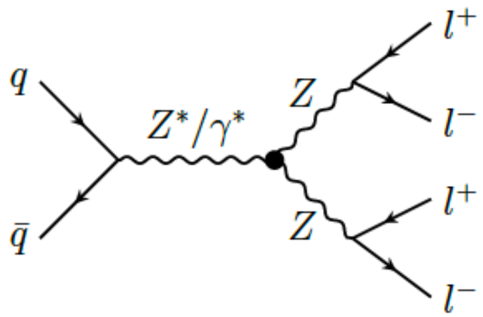
Form factors

- calculate form factors from Wilson coefficients for all models
- we can form two independent ratios of form factors, independent of Λ
- all models lie on a line, but different predictions for all models
- experimentally accessible: ZZ (f_5^γ) and Zγ (h_3^γ) final state
  → ratio of these channels would discriminate the true UV model



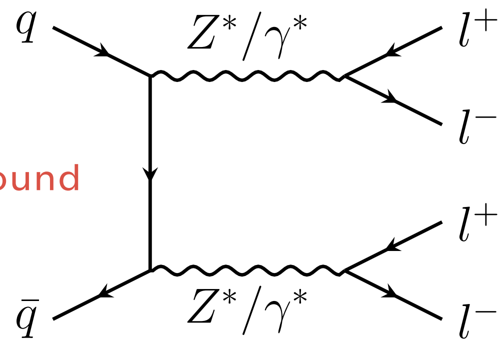
4. Experimental limits

Measuring $ZZ \to 4l$



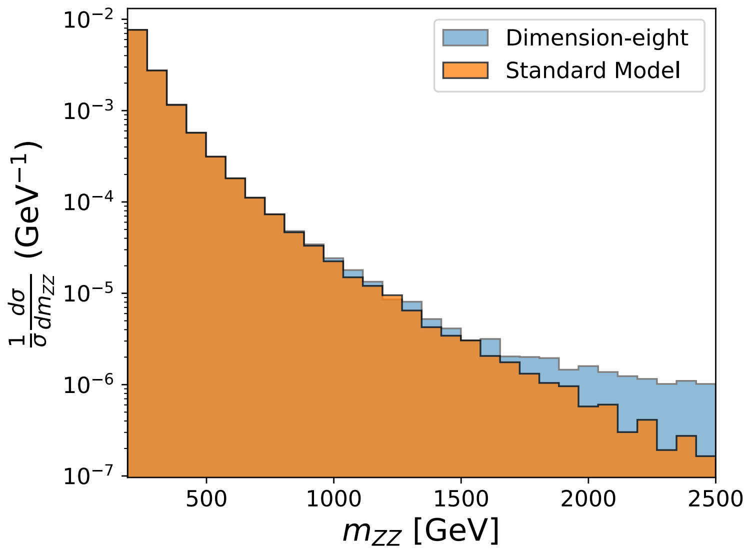
signal at dim-8

SM background



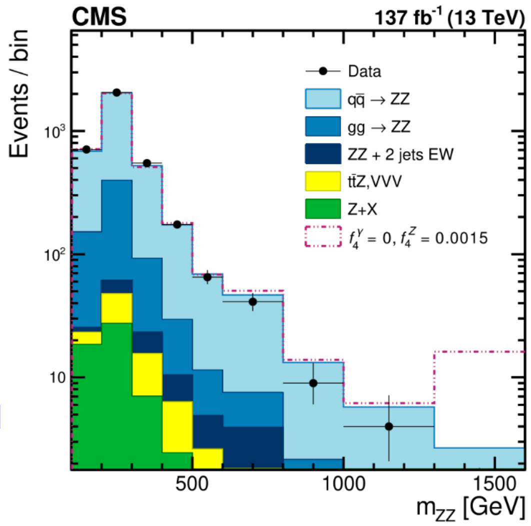
Both ATLAS and CMS search for ZZ and $Z\gamma$ final states, ZZ more sensitive

Dimension-8 growth at high energy, NTGCs are not background limited

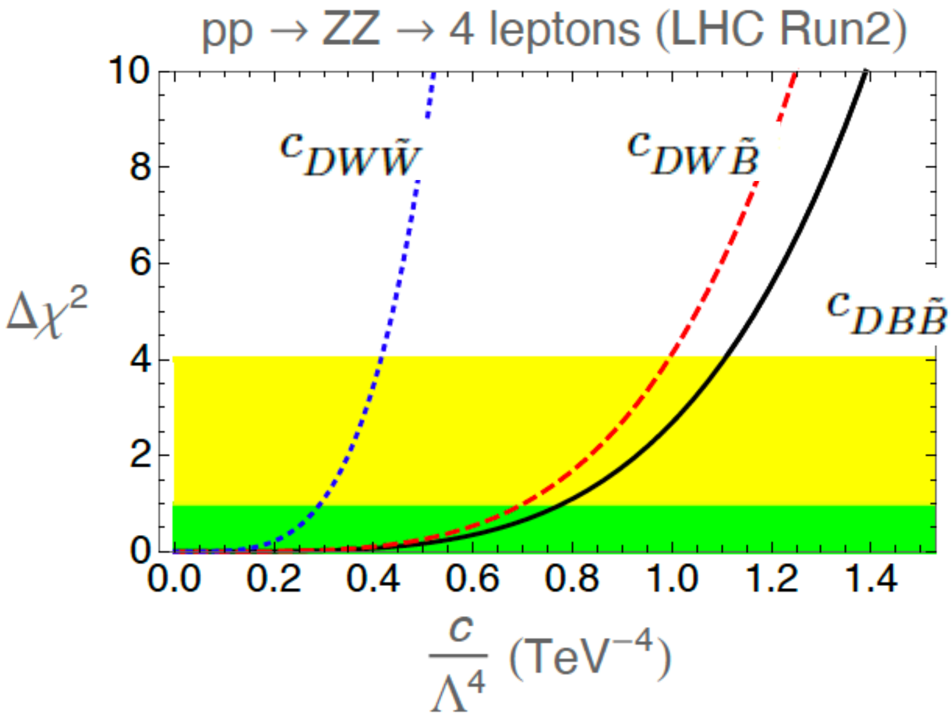


cross section vs. invariant mass

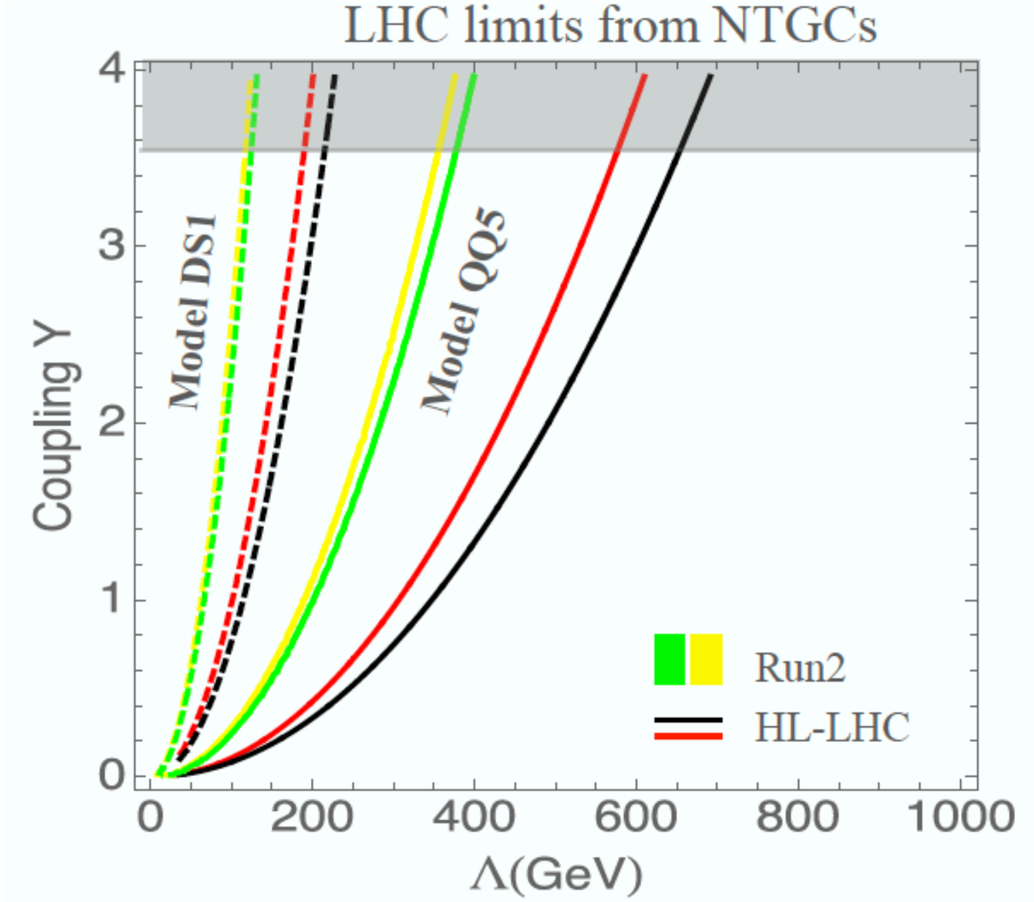
CMS 2021



Limits on NTGCs



Combined ATLAS and CMS
1 and 2 sigma limits on
dim-8 Wilson coefficients



Current limits on models very weak: $\Lambda > 100$ GeV

Prediction for HL-LHC, sensitivity based on projecting the luminosity and using the last bin

Conclusions

- NTGC searches provide important tests for the gauge structure of the SM, clean channel that improves with higher luminosity
- no dim-6 contributions, generated only at dim-8 SMEFT
- We presented a basis of dimension-8 operators that generate all 4 form factors
- We classified specific UV completions: Models with two heavy vector-like leptons that contribute via pentagon diagrams
- Limits derived from $Z \to 4l$ at ATLAS and CMS
- Limits on models very weak, in some cases below $\Lambda = 100$ GeV (EFT assumption not valid)
- Design tailored (direct) searches for these models


Thank you!

Backup slides

Lagrangians for NTGCs

Effective Lagrangian for all NTGC CPC vertices

$$\mathcal{L}_{\rm NP}^{CPC} = \frac{e}{2m_Z^2} \Big[f_5^\gamma (\partial^\sigma F_{\sigma\mu})\tilde{Z}^{\mu\beta} Z_\beta + f_5^Z (\partial^\sigma Z_{\sigma\mu})\tilde{Z}^{\mu\beta} Z_\beta - h_3^\gamma (\partial^\sigma F_{\sigma\mu})\tilde{F}^{\mu\beta} Z_\beta - h_3^Z (\partial^\sigma Z_{\sigma\mu})\tilde{F}^{\mu\beta} Z_\beta + \frac{h_4^\gamma}{2m_Z^2}[\Box(\partial^\sigma F^{\rho\alpha})]\tilde{F}_{\rho\alpha} Z_\sigma + \frac{h_4^Z}{2m_Z^2}[(\Box + m_Z^2)(\partial^\sigma Z^{\rho\alpha})]\tilde{F}_{\rho\alpha} Z_\sigma \Big]$$

[Gounaris et al. 1999]

Why is there a dual field strength in the CPC vertices? $\tilde{X}_{\mu\nu} = 1/2\, \epsilon_{\mu\nu\alpha\beta} X^{\alpha\beta}$

CP-transformations:

$C(Z_\mu) \to -Z_\mu$ and $P(Z_0) \to +Z_0,\ P(Z_i) \to -Z_i$

$P(\partial_0) \to +\partial_0,\ P(\partial_i) \to -\partial_i$ and $P(\epsilon^{\mu\alpha\beta\rho}) \to -\epsilon^{\mu\alpha\beta\rho}$

What type of SMEFT operators can produce this Lagrangian?

Matching for fermionic models

We can derive an analytic formula for the matching
(r: SU(2) representation, y: hypercharge)

$$\tilde{c}_{DB\tilde{B}} = \frac{1}{160}(-1)^{(\mathbf{r_1} \bmod 2)}\mathrm{sgn}\left(y_2^2 - y_1^2\right)\sqrt{2\mathbf{r_1}\mathbf{r_2}}\left(y_1^2 + y_2^2 + \frac{4}{3}y_2 y_1\right),$$

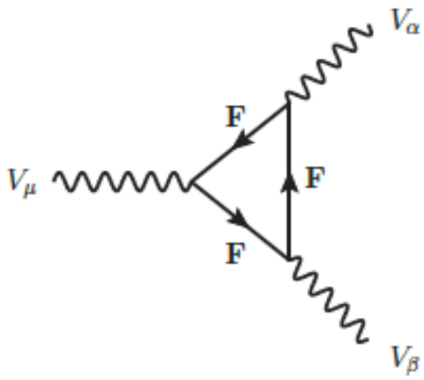
$$\tilde{c}_{DW\tilde{W}} = \frac{1}{160}(-1)^{(\mathbf{r_1} \bmod 2)}\mathrm{sgn}\left(y_2^2 - y_1^2\right)\sqrt{2\mathbf{r_1}\mathbf{r_2}}\,\frac{1}{12}\left[(\mathbf{r_1}^2 - 1) + (\mathbf{r_2}^2 - 1) + \frac{4}{3}(\mathbf{r_1}\mathbf{r_2} - 2)\right],$$

$$\tilde{c}_{DW\tilde{B}} = \frac{1}{48}(-1)^{(\mathbf{r_1} \bmod 2)}\sqrt{2\mathbf{r_1}\mathbf{r_2}}\,\frac{1}{12}(y_1 + y_2)\left[(\mathbf{r_1} + \mathbf{r_2}) + \frac{3}{5}(y_1 - y_2)\right],$$

$$\tilde{c}_{DB\tilde{W}} = \tilde{c}_{DW\tilde{B}}\,.$$

What UV models generate NTGCs at dim-8?

Models that DO NOT generate NTGCs at dim-8:



Triangles with SM or BSM fermions in the mass eigenstate basis (needs EWSB):

could generate NTGCs [Gounaris et al. 2000]

but the contributions quickly vanish with $\sqrt{s}$, they do not correspond to the d=8 EFT limit

Models with scalar states (e.g. 2HDM) can produce CPC and CPV NTGCs, but they appear only at d=12

[Moyotl et al. 2015]

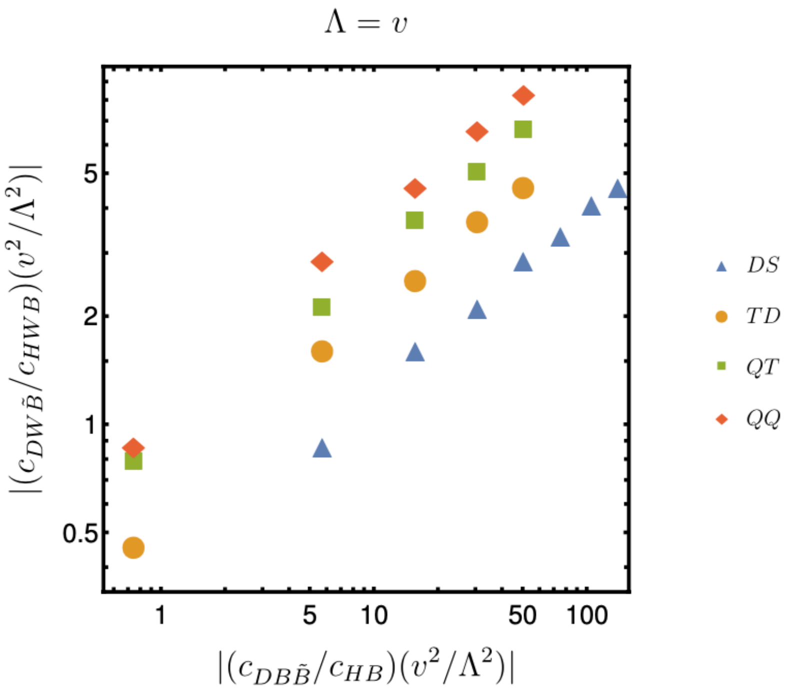
[Belusca-Maito et al 2018]

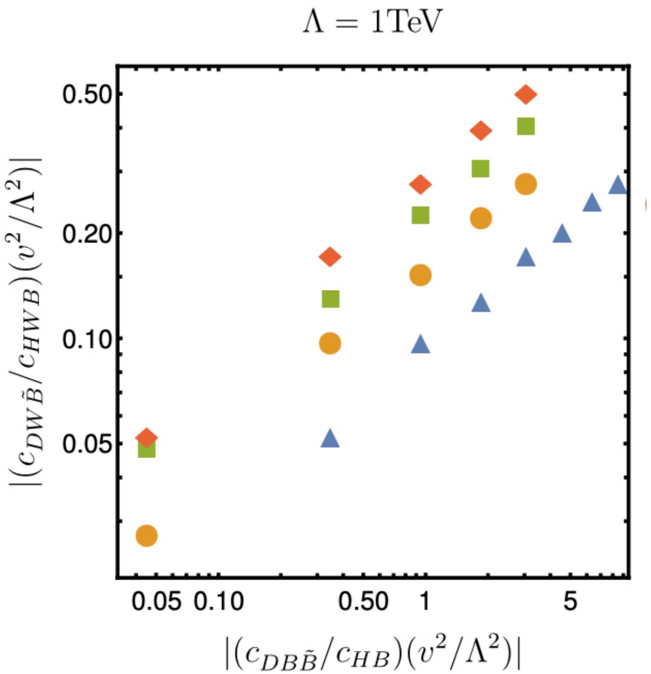
We need two fermions and Higgs insertions in the loop!

dim-6 vs. dim-8

All models that generate NTGCs also will generate the following $d = 6$ operators:

$$\mathcal{O}_{HB} = \frac{c_{HB}}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu},$$
$$\mathcal{O}_{HWB} = \frac{c_{HWB}}{\Lambda^2} H^\dagger H W_{\mu\nu} B^{\mu\nu},$$
$$\mathcal{O}_{HW} = \frac{c_{HW}}{\Lambda^2} H^\dagger H W_{\mu\nu} W^{\mu\nu},$$





d=8 grows fast with energy and will compete with d=6

→ strong gain for ZZ and Zγ searches vs WW and Zjj at large invariant mass

A bottom-up approach to nucleon decay RGEs, correlations and connection to UV

Arnau Bas i Beneito

16th of July, 2024

EFT 2024, Zurich

Based on work in collaboration with J. Gargalionis, J. Herrero-García, M. A. Schmidt. A. Santamaria [2312.13361] (published in JHEP)

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Standard story

$$\mathscr{L}_{SM}$$

$$SU(3)_C \times SU2)_L \times U(1)_Y$$

$$+$$

$$H,\ Q_L^i,\ u_R^i,\ d_R^i,\ L_L^i,\ e_R^i,\ i = 1,\ 2,\ 3$$

Individual Flavour Symmetries

↓ Yukawa couplings

$$U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \times U(1)_B$$

↓ ν oscillations *[Super-K 1999, KamLAND 2003…]*

$$U(1)_L \times U(1)_B$$

B and L accidentally conserved
(B + L violated in 3 units
by sphaleron transitions)

↓

Proton stable

Standard story

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$$SU(3)_C \times SU2)_L \times U(1)_Y$$

$$+$$

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↓ ν oscillations *[Super-K 1999, KamLAND 2003...]*

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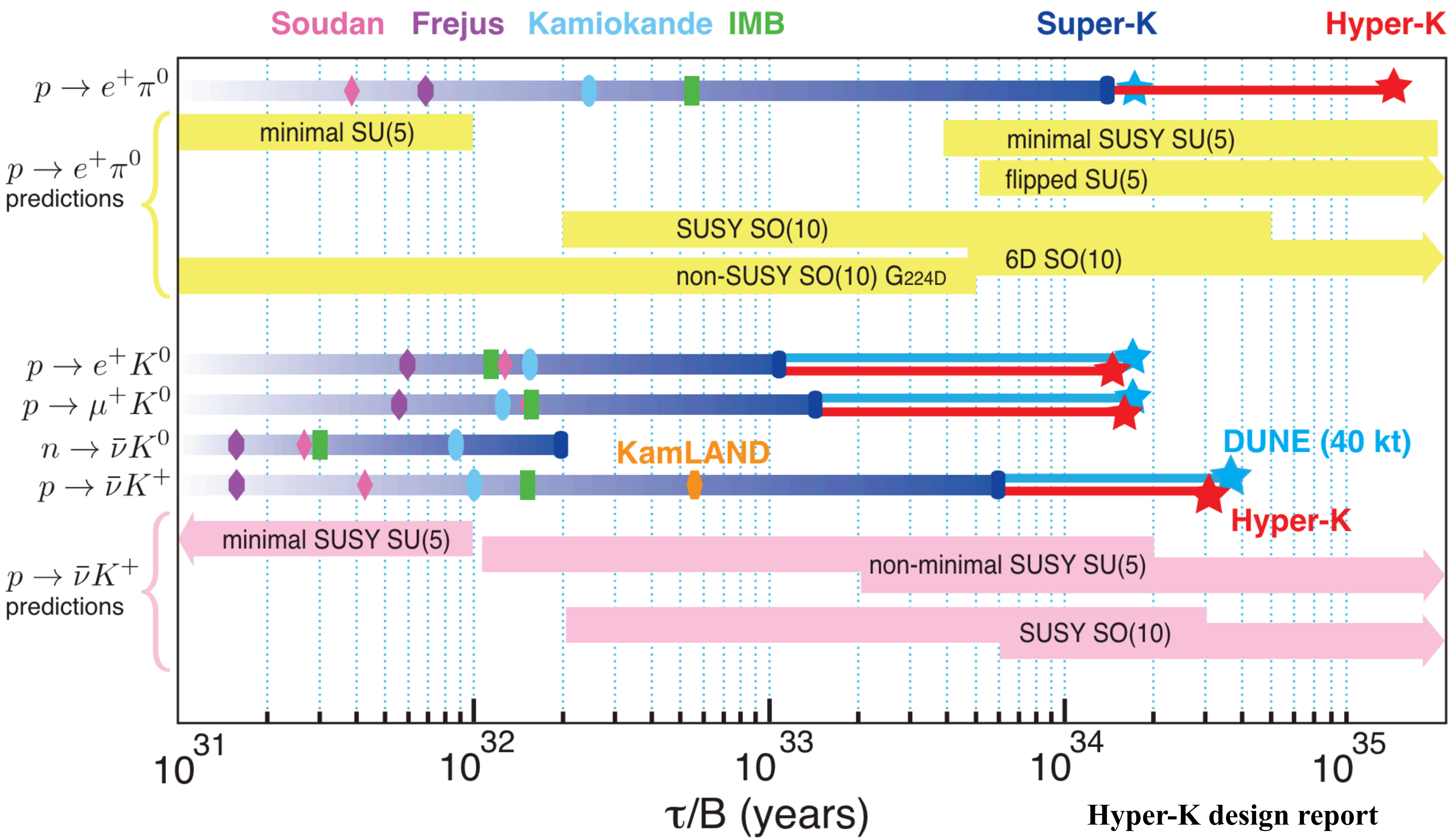
↓

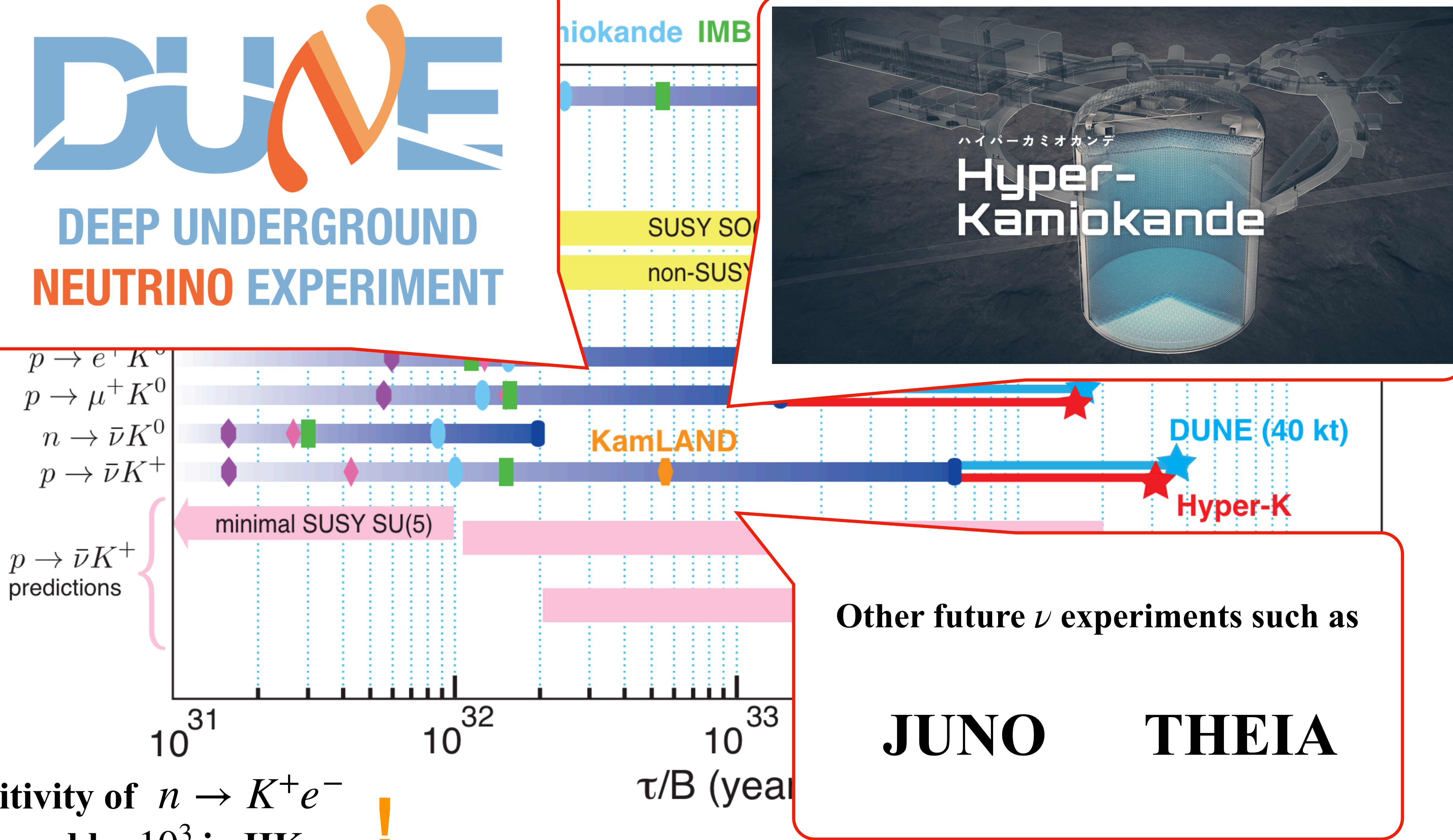
Proton stable

Experimental perspectives



Hyper-K design report

Experimental perspectives



Sensitivity of $n \to K^+ e^-$ **increased by** 10^3 **in HK** !

BNV nucleon decay could be the next big discovery

Then… why nucleon decay?

· **There is** **no fundamental reason** **to have B and L conserved (Leptoquarks, Seesaw particles, SUSY, GUTs…)**

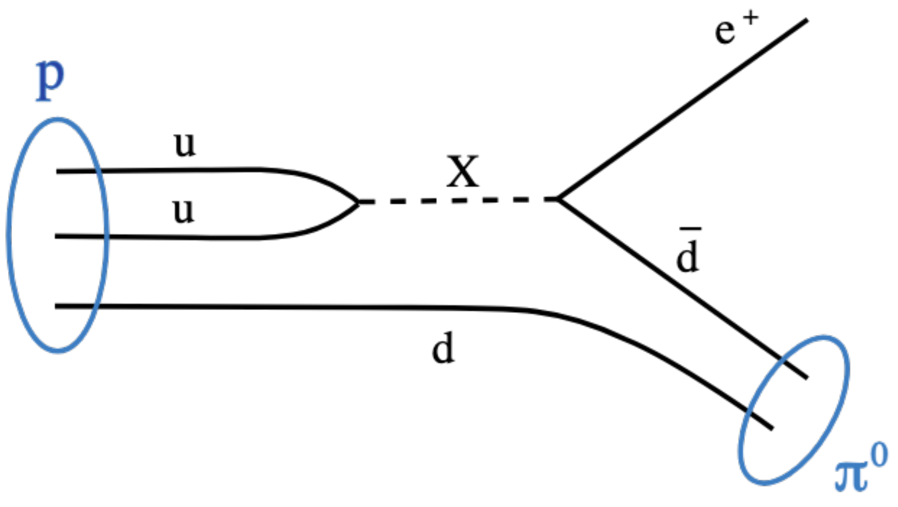
· **Experimental probes of BNV and LNV would constitute one of the** **strongest evidence** **for physics beyond SM (BSM) →PD will be looked for in future experiments (HK, DUNE…)**

Grand Unified Theories (GUTs)
[*Georgi et. al. 1973, H. Fritzsch et al. 1975*]

Baryogenesis
[Sakharov 1967]

→ **Baryon Number violation (BNV)**

E.g. proton decay (PD), neutron-antineutron oscillations



Large number of UV theories predicting PD

↓

Systematic study of PD in a model-independent way (bottom-up)

BNV within the SMEFT

Parametrization of new physics through Effective operators (d > 4)
SM Effective Field Theory (SMEFT)

Bounds on SMEFT WCs serve as a bridge to specific UV models

[S. Weinberg 1979 ,
B. Grzadkowski et al. 2010,
W. Buchmuller et al. 1986,
Brivio et al. 2019,
B. Henning et al. 2016,
De Gouvea et al. 2014]

$\Delta L = 2$ → $\frac{c^{d=5}}{\Lambda}\mathcal{O}_W$; $\Delta(B-L) = 0$ → $\frac{c^{d=6}}{\Lambda^2}\mathcal{O}^{d=6}$; $\Delta(B-L) = 2$ → $\frac{c^{d=7}}{\Lambda^3}\mathcal{O}^{d=7}$

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c^{d=5}}{\Lambda}\mathcal{O}_W + \frac{c^{d=6}}{\Lambda^2}\mathcal{O}^{d=6} + \frac{c^{d=7}}{\Lambda^3}\mathcal{O}^{d=7} + \dots$$

BNV within the SMEFT

Parametrization of new physics through Effective operators (d > 4)
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[S. Weinberg 1979 , B. Grzadkowski et al. 2010, W. Buchmuller et al. 1986, Brivio et al. 2019, B. Henning et al. 2016, De Gouvea et al. 2014]

$\Delta L = 2$ → $\frac{c^{d=5}}{\Lambda}\mathcal{O}_W$

$\Delta(B-L) = 0$ → $\frac{c^{d=6}}{\Lambda^2}\mathcal{O}^{d=6}$

$\Delta(B-L) = 2$ → $\frac{c^{d=7}}{\Lambda^3}\mathcal{O}^{d=7}$

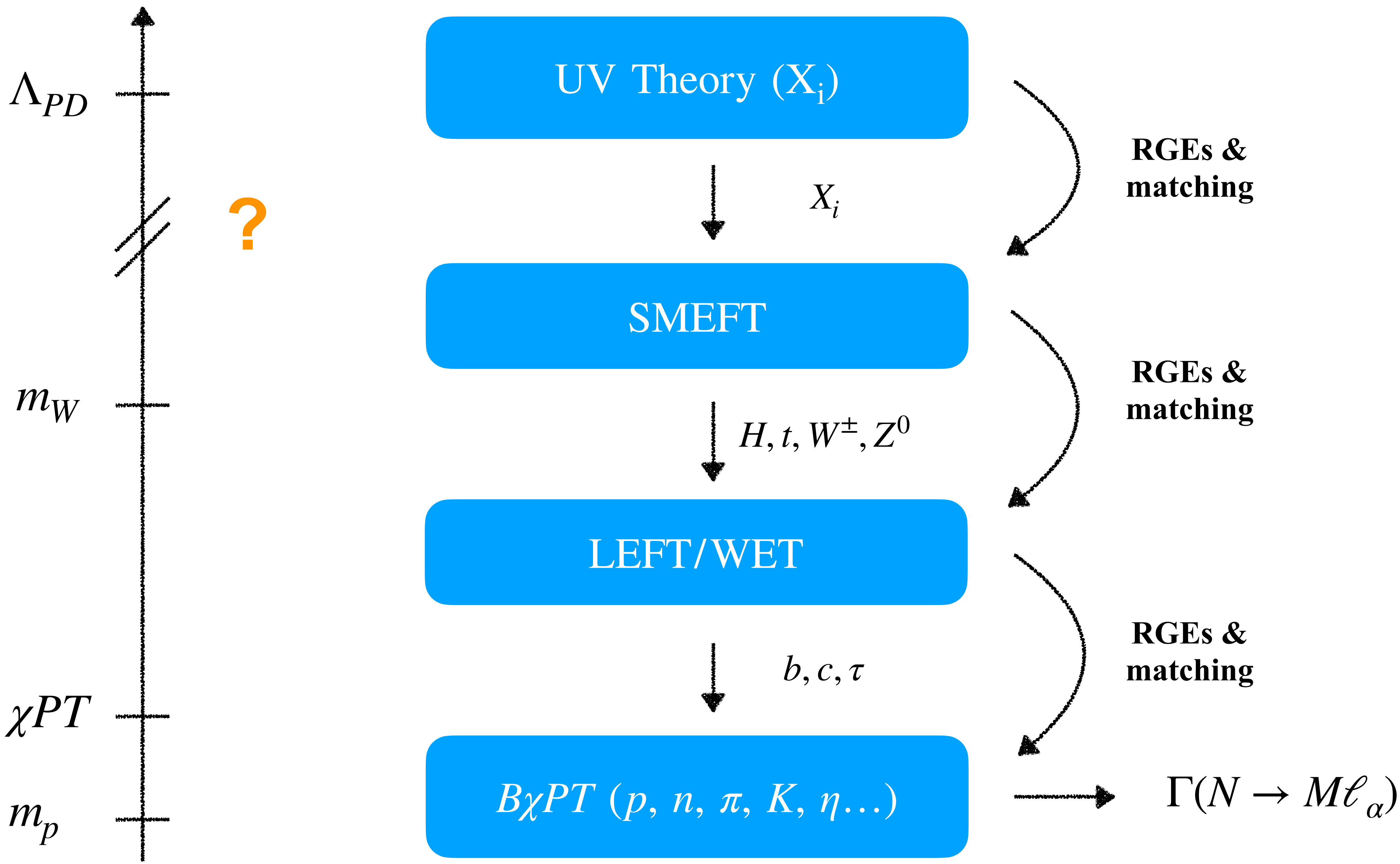
$$\mathscr{L} = \mathscr{L}_{SM} + \frac{c^{d=5}}{\Lambda}\mathcal{O}_W + \frac{c^{d=6}}{\Lambda^2}\mathcal{O}^{d=6} + \frac{c^{d=7}}{\Lambda^3}\mathcal{O}^{d=7} + \ldots$$

Different phenomenology →

$d=6$: $\Lambda \gtrsim 10^{15}$ GeV; $p \to \pi^0 e^+,\ \ p \to K^+ \bar{\nu}$

$d=7$: $\Lambda \gtrsim 10^{10}$ GeV; $n \to \pi^+ e^-,\ \ p \to K^+ \nu$

BNV within the SMEFT



· **Assumptions: Energy Desert and no SUSY in the TeV scale/RpV**

[S. Antusch et al. 2021, H. Dreiner et al. 2020]

SMEFT

$d = 6 \to 4$ (273) operators **[L. F. Abbott et al. 1980, B. Grzadkowski et al. 2010]**

$$\mathcal{O}_{qqql,pqrs} = (Q^i_p Q^j_q)(Q^l_r L^k_s)\epsilon_{ik}\epsilon_{jl} \,, \quad \mathcal{O}_{qque,pqrs} = (Q^i_p Q^j_q)(\bar{u}^\dagger_r \bar{e}^\dagger_s)\epsilon_{ij} \,,$$

$$\mathcal{O}_{duue,pqrs} = (\bar{d}^\dagger_p \bar{u}^\dagger_q)(\bar{u}^\dagger_r \bar{e}^\dagger_s) \,, \quad \mathcal{O}_{duql,pqrs} = (\bar{d}^\dagger_p \bar{u}^\dagger_q)(Q^i_r L^j_s)\epsilon_{ij} \,,$$

$d = 7 \to 6$ (297) operators **[L. Lehman 2014, Yi Liao et al. 2016]**

$$\mathcal{O}_{\bar{l}dddH,pqrs} = (L^\dagger_p \bar{d}^\dagger_q)(\bar{d}^\dagger_r \bar{d}^\dagger_s)H \,, \quad \mathcal{O}_{\bar{l}dqq\tilde{H},pqrs} = (L^\dagger_p \bar{d}^\dagger_q)(Q_r Q^i_s)\tilde{H}^j \epsilon_{ij} \,,$$

$$\mathcal{O}_{\bar{e}qdd\tilde{H},pqrs} = (\bar{e}_p Q^i_q)(\bar{d}^\dagger_r \bar{d}^\dagger_s)\tilde{H}^j \epsilon_{ij} \,, \quad \mathcal{O}_{\bar{l}dud\tilde{H},pqrs} = (L^\dagger_p \bar{d}^\dagger_q)(\bar{u}^\dagger_r \bar{d}^\dagger_s)\tilde{H} \,,$$

$$\mathcal{O}_{\bar{l}qdDd,pqrs} = (L^\dagger_p \bar{\sigma}^\mu Q_q)(\bar{d}^\dagger_r i\overleftrightarrow{D}_\mu \bar{d}^\dagger_s) \,, \quad \mathcal{O}_{\bar{e}dddD,pqrs} = (\bar{e}_p \sigma^\mu \bar{d}^\dagger_q)(\bar{d}^\dagger_r i\overleftrightarrow{D}_\mu \bar{d}^\dagger_s) \,,$$

SMEFT

$d = 6 \rightarrow 4$ (273) operators [L. F. Abbott et al. 1980, B. Grzadkowski et al. 2010]

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$$\mathcal{O}_{\bar{l}qdDd,pqrs} = (L^\dagger_p \bar{\sigma}^\mu Q_q)(\bar{d}^\dagger_r i\overleftrightarrow{D}_\mu \bar{d}^\dagger_s)\,, \quad \mathcal{O}_{\bar{e}dddD,pqrs} = (\bar{e}_p \sigma^\mu \bar{d}^\dagger_q)(\bar{d}^\dagger_r i\overleftrightarrow{D}_\mu \bar{d}^\dagger_s)\,,$$

$$c^{d=6}(m_W) \sim (2-4)\; c^{d=6}(10^{15}\ \text{GeV})$$
$$c^{d=7}(m_W) \sim (1-2)\; c^{d=7}(10^{11}\ \text{GeV})$$

From gauge interactions and y_t
(Operator mixing subdominant)

· RGEs for d = 6 SMEFT [A. Manohar et al. 2014]
· RGEs for d = 7 SMEFT [Yi Liao et al. 2016]

LEFT

$$288\ \Delta(B-L)=0 \text{ operators} \rightarrow 14 \text{ operators}$$

$$228\ \Delta(B+L)=0 \text{ operators} \rightarrow 9 \text{ operators}$$

} LEFT operators involved in nucleon decay at tree-level

Name [52]	SMEFT matching
$[\mathcal{O}^{S,LL}_{udd}]_{pqrs}$	$V_{q'q}V_{r'r}(C_{qqql,r'q'ps} - C_{qqql,q'r'ps} + C_{qqql,q'pr's})$
$[\mathcal{O}^{S,LL}_{duu}]_{pqrs}$	$V_{p'p}(C_{qqql,rqp's} - C_{qqql,qrp's} + C_{qqql,qp'rs})$
$[\mathcal{O}^{S,LR}_{duu}]_{pqrs}$	$-V_{p'p}(C_{qque,p'qrs} + C_{qque,qp'rs})$
$[\mathcal{O}^{S,RL}_{duu}]_{pqrs}$	$C_{duql,pqrs}$
$[\mathcal{O}^{S,RL}_{dud}]_{pqrs}$	$-V_{r'r}C_{duql,pqr's}$
$[\mathcal{O}^{S,RR}_{duu}]_{pqrs}$	$C_{duue,pqrs}$
$[\mathcal{O}^{S,LR}_{udd}]_{pqrs}$	$-V_{q'q}C_{\bar{l}dqq\tilde{H},rspq'}\frac{v}{\sqrt{2}\Lambda}$
$[\mathcal{O}^{S,LR}_{ddd}]_{pqrs}$	$V_{p'p}V_{q'q}(C_{\bar{l}dqq\tilde{H},rsq'p'} - C_{\bar{l}dqq\tilde{H},rsp'q'})\frac{v}{2\sqrt{2}\Lambda}$
$[\mathcal{O}^{S,RL}_{ddd}]_{pqrs}$	$V_{s's}(C_{\bar{e}qdd\tilde{H},rs'qp} - C_{\bar{e}qdd\tilde{H},rs'pq})\frac{v}{\sqrt{2}\Lambda}$
$[\mathcal{O}^{S,RR}_{udd}]_{pqrs}$	$C_{\bar{l}dud\tilde{H},rspq}\frac{v}{\sqrt{2}\Lambda}$
$[\mathcal{O}^{S,RR}_{ddd}]_{pqrs}$	$C_{\bar{l}dddH,rspq}\frac{v}{\sqrt{2}\Lambda}$

Name [52] ([12])	Operator	Flavour
$[\mathcal{O}^{S,LL}_{udd}]_{111r}$ (O^{ν}_{LL})	$(ud)(d\nu_r)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{udd}]_{121r}$ $(\tilde{O}^{\nu}_{LL1})$	$(us)(d\nu_r)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{udd}]_{112r}$ $(\tilde{O}^{\nu}_{LL2})$	$(ud)(s\nu_r)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{duu}]_{111r}$ (O^{e}_{LL})	$(du)(ue_r)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{duu}]_{211r}$ $(\tilde{O}^{e}_{LL})$	$(su)(ue_r)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LR}_{duu}]_{111r}$ (O^{e}_{LR})	$(du)(\bar{u}^\dagger\bar{e}_r^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,LR}_{duu}]_{211r}$ $(\tilde{O}^{e}_{LR})$	$(su)(\bar{u}^\dagger\bar{e}_r^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,RL}_{duu}]_{111r}$ (O^{e}_{RL})	$(\bar{d}^\dagger\bar{u}^\dagger)(ue_r)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{duu}]_{211r}$ $(\tilde{O}^{e}_{RL})$	$(\bar{s}^\dagger\bar{u}^\dagger)(ue_r)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{dud}]_{111r}$ (O^{ν}_{RL})	$(\bar{d}^\dagger\bar{u}^\dagger)(d\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{dud}]_{211r}$ $(\tilde{O}^{\nu}_{RL1})$	$(\bar{s}^\dagger\bar{u}^\dagger)(d\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{dud}]_{112r}$ $(\tilde{O}^{\nu}_{RL2})$	$(\bar{d}^\dagger\bar{u}^\dagger)(s\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$
~~$[\mathcal{O}^{S,RL}_{ddu}]_{[12]1r}$~~	~~$(\bar{d}^\dagger\bar{s}^\dagger)(u\nu_r)$~~	~~$(\mathbf{3},\bar{\mathbf{3}})$~~
$[\mathcal{O}^{S,RR}_{duu}]_{111r}$ (O^{e}_{RR})	$(\bar{d}^\dagger\bar{u}^\dagger)(\bar{u}^\dagger\bar{e}_r^\dagger)$	$(\mathbf{1},\mathbf{8})$
$[\mathcal{O}^{S,RR}_{duu}]_{211r}$ $(\tilde{O}^{e}_{RR})$	$(\bar{s}^\dagger\bar{u}^\dagger)(\bar{u}^\dagger\bar{e}_r^\dagger)$	$(\mathbf{1},\mathbf{8})$

Name	Operator	Flavour
~~$[\mathcal{O}^{S,LL}_{ddd}]_{[12]r1}$~~	~~$(ds)(\bar{e}_r d)$~~	~~$(\mathbf{8},\mathbf{1})$~~
$[\mathcal{O}^{S,LR}_{udd}]_{11r1}$	$(ud)(\nu_r^\dagger\bar{d}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,LR}_{udd}]_{12r1}$	$(us)(\nu_r^\dagger\bar{d}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,LR}_{udd}]_{11r2}$	$(ud)(\nu_r^\dagger\bar{s}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
~~$[\mathcal{O}^{S,LR}_{ddu}]_{[12]r1}$~~	~~$(ds)(\nu_r^\dagger\bar{u}^\dagger)$~~	~~$(\bar{\mathbf{3}},\mathbf{3})$~~
$[\mathcal{O}^{S,LR}_{ddd}]_{[12]r1}$	$(ds)(e_r^\dagger\bar{d}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,RL}_{ddd}]_{[12]r1}$	$(\bar{d}^\dagger\bar{s}^\dagger)(\bar{e}_r d)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RR}_{udd}]_{11r1}$	$(\bar{u}^\dagger\bar{d}^\dagger)(\nu_r^\dagger\bar{d}^\dagger)$	$(\mathbf{1},\mathbf{8})$
$[\mathcal{O}^{S,RR}_{udd}]_{12r1}$	$(\bar{u}^\dagger\bar{s}^\dagger)(\nu_r^\dagger\bar{d}^\dagger)$	$(\mathbf{1},\mathbf{8})$
$[\mathcal{O}^{S,RR}_{udd}]_{11r2}$	$(\bar{u}^\dagger\bar{d}^\dagger)(\nu_r^\dagger\bar{s}^\dagger)$	$(\mathbf{1},\mathbf{8})$
$[\mathcal{O}^{S,RR}_{ddd}]_{[12]r1}$	$(\bar{d}^\dagger\bar{s}^\dagger)(e_r^\dagger\bar{d}^\dagger)$	$(\mathbf{1},\mathbf{8})$

- RG effects universal in the LEFT

$c(2\text{ GeV}) \sim 1.26\ c(m_W)$

(red strike-through): Not generated by D = 6, 7 SMEFT ops.

· RGEs for d = 6 LEFT [A. Manohar et al. 2018]

BχPT

$$M = \sum_{a=1}^{8} M_a \frac{\lambda_a}{\sqrt{2}} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

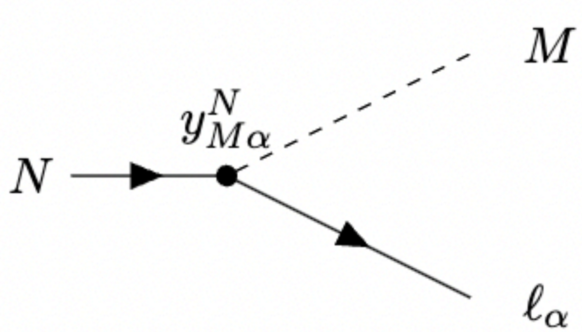
$$B = \sum_{a=1}^{8} B_a \frac{\lambda_a}{\sqrt{2}} = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix}$$

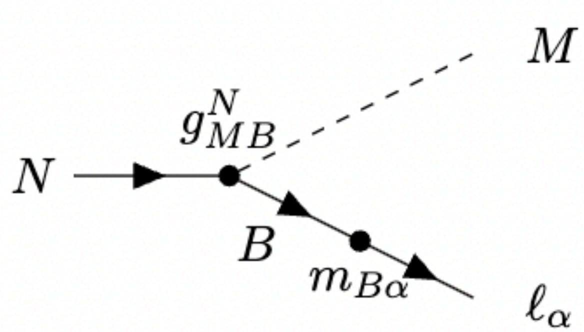
Flavour group $U(3)_L \times U(3)_R$

BNC interactions **BNV interactions**

$$\mathcal{L} = g^N_{MB} \overline{B} \gamma^\mu \gamma_5 N \partial_\mu M + m_{B\alpha,X} \overline{\ell_\alpha} P_{\bar{X}} B + i y^N_{M\alpha,X} \overline{\ell_\alpha} P_{\bar{X}} N M$$

[M. Claudson et al. 1981, P. Nath et al. 2007]





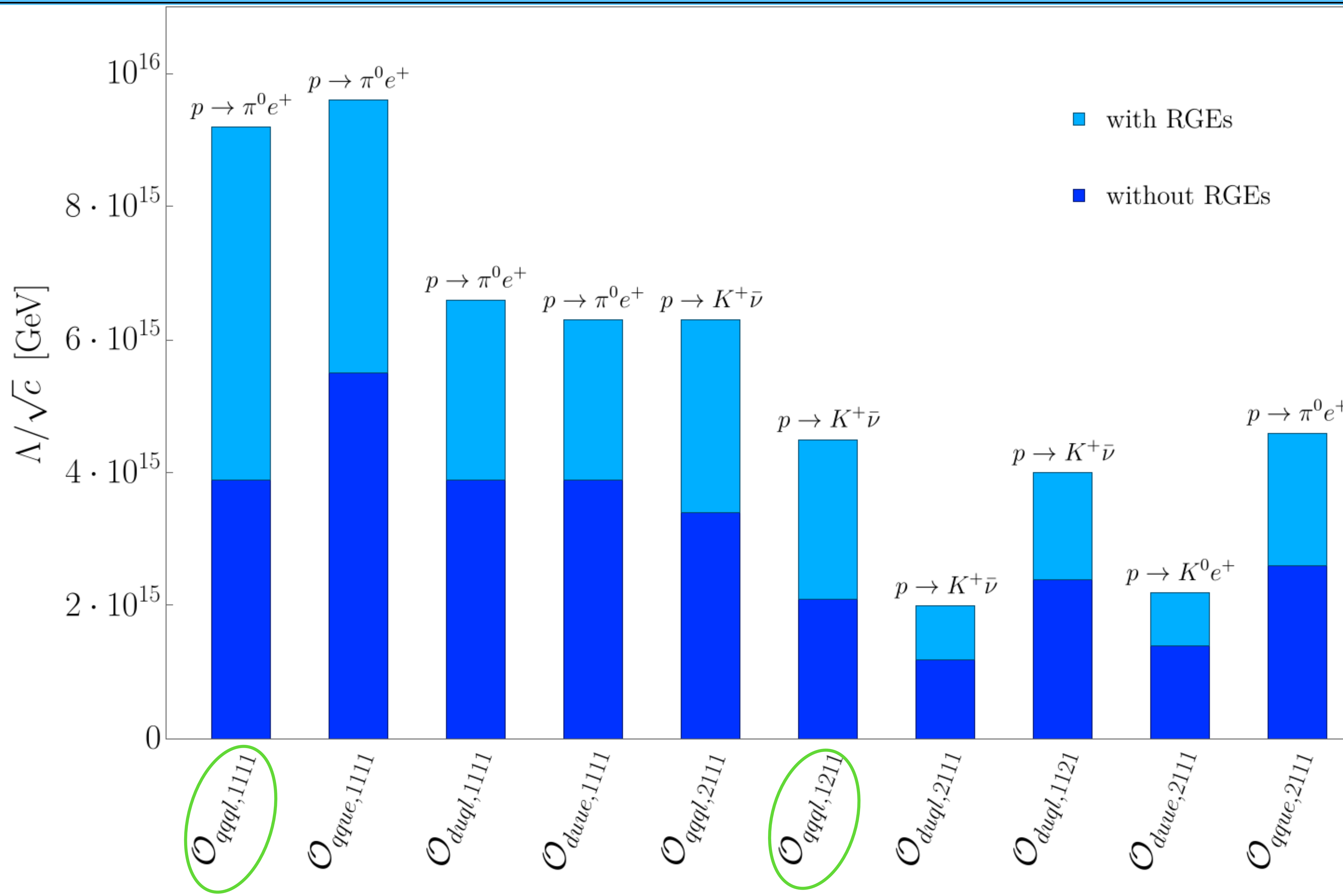
$\Gamma(N \to M\ell_\alpha)$

2 inputs from lattice α, β

[JLQCD 2000, Y. Aoki et al. 2017,]

(First-time computation of $|\Delta(B-L)| = 2$ two-body decays in the BχPT formalism)

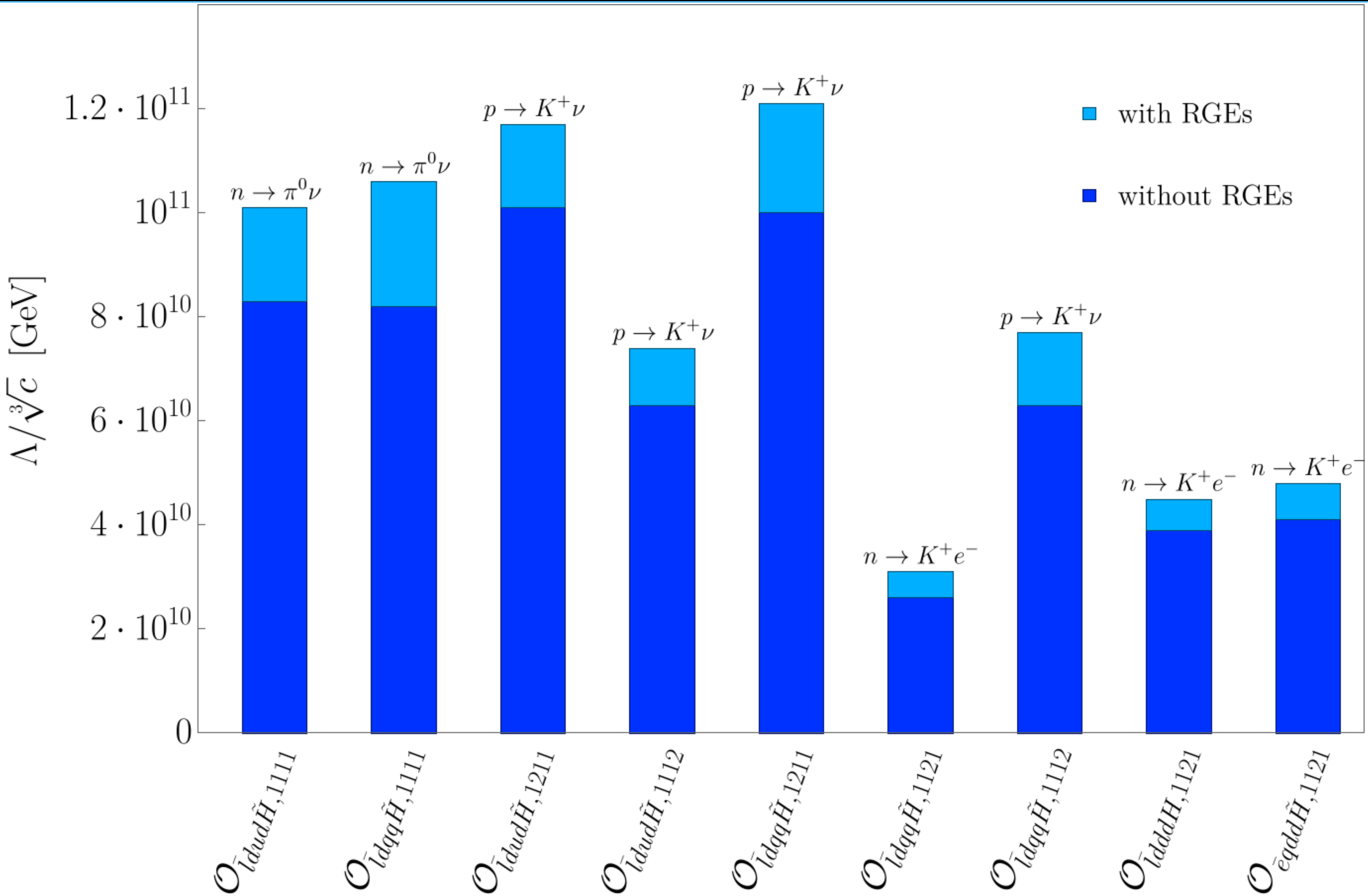
D = 6 limits



$$\Lambda/\sqrt{c} \sim (1 \sim 10) \times 10^{15} \text{ GeV}$$

Bounds on other flavor components can be found in **[J. Gargalionis et al. 2024]**

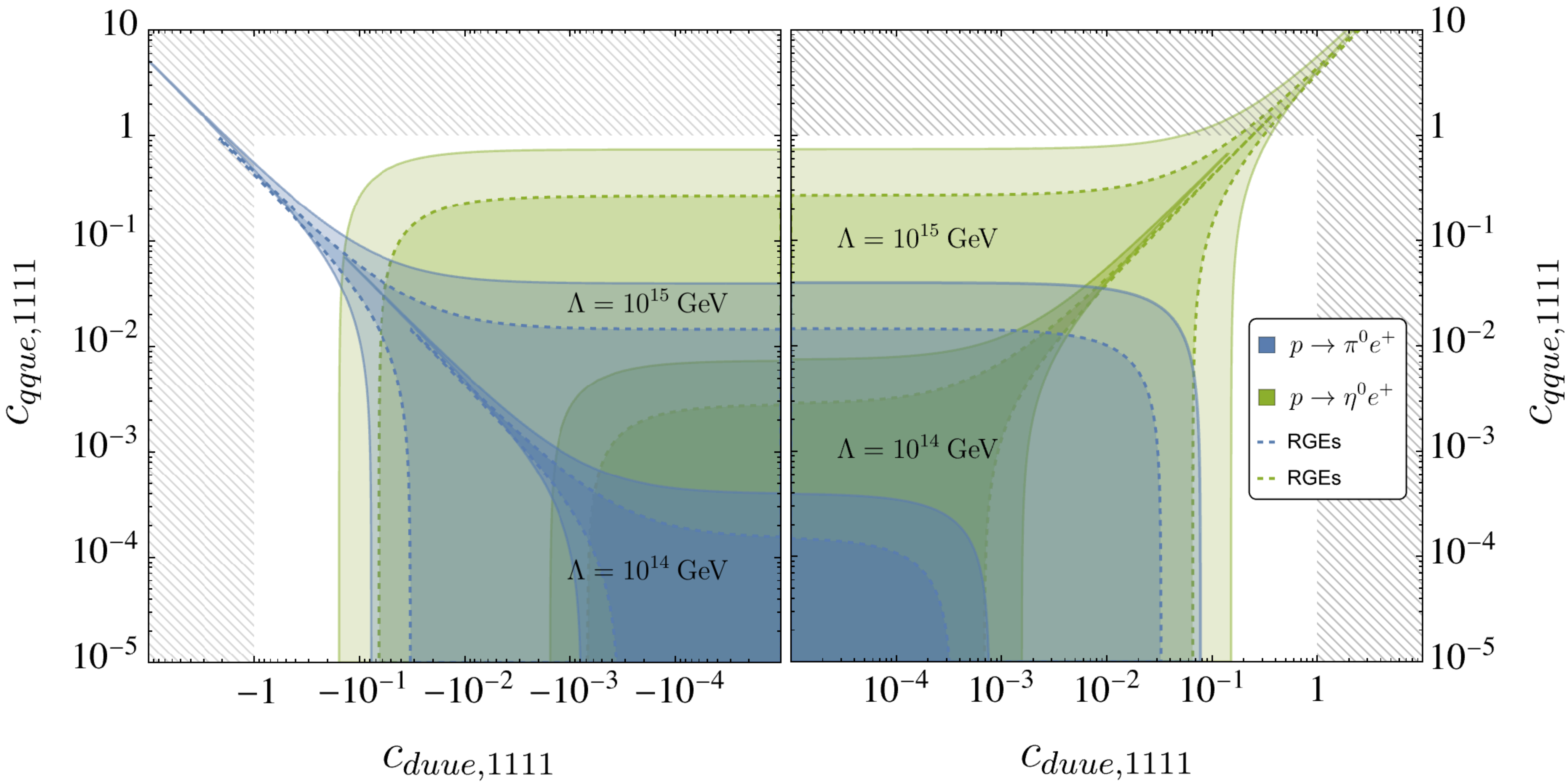
D = 7 limits



$$\Lambda/\sqrt[3]{c} \sim (2 \sim 12) \times 10^{10}\ \mathrm{GeV}$$

Bounds on other flavor components can be found in **[J. Gargalionis et al. 2024]**

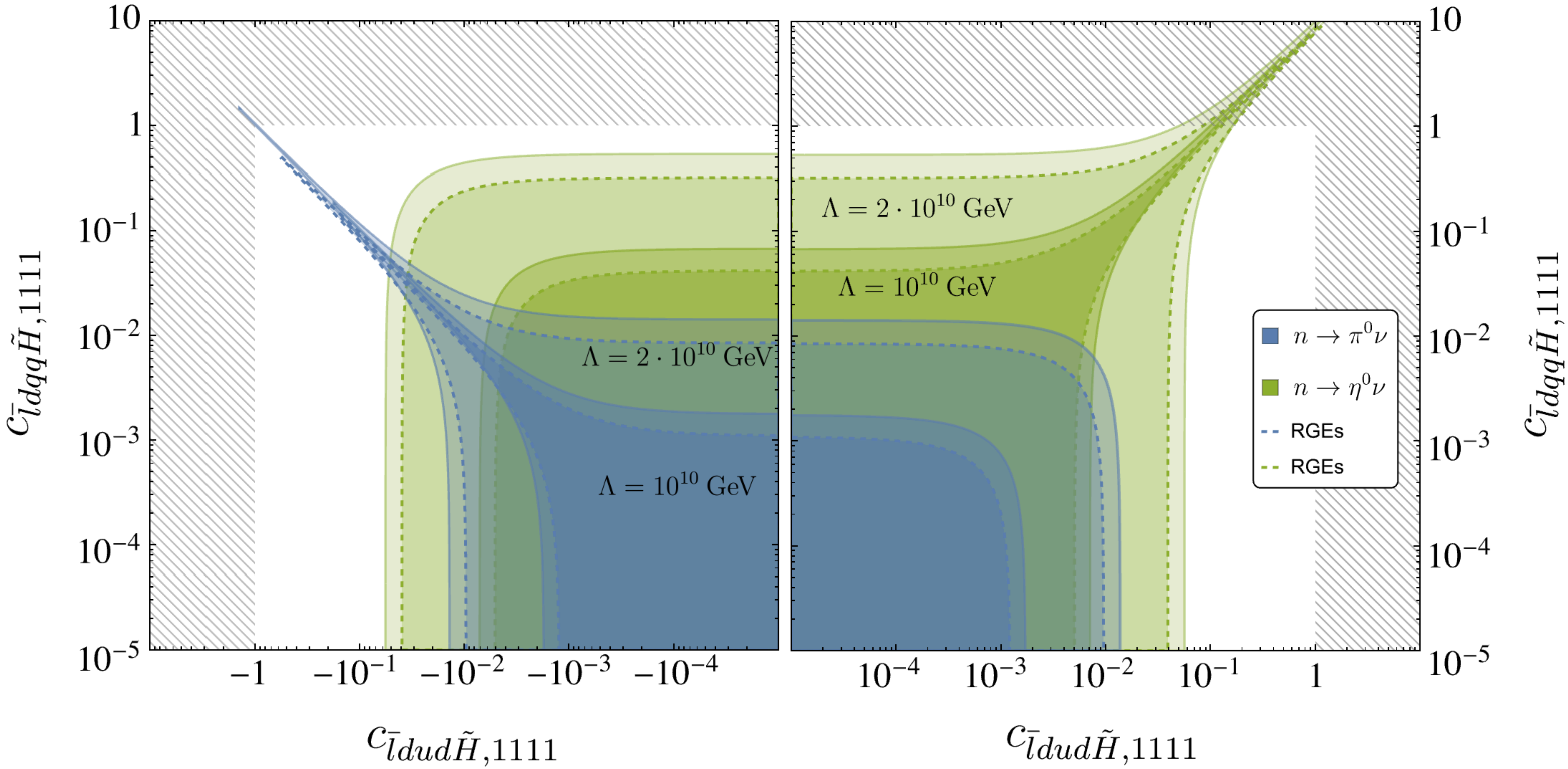
D = 6 pairs of WCs



· Different search channels provide complementary constraints

· No flat directions

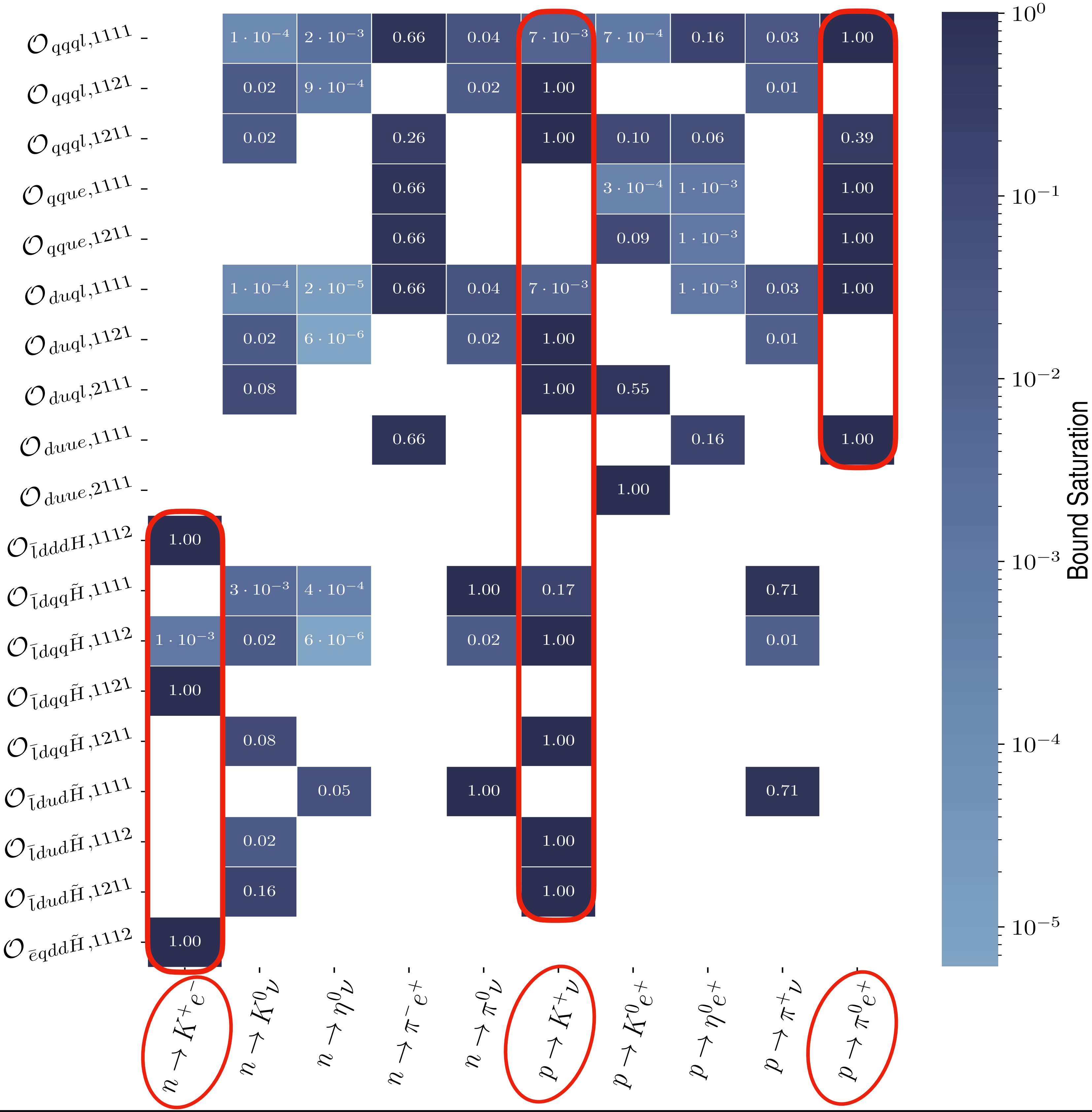
D = 7 pairs of WCs



· Different search channels provide complementary constraints

· No flat directions

Correlations

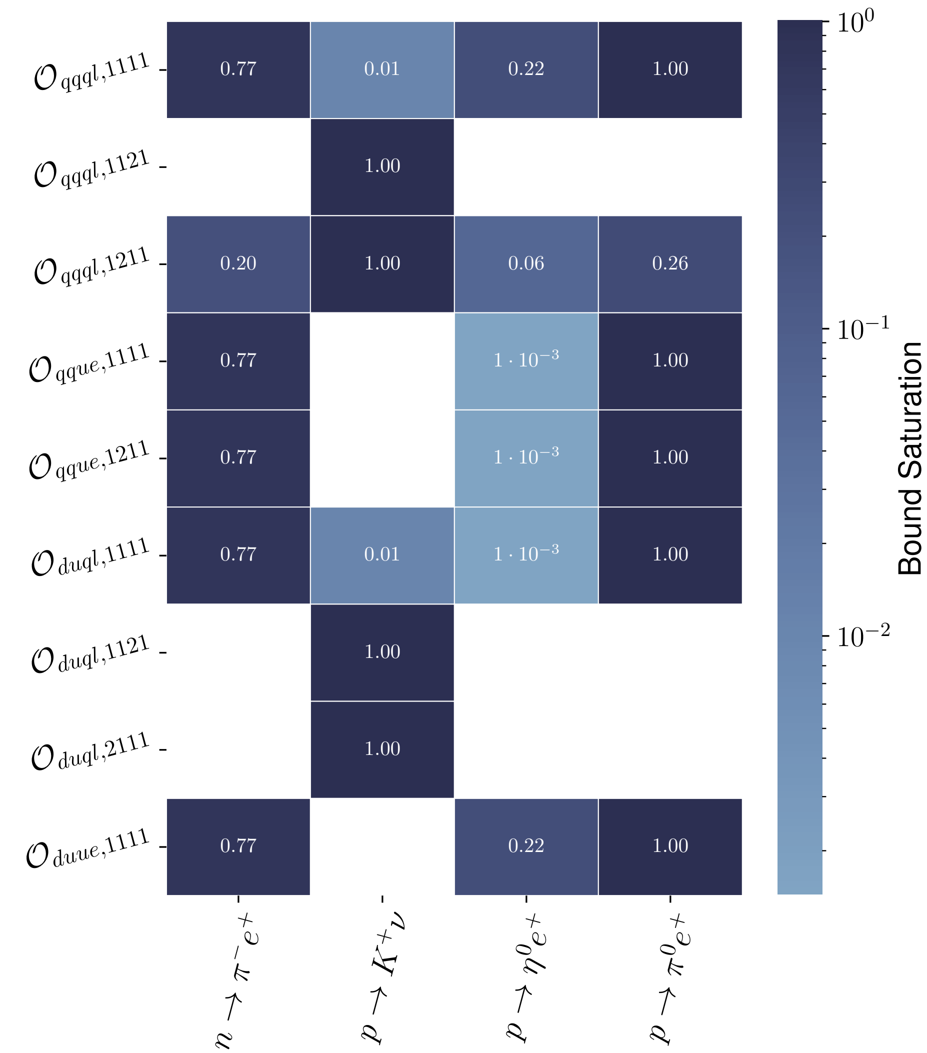


$$\text{Bound Saturation} \equiv \frac{\Gamma_{\text{i,th}}/\Gamma_{\text{max,th}}}{\Gamma_{\text{i,exp}}/\Gamma_{\text{max,exp}}}$$

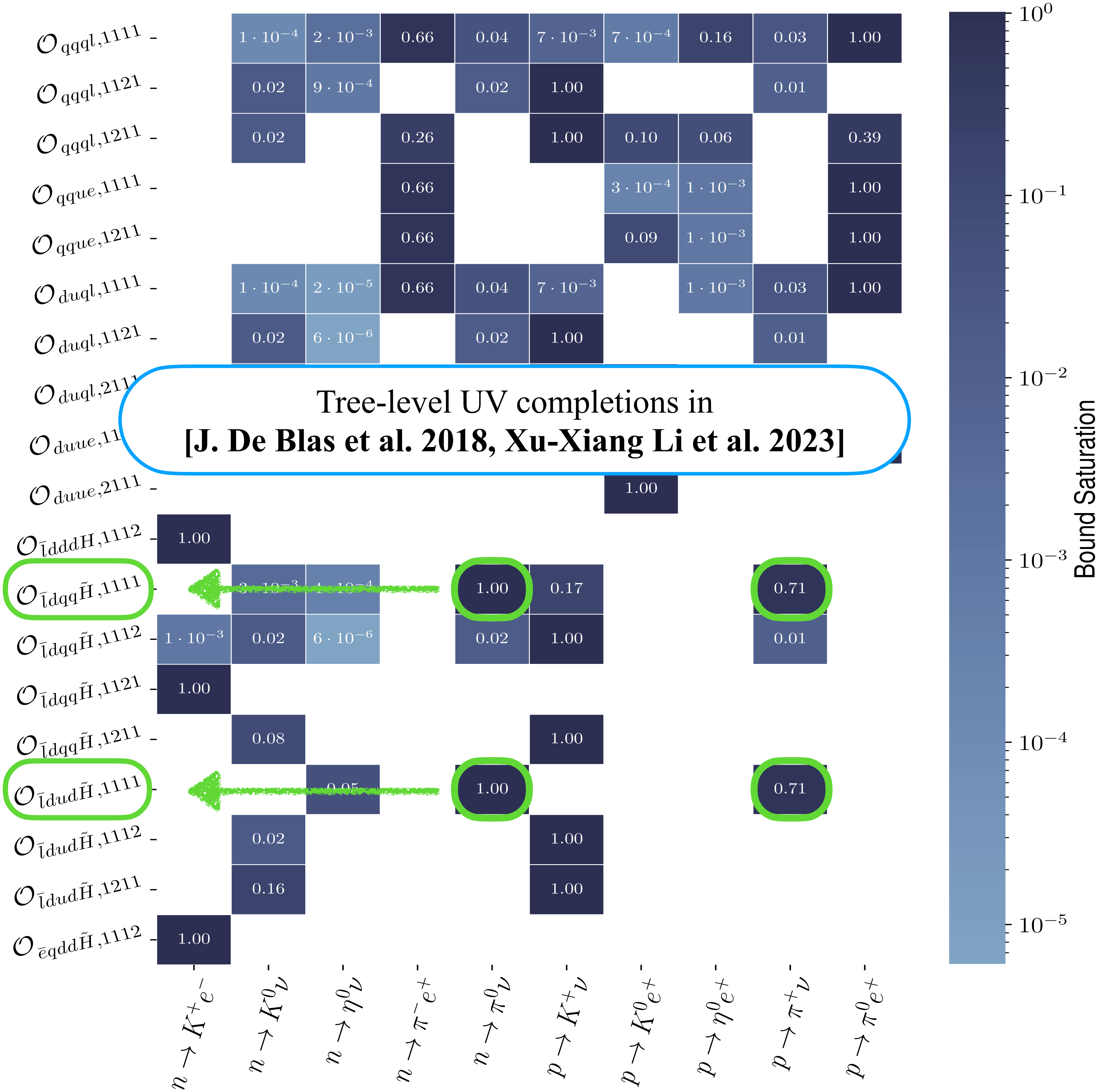
$$\tau_{p\to\pi^0e^+} > 24\cdot10^{33}\ \text{years}$$

$$\tau_{p\to K^+\nu} > 6.61\cdot10^{33}\ \text{years}$$

$$\tau_{n\to K^+e^-} > 0.032\cdot10^{33}\ \text{years}$$



Correlations



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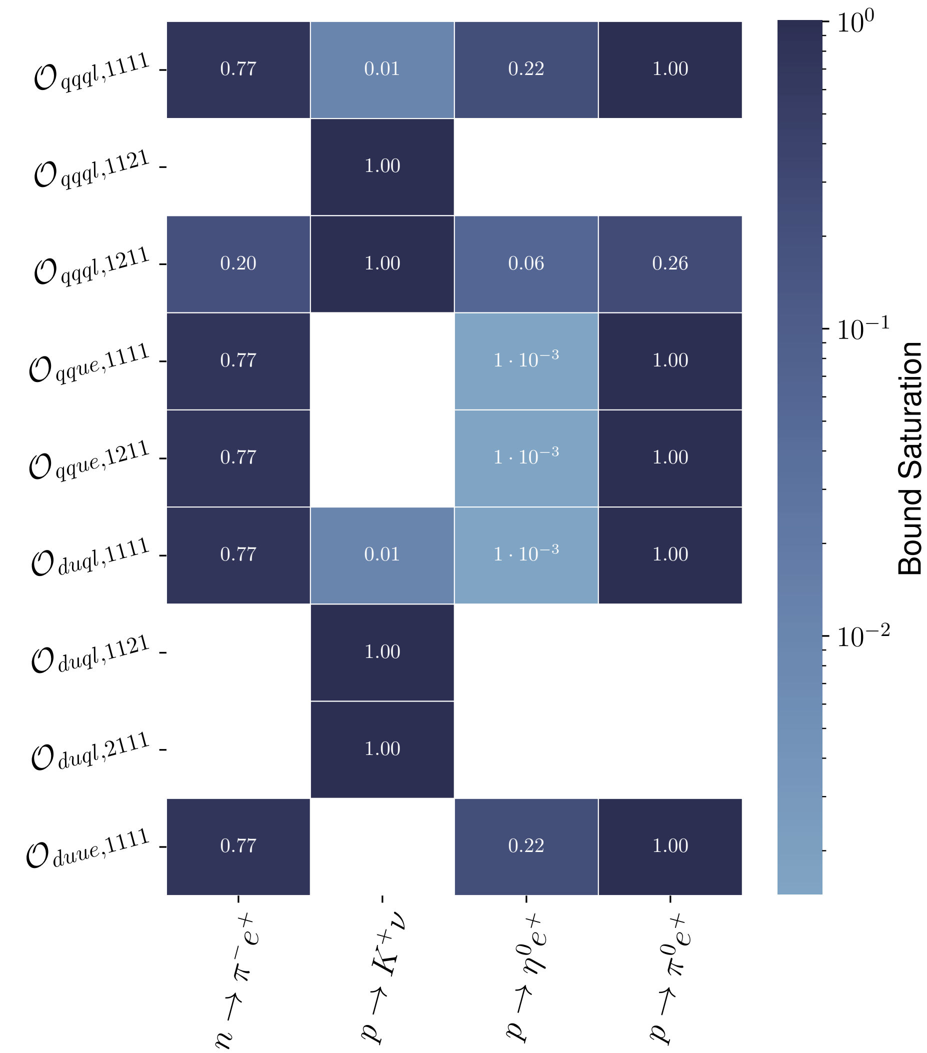

Phenomenological matrices

Numerical κ-matrices available online

$$\Gamma_{(i)}^{\Delta(B-L)=0} \equiv 10^{-4}\, c_j^* \boxed{\kappa_{(i)}^{jk}}\, c_k \frac{m_p^5}{\Lambda^4} \quad \text{for} \quad i = p \to \pi^0 e^+, p \to K^+\nu \ldots \quad \text{(9 matrices)}$$

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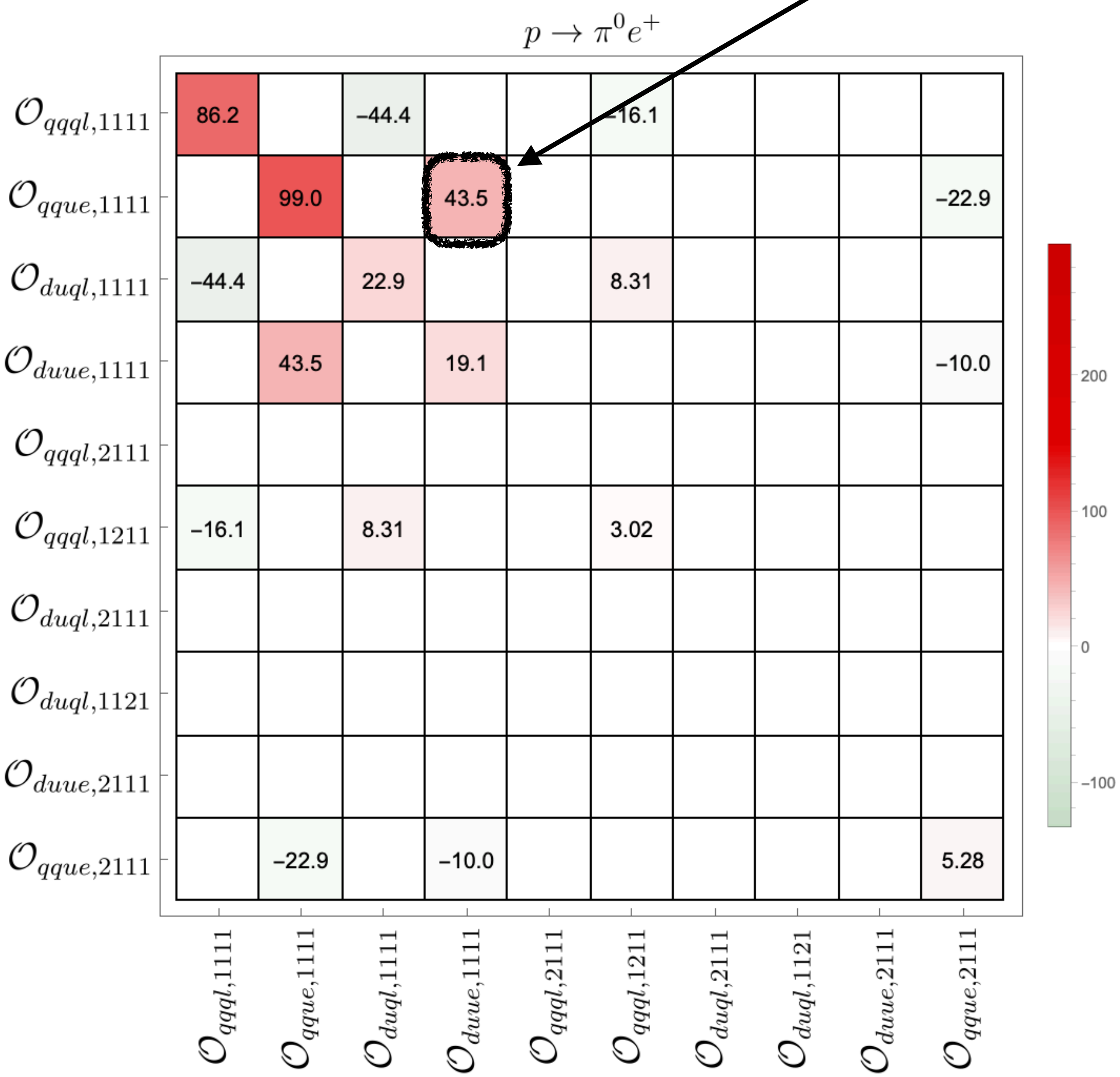
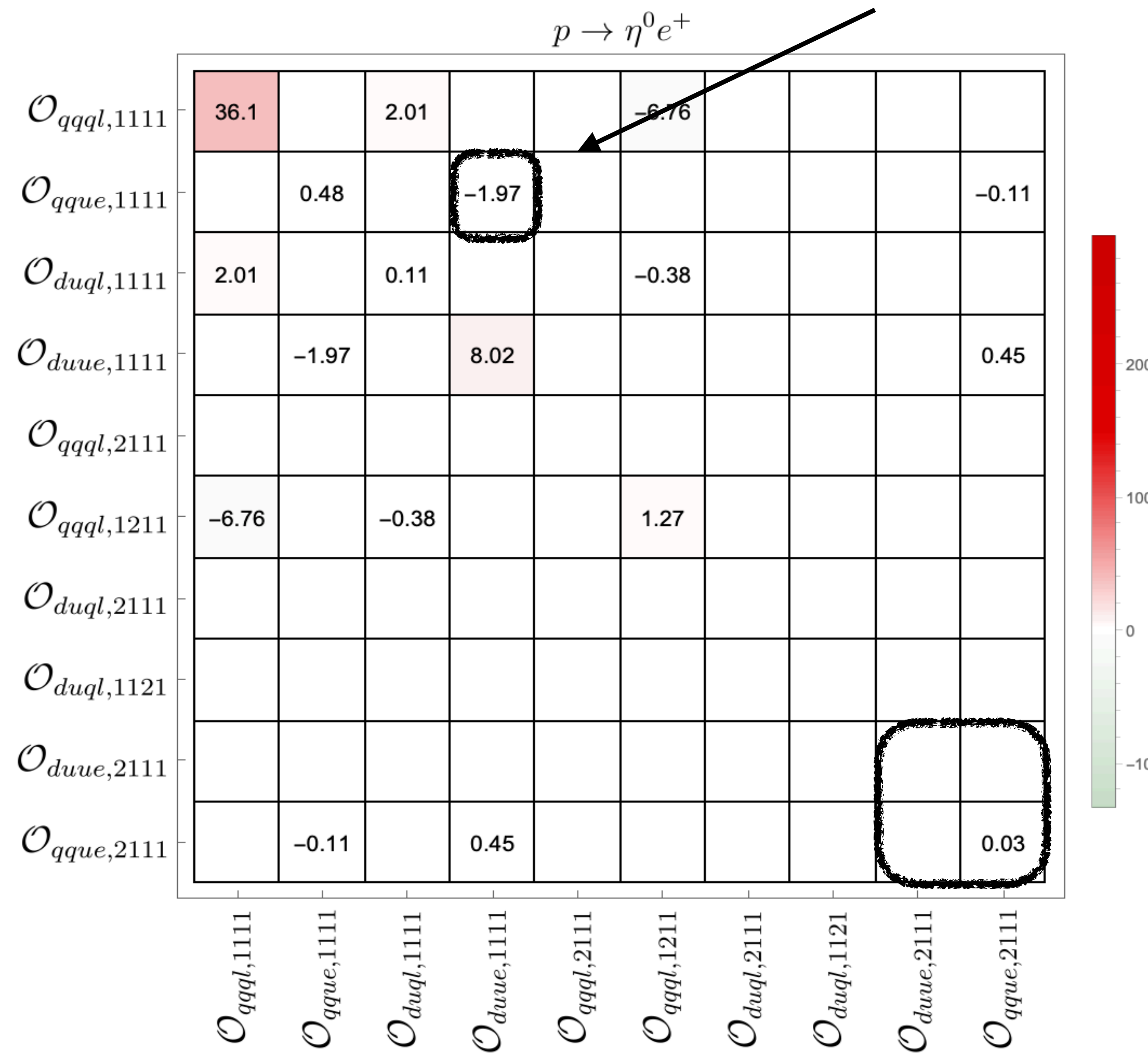
$c_{qque,1111} \sim -0.44\ c_{duue,1111}$

$c_{qque,1111} \sim 4.1\ c_{duue,1111}$





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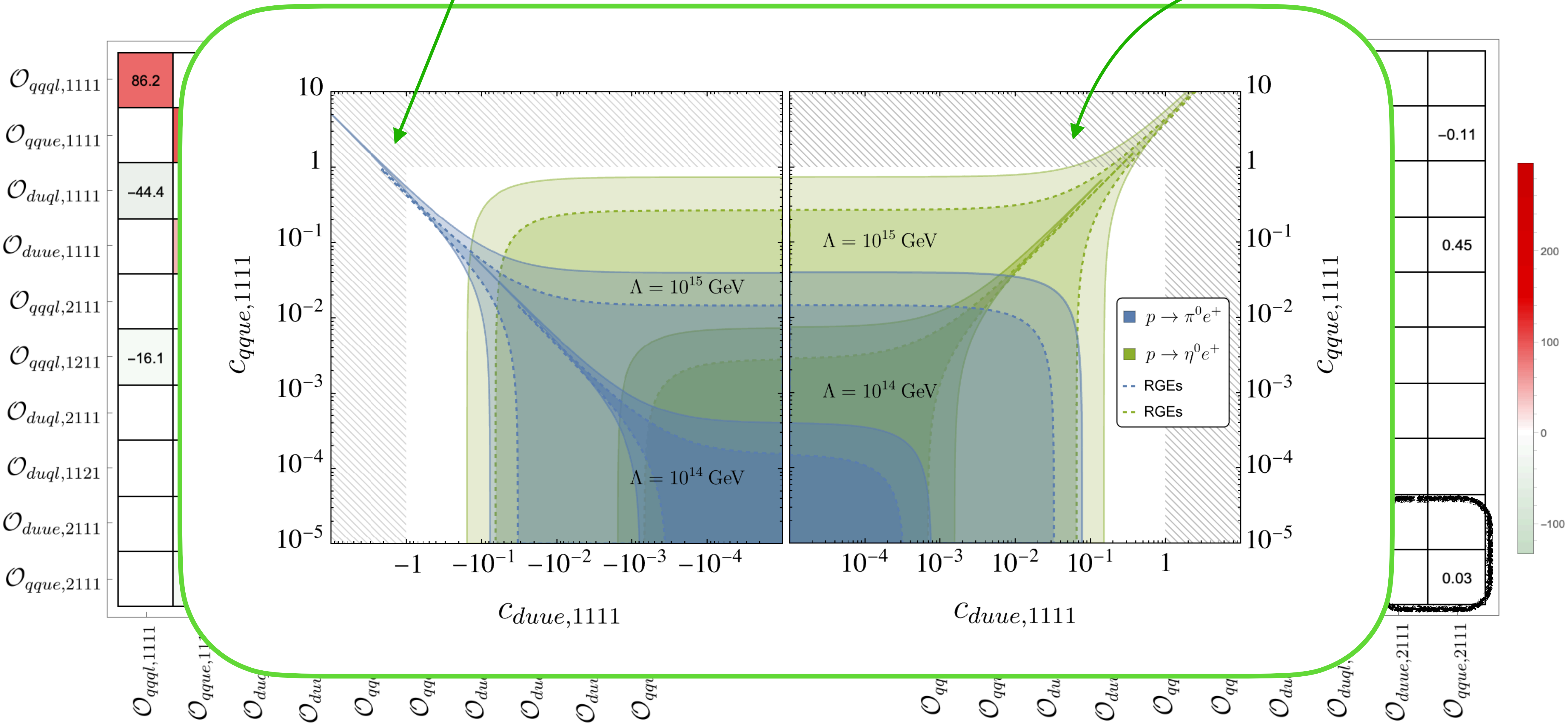
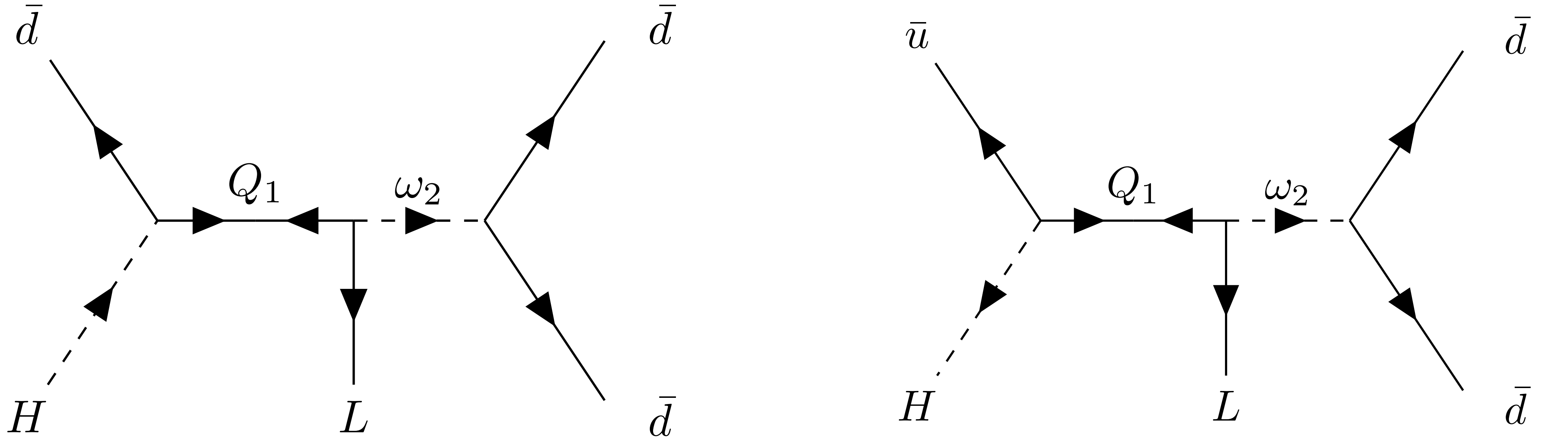


Example UV model

SM enhanced by a scalar LQ ω_2 and a VLF Q_1

$$\omega_2 \sim (3,\ 1,\ 2/3)\,,\ Q_1 + \bar{Q}_1^\dagger \sim (3,\ 2,\ 1/6)$$

$$\mathscr{L}_{\text{int}} = y_{1,ij}\omega_2 \bar{d}^{\dagger i}\bar{d}^{\dagger j} + y_{2,k}H^\dagger Q_1 \bar{d}^k + y_{3,k}\ Q_1 \epsilon H \bar{u}^k + y_{4,l}\ \omega_2 \bar{Q}_1 L^l + \text{h.c.}$$



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$$\downarrow$$

$$\mathscr{L}_{\text{eff}} \supset C_{\bar{l}dddH,pqrs}\mathcal{O}_{\bar{l}dddH,pqrs} + C_{\bar{l}dud\tilde{H},pqrs}\mathcal{O}_{\bar{l}dud\tilde{H},pqrs} + \text{h.c.}$$

y_1 antisymmetric:

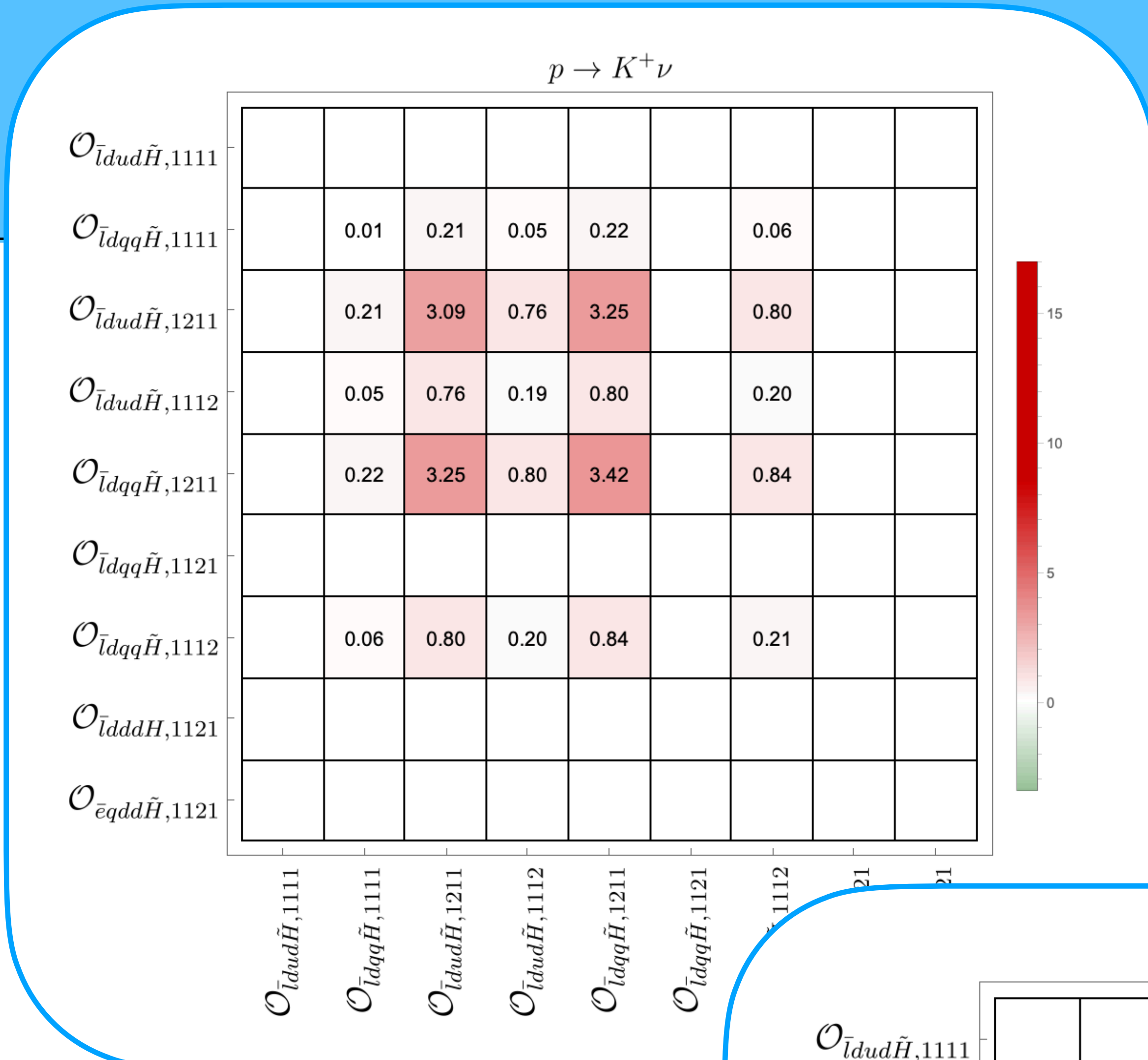
- $p \to K^+\nu \quad n \to K^0\nu \quad n \to K^+e^-$
- $C_{\bar{l}dud\tilde{H},1211} = -C_{\bar{l}dud\tilde{H},1112}$

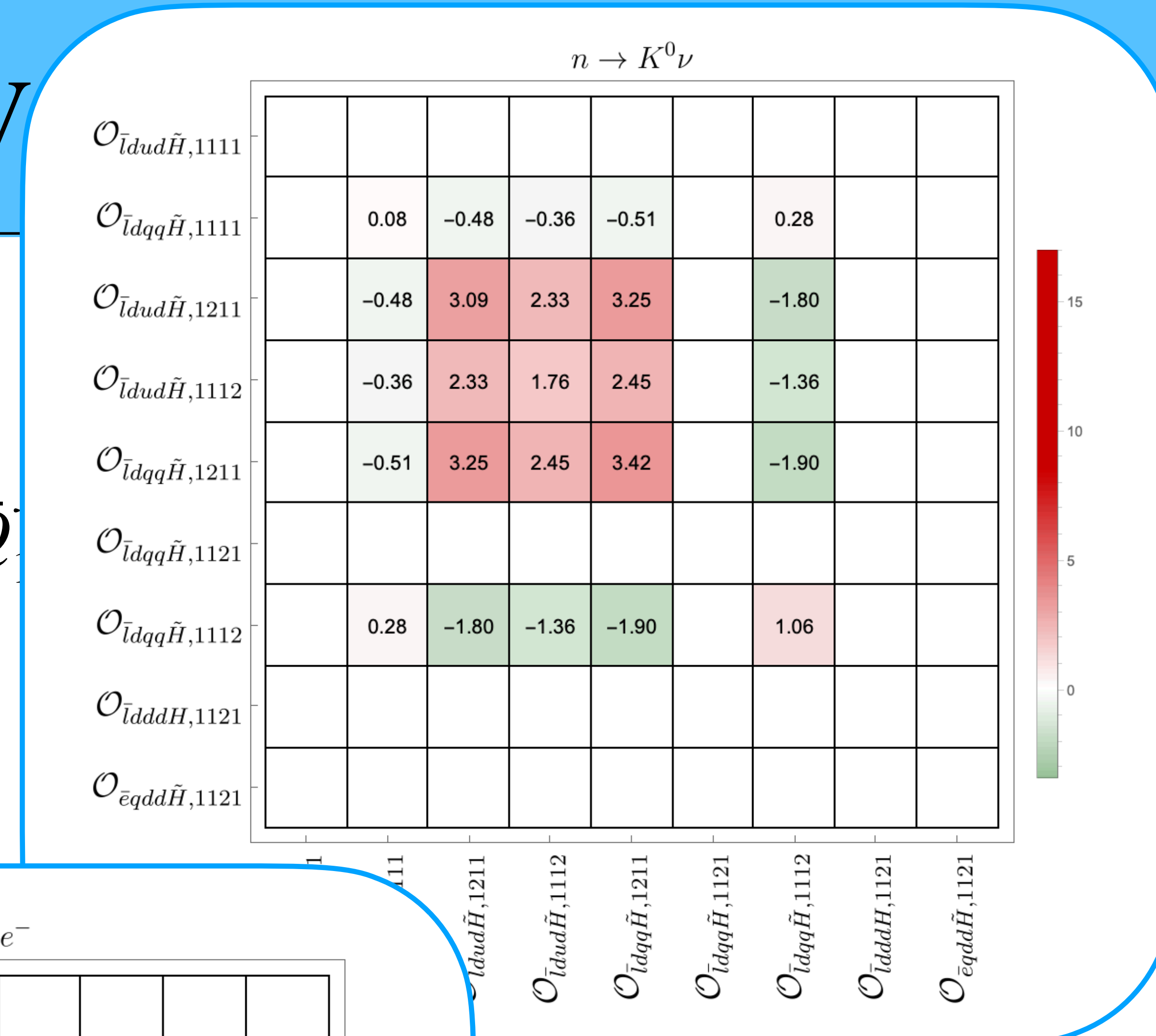
ple UV

scalar LQ

$) \,,\ Q_1 + \bar{Q}$

$Q_1\bar{d}^k + y_{3,k}$

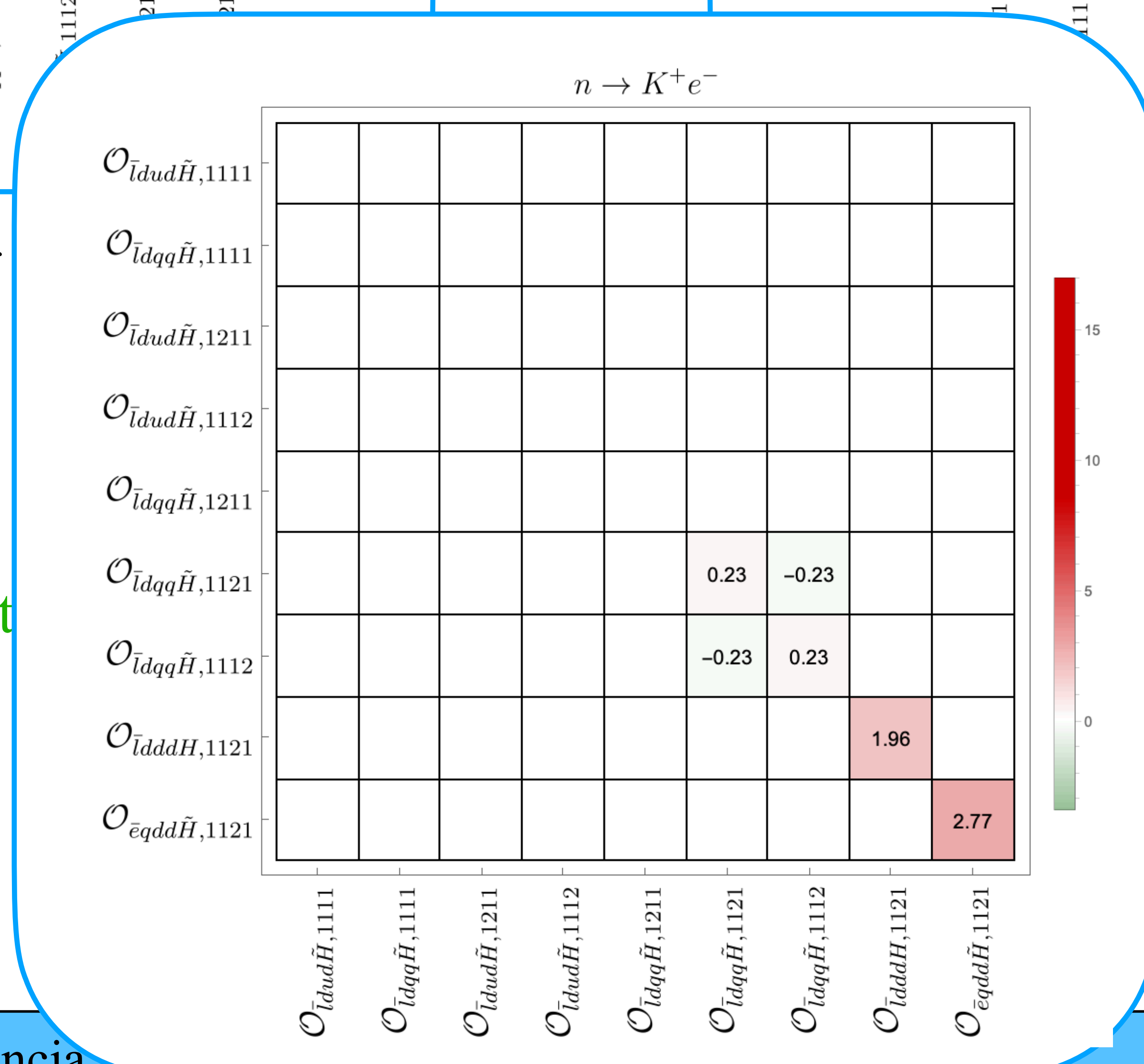




$\mathscr{L}_{\text{eff}}$ + h.c.

K^+e^-

y_1 ant



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- $C_{\bar{l}dud\tilde{H},1211} = -C_{\bar{l}dud\tilde{H},1112}$

κ-matrices for the 3 processes above, compute Γ and compare with Γ^{exp} $\longrightarrow$ **$p \to K^+\nu$ the most constraining**

Main results of this work

· Model-independent analysis on nucleon decay

· RG effects important: limits enhanced by 30% - 130% (d=6) , and 20 - 30% (d=7)

· Complementary analysis → Correlations and flat directions

· κ-matrices: SMEFT WC at $\Lambda \leftrightarrow$ observables at m_p

· Positive signals in 2-3 channels → SMEFT operators → GUT/Models

Main source of uncertainty: nuclear matrix elements α, β

Thank you!

Backup slides

RGEs

$$\left(\dot{C}_i \equiv 16\pi^2\mu\frac{dC_i}{d\mu} = \sum_j \gamma_{ij}C_j\right)$$

$$\dot{C}_{duue,prst} = \left(-4g_3^2 - 2g_1^2\right) C_{duue,prst} - \frac{20}{3} g_1^2\, C_{duue,psrt}$$

$$\dot{C}_{duq\ell,prst} = \left(-4g_3^2 - \frac{9}{2}g_2^2 - \frac{11}{6}g_1^2\right) C_{duq\ell,prst}$$

$$\dot{C}_{qque,prst} = \left(-4g_3^2 - \frac{9}{2}g_2^2 - \frac{23}{6}g_1^2\right) C_{qque,prst}$$

$$\dot{C}_{qqq\ell,prst} = \left(-4g_3^2 - 3g_2^2 - \frac{1}{3}g_1^2\right) C_{qqq\ell,prst} - 4g_2^2\left(C_{qqq\ell,rpst} + C_{qqq\ell,srpt} + C_{qqq\ell,psrt}\right)$$

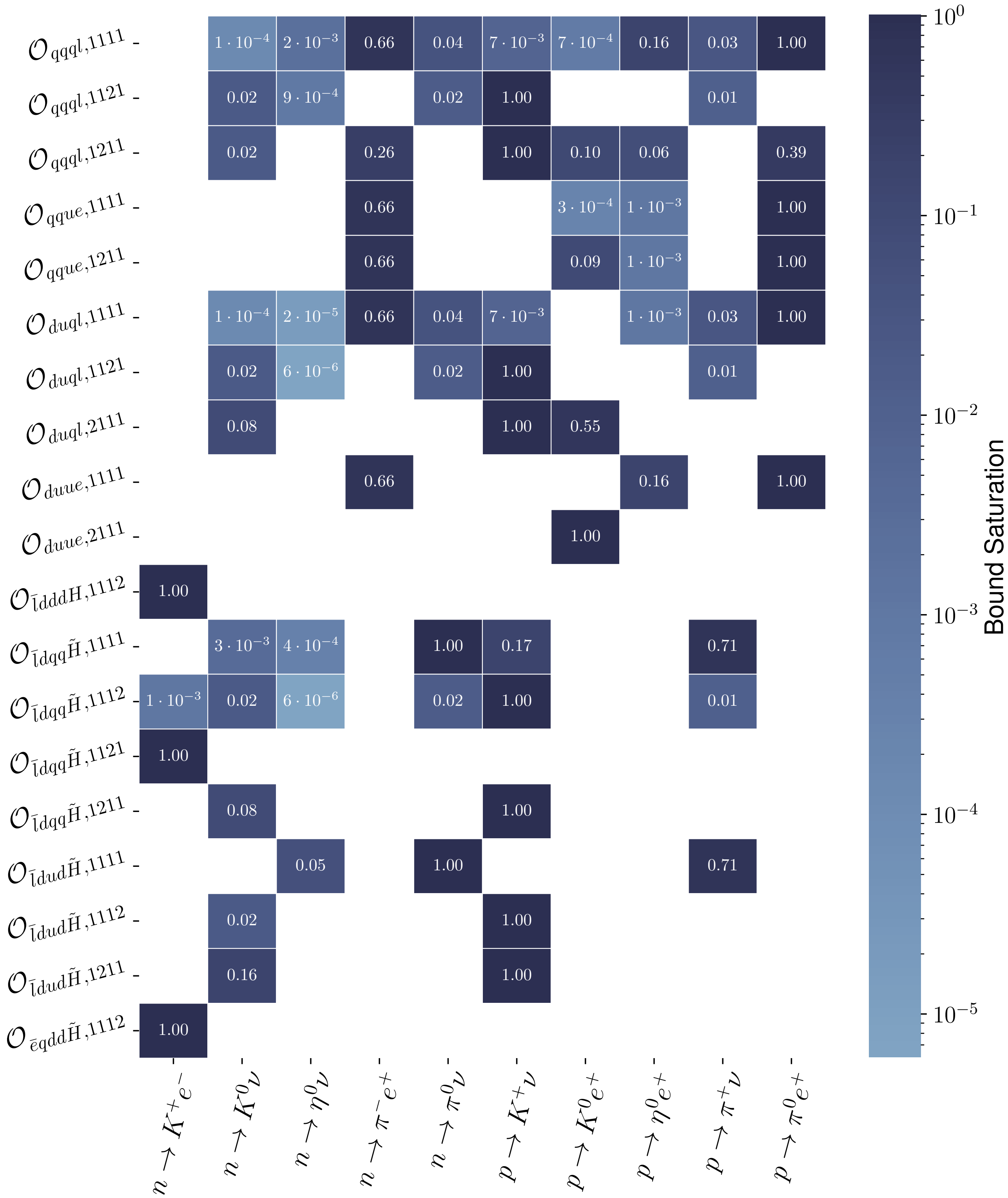
$$\dot{C}_{\bar{l}dud\tilde{H},prst} = \left(-4g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2 + y_t^2\right) C_{\bar{l}dud\tilde{H},prst} - \frac{10}{3} g_1^2\, C_{\bar{l}dud\tilde{H},ptsr}\,,$$

$$\dot{C}_{\bar{l}dddH,prst} = \left(-4g_3^2 - \frac{9}{4}g_2^2 - \frac{13}{12}g_1^2 + y_t^2\right) C_{\bar{l}dddH,prst}\,,$$

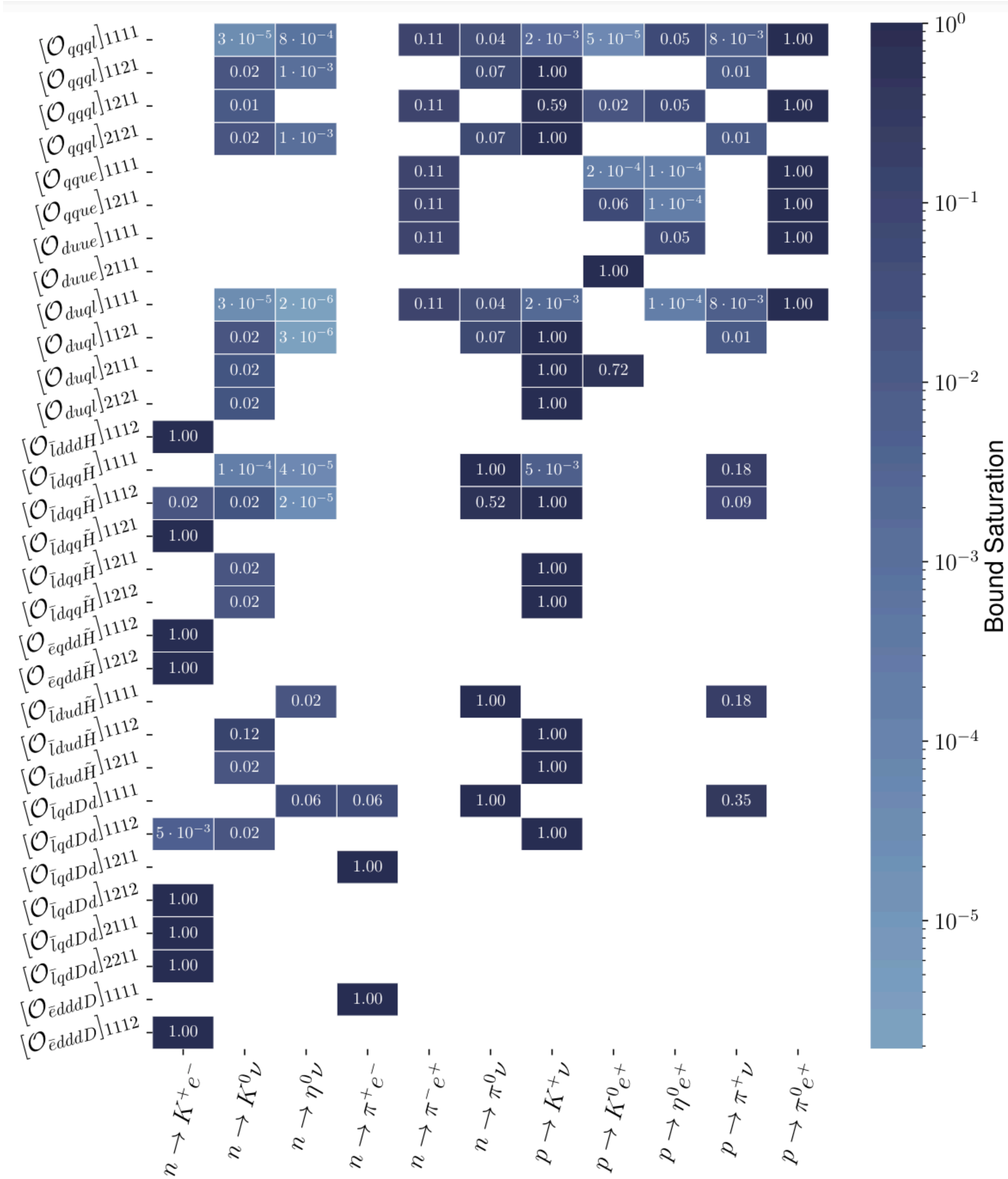
$$\dot{C}_{\bar{e}qdd\tilde{H},prst} = \left(-4g_3^2 - \frac{9}{4}g_2^2 + \frac{11}{12}g_1^2 + y_t^2\right) C_{\bar{e}qdd\tilde{H},prst}\,,$$

$$\dot{C}_{\bar{l}dqq\tilde{H},prst} = \left(-4g_3^2 - \frac{15}{4}g_2^2 - \frac{19}{12}g_1^2 + y_t^2\right) C_{\bar{l}dqq\tilde{H},prst} - 3g_2^2\, C_{\bar{l}dqq\tilde{H},prts}\,.$$

Direct VS Indirect method



BχPT



Direct Method in [J. Gargalionis et al. 2024]

Direct VS Indirect method

$$\Gamma(N \to M + \ell) = \frac{m_N}{32\pi}\left(1 - \frac{m_M^2}{m_N^2}\right)^2 \left|\sum_I C_I W_0^I(N \to M)\right|^2$$

$W_0^I(N \to M)$ computed in the lattice (Several parameters)

$$\Gamma\left(p \to \pi^+\nu_r\right) = (32\pi f_\pi^2 m_p^3)^{-1}(m_p^2 - m_\pi^2)^2 \left|\alpha \left[L_{udd}^{S,LR}\right]_{11r1} + \beta \left[L_{udd}^{S,RR}\right]_{11r1}\right|^2 (1 + D + F)^2$$

$$\Gamma\left(n \to K^+e_r^-\right) = (32\pi f_\pi^2 m_n^3)^{-1}(m_n^2 - m_K^2)^2 \times$$
$$\left\{\left|\beta \left[L_{ddd}^{S,LL}\right]_{12r1} - \alpha \left[L_{ddd}^{S,RL}\right]_{12r1} + \frac{m_n}{m_\Sigma}\left(\alpha \left[L_{ddd}^{S,RL}\right]_{12r1} + \beta \left[L_{ddd}^{S,LL}\right]_{12r1}\right)(D - F)\right|^2\right.$$
$$\left. + \left|\beta \left[L_{ddd}^{S,RR}\right]_{12r1} - \alpha \left[L_{ddd}^{S,LR}\right]_{12r1} + \frac{m_n}{m_\Sigma}\left(\alpha \left[L_{ddd}^{S,LR}\right]_{12r1} + \beta \left[L_{ddd}^{S,RR}\right]_{12r1}\right)(D - F)\right|^2\right\}$$

D, F, f_π

low-energy BχPT constants

$$\Gamma\left(n \to K^0\nu_r\right) = (32\pi f_\pi^2 m_n^3)^{-1}(m_n^2 - m_K^2)^2 \times$$
$$\left|-\alpha \left[L_{udd}^{S,LR}\right]_{12r1} + \beta \left[L_{udd}^{S,RR}\right]_{12r1} + \alpha \left[L_{udd}^{S,LR}\right]_{11r2} + \beta \left[L_{udd}^{S,RR}\right]_{11r2}\right.$$
$$- \frac{m_n}{2m_\Sigma}\left(\alpha \left[L_{udd}^{S,LR}\right]_{12r1} + \beta \left[L_{udd}^{S,RR}\right]_{12r1}\right)(D - F)$$
$$\left. + \frac{m_n}{6m_\Lambda}\left(\alpha \left[L_{udd}^{S,LR}\right]_{12r1} + \beta \left[L_{udd}^{S,RR}\right]_{12r1} + 2\alpha \left[L_{udd}^{S,LR}\right]_{11r2} + 2\beta \left[L_{udd}^{S,RR}\right]_{11r2}\right)(D + 3F)\right|^2$$

α, β

computed in the lattice

κ-matrices

$\Gamma(p \to \pi^+\nu) = 2\,\Gamma(n \to \pi^0\nu)$

$n \to \pi^0\nu$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	81.9		−42.2		−23.5			9.74		
$\mathcal{O}_{qque,1111}$										
$\mathcal{O}_{duql,1111}$	−42.2		21.7		12.1			−5.02		
$\mathcal{O}_{duue,1111}$										
$\mathcal{O}_{qqql,2111}$	−23.5		12.1		6.76			−2.80		
$\mathcal{O}_{qqql,1211}$										
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$	9.74		−5.02		−2.80			1.16		
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$										

$p \to \pi^+\nu$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	163.7		−84.4		−47.0			19.5		
$\mathcal{O}_{qque,1111}$										
$\mathcal{O}_{duql,1111}$	−84.4		43.5		24.2			−10.0		
$\mathcal{O}_{duue,1111}$										
$\mathcal{O}_{qqql,2111}$	−47.0		24.2		13.5			−5.60		
$\mathcal{O}_{qqql,1211}$										
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$	19.5		−10.0		−5.60			2.32		
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$										

$n \to \eta^0\nu$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	34.3		1.91		−9.86			−0.44		
$\mathcal{O}_{qque,1111}$										
$\mathcal{O}_{duql,1111}$	1.91		0.11		−0.55			−0.02		
$\mathcal{O}_{duue,1111}$										
$\mathcal{O}_{qqql,2111}$	−9.86		−0.55		2.83			0.13		
$\mathcal{O}_{qqql,1211}$										
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$	−0.44		−0.02		0.13			0.006		
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$										

$p \to K^0e^+$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	1.59					5.59	2.26			
$\mathcal{O}_{qque,1111}$		0.74							−2.21	3.20
$\mathcal{O}_{duql,1111}$										
$\mathcal{O}_{duue,1111}$										
$\mathcal{O}_{qqql,2111}$										
$\mathcal{O}_{qqql,1211}$	5.59					19.6	7.93			
$\mathcal{O}_{duql,2111}$	2.26					7.93	3.20			
$\mathcal{O}_{duql,1121}$										
$\mathcal{O}_{duue,2111}$		−2.21							6.63	−9.58
$\mathcal{O}_{qque,2111}$		3.20							−9.58	13.9

$\Gamma(n \to \pi^-e^+) = 3\,\Gamma(p \to \pi^0e^+)$

$n \to \pi^-e^+$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	258.5		−133.2			−48.4				
$\mathcal{O}_{qque,1111}$		297.1		130.5						−68.6
$\mathcal{O}_{duql,1111}$	−133.2		68.7			24.9				
$\mathcal{O}_{duue,1111}$		130.5		57.4						−30.1
$\mathcal{O}_{qqql,2111}$										
$\mathcal{O}_{qqql,1211}$	−48.4		24.9			9.06				
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$										
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$		−68.6		−30.1						15.8

$p \to \pi^0e^+$

	$\mathcal{O}_{qqql,1111}$	$\mathcal{O}_{qque,1111}$	$\mathcal{O}_{duql,1111}$	$\mathcal{O}_{duue,1111}$	$\mathcal{O}_{qqql,2111}$	$\mathcal{O}_{qqql,1211}$	$\mathcal{O}_{duql,2111}$	$\mathcal{O}_{duql,1121}$	$\mathcal{O}_{duue,2111}$	$\mathcal{O}_{qque,2111}$
$\mathcal{O}_{qqql,1111}$	86.2		−44.4			−16.1				
$\mathcal{O}_{qque,1111}$		99.0		43.5						−22.9
$\mathcal{O}_{duql,1111}$	−44.4		22.9			8.31				
$\mathcal{O}_{duue,1111}$		43.5		19.1						−10.0
$\mathcal{O}_{qqql,2111}$										
$\mathcal{O}_{qqql,1211}$	−16.1		8.31			3.02				
$\mathcal{O}_{duql,2111}$										
$\mathcal{O}_{duql,1121}$										
$\mathcal{O}_{duue,2111}$										
$\mathcal{O}_{qque,2111}$		−22.9		−10.0						5.28

$$\mathcal{L}_0 \supset \left(\frac{D-F}{f_\pi} \overline{\Sigma^+} \gamma^\mu \gamma_5 p - \frac{D+3F}{\sqrt{6} f_\pi} \overline{\Lambda^0} \gamma^\mu \gamma_5 n - \frac{D-F}{\sqrt{2} f_\pi} \overline{\Sigma^0} \gamma^\mu \gamma_5 n \right) \partial_\mu \bar{K}^0$$
$$+ \left(\frac{D-F}{\sqrt{2} f_\pi} \overline{\Sigma^0} \gamma^\mu \gamma_5 p - \frac{D+3F}{\sqrt{6} f_\pi} \overline{\Lambda^0} \gamma^\mu \gamma_5 p + \frac{D-F}{f_\pi} \overline{\Sigma^-} \gamma^\mu \gamma_5 n \right) \partial_\mu K^-$$
$$+ \frac{3F-D}{2\sqrt{6} f_\pi} \left(\overline{p} \gamma^\mu \gamma_5 p + \overline{n} \gamma^\mu \gamma_5 n \right) \partial_\mu \eta$$
$$+ \frac{D+F}{f_\pi} \overline{p} \gamma^\mu \gamma_5 n \, \partial_\mu \pi^+$$
$$+ \frac{D+F}{2\sqrt{2} f_\pi} \left(\overline{p} \gamma^\mu \gamma_5 p - \overline{n} \gamma^\mu \gamma_5 n \right) \partial_\mu \pi^0 + \text{h.c.}\,.$$

$$\xi B \xi \to L \xi B \xi R^\dagger \qquad \xi^\dagger B \xi^\dagger \to R \xi^\dagger B \xi^\dagger L^\dagger$$
$$\xi B \xi^\dagger \to L \xi B \xi^\dagger L^\dagger \qquad \xi^\dagger B \xi \to R \xi^\dagger B \xi R^\dagger$$

$$\xi B \xi \sim (\mathbf{3}, \bar{\mathbf{3}}), \quad \xi^\dagger B \xi^\dagger \sim (\bar{\mathbf{3}},\ \mathbf{3}), \quad \xi B \xi^\dagger \sim (\mathbf{8}, \mathbf{1}), \quad \xi^\dagger B \xi \sim (\mathbf{1}, \mathbf{8})$$

$$\alpha \cdot \nu \ \text{tr}(\xi B \xi^\dagger P_{32}) = -(du)(d\nu) = [\mathcal{O}_{udd}]^{S,LL}_{1111}$$

$$\langle 0 | \epsilon^{abc} (\bar{u}_a^\dagger \bar{d}_b^\dagger) u_c | p^{(s)} \rangle = \alpha P_L u_p^{(s)}$$

$$\langle 0 | \epsilon^{abc} (u_a d_b) u_c | p^{(s)} \rangle = \beta P_L u_p^{(s)}$$

Name	LEFT	Flavour/BχPT
$[\mathcal{O}^{S,LL}_{udd}]_{rstu}$	$(u_r d_s)(d_t \nu_u)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{udd}]_{111r}$	$(ud)(d\nu_r)$	$-\beta \overline{\nu^c_{Lr}} \text{tr}(\xi B \xi^\dagger P_{32}) \supset -\beta \overline{\nu^c_{Lr}} n - \frac{i\beta}{f_\pi} \overline{\nu^c_{Lr}} \left(\sqrt{\frac{3}{2}} n\eta - \frac{1}{\sqrt{2}} n\pi^0 + p\pi^- \right)$
$[\mathcal{O}^{S,LL}_{udd}]_{121r}$	$(us)(d\nu_r)$	$-\beta \overline{\nu^c_{Lr}} \text{tr}(\xi B \xi^\dagger \tilde{P}_{22}) \supset -\beta \overline{\nu^c_{Lr}} \left(-\frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} \right) - \frac{i\beta}{f_\pi} \overline{\nu^c_{Lr}} n \bar{K}^0$
$[\mathcal{O}^{S,LL}_{udd}]_{112r}$	$(ud)(s\nu_r)$	$-\beta \overline{\nu^c_{Lr}} \text{tr}(\xi B \xi^\dagger P_{33}) \supset \beta \sqrt{\frac{2}{3}} \overline{\nu^c_{Lr}} \Lambda^0 - \frac{i\beta}{f_\pi} \overline{\nu^c_{Lr}} \left(n\bar{K}^0 + pK^- \right)$
$[\mathcal{O}^{S,LL}_{duu}]_{rstu}$	$(d_r u_s)(u_t e_u)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{duu}]_{111r}$	$(du)(ue_r)$	$-\beta \overline{e^c_{Lr}} \text{tr}(\xi B \xi^\dagger \tilde{P}_{31}) \supset \beta \overline{e^c_{Lr}} p + \frac{i\beta}{f_\pi} \overline{e^c_{Lr}} \left(\sqrt{\frac{3}{2}} p\eta + \frac{1}{\sqrt{2}} p\pi^0 + n\pi^+ \right)$
$[\mathcal{O}^{S,LL}_{duu}]_{211r}$	$(su)(ue_r)$	$-\beta \overline{e^c_{Lr}} \text{tr}(\xi B \xi^\dagger P_{21}) \supset -\beta \overline{e^c_{Lr}} \Sigma^+ + \frac{i\beta}{f_\pi} \overline{e^c_{Lr}} p\bar{K}^0$
$[\mathcal{O}^{S,LR}_{uud}]_{[rs]tu}$	$(u_r u_s)(\bar{d}_t^\dagger \bar{e}_u^\dagger)$	—
$[\mathcal{O}^{S,LR}_{duu}]_{rstu}$	$(d_r u_s)(\bar{u}_t^\dagger \bar{e}_u^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,LR}_{duu}]_{111r}$	$(du)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$\alpha \overline{e^c_{Rr}} \text{tr}(\xi^\dagger B \xi^\dagger \tilde{P}_{31}) \supset -\alpha \overline{e^c_{Rr}} p + \frac{i\alpha}{f_\pi} \overline{e^c_{Rr}} \left(-\frac{1}{\sqrt{6}} p\eta + \frac{1}{\sqrt{2}} p\pi^0 + n\pi^+ \right)$
$[\mathcal{O}^{S,LR}_{duu}]_{211r}$	$(su)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$\alpha \overline{e^c_{Rr}} \text{tr}(\xi^\dagger B \xi^\dagger P_{21}) \supset \alpha \overline{e^c_{Rr}} \Sigma^+ - \frac{i\alpha}{f_\pi} \overline{e^c_{Rr}} p\bar{K}^0$
$[\mathcal{O}^{S,RL}_{uud}]_{[rs]tu}$	$(\bar{u}_r^\dagger \bar{u}_s^\dagger)(d_t e_u)$	—
$[\mathcal{O}^{S,RL}_{duu}]_{rstu}$	$(\bar{d}_r^\dagger \bar{u}_s^\dagger)(u_t e_u)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{duu}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(ue_r)$	$-\alpha \overline{e^c_{Lr}} \text{tr}(\xi B \xi \tilde{P}_{31}) \supset \alpha \overline{e^c_{Lr}} p + \frac{i\alpha}{f_\pi} \overline{e^c_{Lr}} \left(-\frac{1}{\sqrt{6}} p\eta + \frac{1}{\sqrt{2}} p\pi^0 + n\pi^+ \right)$
$[\mathcal{O}^{S,RL}_{duu}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(ue_r)$	$-\alpha \overline{e^c_{Lr}} \text{tr}(\xi B \xi P_{21}) \supset -\alpha \overline{e^c_{Lr}} \Sigma^+ - \frac{i\alpha}{f_\pi} \overline{e^c_{Lr}} p\bar{K}^0$
$[\mathcal{O}^{S,RL}_{dud}]_{rstu}$	$(\bar{d}_r^\dagger \bar{u}_s^\dagger)(d_t \nu_u)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{dud}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(d\nu_r)$	$\alpha \overline{\nu^c_{Lr}} \text{tr}(\xi B \xi \tilde{P}_{32}) \supset -\alpha \overline{\nu^c_{Lr}} n + \frac{i\alpha}{f_\pi} \overline{\nu^c_{Lr}} \left(\frac{1}{\sqrt{6}} n\eta + \frac{1}{\sqrt{2}} n\pi^0 - p\pi^- \right)$
$[\mathcal{O}^{S,RL}_{dud}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(d\nu_r)$	$\alpha \overline{\nu^c_{Lr}} \text{tr}(\xi B \xi P_{22}) \supset \alpha \overline{\nu^c_{Lr}} \left(\frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} \right) + \frac{i\alpha}{f_\pi} \overline{\nu^c_{Lr}} n\bar{K}^0$
$[\mathcal{O}^{S,RL}_{dud}]_{112r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(s\nu_r)$	$\alpha \overline{\nu^c_{Lr}} \text{tr}(\xi B \xi \tilde{P}_{33}) \supset \alpha \overline{\nu^c_{Lr}} \sqrt{\frac{2}{3}} \Lambda^0 - \frac{i\alpha}{f_\pi} \overline{\nu^c_{Lr}} \left(n\bar{K}^0 + pK^- \right)$
$[\mathcal{O}^{S,RL}_{dud}]_{212r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(s\nu_r)$	$\alpha \overline{\nu^c_{Lr}} \text{tr}(\xi B \xi P_{23}) \supset \alpha \overline{\nu^c_{Lr}} \Xi^0$
$[\mathcal{O}^{S,RL}_{ddu}]_{[rs]tu}$	$(\bar{d}_r^\dagger \bar{d}_s^\dagger)(u_t \nu_u)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{ddu}]_{[12]1r}$	$(\bar{d}^\dagger \bar{s}^\dagger)(u\nu_r)$	$-\alpha \overline{\nu^c_{Lr}} \text{tr}(\xi B \xi P_{11}) \supset \alpha \overline{\nu^c_{Lr}} \left(\frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} \right) - \frac{i\alpha}{f_\pi} \overline{\nu^c_{Lr}} pK^-$
$[\mathcal{O}^{S,RR}_{duu}]_{rstu}$	$(\bar{d}_r^\dagger \bar{u}_s^\dagger)(\bar{u}_t^\dagger \bar{e}_u^\dagger)$	$(\mathbf{1},\mathbf{8})$
$[\mathcal{O}^{S,RR}_{duu}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$\beta \overline{e^c_{Rr}} \text{tr}(\xi^\dagger B \xi \tilde{P}_{31}) \supset -\beta \overline{e^c_{Rr}} p + \frac{i\beta}{f_\pi} \overline{e^c_{Rr}} \left(\sqrt{\frac{3}{2}} p\eta + \frac{1}{\sqrt{2}} p\pi^0 + n\pi^+ \right)$
$[\mathcal{O}^{S,RR}_{duu}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$\beta \overline{e^c_{Rr}} \text{tr}(\xi^\dagger B \xi P_{21}) \supset \beta \overline{e^c_{Rr}} \Sigma^+ + \frac{i\beta}{f_\pi} \overline{e^c_{Rr}} p\bar{K}^0$

Name	LEFT	Flavour/BχPT
$[\mathcal{O}^{S,LL}_{udd}]_{rstu}$	$(u_r d_s)(d_t \nu_u)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{udd}]_{111r}$	$(ud)(d\nu_r)$	$-\beta\overline{\nu^c_{Lr}}\mathrm{tr}(\xi B\xi^\dagger P_{32}) \supset -\beta\overline{\nu^c_{Lr}}n - \frac{i\beta}{f_\pi}\overline{\nu^c_{Lr}}\left(\sqrt{\frac{3}{2}}n\eta - \frac{1}{\sqrt{2}}n\pi^0 + p\pi^-\right)$
$[\mathcal{O}^{S,LL}_{udd}]_{121r}$	$(us)(d\nu_r)$	$-\beta\overline{\nu^c_{Lr}}\mathrm{tr}(\xi B\xi^\dagger \tilde{P}_{22}) \supset -\beta\overline{\nu^c_{Lr}}\left(-\frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}}\right) - \frac{i\beta}{f_\pi}\overline{\nu^c_{Lr}}n\bar{K}^0$
$[\mathcal{O}^{S,LL}_{udd}]_{112r}$	$(ud)(s\nu_r)$	$-\beta\overline{\nu^c_{Lr}}\mathrm{tr}(\xi B\xi^\dagger P_{33}) \supset \beta\sqrt{\frac{2}{3}}\overline{\nu^c_{Lr}}\Lambda^0 - \frac{i\beta}{f_\pi}\overline{\nu^c_{Lr}}\left(n\bar{K}^0 + pK^-\right)$
$[\mathcal{O}^{S,LL}_{duu}]_{rstu}$	$(d_r u_s)(u_t e_u)$	$(\mathbf{8},\mathbf{1})$
$[\mathcal{O}^{S,LL}_{duu}]_{111r}$	$(du)(ue_r)$	$-\beta\overline{e^c_{Lr}}\mathrm{tr}(\xi B\xi^\dagger \tilde{P}_{31}) \supset \beta\overline{e^c_{Lr}}p + \frac{i\beta}{f_\pi}\overline{e^c_{Lr}}\left(\sqrt{\frac{3}{2}}p\eta + \frac{1}{\sqrt{2}}p\pi^0 + n\pi^+\right)$
$[\mathcal{O}^{S,LL}_{duu}]_{211r}$	$(su)(ue_r)$	$-\beta\overline{e^c_{Lr}}\mathrm{tr}(\xi B\xi^\dagger P_{21}) \supset -\beta\overline{e^c_{Lr}}\Sigma^+ + \frac{i\beta}{f_\pi}\overline{e^c_{Lr}}p\bar{K}^0$
$[\mathcal{O}^{S,LR}_{uud}]_{[rs]tu}$	$(u_r u_s)(\bar{d}^\dagger_t \bar{e}^\dagger_u)$	—
$[\mathcal{O}^{S,LR}_{duu}]_{rstu}$	$(d_r u_s)(\bar{u}^\dagger_t \bar{e}^\dagger_u)$	$(\bar{\mathbf{3}},\mathbf{3})$
$[\mathcal{O}^{S,LR}_{duu}]_{111r}$	$(du)(\bar{u}^\dagger \bar{e}^\dagger_r)$	$\alpha\overline{e^c_{Rr}}\mathrm{tr}(\xi^\dagger B\xi^\dagger \tilde{P}_{31}) \supset -\alpha\overline{e^c_{Rr}}p + \frac{i\alpha}{f_\pi}\overline{e^c_{Rr}}\left(-\frac{1}{\sqrt{6}}p\eta + \frac{1}{\sqrt{2}}p\pi^0 + n\pi^+\right)$
$[\mathcal{O}^{S,LR}_{duu}]_{211r}$	$(su)(\bar{u}^\dagger \bar{e}^\dagger_r)$	$\alpha\overline{e^c_{Rr}}\mathrm{tr}(\xi^\dagger B\xi^\dagger P_{21}) \supset \alpha\overline{e^c_{Rr}}\Sigma^+ - \frac{i\alpha}{f_\pi}\overline{e^c_{Rr}}p\bar{K}^0$
$[\mathcal{O}^{S,RL}_{uud}]_{[rs]tu}$	$(\bar{u}^\dagger_r \bar{u}^\dagger_s)(d_t e_u)$	—
$[\mathcal{O}^{S,RL}_{duu}]_{rstu}$	$(\bar{d}^\dagger_r \bar{u}^\dagger_s)(u_t e_u)$	$(\mathbf{3},\bar{\mathbf{3}})$
$[\mathcal{O}^{S,RL}_{duu}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(ue_r)$	$-\alpha\overline{e^c_{Lr}}\mathrm{tr}(\xi B\xi \tilde{P}_{31}) \supset \alpha\overline{e^c_{Lr}}p + \frac{i\alpha}{f_\pi}\overline{e^c_{Lr}}\left(-\frac{1}{\sqrt{6}}p\eta + \frac{1}{\sqrt{2}}p\pi^0 + n\pi^+\right)$
$[\mathcal{O}^{S,RL}_{duu}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(ue_r)$	$-\alpha\overline{e^c_{Lr}}\mathrm{tr}(\xi B\xi P_{21}) \supset -\alpha\overline{e^c_{Lr}}\Sigma^+ - \frac{i\alpha}{f_\pi}\overline{e^c_{Lr}}p\bar{K}^0$

Nucleon decay channels

$$\Gamma(N \to M\ell_\alpha) \quad \Delta(B-L)=0 \quad \begin{cases} n \to \eta^0\nu \\ n \to \pi^0\nu \\ p \to \pi^+\nu \\ n \to \pi^- e^+ \\ p \to \eta^0 e^+ \\ p \to \pi^0 e^+ \\ p \to K^0 e^+ \\ n \to K^0\nu \\ p \to K^+\nu \end{cases}$$

$$\Gamma(N \to M\ell_\alpha) \quad |\Delta(B-L)|=2 \quad \begin{cases} n \to \eta^0\nu \\ n \to \pi^0\nu \\ p \to \pi^+\nu \\ n \to K^0\nu \\ p \to K^+\nu \\ n \to K^+ e^- \end{cases}$$

- All 2-body PS decays except for $p \to \bar{K}^0 e^+$ $n \to \bar{K}^0\nu$ $n \to K^- e^+$ $n \to \pi^+ e^-$

(No BχPT formalism developed for PD into vector mesons, e.g. $p \to \rho^0 e^+$)

SMEFT

- RGEs dominated by the SM gauge couplings → enhancement

- QCD contributions universal and dominant, BUT electroweak contributions are relevant for $\mathcal{O}_{qqql,1111}$

- top-loop contributions universal and suppressive for d = 7 WCs

- Operator Mixing subdominant for proton decay

$$c(m_W) \sim (2-4)\, c(10^{15}\ \mathrm{GeV})$$

- RGEs for d = 6 SMEFT Manohar et. al. [2014]
- RGEs for d = 7 SMEFT Yi Liao et. al. [2016]

Phenomenological matrices

$$\Gamma_{(i)}^{\Delta(B-L)=0} \equiv 10^{-4}\, c_j^* \boxed{\boldsymbol{\kappa}_{(i)}^{jk}}\, c_k \frac{m_p^5}{\Lambda^4}$$ for $i = p \to \pi^0 e^+, p \to K^+\nu \ldots$ (9 matrices)

$$\Gamma_{(i)}^{|\Delta(B-L)|=2} \equiv 10^{-4}\, c_j^* \boxed{\boldsymbol{\kappa}_{(i)}^{jk}}\, c_k \frac{m_p^7}{\Lambda^6}$$ for $i = p \to K^+\nu, n \to K^+ e^- \ldots$ (6 matrices)

$n \to K^0 \nu$

	$\mathcal{O}_{\bar{l}dud\tilde{H},1111}$	$\mathcal{O}_{\bar{l}dqq\tilde{H},1111}$	$\mathcal{O}_{\bar{l}dud\tilde{H},1211}$	$\mathcal{O}_{\bar{l}dud\tilde{H},1112}$	$\mathcal{O}_{\bar{l}dqq\tilde{H},1211}$	$\mathcal{O}_{\bar{l}dqq\tilde{H},1121}$	$\mathcal{O}_{\bar{l}dqq\tilde{H},1112}$	$\mathcal{O}_{\bar{l}dddH,1121}$	$\mathcal{O}_{\bar{e}qdd\tilde{H},1121}$
$\mathcal{O}_{\bar{l}dud\tilde{H},1111}$									
$\mathcal{O}_{\bar{l}dqq\tilde{H},1111}$		0.08	−0.48	−0.36	−0.51		0.28		
$\mathcal{O}_{\bar{l}dud\tilde{H},1211}$		−0.48	3.09	2.33	3.25		−1.80		
$\mathcal{O}_{\bar{l}dud\tilde{H},1112}$		−0.36	2.33	1.76	2.45		−1.36		
$\mathcal{O}_{\bar{l}dqq\tilde{H},1211}$		−0.51	3.25	2.45	3.42		−1.90		
$\mathcal{O}_{\bar{l}dqq\tilde{H},1121}$									
$\mathcal{O}_{\bar{l}dqq\tilde{H},1112}$		0.28	−1.80	−1.36	−1.90		1.06		
$\mathcal{O}_{\bar{l}dddH,1121}$									
$\mathcal{O}_{\bar{e}qdd\tilde{H},1121}$									

$p \to K^+ \nu$

	$\mathcal{O}_{\bar{l}dud\tilde{H},1111}$	$\mathcal{O}_{\bar{l}dqq\tilde{H},1111}$	$\mathcal{O}_{\bar{l}dud\tilde{H},1211}$	$\mathcal{O}_{\bar{l}dud\tilde{H},1112}$	$\mathcal{O}_{\bar{l}dqq\tilde{H},1211}$	$\mathcal{O}_{\bar{l}dqq\tilde{H},1121}$	$\mathcal{O}_{\bar{l}dqq\tilde{H},1112}$	$\mathcal{O}_{\bar{l}dddH,1121}$	$\mathcal{O}_{\bar{e}qdd\tilde{H},1121}$
$\mathcal{O}_{\bar{l}dud\tilde{H},1111}$									
$\mathcal{O}_{\bar{l}dqq\tilde{H},1111}$		0.01	0.21	0.05	0.22		0.06		
$\mathcal{O}_{\bar{l}dud\tilde{H},1211}$		0.21	3.09	0.76	3.25		0.80		
$\mathcal{O}_{\bar{l}dud\tilde{H},1112}$		0.05	0.76	0.19	0.80		0.20		
$\mathcal{O}_{\bar{l}dqq\tilde{H},1211}$		0.22	3.25	0.80	3.42		0.84		
$\mathcal{O}_{\bar{l}dqq\tilde{H},1121}$									
$\mathcal{O}_{\bar{l}dqq\tilde{H},1112}$		0.06	0.80	0.20	0.84		0.21		
$\mathcal{O}_{\bar{l}dddH,1121}$									
$\mathcal{O}_{\bar{e}qdd\tilde{H},1121}$									