EFT Approach to $(g-2)_\mu$ in the 2-Higgs-Doublet and Vector-like Lepton Model

EFT 2024 - University of Zürich

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TU Dresden, Institut für Kern- und Teilchenphysik

Zürich, 18.07.2024

Standard Model

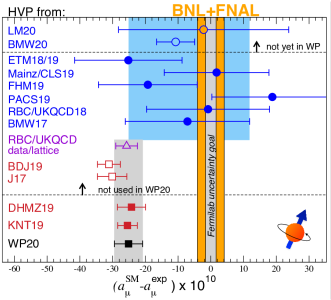

Figure: Current status theory vs experiment (Colangelo et al. [2203.15810])

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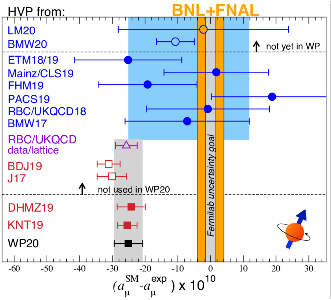

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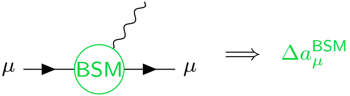
Many extensions of the SM result in additional contributions to a_μ



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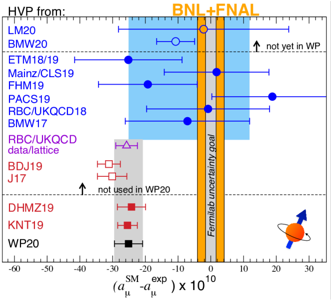

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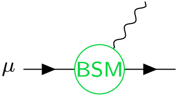

$\Longrightarrow \Delta a_\mu^{\mathrm{BSM}}$

in EFT: dim-5 dipole operator

$$H_\mu \equiv \bar{\mu}\sigma^{\alpha\beta}\mu F_{\alpha\beta}$$

$\rightarrow$ chirality flipping (connects μ_L and μ_R)

$$C_{H_\mu} \sim \Delta a_\mu^{\mathrm{BSM}} \propto m_\mu \frac{y^{\mathrm{BSM}} v}{M_{\mathrm{BSM}}^2}$$

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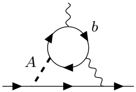
2HDM: leading contribution from **2-loop**
Barr-Zee Diagram



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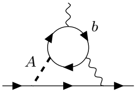

Multiple mass scales

$$m_\mu \ll m_b \ll M_A$$

$\rightarrow$ logarithmic enhancement

$$\Delta a_\mu^{2l,b} \sim m_\mu e^2 \frac{y_\mu^A y_b^A m_b}{M_A^2} \ln^2 \left(\frac{M_A}{m_b} \right)$$

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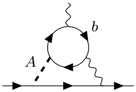

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→ "NLO" from 3-loop

instead: estimate of leading logs (LL) from **EFT Renormalization Group**

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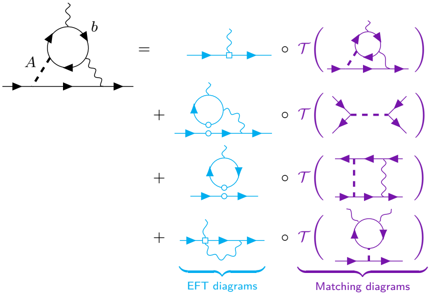

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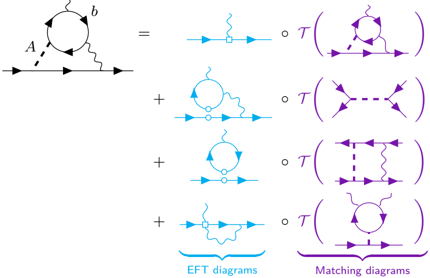

$$\underbrace{\ln\left(\frac{M_A}{m_b}\right)}_{\text{phys. log.}} \rightarrow \underbrace{\ln\left(\frac{\mu}{m_b}\right)}_{\text{EFT diagrams}} + \underbrace{\ln\left(\frac{M_A}{\mu}\right)}_{C_i(\mu)}$$

Leading Logarithmic contributions to $(g-2)_\mu$

EFT RGE:

$$\mu \frac{dC_{H_\mu}}{d\mu} \approx \sum_i C_i(\mu) \gamma_{\mathcal{O}_i H_\mu}(\mu) \tag{1}$$

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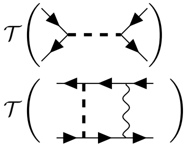

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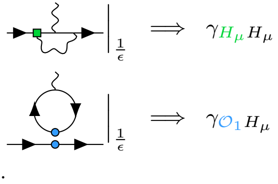

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After choosing parametrization of 2-loop result (e.g. $\overline{\text{MS}}$ with $e(m_\mu),\ g_s(m_b)$), 3-loop LL are fixed by solution of RGE

$$\Delta a_\mu^{\text{LL},3} = \Delta a_\mu^{\text{LL},2} \cdot \left\{ \left(2\beta_e - \frac{1}{\pi^2}\right)\frac{e^2}{3}\ln\left(\frac{M_A^2 m_b}{m_\mu^3}\right) - \frac{7g_s^2 C_F + (7Q_b^2 - 5)e^2}{12\pi^2}\ln\left(\frac{M_A}{m_\tau}\right)\right\}$$

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$\Longrightarrow$ complicated 3-loop calculation split into 2 one-loop calculations + RGE running.

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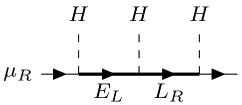

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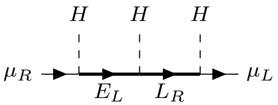

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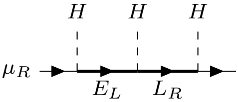

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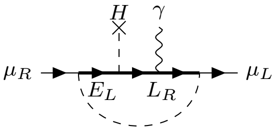

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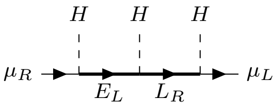

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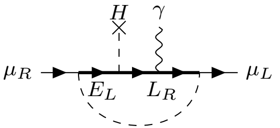

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total contribution

$$\Delta a_\mu \simeq -\frac{m_\mu^{LE}}{m_\mu} \times 22.5 \times 10^{-10}$$

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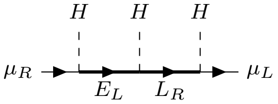

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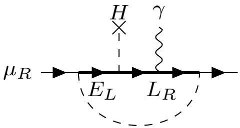

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Correlation

$$\boxed{\frac{\lambda_{\mu\mu}}{\lambda_{\mu\mu}^{\text{SM}}} \simeq 1 - \frac{2\Delta a_\mu}{22.5 \times 10^{-10}}}$$

$\implies$ $(g-2)_\mu$ explained for $\lambda_{\mu\mu} \approx -\lambda_{\mu\mu}^{\text{SM}}$

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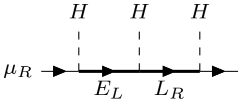

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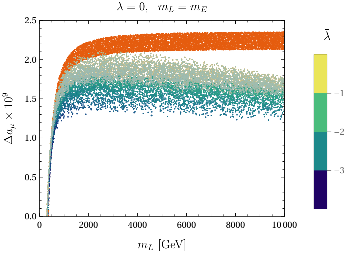

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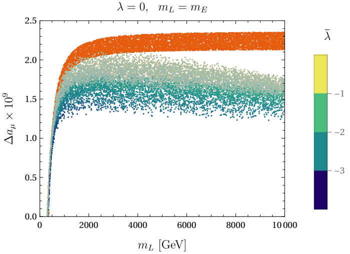

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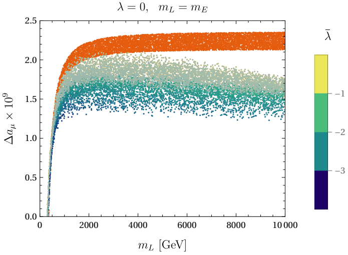

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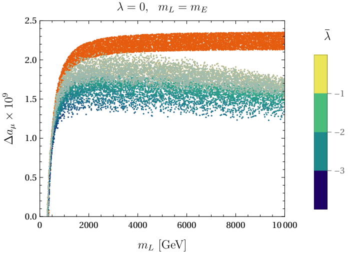


Thanks for your attention!

CP-violating portal to the Dark Sector

Nicola Valori

University of Valencia & IFIC

EFT 2024
Zurich 07/2024

with **M. Ardu, M.H. Rahat, O.Vives**

IFIC INSTITUT DE FÍSICA CORPUSCULAR

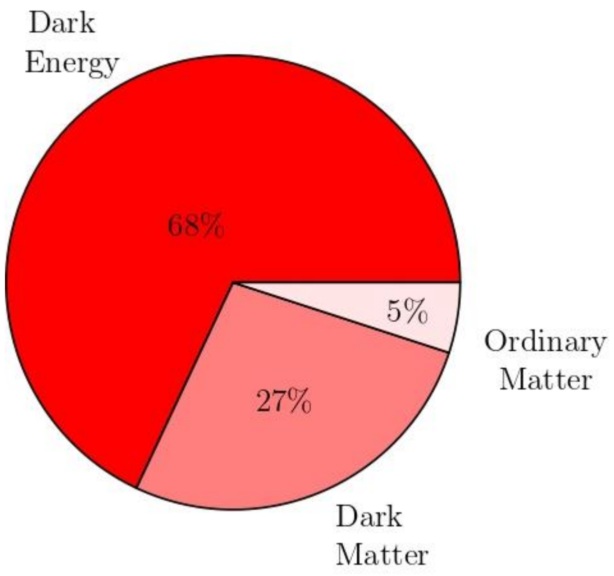
CSIC

VNIVERSITAT DE VALÈNCIA

Motivation

Particle Dark Matter:

- **Dark Matter** comprises almost ¼ of the whole energy budget.
- **Dark Matter Production** (usually) requires interaction with SM.
- If DM ∈ DS: **Portals** between the visible and dark sector.



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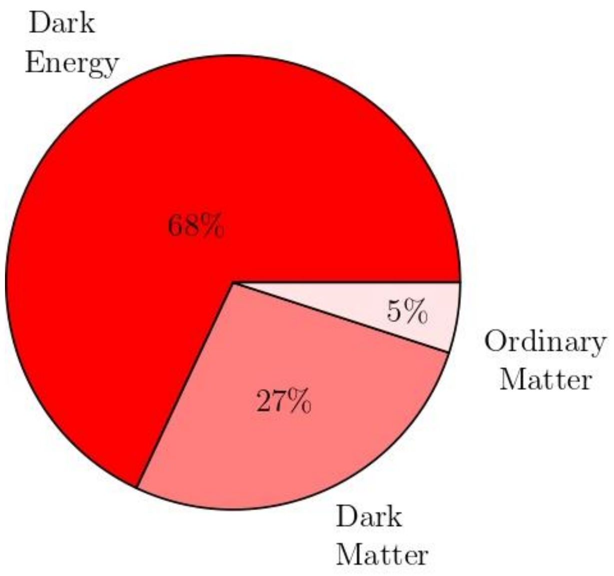


CP-violation :

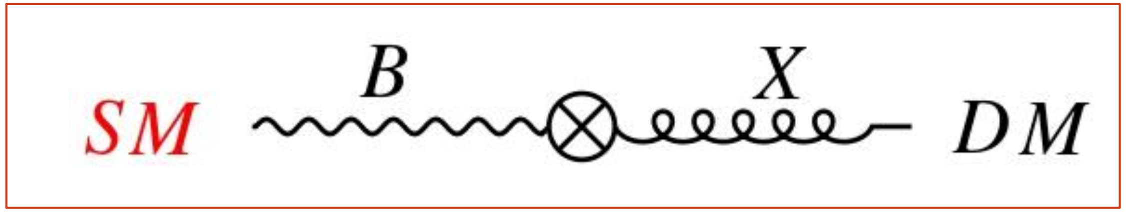
- The SM of particle physics allows for **CP-violation** (CKM matrix)
- CP-violation in the SM is not enough to explain matter-antimatter asymmetry
- CP-violation in Hidden sectors or **Portals** ?

Portals

Portals

Abelian Kinetic Mixing:

- Additional **U(1) abelian** dark gauge group
- **Kinetic Mixing** at renormalizable level: $\frac{\epsilon}{2} B^{\mu\nu} X_{\mu\nu}$
- ϵ can naturally be O(1) but experiments yields $\epsilon << 1$



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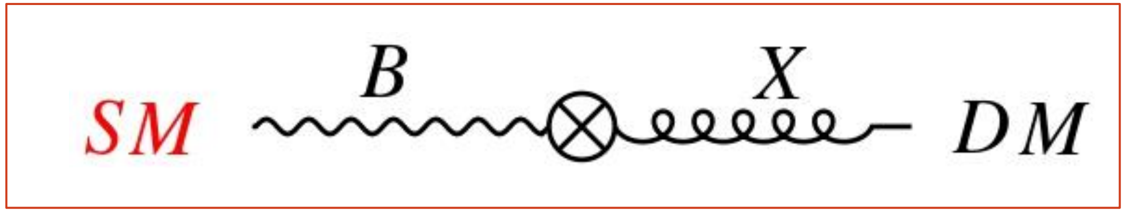
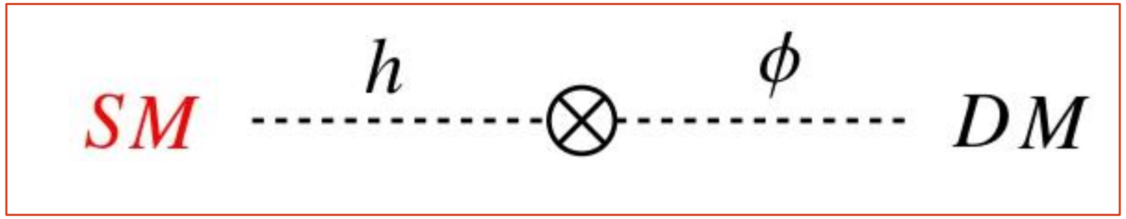


Scalar Portal:

- Additional **Dark Scalar** neutral under SM
- **Interaction** at renormalizable level: $k\,|H|^2|S|^2$
- SSB ($\langle S \rangle \neq 0$) and mixing.



Non Abelian Kinetic Mixing

- Introduction of a SU(N) **Non Abelian Dark Sector** $\supset$
  - Σ_{a} : Scalar fields in the adjoint of SU(N)
  - X_a^{μ}: $N^2 - 1$ gauge bosons

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$$-\frac{\epsilon_a}{2} X_a^{\mu\nu} B_{\mu\nu} - \frac{\tilde{\epsilon}}{2} \phi^a X_a^{\mu\nu} \tilde{B}_{\mu\nu}$$

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→

- Kinetic Mixing parameters **naturally** small
- New source of **CP-violation**

EDM

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- CPV interaction of spin 1/2 particles with EM fields
- QFT description: $\mathcal{L} = -\frac{i}{2} d\, \overline{\psi} \sigma^{\mu\nu} \gamma_5 \psi F_{\mu\nu}$

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- eEDMs is the most sensitive to CPV
- CPV in the SM predicts: $d_e^{eq} = 10^{-35} e\ cm$

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Upper bound on $|d_e|$ (e · cm)

JILAeEDM	4.1×10^{-30}
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Expect significant improvements of the current JILAeEDM sensitivity in the coming years!

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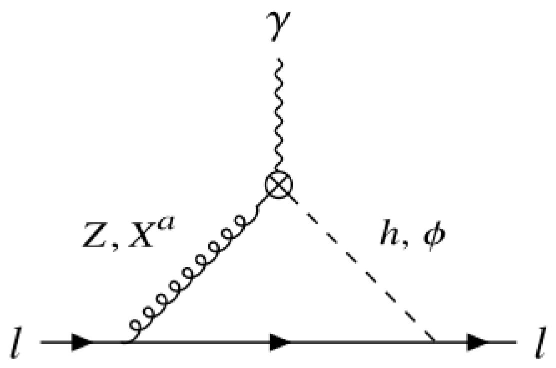

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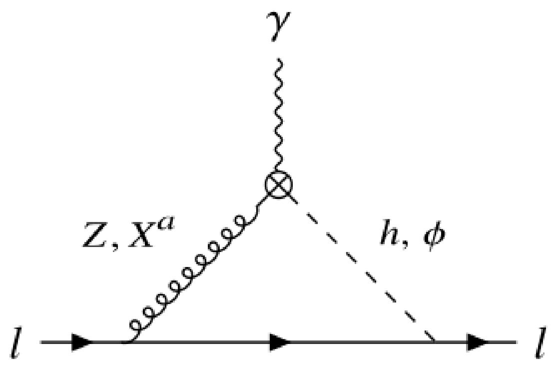

$$d_e = \frac{3Y_e}{32\pi^2 v}\,\epsilon^2\,\beta\tan\chi\, e\, f(M_X, m_\phi, m_h)$$

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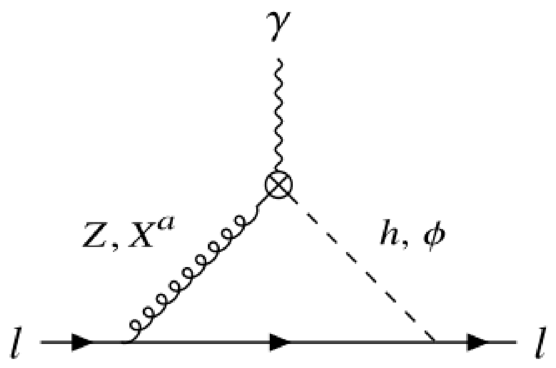

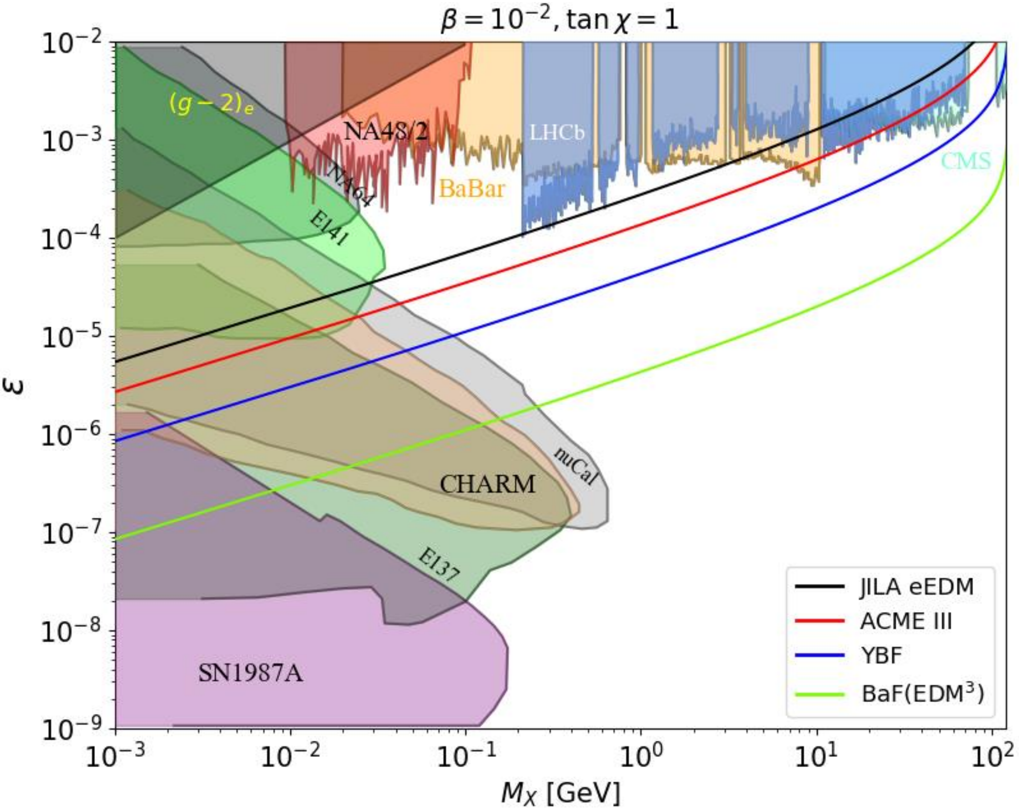

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- Parameter space probed by eEDM sens.
- Scalar mixing parameter $\beta \lesssim 10^{-2}$ [T.Ferber et al. (2024)]
- Constraints on ϵ from colliders and beam dump

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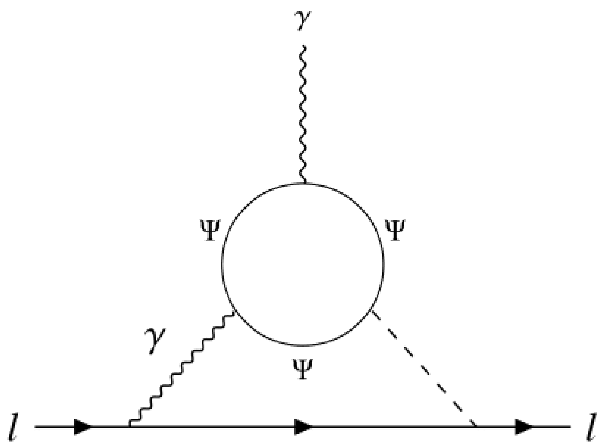
UV-completion → Barr-Zee diagram



$$d_e = \frac{e\alpha}{24\pi^3}\frac{Y_e \mathrm{Im}[\mathcal{Y}]}{\Lambda}\beta \log\left(\frac{m_h^2}{m_\phi^2}\right)$$

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- D-D severly constrains ϵ for DM > few GeV

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- $m_{\chi_S} \sim m_{\chi_H} \lesssim M_X$ ~ 1-10 GeV scale
- DM prod. via coannihilation $\chi_H \chi_S$ → SM
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- Cosmo bounds inefficient
- PS not covered by $X \to$ inv. decay searches at labs
- Future eEDM sensitivities can probe the model

Summary

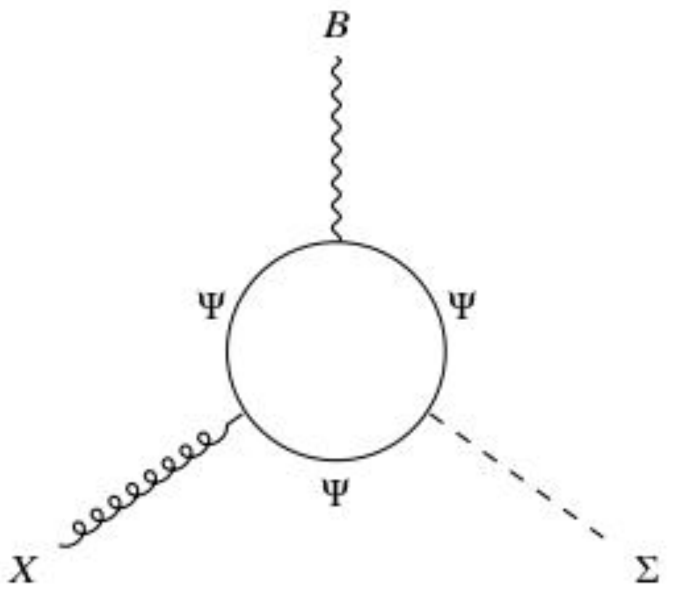
- Non-abelian Dark sector allows for kinetic portals with small ϵ
- Non-abelian Dark sector allows for a CP-violating phase
- Scalar and kinetic mixing + CP-violation signals can be traced in eEDM
- Model of iDM can be probed by the future searches for a permanent eEDM!

Thank you for your attention!

BACK UP

UV completion

- **EFT** call for UV completion
- Heavy vector-like fermion Ψ charged under $SU(N) \otimes U(1)_Y$
- Physical phase χ in Yukawa-like scalar couplings $\mathcal{Y}$



UV Lagrangian:

$$\mathcal{L}_\Psi \supset \; igY\bar{\Psi}\gamma_\mu\Psi B^\mu + ig_D\bar{\Psi}X^\mu\gamma_\mu\Psi - \Lambda\bar{\Psi}_R\Psi_L - \mathcal{Y}\bar{\Psi}_R\Sigma\Psi_L + h.c$$

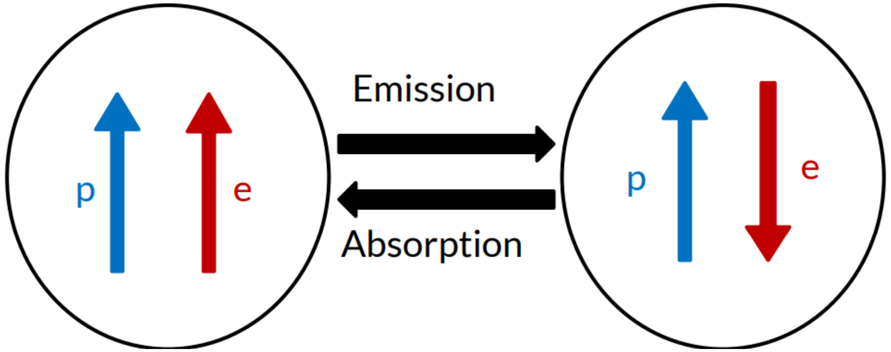
UV-EFT matching

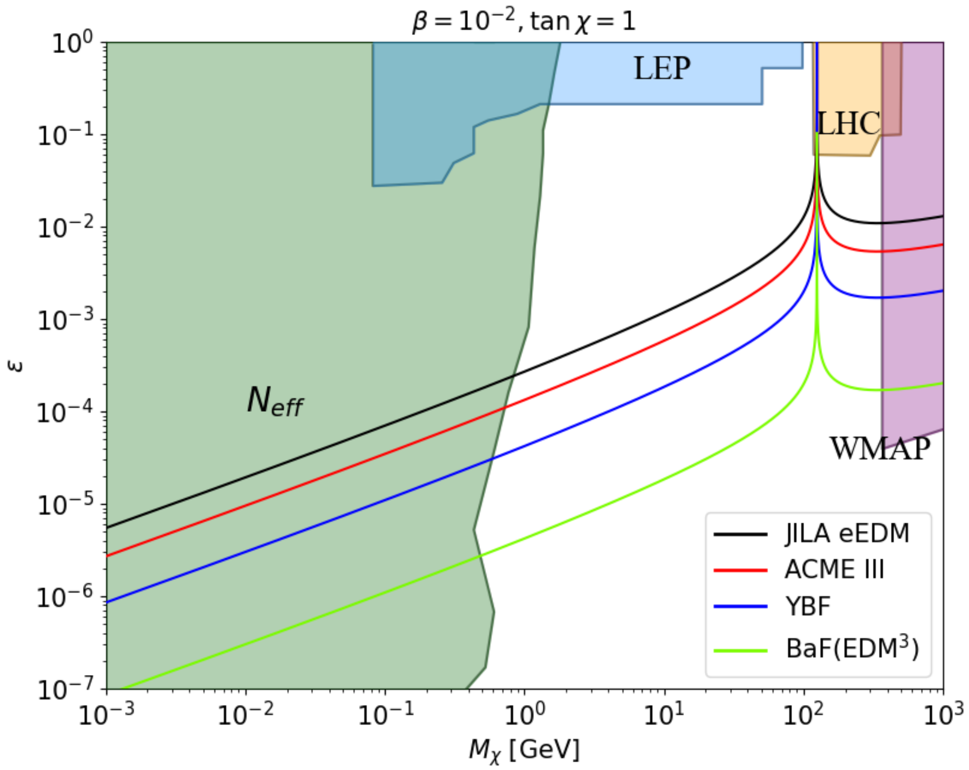
$$C \sim \frac{g_d Y g \mathrm{Re}[\mathcal{Y}]}{16\pi^2}; \qquad \tilde{C} \sim \frac{g_d Y g \mathrm{Im}[\mathcal{Y}]}{16\pi^2}$$

$$\tilde{\epsilon} = \frac{\tan\chi}{v_a}\epsilon_a$$

EDGES anomaly and milli-charged particles

- Spin flip of an electron after recombination epoch results in emission/absorption of 21-cm radiation
- This can give important information on the Universe
- EDGES has detected a primordial abosorption corresponding to to a 21 cm radiation at z ~15-20
- This would suggest a lower baryons temperature
- Baryons-mDM could cool T_B through Rutherfor scattering
- Small fraction of DM can cool the gas efficiently over a wide range of mass

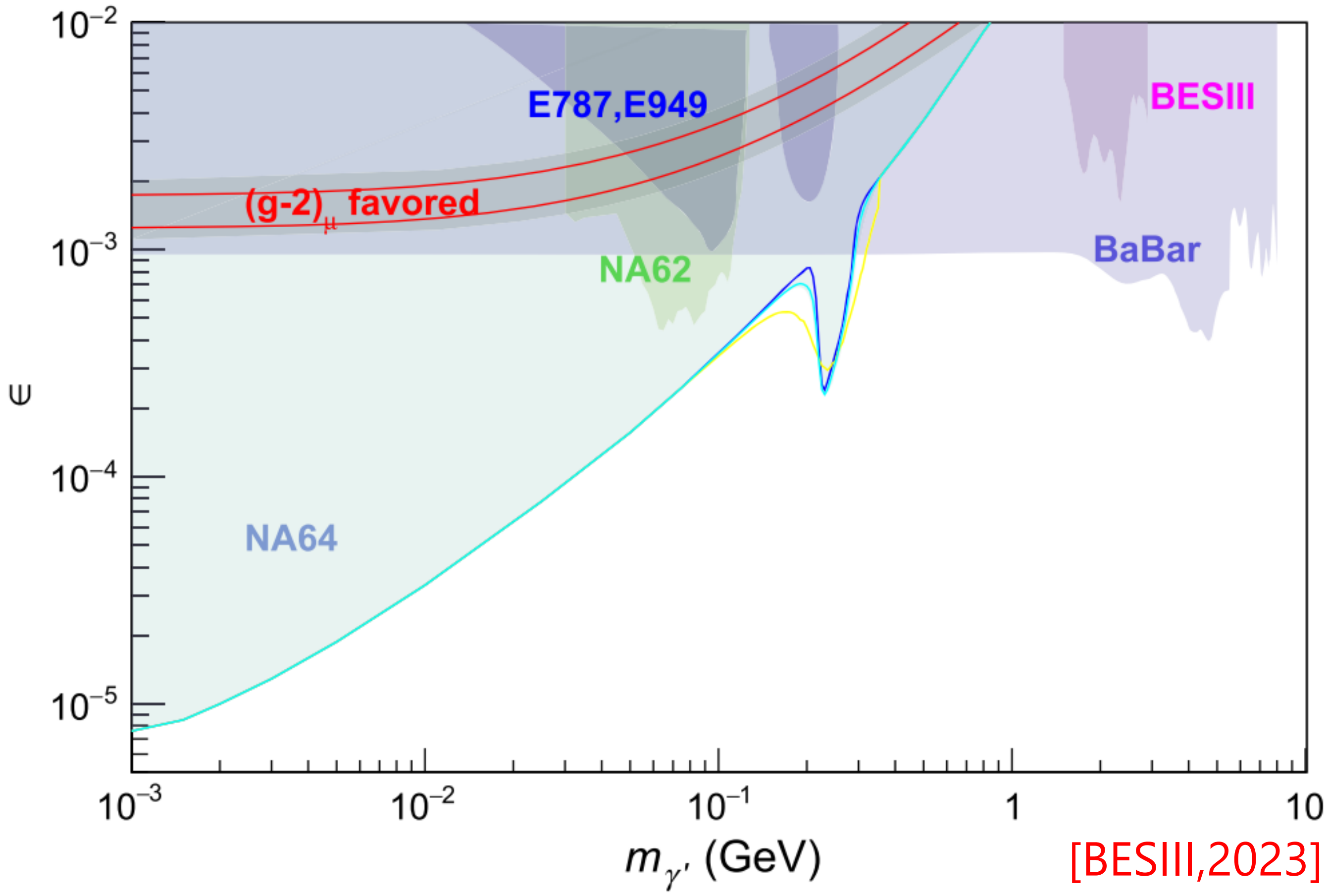




A model for Inelastic Dark Matter

- SU(2) Dark group with matter content:
  - 3 gauge fields X_i^{μ}
  - 2 scalar fields in the adj. Σ_2^a, Σ_3^a
  - 2 Majorana SU(2) doublet
    - $\chi_L = (\chi_L^1, \chi_L^2)$
    - $\psi_R = (\psi_R^1, \psi_R^2)$
- Mass term: $$-m_D \overline{\chi_L} \psi_R - \sum_{i=1,2} Y_{L,i} \overline{\chi_L^c} i\sigma_2 \Sigma_i \chi_L - \sum_{i=1,2} Y_{R,i} \overline{\psi_R^c} i\sigma_2 \Sigma_i \psi_R + \text{h.c.}$$
- SU(2) fully broken by: $\langle \Sigma_2 \rangle = (0, v_2, 0); \langle \Sigma_3 \rangle = (0, 0, v_3)$
- Dirac masses: $M_1 = m_D + vY_1 - vY_2; M_2 = m_D + vY_1 + vY_2$
- Off-diagonal currents with X_2 and X_3 and inelatic dark matter scenario.

Laboratory bounds



[BESIII,2023]

- M_X between 1-10 Gev
- Eps. Between $10^{-5} \div 10^{-3}$
- Parameter space can be probed by eEDM

Scalar leptoquarks for $R_{D^{(*)}}$

(based on 2404.16772)

EFT 2024 Summer School

UNIVERZA V LJUBLJANI

FMF

Fakulteta za matematiko in fiziko

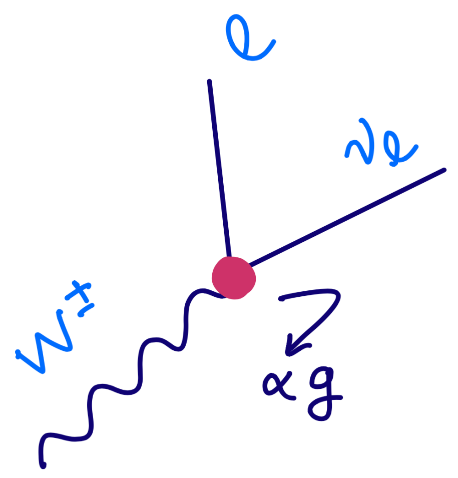
Jožef Stefan Institute
Ljubljana, Slovenia

Lovre Pavičić 18.7.2024

Motivation

- Standard Model cannot address Dark Matter, BAU, Neutrino masses...

  ⇒ Need for **New Physics**: Direct searches at LHC - **Indirect searches** at low energy

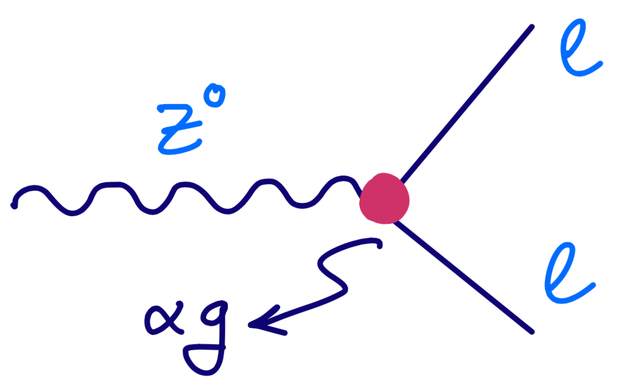


- Indirect searches - Test SM (accidental) symmetries

Flavour physics: **test lepton flavour universality**

W^+ DECAY MODES		Fraction (Γ_i/Γ)
$\ell^+\nu$	[b]	(10.86 ± 0.09) %
$e^+\nu$		(10.71 ± 0.16) %
$\mu^+\nu$		(10.63 ± 0.15) %
$\tau^+\nu$		(11.38 ± 0.21) %
hadrons		(67.41 ± 0.27) %

Z DECAY MODES		Fraction (Γ_i/Γ)
e^+e^-	[h]	(3.3632 ± 0.0042) %
$\mu^+\mu^-$	[h]	(3.3662 ± 0.0066) %
$\tau^+\tau^-$	[h]	(3.3696 ± 0.0083) %
$\ell^+\ell^-$	[b,h]	(3.3658 ± 0.0023) %



2 [PDG 2024]

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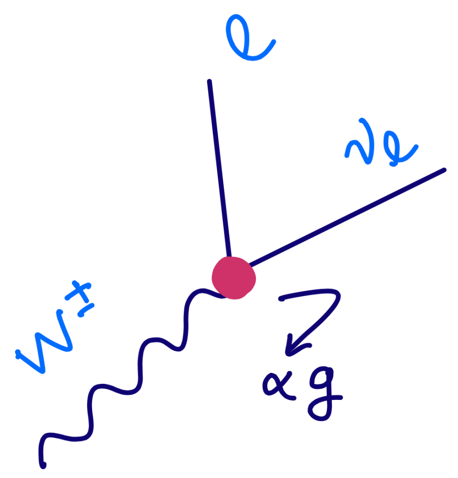

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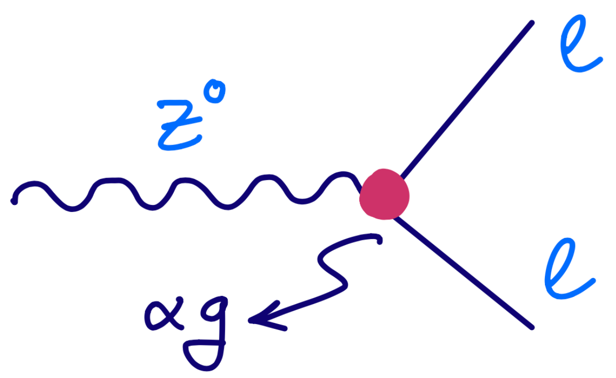
[PDG 2024]

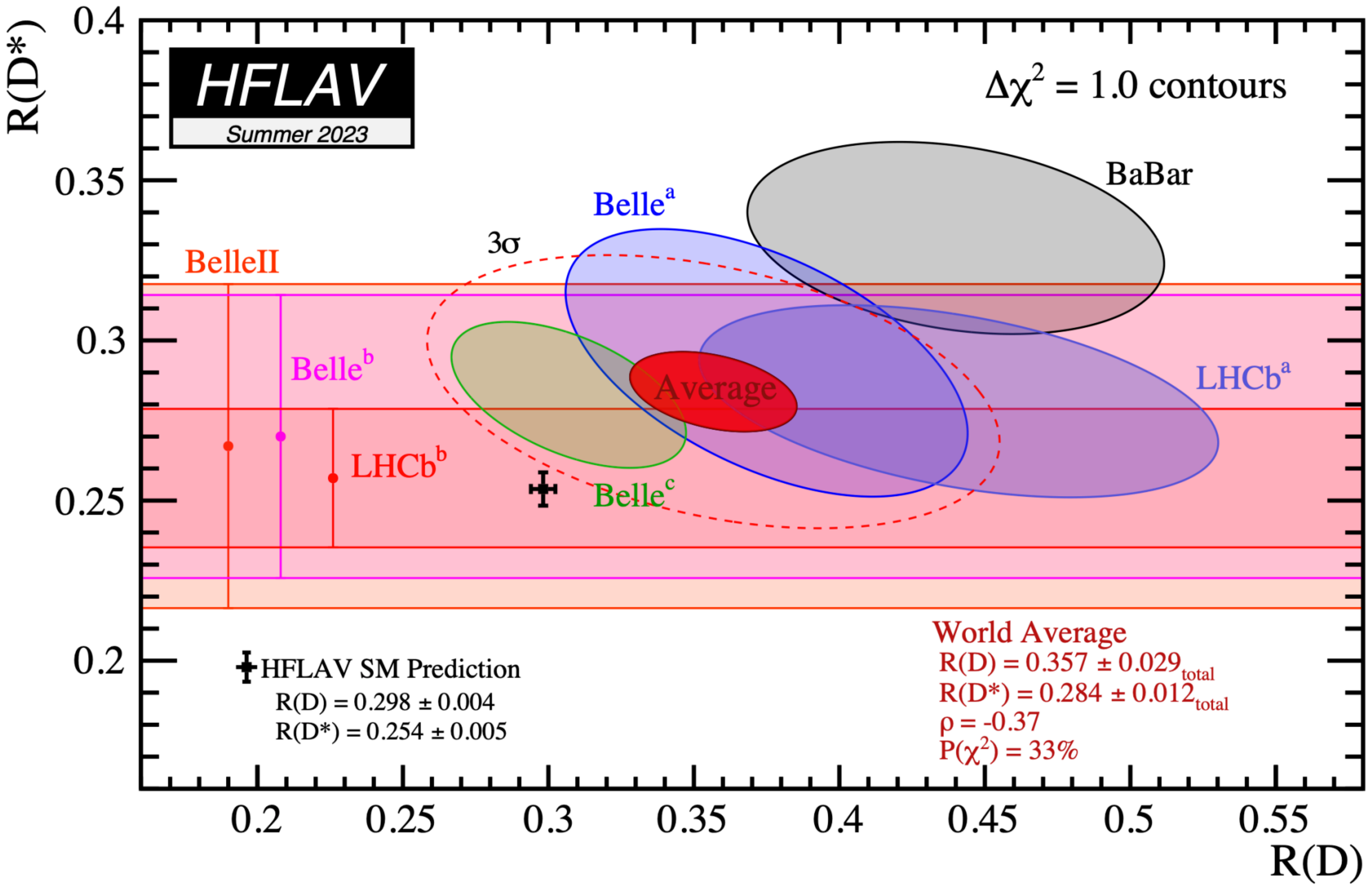
- BUT: current measurements of **semi-leptonic B-meson** decays appear to tell a **different story**!



Observables in $b \to c\ell\nu$

$$R_{D^{(*)}} = \frac{\Gamma(B \to D^{(*)}\tau\nu)}{\Gamma(B \to D^{(*)}\ell\nu)}, \quad \ell = e, \mu$$

- Test of lepton flavour universality
- Theoretically clean; **hadronic uncertainties cancel** in the ratio
- SM predictions significantly smaller than experiment, **combined deviation:** $\sim 3.3\,\sigma$



$\Rightarrow$ Violation of LFU? **New Physics** coupled to b and τ?

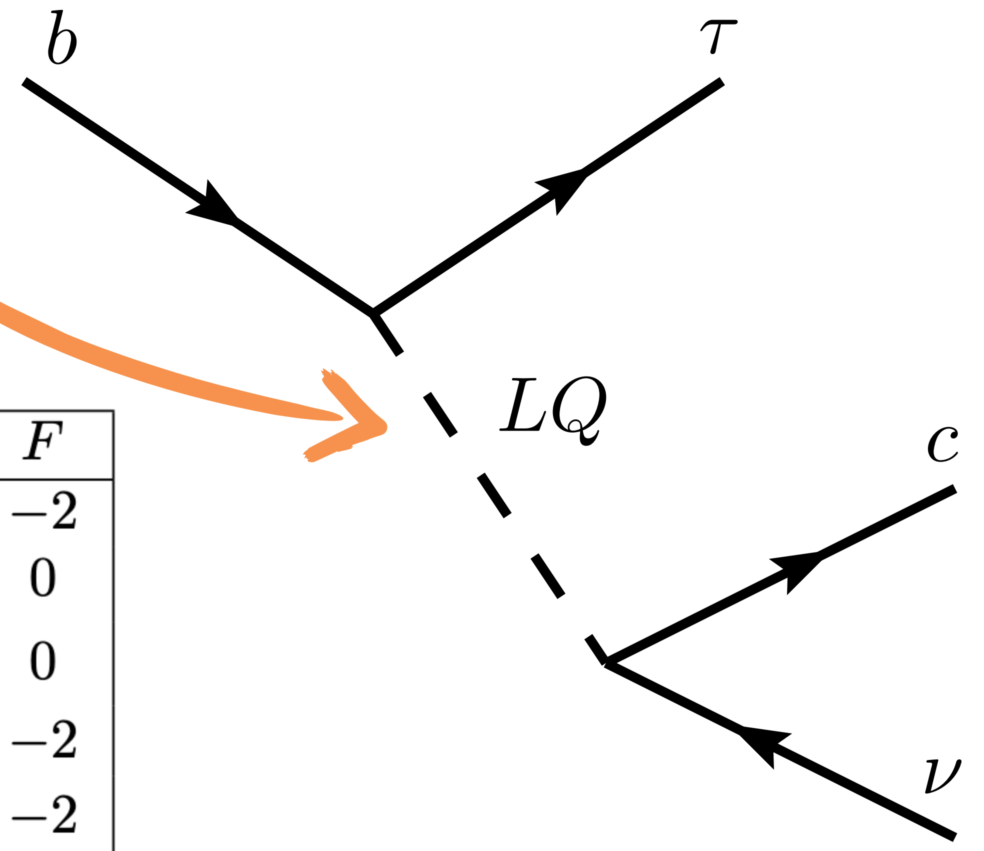
Possible explanations

$$\mathcal{L}_{b\to c\tau\nu} = -2\sqrt{2}G_F V_{cb}\Big[\left(1+g_{V_L}\right)\left(\bar{c}_L\gamma^\mu b_L\right)\left(\bar{\tau}_L\gamma_\mu\nu_{\tau L}\right) + g_{V_R}\left(\bar{c}_R\gamma^\mu b_R\right)\left(\bar{\tau}_L\gamma_\mu\nu_{\tau L}\right) + g_{S_L}\left(\bar{c}_R b_L\right)\left(\bar{\tau}_R\nu_{\tau L}\right) + g_T\left(\bar{c}_R\sigma^{\mu\nu}b_L\right)\left(\bar{\tau}_R\sigma_{\mu\nu}\nu_{\tau L}\right)\Big]$$

EFT study - $\Lambda_{NP} \simeq m_{NP}/C_{NP} \sim \mathcal{O}(1-3)\text{TeV}$

- Possible NP solutions: W', Charged Higgses, Exotic neutrino interactions...

- **Or Leptoquarks!**



$(SU(3), SU(2), U(1))$	Spin	Symbol	Type	F
$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	0	S_3	$LL\,(S_1^L)$	-2
$(\mathbf{3}, \mathbf{2}, 7/6)$	0	R_2	$RL\,(S_{1/2}^L)$, $LR\,(S_{1/2}^R)$	0
$(\mathbf{3}, \mathbf{2}, 1/6)$	0	$\tilde{R}_2$	$RL\,(\tilde{S}_{1/2}^L)$, $\overline{LR}\,(\tilde{S}_{1/2}^{\overline{L}})$	0
$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	0	$\tilde{S}_1$	$RR\,(\tilde{S}_0^R)$	-2
$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	0	S_1	$LL\,(S_0^L)$, $RR\,(S_0^R)$, $\overline{RR}\,(S_0^{\overline{R}})$	-2
$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$	0	$\bar{S}_1$	$\overline{RR}\,(\bar{S}_0^R)$	-2
$(\mathbf{3}, \mathbf{3}, 2/3)$	1	U_3	$LL\,(V_1^L)$	0
$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$	1	V_2	$RL\,(V_{1/2}^L)$, $LR\,(V_{1/2}^R)$	-2
$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	1	$\tilde{V}_2$	$RL\,(\tilde{V}_{1/2}^L)$, $\overline{LR}\,(\tilde{V}_{1/2}^{\overline{R}})$	-2
$(\mathbf{3}, \mathbf{1}, 5/3)$	1	$\tilde{U}_1$	$RR\,(\tilde{V}_0^R)$	0
$(\mathbf{3}, \mathbf{1}, 2/3)$	1	U_1	$LL\,(V_0^L)$, $RR\,(V_0^R)$, $\overline{RR}\,(V_0^{\overline{R}})$	0
$(\mathbf{3}, \mathbf{1}, -1/3)$	1	$\bar{U}_1$	$\overline{RR}\,(\bar{V}_0^R)$	0

(1603.04993)

Possible explanations

$$\mathcal{L}_{b\to c\tau\nu} = -2\sqrt{2}G_F V_{cb}\Big[(1+g_{V_L})(\bar{c}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma_\mu\nu_{\tau L}) + g_{V_R}(\bar{c}_R\gamma^\mu b_R)(\bar{\tau}_L\gamma_\mu\nu_{\tau L}) + g_{S_L}(\bar{c}_R b_L)(\bar{\tau}_R\nu_{\tau L}) + g_T(\bar{c}_R\sigma^{\mu\nu}b_L)(\bar{\tau}_R\sigma_{\mu\nu}\nu_{\tau L})\Big]$$

EFT study - $\Lambda_{NP} \simeq m_{NP}/C_{NP} \sim \mathcal{O}(1-3)\text{TeV}$

- Possible NP solutions: W', Charged Higgses, Exotic neutrino interactions

- **Or Leptoquarks**

$(SU(3), SU(2), U(1))$	Spin	Symbol	Type	F
$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	0	S_3	$LL\,(S_1^L)$	-2
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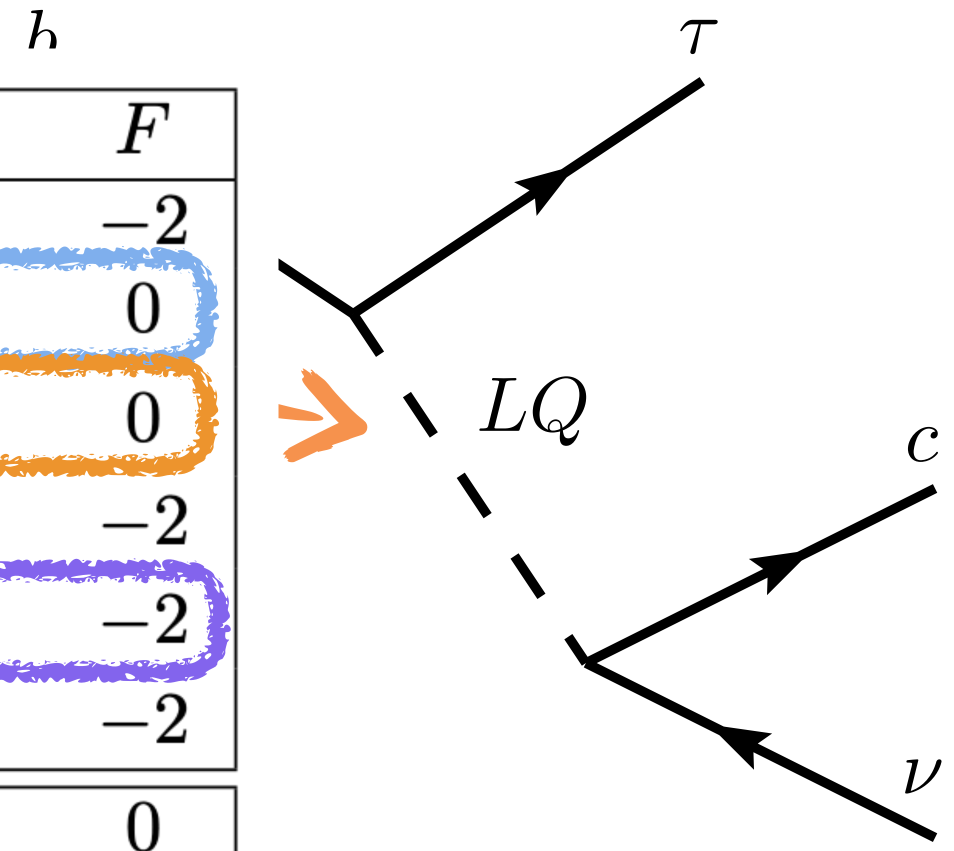
$(\mathbf{3}, \mathbf{2}, 1/6)$	0	$\tilde{R}_2$	$RL\,(\tilde{S}_{1/2}^L),\ \overline{LR}\,(\tilde{S}_{1/2}^L)$	0
$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	0	$\tilde{S}_1$	$RR\,(\tilde{S}_0^R)$	-2
$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	0	S_1	$LL\,(S_0^L),\ RR\,(S_0^R),\ \overline{RR}\,(S_0^R)$	-2
$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$	0	$\bar{S}_1$	$\overline{RR}\,(\bar{S}_0^R)$	-2
$(\mathbf{3}, \mathbf{3}, 2/3)$	1	U_3	$LL\,(V_1^L)$	0
$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$	1	V_2	$RL\,(V_{1/2}^L),\ LR\,(V_{1/2}^R)$	-2
$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	1	$\tilde{V}_2$	$RL\,(\tilde{V}_{1/2}^L),\ \overline{LR}\,(\tilde{V}_{1/2}^R)$	-2
$(\mathbf{3}, \mathbf{1}, 5/3)$	1	$\tilde{U}_1$	$RR\,(\tilde{V}_0^R)$	0
$(\mathbf{3}, \mathbf{1}, 2/3)$	1	U_1	$LL\,(V_0^L),\ RR\,(V_0^R),\ \overline{RR}\,(V_0^R)$	0
$(\mathbf{3}, \mathbf{1}, -1/3)$	1	$\bar{U}_1$	$\overline{RR}\,(\bar{V}_0^R)$	0



(1603.04993)

Constraints on LQ models

- **Collider bounds**: Direct searches ($M_{LQ} \gtrsim 1.5$ TeV), **high-p_T** tails in $pp \to \tau\tau$, $pp \to \tau\nu$

- **Electroweak precision:** $Z \to \tau\tau$, $Z \to \nu\nu$, $\tau \to \ell\nu\bar{\nu}$

- **B-physics observables**: $B_s - \overline{B}_s$ mixing, $B \to K\nu\bar{\nu}$, $B_c \to \tau\nu$, $B_s \to \tau\tau$, $B \to K\tau\tau$, angular observables

R_2

- Consider minimal coupling texture

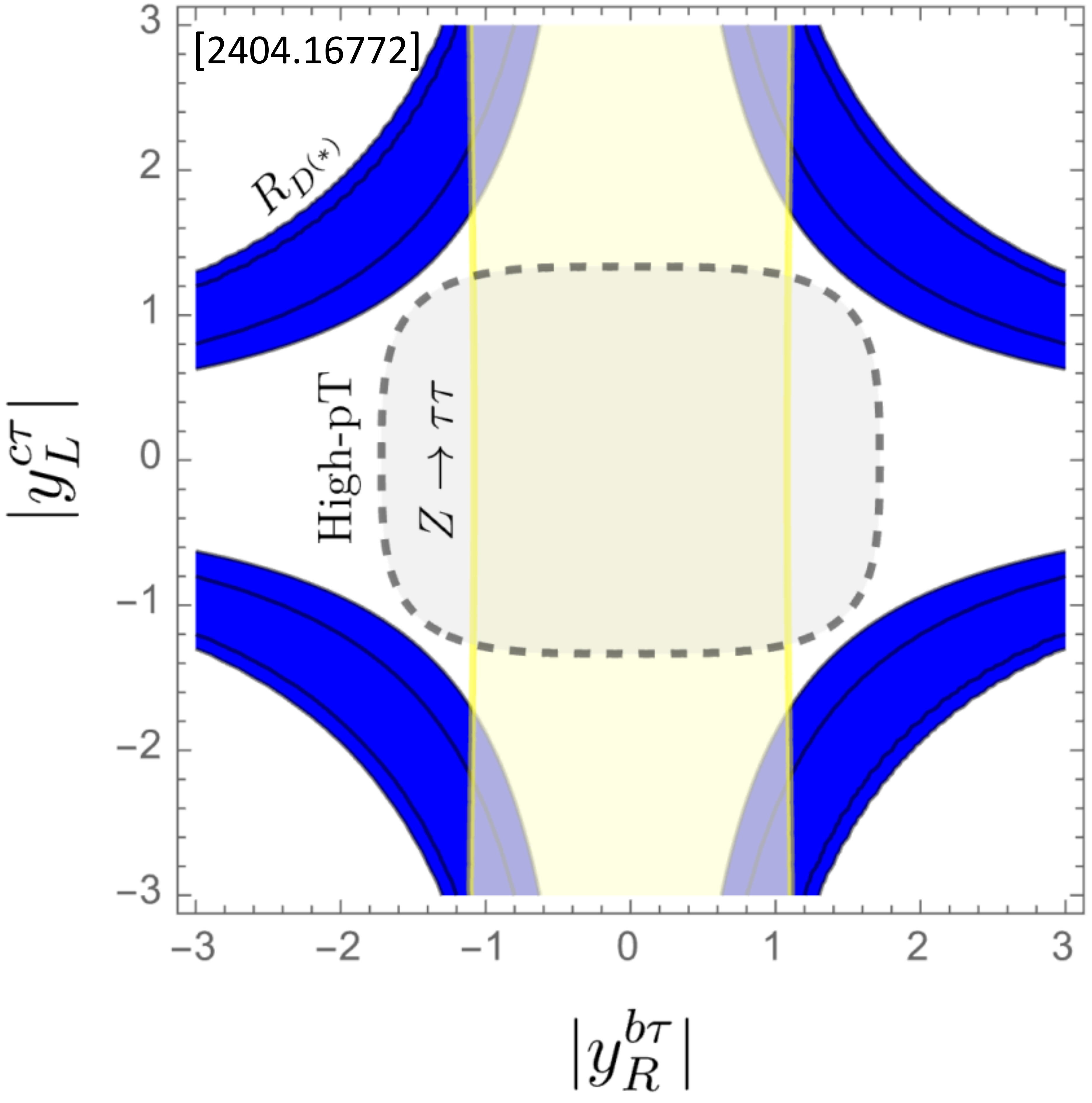
$$y_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_R^{b\tau} \end{pmatrix}, \qquad y_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_L^{c\tau} \\ 0 & 0 & 0 \end{pmatrix}$$

- $R_{D^{(*)}}$ can be accommodated :)

- But: high-p_T - data excludes the viable parameter space :(

$$\mathcal{L}_{R_2} = y_R^{ij}\,\bar{Q}_i^a e_j R_2^a \;+\; y_L^{ij}\,\bar{u}_{Ri} R_2^{T,a} \epsilon^{ab} L_j^b + \text{h.c.}$$

$m_{R_2} = 1.5\,\text{TeV}$

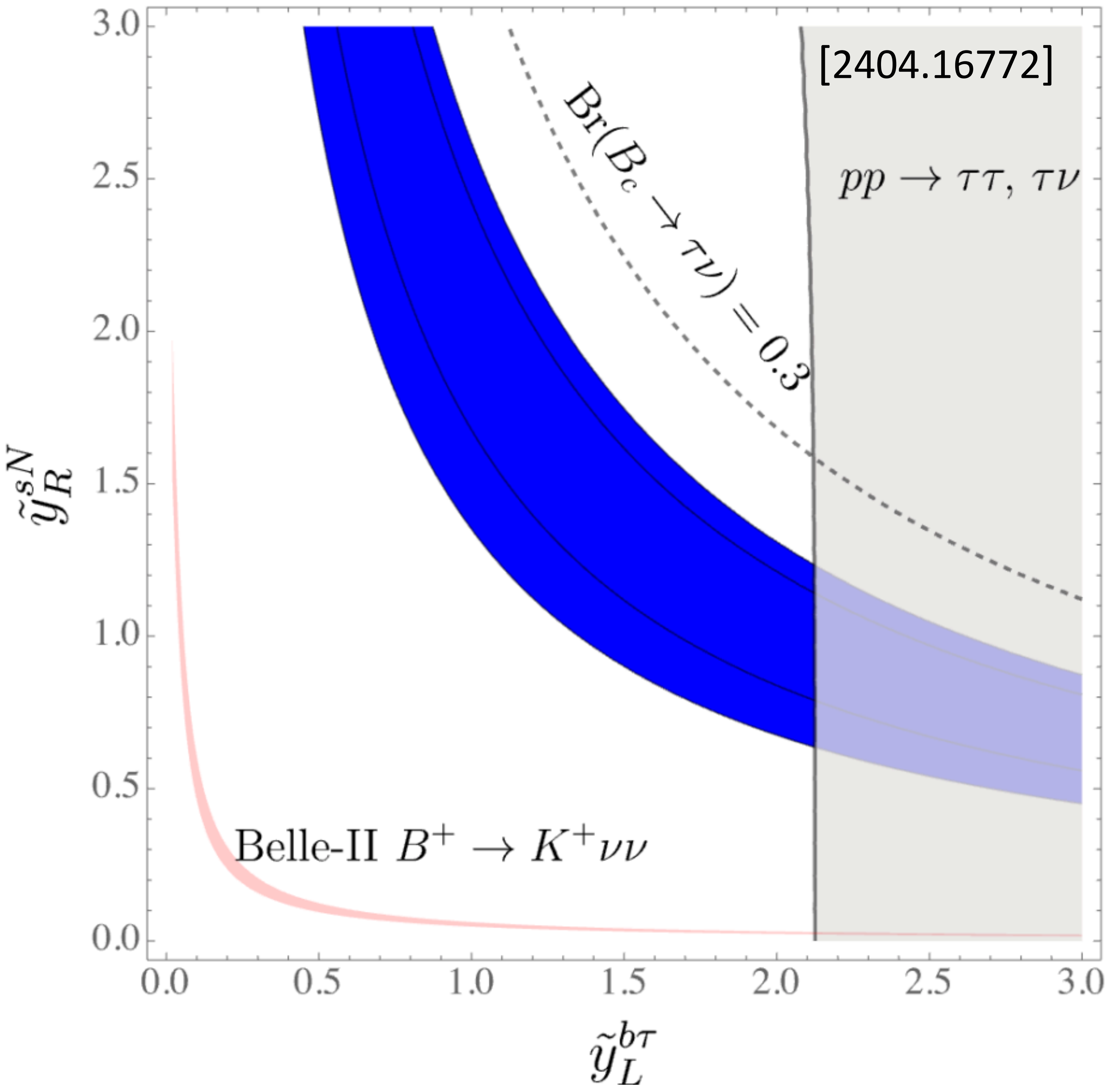


$\tilde{R}_2$

$$\mathcal{L} = -\widetilde{y}_L^{ij}\overline{d}^i\,\widetilde{R}_2^a\epsilon^{ab}L^{j,b} + \widetilde{y}_R^{iN}\overline{Q}^{i,a}\,\widetilde{R}_2^a N_R + \text{h.c.}$$

$$\widetilde{y}_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \widetilde{y}_L^{b\tau} \end{pmatrix}, \qquad \widetilde{y}_R = \begin{pmatrix} 0 \\ \widetilde{y}_R^{sN} \\ 0 \end{pmatrix}$$

$m_{\tilde{R}_2} = 1.5\,\text{TeV}$



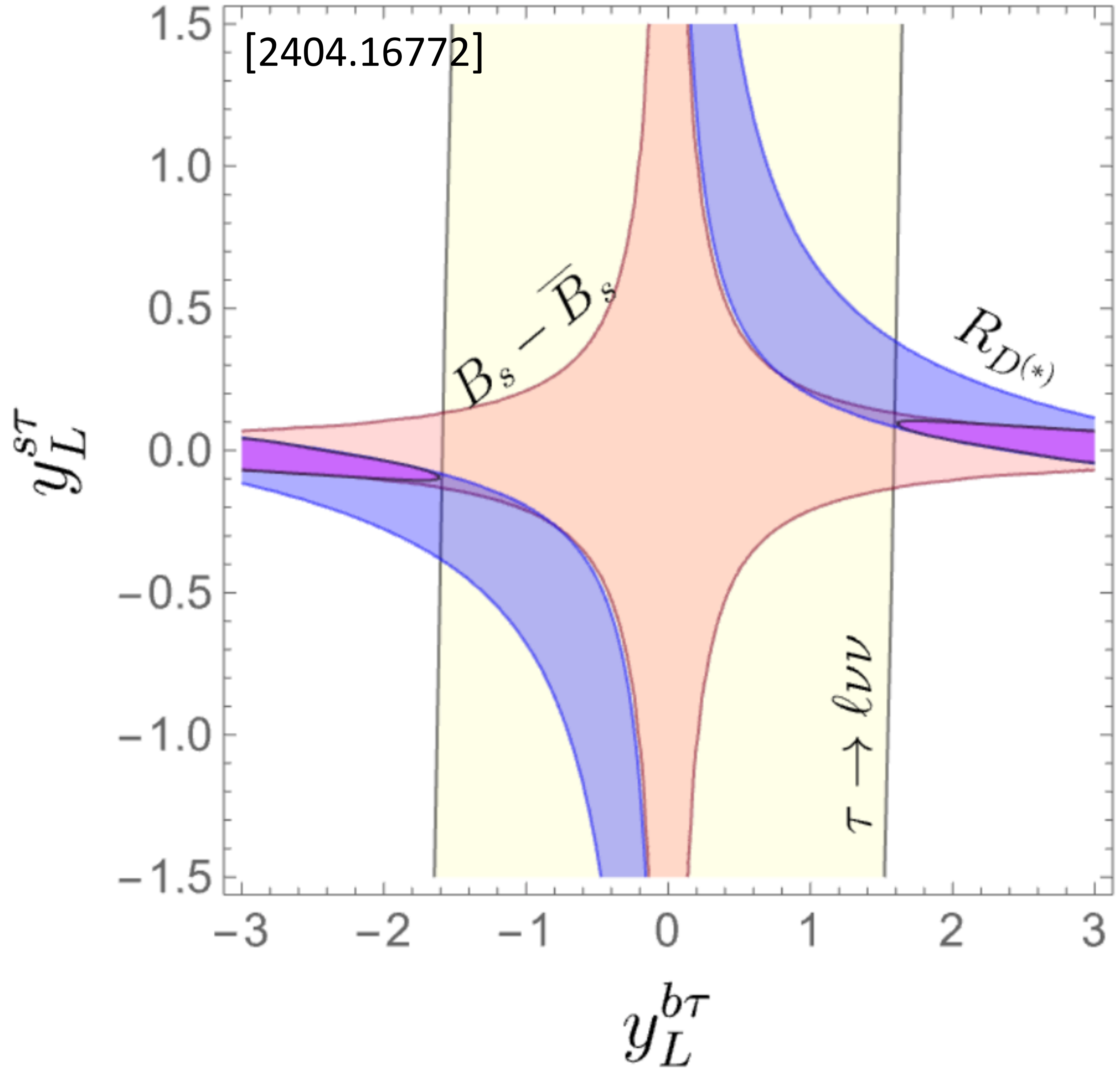
- Again, $R_{D^{(*)}}$ can be accommodated :)
- But $B \to K\nu\nu$ is too severely affected

S_1 - part I.

$$\mathcal{L}_{S_1} = y_L^{ij}\,\overline{Q_i^{C,a}}\,\epsilon^{ab}\,L_j^b\,S_1 + y_R^{ij}\,\overline{u_i^C}e_j\,S_1 + \text{h.c.}$$

$$m_{S_1} = 1.5\ \text{TeV}$$

$$y_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_L^{s\tau} \\ 0 & 0 & y_L^{b\tau} \end{pmatrix}, \qquad y_R = 0$$



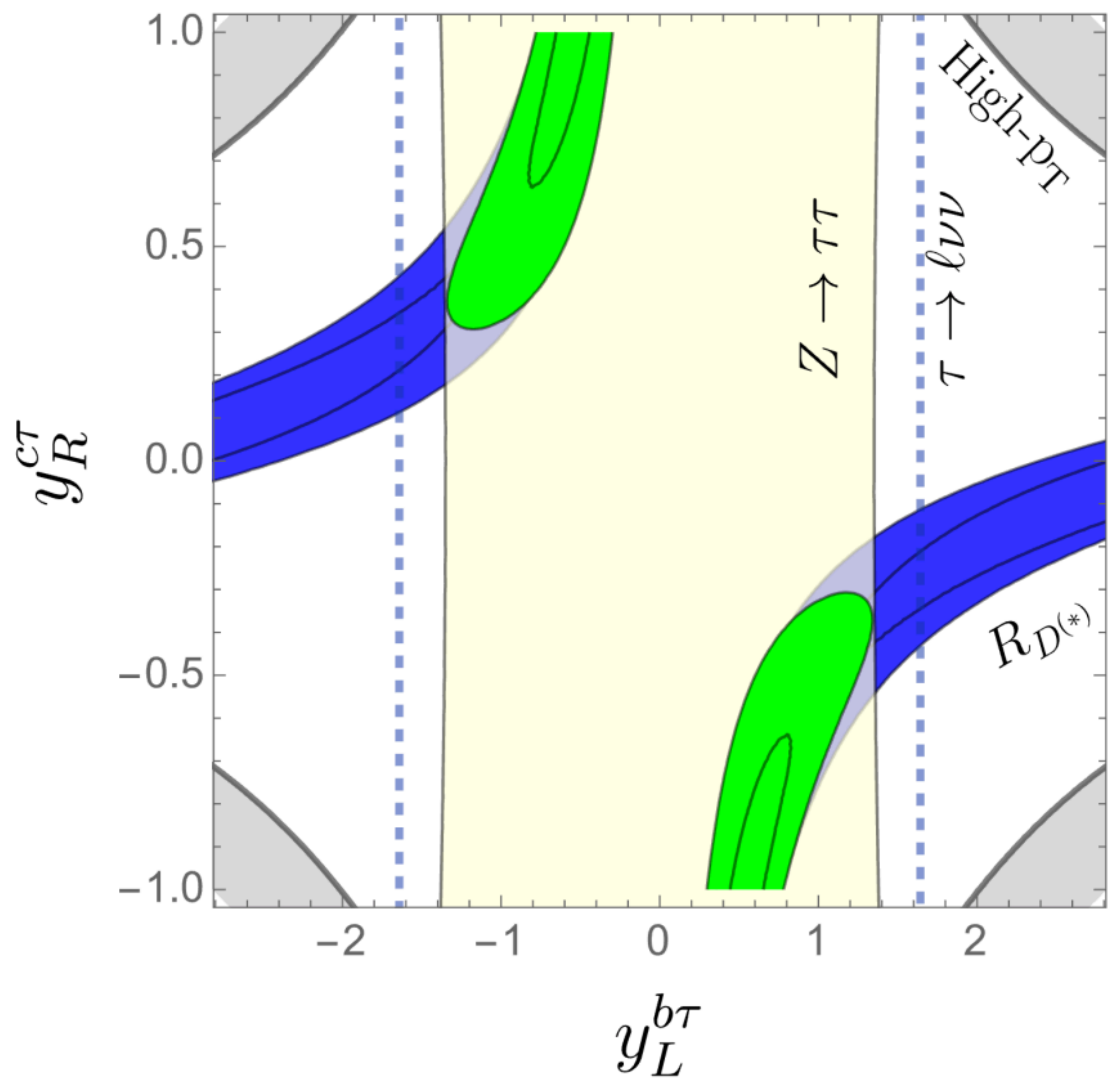
- Once again, $R_{D^{(*)}}$ can be accommodated
- But this time the effect in $B_s - \overline{B}_s$ is slightly too large

S_1 - part II.

$$\mathcal{L}_{S_1} = y_L^{ij}\, \overline{Q_i^{C,a}}\, \epsilon^{ab}\, L_j^b\, S_1 + y_R^{ij}\, \overline{u_i^C} e_j\, S_1 + \text{h.c.}$$

$$m_{S_1} = 1.5 \text{ TeV}$$

$$y_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_L^{b\tau} \end{pmatrix}, \qquad y_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_R^{c\tau} \\ 0 & 0 & 0 \end{pmatrix}$$



- Need right-handed couplings

  $\Rightarrow$evade $B_s - \overline{B}_s$ mixing constraint

- Successfully accommodate $R_{D^{(*)}}$ and consistent with other observables :)

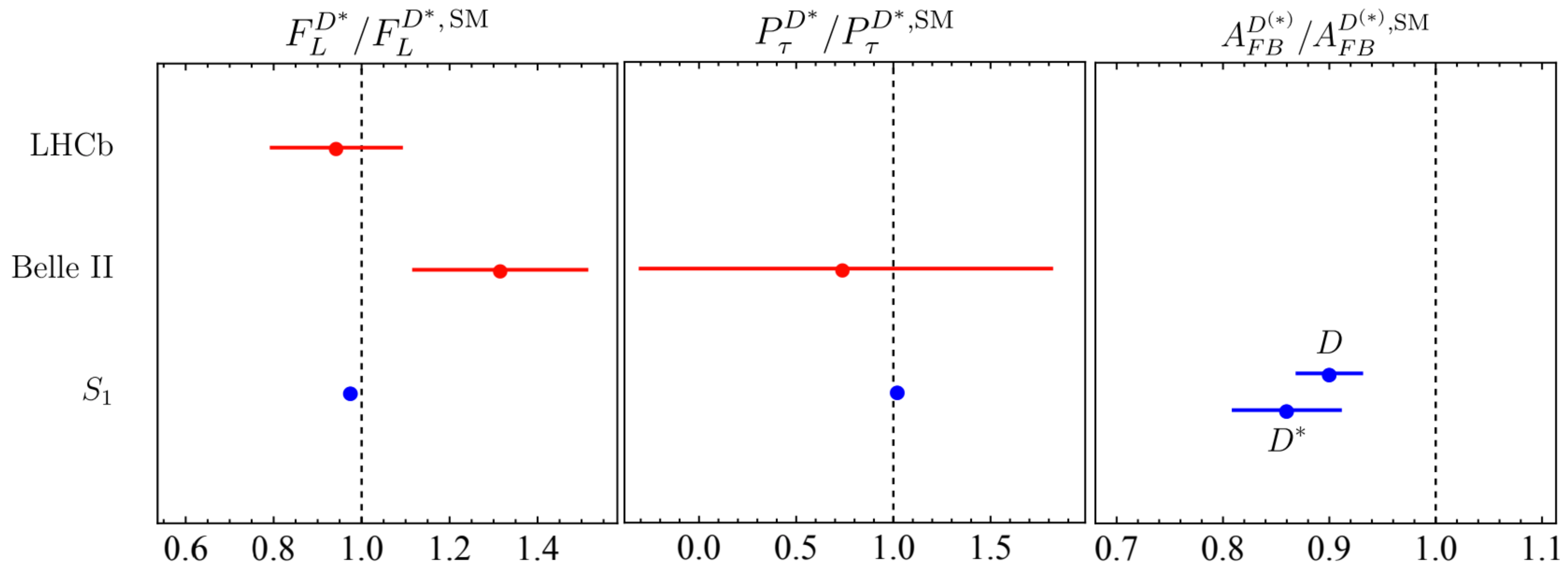
Summary and conclusion

- Hint for the New Physics in $b \to c\ell\nu$ transitions

- Explored 3 different minimal TeV-scale LQ models

  $\Rightarrow$Only S_1 with left and right-handed couplings **phenomenologically viable**

- **Can be tested** in $B \to D^{(*)}\tau\nu$ angular observables



Thank you for your attention!

Theoretical predictions of the width difference and semileptonic CP asymmetry of B mesons in the Unitarity Triangle analysis

EFT school 2024 | Zürich

Josua Scholze | supervisors: Prof. Luca Silvestrini and Prof. Tobias Hurth

18.07.2024

MPA
MAINZ PHYSICS ACADEMY

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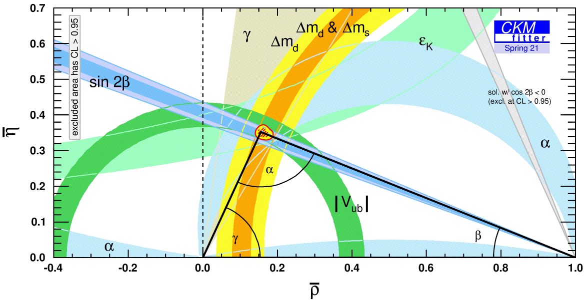
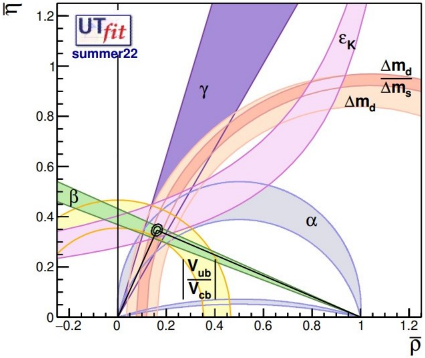
SAPIENZA
UNIVERSITÀ DI ROMA

Testing the Standard Model

- Flavor observables (e.g.: ΔM_d, ΔM_s) put strong constrains on the Standard Model
- Unitarity triangle by CKMfitter, UTfit:





Content

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- Theory for Γ_{12}
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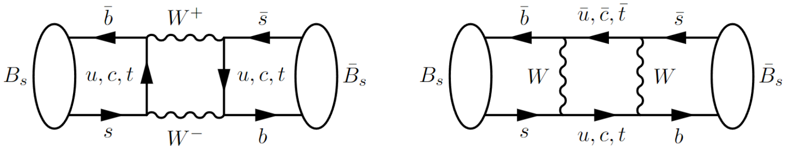
Mixing of neutral B mesons

- Weak interaction allows mixing:



- Hamiltonian: $\hat{H} = \hat{M} - i\hat{\Gamma}/2 = \begin{pmatrix} M_{11} - i\frac{\Gamma_{11}}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{21} - i\frac{\Gamma_{21}}{2} & M_{22} - i\frac{\Gamma_{22}}{2} \end{pmatrix}$ for the states $\begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix}$
- Diagonalization of $\hat{H}$ gives mass states: $|B_{\text{H}}\rangle = p\,|B\rangle + q\,|\bar{B}\rangle\,, |B_{\text{L}}\rangle = p\,|B\rangle - q\,|\bar{B}\rangle$
- M_{12}: off-shell contribution from: u, c, t, W
- Γ_{12}: on-shell contribution from: u, c

Physical observables

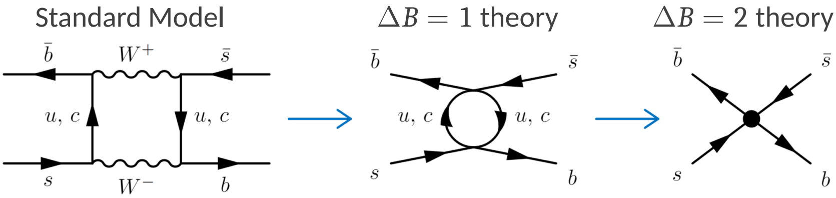
- Three independent observables: (in B system: $|\Gamma_{12}| \ll |M_{12}|$) — SM pred. for B_s
  - Mass difference: $\Delta M = M_{\mathrm{H}} - M_{\mathrm{L}} \approx 2|M_{12}|$ — $\sim 18\,\mathrm{ps}^{-1}$
  - Decay width difference: $\Delta\Gamma = \Gamma_{\mathrm{L}} - \Gamma_{\mathrm{H}} = -\mathrm{Re}\left(\frac{\Gamma_{12}}{M_{12}}\right)\Delta M$ — $\sim 0.1\,\mathrm{ps}^{-1}$
  - Semileptonic CP asymmetry: $a_{\mathrm{sl}} = \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)$ — $\sim 2\cdot 10^{-5}$

    Experimentally from semileptonic decays: $\frac{\Gamma\left(\bar{B}(t)\to\bar{l}\nu_l X\right)-\Gamma\left(B(t)\to l\bar{\nu}_l X\right)}{\Gamma\left(\bar{B}(t)\to\bar{l}\nu_l X\right)+\Gamma\left(B(t)\to l\bar{\nu}_l X\right)}$
- Up to now measured precisely: ΔM_s, ΔM_d, $\Delta\Gamma_s$
- Need to improve prediction of $\Delta\Gamma_s$: by a factor 3

Content

Obtaining Γ_{12}

- General procedure:



- $\Delta B = 2$: Heavy Quark Expansion in $\Lambda/m_b \approx 0.05$
- $\Delta B = 1$: Decomposition of Γ_{12}:

$$\Gamma_{12} = -\left[\lambda_c^2\,\Gamma_{12}^{cc} + 2\,\lambda_c\,\lambda_u\,\Gamma_{12}^{uc} + \lambda_u^2\,\Gamma_{12}^{uu}\right]$$
$$= -\lambda_t^2\left[\Gamma_{12}^{cc} + 2\,\frac{\lambda_u}{\lambda_t}\,(\Gamma_{12}^{cc} - \Gamma_{12}^{uc}) + \frac{\lambda_u^2}{\lambda_t^2}\,(\Gamma_{12}^{cc} - 2\Gamma_{12}^{uc} + \Gamma_{12}^{uu})\right]$$

Why do we calculate the ratio Γ_{12}/M_{12}?

- Decomposition of Γ_{12}:

$$\Gamma_{12} = -\lambda_t^2 \left[\Gamma_{12}^{cc} + 2\,\frac{\lambda_u}{\lambda_t}\,(\Gamma_{12}^{cc} - \Gamma_{12}^{uc}) + \frac{\lambda_u^2}{\lambda_t^2}\,(\Gamma_{12}^{cc} - 2\Gamma_{12}^{uc} + \Gamma_{12}^{uu}) \right]$$

- Factor λ_t^2 appears also in $M_{12} \Rightarrow$ cancels in the ratio Γ_{12}/M_{12}
- For $a_{\text{sl}} = \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)$: Γ_{12}^{cc} doesn't contribute $\Rightarrow$ dependence on m_c

Why do we calculate the ratio Γ_{12}/M_{12}?

- Decomposition of Γ_{12}:

$$\Gamma_{12} = -\lambda_t^2 \left[\Gamma_{12}^{cc} \; + \; 2\,\frac{\lambda_u}{\lambda_t}\; (\Gamma_{12}^{cc} - \Gamma_{12}^{uc}) \; + \; \frac{\lambda_u^2}{\lambda_t^2}\; (\Gamma_{12}^{cc} - 2\Gamma_{12}^{uc} + \Gamma_{12}^{uu}) \right]$$

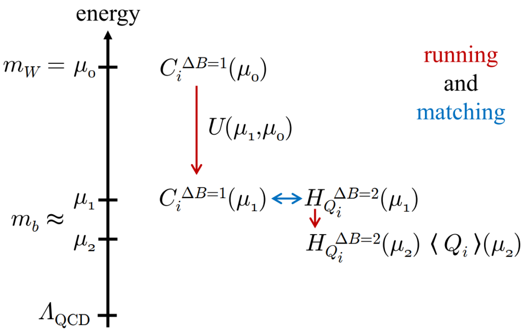
- Factor λ_t^2 appears also in $M_{12} \Rightarrow$ cancels in the ratio Γ_{12}/M_{12}
- For $a_{\mathrm{sl}} = \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)$: Γ_{12}^{cc} doesn't contribute $\Rightarrow$ dependence on m_c
- With $\Delta B = 2$ Wilson coefficients H_{Q_i} that contain $\Delta B = 1$ Wilson coefficients:

$$\Gamma_{12} \; \propto \; \sum_i H_{Q_i} \, \langle B | Q_i^{\Delta B=2} | \overline{B} \rangle$$

- M_{12} contains just one factor $\langle B | Q^{\Delta B=2} | \overline{B} \rangle \Rightarrow$ cancellation with Γ_{12} possible

Matching procedure for Γ_{12}

- Goal: obtain $\Delta B = 2$ Wilson coefficients at μ_2 used by lattice QCD
- use of Renormalization Group Equation (RGE): $\mu \frac{d}{d\mu}\vec{C}(\mu) = \vec{\gamma}\vec{C}(\mu)$



1. Match SM to 5-quark $\Delta B = 1$ theory: integrate out $W^{\pm}$, Z and top-quark
2. Run down to μ_1
3. Match $\Delta B = 2$ to $\Delta B = 1$ theory
4. Adapt to μ_2

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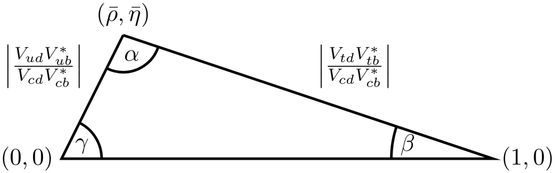
Unitarity Triangle: A geometric picture for CP violation

- CKM matrix is unitary: $V^\dagger V = 1$

$\rightarrow$ $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

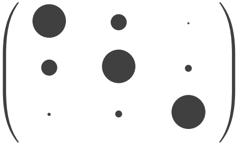
- Area predicts amount of CP violation



The Unitarity Triangle

- $[\text{quark}]_{\text{flavour}} = V\,[\text{quark}]_{\text{mass}}$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$\sim$ 

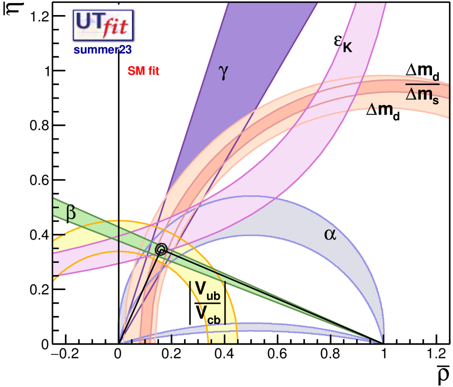
Current status of the Unitarity Triangle

- Assume the Standard model
- Use all available information: global fit

→ Shows good overall consistency

→ Favours inclusive determination of $|V_{cb}|$

- For scenarios with New Physics:
  - Tree-only Unitarity Triangle
  - Universal Unitarity Triangle



Fit results from UTfit collaboration

Content

What is HEPfit?

- Fitter for High Energy Physics
- Choose your model (e.g. Standard Model):
  - fit Model parameters to experimental constraints
  - predict observables
- Calculates probabilities with Bayesian statistics
- Optimized for Monte Carlo analysis
- Broad usage in phenomenology: Flavour and BSM physics…

©HEPfit collaboration

What did I add?

- Input parameters:
  - Subleading bag parameters for non-perturbative matrix elements: for Γ_{1/m_b}
  - Experimental values of ΔM: for prediction of $\Delta\Gamma$

- Calculation of Γ_{12}/M_{12}

- Observables for $\Delta\Gamma$ and a_{sl}
  - Taking different orders in α_s: LO, NLO, NNLO
  - For different mass schemes: pole, $\overline{\text{MS}}$, Potential Subtracted
  - Using the Renormalization independent scheme
  - Including partial contributions of higher orders

The C++ code for Γ_{12}

- Too long for my slides ... but available over `HEPfit` on GitHub

```cpp
/*******************************************************************************
 *  @f$\Gamma_{21}@f$ in NNLO from Marvin Gerlach (2205.07907 and thesis)      *
 * ****************************************************************************/

// Values of the products of CKM elements
gslpp::complex lambda_c_d; /* V_cd* V_cb  */
gslpp::complex lambda_u_d; /* V_ud* V_ub  */
gslpp::complex lambda_c_s; /* V_cs* V_cb  */
gslpp::complex lambda_u_s; /* V_us* V_ub  */

gslpp::vector<gslpp::complex> transformation(gslpp::vector< gslpp::complex > result, orders order);

//Values of DB=2 Wilson coefficients (Gerlach thesis)
gslpp::vector<gslpp::complex> c_H(quark q, orders order); //require compute_pp_s and Wilson coefficients in Misiak basis
gslpp::complex H(quarks qq, orders order); /*Values of contributions to the DB=2 Wilson coefficients for B_d (Gerlach thesis) */
gslpp::complex H_s(quarks qq, orders order); /*Values of contributions to the DB=2 Wilson coefficients for B_s (Gerlach thesis) */

// Values of DB=2 Wilson coefficients (Gerlach thesis) separated for
// C-12-12 (LO, NLO, NNLO), C-12-36 (LO, NLO), C-36-36 (LO, NLO),C-12-8 (LO, NLO), C-36-8 (LO, NLO), C-8-8 (LO)
gslpp::vector<gslpp::complex> c_H_partial(quark q, int i);
gslpp::vector<gslpp::complex> H_allpartial(quarks qq);   /*Values of partial contributions to the DB=2 Wilson coefficients for B_d (Gerlach thesis) */
gslpp::vector<gslpp::complex> H_s_allpartial(quarks qq); /*Values of partial contributions to the DB=2 Wilson coefficients for B_s (Gerlach thesis) */
gslpp::complex H_partial(quarks qq, int i_start, int i_end, int j_start, int j_end, int n);
gslpp::complex H_s_partial(quarks qq, int i_start, int i_end, int j_start, int j_end, int n);

// Values of the coefficient functions needed for DB=2 Wilson coefficients (Gerlach thesis)
double p(quarks qq, int i, int j, int n, bool flag_LOz = false);
double p_s(quarks qq, int i, int j, int n, bool flag_LOz = false);
double lastInput_compute_pp_s[4] = {NAN, NAN, NAN, NAN};

//Values of the coefficient functions needed for DB=2 Wilson coefficients (Gerlach thesis)
double cache_p[768] = { 0. };
double cache_ps[768] = { 0. };
//Values of the coefficient functions in LO in z needed for DB=2 Wilson coefficients (Gerlach thesis)
bool flag_LOz = true;
double cache_p_LO[576] = {0.};
double cache_ps_LO[576] = {0.};

// Method to compute coefficient functions needed for DB=2 Wilson coefficients (Gerlach thesis)
void compute_pp_s();
```

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Comparison with measurement

Predictions using the UT analysis

$$\Delta\Gamma_s = (0.071 \pm 0.011)\,\text{ps}^{-1}$$

$$a_{\text{sl}}^s = (2.27 \pm 0.13) \times 10^{-5}$$

$$\Delta\Gamma_d = (2.11 \pm 0.33) \times 10^{-3}\,\text{ps}^{-1}$$

$$a_{\text{sl}}^d = (-5.26 \pm 0.30) \times 10^{-4}$$

Experiment (HFLAV [2206.07501])

$$\Delta\Gamma_s = (0.083 \pm 0.005)\,\text{ps}^{-1}$$

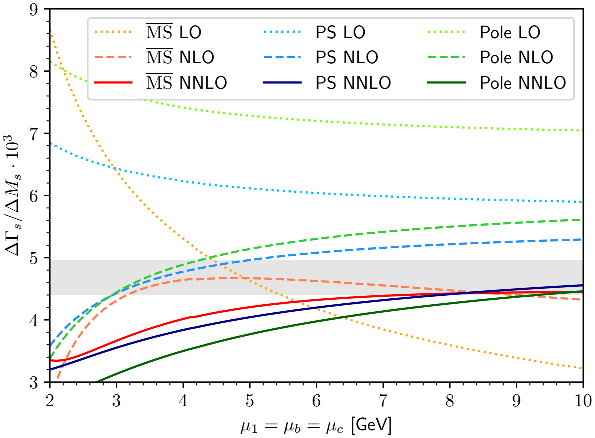
$$a_{\text{sl}}^s = (-60 \pm 280) \times 10^{-5}$$

$$\Delta\Gamma_d = (0.7 \pm 6.6) \times 10^{-3}\,\text{ps}^{-1}$$

$$a_{\text{sl}}^d = (-21 \pm 17) \times 10^{-4}$$

- Agreement between theory and experiment within 1 sigma
- Smaller theory uncertainties in $\Delta\Gamma$ than without UT analysis ($0.017\,\text{ps}^{-1}$ for $\Delta\Gamma_s$)

Renormalization scale dependence: $\Delta\Gamma_s$



- important consistency check ✓
- known characteristics:
  - μ_1 scale dependence shrinks by including higher orders
  - Potential Subtracted (PS) and $\overline{\text{MS}}$ scheme behave better than the pole scheme
  - consistent with experimental measurement (grey band: 1σ)

Renormalization independent scheme

- Renormalization prescription for the RI scheme:

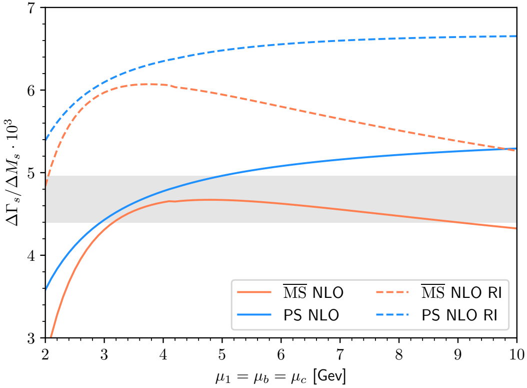
$$\langle F|Q_i|I\rangle_\lambda = \langle F|Q_i|I\rangle_{\text{tree}}$$

- Ensures to all orders:

$$\langle B|R_0|\bar{B}\rangle = \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$$

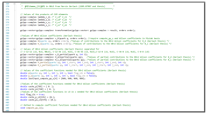
- Conversion only known to NLO

→ No significant improvement through the RI scheme

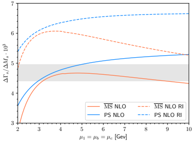


Conclusions

- Implemented $\Delta\Gamma$ and a_{sl} for different mass schemes in `HEPfit`
- Updated theory predictions within the UT analysis
- Compared scale dependence of different schemes and orders
- Future: Phenomenology studies of rare processes
  Extension to physics beyond the SM



$\Delta\Gamma_s = (0.071 \pm 0.011)\ \mathrm{ps}^{-1}$



Thank you for your attention.

Any questions?

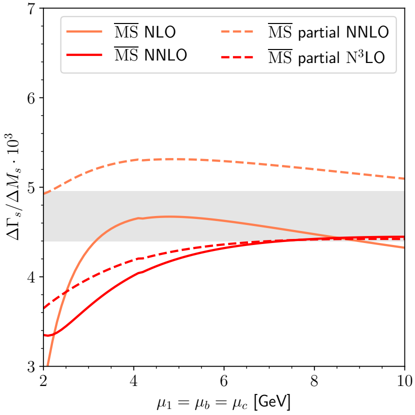
General sources

- "Effective Theories for Quark Flavour Physics" by Silvestrini
- "Three Lectures on Meson Mixing and CKM phenomenology" by Nierste
- "Meson width differences and asymmetries", thesis by Gerlach
- "CP violation in the B_s^0 system" by Artuso et al.
- "Gauge Theory of Weak Decays" by Buras
- "HEPfit Manual" by de Blas et al.

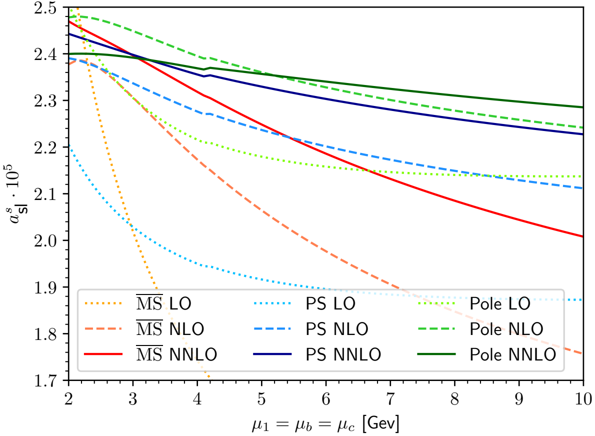
Renormalization scale dependence: partial N^3LO effects

- N^3LO pieces from products of NLO and NNLO factors
- No large shift of the central value
- Not RGE invariant

→ Only a first impression of N^3LO effects



Renormalization scale dependence: a_{sl}



- μ_1 scale dependence shrinks by including higher orders
- $\overline{\mathrm{MS}}$ scheme behaves worse than the other schemes
-

$\Delta B = 1$ Effective Hamiltonian

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \left\{ \left[\left(V_{cb}^* V_{ud} \left(C_1 Q_1 + C_2 Q_2 \right) + V_{cb}^* V_{cd} \left(C_1 Q_1^c + C_2 Q_2^c \right) + \left(c \leftrightarrow u \right) \right) \right. \right.$$
$$\left. \left. - V_{tb}^* V_{td} \left(\sum_{i=3}^{6} C_i Q_i + C_{8G} Q_{8G} \right) \right] + \left[d \rightarrow s \right] \right\} + h.c.$$

- operator in traditional basis [hep-ph/9211304], [hep-ph/0308029]:

$$Q_1 = (\bar{b}_i c_j)_{V-A} (\bar{u}_j d_i)_{V-A} \,, \qquad Q_2 = (\bar{b}_i c_i)_{V-A} (\bar{u}_j d_j)_{V-A} \,,$$
$$Q_1^c = (\bar{b}_i c_j)_{V-A} (\bar{c}_j d_i)_{V-A} \,, \qquad Q_2^c = (\bar{b}_i c_i)_{V-A} (\bar{c}_j d_j)_{V-A} \,,$$
$$Q_3 = (\bar{b}_i d_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A} \,, \qquad Q_4 = (\bar{b}_i d_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A} \,,$$
$$Q_5 = (\bar{b}_i d_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A} \,, \qquad Q_6 = (\bar{b}_i d_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A} \,,$$
$$Q_{8G} = \frac{g_s}{8\pi^2} m_b \bar{b}_i \sigma^{\mu\nu} \left(1 - \gamma^5 \right) t_{ij}^a d_j G_{\mu\nu}^a$$

$\Delta B = 1$ Effective Hamiltonian

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \Bigg\{ \Bigg[\Bigg(V_{cb}^* V_{ud} \left(C_1 Q_1 + C_2 Q_2 \right) + V_{cb}^* V_{cd} \left(C_1 Q_1^c + C_2 Q_2^c \right) + \left(c \leftrightarrow u \right) \Bigg)$$

$$- V_{tb}^* V_{td} \left(\sum_{i=3}^{6} C_i Q_i + C_{8G} Q_{8G} \right) \Bigg] + \Big[d \to s \Big] \Bigg\} + h.c.$$

- to diminish problems with γ_5:

  alternative basis by Chetyrkin, Misiak and Münz [hep-ph/9711280]
  known up to NNLO and transformation to traditional basis up to NLO

Operator basis for $\Delta B = 2$

- Result: $\Gamma_{12} = \frac{G_F^2 m_b^2}{24\pi M_B}\left[H(z)\langle B|Q|\bar{B}\rangle + \cancel{H_S(z)\langle B|Q_S|\bar{B}\rangle} + \widetilde{H}_S(z)\langle B|\widetilde{Q}_S|\bar{B}\rangle\right] + \Gamma_{1/m_b}$
- with dimension 6 operators:

$$Q = \bar{s}_i\gamma^\mu\left(1-\gamma^5\right)b_i\,\bar{s}_j\gamma_\mu\left(1-\gamma^5\right)b_j$$
$$Q_S = \bar{s}_i\left(1+\gamma^5\right)b_i\,\bar{s}_j\left(1+\gamma^5\right)b_j$$
$$\widetilde{Q}_S = \bar{s}_i\left(1+\gamma^5\right)b_j\,\bar{s}_j\left(1+\gamma^5\right)b_i$$
$$R_0 = \frac{1}{2}\alpha_1 Q + Q_S + \alpha_2\widetilde{Q}_S = \mathcal{O}\left(\frac{\Lambda}{m_b}\right)\text{, at LO in }\alpha_s\text{: }\alpha_1 = \alpha_2 = 1$$

- old choice: use Q and Q_S [hep-ph/9808385], implemented from [hep-ph/0308029]
- better alternative: use Q and $\widetilde{Q}_S$ [hep-ph/0612167] to cancel $\langle B|Q|\bar{B}\rangle$ in $\Delta\Gamma/\Delta M$

Switch to the RI scheme for $\Delta B = 2$ operators

- renormalization prescription for the RI scheme [hep-ph/9501265]:

$$\langle F|Q_i|I\rangle_\lambda = \langle F|Q_i|I\rangle_{\text{tree}}$$

- ensures to all orders: $\langle B|R_0|\bar{B}\rangle = \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$
- conversion only known to NLO [hep-lat/0110091]:

$$\begin{pmatrix} \langle Q(\mu)\rangle \\ \langle Q_S(\mu)\rangle \\ \langle \widetilde{Q}_S(\mu)\rangle \end{pmatrix}_{\overline{\text{MS}}} = \left[\mathbb{1} + r_{123}\frac{\alpha_s(\mu)}{4\pi}\right] \begin{pmatrix} \langle Q(\mu)\rangle \\ \langle Q_S(\mu)\rangle \\ \langle \widetilde{Q}_S(\mu)\rangle \end{pmatrix}_{\text{RI}}, r_{123} = \frac{1}{9}\begin{pmatrix} -42+72\log 2 & 0 & 0 \\ 0 & 61+44\log 2 & -7+28\log 2 \\ 0 & -25+28\log 2 & -29+44\log 2 \end{pmatrix}$$

Resummation of logarithms

- dominant z-dependent contribution at order α_s^n from $\alpha_s^n z \ln^n z$
- change renormalisation scheme [hep-ph/0307344]:

$$z = \frac{\overline{m}_c^2(\overline{m}_c)}{\overline{m}_b^2(\overline{m}_b)} \to \bar{z} = \frac{\overline{m}_c^2(\overline{m}_b)}{\overline{m}_b^2(\overline{m}_b)} \approx \frac{z}{2}$$

- important for semileptonic asymmetry (of order z)

Jet Bundle Geometry of Scalar EFTs

Mohammad Alminawi (Speaker) - University of Zurich
Ilaria Brivio – University of Bologna
Joe Davighi – CERN

Zurich EFT School 2024

Scalar Effective Field Theories

$$L = V + \frac{1}{2} g_{ij}(\phi)\partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4)$$

Outline

- Motivation for geometric formalism
- Motivation for bundle formalism
- Introduction to bundles and jets
- Non-derivative field redefinitions as diffeomorphisms
- Amplitude calculations on 0-Jet bundle

Motivation for Geometric Formalism

- SMEFT and HEFT are the main way to extend the standard model

$$SM \subset SMEFT \subset HEFT$$

- Map from SMEFT to HEFT is well defined. Inverse is tricky.

- Exploit geometric techniques to identify when HEFT is needed.

[T. Cohen, N. Craig, X. Lu and D. Sutherland, arXiv:2008.08597]
[R. Alonso, E.E. Jenkins and A.V. Manohar, arXiv:1602.00706]
[R. Gomez-Ambrosio et al., arXiv:2204.01763]

Motivation for Bundle Geometry

- Previous geometric formulations

[R. Alonso, E.E. Jenkins and A.V. Manohar, arXiv:1605.03602]

$$L = \frac{1}{2} g_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - V + O(\partial^4)$$

[A. Helset, A. Martin and M. Trott, arXiv:2001.01453]

- Using jet bundles

$$L = \frac{1}{2} g_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - V + O(\partial^4)$$

[M. Alminawi, I. Brivio and J. Davighi , arXiv:2308.00017]

Motivation for Bundle Geometry

- Full Lagrangian obtained from geometry

$$L = \frac{1}{2}\langle \eta^{-1} , \ (j^n \phi)^* \, g \, \rangle$$

- Transformation rules of physical amplitudes indicate that they are combinations of momenta and tensors

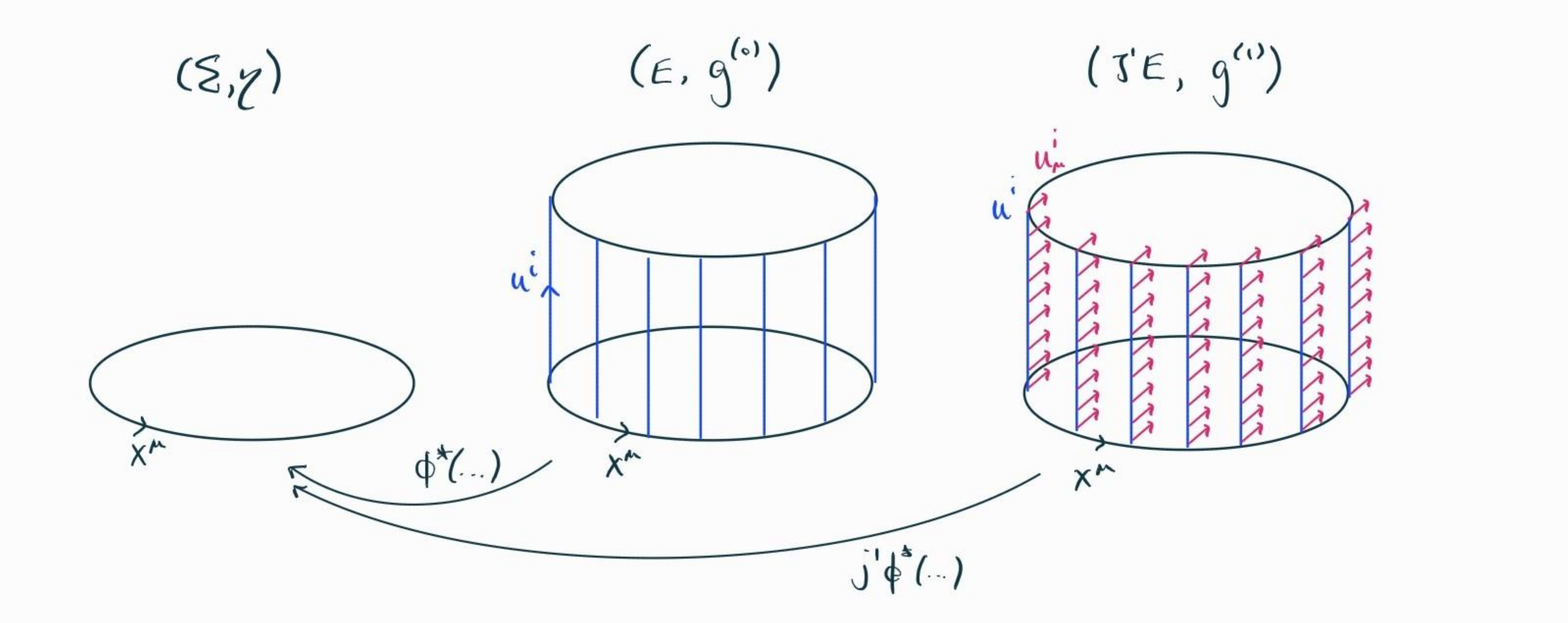
$$\overline{V}_{;(\alpha_1\alpha_2\alpha_3\alpha_4)} + \frac{2}{3}\left(s_{12}\overline{R}_{\alpha_1(\alpha_3\alpha_4)\alpha_2} + s_{13}\overline{R}_{\alpha_1(\alpha_2\alpha_4)\alpha_3} + s_{14}\overline{R}_{\alpha_1(\alpha_2\alpha_3)\alpha_4}\right)$$

[T. Cohen, N. Craig, X. Lu and D. Sutherland, arXiv:2108.03240]

- Only tensors that can be constructed from a metric with a torsion free connection are of the form $\nabla^n R^m$ where n, m are integers

[M. Alminawi, I. Brivio and J. Davighi , *in progress*]

What is a Bundle?



What is a Bundle?

- Consider two manifolds Σ and E with coordinate charts $\{x^\mu\}$ and $\{x^\mu, u^i\}$ and a map $\pi: \Sigma \to E$ then the triple (Σ, E, π) forms a bundle

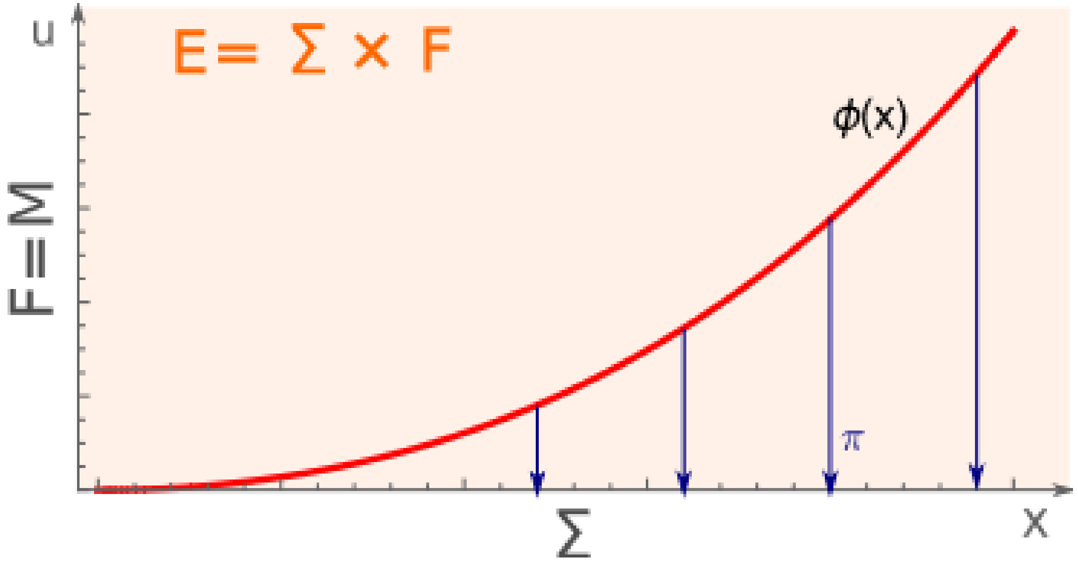
- Local inverses to the map π are called sections ϕ and they are defined by

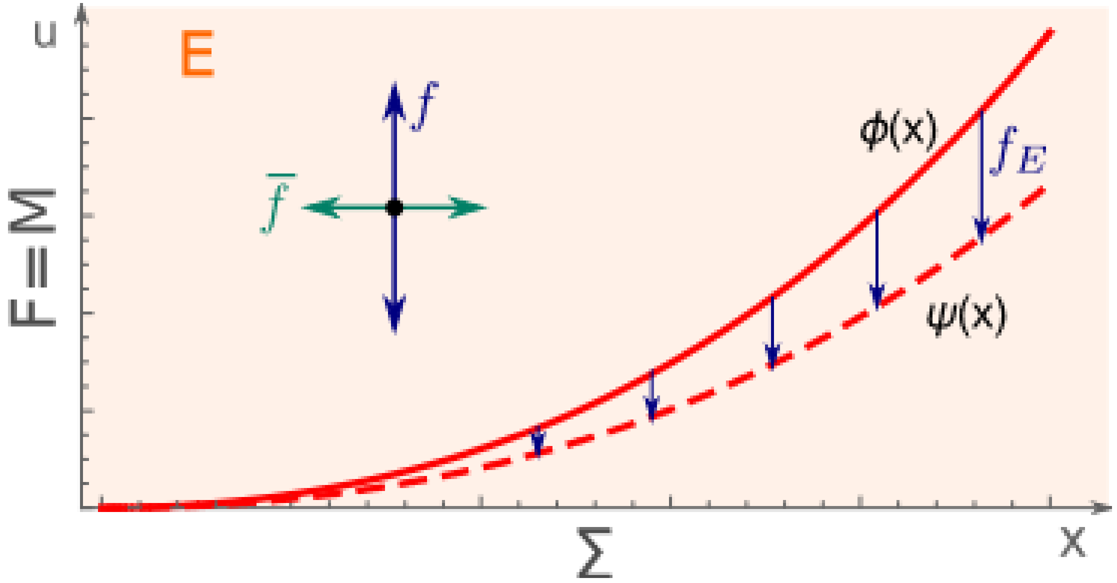
$$\phi \circ x^\mu = x^\mu$$
$$\phi \circ u^i = \phi^i$$

- Sections give us the tools to obtain fields and their derivatives from coordinates on bundles

D. J. Saunders, The Geometry of Jet Bundles, doi:10.1017/CBO9780511526411

What is a Bundle?





What is a Jet?

- Two sections ϕ, ψ are called 1-equivalent at some point $p \in E$ if we have

$$\phi(p) = \psi(p) \qquad \left.\frac{\partial(\phi \circ u^i)}{\partial x^\mu}\right|_p = \left.\frac{\partial(\psi \circ u^i)}{\partial x^\mu}\right|_p$$

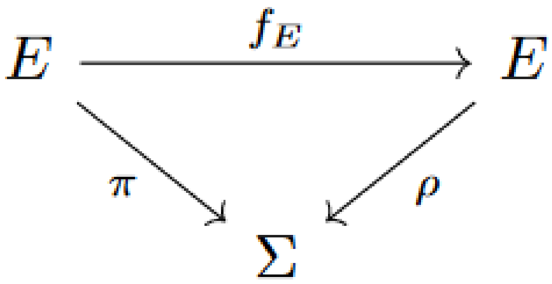
- The equivalence class containing ϕ at p is called the 1-jet and is denoted $j_p^1\phi$

- The set of all 1-jets is referred to as the 1-jet bundle and it naturally has the structure of a smooth manifold

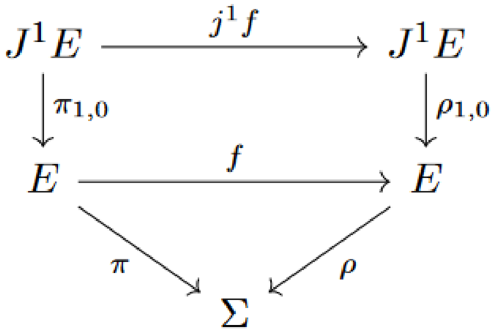
Field Redefinitions on Bundles

- A non-derivative field redefinition in the Lagrangian is equivalent to a diffeomorphism on the bundle

- Consider transformations that leave spacetime unchanged



Morphism on 0-Jet Bundle
Equivalent to $\psi = f_E \circ \phi$



Morphism on 1-Jet Bundle
Equivalent to $j^1\psi = j^1f \circ j^1\phi$

Diffeomorphism vs. Coordinate Transformation

- Tensors are coordinate independent, thus a coordinate transformation $x \to y(x)$ leaves the metric unchanged

$$g = g_{ij}(x)dx^i dx^j \to g'_{ab}(y(x))dy^a dy^b = g_{ij}(x)\frac{\partial x^i}{\partial y^a}\frac{\partial x^j}{\partial y^b}dy^a dy^b = g$$

- In contrast a diffeomorphism of the form $x \to y(x)$ transforms the metric as follows

$$g = g_{ij}(x)dx^i dx^j \to g_{ab}\big(y(x)\big)\frac{\partial y^a}{\partial x^i}\frac{\partial y^b}{\partial x^j}dx^i dx^j$$

- Where now $g_{ab}\big(y(x)\big) \neq g_{ij}(x)\frac{\partial x^i}{\partial y^a}\frac{\partial x^j}{\partial y^b}$

Riemannian Metric on Jet Bundle

$$L = \frac{1}{2}\langle \eta^{-1} , (j^n\phi)^* g \rangle$$

$$(j^1\phi)^*g = g_{\mu\nu}dx^\mu \otimes dx^\nu + g_{ij}d\phi^i \otimes d\phi^j + g_{ij}^{\mu\nu}d\phi_\mu^i \otimes d\phi_\nu^j$$
$$+g_{i\mu}d\phi^i \otimes dx^\mu + g_{ij}^{\mu}d\phi_\mu^i \otimes d\phi^j + g_{i\nu}^{\mu}d\phi_\mu^i \otimes dx^\nu$$

$$g_{ij}\eta^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j \subset L$$

$$g_{\mu\nu}\eta^{\mu\nu} = V(\phi) + \cdots \subset L$$

$$g_{ij}^{\mu\nu}\eta^{\rho\sigma}\partial_\rho\partial_\mu\phi^i\partial_\sigma\partial_\nu\phi^j \subset L$$

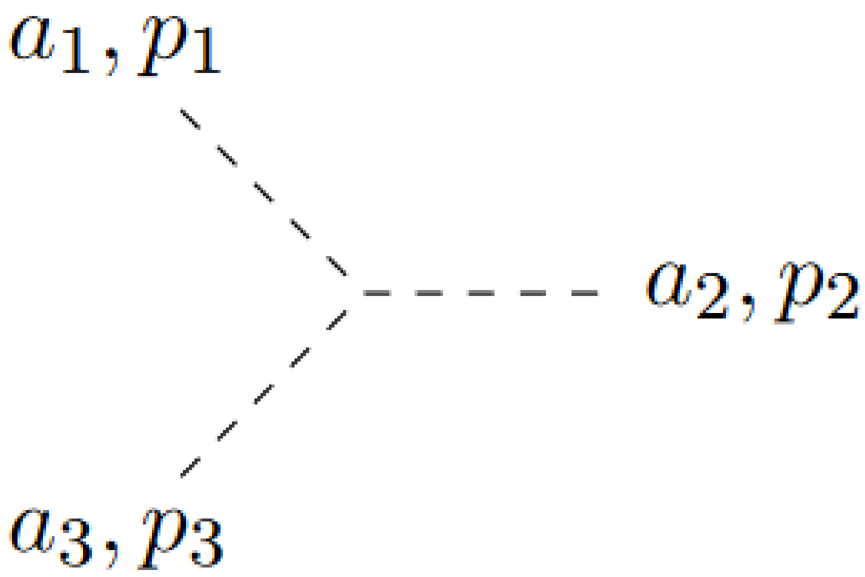
Amplitudes on 0-Jet

- Poincare invariance implies that our metric is block diagonal

$$\begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & g_{ij} \end{pmatrix}$$

- Where $g_{\mu\nu} = -\frac{1}{2}\eta_{\mu\nu}V$ has dimensions determined by spacetime and g_{ij} has dimensions determined by the number of fields

Three Point Amplitude on 0-Jet Bundle



Three Point Amplitude on 0-Jet Bundle

- Label the particles 1,2,3 and their flavors by a_1, a_2, a_3
- Label quantities evaluated at the vacuum (typically $u^i = 0$) with a bar $\overline{g_{ij}} = g_{ij}(0)$. Derivatives denoted by a comma $\partial_k g_{ij} = g_{ij,k}$
- The Feynman rule for a 3-point interaction is given by

$$\frac{1}{12}\eta^{\mu\nu}\bar{g}_{\mu\nu,a_1a_2a_3} + \frac{1}{2}\bar{g}_{a_1a_2,a_3}p_1 \cdot p_2 + \frac{1}{2}\bar{g}_{a_1a_3,a_2}p_1 \cdot p_3 + \frac{1}{2}\bar{g}_{a_2a_3,a_1}p_2 \cdot p_3$$

Three Point Amplitude on 0-Jet Bundle

- The momenta fulfill

$$p_3^2 = (p_1 + p_2)^2$$

- The Christoffel symbols are defined as

$$\Gamma_{IJK} = \frac{1}{2}(g_{IJ,K} + g_{IK,J} - g_{JK,I})$$

- For the momentum independent term

$$\eta^{\mu\nu}\bar{g}_{\mu\nu,a_1a_2a_3} = \overline{\nabla_{a_3}R^{\mu}_{a_1\mu\,a_2}} - 2(m_1^2\,\overline{\Gamma_{a_1a_2a_3}} + m_2^2\,\overline{\Gamma_{a_2a_1a_3}} + m_3^2\,\overline{\Gamma_{a_3a_1a_2}})$$

Three Point Amplitude on 0-Jet Bundle

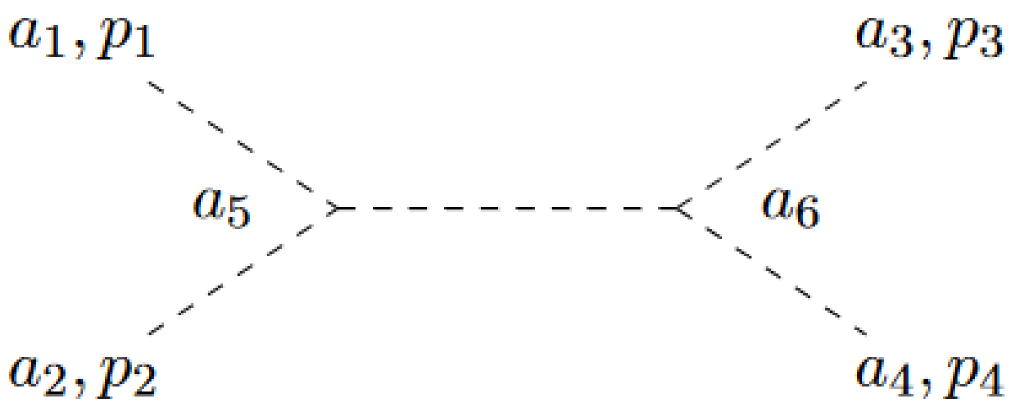
- Accounting for the symmetry factors the three-point amplitude is given by

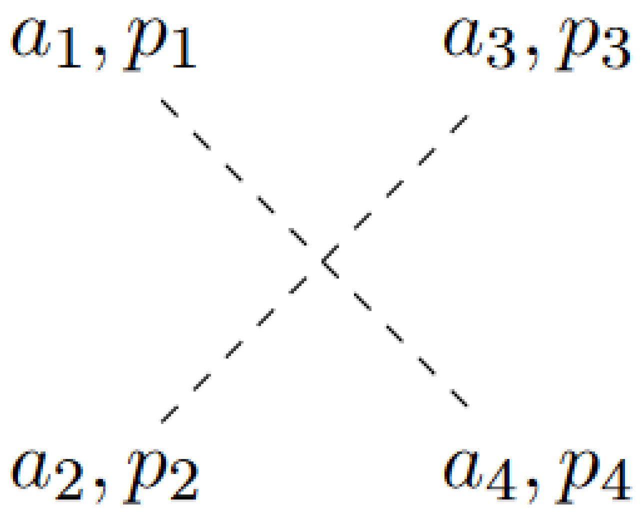
$$-i\left(\frac{1}{6}(\overline{\nabla_{a_3}R^{\mu}_{a_1\mu a_2}}+\overline{\nabla_{a_2}R^{\mu}_{a_1\mu a_3}}+\overline{\nabla_{a_1}R^{\mu}_{a_2\mu a_3}}) \right.$$

$$\left. +(p_1^2-m_1^2)\overline{\Gamma_{a_1a_2a_3}}+(p_2^2-m_2^2)\overline{\Gamma_{a_2a_1a_3}}+(p_3^2-m_3^2)\overline{\Gamma_{a_3a_2a_1}}\right)$$

- On-shell only the tensorial piece survives

Two to Two Scattering





Two to Two Scattering

- Contributions from gluing of three-point interactions and from contact terms

- Momenta degrees of freedom exist unlike the three-point amplitude

- On-shell the amplitude should be given by products of s_{12}, s_{13}, s_{14} and $\nabla^n R^m$ with $n, m \leq 2$

Two to Two Scattering

$$
= i\Bigg(\frac{1}{24}\Bigg(\overline{\nabla_{a_1}\nabla_{a_2}R^{\mu}_{a_3\mu a_4}} + \overline{\nabla_{a_2}\nabla_{a_1}R^{\mu}_{a_3\mu a_4}} + \overline{\nabla_{a_1}\nabla_{a_3}R^{\mu}_{a_4\mu a_2}} + \overline{\nabla_{a_3}\nabla_{a_1}R^{\mu}_{a_4\mu a_2}} + \overline{\nabla_{a_1}\nabla_{a_4}R^{\mu}_{a_3\mu a_2}}
$$

$$
+ \overline{\nabla_{a_4}\nabla_{a_1}R^{\mu}_{a_3\mu a_2}} + \overline{\nabla_{a_2}\nabla_{a_3}R^{\mu}_{a_1\mu a_4}} + \overline{\nabla_{a_3}\nabla_{a_2}R^{\mu}_{a_1\mu a_4}} + \overline{\nabla_{a_2}\nabla_{a_4}R^{\mu}_{a_1\mu a_3}} + \overline{\nabla_{a_4}\nabla_{a_2}R^{\mu}_{a_1\mu a_3}}
$$

$$
+ \overline{\nabla_{a_3}\nabla_{a_4}R^{\mu}_{a_1\mu a_2}} + \overline{\nabla_{a_4}\nabla_{a_3}R^{\mu}_{a_1\mu a_2}}\Bigg) + \frac{5}{6}\Bigg(\overline{R^{\mu}_{a_1\nu a_2}R^{\nu}_{a_3\mu a_4}} + \overline{R^{\mu}_{a_1\nu a_3}R^{\nu}_{a_2\mu a_4}} + \overline{R^{\mu}_{a_1\nu a_4}R^{\nu}_{a_2\mu a_3}}\Bigg)
$$

$$
+ \frac{1}{3}\Bigg(s_{12}\Bigg(\overline{R_{a_1a_4a_3a_2}} + \overline{R_{a_2a_4a_3a_1}}\Bigg) + s_{13}\Bigg(\overline{R_{a_1a_4a_2a_3}} + \overline{R_{a_3a_4a_2a_1}}\Bigg) + s_{14}\Bigg(\overline{R_{a_1a_2a_3a_4}} + \overline{R_{a_4a_2a_3a_1}}\Bigg)\Bigg)
$$

$$
+ \frac{1}{36}\Bigg(\frac{\overline{g^{a_5a_6}}}{s_{12}-m_5^2}(\overline{\nabla_{a_5}R^{\mu}_{a_1\mu a_2}} + \overline{\nabla_{a_1}R^{\mu}_{a_5\mu a_2}} + \overline{\nabla_{a_2}R^{\mu}_{a_1\mu a_5}})(\overline{\nabla_{a_6}R^{\mu}_{a_3\mu a_4}} + \overline{\nabla_{a_3}R^{\mu}_{a_6\mu a_4}} + \overline{\nabla_{a_4}R^{\mu}_{a_3\mu a_6}})
$$

$$
+ (2 \leftrightarrow 3) + (2 \leftrightarrow 4)\Bigg)\Bigg)
$$

Diffeomorphisms and Tensors

- Under a general diffeomorphism f the Riemann tensor is not invariant

$$R_{IJKL}(x)dx^I dx^J dx^K dx^L \to R_{IJKL}(f(x))\frac{\partial(f\circ x^I)}{\partial x^A}\frac{\partial(f\circ x^J)}{\partial x^B}\frac{\partial(f\circ x^K)}{\partial x^C}\frac{\partial(f\circ x^L)}{\partial x^D}dx^A dx^B dx^C dx^D$$

- A diffeomorphism of the form $u \to f(u) = u + c_n u^n$ with $n \geq 2$ is special since at the point $u = 0$ we have

$$\lim_{u\to 0} f(u) = \lim_{u\to 0} u = 0 \qquad \lim_{u\to 0}\frac{\partial(f\circ u^i)}{\partial u^j} = \delta^i_j$$

- Tensors are invariant under such a transformation at the vacuum just like amplitudes

Scalar Curvature

- The Ricci Scalar R is also not invariant under a diffeomorphism f. It transforms according to

$$R(u) \rightarrow R(f(u))$$

- At the vacuum, a diffeomorphism of the form discussed earlier leaves the scalar invariant since

$$\lim_{u \to 0} R(f(u)) = \lim_{u \to 0} R(u)$$

- Disagreement of Ricci scalars at the vacuum indicates that the physical amplitudes are different.

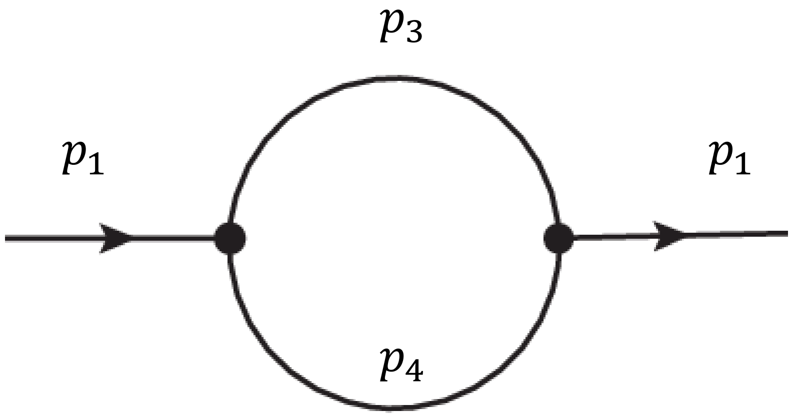
Conclusion and Outlook

- Jet bundles offer a path to write a Lagrangian of any derivative order in terms of geometry
- Amplitudes are combinations of geometric tensors
- Non-derivative field redefinitions are diffeomorphisms on bundle
- Derivative field redefinitions as maps between jet bundle orders (in progress)
- Incorporating gauge fields and fermions (future goal)

Thank you

Loop Diagrams

- Consider the 1-loop correction to the propagator



$$\int \frac{d^4p_3}{(2\pi)^4}\frac{\overline{g}^{a_3a_5}\overline{g}^{a_4a_6}}{(p_3^2-m_3^2)((p_1+p_3)^2-m_4^2)}$$

$$\left(\frac{1}{6}(\overline{\nabla_{a_5}R^{\mu}_{a_2\mu a_6}}+\overline{\nabla_{a_2}R^{\mu}_{a_5\mu a_6}}+\overline{\nabla_{a_6}R^{\mu}_{a_2\mu a_5}})+(p_3^2-m_3^2)\bar{\Gamma}_{a_5a_2a_6}+((p_1+p_3)^2-m_4^2)\bar{\Gamma}_{a_6a_2a_5}\right)$$

$$\left(\frac{1}{6}(\overline{\nabla_{a_3}R^{\mu}_{a_1\mu a_4}}+\overline{\nabla_{a_1}R^{\mu}_{a_3\mu a_4}}+\overline{\nabla_{a_4}R^{\mu}_{a_1\mu a_3}})+(p_3^2-m_3^2)\bar{\Gamma}_{a_3a_1a_4}+((p_1+p_3)^2-m_4^2)\bar{\Gamma}_{a_4a_1a_3}\right)$$