

Particle escape from SNR shocks: gamma-ray and cosmic-ray signatures



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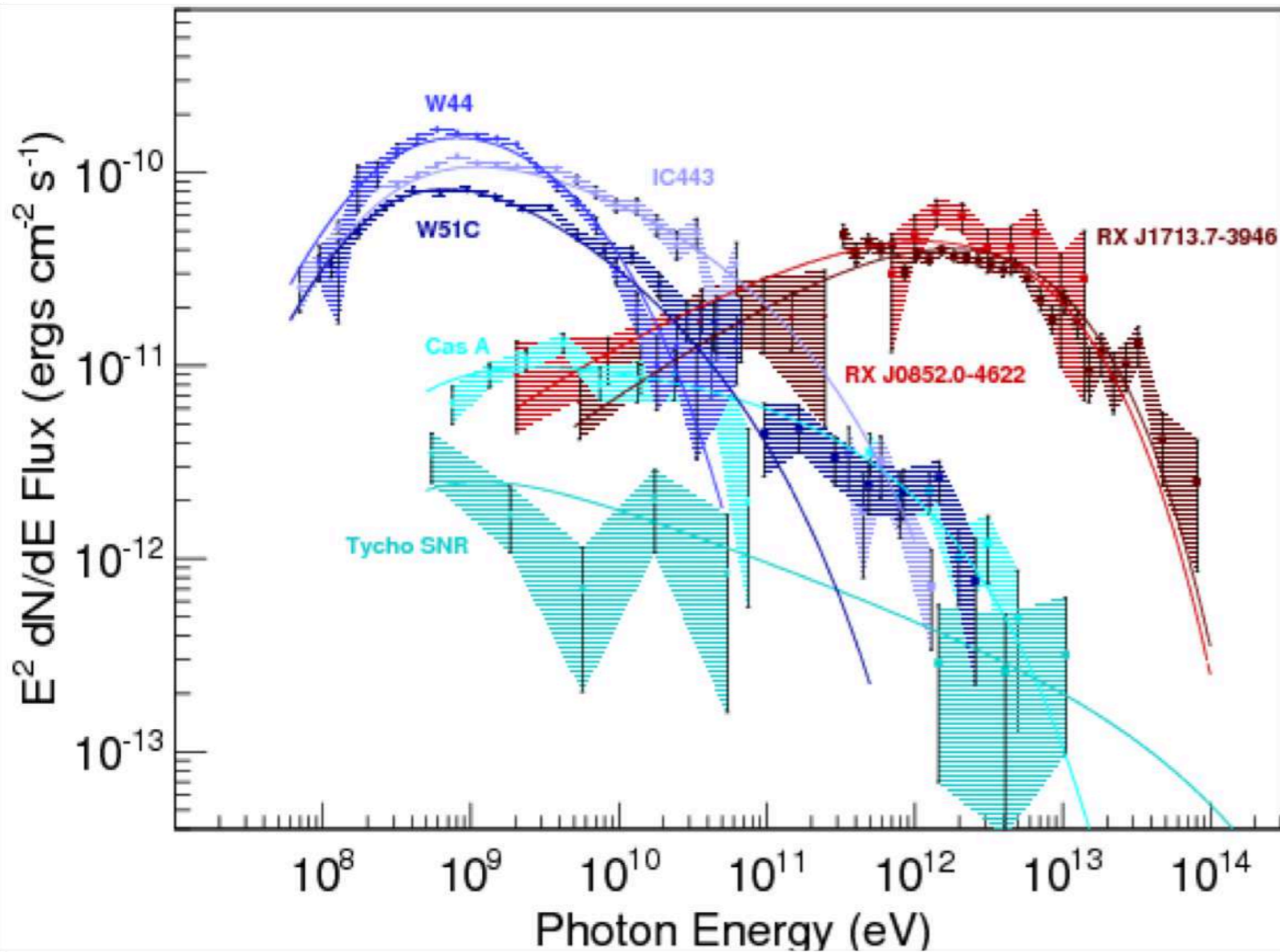
Sapienza University of Rome & Max Planck Institute für Kernphysik



July 7th 2022, Barcelona



SNRs in the HE and VHE domain



Middle-aged SNRs (20000 yrs)


- hadronic emission
- steep spectra
- $E_{\text{max}} < 1 \text{ TeV}$


Young SNRs (2000 yrs)

- hadronic/leptonic ?
- hard spectra
- $E_{\text{max}} = 10 - 100 \text{ TeV}$

Very young SNRs (300 yrs)

- hadronic ?
- steep spectra $E^{-2.3}$
- $E_{\text{max}} = 10 - 100 \text{ TeV}$

 Drury et al., A&A 287 (1994) 959

 Tsuguya & Fumio, J. Phys. G 20 (1994) 477

 Funk et al., ARNPS 65 (2015) 245F

How do accelerated particles become CRs?

Acceleration at the shock: $f_0(p)$



Escape from the shock: $f_{esc}(p)$



Ptuskin & Zirakashvili, A&A 429 (2005) 755



Gabici, Aharonian & Casanova, MNRAS (2009)



Ohira, Murase & Yamakazi, A&A (2010) 513



Bell & Shure, MNRAS 437 (2014) 2802



Cardillo, Amato & Blasi, APh 69 (2015) 1



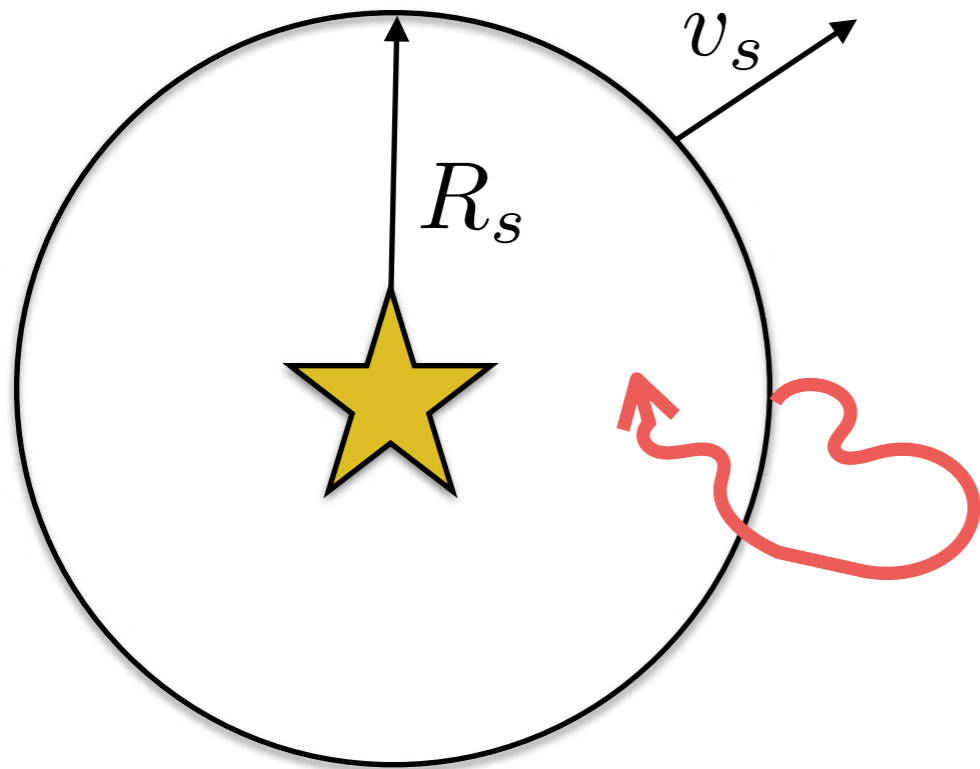
Cristofari, Blasi & Caprioli, A&A 650A (2021) 62C

- Connect the **CR spectrum** observed on Earth with the spectrum of particles released at the sources;
- Understand observations of **SNR spectra** → unveil the presence of **PeV particle accelerators**.

A **phenomenological** model to investigate the particle **escape** through spectral and morphological features of SNRs in the **HE** and **VHE** domain.



Proton maximum energy in SNRs



$$t_{\text{acc}} = t_{\text{age}}$$

acceleration
limited by
remnant age

$$\frac{D(p_{\text{max}})}{v_s^2(t)} = t$$

$$p_{\text{max},0} \propto \mathcal{F}(t) v_s^2(t) t$$

$$\left(\frac{\delta B(\mathbf{x}, t)}{B_0} \right)^2 = \int \mathcal{F}(k, \mathbf{x}, t) d \ln k$$

ST stage: $v_s(t) \propto t^{-3/5} \longrightarrow p_{\text{max},0} \propto \mathcal{F}(t) t^{-1/5}$

$$p_{\text{max},0}(t) = p_M \left(\frac{t}{t_{\text{Sed}}} \right)^{-\delta}$$

$$\delta > 0$$

$$t_{\text{esc}}(p) = t_{\text{Sed}} \left(\frac{p}{p_M} \right)^{-1/\delta}$$



Ptuskin & Zirakashvili, A&A 429 (2005) 755

$$t_{\text{Sed}} \simeq 1.6 \times 10^3 \text{ yr} \left(\frac{E_{\text{SN}}}{10^{51} \text{ erg}} \right)^{-1/2} \left(\frac{M_{\text{ej}}}{10 M_{\odot}} \right)^{5/6} \left(\frac{\rho_0}{1 m_p / \text{cm}^3} \right)^{-1/3}$$

A model for particle propagation

Analytical solution of the accelerated **proton** transport

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = \frac{p}{3} \frac{\partial f}{\partial p} \nabla \cdot \mathbf{v} + \nabla \cdot [D \nabla f]$$

$$v(t, r) = \left(1 - \frac{1}{\sigma}\right) \frac{v_{\text{sh}}(t)}{R_{\text{sh}}(t)} r$$

Particles confined inside the SNR

$$\frac{\partial f_{\text{conf}}}{\partial t} + \mathbf{v} \cdot \nabla f_{\text{conf}} = \frac{p}{3} \frac{\partial f_{\text{conf}}}{\partial p} \nabla \cdot \mathbf{v}$$

$$p \leq p_{\text{max},0}(t)$$

Escaped particles

$$\frac{\partial f_{\text{esc}}}{\partial t} = \nabla \cdot [D \nabla f_{\text{esc}}]$$

$$p > p_{\text{max},0}(t)$$

Matching condition: $f_{\text{esc}}(t_{\text{esc}}) = f_{\text{conf}}(t_{\text{esc}})$

A model for particle propagation

Analytical solution of the accelerated **proton** transport

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = \frac{p}{3} \frac{\partial f}{\partial p} \nabla \cdot \mathbf{v} + \nabla \cdot [D \nabla f]$$

$$v(t, r) = \left(1 - \frac{1}{\sigma}\right) \frac{v_{\text{sh}}(t)}{R_{\text{sh}}(t)} r$$

Particles confined inside the SNR

$$\frac{\partial f_{\text{conf}}}{\partial t} + \mathbf{v} \cdot \nabla f_{\text{conf}} = \frac{p}{3} \frac{\partial f_{\text{conf}}}{\partial p} \nabla \cdot \mathbf{v}$$

Escaped particles

$$\frac{\partial f_{\text{esc}}}{\partial t} = \nabla \cdot [D \nabla f_{\text{esc}}]$$

Assumption 1: spherical symmetry $\mathbf{f}=\mathbf{f}(\mathbf{t},\mathbf{r},\mathbf{p})$;

Assumption 2: stationary homogeneous diffusion coefficient is assumed inside and outside the remnant

$$D_{\text{in}}(p) = D_{\text{out}}(p) \equiv \chi D_{\text{Gal}}(p) = \chi 10^{28} \left(\frac{pc}{10 \text{ GeV}}\right)^{1/3} \text{ cm}^2 \text{ s}^{-1}$$

A model for particle propagation

Assumption 3: at every time, a constant fraction ξ_{CR} of the shock ram pressure is converted into CR pressure, such that the acceleration spectrum reads as

$$f_{0,p}(t, p) = \frac{3 \xi_{\text{CR}} \rho_0 v_{\text{sh}}^2(t)}{4\pi c (m_p c)^4 \Lambda(p_{\text{max},0}(t))} \left(\frac{p}{m_p c} \right)^{-\alpha} \Theta [p_{\text{max},0}(t) - p]$$

acceleration
efficiency
constant in time

normalization factor
such that

$$P_{\text{CR}} = \xi_{\text{CR}} \rho_0 v_{\text{sh}}^2(t)$$

acceleration spectrum
($\alpha \sim 4$ from DSA)



Ptuskin & Zirakashvili, A&A 429 (2005) 755

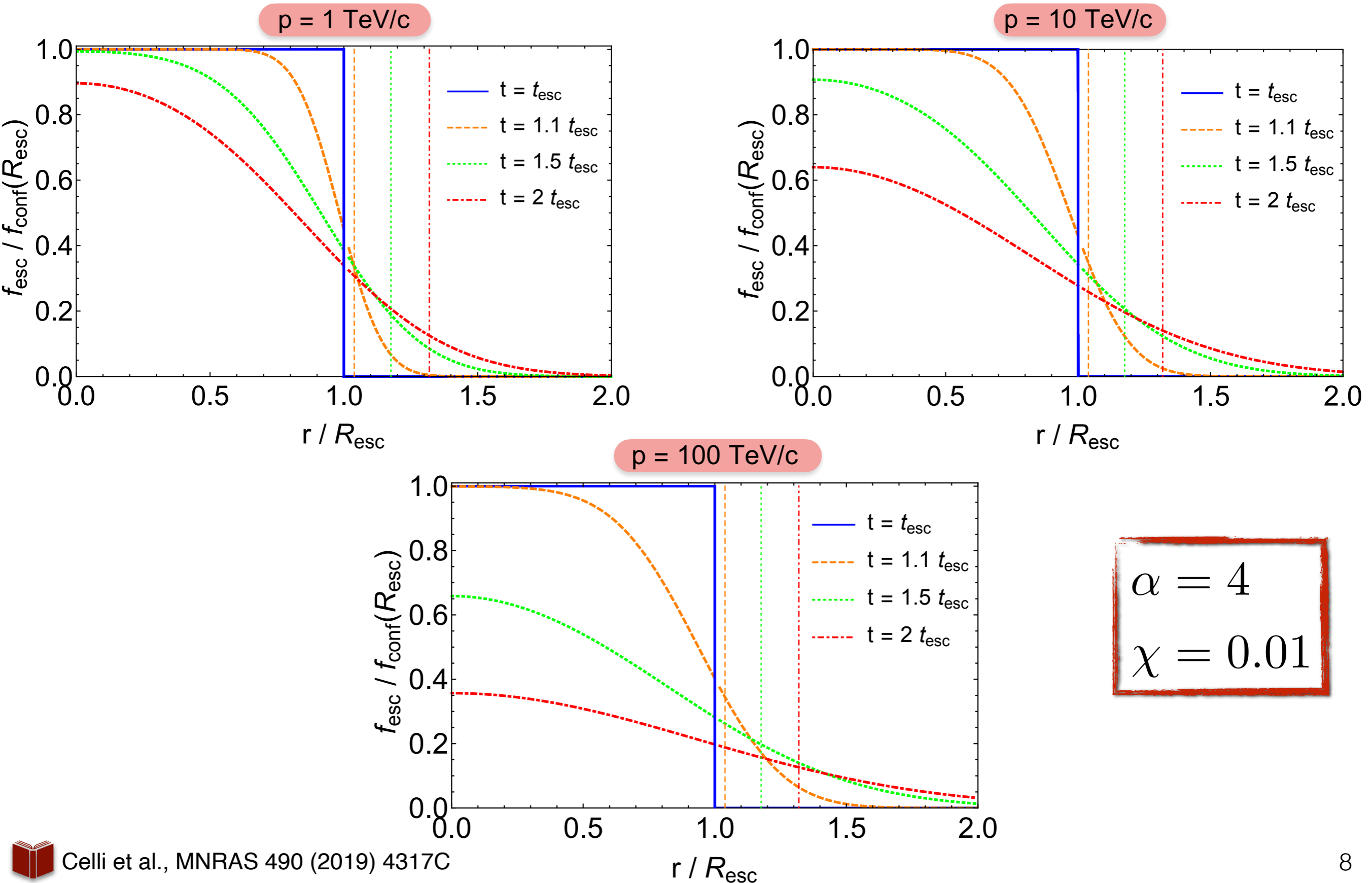
Assumption 4: the shock is evolving through the ST phase

$$R_{\text{sh}}(t) \propto t^{2/5}$$

$$v_{\text{sh}}(t) \propto t^{-3/5}$$

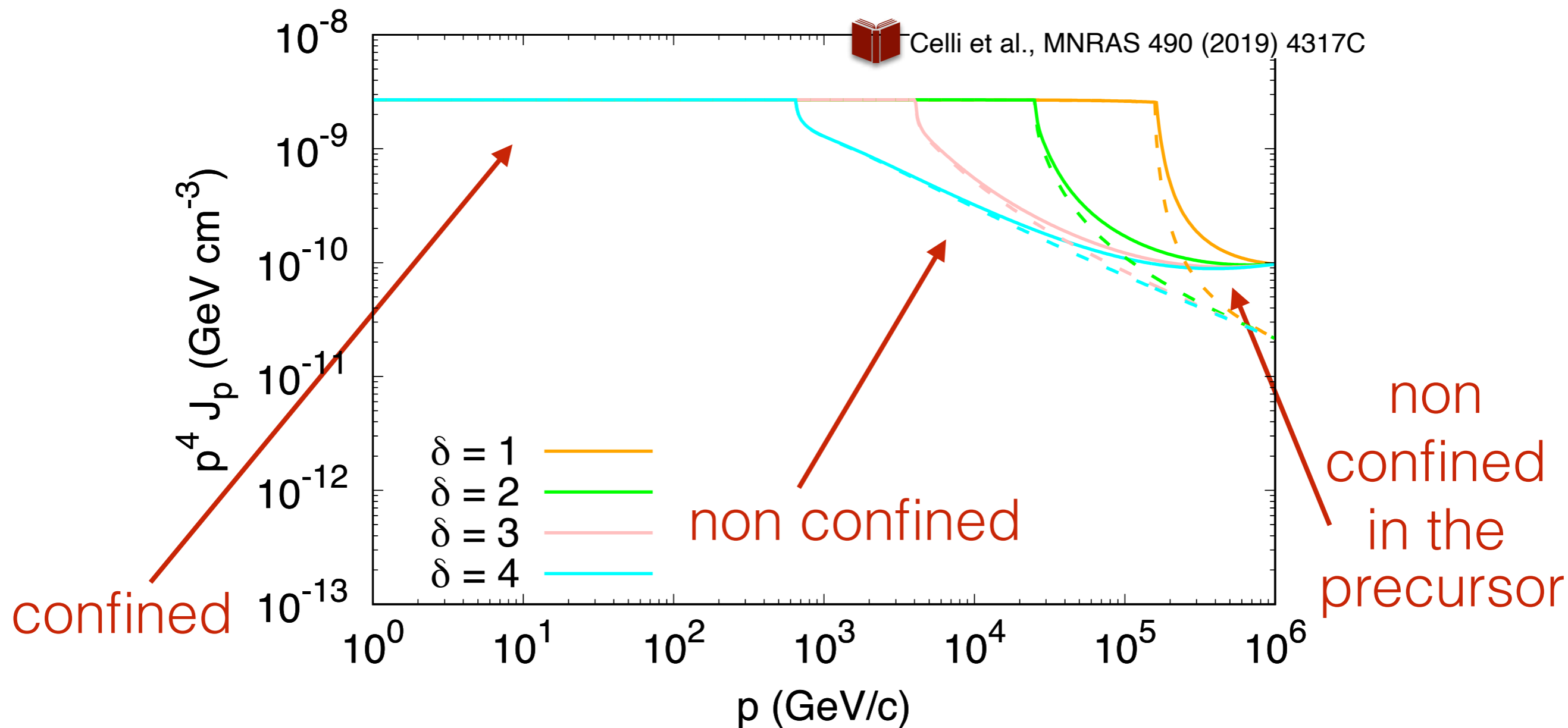


Density of non-confined protons



The spectrum of protons inside the SNR

$$J_p^{\text{in}}(t, p) = \frac{4\pi}{V_{\text{SNR}}} \int_0^{R_{\text{sh}}(t)} [f_{\text{esc}}(t, r, p) + f_{p,\text{esc}}(t, r, p) + f_{\text{conf}}(t, r, p)] r^2 dr$$



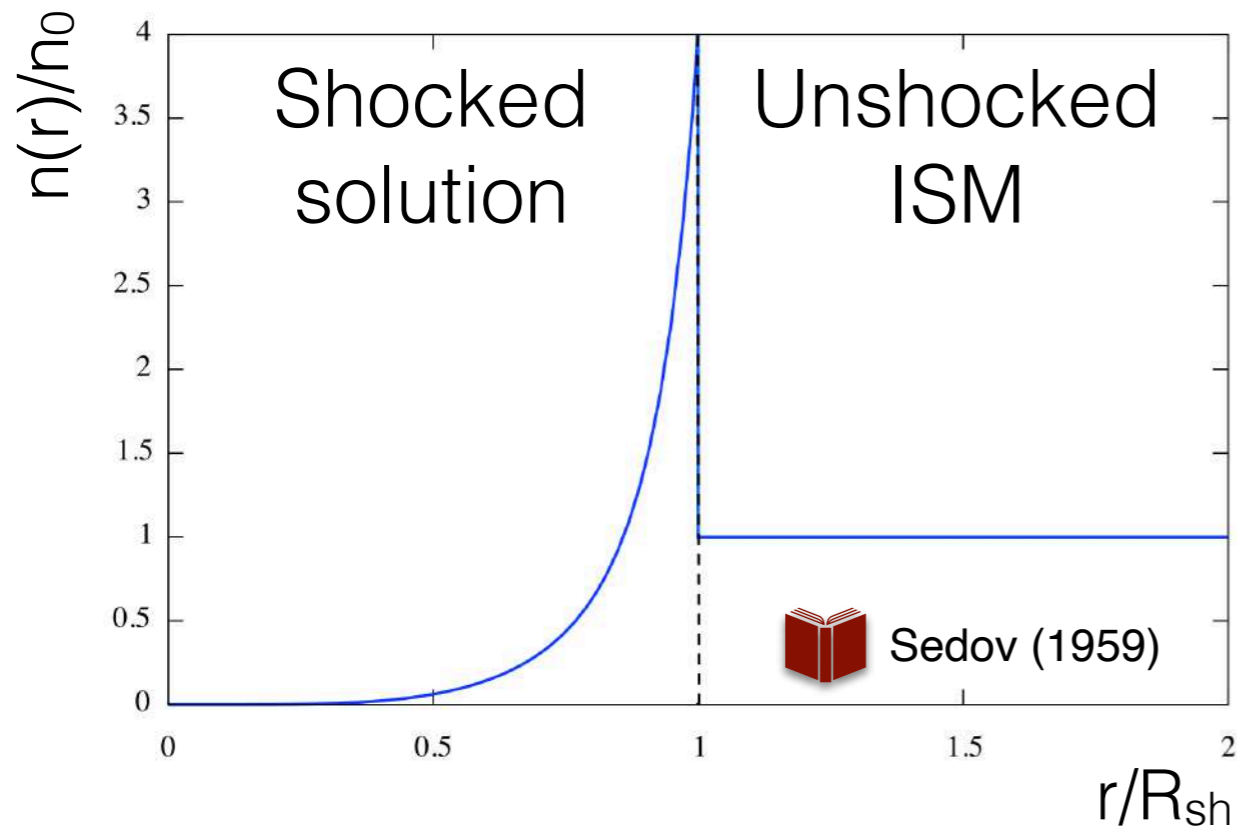
$$D(p) = 10^{27} \left(\frac{pc}{10 \text{ GeV}} \right)^{1/3} \text{cm}^2 \text{s}^{-1}$$

$$T_{\text{SNR}} = 10^4 \text{ yr}$$

$$\xi_{\text{CR}} = 10\%$$

$$n_0 = 1 \text{ cm}^{-3}$$

Volume integrated gamma-ray emission from hadronic (pp) interactions



$$f_0(p) \propto p^{-4}$$

$$D(10 \text{ GeV}/c) = 10^{27} \text{ cm}^2 \text{ s}^{-1}$$

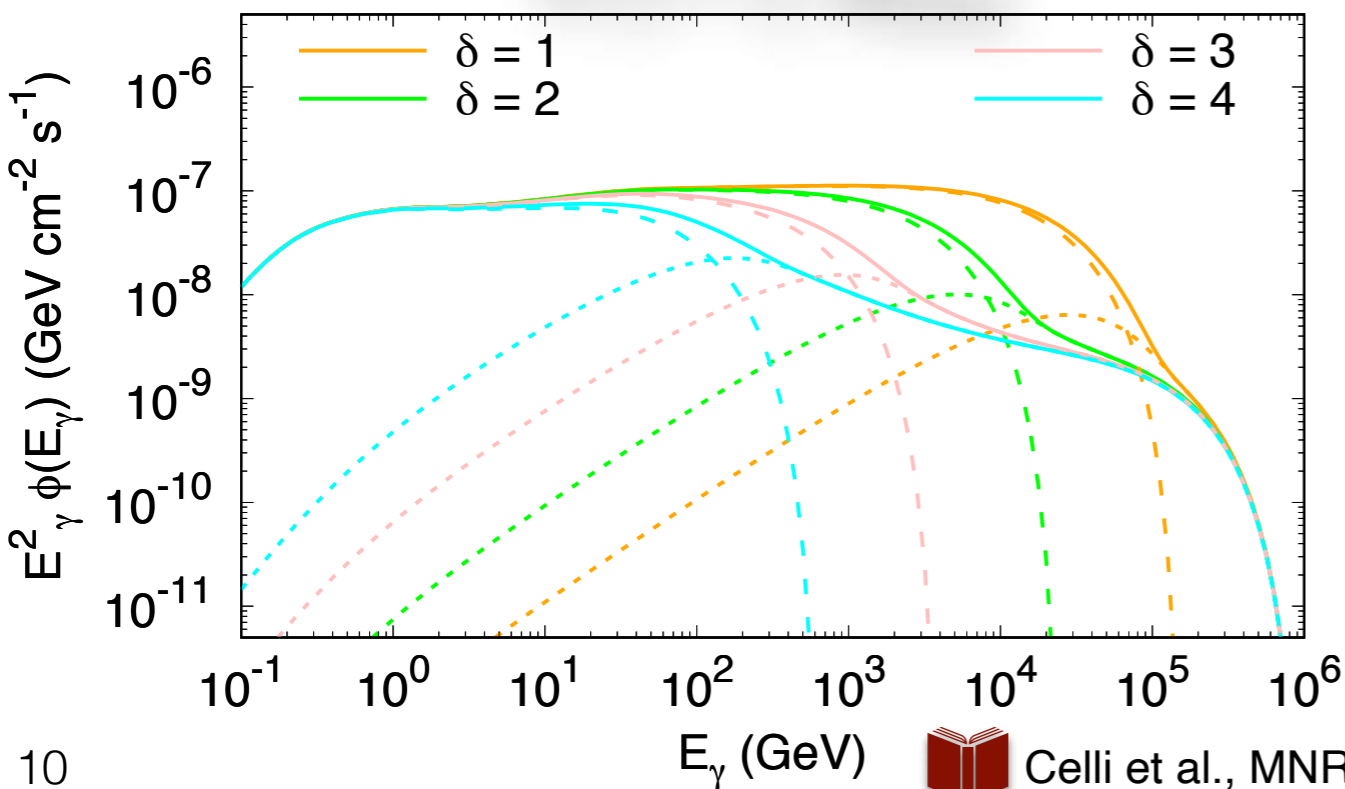
$$T_{\text{SNR}} = 10^4 \text{ yr}$$

$$n_0 = 1 \text{ cm}^{-3}$$

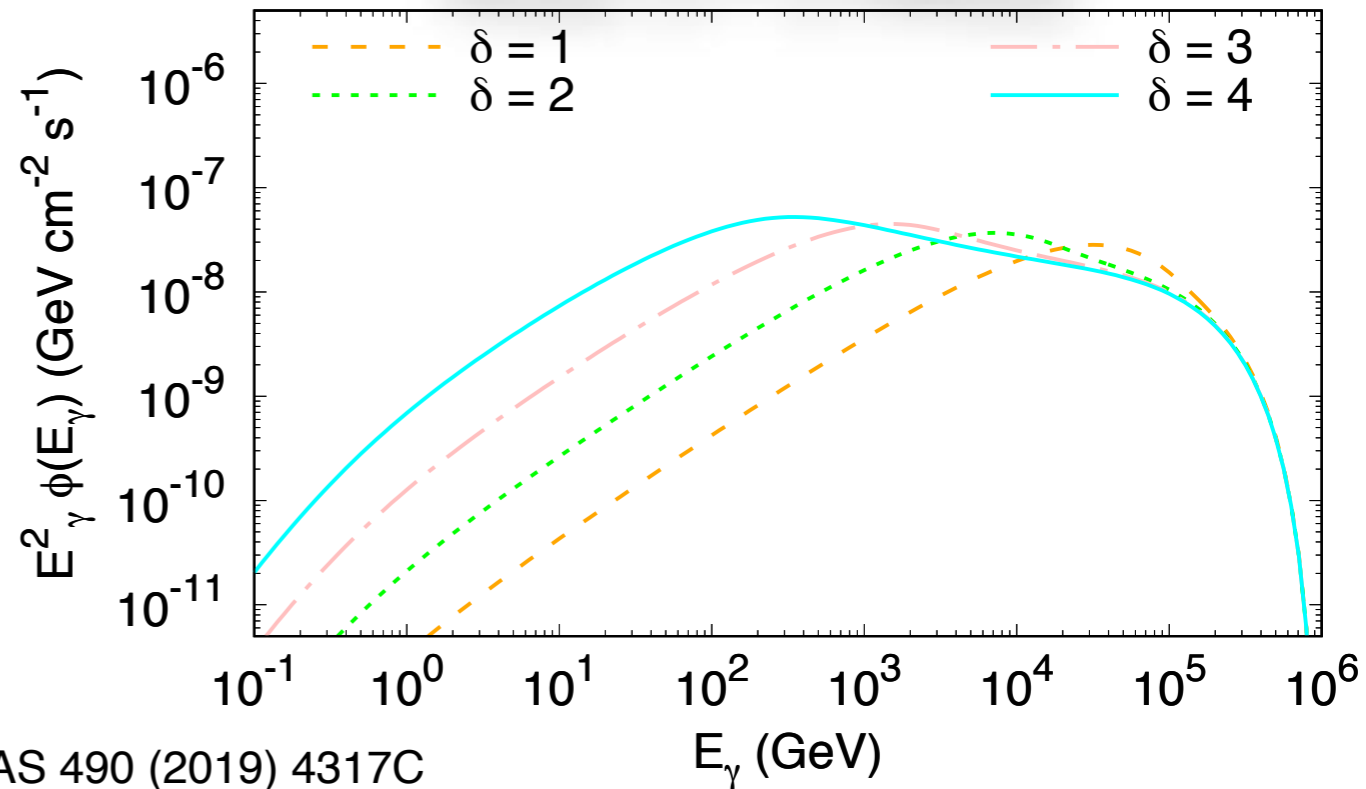
$$\xi_{\text{CR}} = 10\%$$

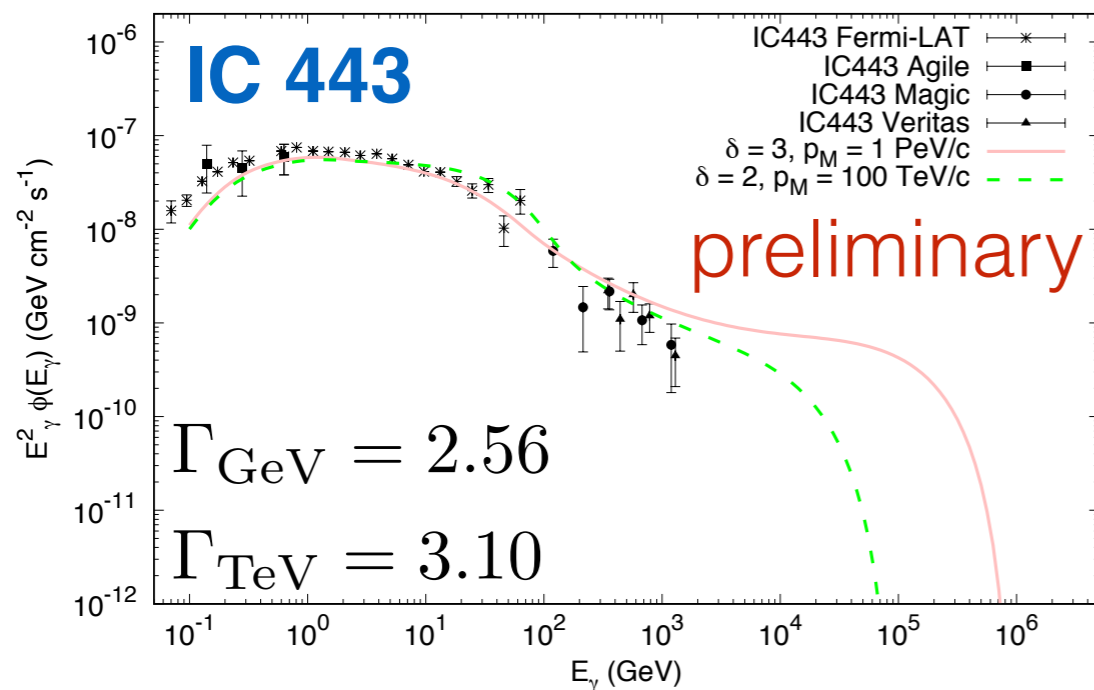
$$d = 1 \text{ kpc}$$

$$0 \leq r \leq R_{\text{SNR}}$$



$$R_{\text{SNR}} \leq r \leq 2R_{\text{SNR}}$$



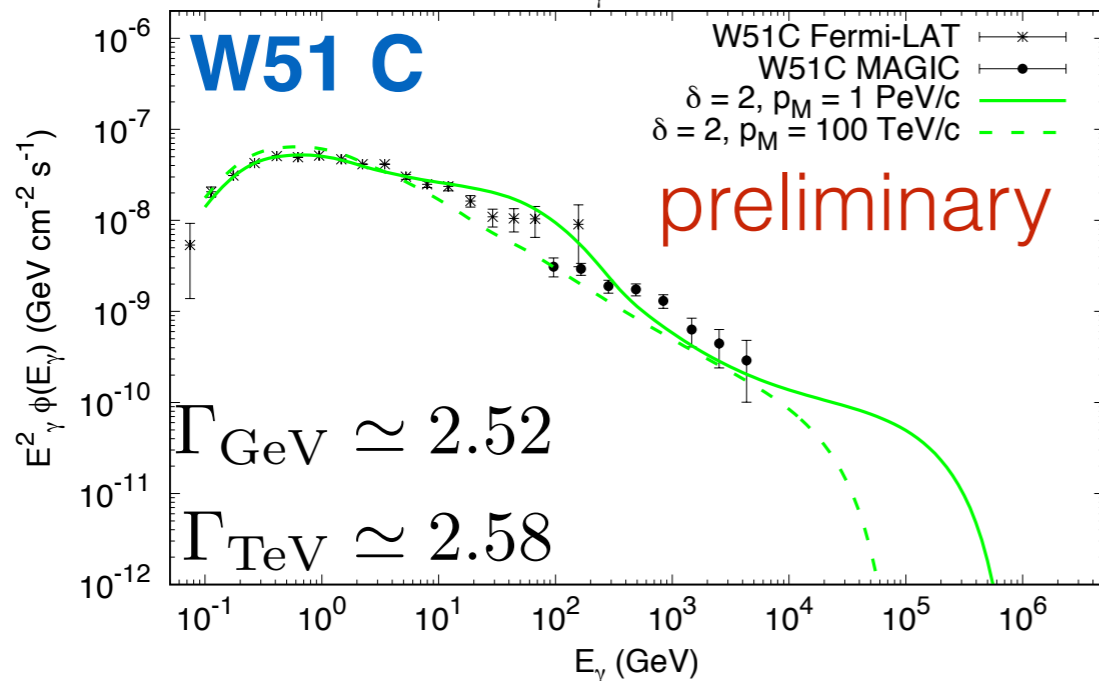


$$f_0(p) \propto p^{-4}$$

$$T_{\text{SNR}} = 1.5 \times 10^4 \text{ yr}, n_{\text{up}} = 10 \text{ cm}^{-3}$$

$$d = 1.5 \text{ kpc}, \xi_{\text{CR}} = 2\%$$

$$D(10 \text{ GeV}/c) = 10^{27} \text{ cm}^2 \text{ s}^{-1}$$

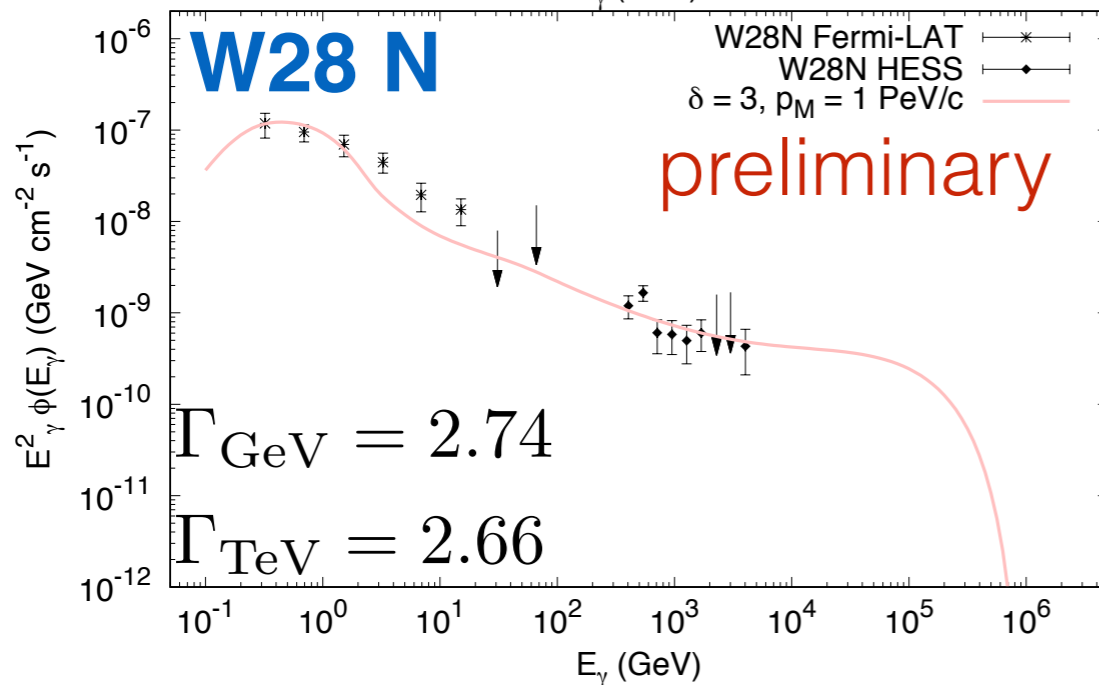


$$f_0(p) \propto p^{-(4+1/3)}$$

$$T_{\text{SNR}} = 3 \times 10^4 \text{ yr}, n_{\text{up}} = 10 \text{ cm}^{-3}$$

$$d = 5.4 \text{ kpc}, \xi_{\text{CR}} = 12\% - 15\%$$

$$D(10 \text{ GeV}/c) = 3 \times 10^{26} \text{ cm}^2 \text{ s}^{-1}$$



$$f_0(p) \propto p^{-4}$$

$$T_{\text{SNR}} = 4 \times 10^4 \text{ yr}, n_{\text{up}} = 10 \text{ cm}^{-3}$$

$$d = 2.0 \text{ kpc}, \xi_{\text{CR}} = 15\%$$

$$D(10 \text{ GeV}/c) = 3 \times 10^{27} \text{ cm}^2 \text{ s}^{-1}$$

The CR-p spectrum injected into the Galaxy

$$f_{\text{inj}}(p) = 4\pi \int_0^{R_{\text{esc}}(p)} r^2 f_{\text{conf}}(t_{\text{esc}}(p), r, p) dr$$

$$\longrightarrow f_{\text{inj}}(p) \propto v_{\text{esc}}^2(p) R_{\text{esc}}^3(p) \frac{p^{-\alpha}}{\Lambda(p)}$$

$$\longrightarrow f_{\text{inj}}(p) \propto \frac{p^{-\alpha}}{\Lambda(p)}$$


**Exact balance
between v_{esc}^2 and R_{esc}^3
during the ST phase**


- Ultra-relativistic limit ($p \gg m_p c$):

$$f_{\text{inj}}(p) \propto \begin{cases} p^{-\alpha} & \alpha > 4 \\ p^{-4} & \alpha < 4 \end{cases}$$



 Bell & Shure, MNRAS 437 (2014) 2802

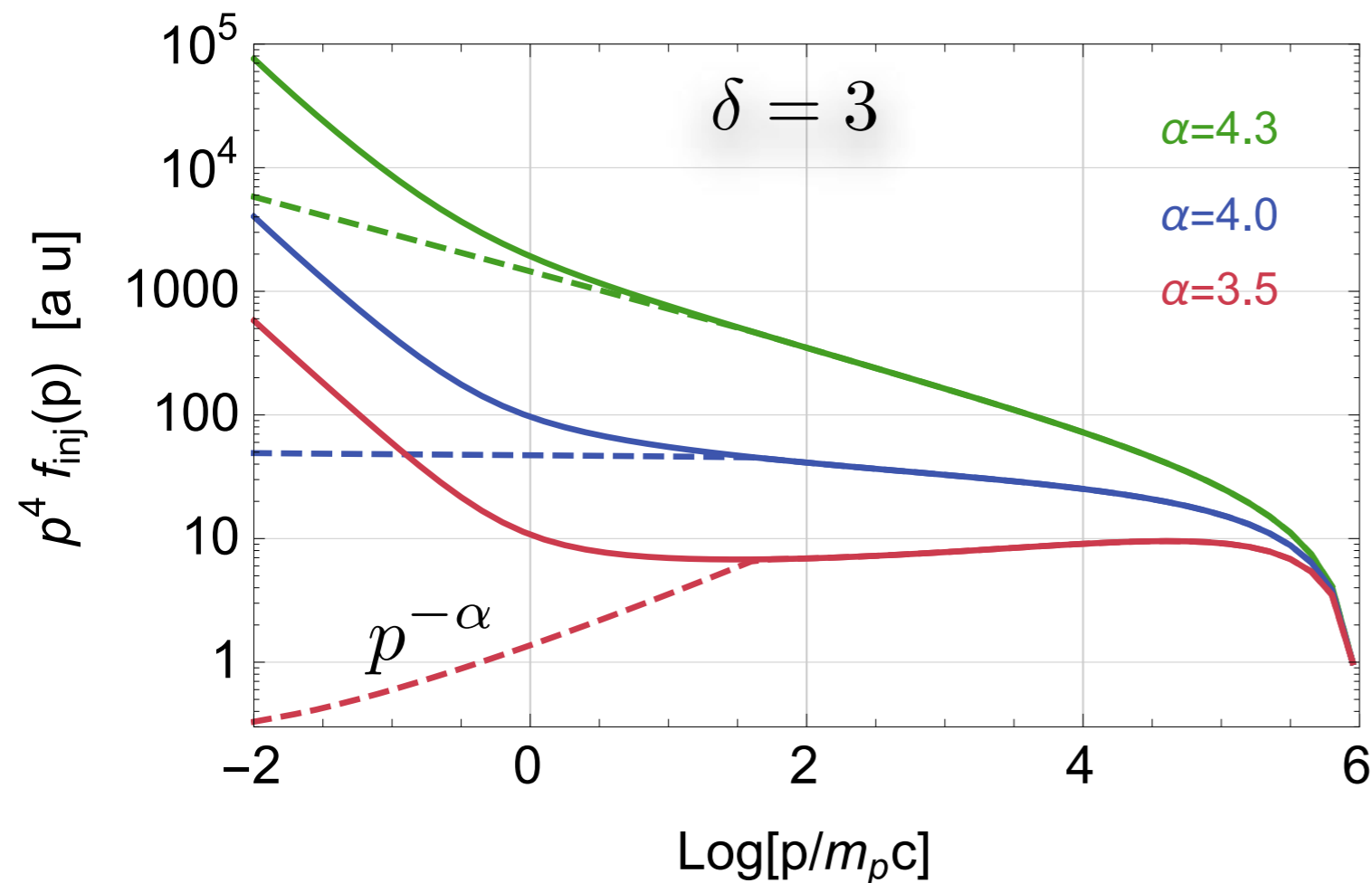
 Cardillo, Amato & Blasi, APh 69 (2015) 1

 Celli et al., MNRAS 490 (2019) 4317C

The CR-p spectrum injected into the Galaxy

The end of the acceleration phase produces visible signatures on the spectrum of the injected particles. Assuming this happens at t_{SP}

$$T \leq 10^6 \text{ K}, v_s \simeq 200 \text{ Km/s} \longrightarrow p_{\text{max},0}(t_{\text{SP}}) \simeq 40 \text{ GeV/c}$$



→ particles with $p < p_{\text{max},0}(t_{\text{SP}})$ do not suffer further adiabatic losses as they are instantaneously released into the ISM

 Celli et al., MNRAS 490 (2019) 4317C

* $t_{\text{SP}} \simeq 47 \text{ kyr}$

Electron transport and E_{\max} in SNRs

**Radiative +
adiabatic losses**

$$\frac{dE}{dt} = \left(\frac{dE}{dt} \right)_{\text{syn+IC}} + \frac{E}{L} \frac{dL}{dt}$$



Reynolds, ApJ 493 (1998) 375

Morlino & Caprioli, A&A 538 (2012) 381

$$\longrightarrow f_{e,\text{conf}}(E, r, t) = f_{e,0} \left(\frac{E}{L(t', t) - IE}, t' \right) \frac{L^4}{(L - IE)^2}$$

TIME-LIMITED ACCELERATION: $f_{e,0}(p) = K_{\text{ep}} f_{p,0}(p) e^{-\left(\frac{p}{p_{\max,e,0}}\right)}$

LOSS-LIMITED ACCELERATION: $f_{e,0}(p) = K_{\text{ep}} f_{p,0}(p) \left[1 + 0.523 \left(p/p_{\max,e,0} \right)^{\frac{9}{4}} \right]^2 e^{-\left(\frac{p}{p_{\max,e,0}}\right)^2}$



Aharonian et al., A&A 465 (2007) 695



Blasi, MNRAS 402 (2010) 2807

Radiative losses in the proton self-amplified magnetic field and radiation fields strongly affect the electron **maximum energy**:

$$\left(\frac{dE}{dt} \right)_{\text{syn+IC}} = - \frac{\sigma_{\text{T}} c}{6\pi} \left(\frac{E}{m_e c^2} \right)^2 \left(B^2 + B_{\text{eq}}^2 \right)$$

$$t_{\text{acc}} = t_{\text{loss}} \longrightarrow \frac{E_{\max,0,e}(t)}{m_e c^2} = \sqrt{\frac{(\sigma - 1) r_B}{\sigma \left[r_B (1 + \sigma_{\text{eq}}^2) + \sigma (r_B^2 + \sigma_{\text{eq}}^2) \right]} \frac{6\pi e B_0 \mathcal{F}(t)}{\sigma_{\text{T}} B_{1,\text{tot}}^2(t)} \frac{v_{\text{sh}}(t)}{c}}$$

Electron transport and E_{\max} in SNRs

The **CR self-amplified** magnetic field at the shock is given by:

$$t_{\text{acc}} = t_{\text{SNR}} \longrightarrow \mathcal{F}(t) = \frac{8 p_{\text{MC}}}{3 e B_0 c t_{\text{Sed}}} \begin{cases} \left(\frac{v_{\text{sh}}}{c}\right)^{-2} & t < t_{\text{Sed}} \\ \left(\frac{v_{\text{sh}}}{c}\right)^{-2} \left(\frac{t}{t_{\text{Sed}}}\right)^{-\delta-1} & t \geq t_{\text{Sed}} \end{cases}$$

$$\longrightarrow \delta B_1(t) = \frac{B_0}{2} \left(\mathcal{F}(t) + \sqrt{4\mathcal{F}(t) + \mathcal{F}^2(t)} \right)$$

In the shock **downstream**, magnetic field compression and adiabatic losses are included such that

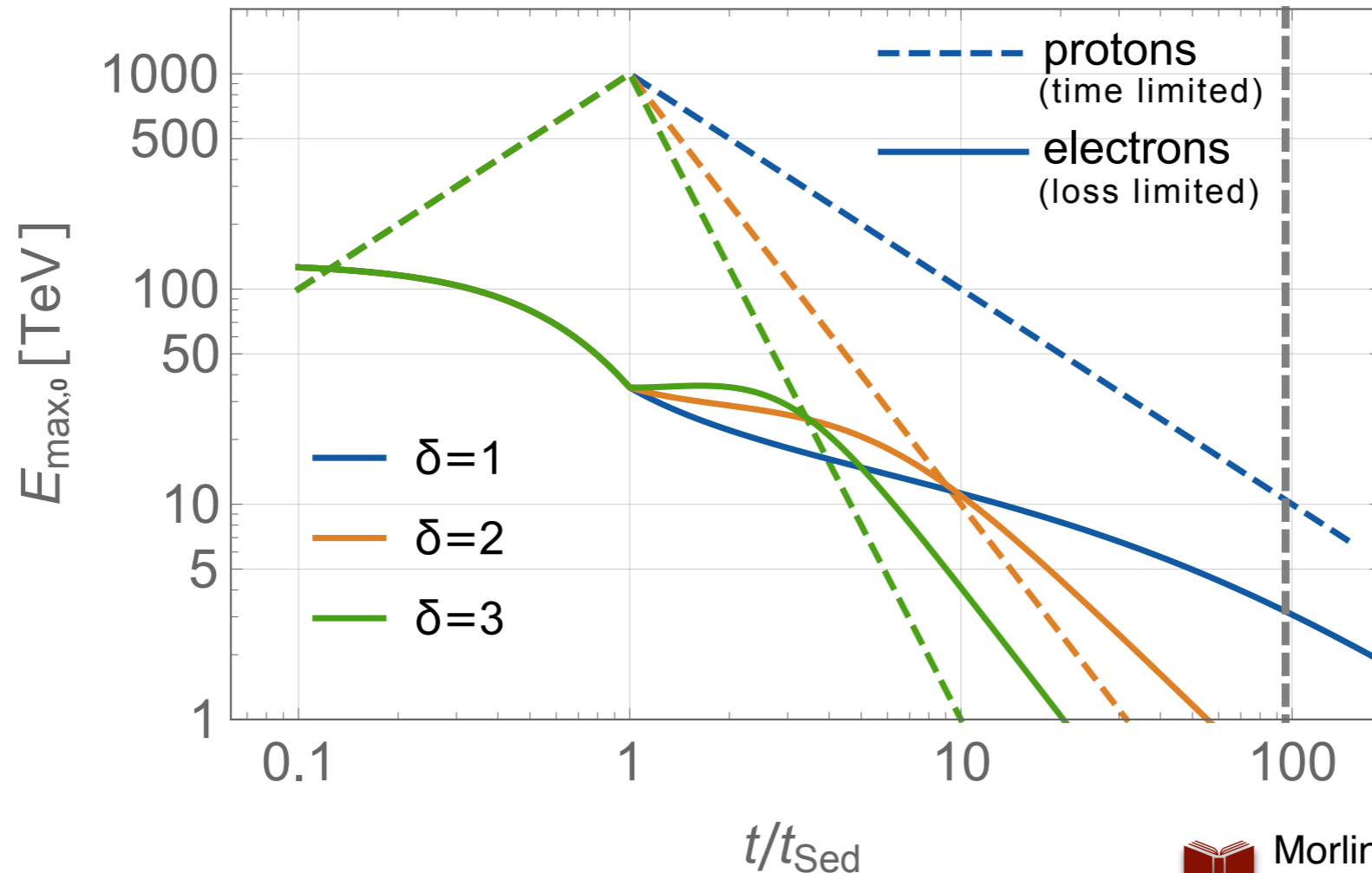
$$B_2^2(r, t) = \frac{B_0^2}{3} \left[\left(\frac{R_{\text{sh}}(t)}{r}\right)^4 + 2\sigma^2 L^6(t', t) \left(\frac{R_{\text{sh}}(t)}{r}\right)^2 \right]$$


where $L(t', t) = \left[\frac{\rho_2(t, r)}{\rho_2(t'(t, r))} \right]^{1/3} \implies L(t', t) = \left[\frac{R_{\text{sh}}(t')}{R_{\text{sh}}(t)} \right]^{3/4}$ accounts for

continuous adiabatic energy losses between t' and t .

Solving electron propagation

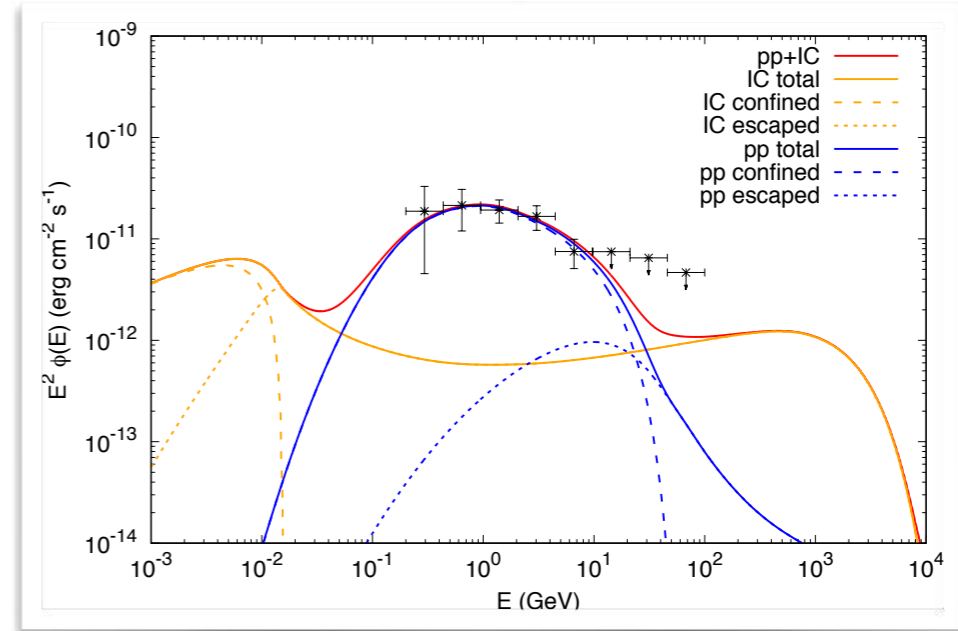
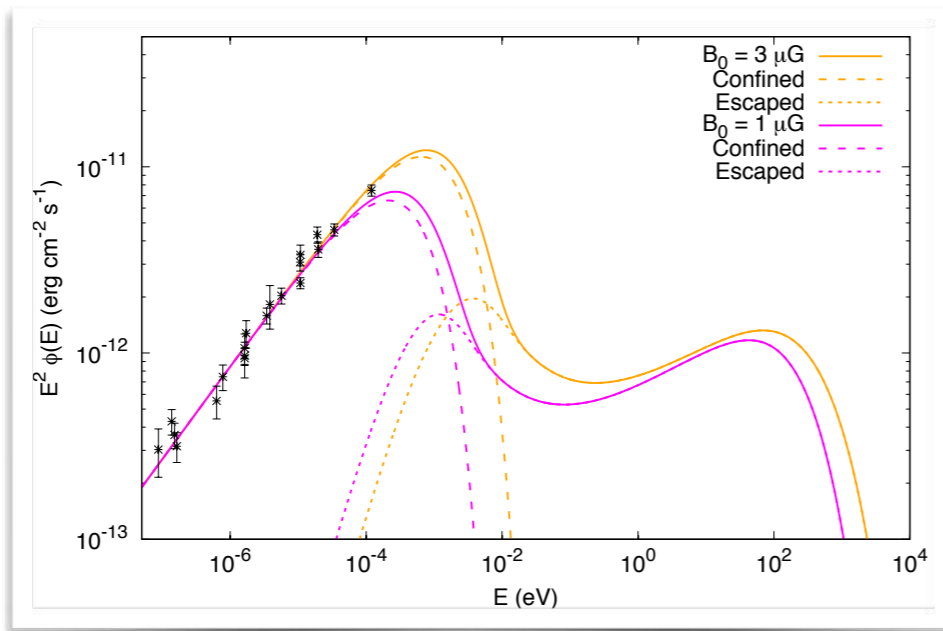
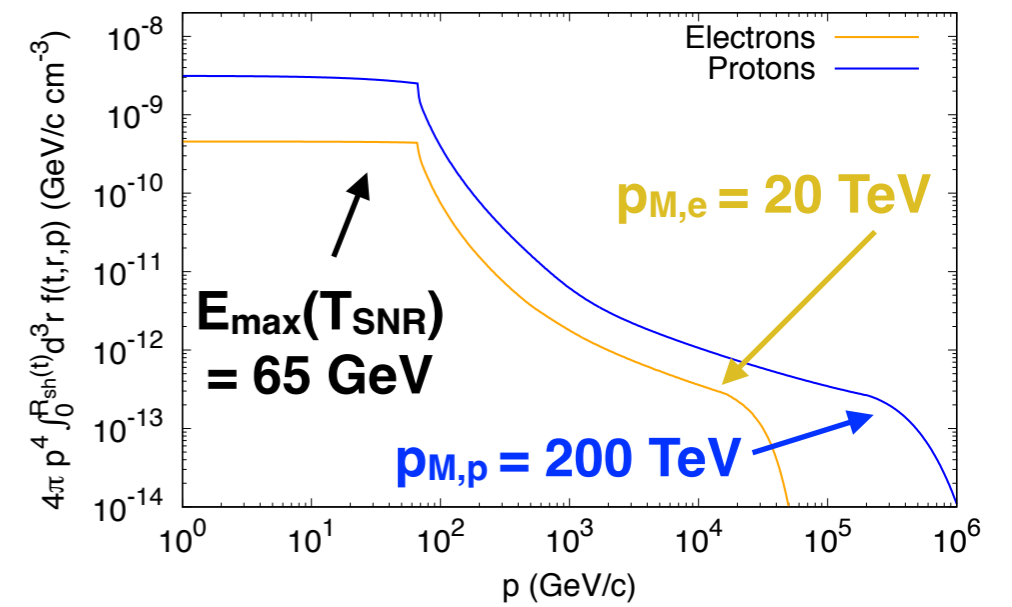
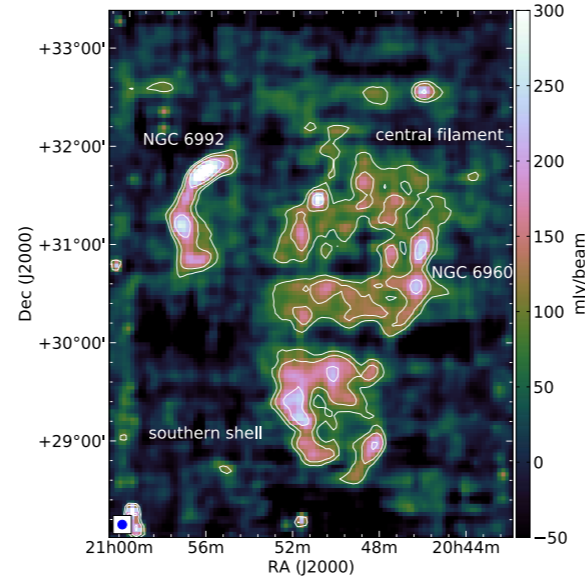
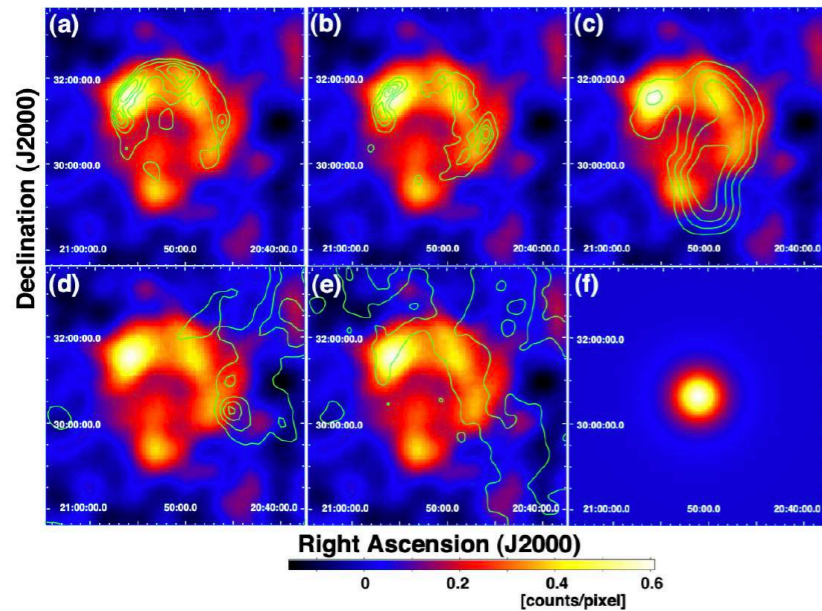
Numerical solution of the transport equation for accelerated **electrons**, including radiative and adiabatic losses



 Morlino & Celli,
MNRAS 508 (2021) 6142M

$$\frac{E_{\max,0,e}(t)}{m_e c^2} = \sqrt{\frac{(\sigma - 1)r_B}{\sigma \left[r_B(1 + \sigma_{\text{eq}}^2) + \sigma(r_B^2 + \sigma_{\text{eq}}^2) \right]} \frac{6\pi e B_0 \mathcal{F}(t)}{\sigma_T B_{1,\text{tot}}^2(t)} \frac{v_{\text{sh}}(t)}{c}}$$

Cygnus Loop: particles and radiation



Loru et al., MNRAS 500 (2020) 5177

Cygnus Loop properties

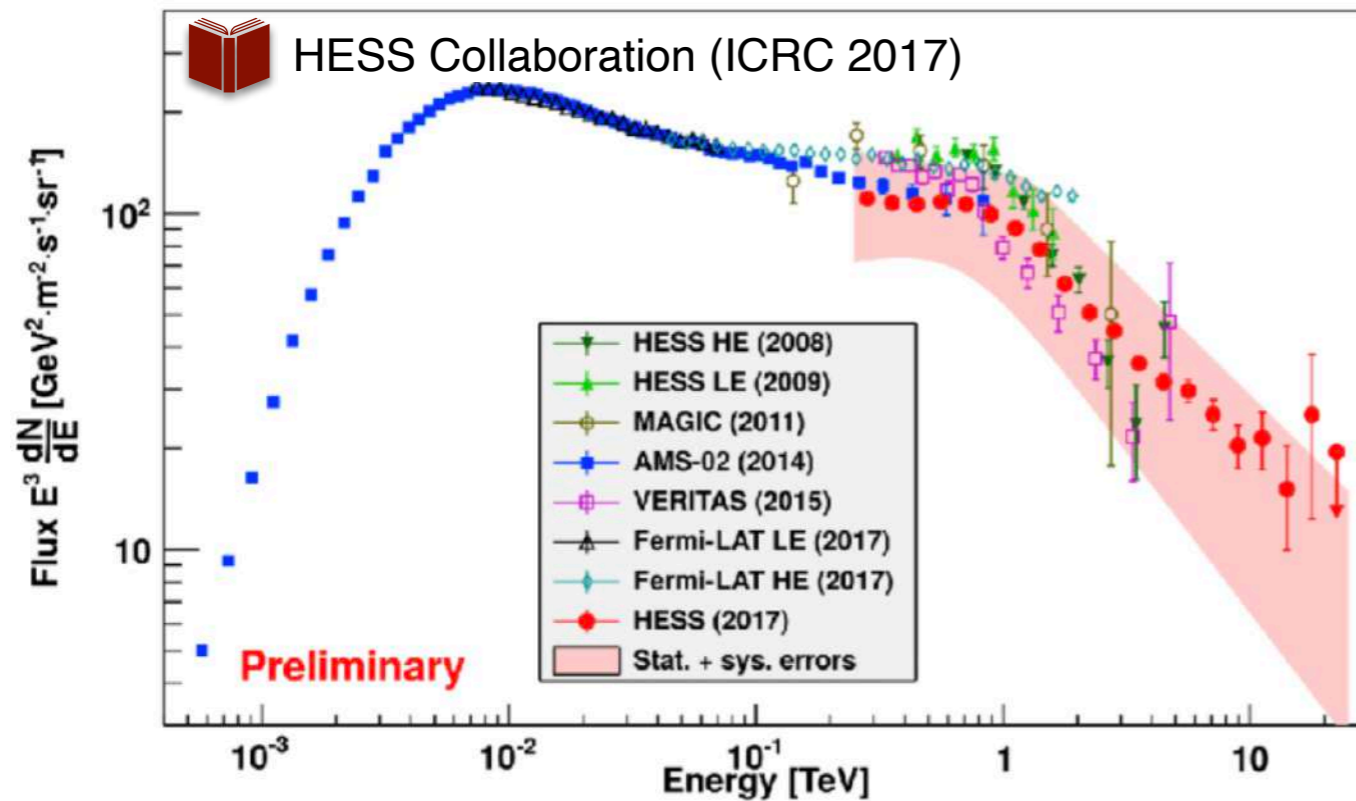
Acceleration model parameters

Assumed

Derived

E_{SN}	M_{ej}	t_{age}	d	n_0	R_{sh}	u_{sh}	ξ_{CR}	s	E_M	δ	K_{ep}	B_0	χ
7×10^{50} erg	$5 M_{\odot}$	2.1×10^4 yr	735 pc	0.4 cm^{-3}	20 pc	380 km s^{-1}	0.07	4.0	200 TeV	3	0.15	$3 \mu\text{G}$	1

The observed CR-e spectrum

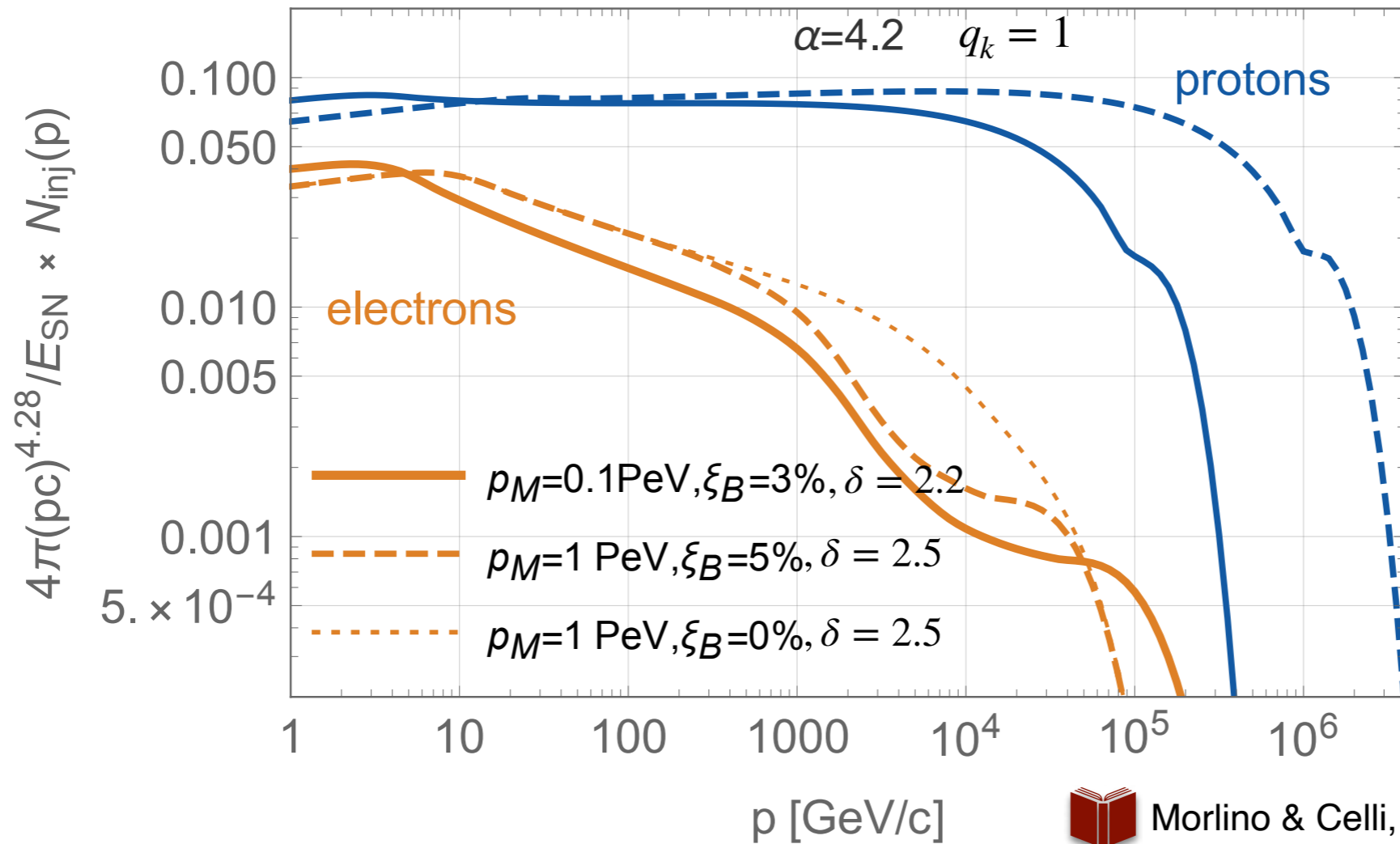


Many works on theoretical interpretation:

- Manconi et al., JCAP (2019) 024
- Diesing & Caprioli, PRL 123 (2019) 071101
- Recchia et al., PRD 99 (2019) 103022
- Brose et al., A&A 634 (2020) 359
- Di Mauro et al., PRD 8 (2020) 083012
- Fornieri et al., JCAP (2020) 009
- Cristofari, Blasi & Caprioli, A&A 650A (2021) 62C
- Evoli et al., PRD 103 (2021) 083010
- Morlino & Celli, MNRAS 508 (2021) 6142M

- Origin of the **spectral steepening** of the CR-electron spectrum above 10 GeV?
- Origin of the **TeV suppression** in the CR-electron spectrum?

The CR-e spectrum injected into the Galaxy



Morlino & Celli, MNRAS 508 (2021) 6142M

Turbulent MHD amplification:

Time dependent e/p injection:

$$\frac{\delta B_{2,\text{tur}}^2}{8\pi} = \xi_B \frac{1}{2} \rho v_{\text{sh}}^2$$

$$\frac{\xi_{\text{CRe}}}{\xi_{\text{CRp}}} = v_{\text{esc}}(p)^{-q_k} \propto p^{-3q_k/(5\delta)} \equiv p^{-\Delta S_{\text{ep}}}$$

Conclusions

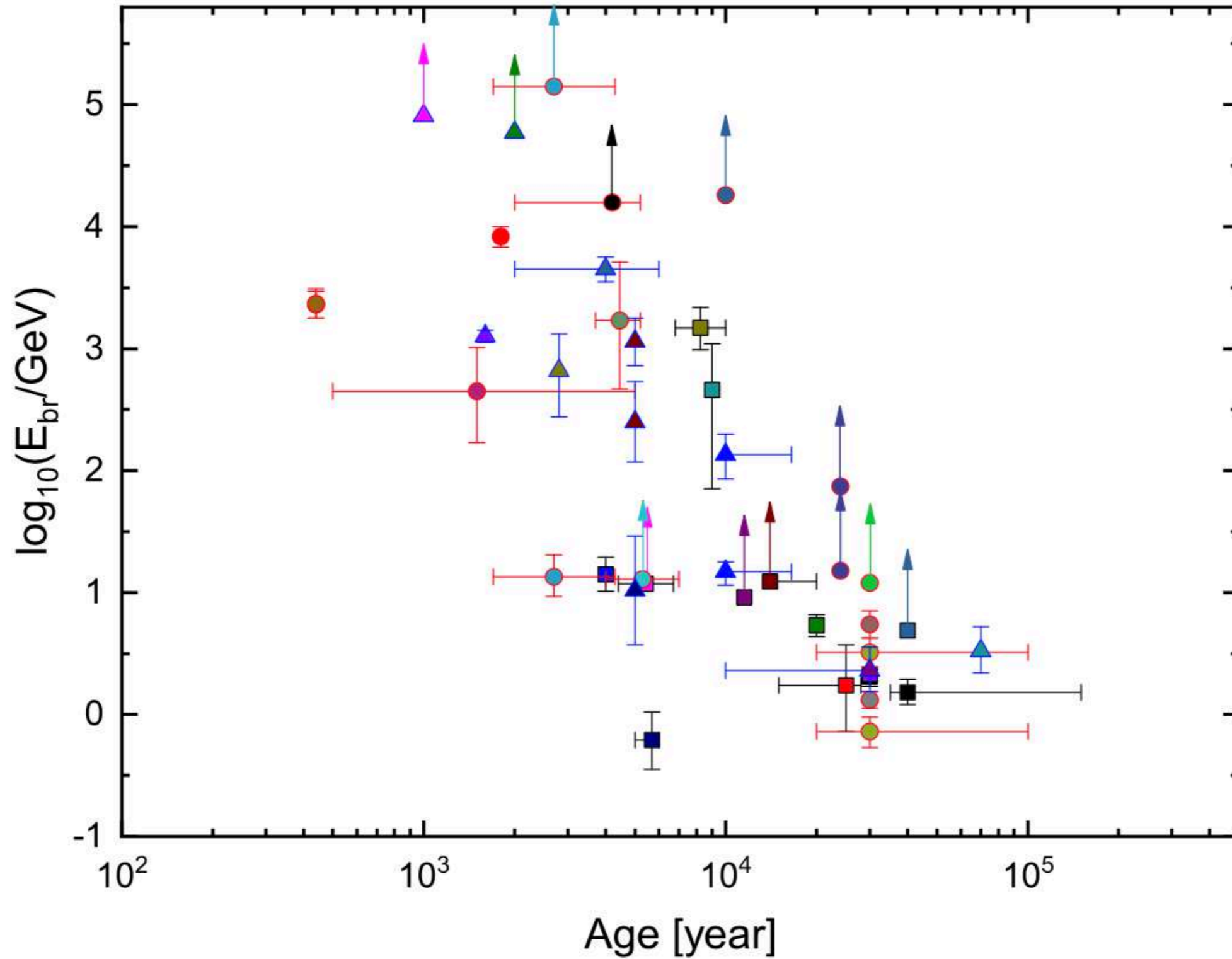
- Particle **escape** is still nowadays a poorly understood mechanism, though strongly embedded in the process of particle acceleration, mainly because of its dependence on the temporal evolution of the magnetic turbulence.
- Modelling is needed for interpreting the **steep spectra** and low **maximum energy** observed in the **HE** and **VHE** emission of many middle-aged SNRs (e.g. IC443, W28, W51C, Cygnus Loop).
- Escape studies can also shed light on the nature of **magnetic field amplification** and **particle injection** in CR accelerators.
- Results obtained can be used as a strategy to search for **PeV CR-proton accelerators**:
 - TeV halos around young-middle aged SNRs (LHAASO, CTA);
 - Passive molecular clouds illuminated by PeVatrons.

A bright pink sticky note is placed on a laptop keyboard. The note has the words "Thank You!" written in a bold, black, sans-serif font. The keyboard keys visible include 'A', 'Z', 'X', 'C', and 'alt'. The note is slightly tilted and casts a soft shadow on the keyboard surface.

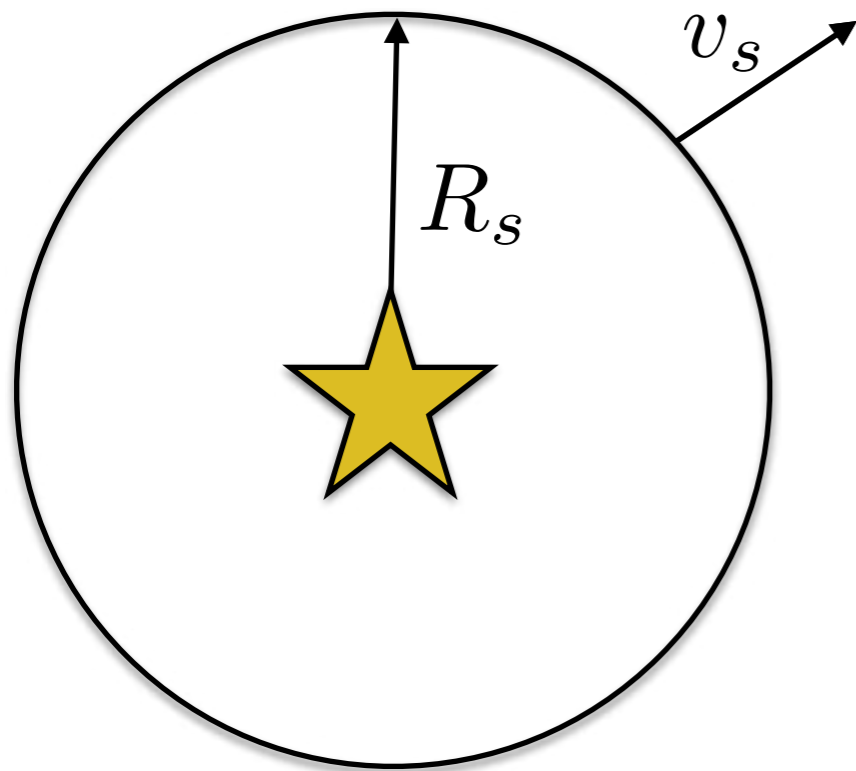
**Thank
You!**

Backup slides

A population study of evolved SNRs



The hydrodynamical evolution of an SNR



I. Ejecta-dominated stage

$$M_{\text{ej}} \gg \frac{4}{3} \pi \rho R_s^3(t)$$

→ free expansion

II. Sedov-Taylor stage

$$M_{\text{ej}} \sim \frac{4}{3} \pi \rho R_s^3(t)$$

→ energy conservation

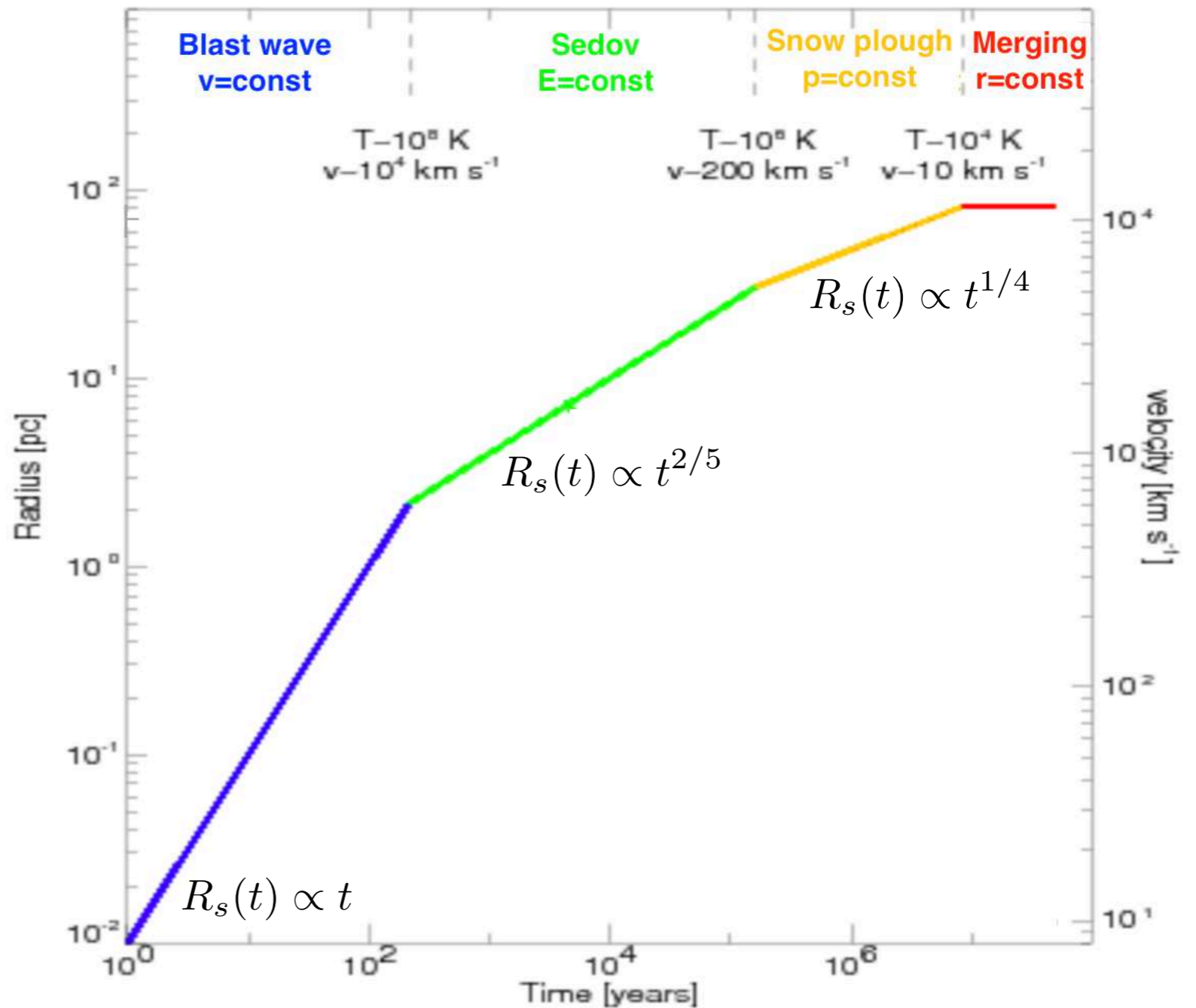
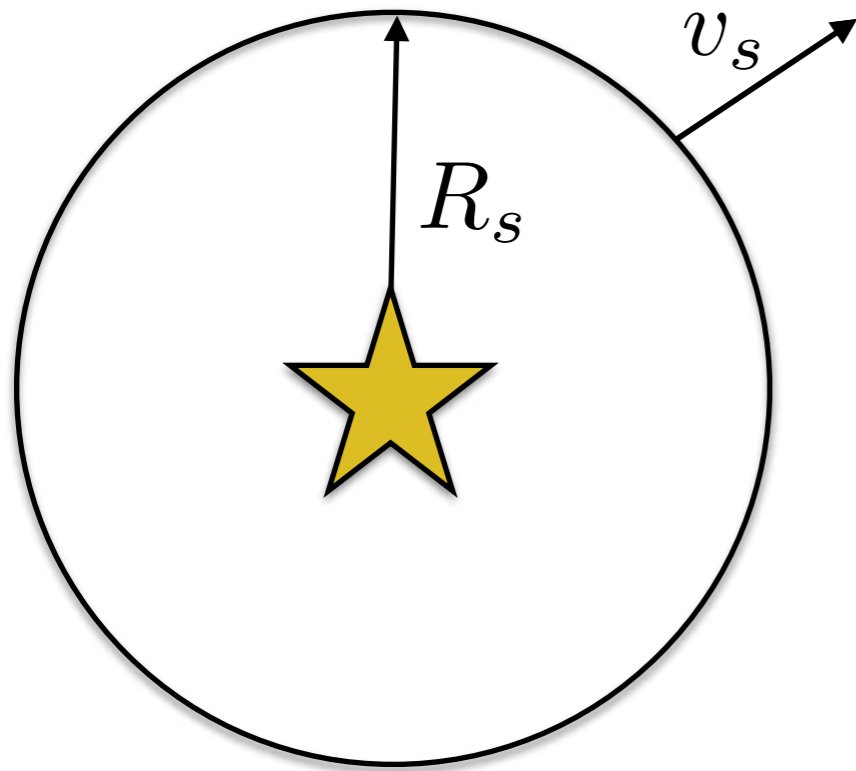
III. Radiative stage

→ momentum conservation

IV. Merging phase

→ pressure comparable to ISM

The hydrodynamical evolution of an SNR



$$t_{\text{Sed}} \simeq 1.6 \times 10^3 \text{ yr} \left(\frac{E_{\text{SN}}}{10^{51} \text{ erg}} \right)^{-1/2} \left(\frac{M_{\text{ej}}}{10 M_{\odot}} \right)^{5/6} \left(\frac{\rho_0}{1 m_{\text{p}}/\text{cm}^3} \right)^{-1/3}$$

Maximum energy in SNRs

In the scenario where the maximum momentum confined by the shock is a decreasing function of time, i.e.

$$p_{\max,0}(t) = p_M \left(\frac{t}{t_{\text{Sed}}} \right)^{-\delta} \quad (t \geq t_{\text{Sed}})$$



Ptuskin & Zirakashvili, A&A 429 (2005) 755

- Magnetic field not amplified

$$p_{\max,0}(t) \propto t^{-1/5}$$

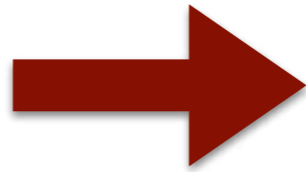
- Magnetic field amplification driven by resonant waves

$$p_{\max,0}(t) \propto t^{-7/5}$$

- Magnetic field amplification driven by non-resonant waves

$$p_{\max,0}(t) \propto t^{-2}$$

$$D/D_{\text{Gal}} \leq 0.3$$



Suppression of diffusion coefficient required:

- local turbulence?
- CR-induced turbulence (streaming instability)?



Malkov et al., ApJ 768 (2013) 63

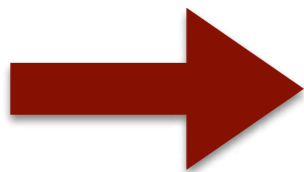


Nava et al., MNRAS 461 (2016) 3552N



D'Angelo et al., MNRAS 474 (2018) 1944D

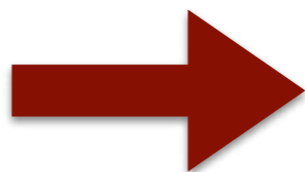
$$\delta \geq 2$$



How does magnetic turbulence evolve with time?

Needs to include damping effects (MHD cascade, ion-neutral friction).

$$\xi_{\text{CR}} \simeq 2 - 20\%$$

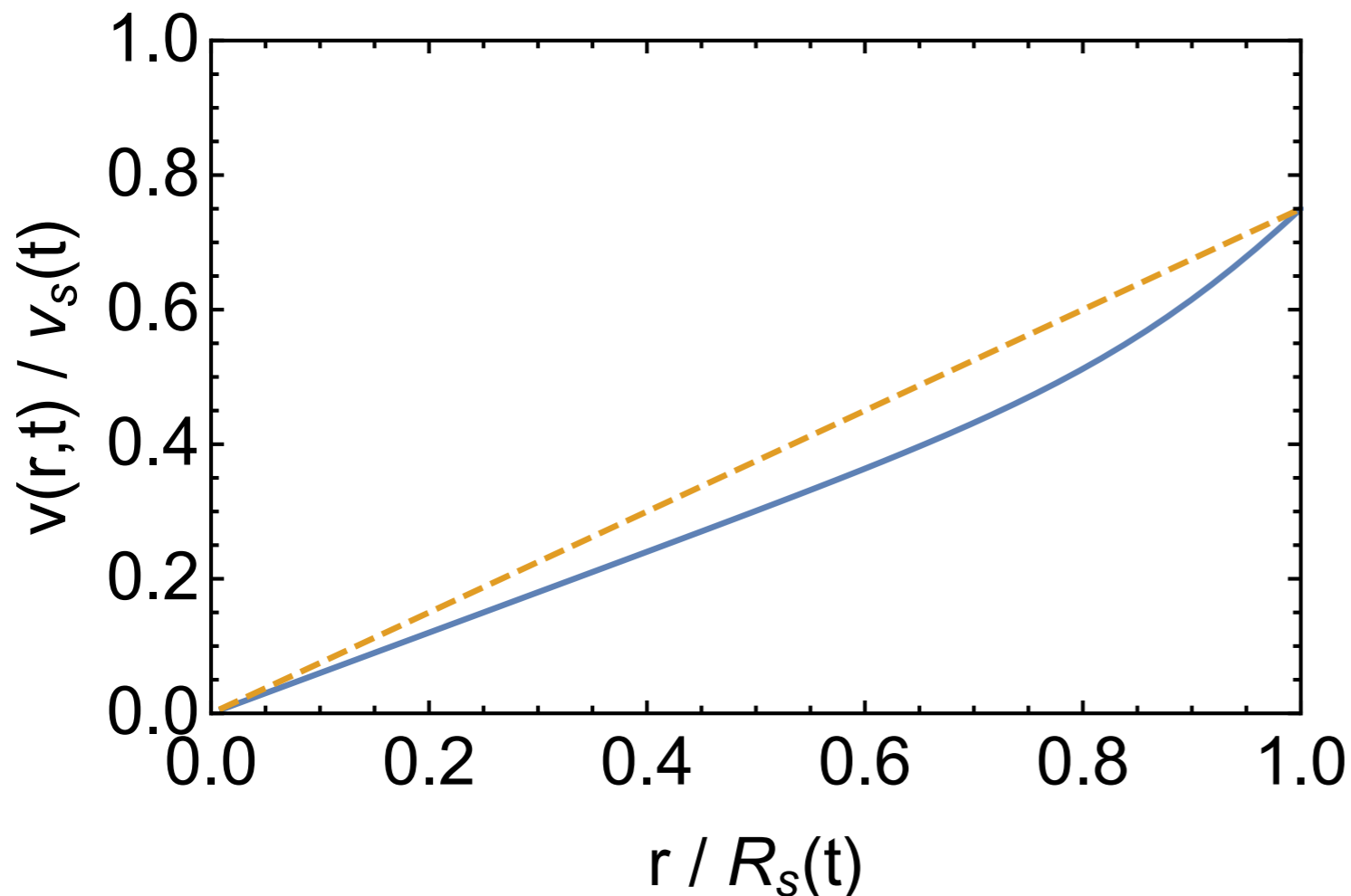


Standard assumption in the SNR paradigm for the origin of GCRs.

The velocity profile in the downstream

The **velocity field** in the downstream plasma, adopted for solution of the confined particle equation, follows from the ST solution in a homogeneous medium

→ linear approximation:
$$v(r, t) = \left(1 - \frac{1}{\sigma}\right) \frac{r}{R_s(t)} v_s(t)$$



Ostriker & McKee, RMP 60 (1988) 1



Ptuskin & Zirakashvili, A&A 429 (2005) 755

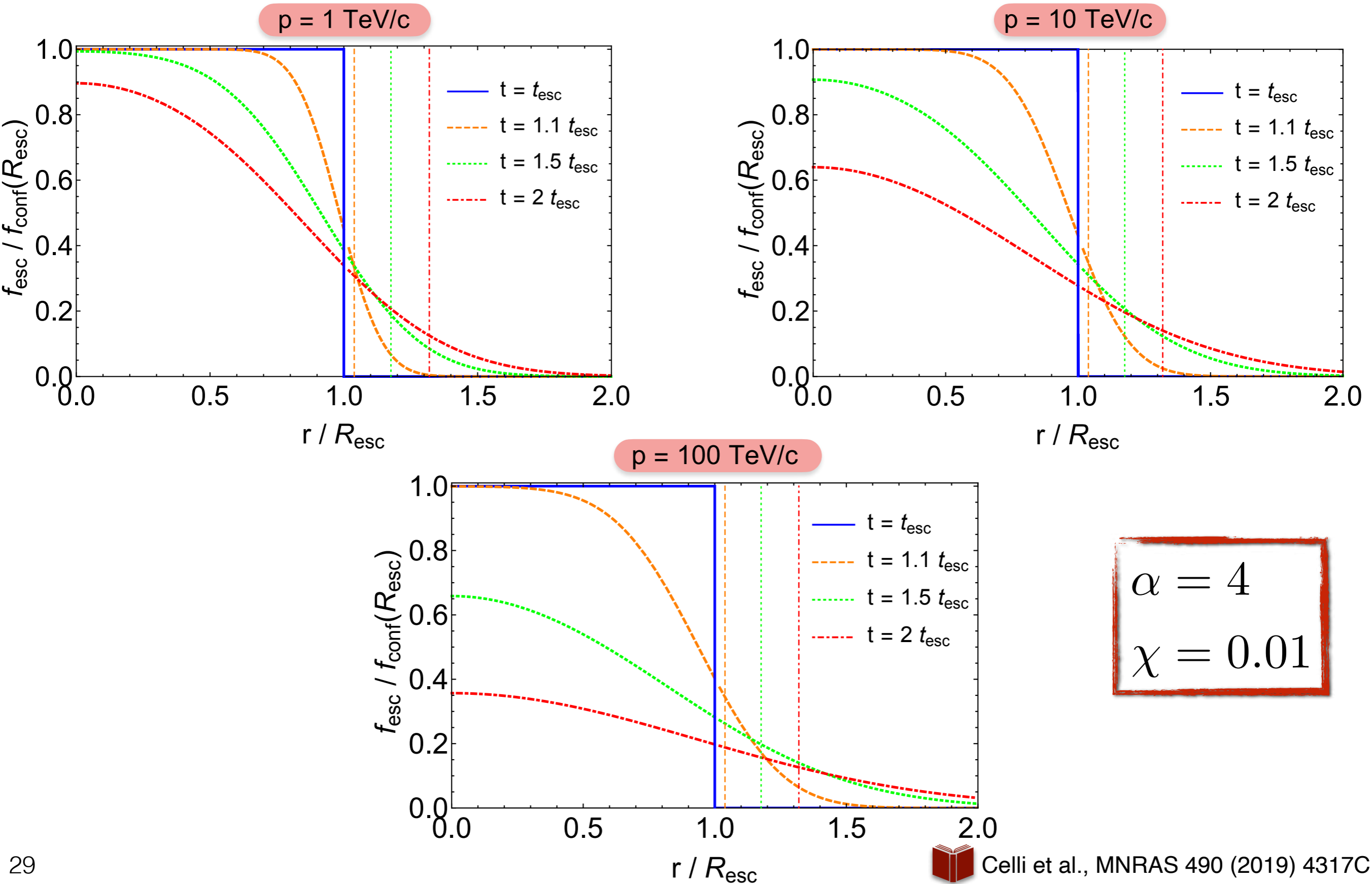


Sedov



Linear approx.

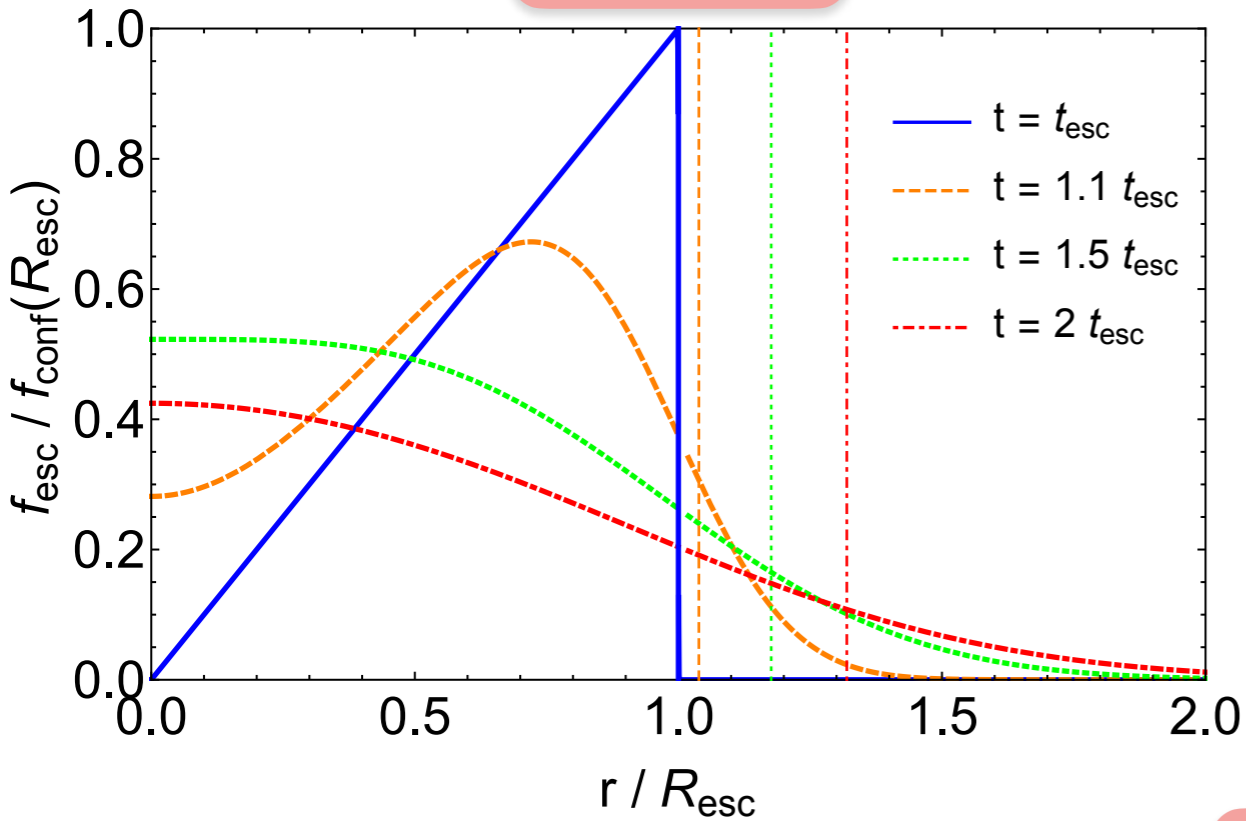
Density of non-confined protons



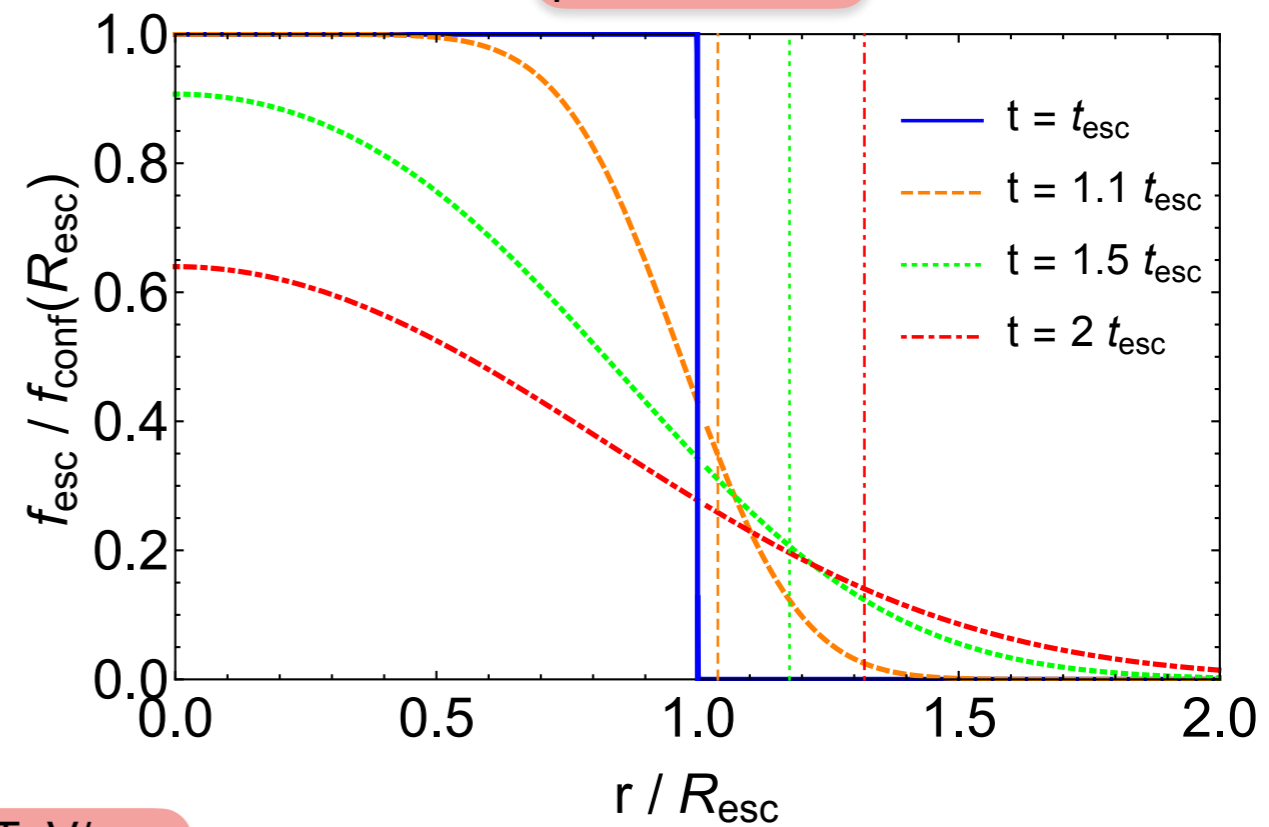
$$\alpha = 4 + 1/3$$

$$\alpha = 4$$

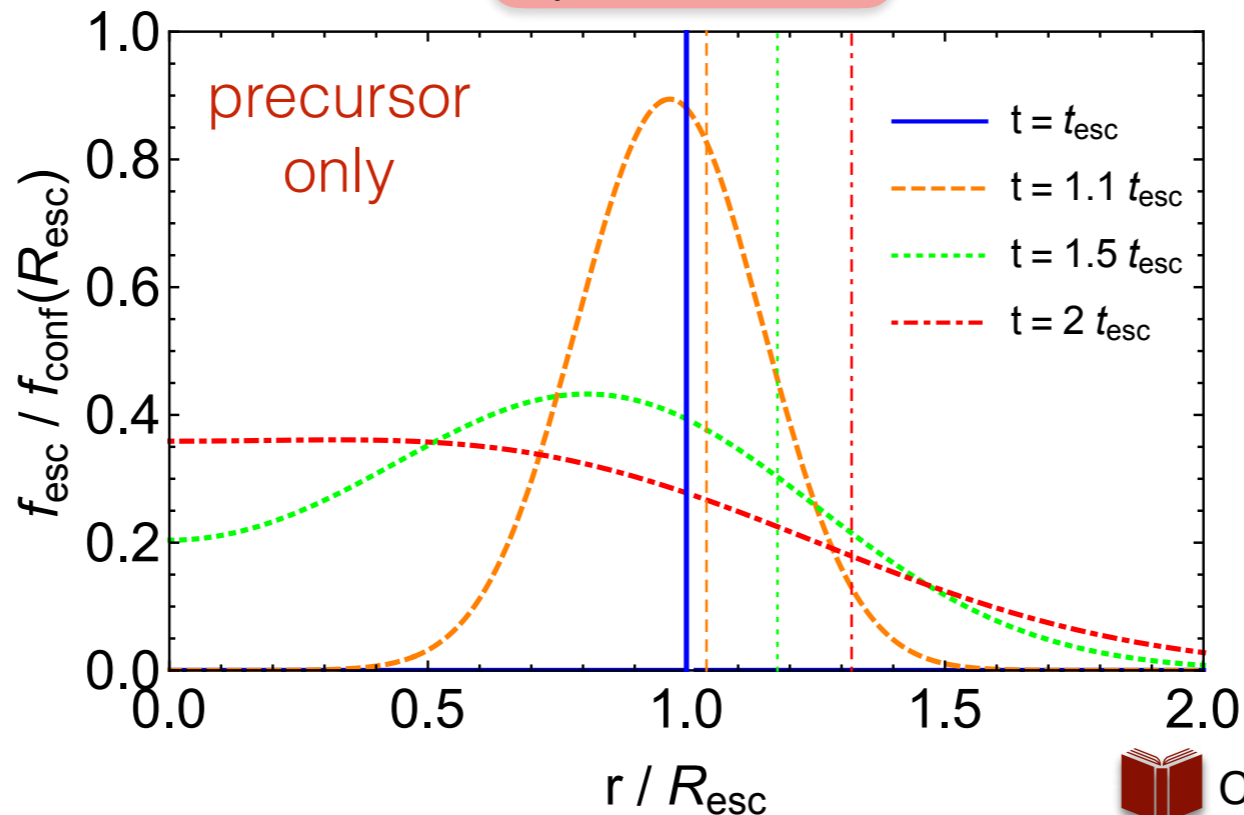
$p = 10 \text{ TeV}/c$



$p = 10 \text{ TeV}/c$



$p = 10 \text{ TeV}/c$

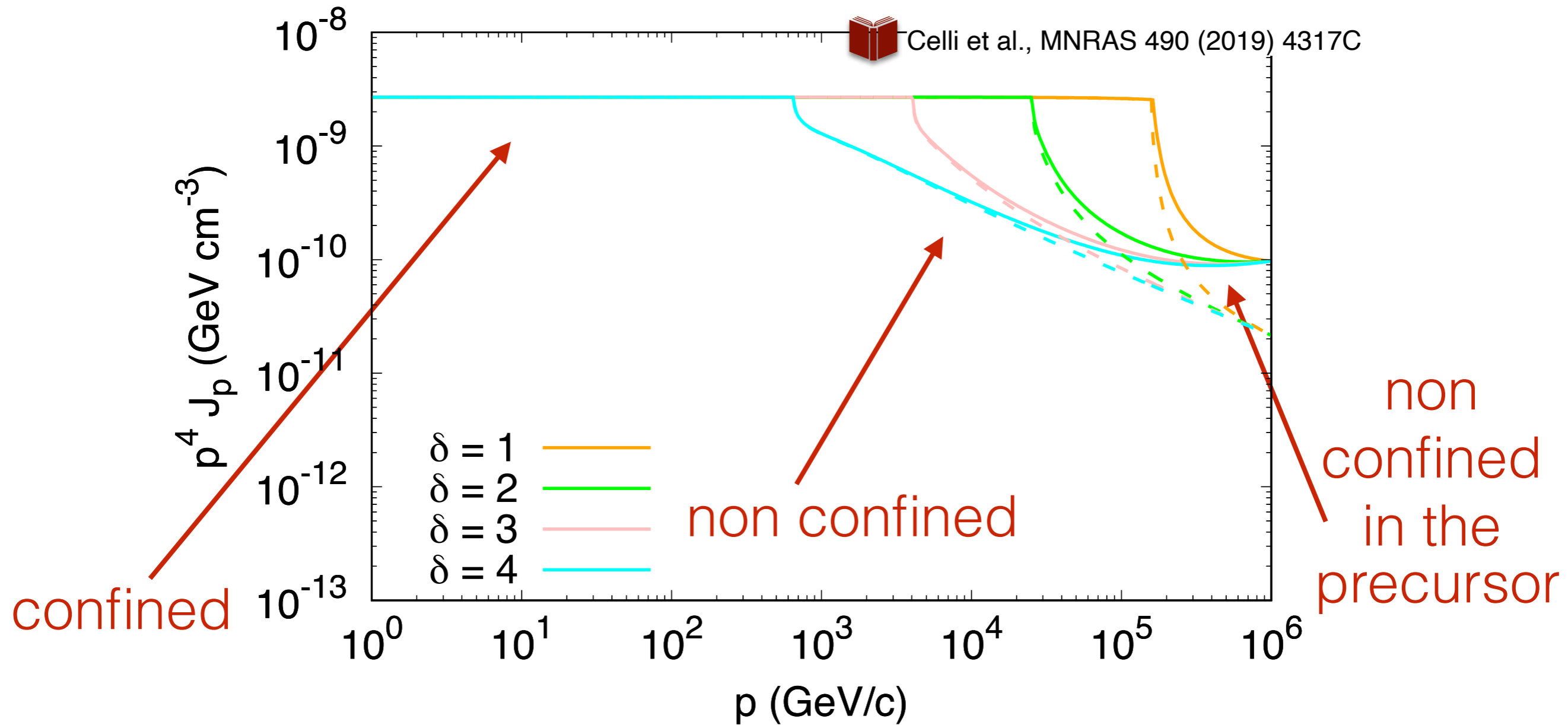


$$\chi = 0.01$$



The spectrum of protons inside the SNR

$$J_p^{\text{in}}(t, p) = \frac{4\pi}{V_{\text{SNR}}} \int_0^{R_{\text{sh}}(t)} [f_{\text{esc}}(t, r, p) + f_{p,\text{esc}}(t, r, p) + f_{\text{conf}}(t, r, p)] r^2 dr$$



$$D(p) = 10^{27} \left(\frac{pc}{10 \text{ GeV}} \right)^{1/3} \text{cm}^2 \text{s}^{-1}$$

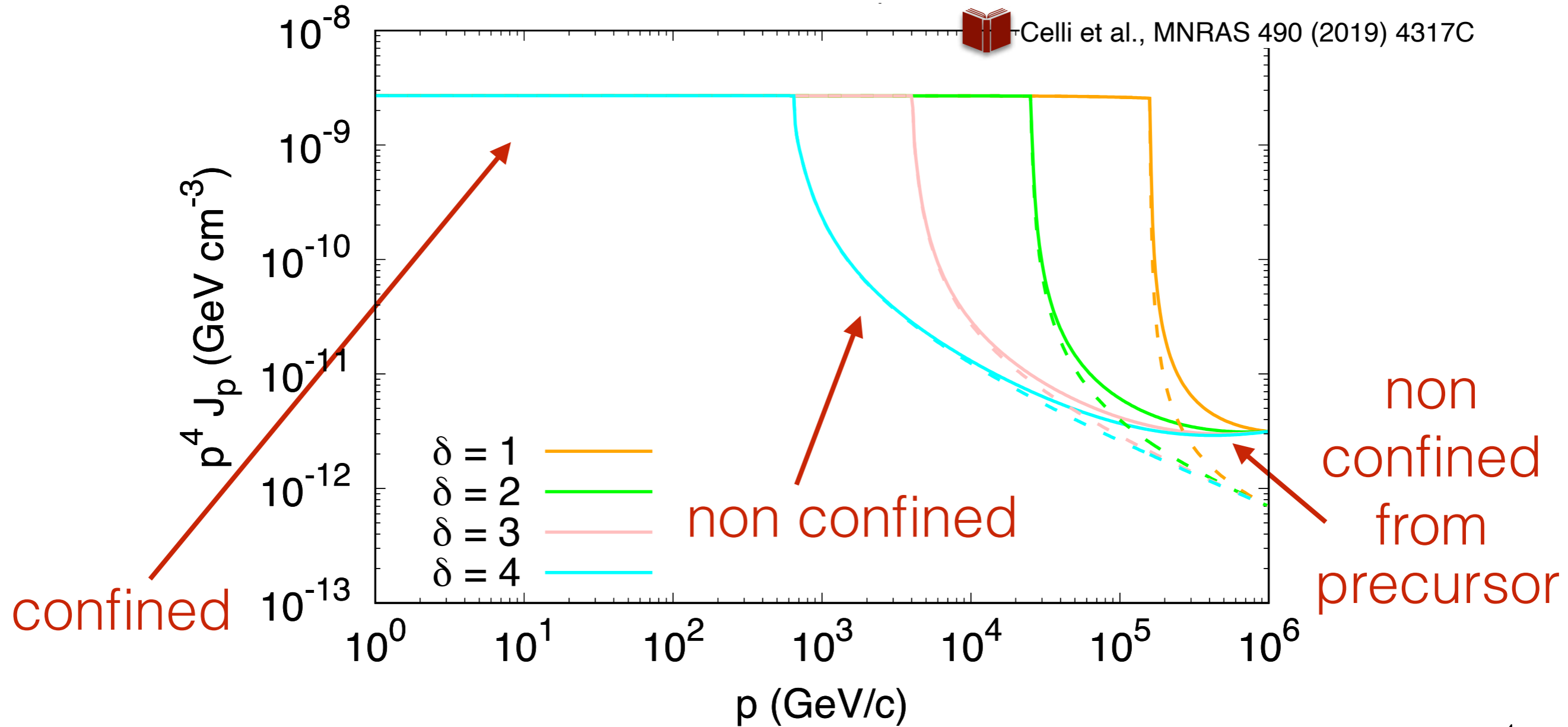
$$T_{\text{SNR}} = 10^4 \text{ yr}$$

$$\xi_{\text{CR}} = 10\%$$

$$n_0 = 1 \text{ cm}^{-3}$$

The spectrum of protons inside the SNR

$$J_p^{\text{in}}(t, p) = \frac{4\pi}{V_{\text{SNR}}} \int_0^{R_{\text{sh}}(t)} [f_{\text{esc}}(t, r, p) + f_{p,\text{esc}}(t, r, p) + f_{\text{conf}}(t, r, p)] r^2 dr$$



$$D(p) = 10^{28} \left(\frac{pc}{10 \text{ GeV}} \right)^{1/3} \text{cm}^2 \text{s}^{-1}$$

$$T_{\text{SNR}} = 10^4 \text{ yr}$$

$$\xi_{\text{CR}} = 10\%$$

$$n_0 = 1 \text{ cm}^{-3}$$

A model for particle propagation

1. Confined particle density \longrightarrow Method of characteristics

$$f_{\text{conf}}(t, r, p) = f_0 \left(t_0(t, r), p \left(\frac{R_s(t)}{R_s(t_0)} \right)^{3/4} \right) \quad \text{adiabatic losses}$$

2. Escaped particle density \longrightarrow Laplace transformation

$$\frac{f_{\text{esc}}(r, t, p)}{f_{\text{conf}}(t_{\text{esc}}, p)} = \frac{1}{2} \left[\text{Erf} \left[\frac{R_+}{R_d} \right] + \text{Erf} \left[\frac{R_-}{R_d} \right] + \frac{R_d}{\sqrt{\pi r}} \left(e^{-\left(\frac{R_+}{R_d}\right)^2} - e^{-\left(\frac{R_-}{R_d}\right)^2} \right) \right] \theta[t - t_{\text{esc}}(p)]$$

$$R_+ = (R_{\text{esc}} + r) \quad R_- = (R_{\text{esc}} - r) \quad R_d(t, p) = 2D(p) \sqrt{t - t_{\text{esc}}(p)}$$

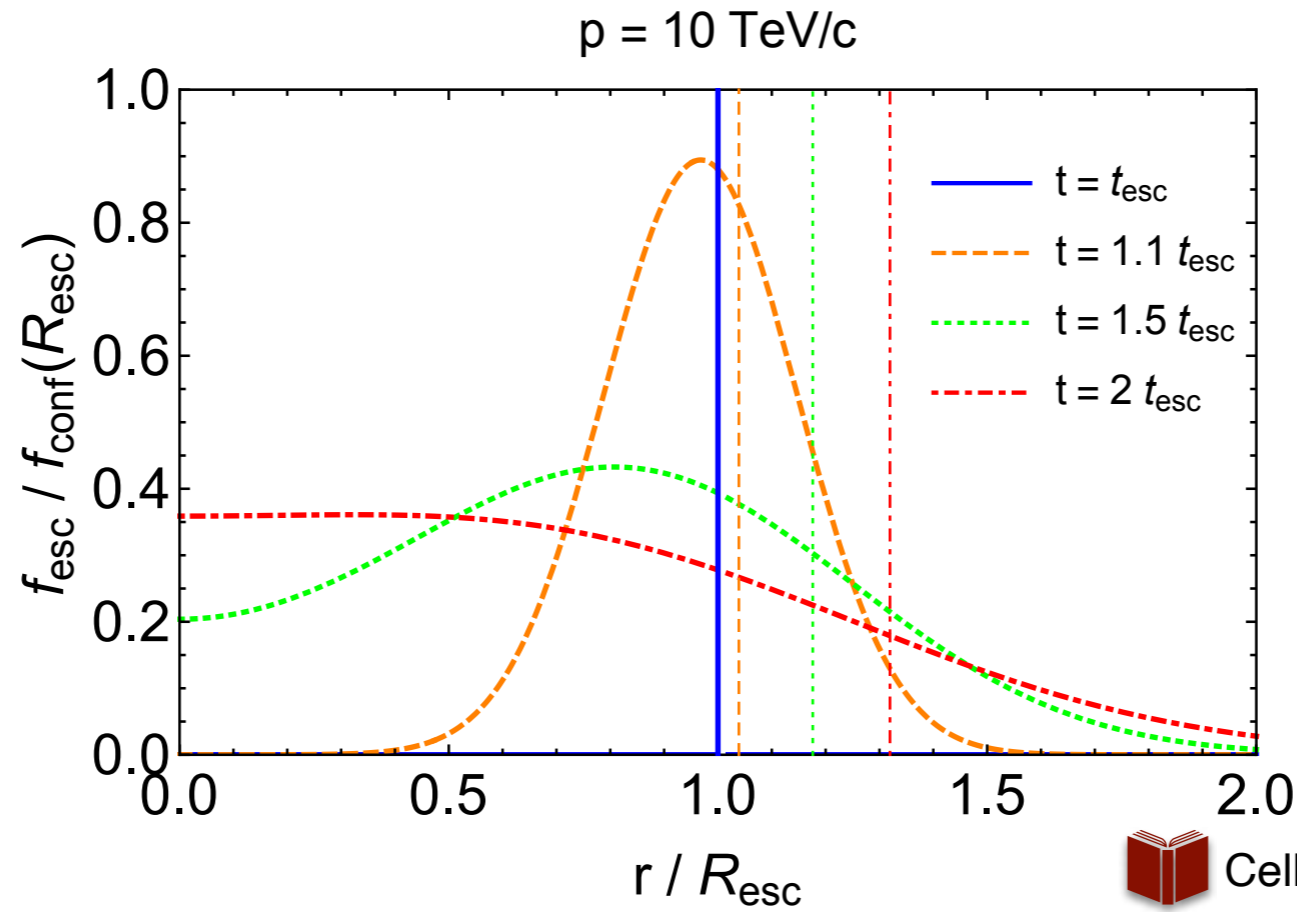
$$\alpha = 4$$

The precursor contribution to the density of non-confined particles

$$f_p(t, r, p) = f_0(t, p) \exp \left[-\frac{u_{\text{sh}}(t)}{D_p(p)} (r - R_{\text{sh}}) \right]$$

$$f_{p,\text{conf}}(t, r, p) \simeq f_0(t, p) \frac{D_p(p)}{u_{\text{sh}}(t)} \delta(r - R_{\text{sh}})$$

$$\longrightarrow \frac{f_{p,\text{esc}}(r, t, p)}{f_0(p, t_{\text{esc}})} = \frac{1}{\sqrt{\pi}} \frac{R_{\text{esc}}}{R_d} \frac{D_p(p)}{v_s(t_{\text{esc}})r} \left[e^{-\left(\frac{R_-}{R_d}\right)^2} - e^{-\left(\frac{R_+}{R_d}\right)^2} \right] \theta[t - t_{\text{esc}}(p)]$$



Self-generated turbulence

$$\Gamma_{\text{CR}}(k) = \frac{16\pi^2}{3} \frac{v_A}{B_0^2 \mathcal{F}(k)} \left[p^4 v(p) \frac{\partial f}{\partial r} \right]_{p=p_{\text{res}}}$$

growth rate by resonant streaming instability



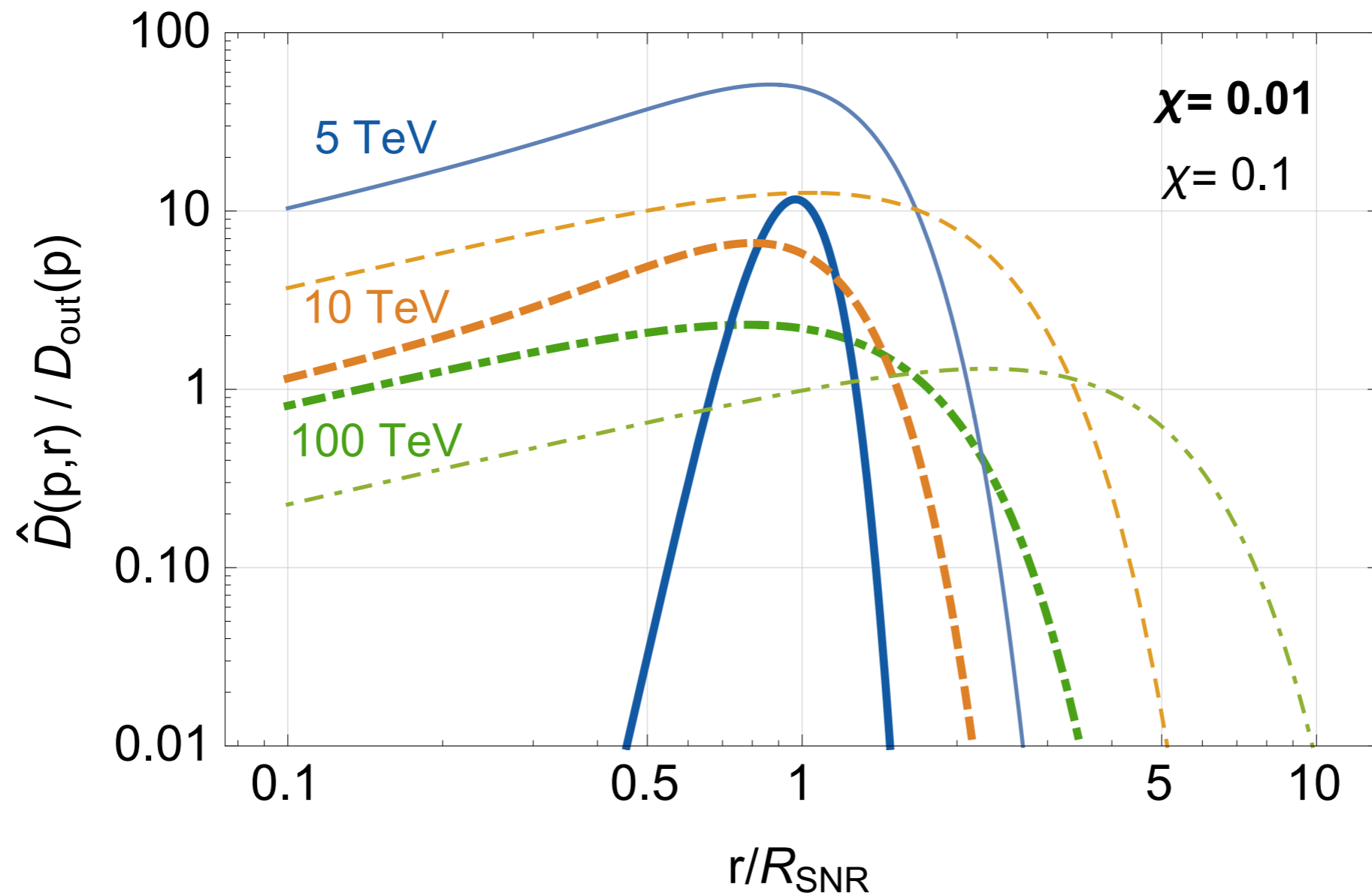
Skilling, ApJ 170 (1971) 265

$$\Gamma_{\text{NLD}}(k) = (2c_k)^{-3/2} k v_A \sqrt{\mathcal{F}(k)}$$

non-linear damping rate



Ptuskin & Zirakashvini, A&A 403 (2003) 1



$\chi = 0.01$

$\chi = 0.1$

5 TeV

10 TeV

100 TeV

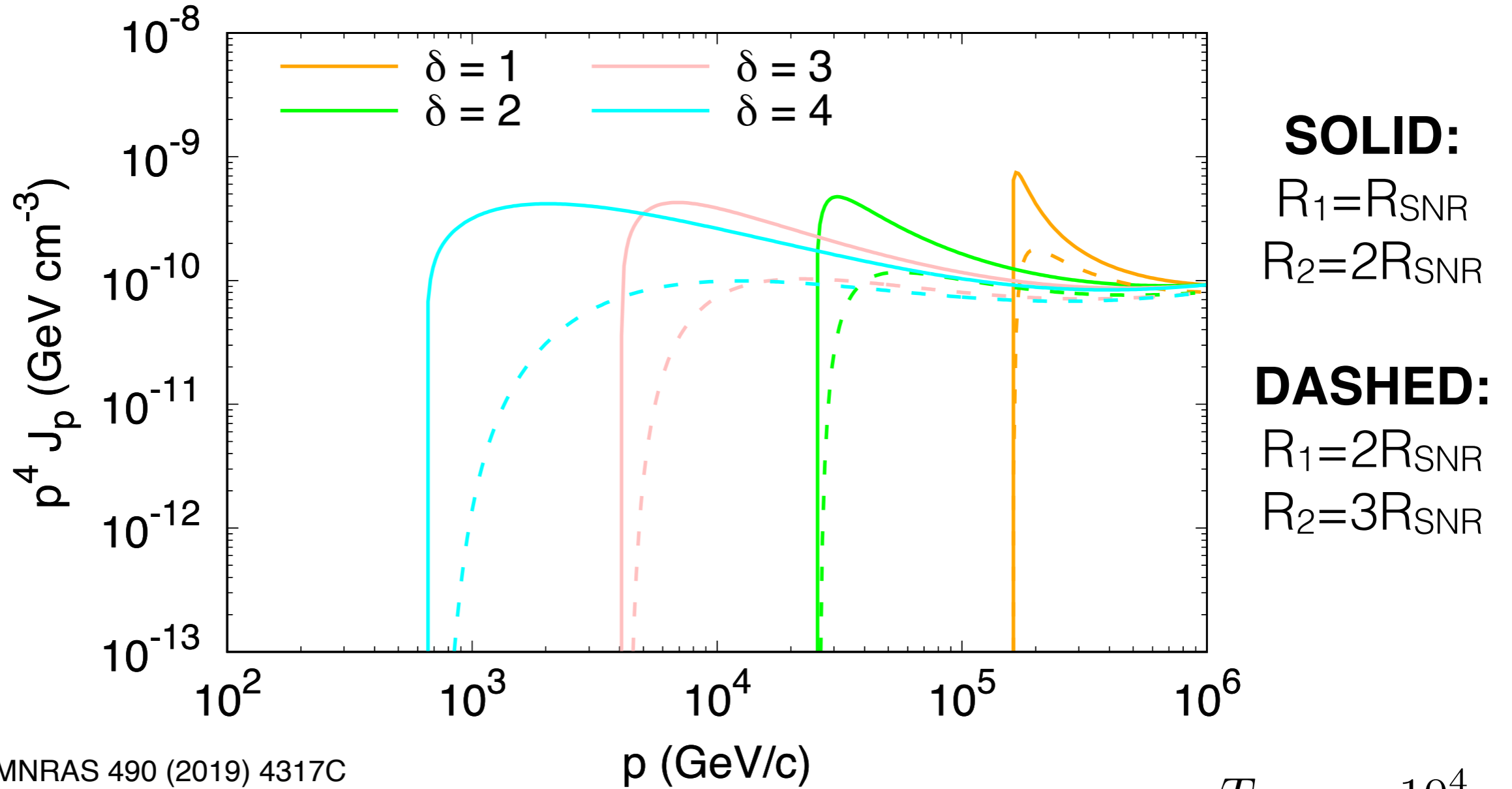
$$\xi_{\text{CR}} = 10\%$$

$$n_{\text{up}} = 1 \text{ cm}^{-3}$$

$$T_{\text{SNR}} = 10^4 \text{ yr}$$

The spectrum of protons outside the SNR

$$J_p^{\text{out}}(t, p) = \frac{3}{R_2^3 - R_1^3} \int_{R_1}^{R_2} [f_{\text{esc}}(t, r, p) + f_{p,\text{esc}}(t, r, p)] r^2 dr$$



Celli et al., MNRAS 490 (2019) 4317C

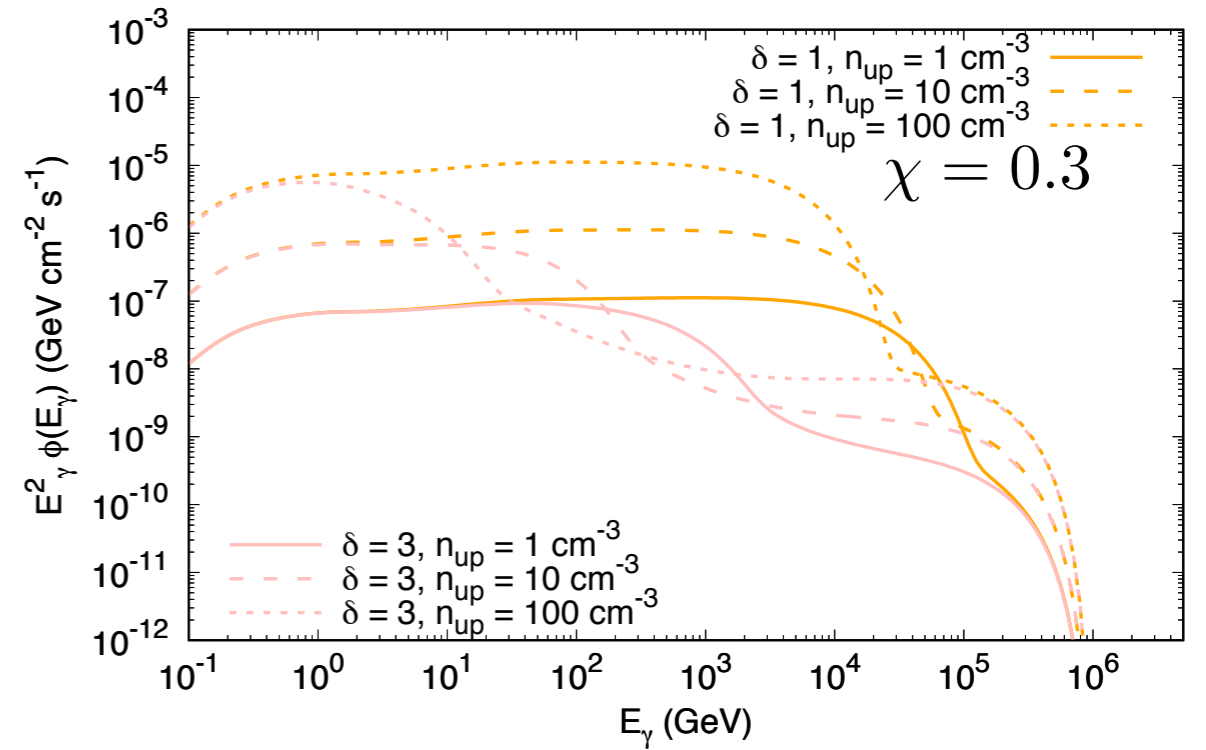
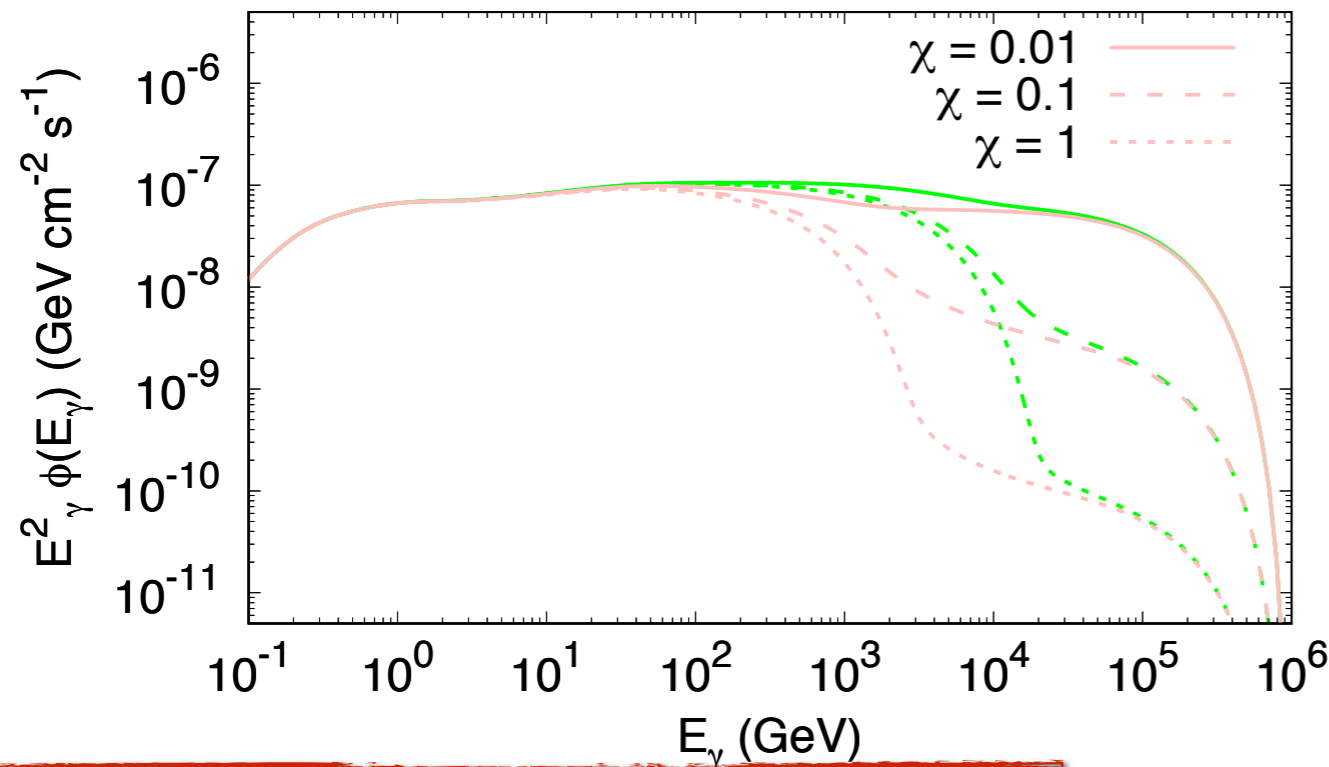
$$D(p) = 10^{27} \left(\frac{pc}{10 \text{ GeV}} \right)^{1/3} \text{ cm}^2 \text{ s}^{-1}$$

$$T_{\text{SNR}} = 10^4 \text{ yr}$$

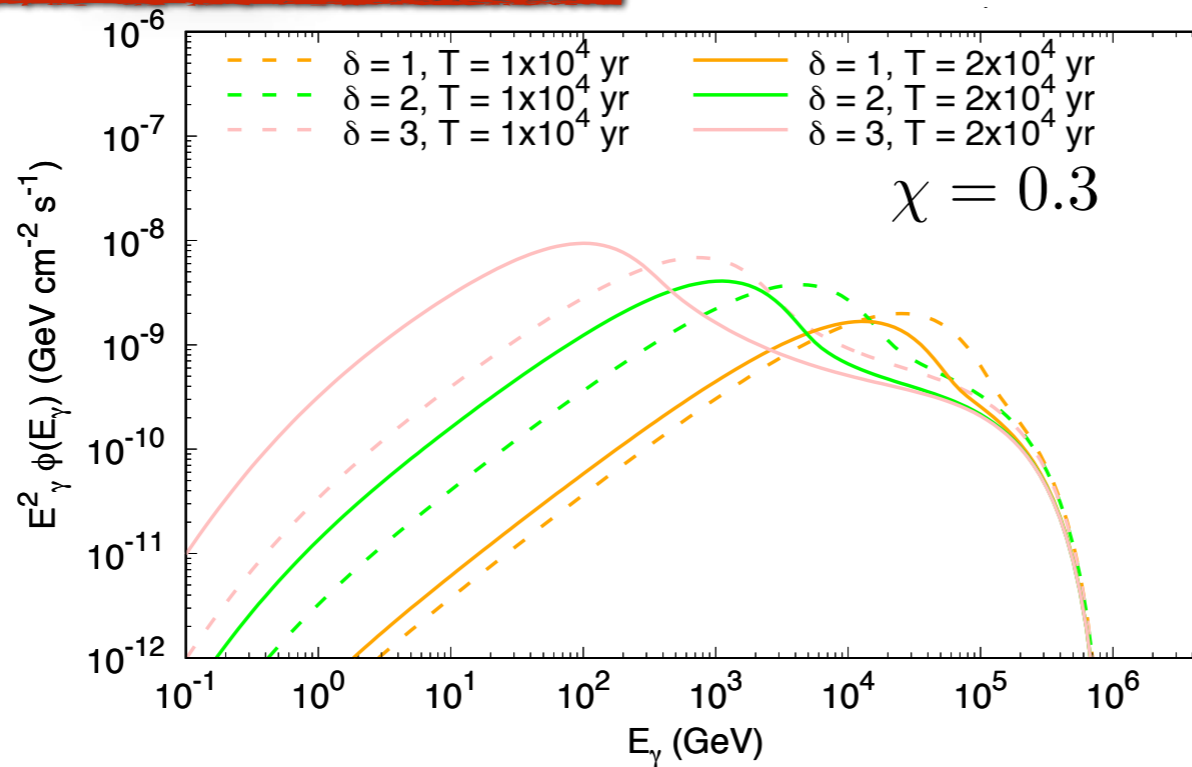
$$\xi_{\text{CR}} = 10\%$$

$$n_{\text{up}} = 1 \text{ cm}^{-3}$$

Volume integrated gamma-ray emission from hadronic interactions



$$D(p) \equiv \chi D_{\text{Gal}} = \chi 10^{28} \left(\frac{pc}{10 \text{ GeV}} \right)^{1/3} \text{ cm}^2 \text{ s}^{-1}$$



$$\xi_{\text{CR}} = 10\%$$

$$n_{\text{up}} = 1 \text{ cm}^{-3}$$

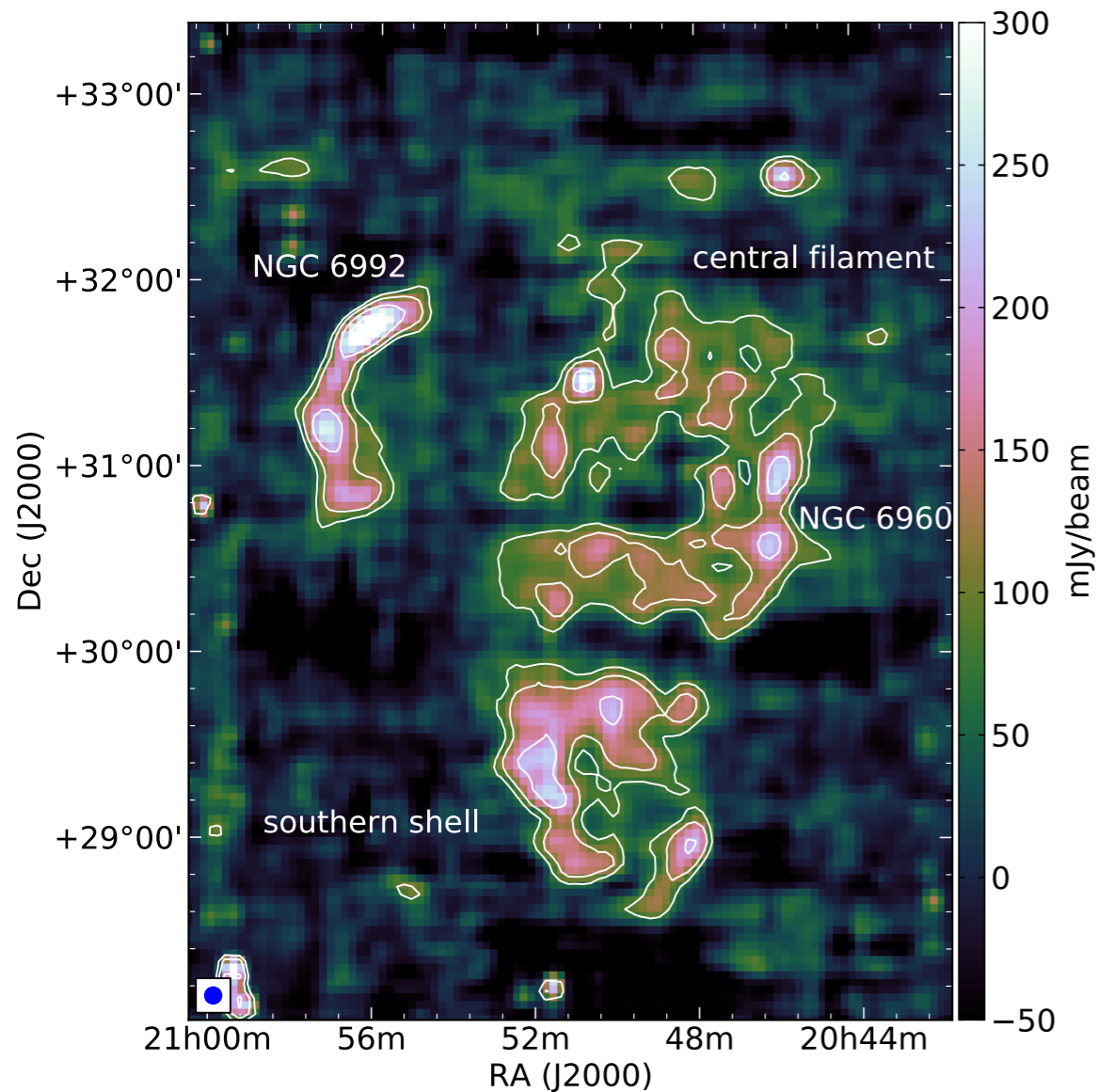
$$T_{\text{SNR}} = 10^4 \text{ yr}$$

$$d = 1 \text{ kpc}$$



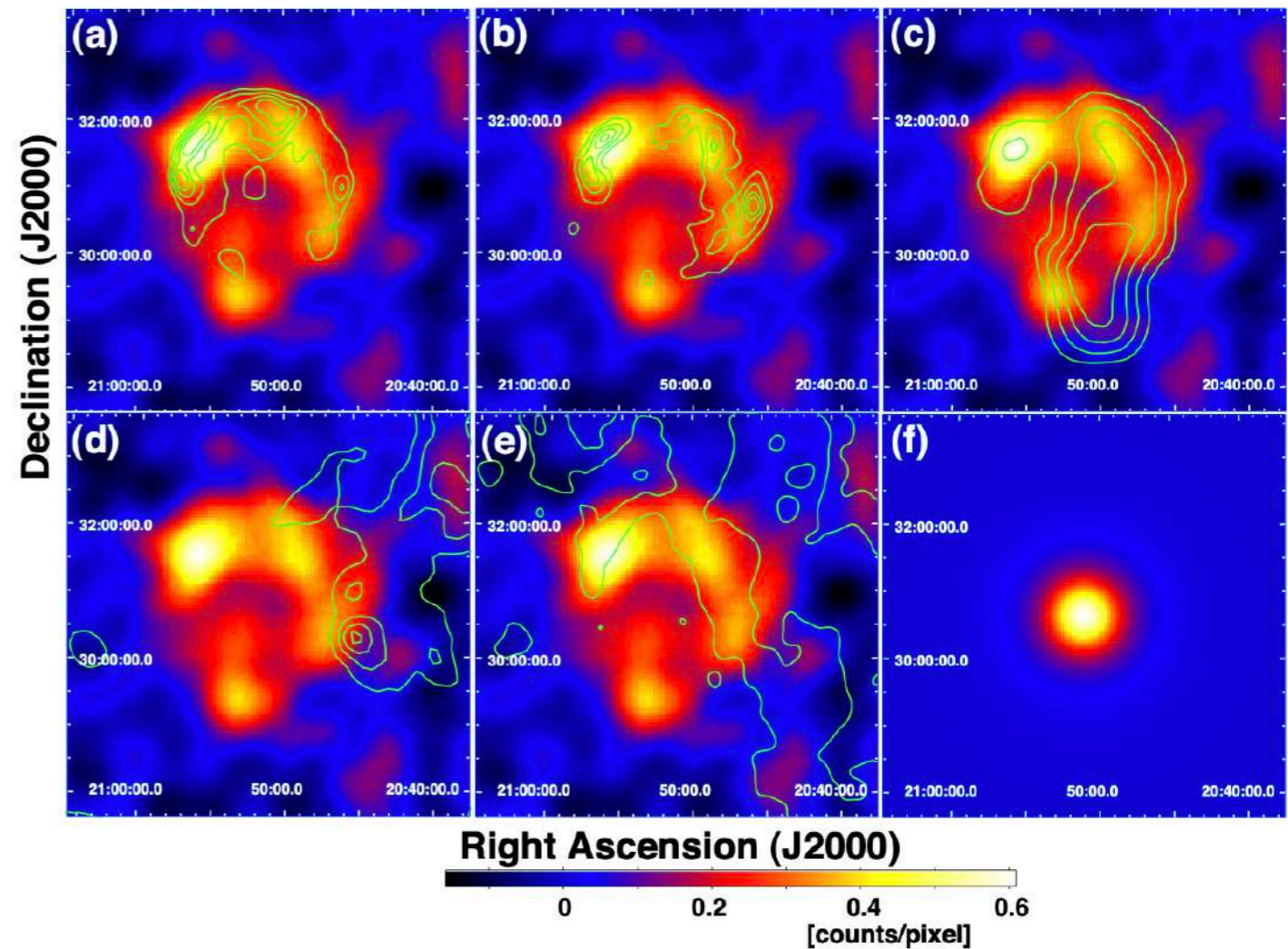
The Cygnus Loop SNR

Middle-aged SNR ($\sim 2 \times 10^4$ yr) located at high latitudes




Medicina radio telescope map @ 8.5 GHz

 Loru et al., MNRAS 500 (2020) 5177

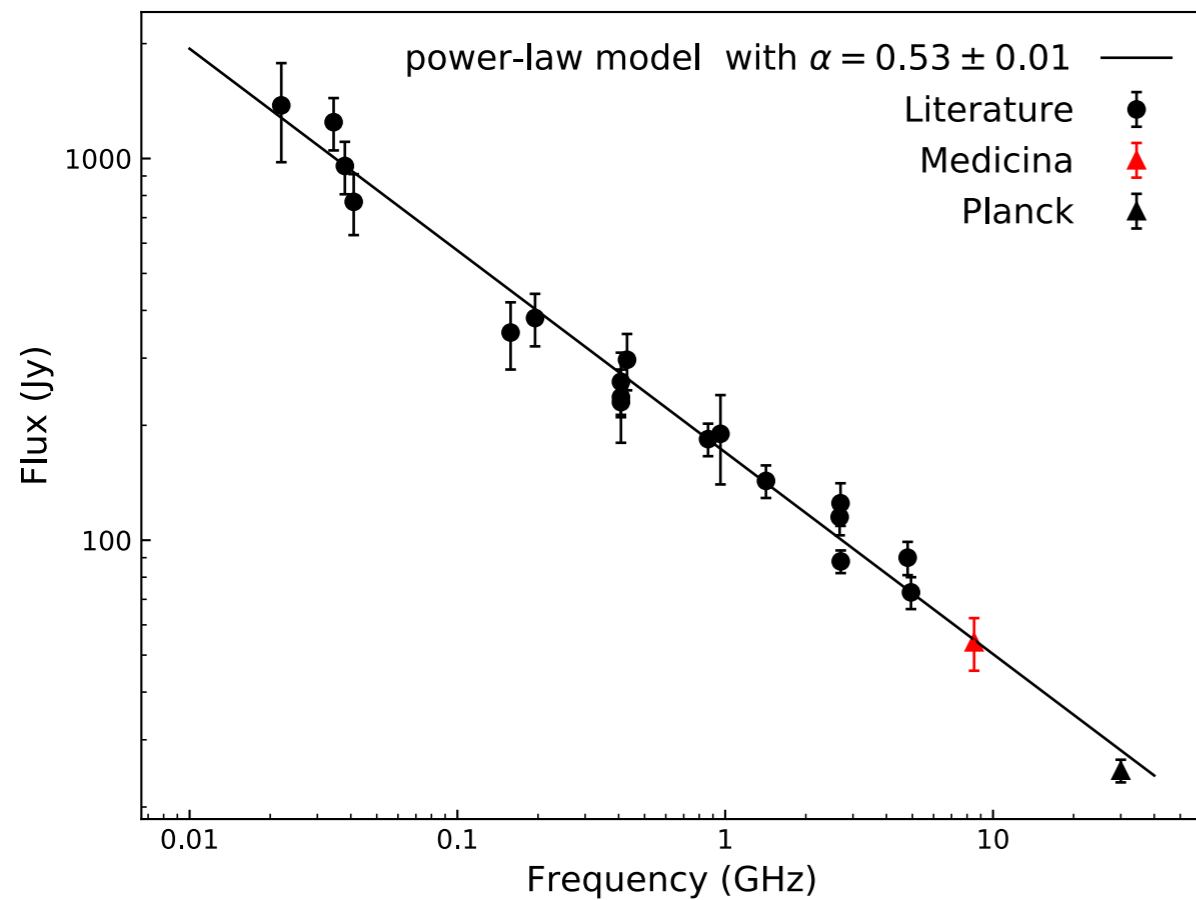


Fermi-LAT count map in [0.5 - 10] GeV

 Katagiri et al., ApJ 741 (2011) 44

The Cygnus Loop SNR

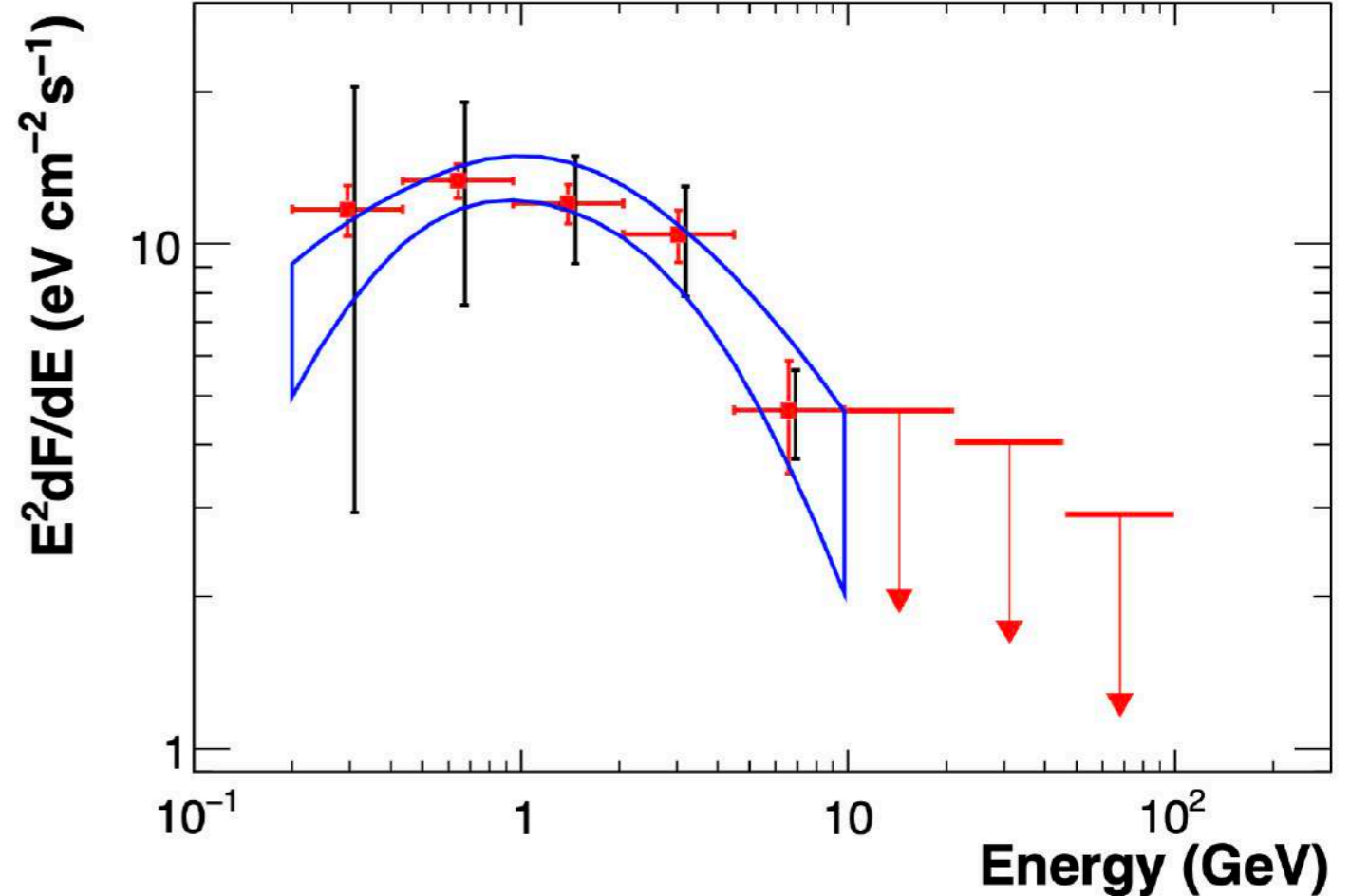
Middle-aged SNR ($\sim 2 \times 10^4$ yr) located at high latitudes




 Loru et al., MNRAS 500 (2020) 5177

$$s \simeq 0.5 \implies \alpha \simeq 4$$

→ constraints over K_{ep} and B_0



 Katagiri et al., ApJ 741 (2011) 44

$$n = 0.4 \text{ cm}^{-3} \implies \xi_{\text{CR}} \simeq 7\%$$

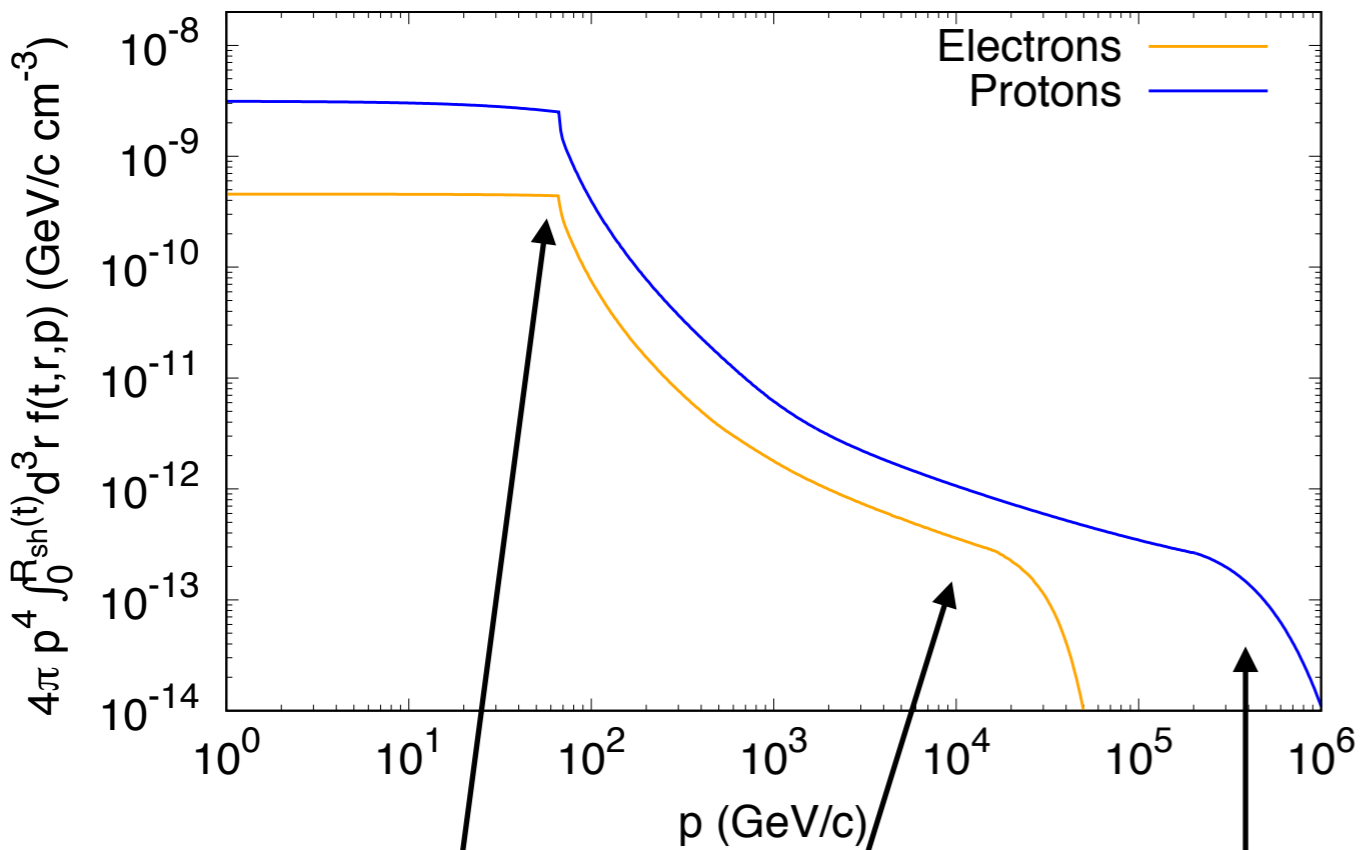
→ constraints over p_M , δ and χ

The Cygnus Loop SNR

Middle-aged SNR ($\sim 2 \times 10^4$ yr) located at high latitudes



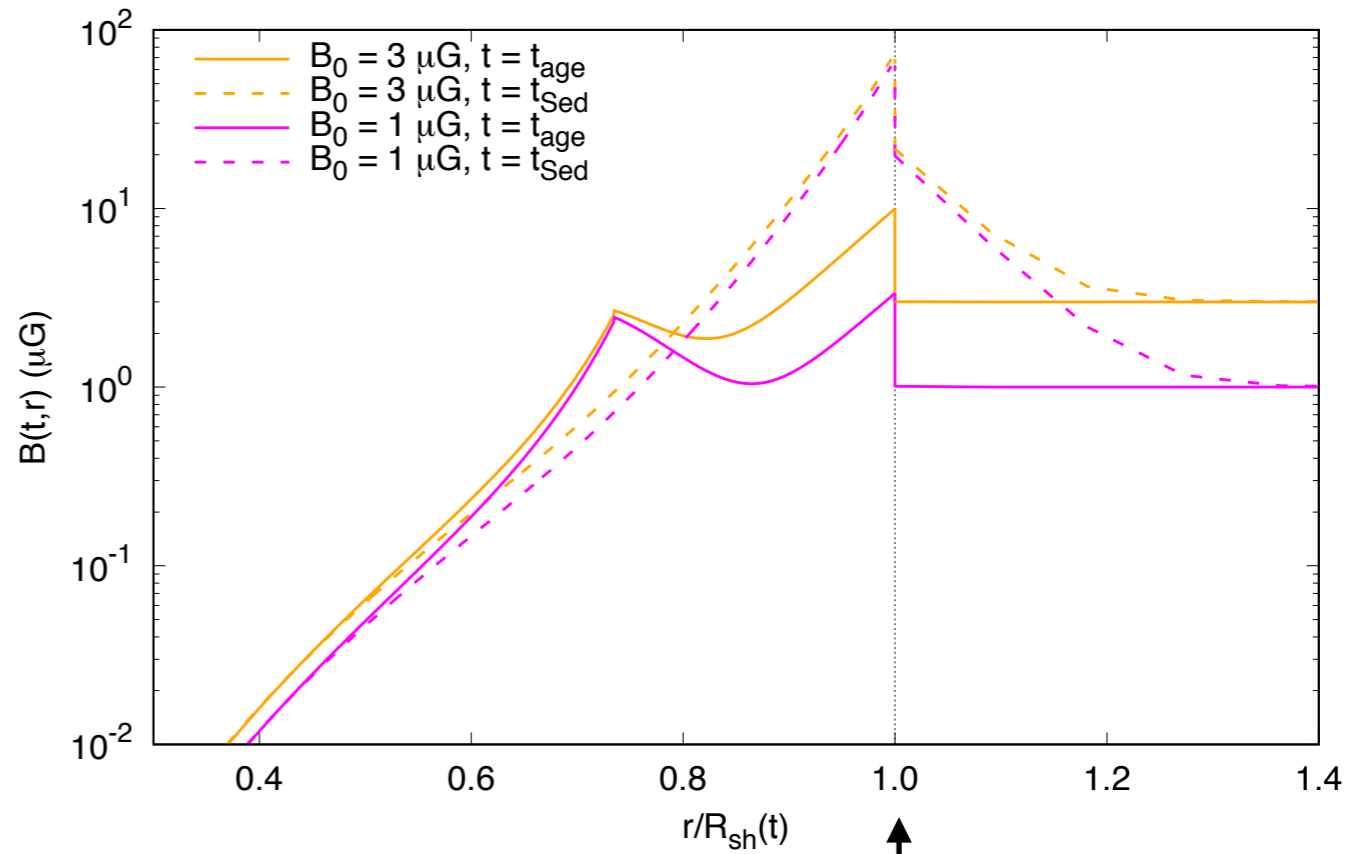
Loru et al., MNRAS 500 (2020) 5177



$p_{M,e} = 20$ TeV

$p_{M,p} = 200$ TeV

$E_{\max}(T_{\text{SNR}}) = 65$ GeV



Magnetic field at the shock today at the level of $10 \mu\text{G}$, compatible with compression

The Cygnus Loop SNR

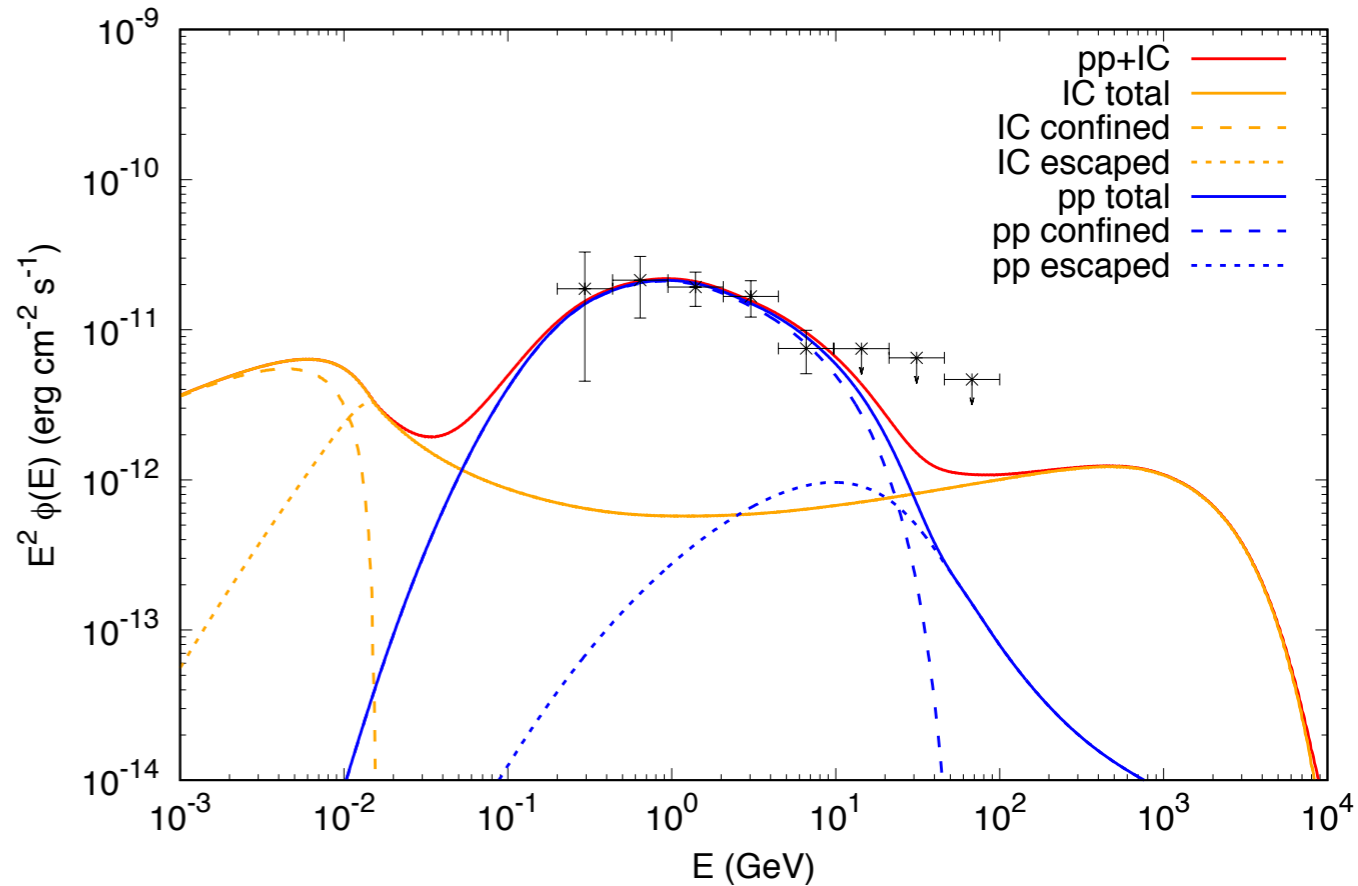
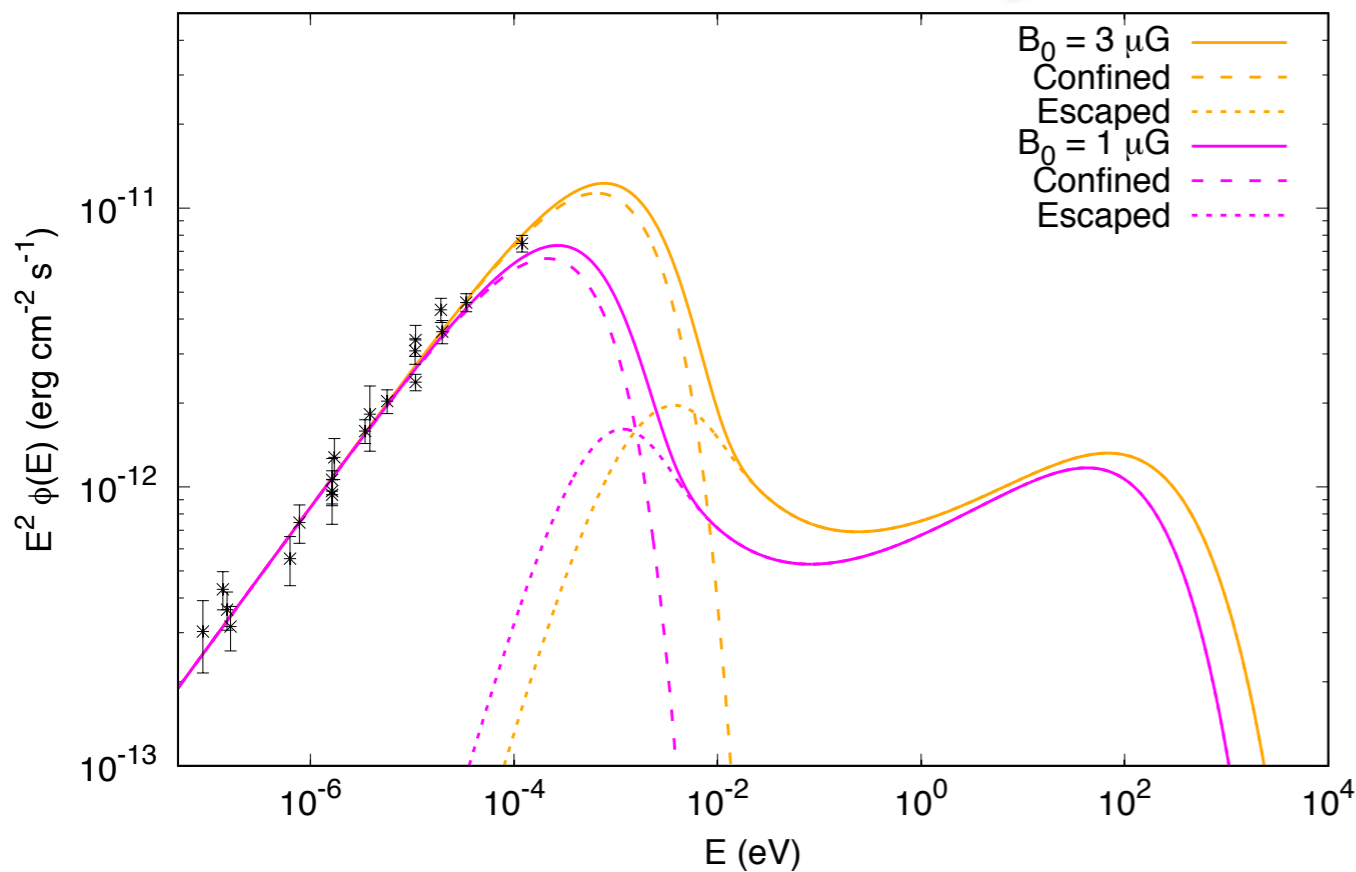
Cygnus Loop properties

Acceleration model parameters

Assumed					Derived		Acceleration model parameters						
E_{SN}	M_{ej}	t_{age}	d	n_0	R_{sh}	u_{sh}	ξ_{CR}	s	E_M	δ	K_{ep}	B_0	χ
7×10^{50} erg	$5 M_{\odot}$	2.1×10^4 yr	735 pc	0.4 cm^{-3}	20 pc	380 km s^{-1}	0.07	4.0	200 TeV	3	0.15	$3 \mu\text{G}$	1



Loru et al., MNRAS 500 (2020) 5177



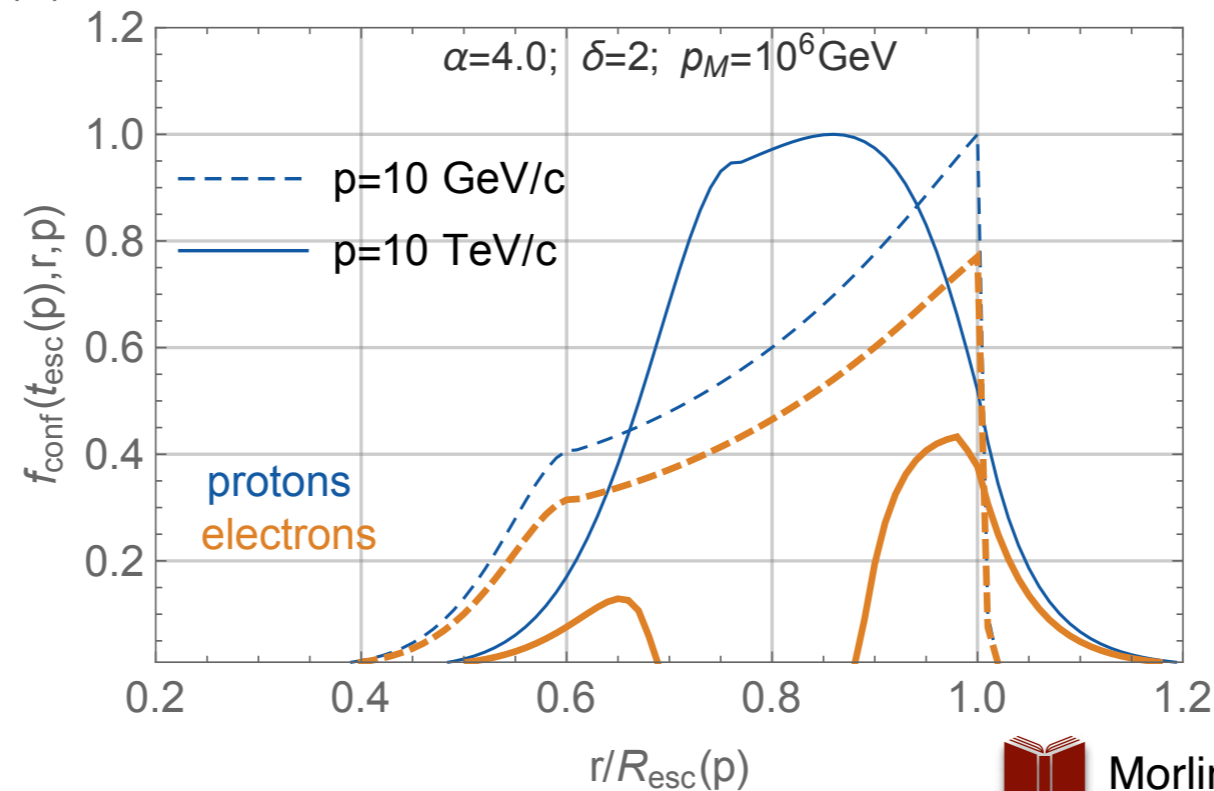
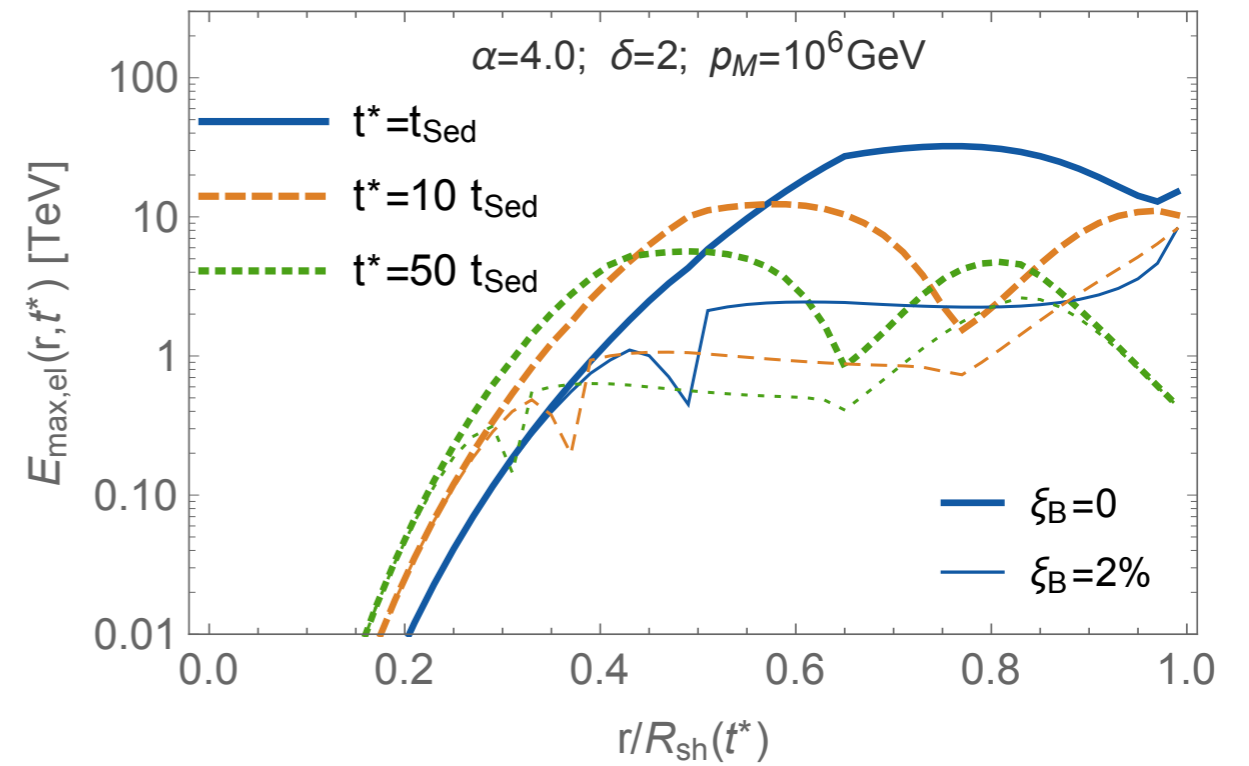
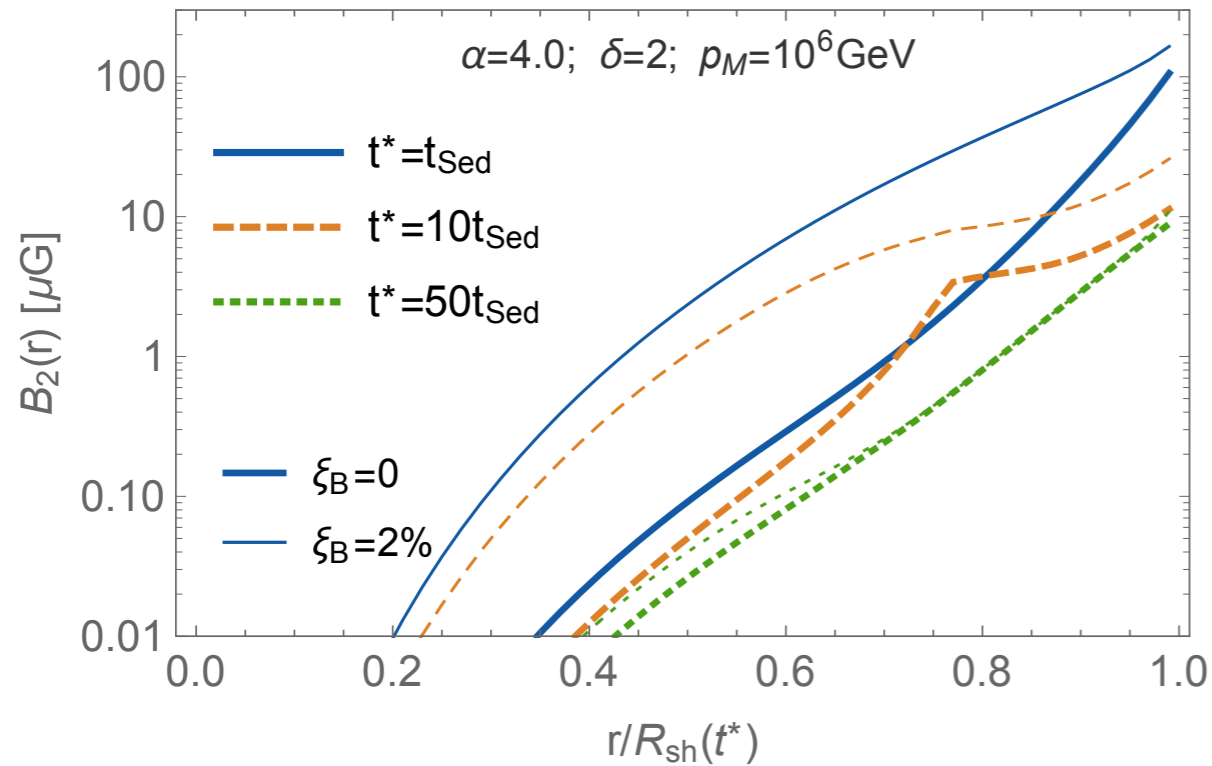
$\delta \geq 2$ → **Temporal evolution of magnetic turbulence:** MFA + damping effects?

$K_{\text{ep}} \simeq 0.15$ → **Electron to proton injection ratio:** hints for K_{ep} increasing with decreasing shock speed?

$\xi_{\text{CR}} \simeq 10\%$ → **CR acceleration efficiency:** Standard value in the SNR theory.

Solving electron propagation

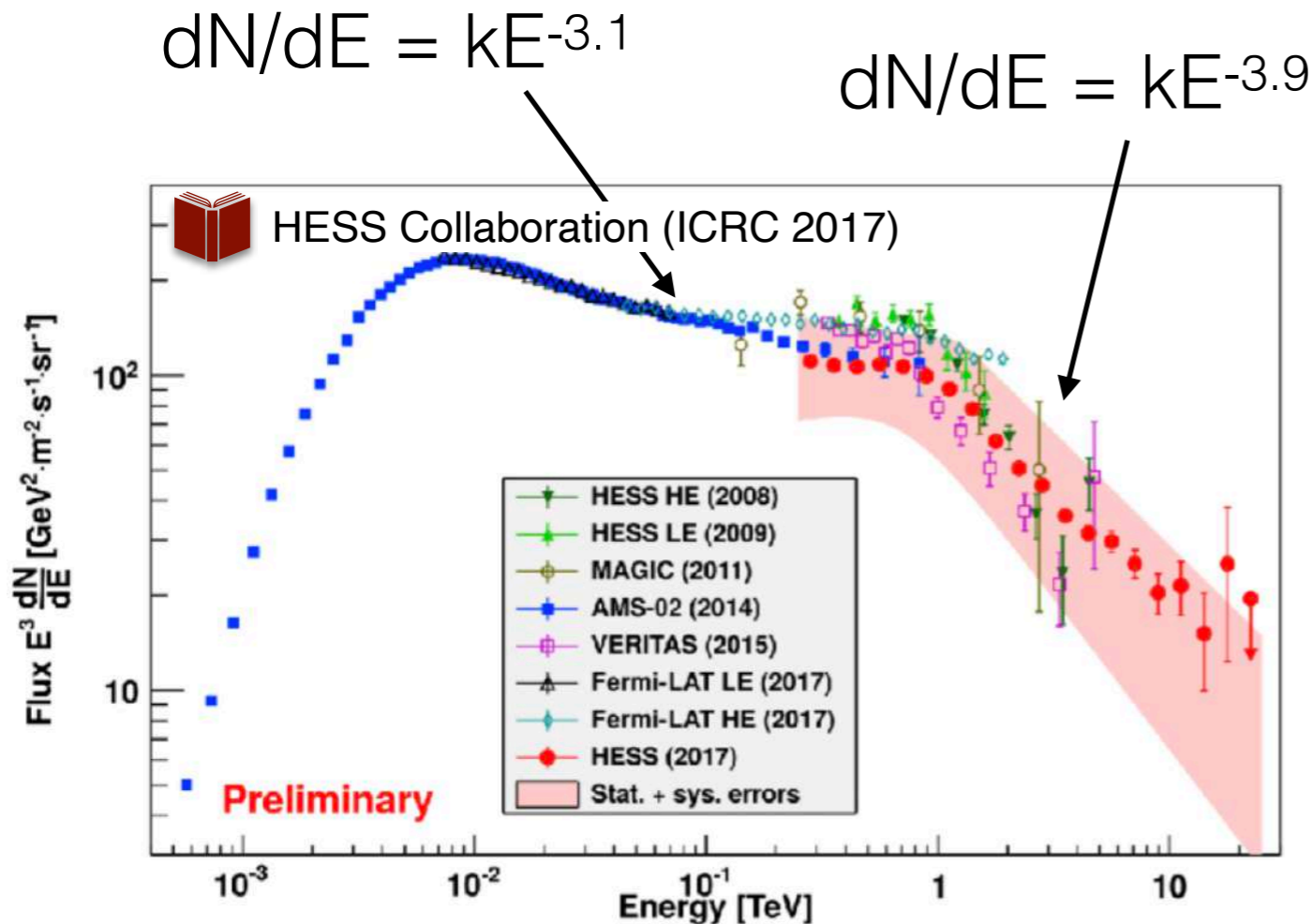
Numerical solution of the transport equation for accelerated **electrons**, including radiative and adiabatic losses



$*t_{\text{esc}}(10 \text{ GeV})=t_{\text{SP}}$
 $t_{\text{esc}}(10 \text{ TeV})=10t_{\text{Sed}}$



The observed CR-e spectrum



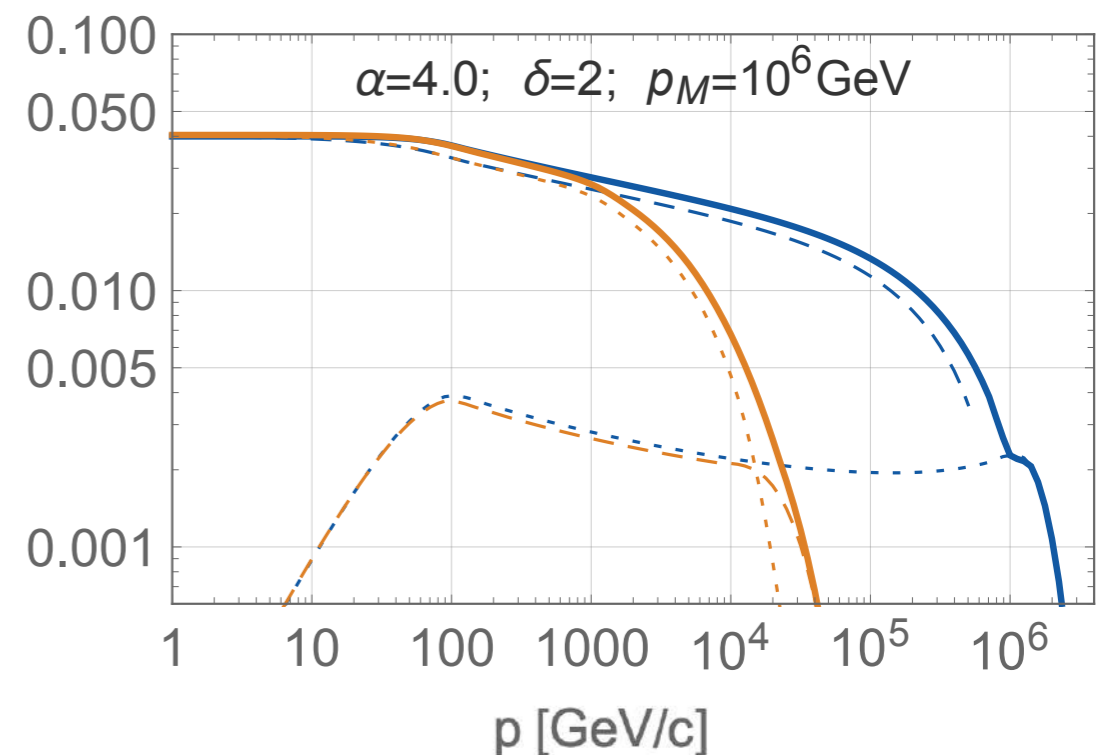
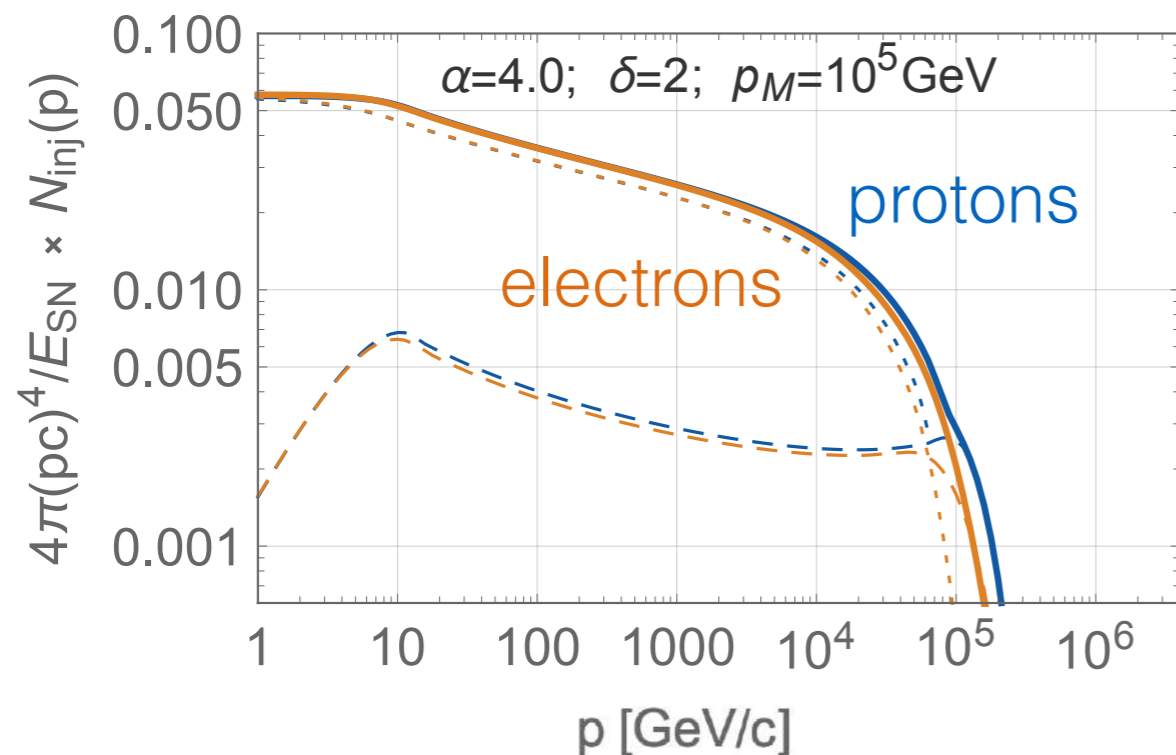
- Origin of the **spectral steepening** of the CR-electron spectrum below 1 TeV?
- Origin of the **TeV break** in the CR-electron spectrum?

- In the sub-TeV domain, after accounting for energy losses during propagation to Earth, **proton and electron injection spectra** are characterized by **different slopes**: energy losses within the source? Other mechanism?


$$\frac{Q_e}{Q_p} \propto \frac{N_e}{N_p} \frac{1}{\sqrt{D\tau_{\text{loss}}}} \propto E^{-0.4} \frac{1}{\sqrt{E^{0.54} E^{-0.77}}} \propto E^{-0.28}$$


The CR-e spectrum injected into the Galaxy

1. Self generated turbulence




- Spectral steepening in both species of **~ 0.15 above $p_{\text{max}}(t_{\text{SP}})$** ;
- Proton and electron spectra only differ if significant MFA is effective
→ large p_M ($\sim \text{PeV}$) or δ (>2);
- However, even in the **PeVatron** scenario, **self-amplified magnetic field can explain spectral differences only above $\sim 1 \text{ TeV}$** .

 Diesing & Caprioli, PRL 123 (2019) 071101

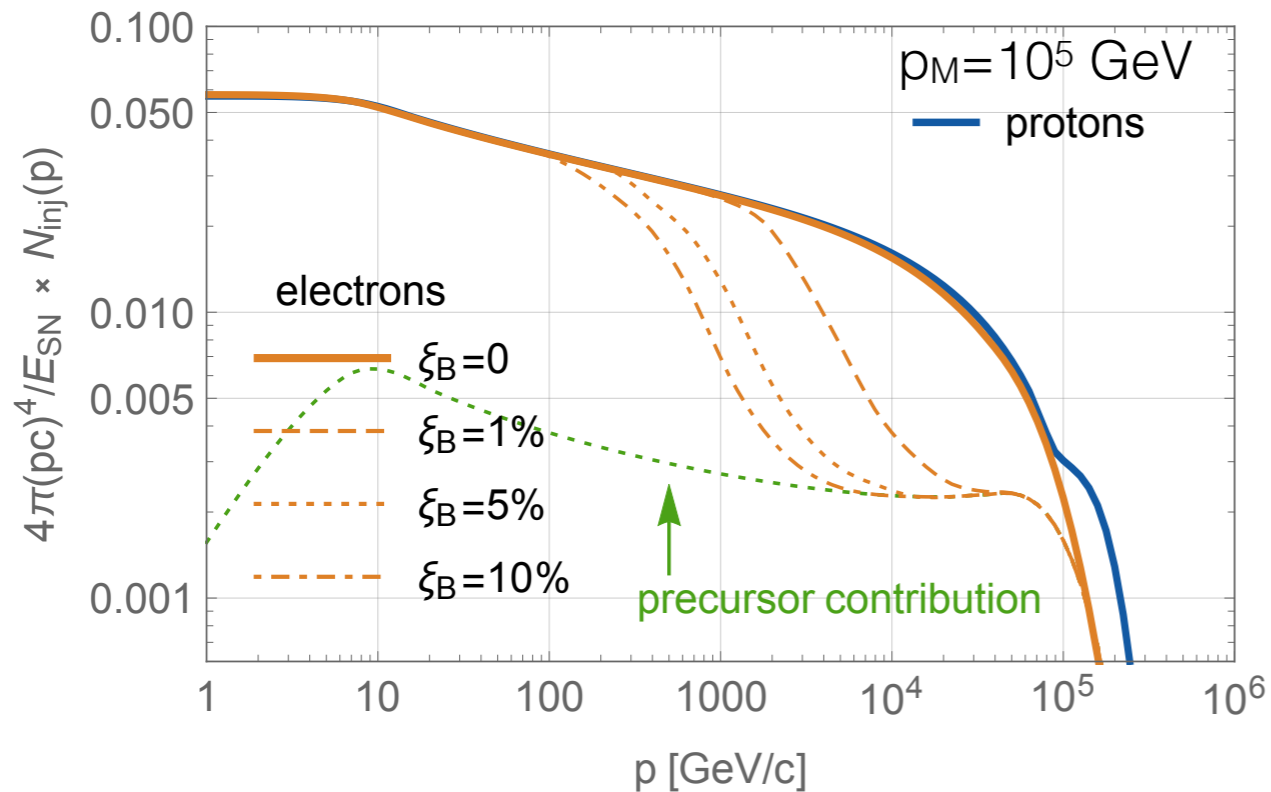
 Brose et al., A&A 634 (2020) 359

 Morlino & Celli, MNRAS 508 (2021) 6142M

 Cristofari, Blasi & Caprioli, A&A 650A (2021) 62C

The CR-e spectrum injected into the Galaxy

2. Turbulent MHD amplification



$$\frac{\delta B_{2,tur}^2}{8\pi} = \xi_B \frac{1}{2} \rho v_{sh}^2$$

 Morlino & Celli, MNRAS 508 (2021) 6142M

- It only affects electron losses **downstream**, not the maximum energy reached at the shock;
- **$\xi_B=1\%$** : efficient losses above 1 TeV, produce a steepening in electron spectrum amounting to **0.8 up to 20 TeV**;
- **→ $\xi_B \gg 10\%$ values required to get steepening down to ~ 10 GeV.**

The CR-e spectrum injected into the Galaxy

3. Time-dependent electron-to-proton injection

$$N_{i,\text{inj}}(p) \simeq \xi_{\text{CR}i} (t_{\text{esc}}(p)) v_{\text{esc}}(p)^2 R_{\text{esc}}(p)^3 p^{-\alpha}$$

$$\longrightarrow \frac{N_{e,\text{inj}}}{N_{p,\text{inj}}} = \frac{\xi_{\text{CR}e}}{\xi_{\text{CR}p}} = v_{\text{esc}}(p)^{-q_k} \propto p^{-3q_k/(5\delta)} \equiv p^{-\Delta s_{ep}}$$

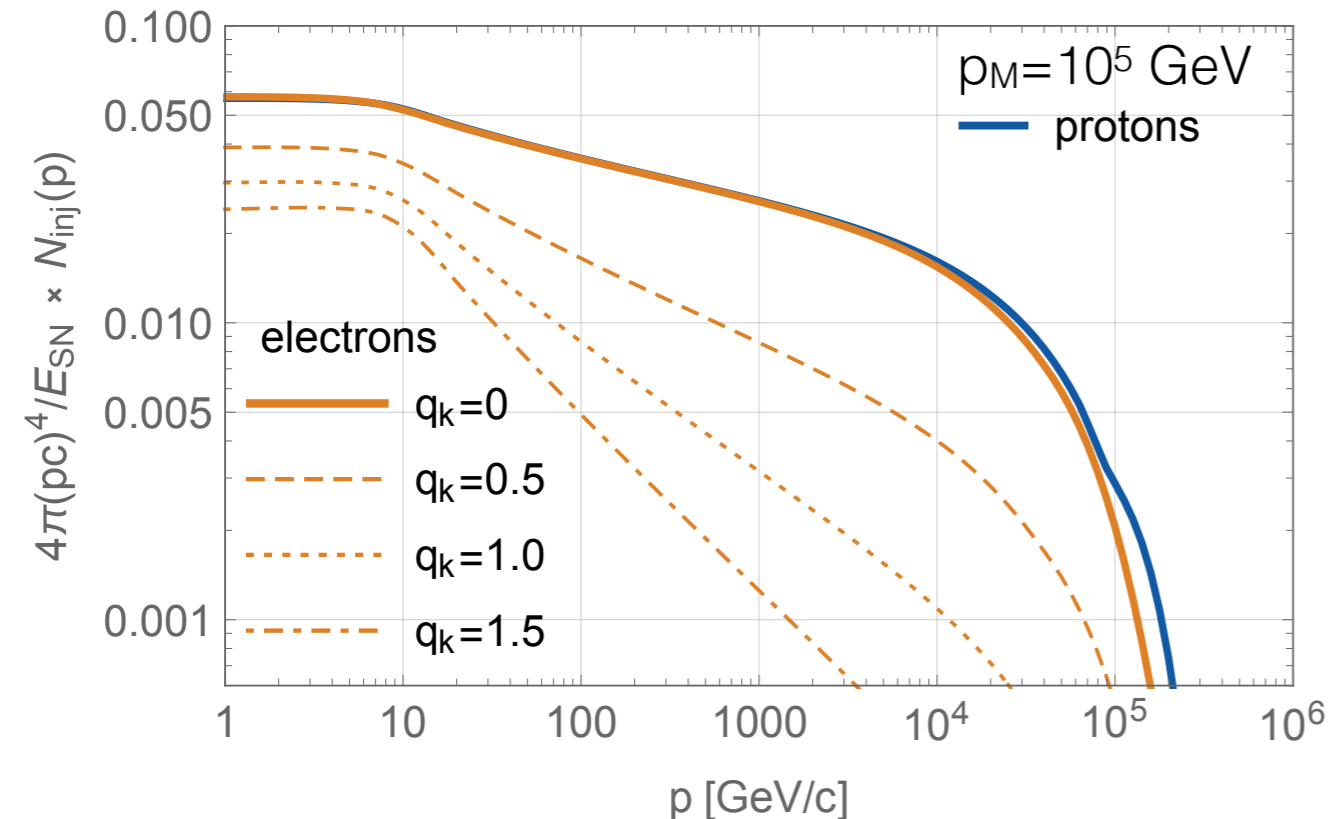
Hp

$$q_k = 5 \delta \Delta s_{ep} / 3$$

- $\Delta s_{ep} \simeq 0.3 \longrightarrow q_k \simeq 1$;
- Steepening down to 10 GeV consistent with observations only if $p_{\text{max}}(t_{\text{SP}}) \leq 10 \text{ GeV}$.



Morlino & Celli, MNRAS 508 (2021) 6142M




On electron and proton injection

Time-dependent electron-to-proton injection might be connected to temperature ratio evolution:

- T_e / T_p estimated from Balmer lines in collisionless shocks propagating in neutral media

$$T_e / T_p \propto v_{sh}^{-2} \quad \text{Van Adelberg et al., ApJ 689 (2008) 1089}$$


also confirmed by PIC simulations up to Mach number ~ 20 .


 Tran & Sironi, ApJ 900 (2020) L26

- Protons are heated by pure randomization of their kinetic energy $\longrightarrow T_p \propto m_p v_{sh}^2$

- Electron temperature is almost constant with shock speed,

$$T_e \sim 0.3 \text{ keV}$$

 Ghavamian et al., ApJ 654 (2007) L69

 Rakowski et al., ApJ 686 (2009) 2195

On electron and proton injection

If electron injection into DSA is connected to their heating, then injection efficiency would scale inversely with v_{sh} .

In fact, electrons and protons develop a non-thermal tail at

$$P_{inj,i} = \xi_i P_{th,i}$$

From PIC simulations, it is found that $\xi_e = \xi_p$



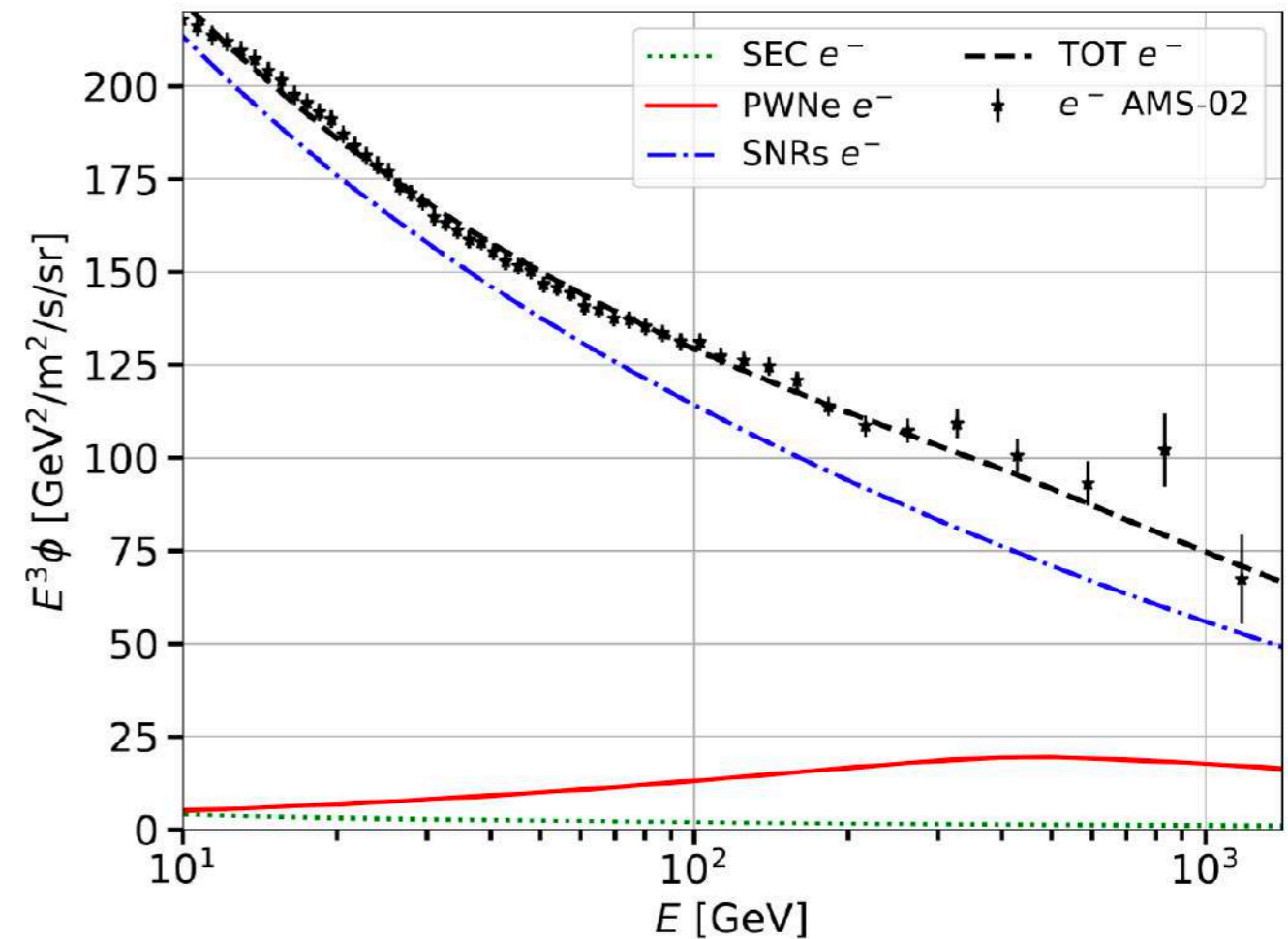
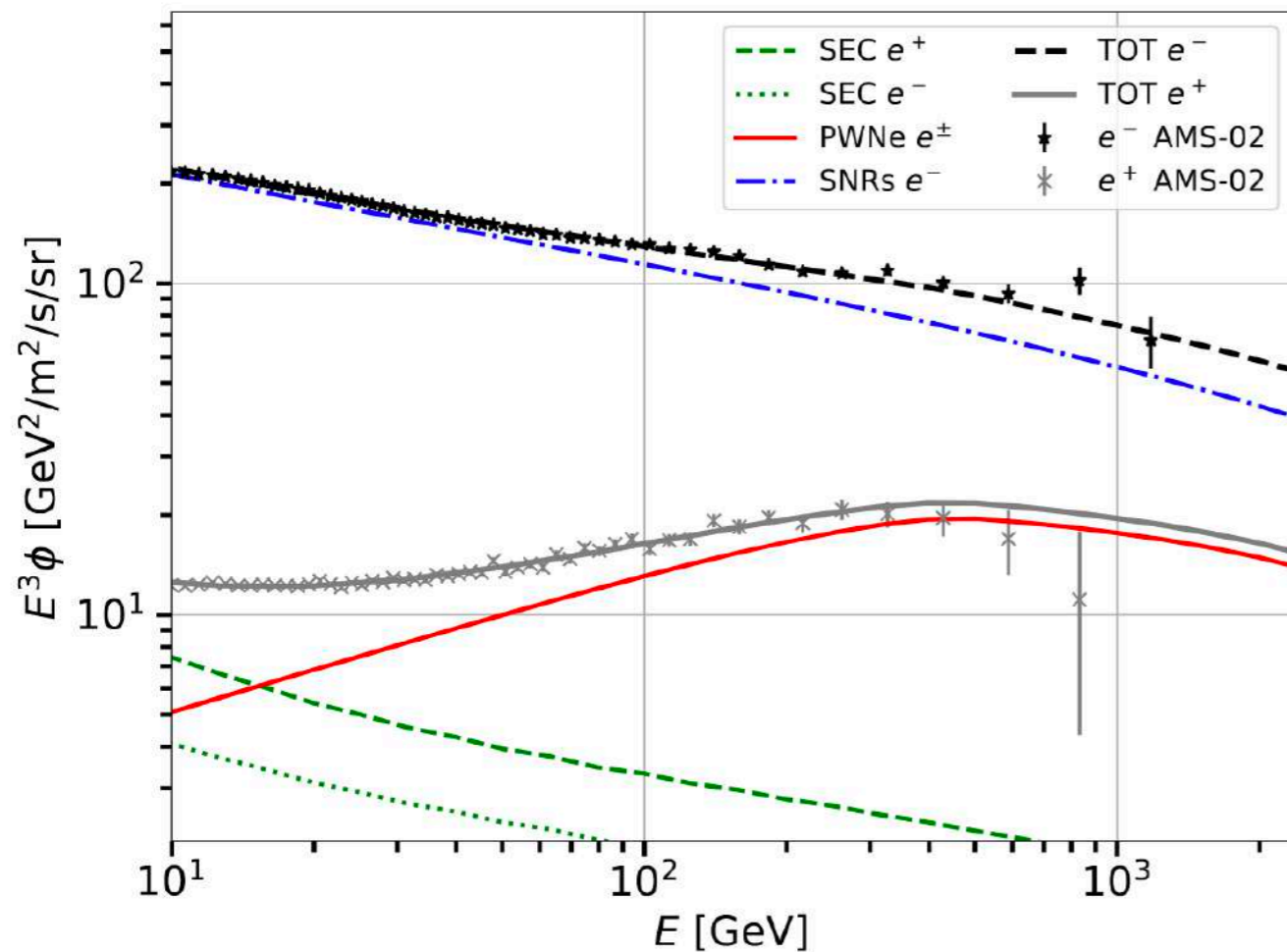
Arbutina & Zekovic,
Astropart. Phys. 127 (2021) 102546

$$K_{ep} \simeq \left(\frac{m_e}{m_p} \frac{\Delta E}{\frac{3}{16} m_p v_{sh}^2} \right)^{\frac{3}{2(R_{sub} - 1)}} \propto v_{sh}^{-\frac{3}{(R_{sub} - 1)}}$$

Hence, for $R_{sub} < 4$, $K_{ep} \sim v_{sh}^{-1}$ and $\Delta s_{ep} \sim 0.3$

The PWN contribution

Break at 40 GeV in electron spectrum also consistent with change in main source contributor, from SNRs to PWNe



Di Mauro et al., PRD 104 (2021) 083012

PWNe contribution is maximal at 500 GeV, ~21%.