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# On the Propagation of Relativistic Shocks in Conductive Media

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## 1. Abstract

Relativistic shocks propagating in perfectly conductive plasmas have been extensively studied due to their central role in high energy astrophysical phenomena, with Gamma-Ray Bursts being the most prominent example. In the present work, assuming a finite electrical conductivity for the propagation medium and a finite length for the shock front, we investigate the mechanism by which a relativistic shock interacts with the propagation medium's electromagnetic field. These two assumptions necessitate the inclusion of one more jump condition derived through the covariant Gauss-Ampère Law and introduce a dimensionless parameter dependent on the magnetic diffusivity of the plasma in the shock front, the shock front's length, as well as on the shock's propagation four-velocity. We investigate the effects of this parameter's value on shock dynamics and discuss possible applications of this model in the study of Gamma-Ray Bursts.

**Keywords:**
**MHD - shockwaves - gamma-ray bursts**

## 2. Introduction

The jump conditions for relativistic shocks propagating in electrically conductive media are derived by projecting the particle number four-current $N^\mu$, energy-momentum tensor $T^{\mu\nu}$, electromagnetic tensor $F^{\mu\nu}$ and its Hodge dual ${}^*F^{\mu\nu}$ on a spacelike four-vector $S^\mu$ perpendicular to the timelike shock hypersurface in Minkowski space [1]. These covariant relations are:

- $$[N^\mu]S_\mu = 0 \tag{1}$$
- $$[T^{\mu\nu}]S_\nu = 0 \tag{2}$$
- $$[{}^*F^{\mu\nu}]S_\mu = 0 \tag{3}$$
- $$[F^{\mu\nu}]S_\mu + \frac{4\pi\mathcal{L}_{sh}}{c}\tilde{J}^\nu = 0 \tag{4}$$

where $[\mathcal{Q}] = \mathcal{Q}_2 - \mathcal{Q}_1$, with the subscripts 1 and 2 denoting the propagation and shocked media respectively. In order to study the effects of the shock front plasma's finite electrical conductivity on the properties of the shocked medium, one needs to assume that the shock front is a hypervolume of finite length $\mathcal{L}_{sh}$ along the shock normal $S^\mu$, instead of a hypersurface normal to $S^\mu$. $\tilde{J}^\nu$ then is average four-current over $\mathcal{L}_{sh}$, defined as:

- $$\tilde{J}^\nu = \xi\,(J_1^\nu + J_2^\nu) \tag{5}$$

with $\xi$ of order unity. The four-current is determined by the special relativistic generalization of Ohm's Law:

- $$J^\mu + J_\nu\frac{U^\nu U^\mu}{c^2} = \frac{\sigma}{c}F^{\mu\nu}U_\nu \tag{6}$$

where $U^\mu$ is the medium's bulk four-velocity.
The jump conditions are solved numerically, assuming the Taub-Matthews Equation of State [2]:

- $$\rho h c^2 = \frac{5}{2}P + \sqrt{\frac{9}{4}P^2 + \rho^2 c^4} \tag{7}$$

with $\rho, h, P$ the plasma's rest mass density, specific enthalpy and thermal pressure respectively.
The following field configuration is assumed, in the propagation medium's frame:

- $$\boldsymbol{\beta}_2 = \beta_2\hat{x}, \boldsymbol{E}_2 = E_2\hat{y}, \boldsymbol{B}_{1,2} = B_{1,2}\hat{z} \tag{8}$$

All other field components are taken to be equal to zero.

## 3. Results

The 3+1 decomposition of Eqs. 1-4 reveals that the properties of the shocked medium are determined by a dimensionless parameter:

- $$\alpha = \frac{\xi}{\kappa}\frac{c\tilde{\Gamma}_{sh}^2\tilde{\beta}_{sh}\tilde{\mathcal{L}}_{sh}}{\eta} \tag{9}$$

with $\kappa$ of order unity, as determined through the relation:

- $$\tilde{\Gamma}_{sh}^2\tilde{\beta}_{sh} = \kappa\Gamma_{sh,2}\beta_{sh,2} \tag{10}$$

where $\Gamma_{sh,2}\beta_{sh,2}$ is the four-velocity of the shock front in the shocked medium's frame. $\tilde{\Gamma}_{sh}\tilde{\beta}_{sh}$, $\tilde{\mathcal{L}}_{sh}$ are the shock front's propagation four-velocity and length, as measured in the frame of the plasma inside the shock front, and $\eta$ the magnetic diffusivity of the plasma in the shock front's volume. As the value of $\alpha$ increases, the solutions to the jump conditions 1-4 approach those to the Ideal MHD jump conditions [3], [4]. For $\alpha \ll 1$ the medium's electromagnetic field is unaffected by the shock, while its hydrodynamic quantities satisfy the hydrodynamic jump conditions.

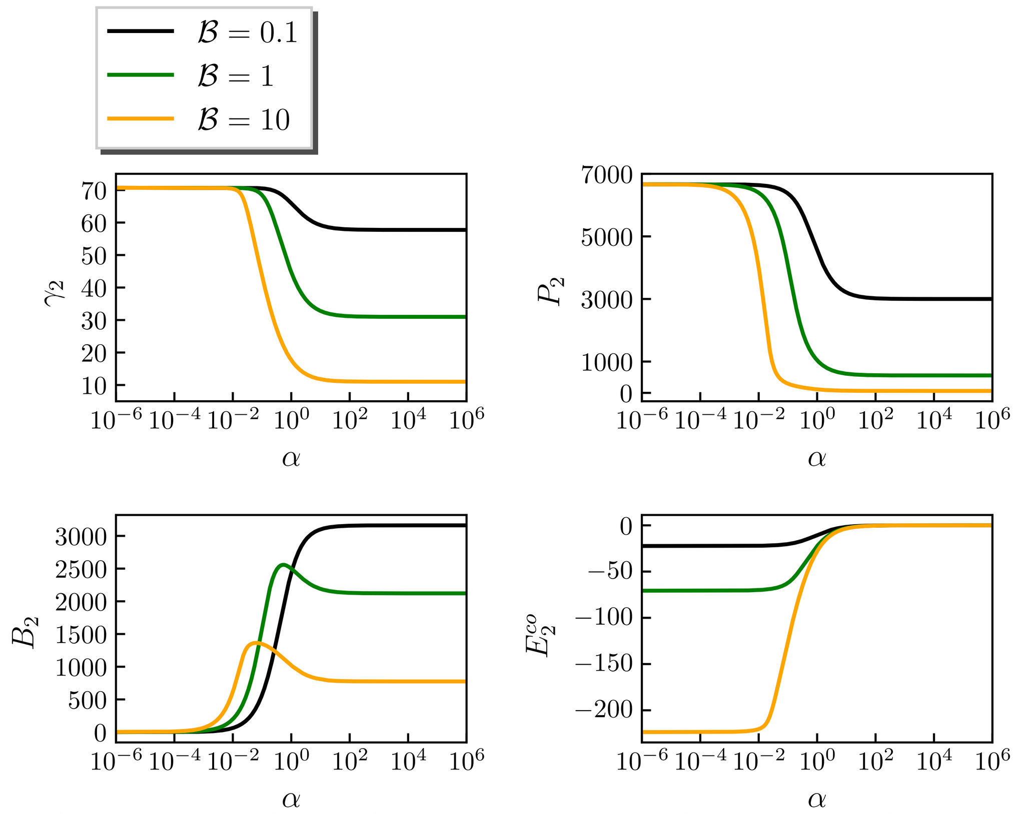


**Figure 1:** The shocked medium's Lorentz factor $\gamma_2$, thermal pressure $P_2$, magnetic field $B_2$, and comoving electric field $E_2^{co}$ for $\Gamma_{sh} = 100$.
$\mathcal{B}$ is defined as $\frac{B_1^2}{8\pi\rho_1 c^2}$.
The units used are: $[P_2] = [\rho_1 c^2]$, $[B_2] = \left[\sqrt{8\pi\rho_1 c^2}\right]$.

## 4. Conclusions

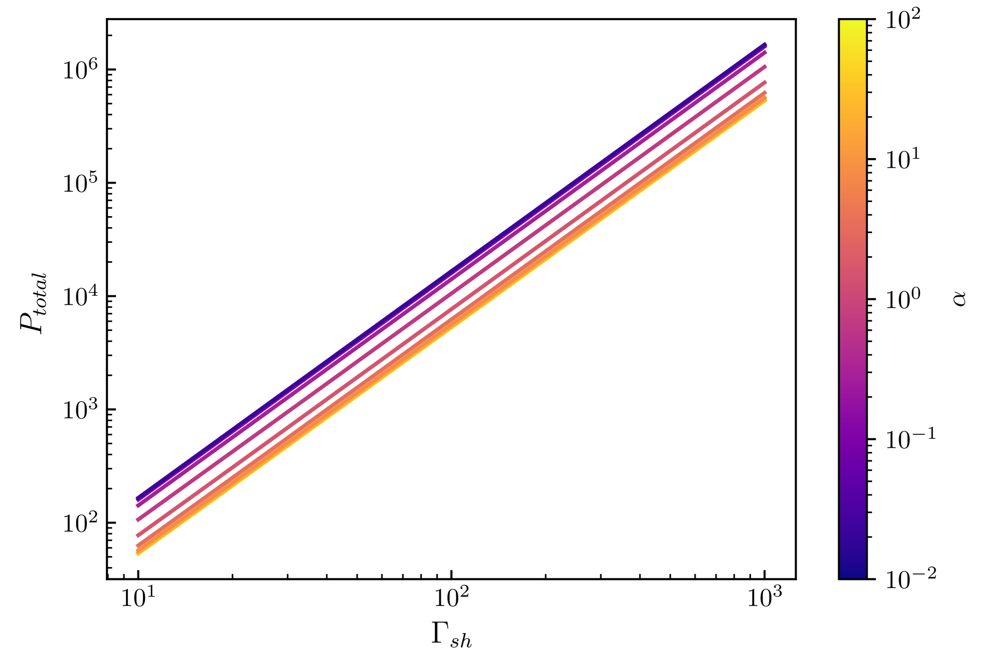


**Figure 2:** Total pressure generated by the shock for a propagation medium with $\mathcal{B} = 1$.

- The jump conditions for relativistic shocks are supplemented with a new covariant relation derived through the Gauss-Ampère Law. The properties of the shocked medium are fully determined by the value of the dimensionless parameter $\alpha$ (Eq. 9).
- The value of $\alpha$ strongly affects the shocked medium. The results presented demonstrate that as $\alpha$ increases, the degree to which the medium's EM field is affected by the shock also increases. For $\alpha \gg 1$ the Ideal MHD jump conditions are retrieved.

## 5. Future Research

The present analysis expands the parameter space of relativistic shock propagation in magnetized media, shedding light on the as of yet largely uninvestigated dynamics of relativistic shock propagation in plasmas with a finite electrical conductivity and paves the way for future applications in the study of GRBs. In future works the forward-shock-reverse shock problem is considered, through which the conditions inside the blastwave as well as its evolution are determined, in the spirit of [5].

## 6. References

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