

# Radio- $\gamma$ response in blazars as a signature of adiabatic blob expansion: a self-consistent approach

[https://github.com/andreatramacere/adiabatic\\_exp\\_radio\\_gamma\\_delay](https://github.com/andreatramacere/adiabatic_exp_radio_gamma_delay)

<https://jetset.readthedocs.io/en/1.2.2/>

## Andrea Tramacere

Vitalii Sliusar, Roland Walter, Jakub Jurysek, Matteo Balbo

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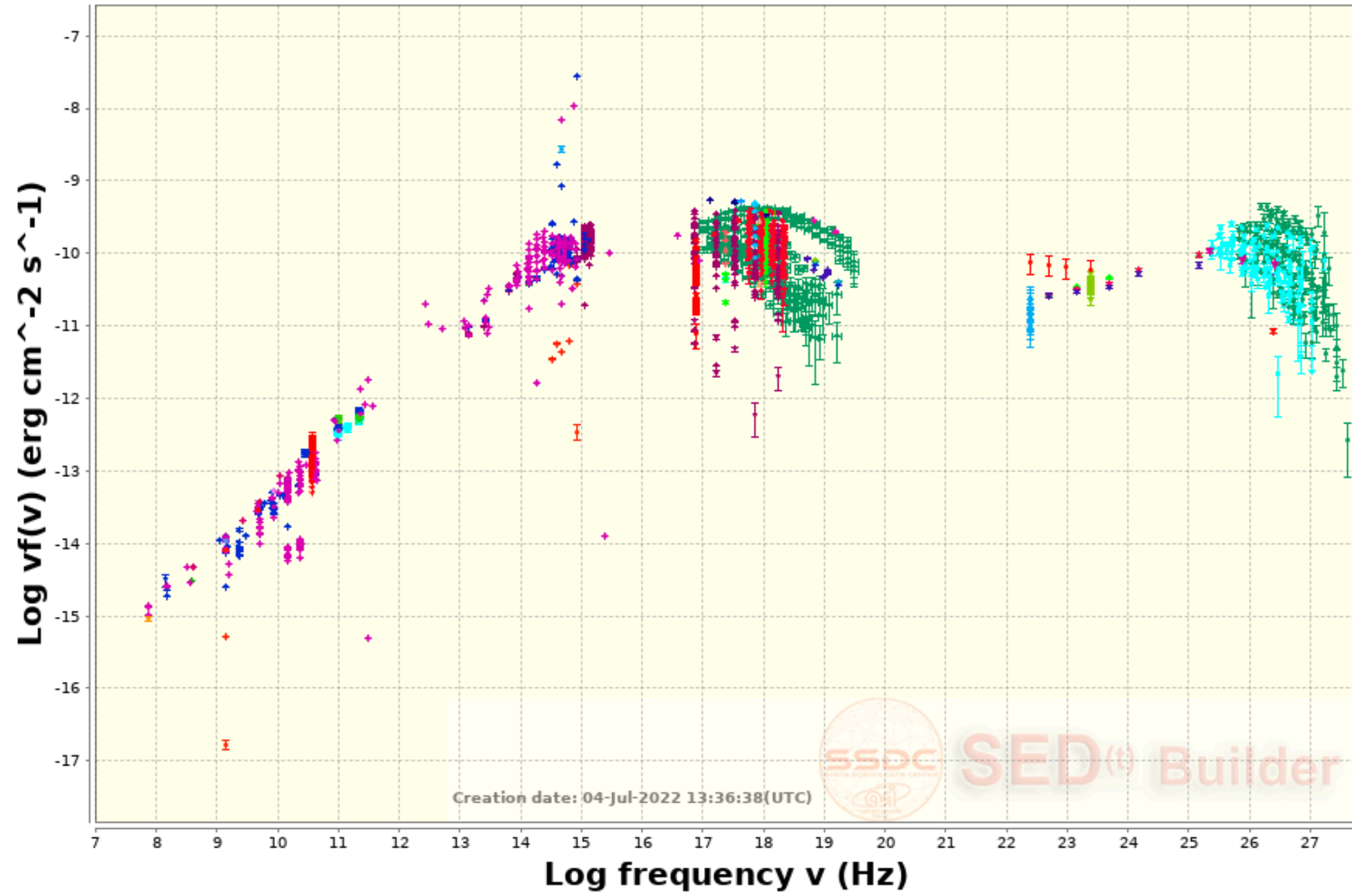
Astronomy  
&  
Astrophysics

Radio- $\gamma$ -ray response in blazars as a signature of adiabatic blob  
expansion

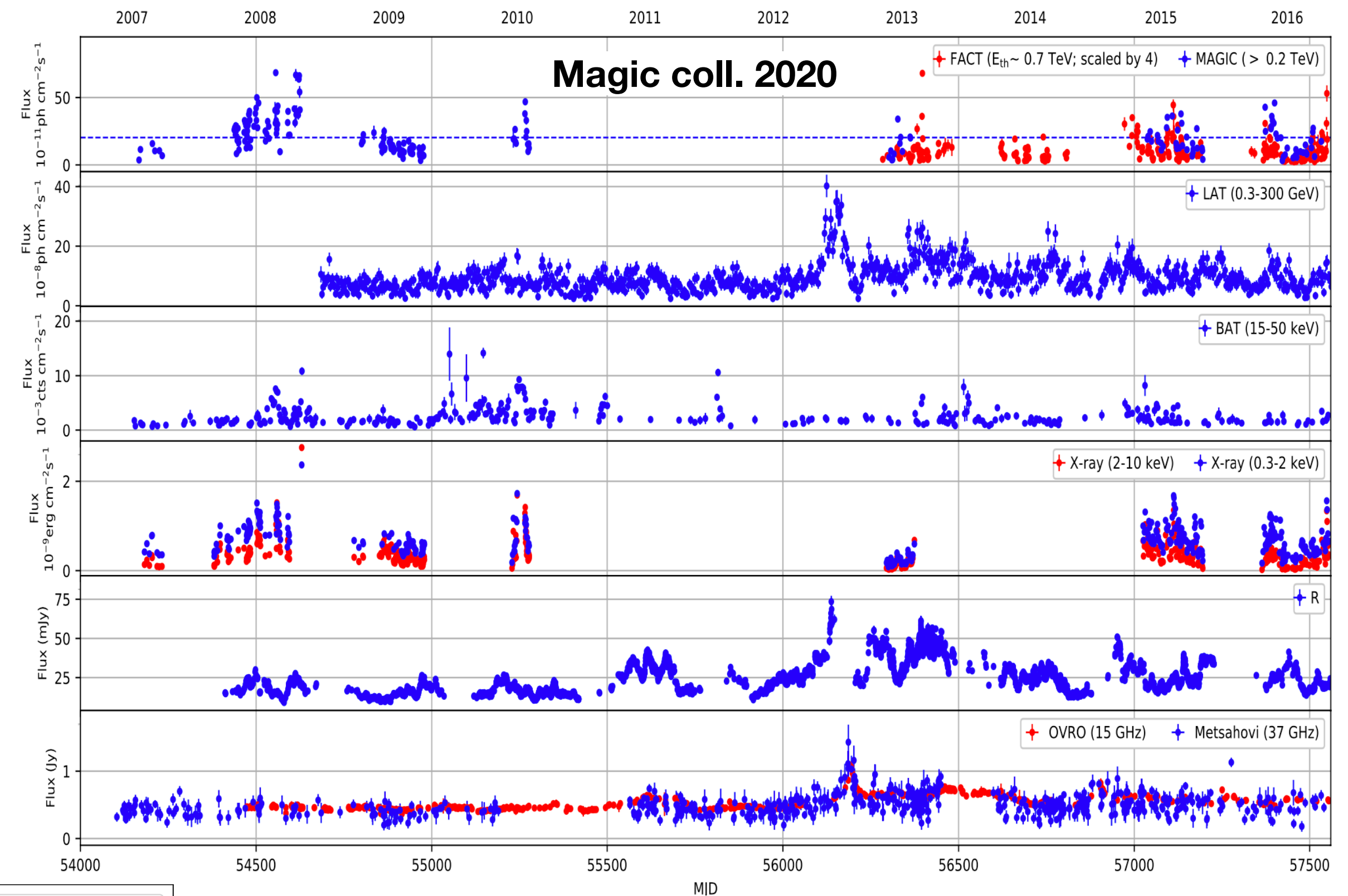
A. Tramacere<sup>✉</sup>, V. Sliusar, R. Walter, J. Jurysek, and M. Balbo



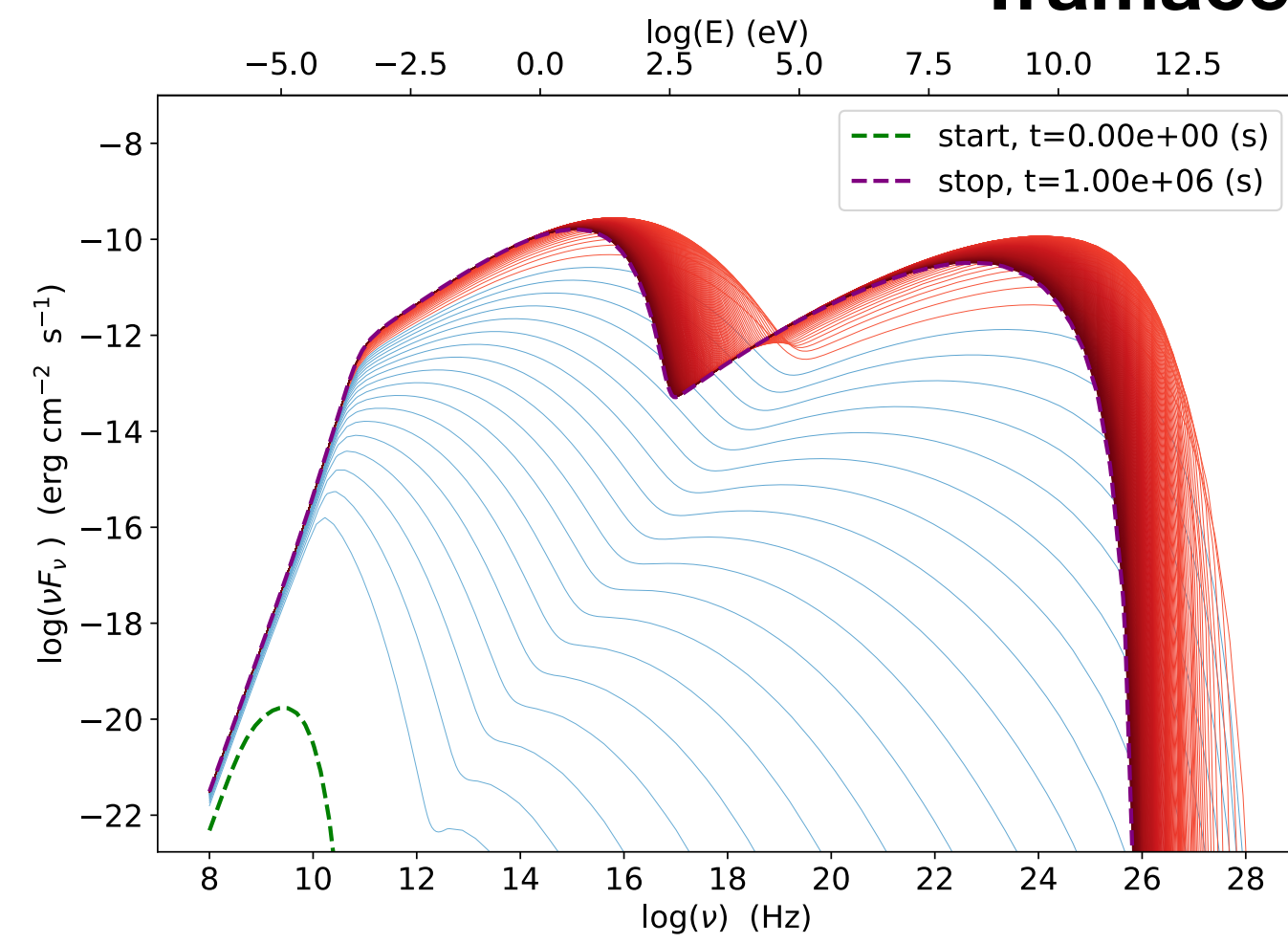
<https://tools.ssdc.asi.it/SED> Mrk 421



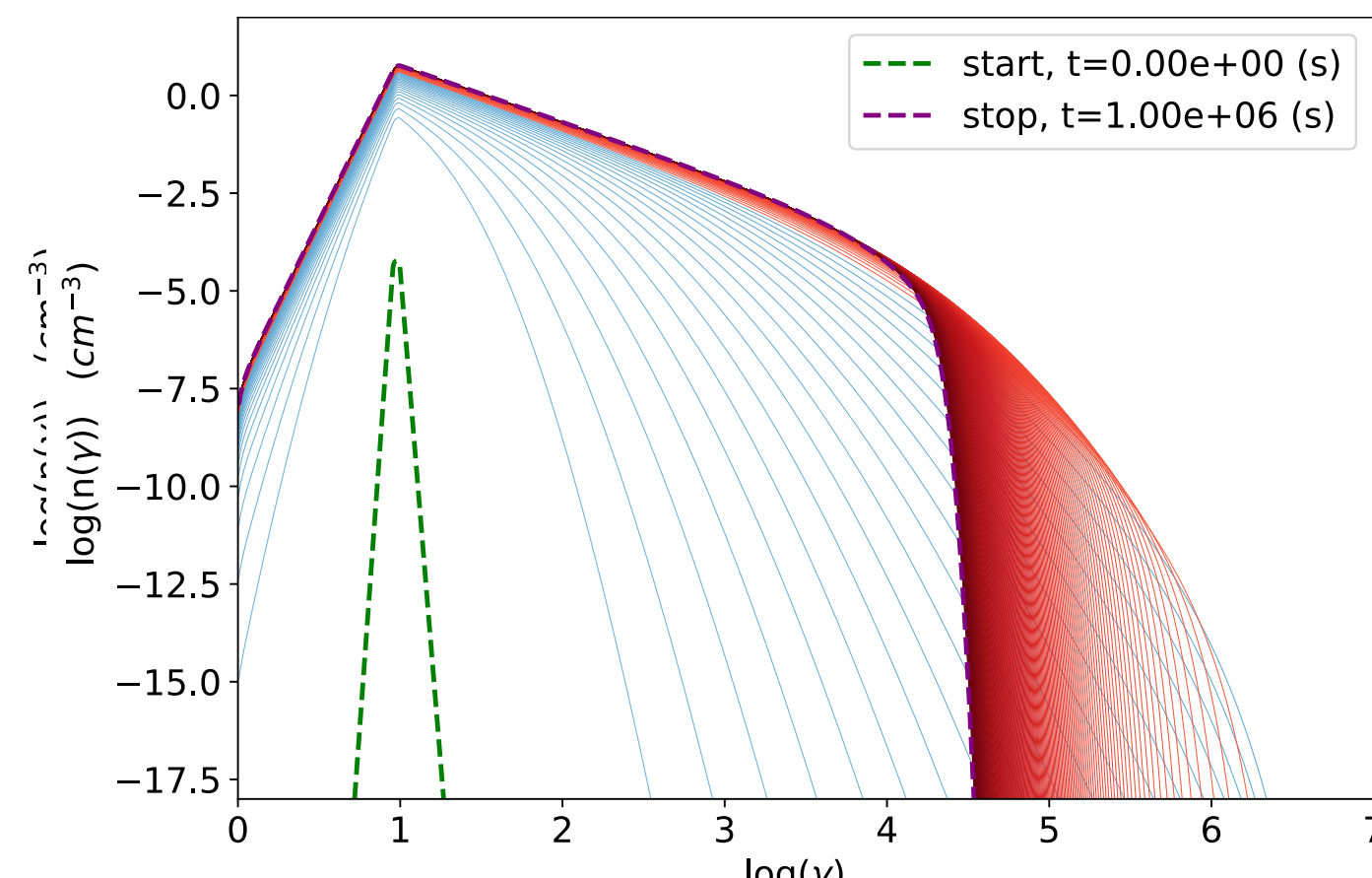
$t_{var}$  depends on  $t_{acc, inj.}$  and properties of radiative process plus geometry



Acc+cooling  
Tramacere+ 2022, 2011

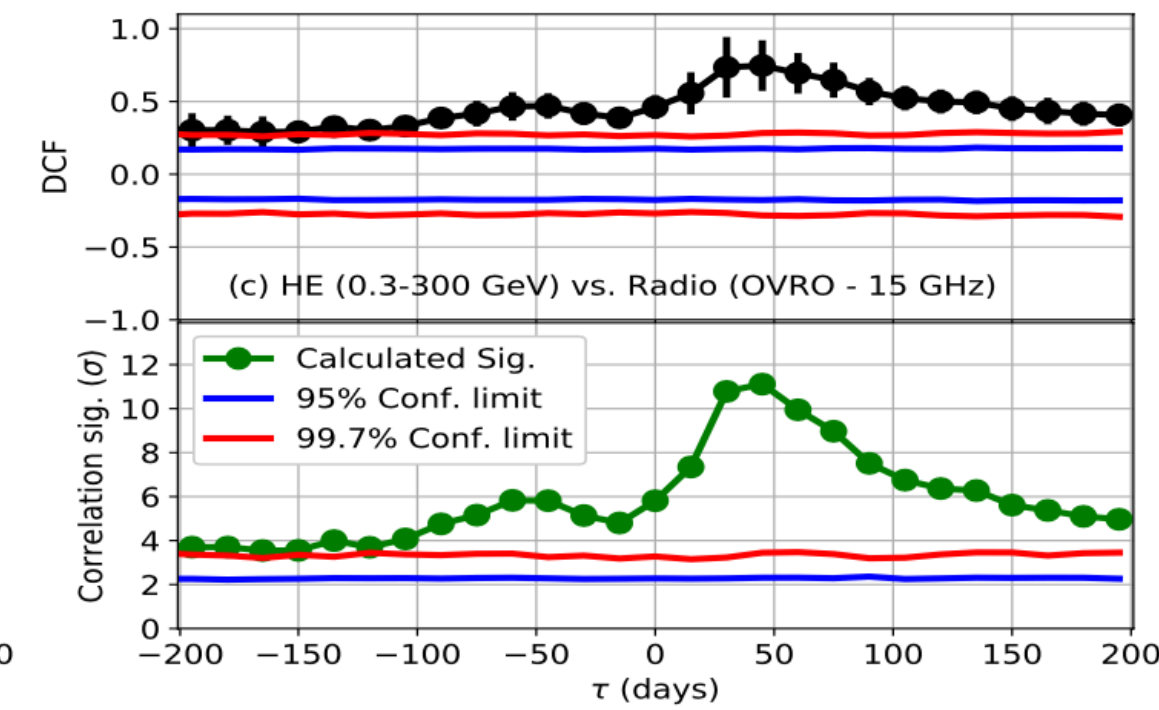
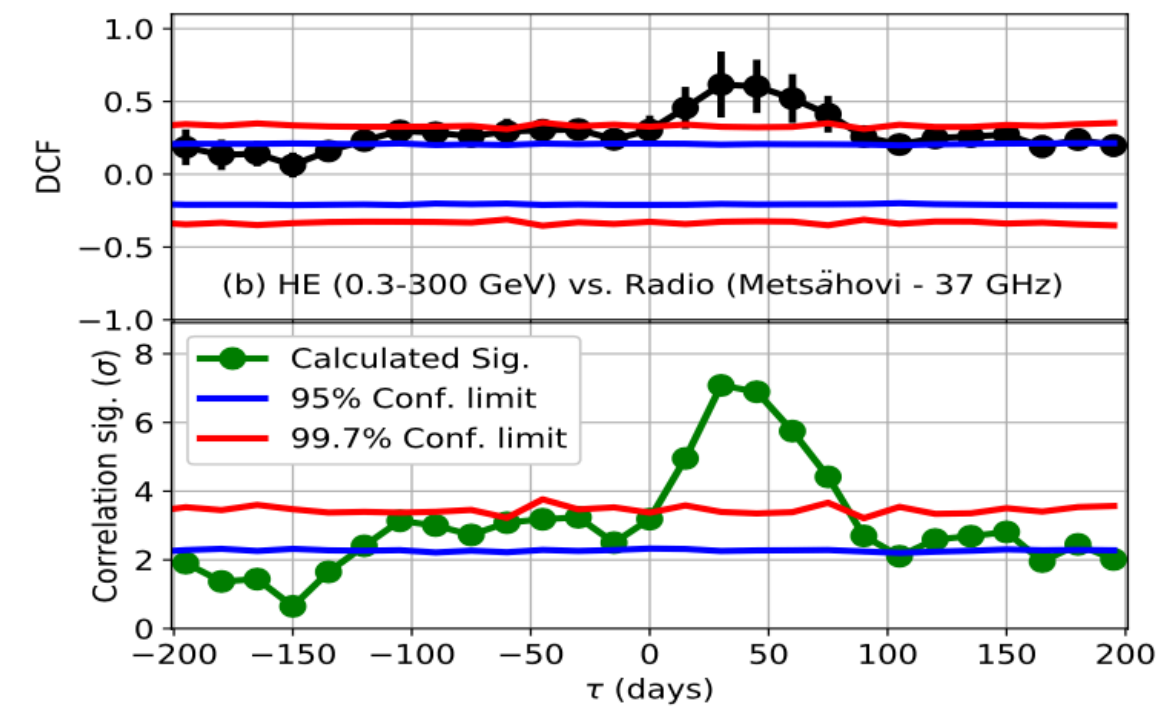
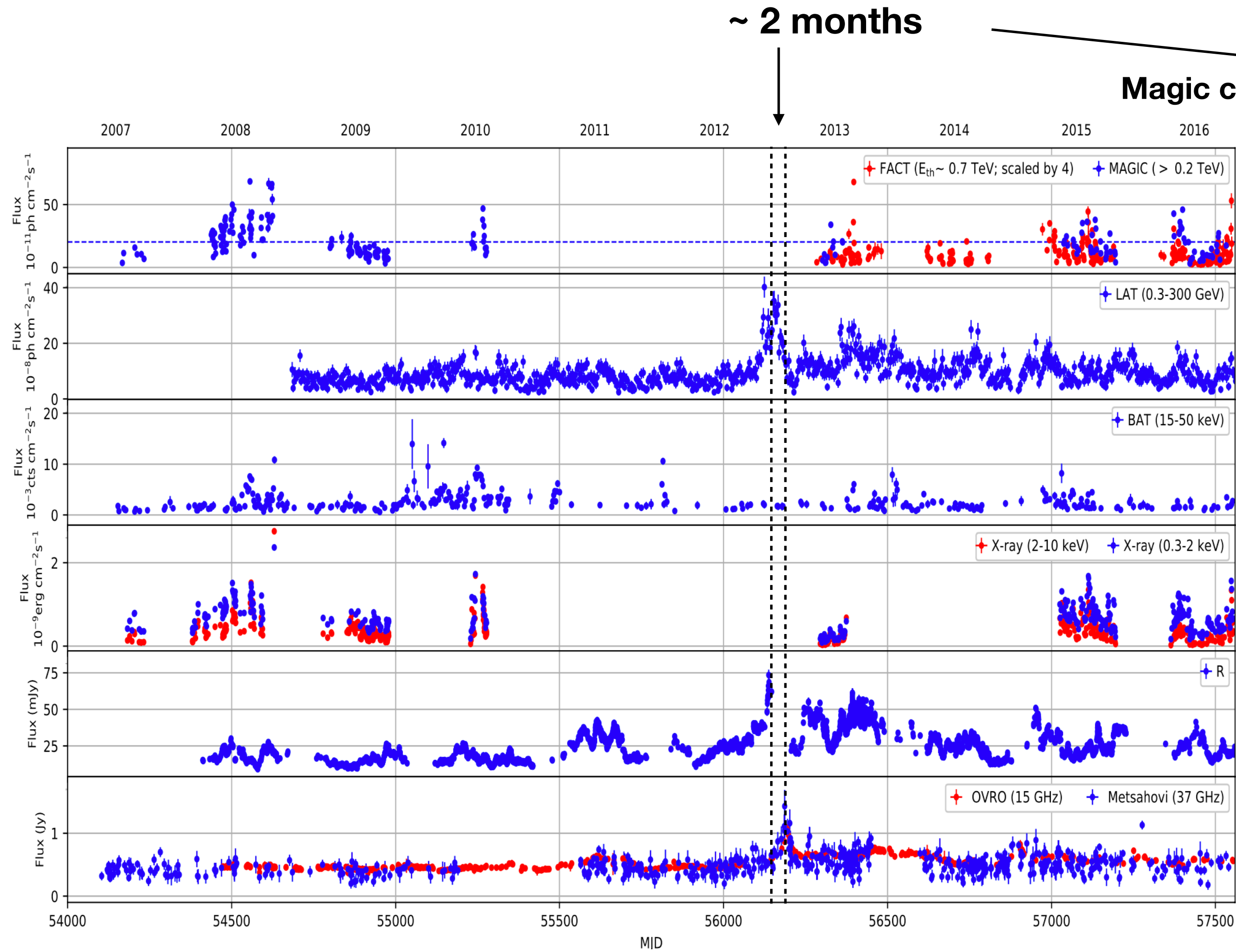


FI+FI



MW variability and correlation studies of Mrk 421 during historically low X-ray and  $\gamma$ -ray activity in 2015-2016

## Radio- $\gamma$ delay in Mrk 421 (months)

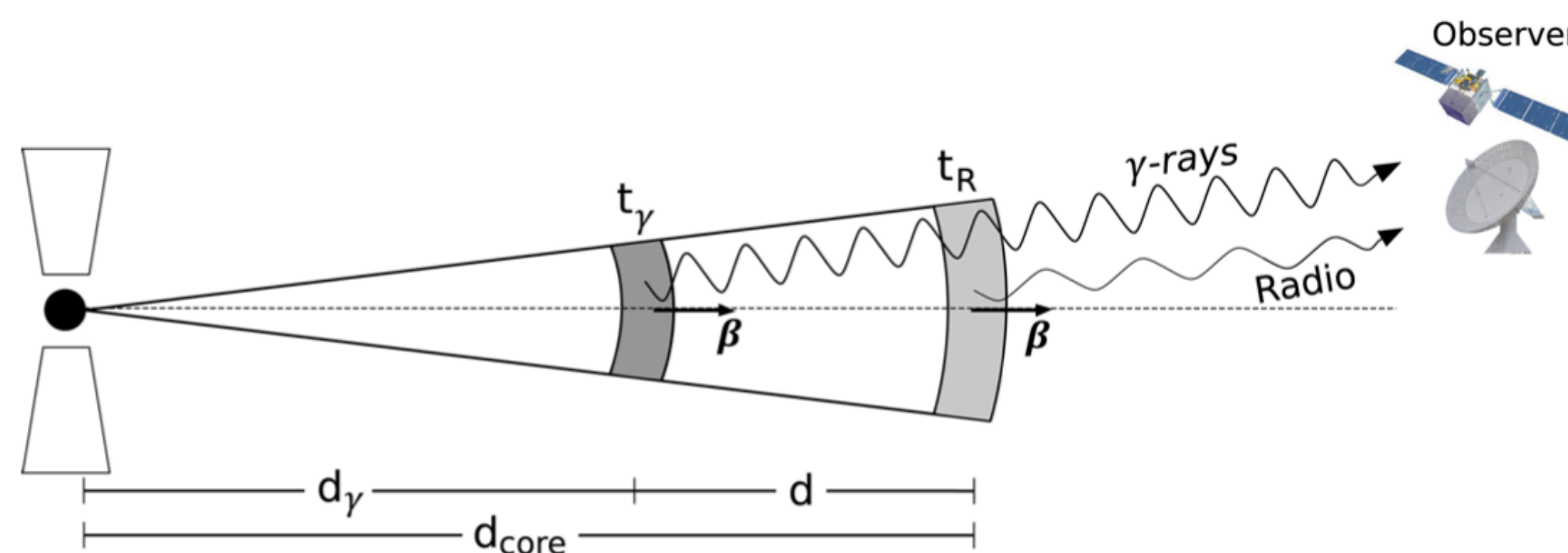


Radio- $\gamma$  delay  $\sim 1/\nu$



- Observed lags are not compatible with cooling, acc., crossing (unless strong fine tuning)
- Explanations based on reacceleration, would be challenging due to MW observations

**We want to test if it is possible to reproduce a radio- $\gamma$  due ( $T \gg d$ ) due to blob expansion**

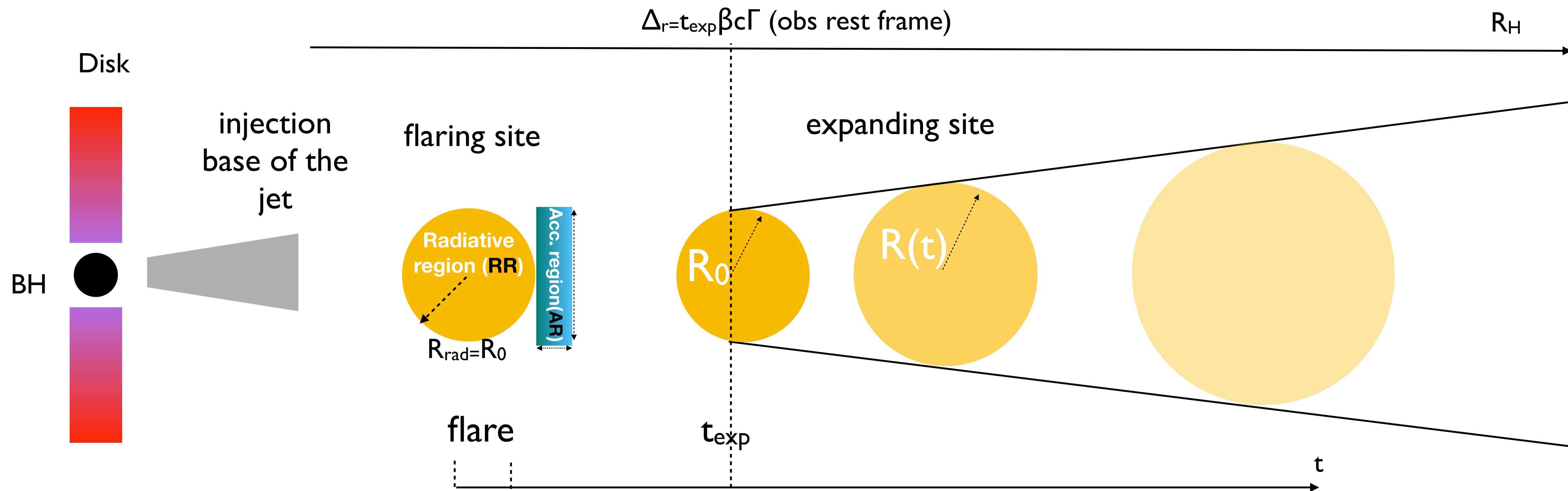


**W. Max-Moerbeck+ 2014**

**B. Pushkarev+ 2010**

**McCray, R. 1968**

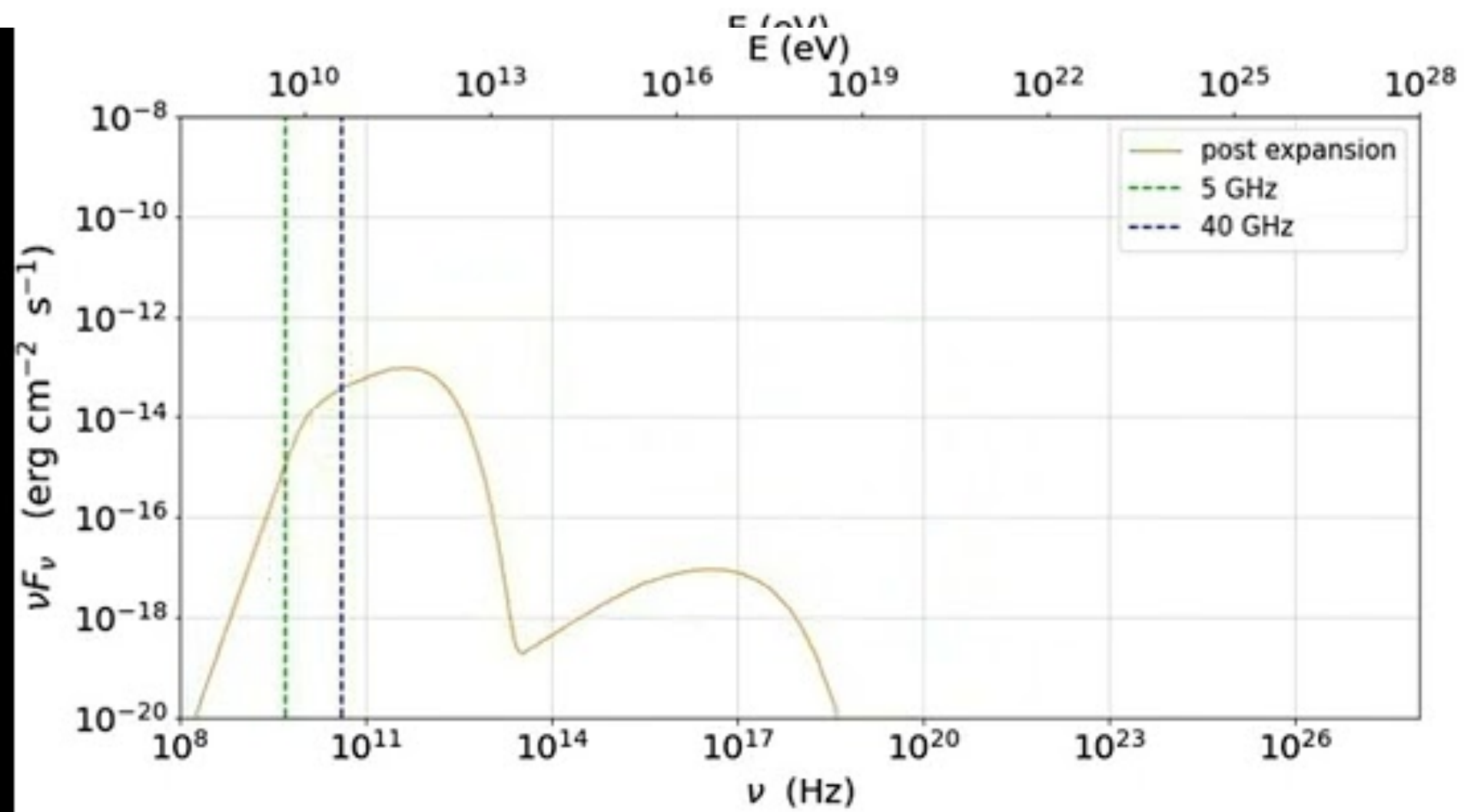




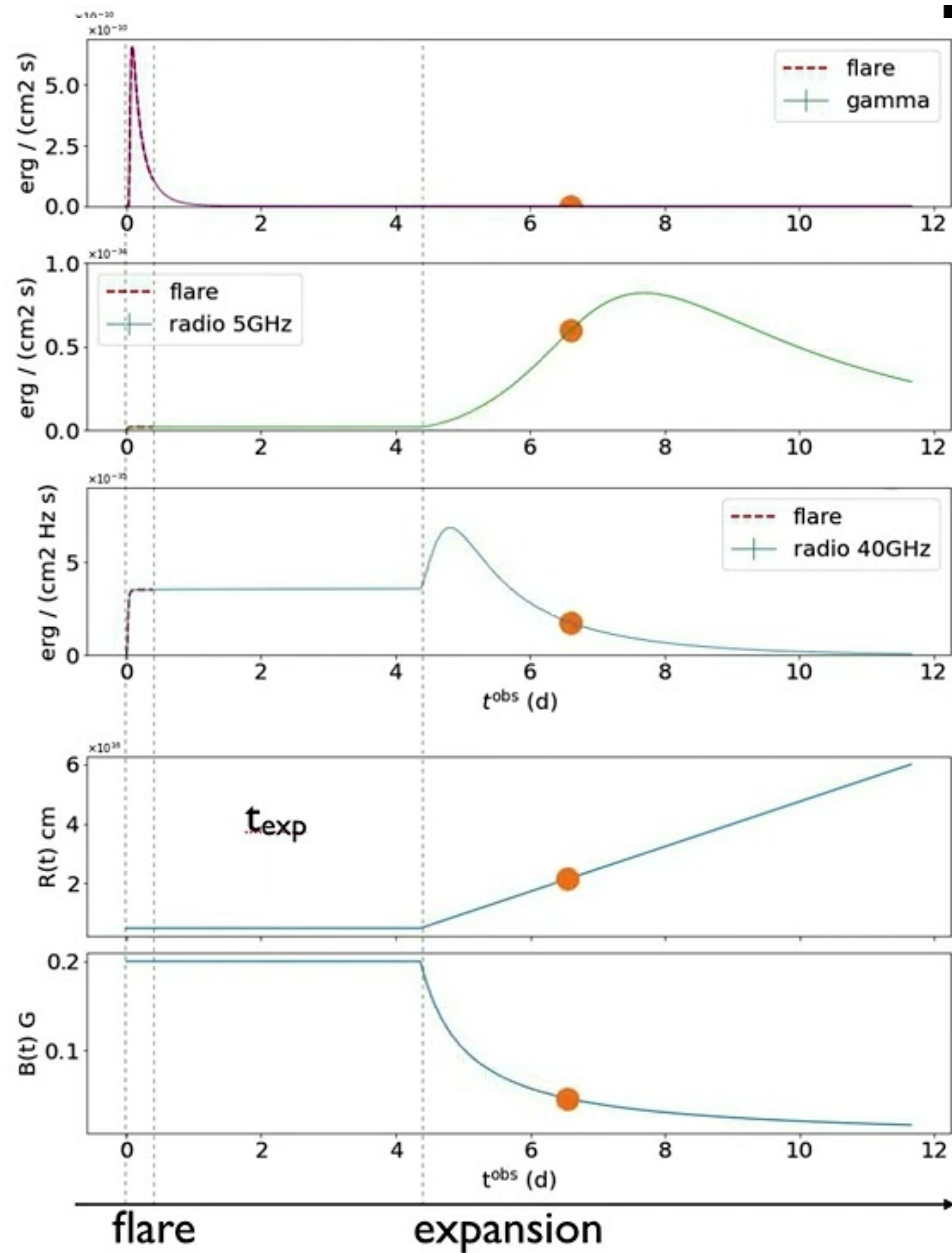
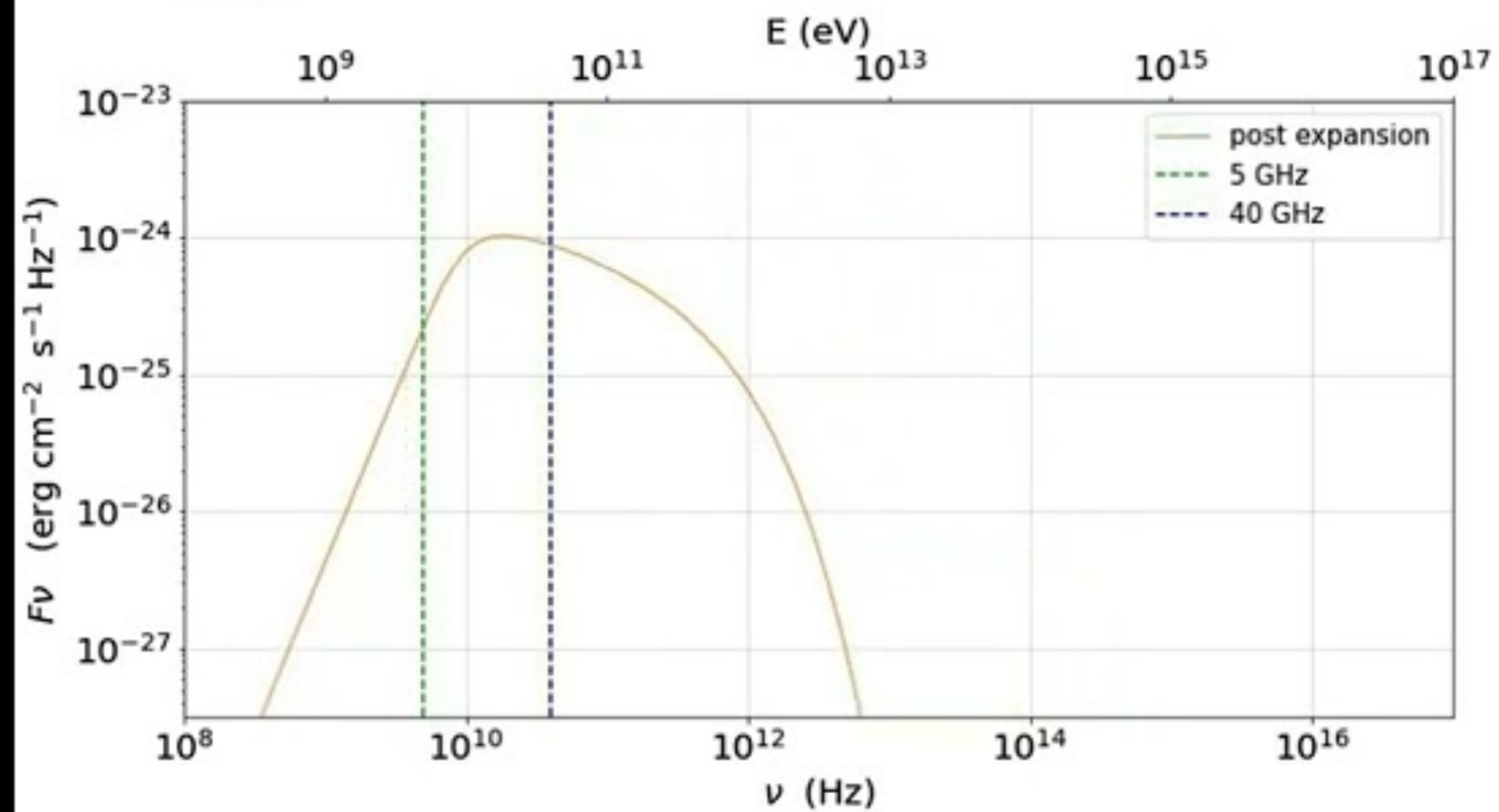
**Numerical solution of FP equation taking into account:**

- **FI+FI**(first order and stochastic acceleration)
- **Radiative cooling: Sync+IC(SSC)**
- **Adiabatic expansion/cooling**



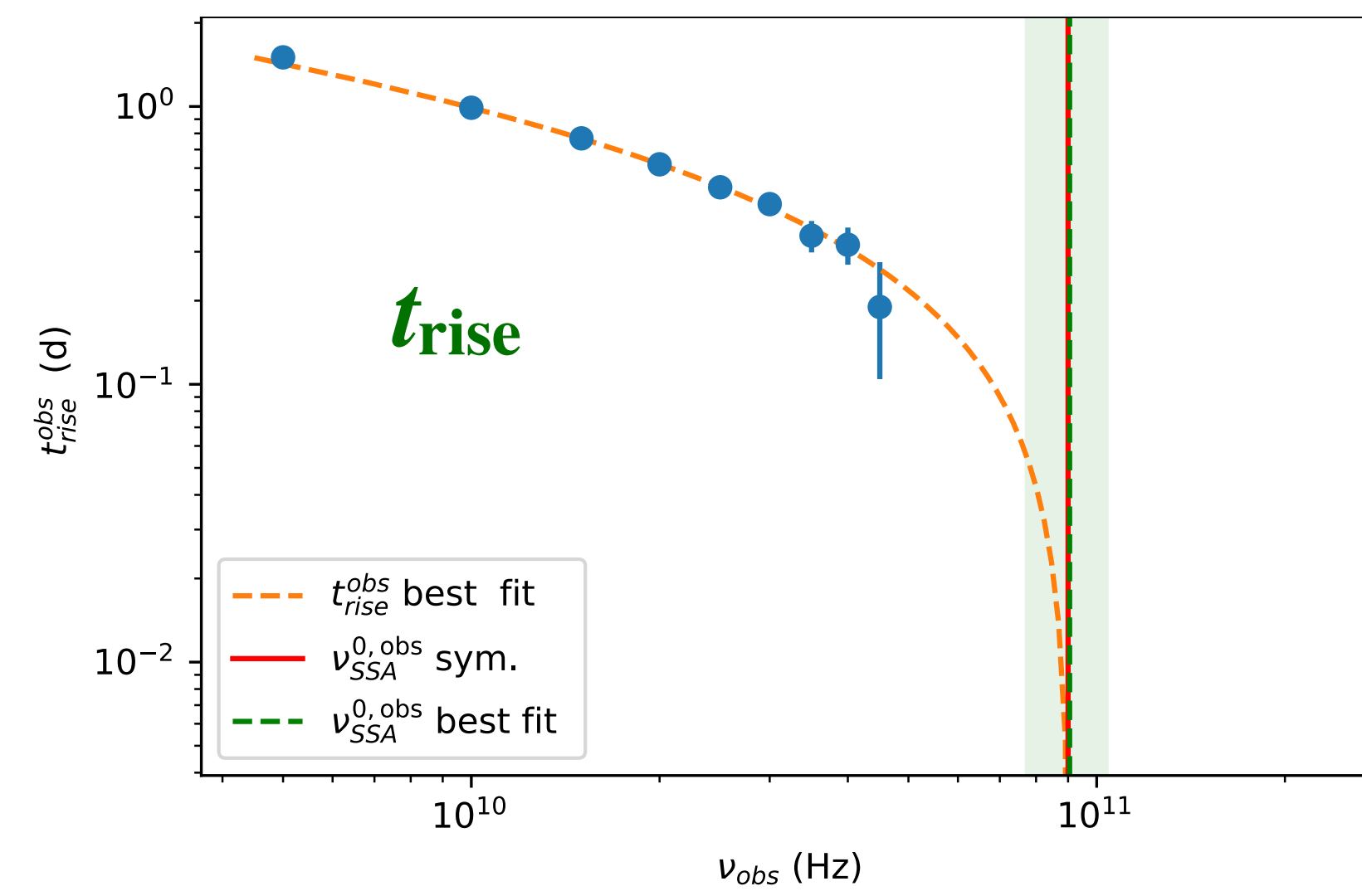
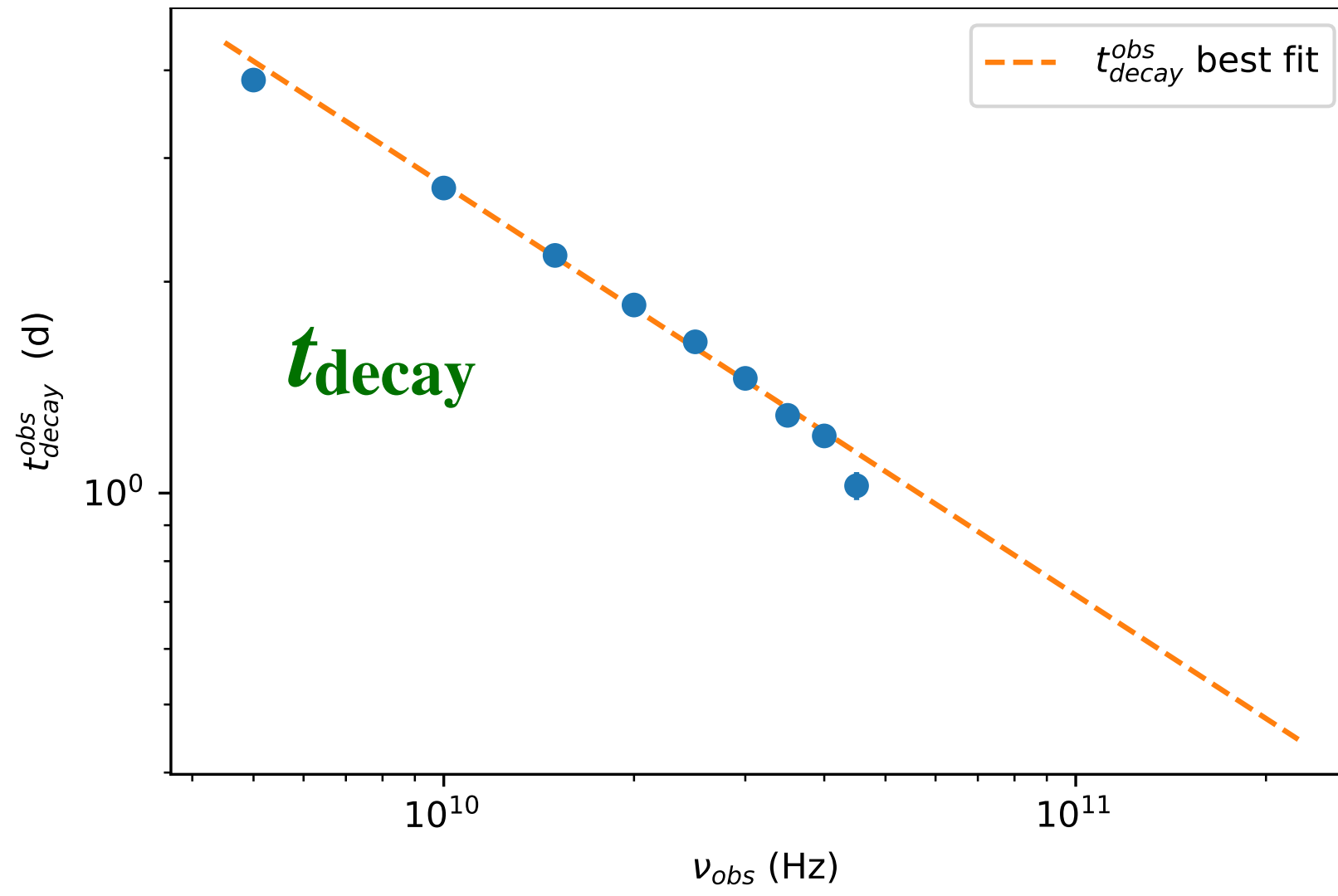


- duration  $\sim 3 \times 10^7$  s (blob frame)  $\sim 11$  d obs
- $t_{\text{exp}} = 1 \times 10^7$  s
- $\beta_{\text{exp}} = 0.1c$



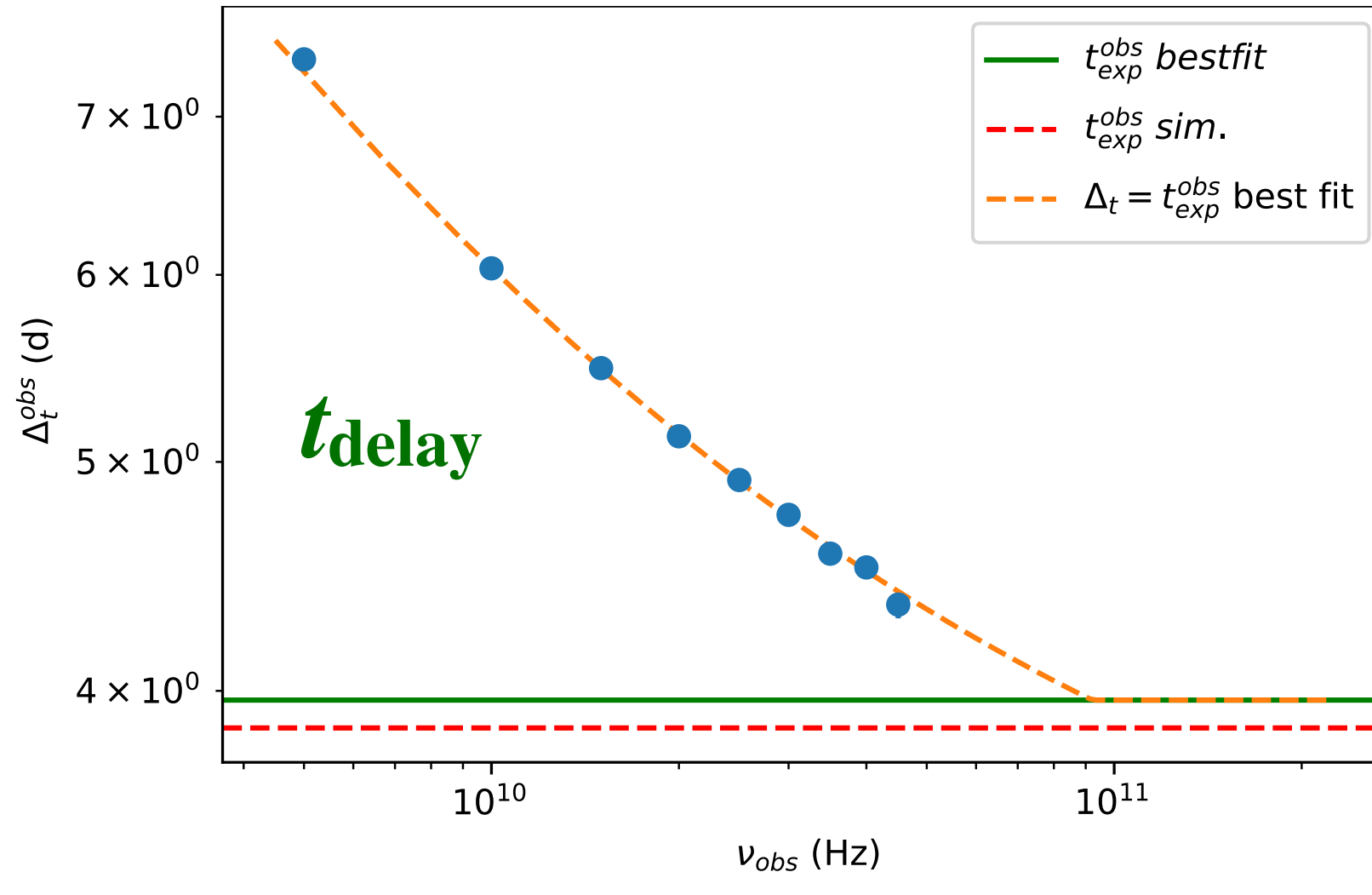


Tramacere+ 2022



$\beta_{exp}=[0.1]$

- results from simulations
- phenomenological trend



$$t_{[rise, decay, delay]} = f(m_B, \beta_{exp}, \nu_{ssa}, B, R, p)$$

link the **observables** to **physics**

$$\nu_{SSA}^0 / \nu_{SSA}^{obs}$$

$$t_{[rise, decay, delay]} \sim$$

$$1 / \beta_{exp}$$

$$R(t) = R_0 + \beta_{exp} c(t - t_{exp}) H(t - t_{exp})$$

$$\nu_{SSA}(t) = \nu_L(t) \left[ \frac{\pi \sqrt{\pi}}{4} \frac{e R(t) N(t)}{B(t)} f_k(p) \right]^{\frac{2}{p+4}}$$

$m_B=[1,2]$  (tor., pol.)  $B(t) = B_0 \left( \frac{R_0}{R(t)} \right)^{m_B}$



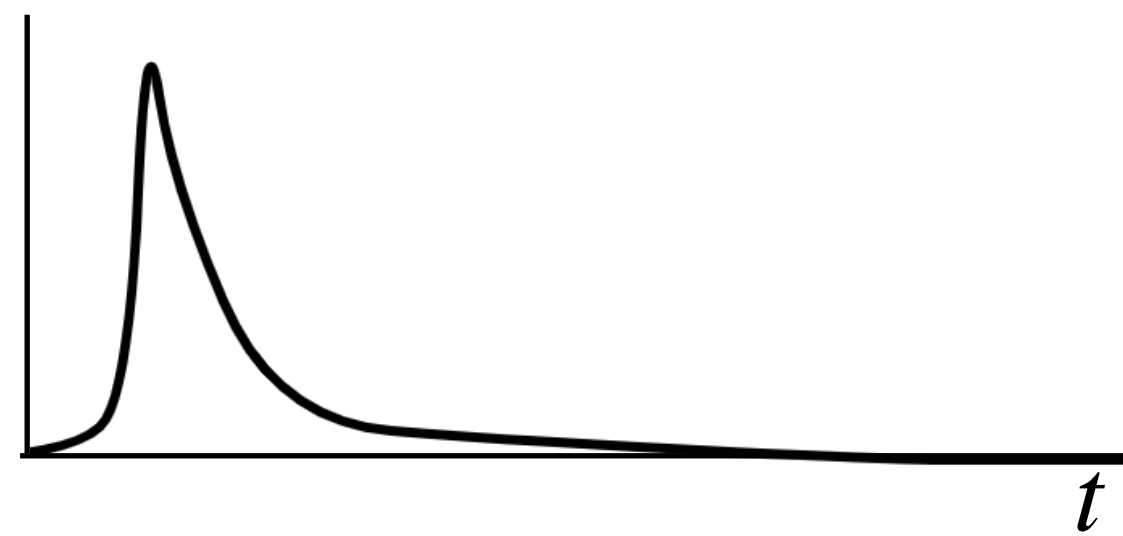
✓ **Numerical self-consistent simulations reproduce the radio-gamma delay due to adiabatic expansion!**

✓ **phenomenological trends, derived for adiabatic expansion match numerical simulation!**

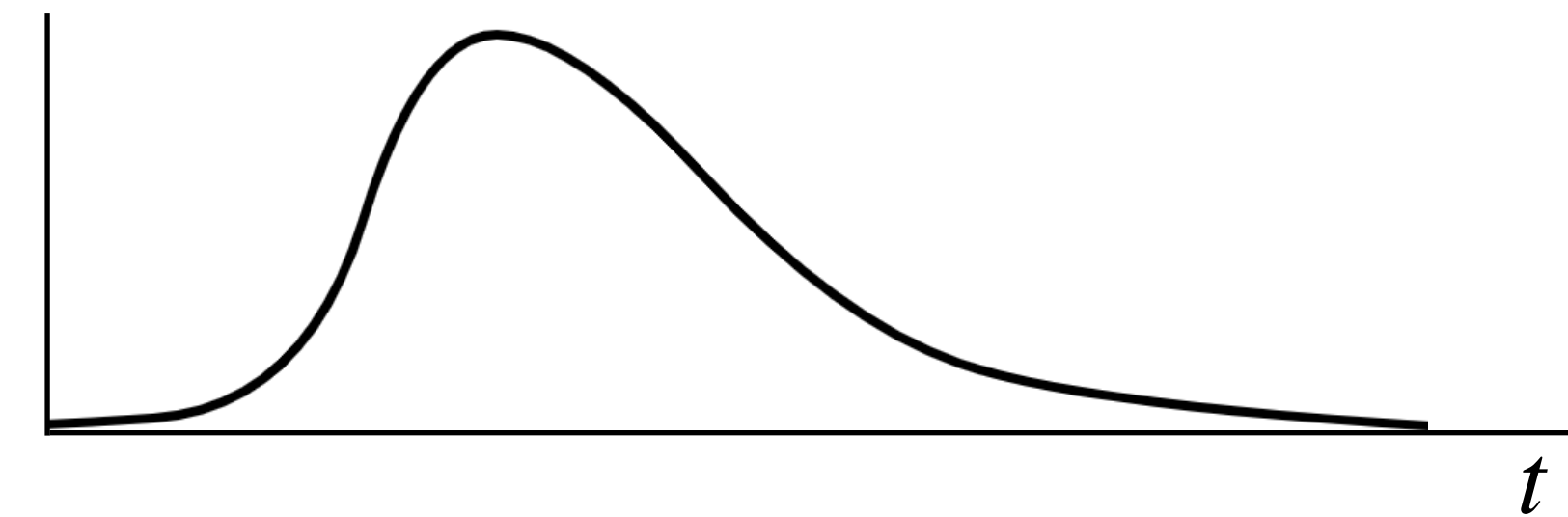
**let's extract some physics from the data!**

**single flare fit or....**

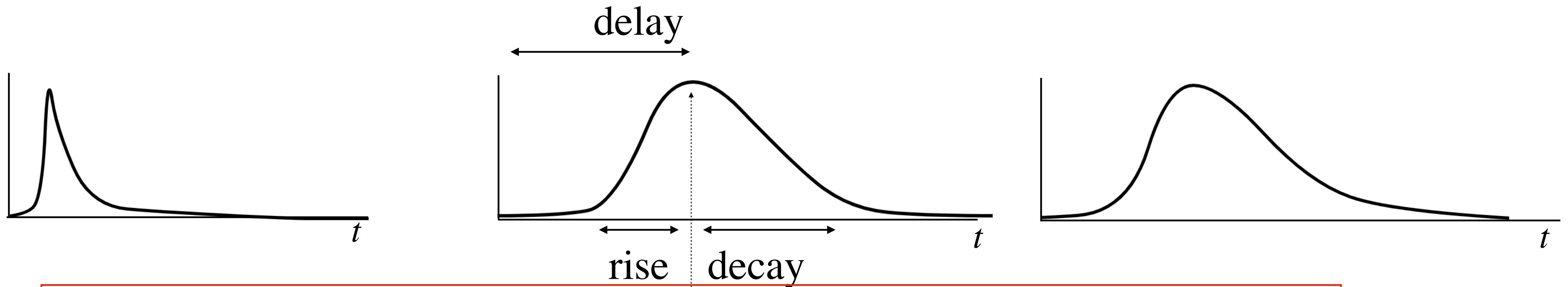


 $l_\gamma(t)$ 

phenomenology  
links the  
**observables** to  
**physics**

 $t$  $l_R(t)$

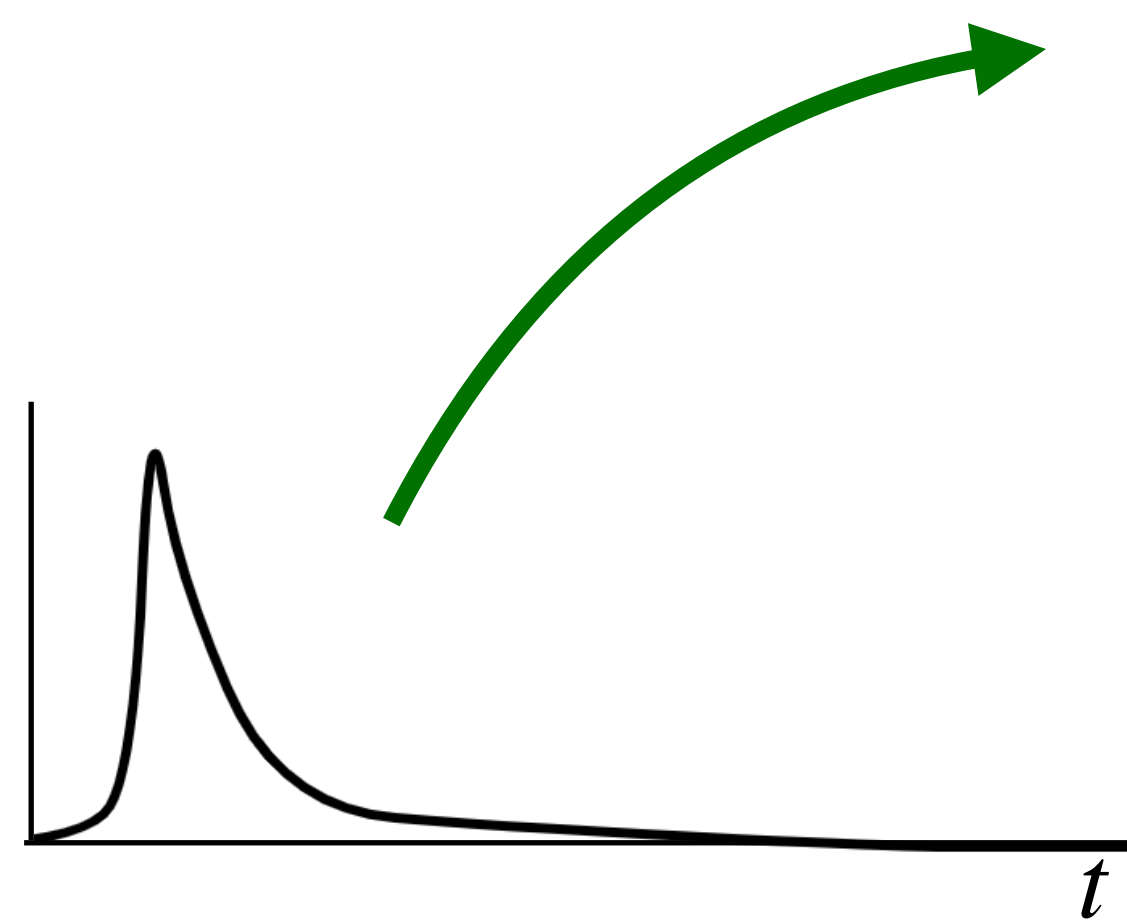




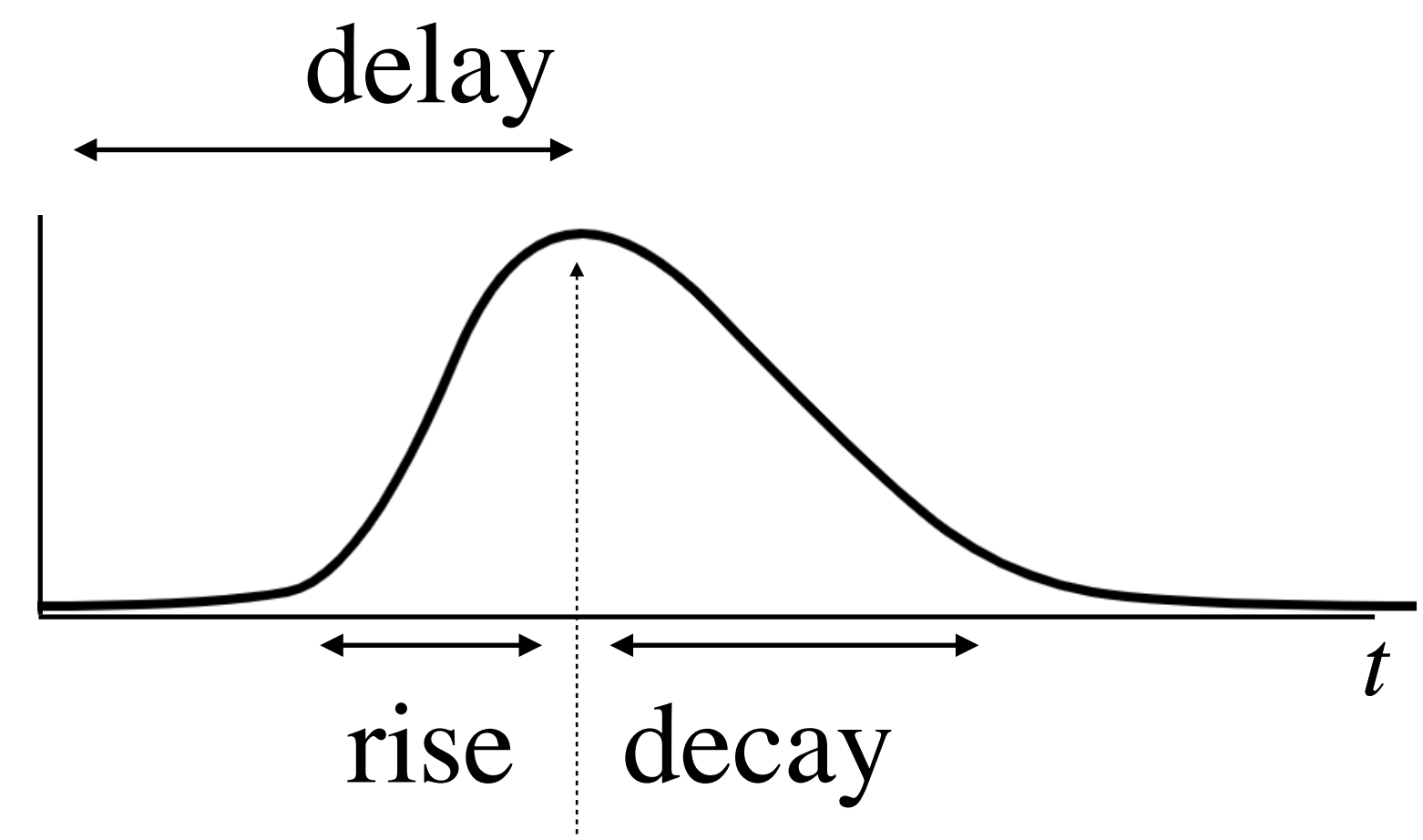
$$l_{\gamma}(t) * S(t, t_{\text{rise}}, t_{\text{decay}}, t_{\text{delay}}) = l_R(t)$$



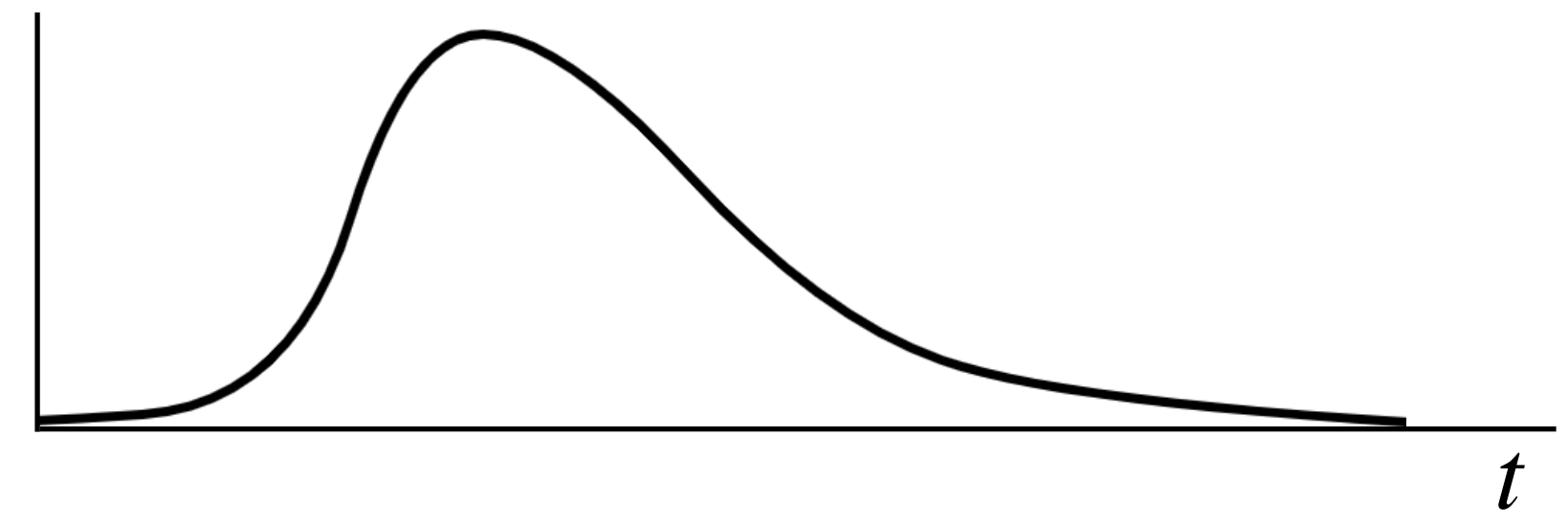
optimise  $S$  to match  $l_R$



$l_\gamma(t)$

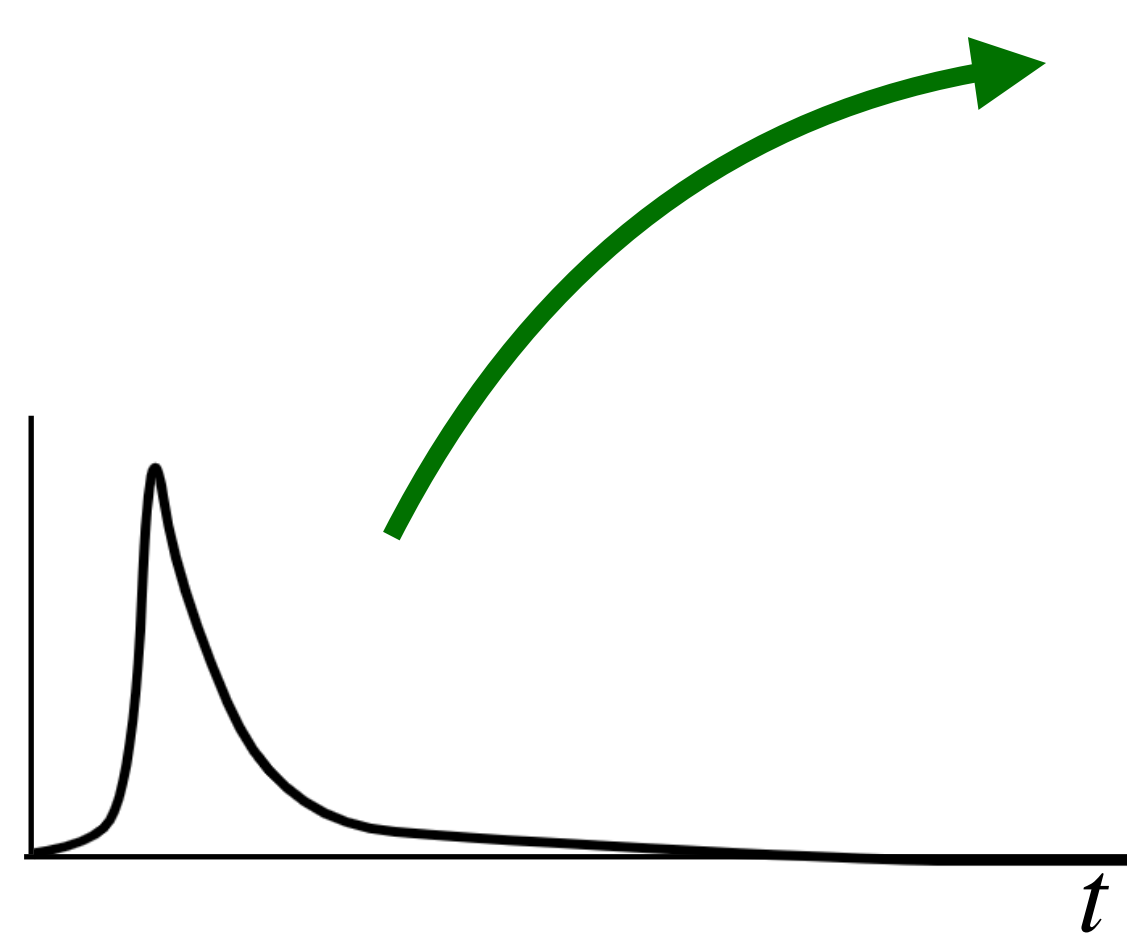


$$* S(t, t_{rise}, t_{decay}, t_{delay}) = l_R(t)$$



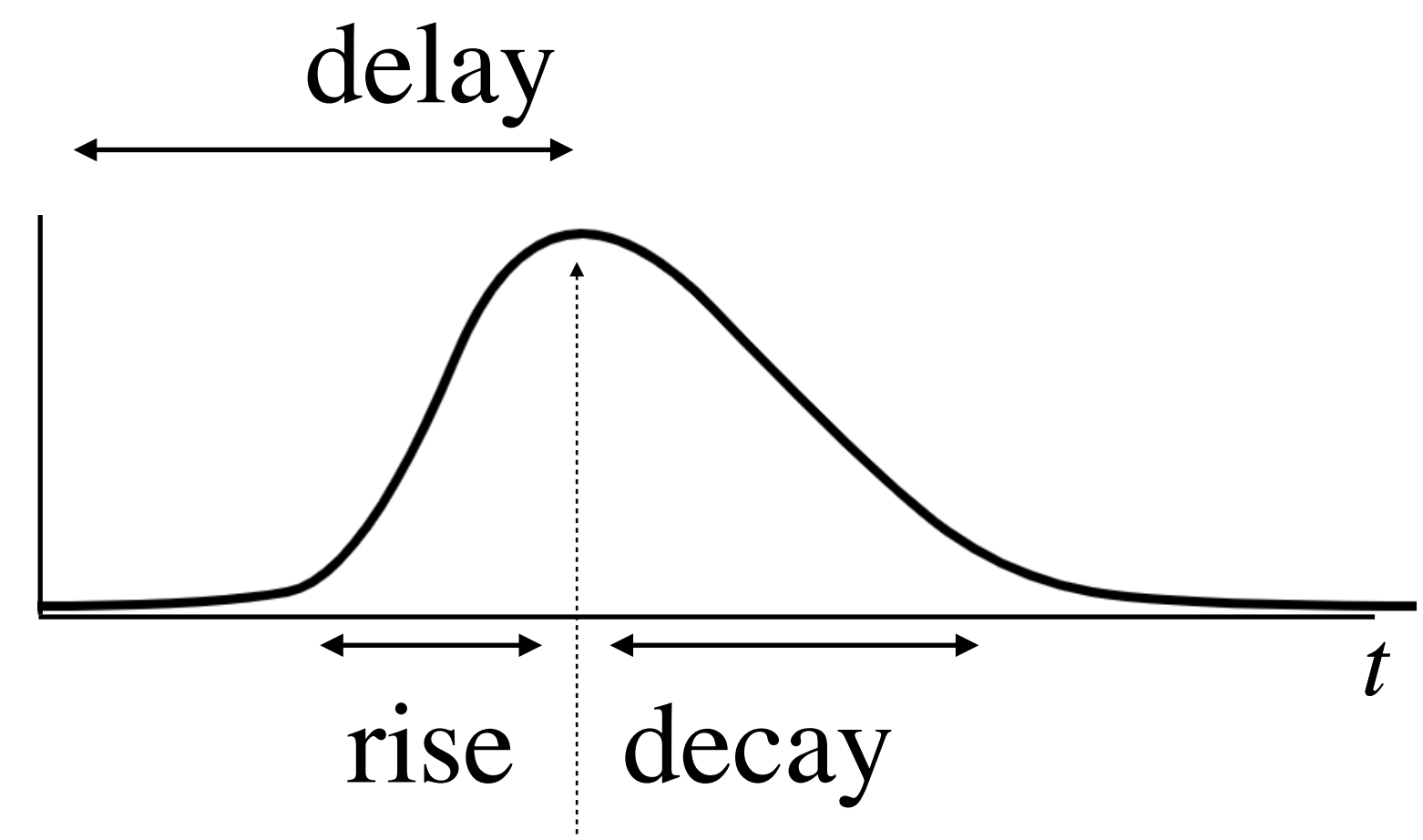
$l_R(t)$



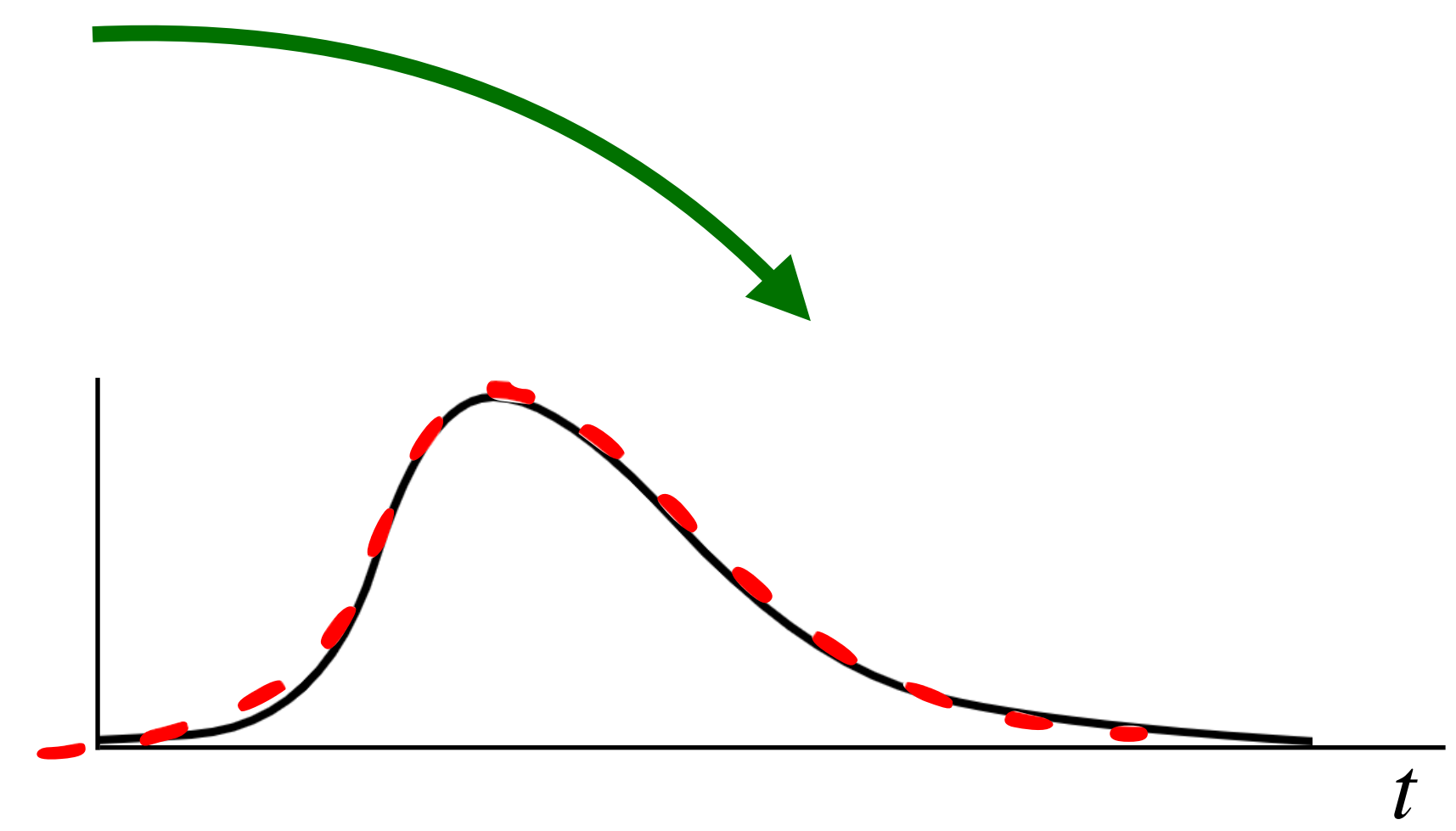


$l_\gamma(t)$

optimise  $S$  to match  $l_R$



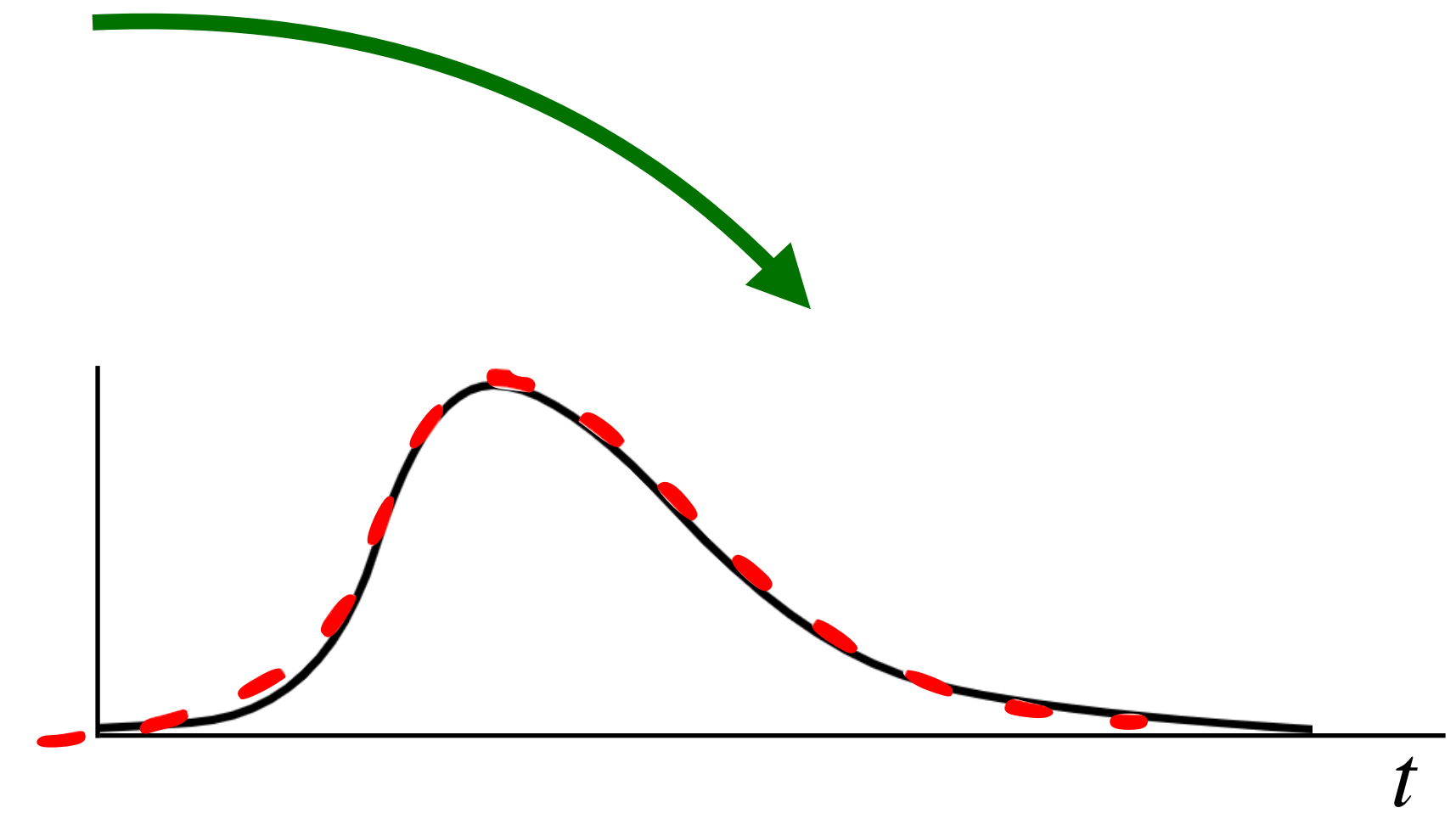
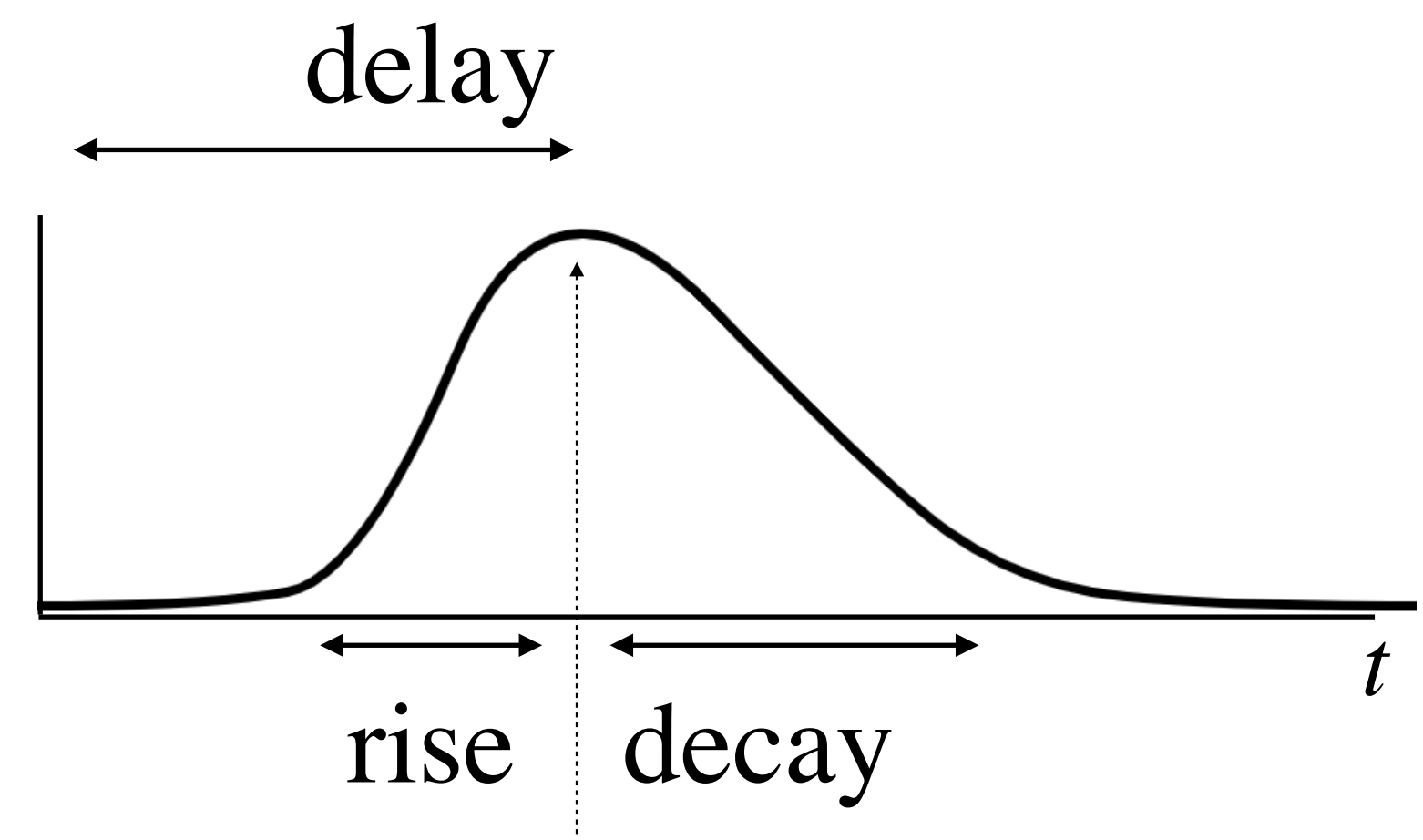
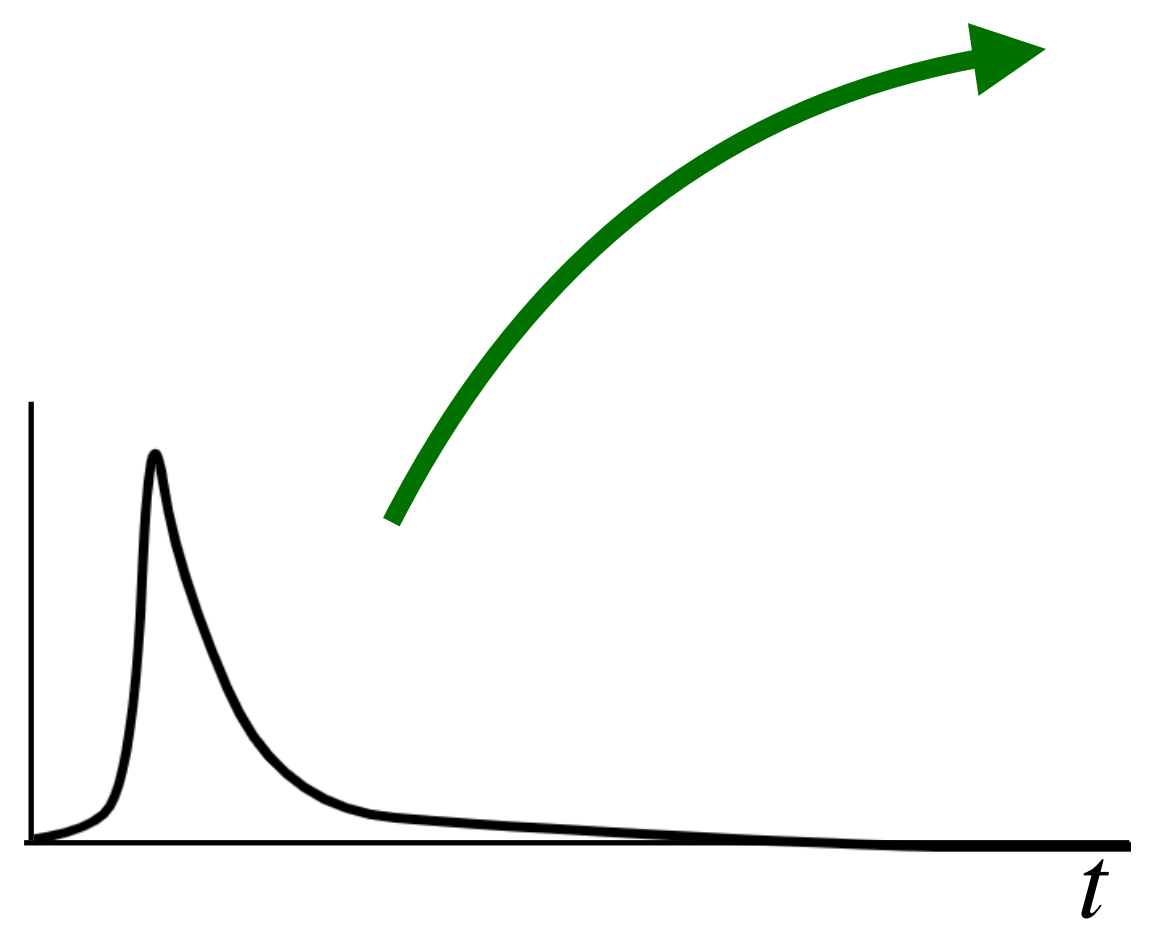
\*  $S(t, t_{rise}, t_{decay}, t_{delay})$



=  $l_R(t)$



optimise  $S$  to match  $l_R$

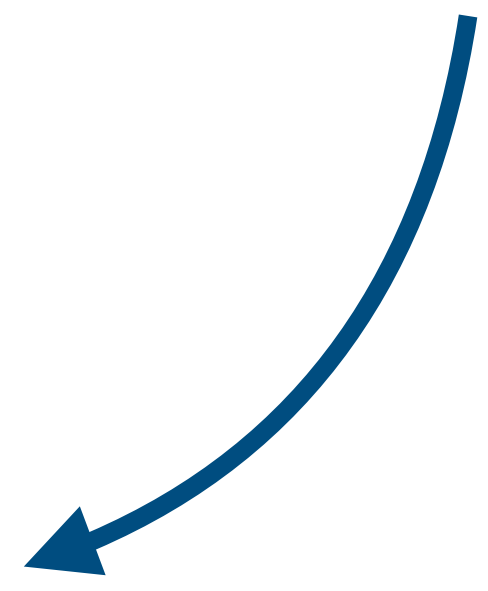
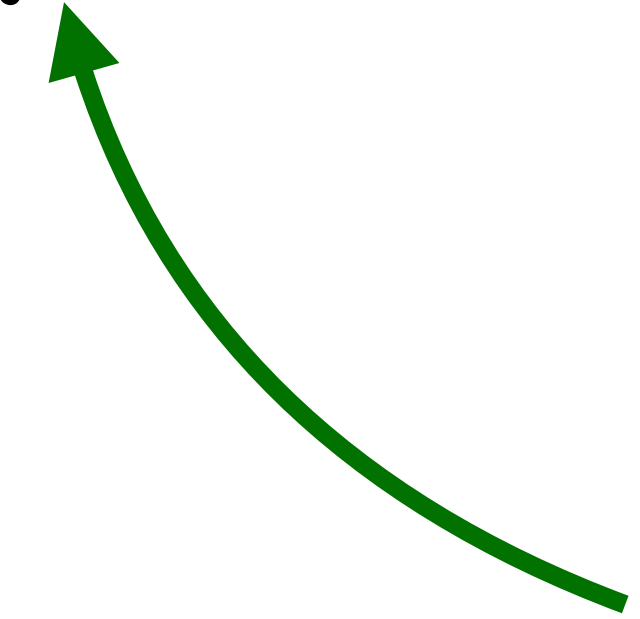


$l_\gamma(t)$

$$* S(t, t_{rise}, t_{decay}, t_{delay}) = l_R(t)$$

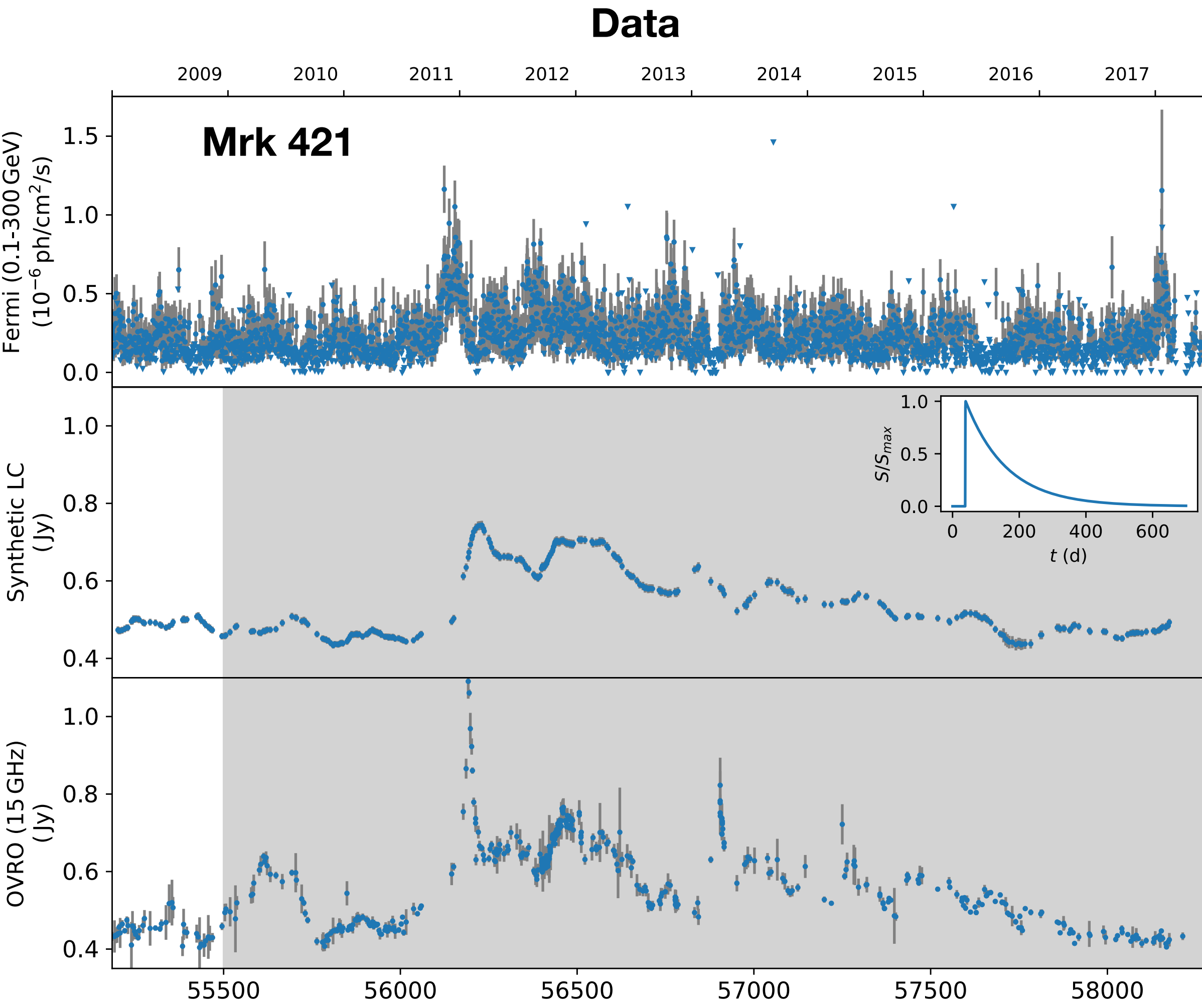
phenomenology:  
observables -> physics

$$t_{[rise, decay, delay]} = f(m_B, \beta_{exp}, \nu_{ssa}, B, R, p)$$

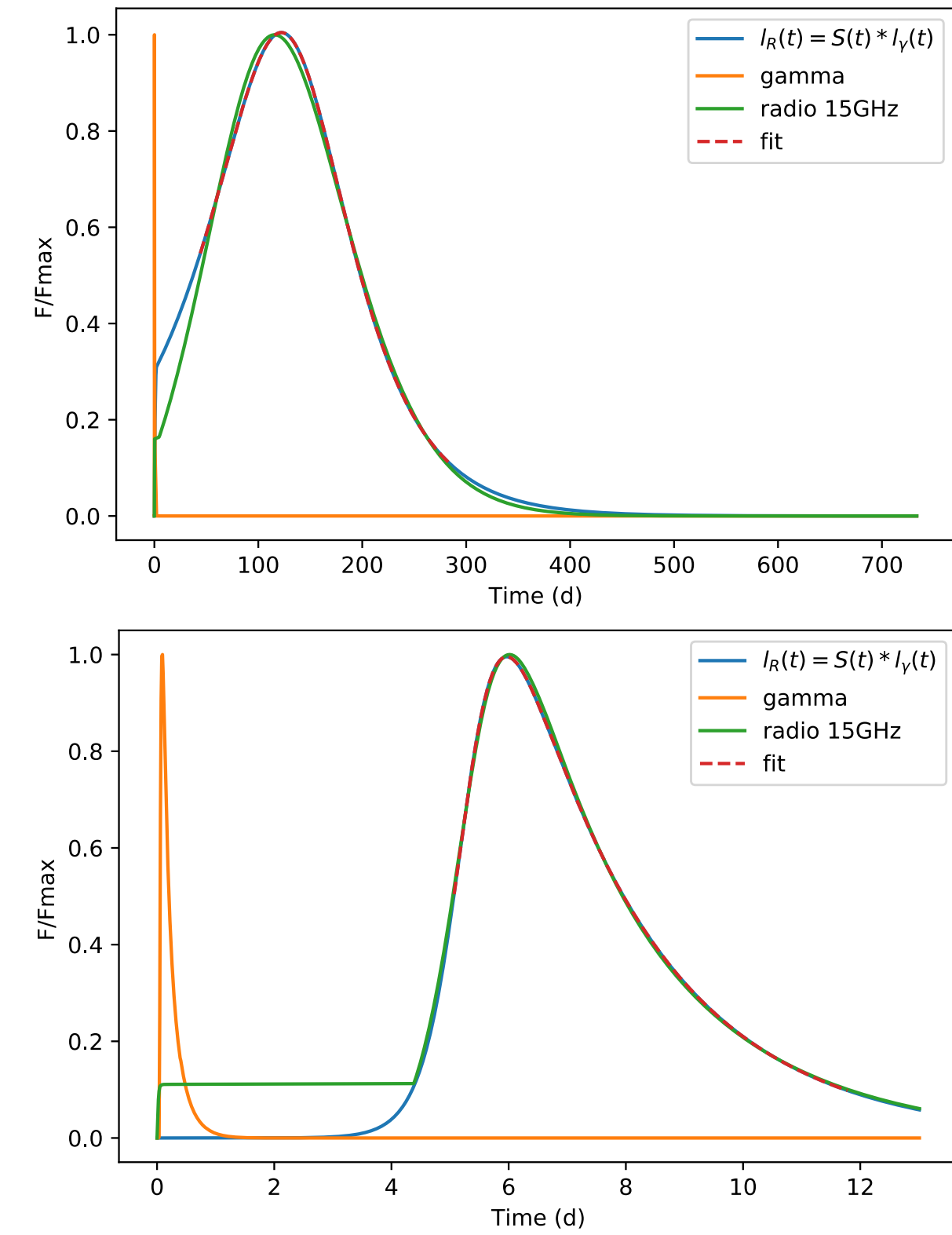




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## Numerical simulation



### exp-logistic

$$S(t) = A \frac{\exp^{-(t-\Delta_t)/t_{decay}^*}}{1 + \exp^{-(t-\Delta_t)/t_{rise}}}$$

$t_{decay}^{obs}$

$t_{rise}^{obs}$

$\Delta t_{SSA}^{obs} \rightarrow \nu_{SSA}^{*,obs}$

Parameter	Value
$A$	$12.5^{+0.5}_{-0.013} \times 10^3 \text{ Jy cm}^2 \text{ s/ph}$
$t_{rise}$	$\lesssim 1 \text{ day}$
$t_{decay}$	$126.5^{+1.3}_{-1.3} \text{ days}$
$\Delta t$	$37.58^{+0.13}_{-0.13} \text{ days}$
$F_{background}$	$0.18^{+0.008}_{-0.0004} \text{ Jy}$

$$\mathcal{L} = \mathcal{L}_{\text{rise}} + \mathcal{L}_{\text{decay}} + \mathcal{L}_{\text{delay}}$$

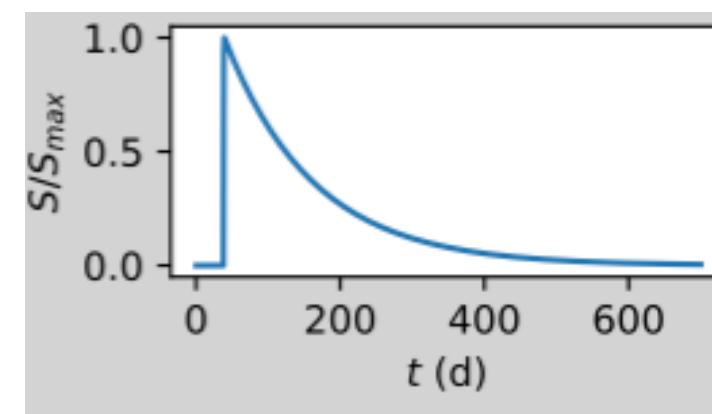
$$t_{[\text{rise}, \text{decay}, \text{delay}]} = f(m_B, \beta_{\text{exp}}, \nu_{\text{ssa}}, B, R, p)$$

$$\mathcal{L} \propto \sum_{i=[1,2,3]} -\frac{1}{2} \frac{(x_i - \mu_i)^2}{2\sigma_i^2} - \frac{1}{2} \ln(\sigma_i^2)$$

conv. analysis best fit values

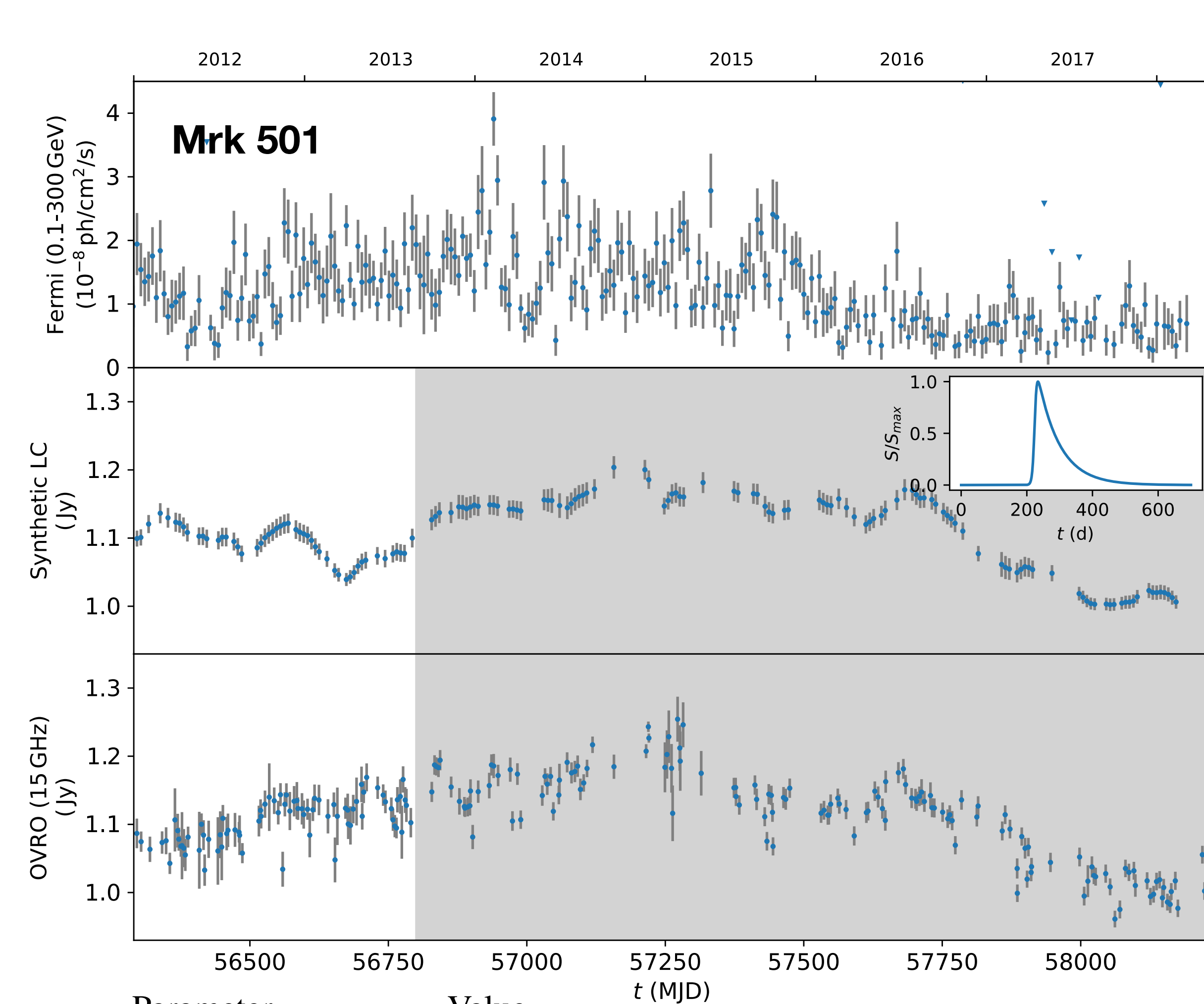
$$\Delta_t, t_{\text{rise}}^{\text{obs}}, t_{\text{decay}}^{\text{obs}}$$

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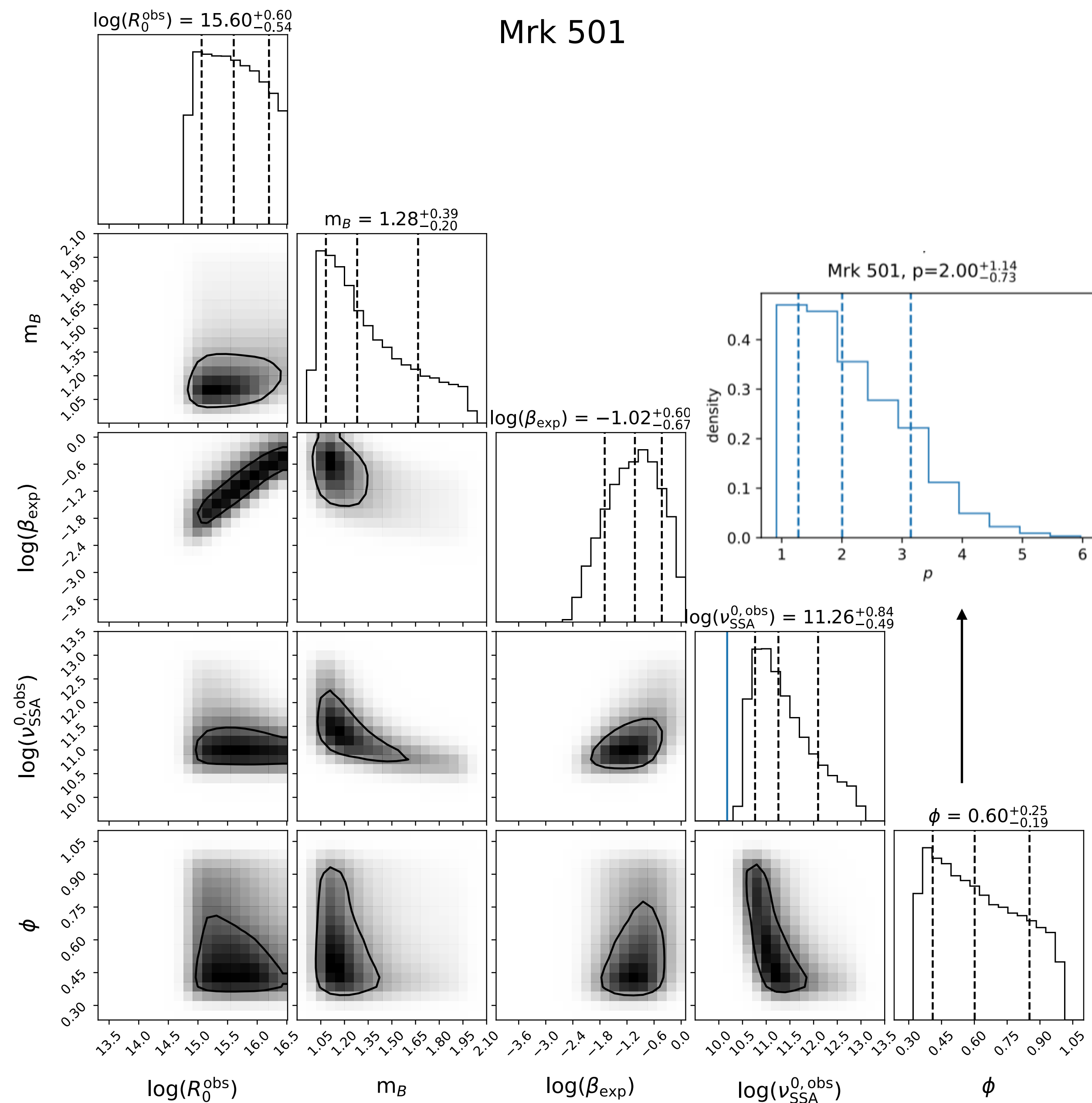




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Parameter	Value
$A$	$166^{+5}_3 \times 10^4 \text{ Jy cm}^2 \text{ s/ph}$
$t_{rise}$	$12^{+4}_{-4} \text{ days}$
$t_{decay}$	$73^{+3.6}_{-3.6} \text{ days}$
$\Delta t$	$234^{+10}_{-10} \text{ days}$
$F_{background}$	$0.915^{+0.004}_{-0.004} \text{ Jy}$



- Radio-gamma delays on weeks to year timescales can be self consistently reproduced by adiabatic blob expansion (consistent with Potter 2018, Boula 2022)
- We derived phenomenological relations, validated via accurate numerical simulations, and plugged to a response function, providing a direct link between the radio delay timescales and physics of the jet
- Implication on structure of magnetic fields, jet expansion, and MW connection open an interesting path to a deeper understanding of the how the engine of the jets work, and how jets evolve on larger scales, providing connection between micro and macro physics in relativistic jets
- Next: Plugging a realistic jet model with parabolic-to-conical transition, plus EC and crossing time effect, and application to a large sample of BL Lacs and FSRQs
- **Analysis fully reproducible with JetSeT and convolution tool:**

[https://github.com/andreatramacere/adiabatic\\_exp\\_radio\\_gamma\\_delay](https://github.com/andreatramacere/adiabatic_exp_radio_gamma_delay)

<https://jetset.readthedocs.io/en/1.2.2/>







no crossing time

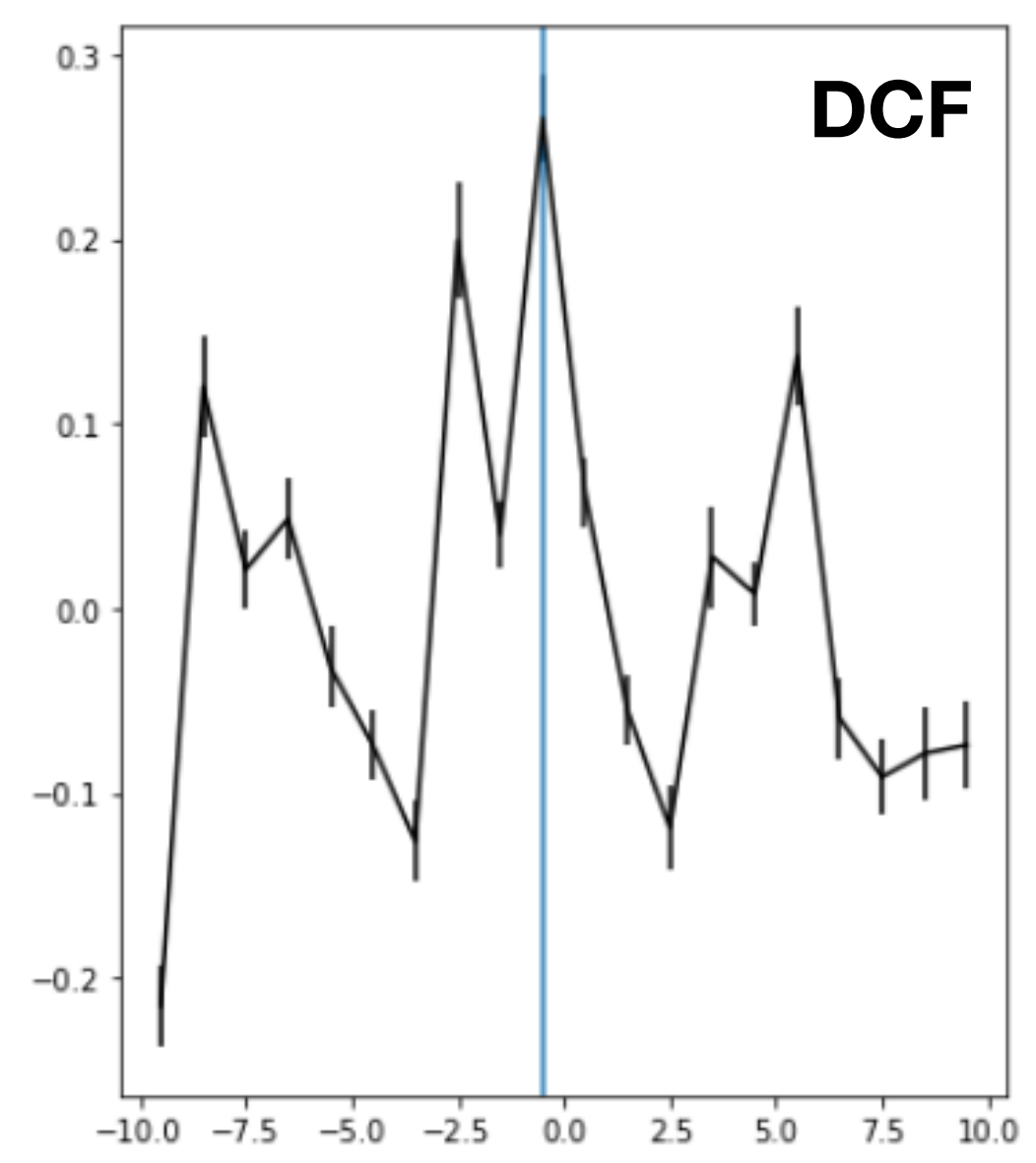
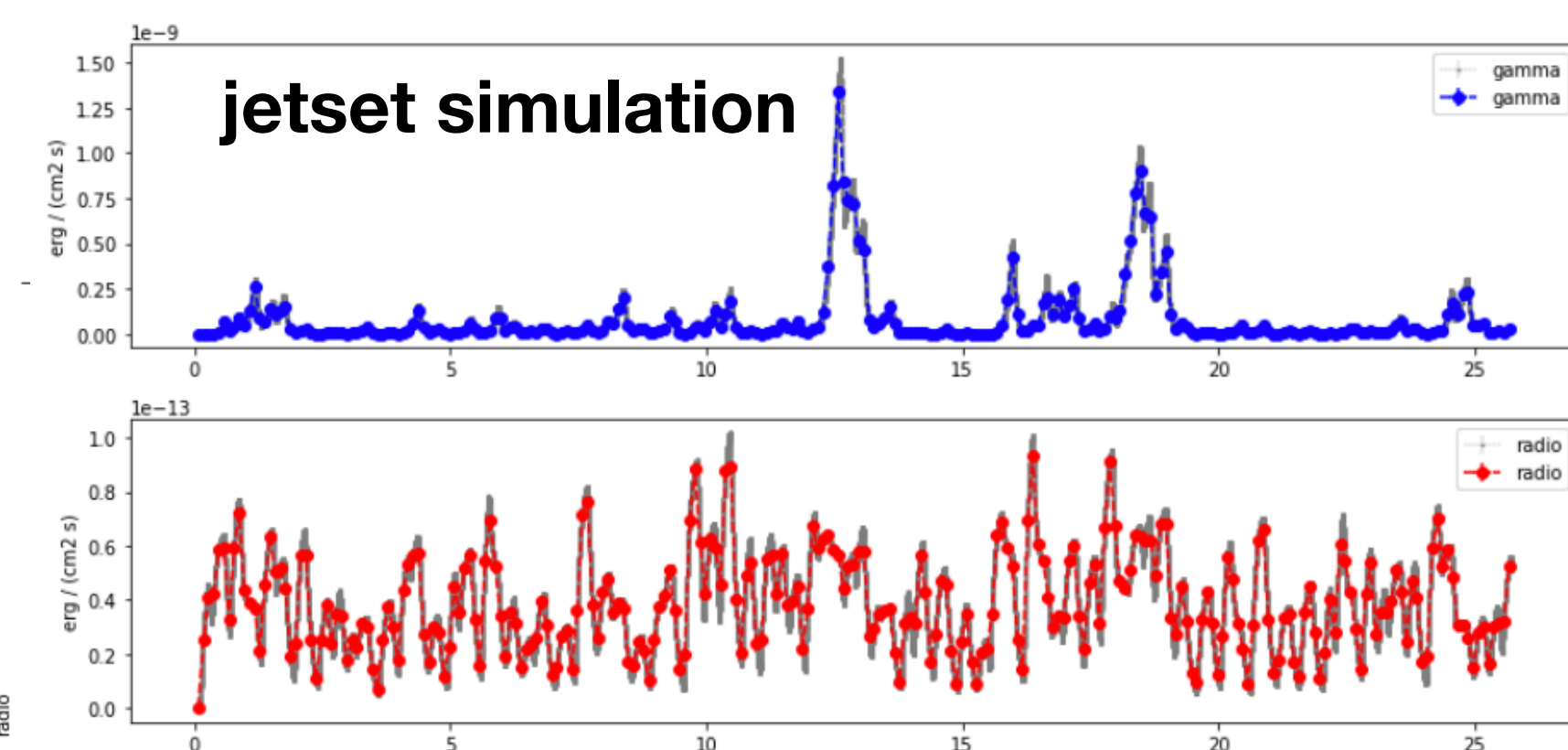
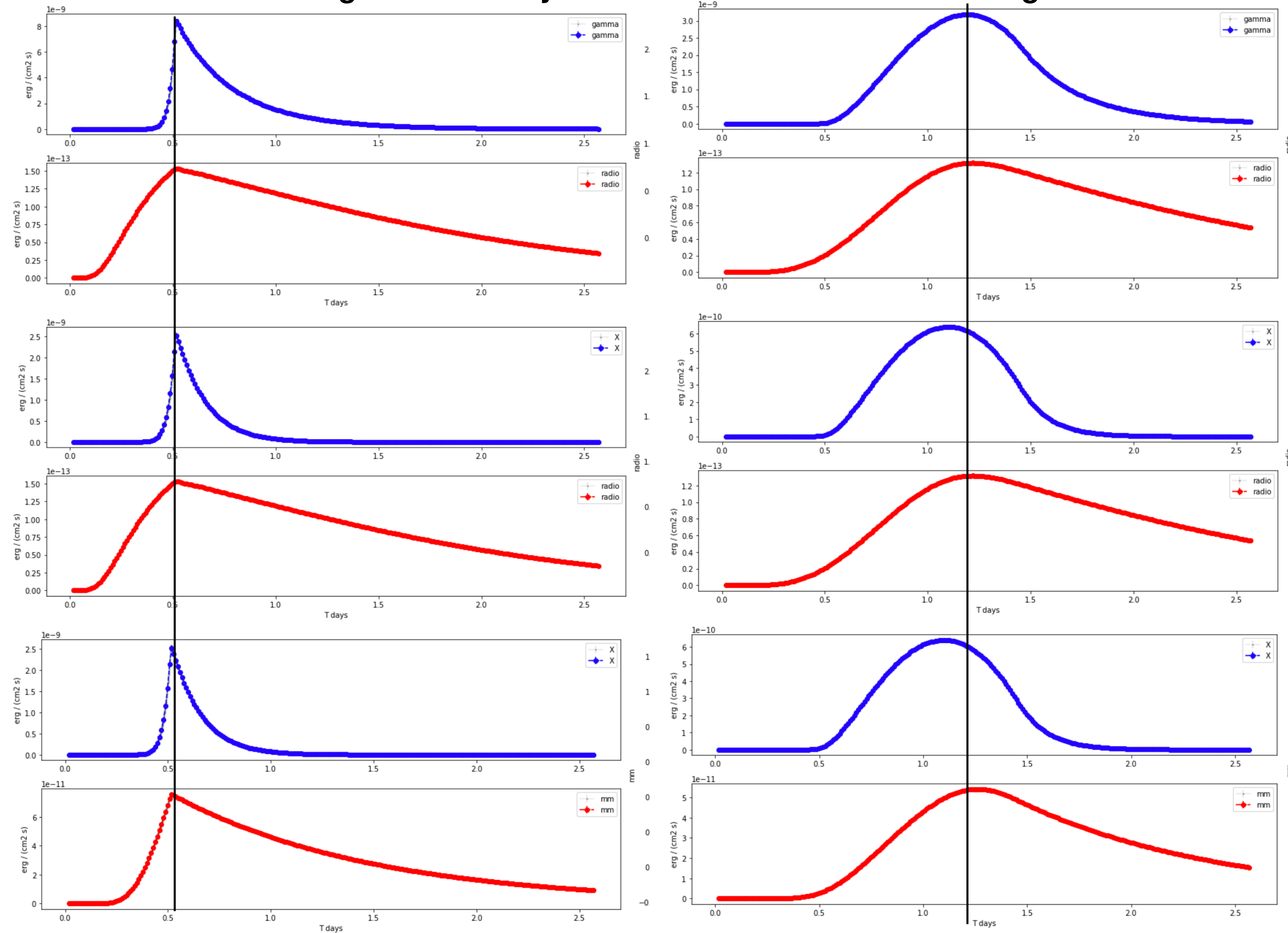
jetset simulation

with crossing time

hours to days

$$\Delta_T \sim R/c, t_{cool}$$

dispersion on  $\Delta_T \sim R/c, t_{cool}$

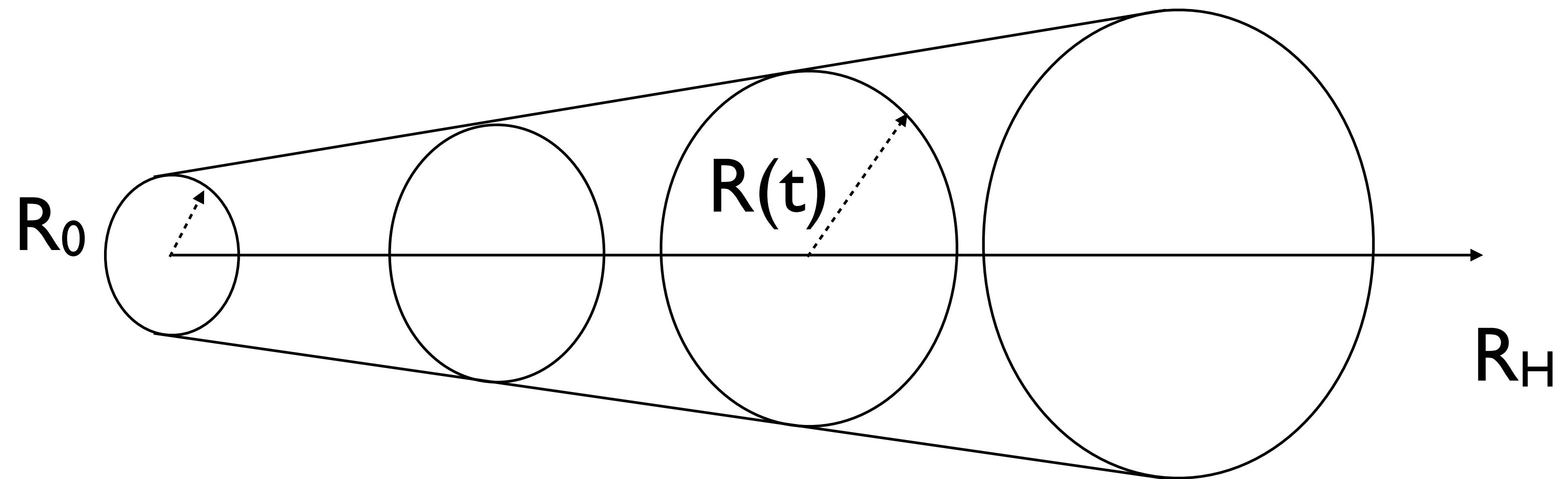




$$\beta_{\text{exp}} = v_{\text{exp}}/c$$

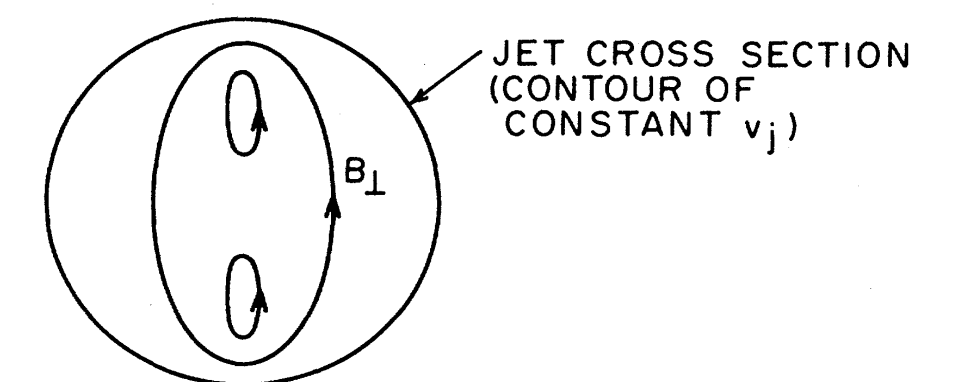
$$R(t) = R_0 + \beta_{\text{exp}} c (t - t_{\text{exp}}) H(t - t_{\text{exp}})$$

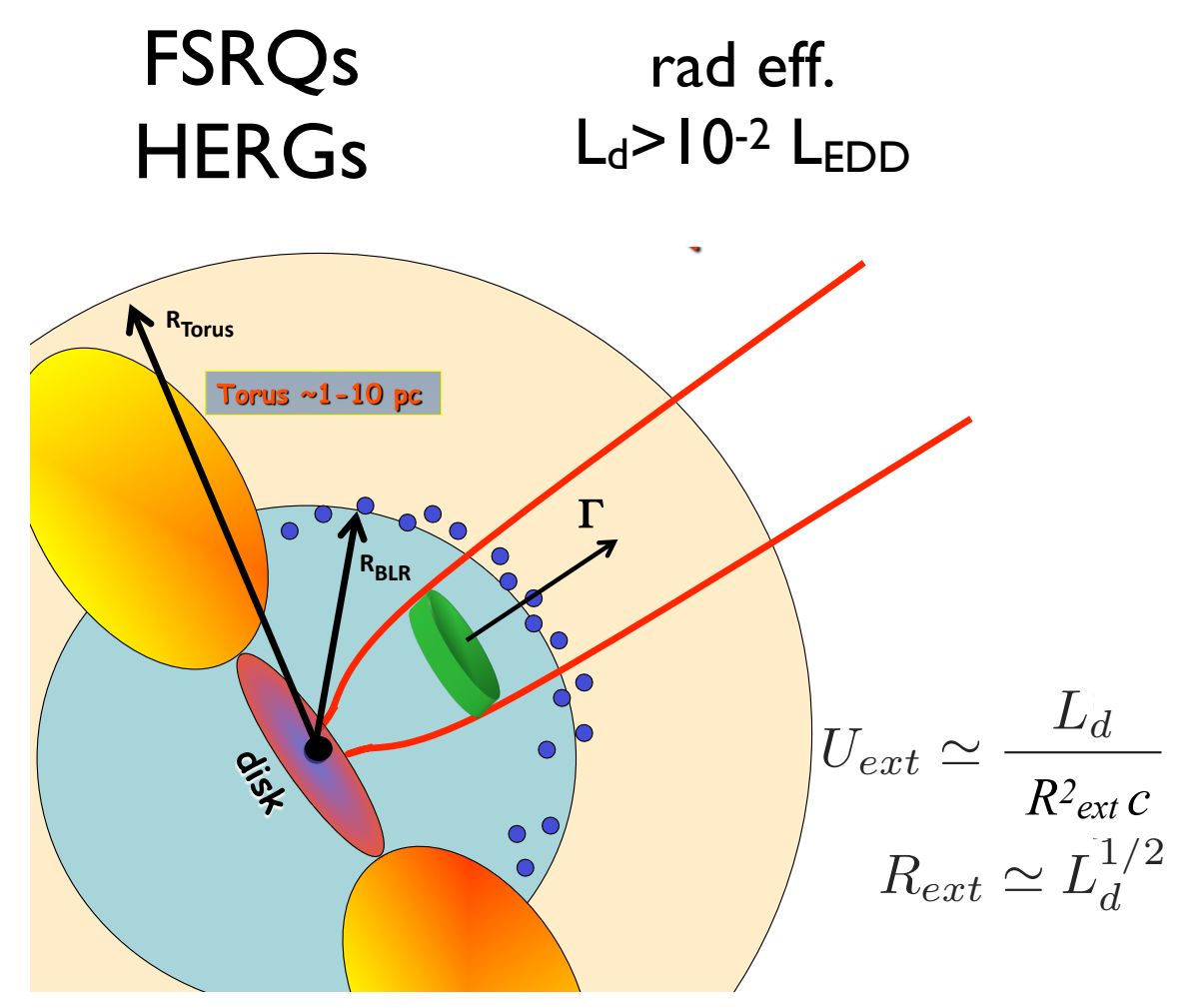
$$B(t) = B_0 \left( \frac{R_0}{R(t)} \right)^{m_B} \quad m_B \in [1, 2]$$



**Magnetic field** (flux freezing and conservation): (Begelman, Blandford, and Rees 1984)

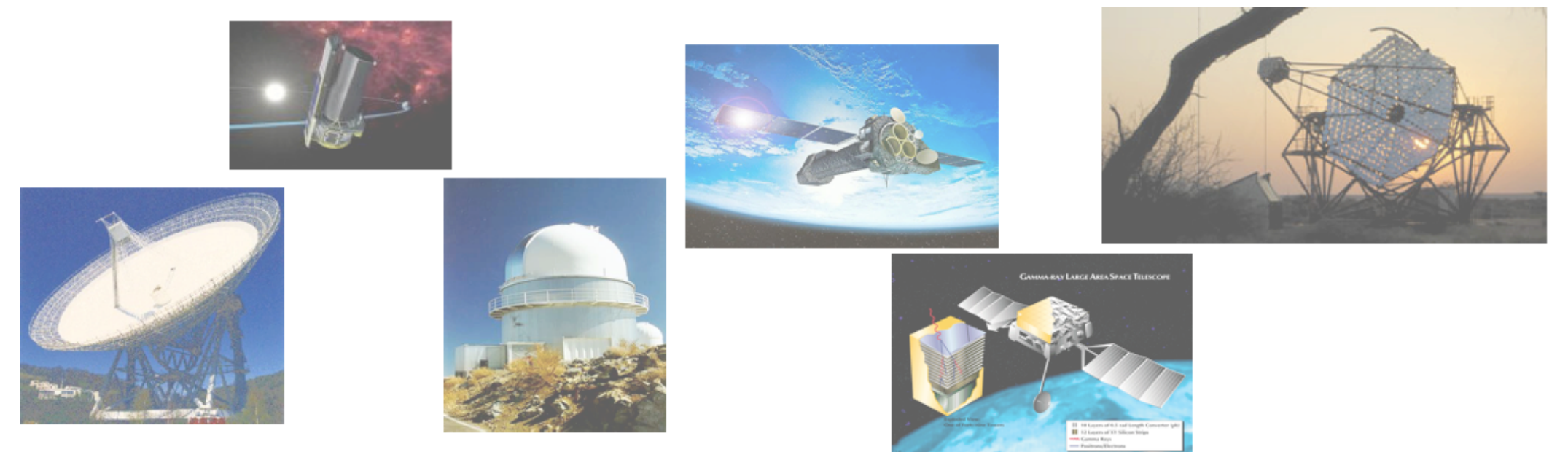
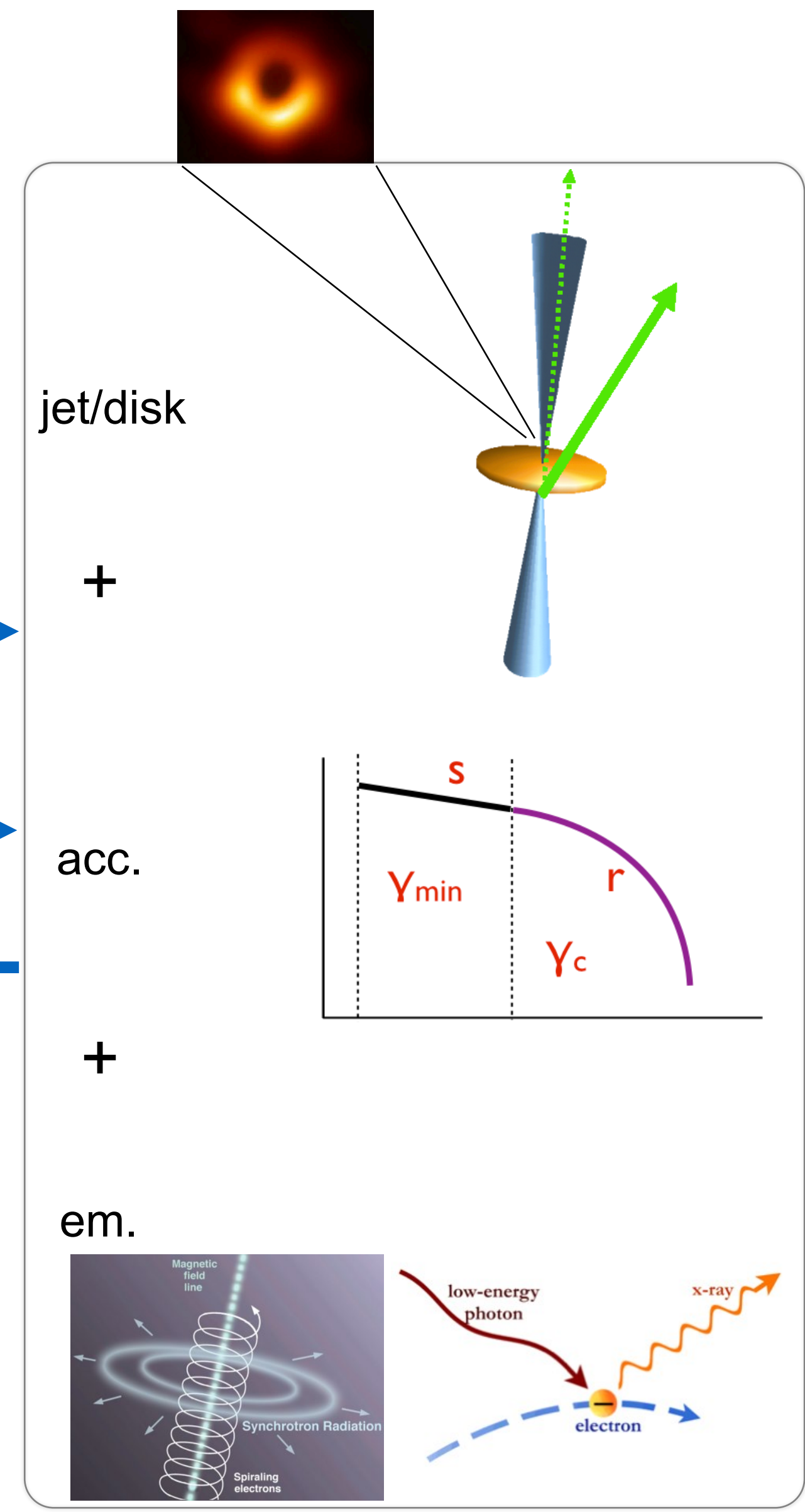
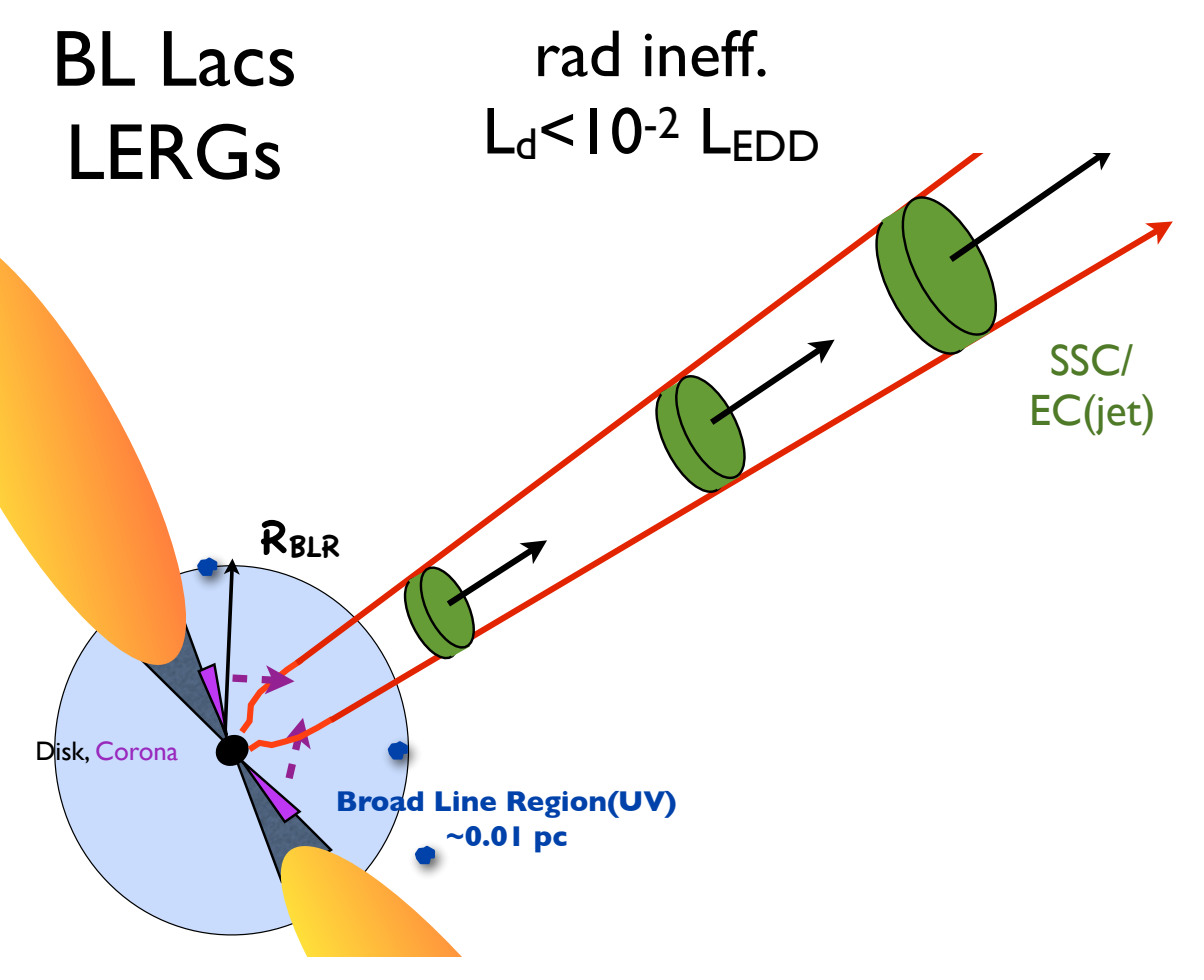
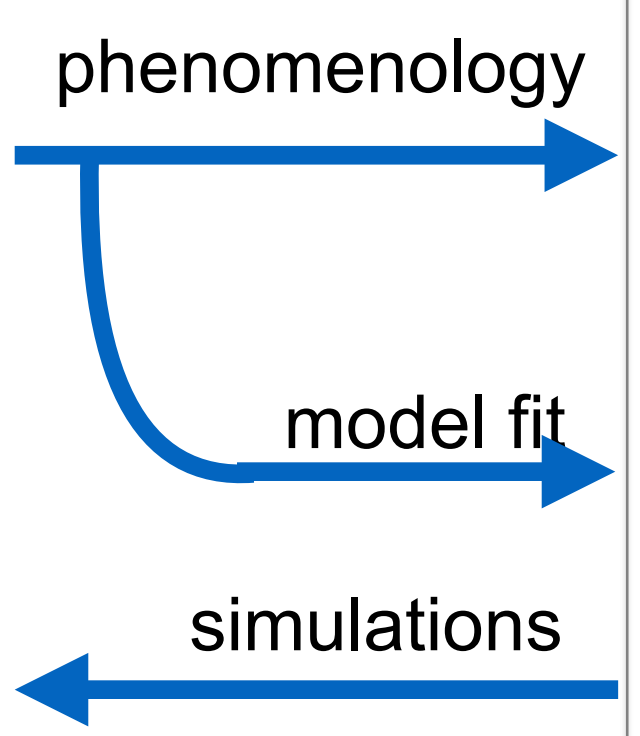
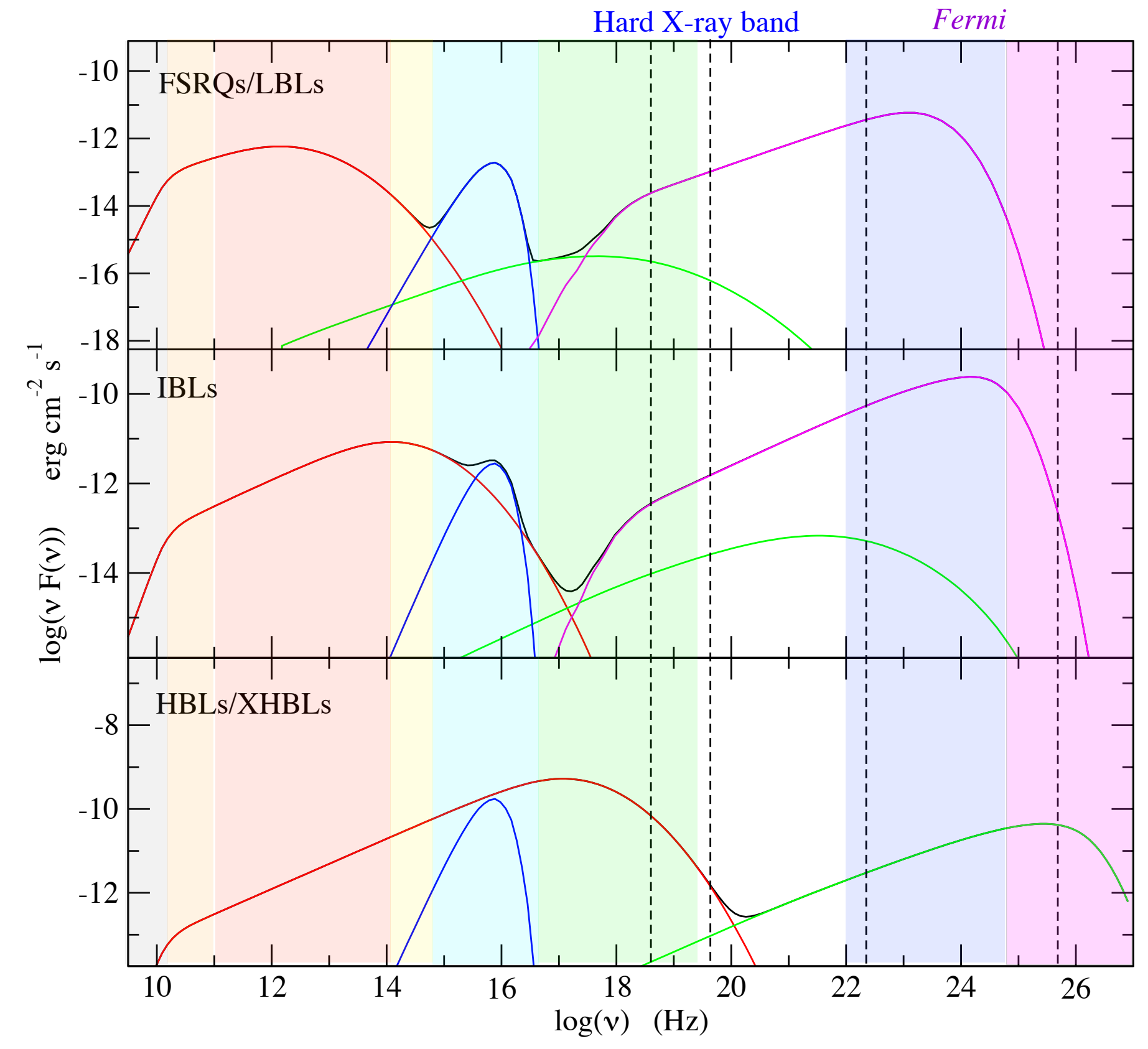
- $B_{||} \propto R^{-2}$  (poloidal)  $m_B=2$
- $B_{\perp} \propto R^{-1}$  (toroidal)  $m_B=1$
- for initial mixed configuration, **and no velocity gradient**,  $B_{\perp}$  will dominate with  $m_B \sim m_R$



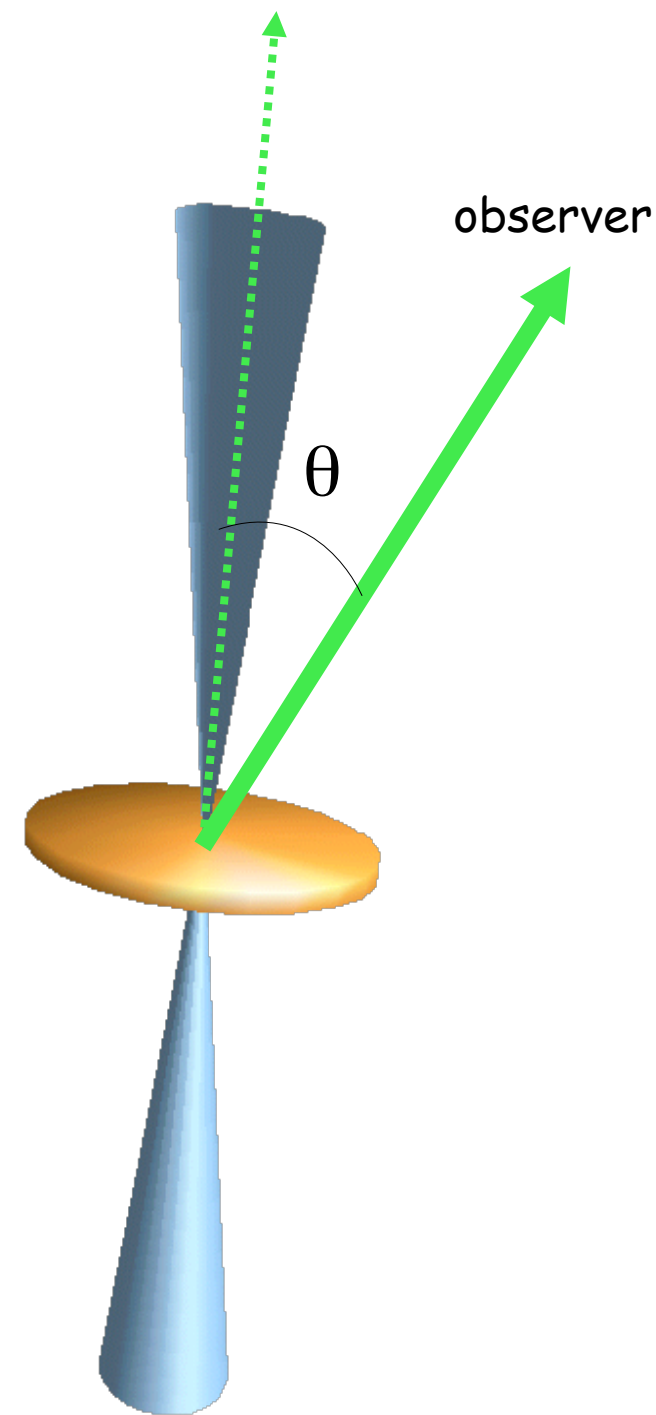
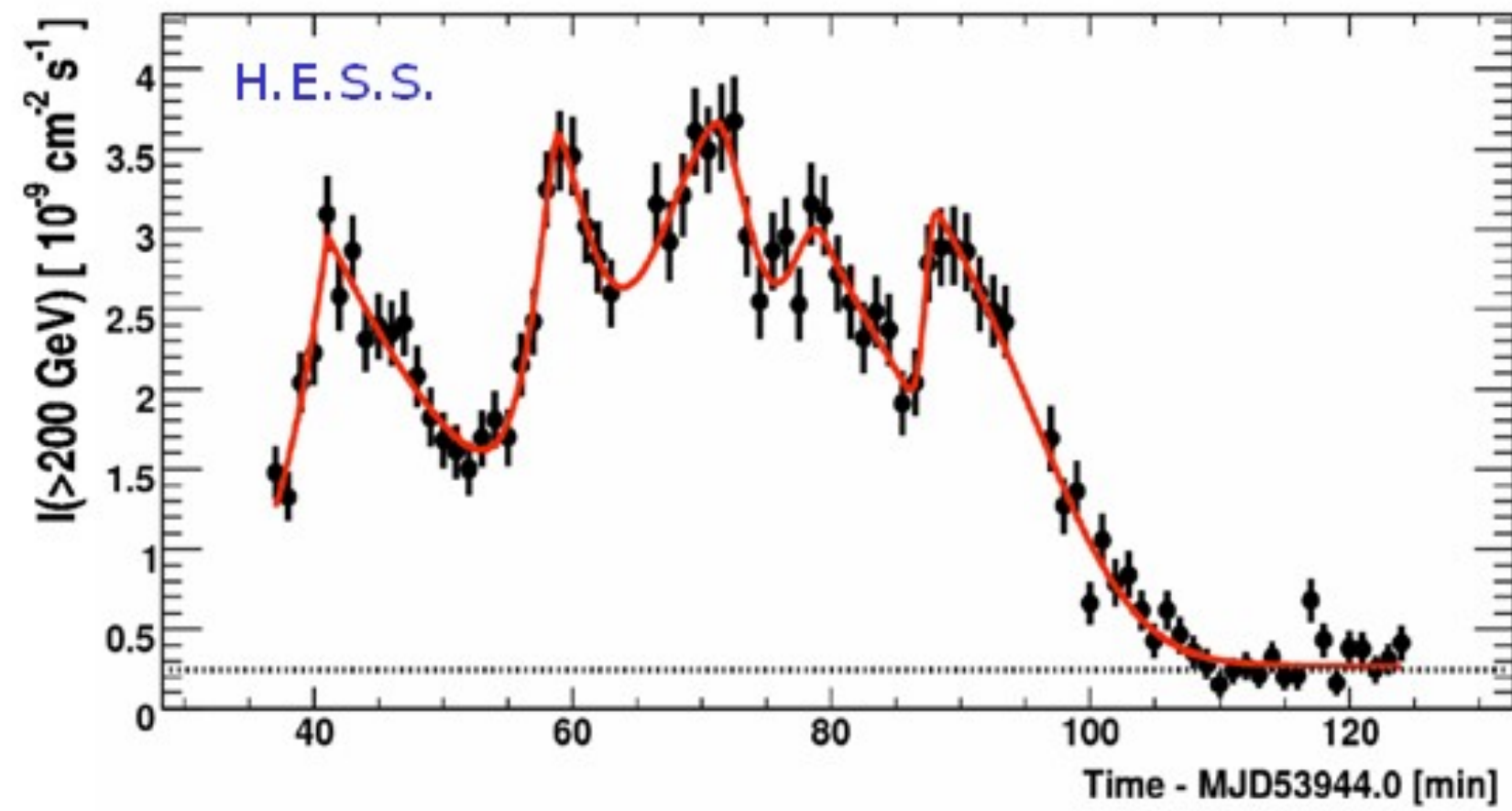


$$U_{\text{ext}} \approx \frac{L_d}{R_{\text{ext}}^2 c}$$

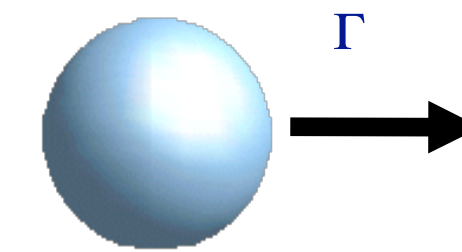
$$R_{\text{ext}} \approx L_d^{1/2}$$



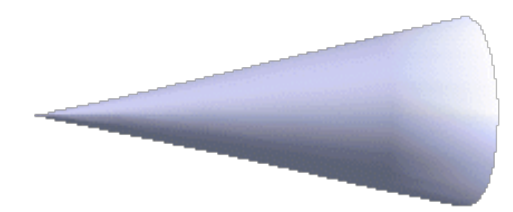




rest frame :  
isotropic emission

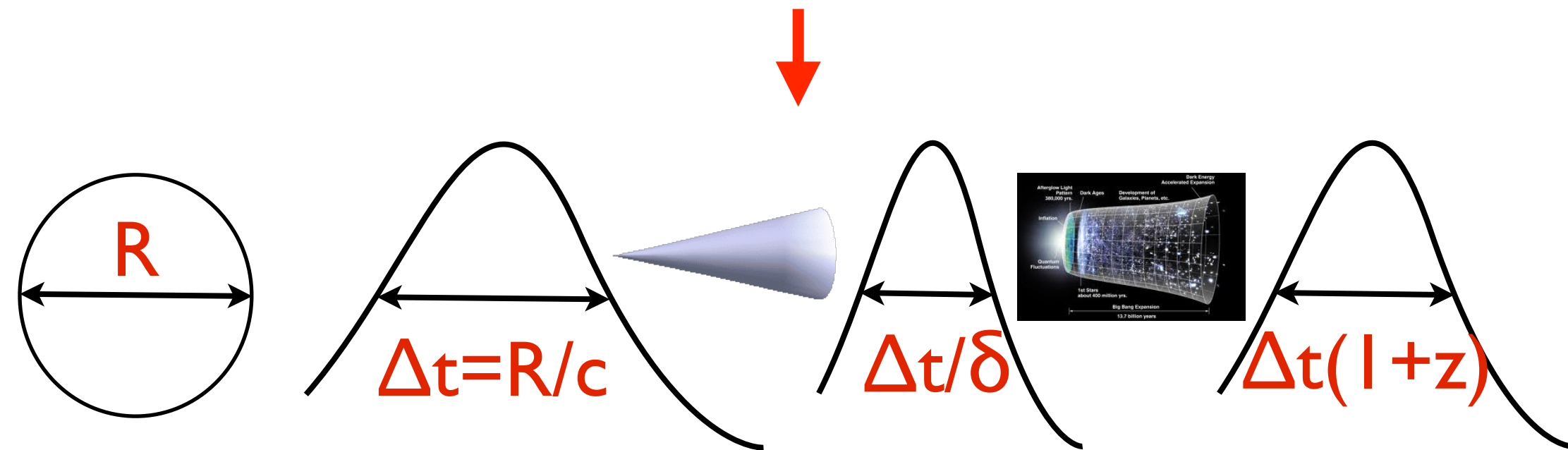


Observer frame: beamed

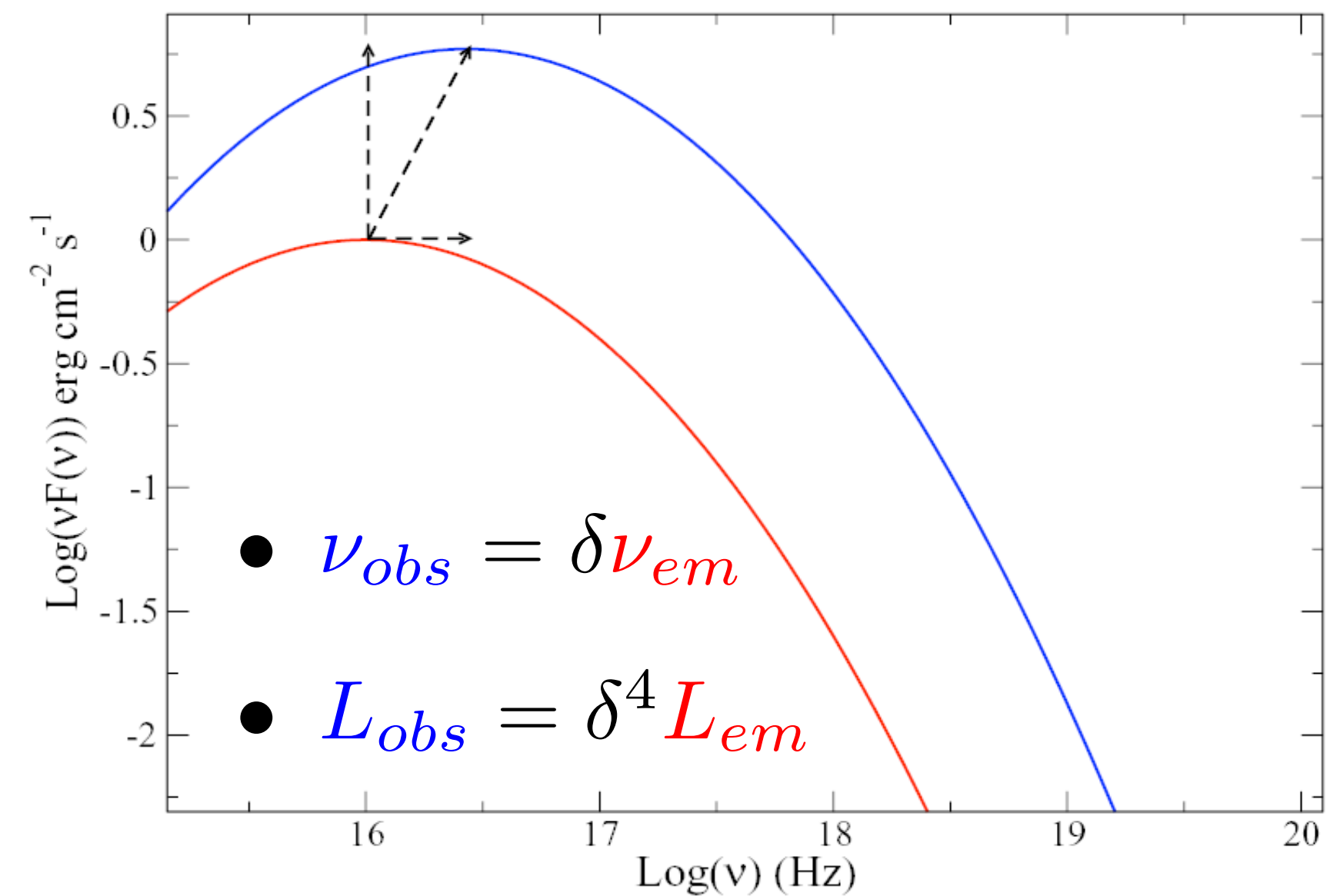


Beaming factor:

- $\delta = \frac{1}{\Gamma(1-\beta \cos(\theta))}$
- $\theta = 1/\Gamma$



$$R \leq c \Delta t \delta / (1+z)$$



$$R(t) = R_0 + \beta_{\text{exp}} c(t - t_{\text{exp}}) H(t - t_{\text{exp}})$$

$$B(t) = B_0 \left(\frac{R_0}{R(t)}\right)^{m_B} \quad m_B \in [1, 2]$$

**Geom.**

$$|\dot{\gamma}_{\text{synch}}(t)| = \frac{4\sigma_{TC}}{3m_e c^2} \gamma^2 U_B(t) = C_0 \gamma^2 U_B(t)$$

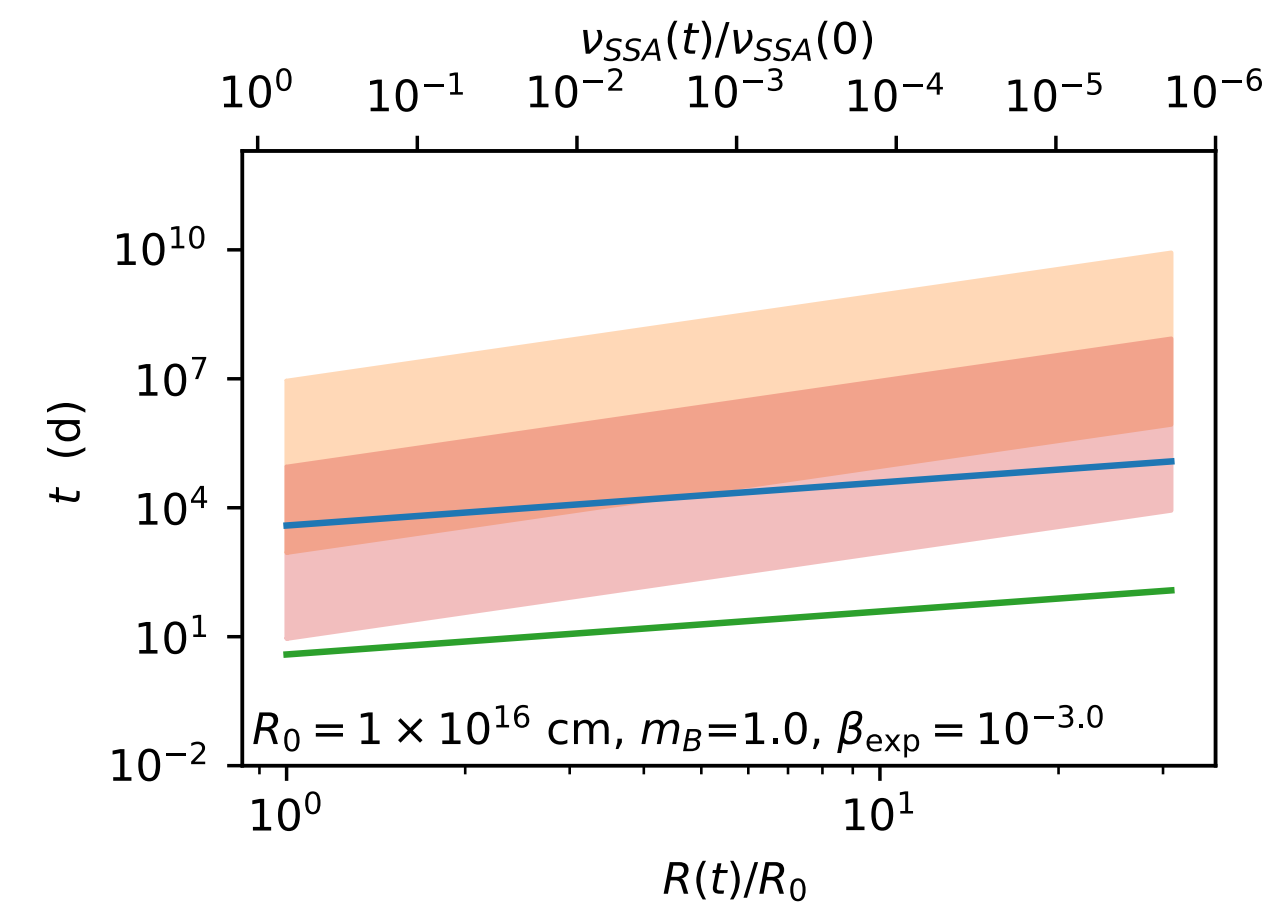
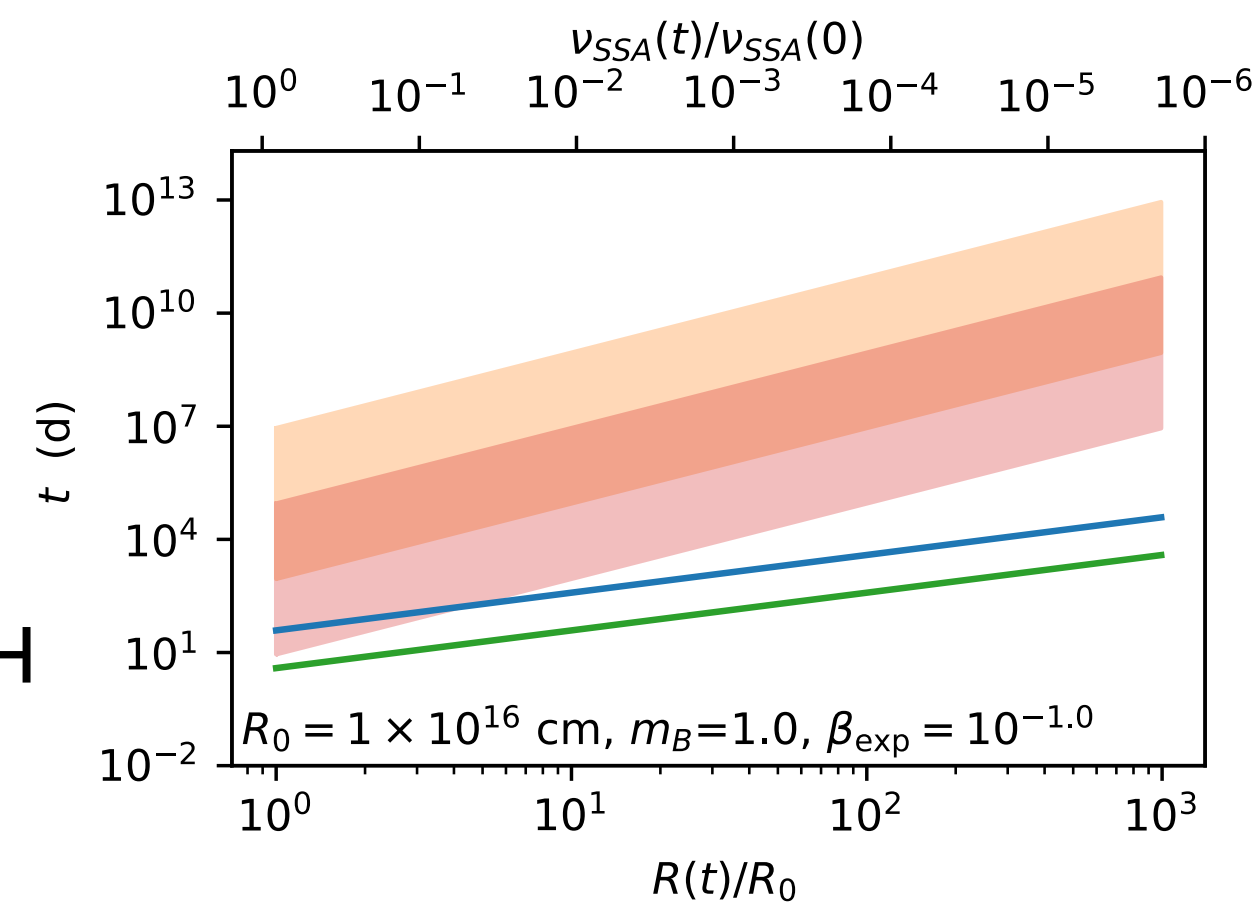
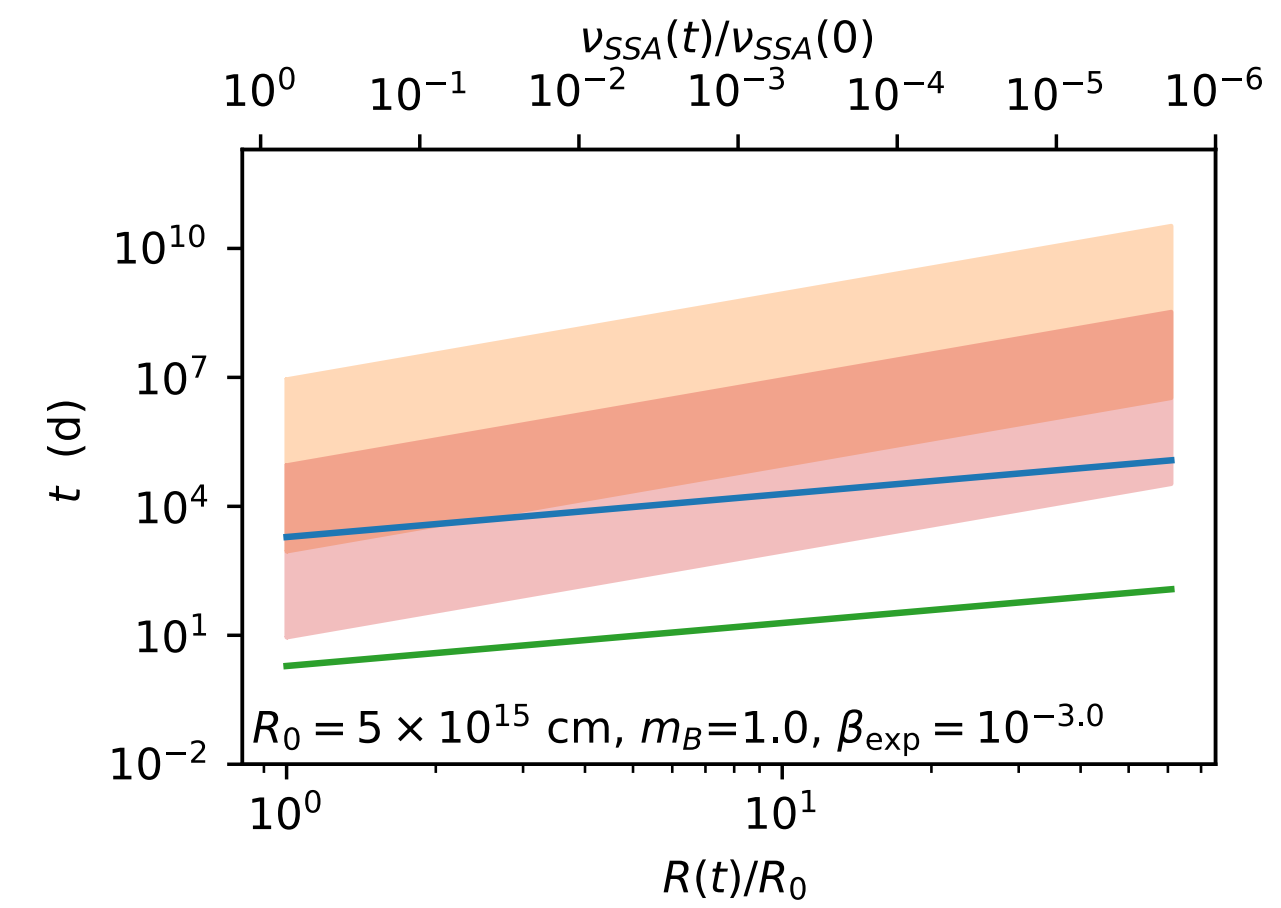
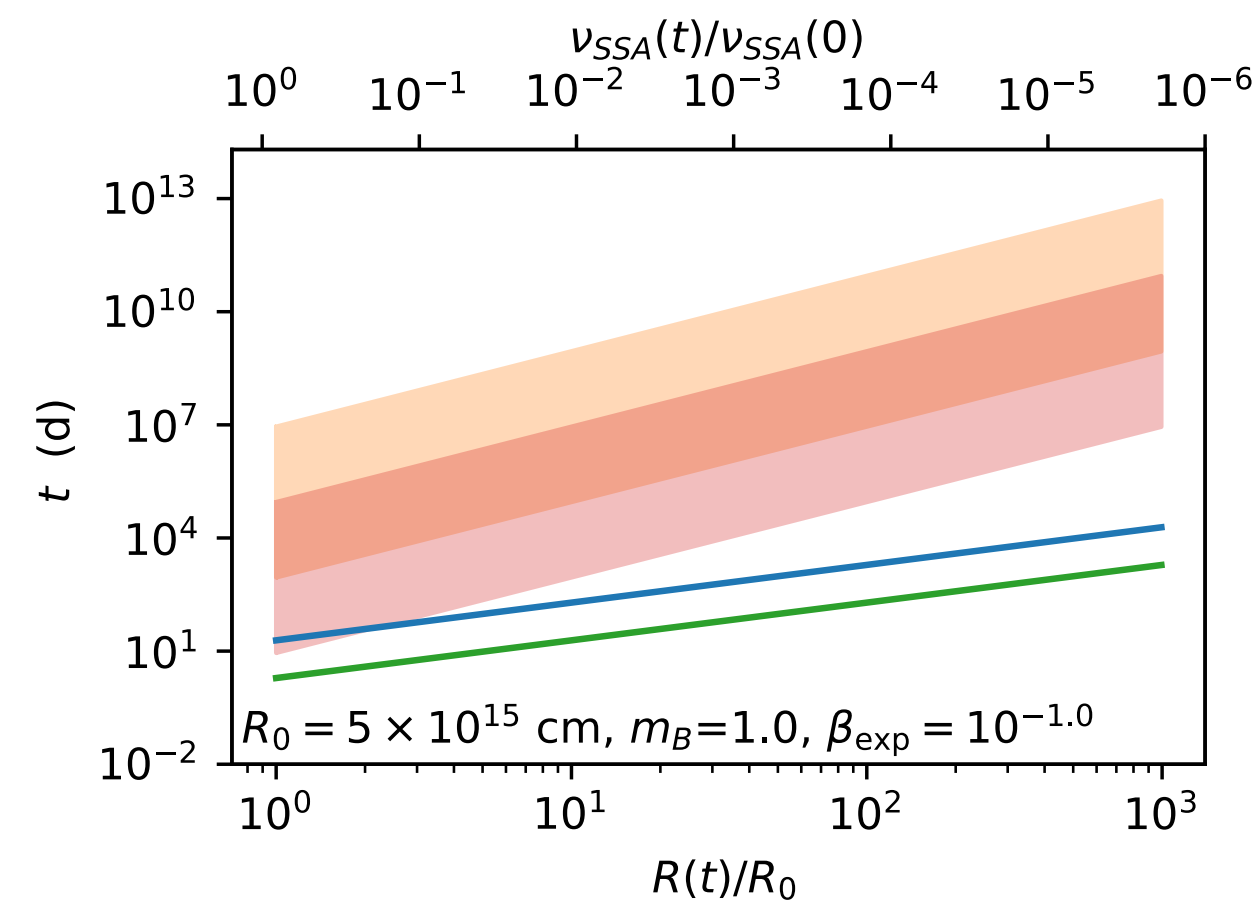
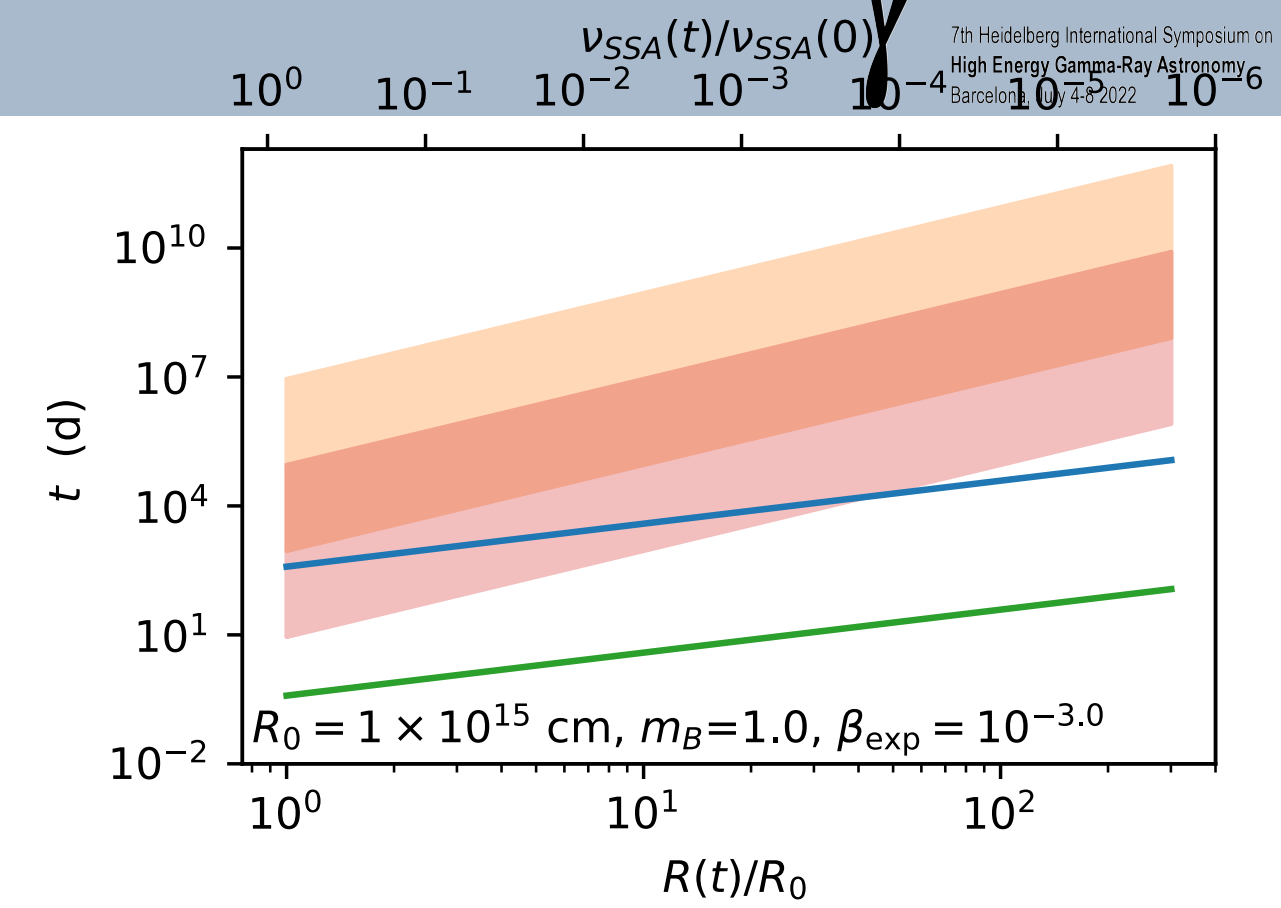
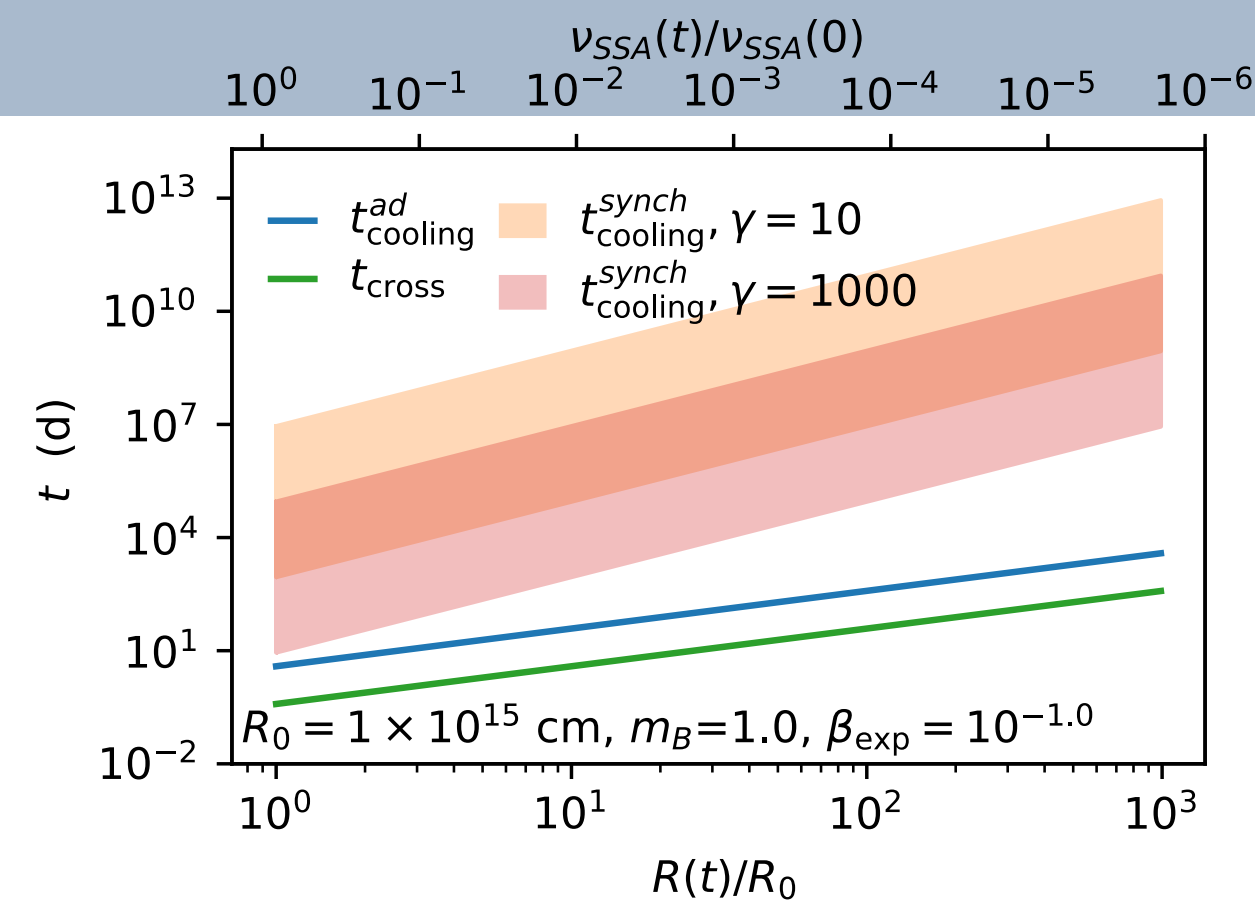
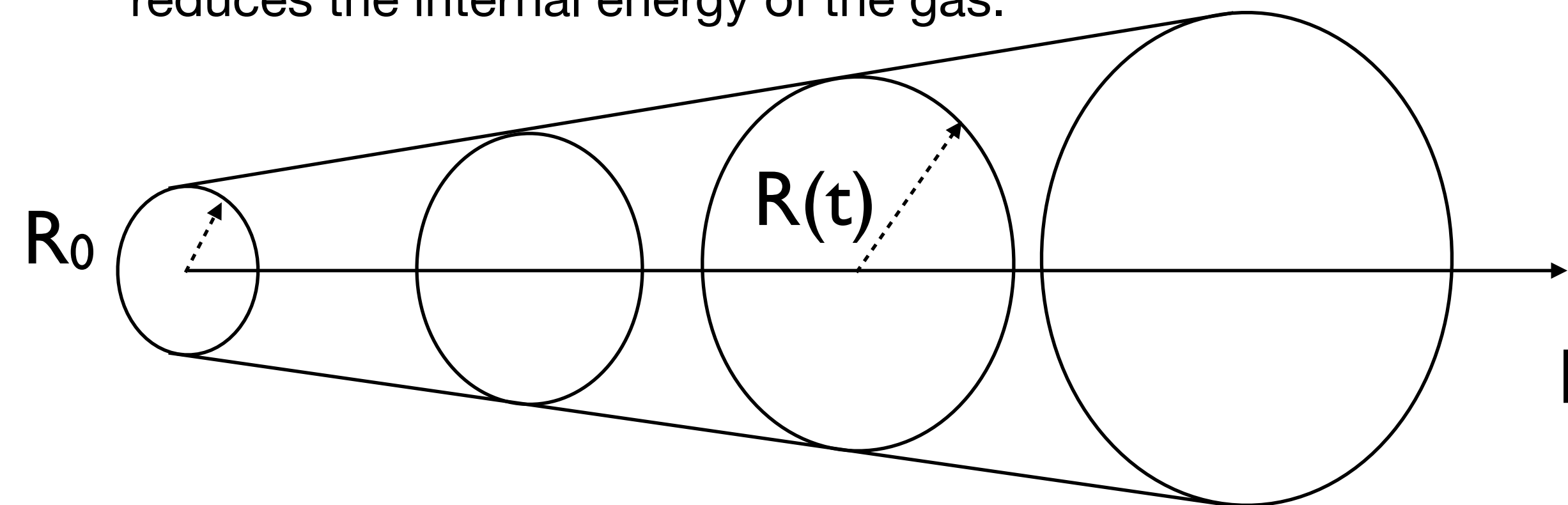
$$|\dot{\gamma}_{IC}(t)| = \frac{4\sigma_{TC}}{3m_e c^2} \gamma^2 \int f_{KN}(4\gamma\epsilon_0) \epsilon_0 n_{ph}(\epsilon_0, t) d\epsilon_0$$

$$= C_0 \gamma^2 F_{KN}(\gamma, t)$$

$$|\dot{\gamma}_{ad}(t)| = \frac{1}{3} \frac{\dot{V}}{V} \gamma = \frac{\dot{R}(t)}{R(t)} \gamma = \frac{\beta_{\text{exp}} c}{R(t)} \gamma$$

**Cooling**

If the electrons are confined within an expanding volume, they are subject to adiabatic losses as they do work which reduces the internal energy of the gas.





here we add adiabatic cooling  
(t=time elapsed from the expansion)

$$|\dot{\gamma}_{ad}| = \frac{1}{3} \frac{\dot{V}}{V} \gamma = \frac{\dot{R}(t)}{R(t)} \gamma = \frac{\beta_{exp} c}{R(t)} \gamma$$

Tramacere+2022, Tramacere+2011

**injection term**

$$L_{inj} = V_{acc} \int \gamma m_e c^2 Q(\gamma, t) d\gamma \quad (erg/s)$$

**systematic term**

$$S(\gamma, t) = -C(\gamma, t) + A(\gamma, t)$$

**cooling term**

$$C(\gamma) = |\dot{\gamma}_{synch}| + |\dot{\gamma}_{IC}| + |\dot{\gamma}_{ad}|$$

**sys. acc. term**

$$A(\gamma) = A_{p0} \gamma, \quad t_A = \frac{1}{A_0}$$

$$\frac{\partial n(\gamma, t)}{\partial t} = \frac{\partial}{\partial \gamma} \left\{ - [S(\gamma, t) + D_A(\gamma, t)] n(\gamma, t) \right\} + \frac{\partial}{\partial \gamma} \left\{ D_p(\gamma, t) \frac{\partial n(\gamma, t)}{\partial \gamma} \right\} - \frac{n(\gamma, t)}{T_{esc}(\gamma)} - \frac{n(\gamma, t)}{T_{ad}} + Q(\gamma, t)$$

$$T_{ad} = \frac{1}{3} \frac{R(t)}{\beta_{exp} c} \quad (\text{Gould 1975})$$

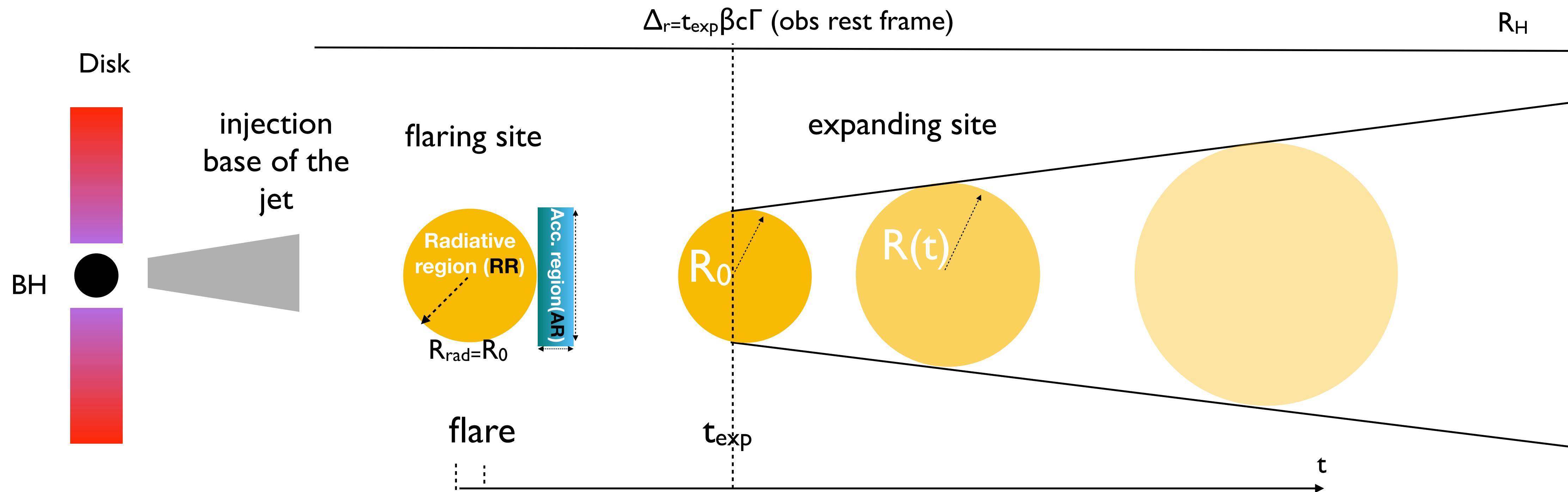
**Turbulent magnetic field**

$$W(k) = \frac{\delta B(k_0^2)}{8\pi} \left( \frac{k}{k_0} \right)^{-q}$$



**momentum diffusion term**

$$D_p \approx \beta_A^2 \left( \frac{\delta B}{B_0} \right)^2 \left( \frac{\rho_g}{\lambda_{max}} \right)^{q-1} \frac{p^2 c^2}{\rho_g c}$$

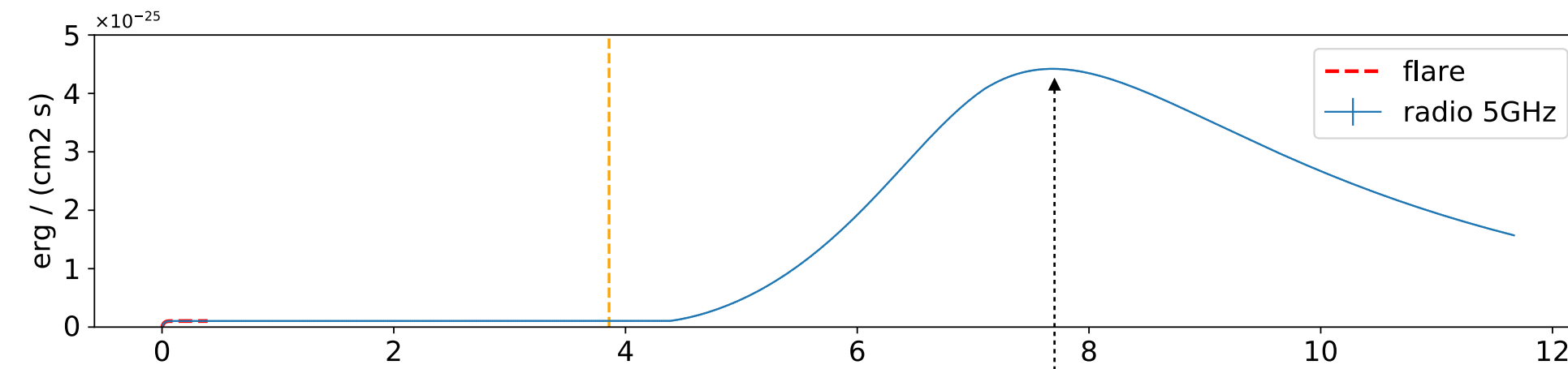
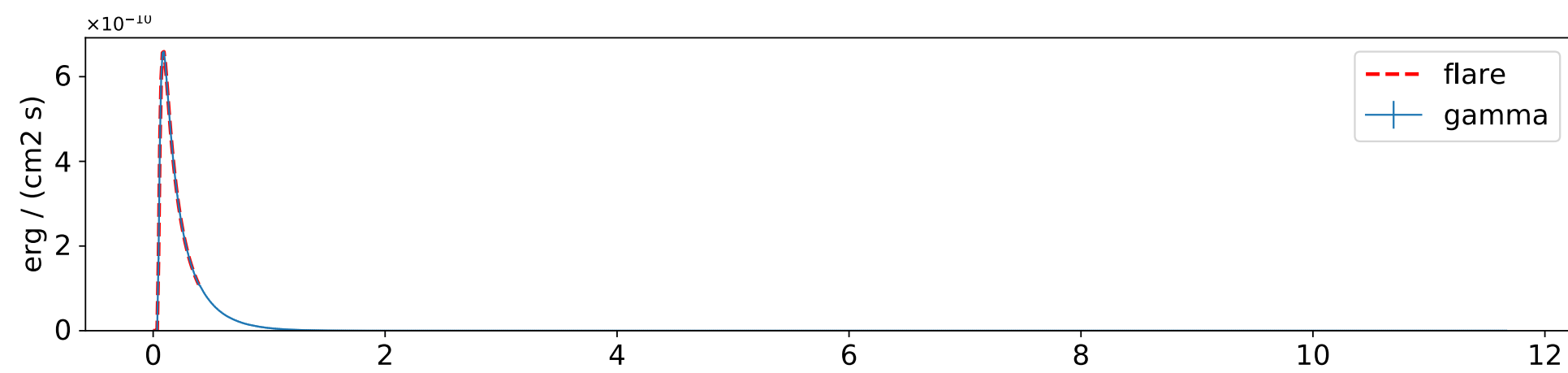
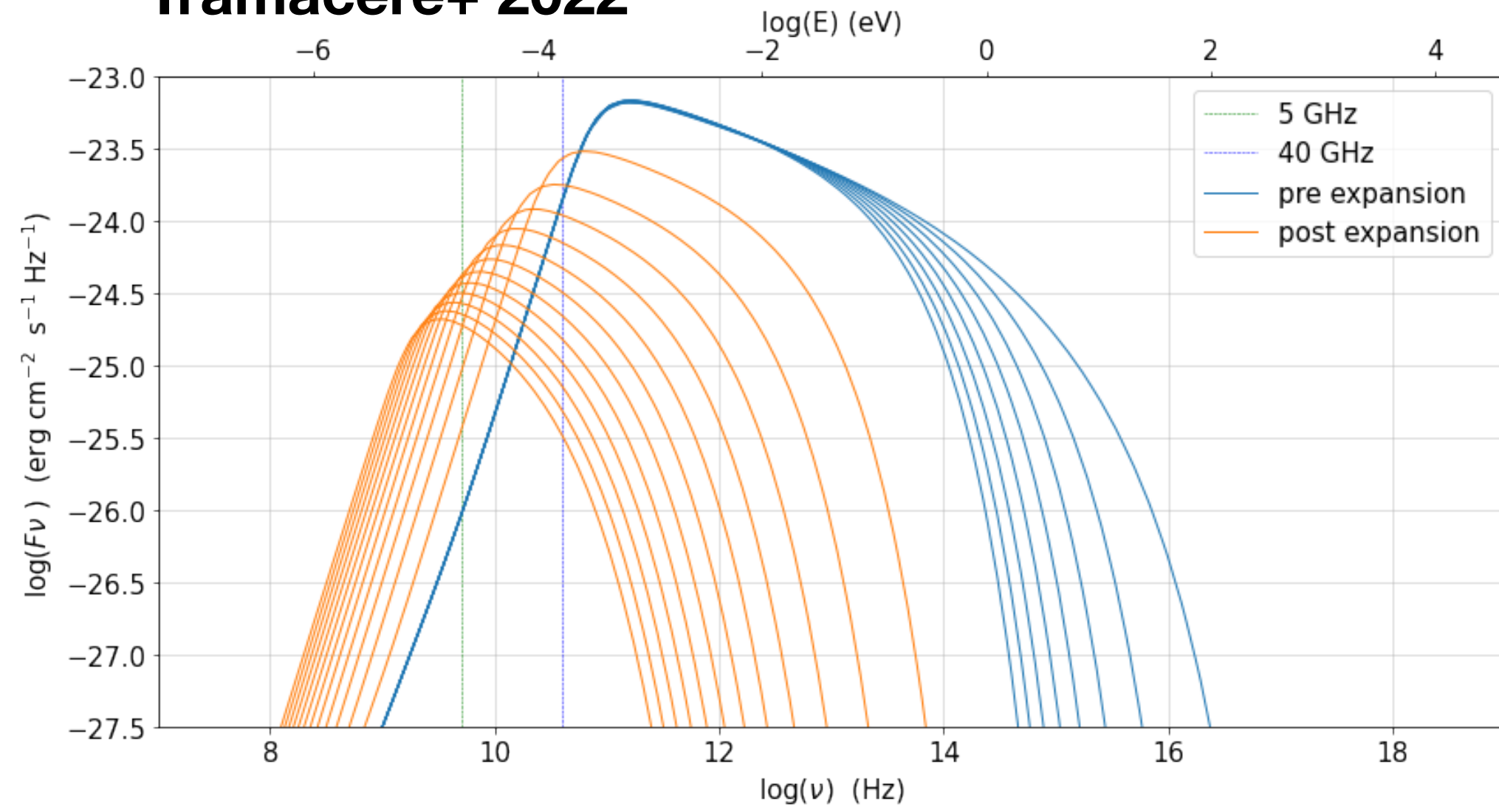


- Acc. region+Rad. region
- SSC scenario ( $R, B$  similar to those observed in HBLs from MW model fitting)
- Particles are confined in Rad. region
- we limit to  $m_B=1$ , beaming factor constant across the jet
- we ignore crossing time ( $\ll$  other time scales in the expanding site)





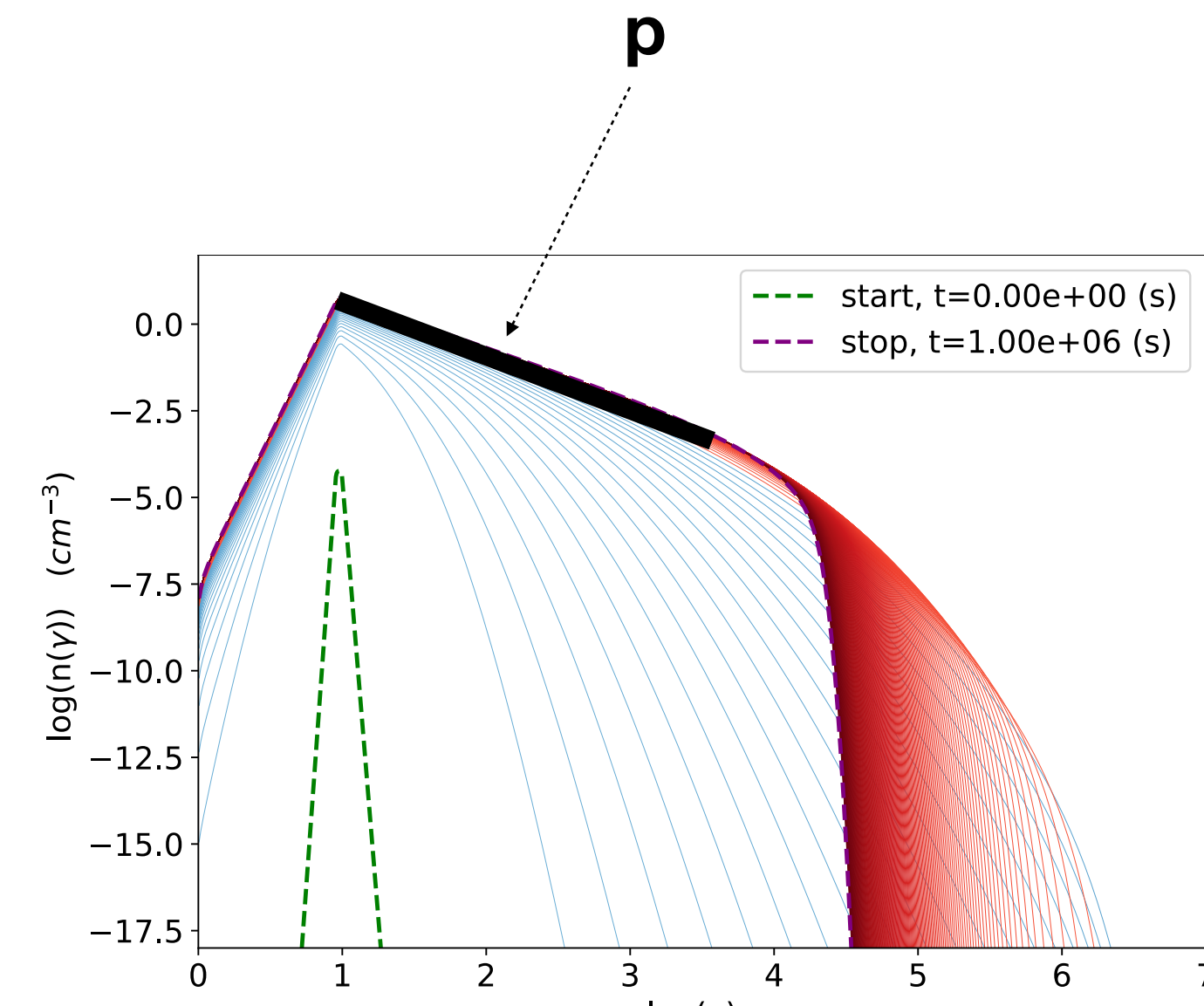
## Tramacere+ 2022



src transp  
at  $\nu^*_{\text{SSA}}$   
 $R=R^*$

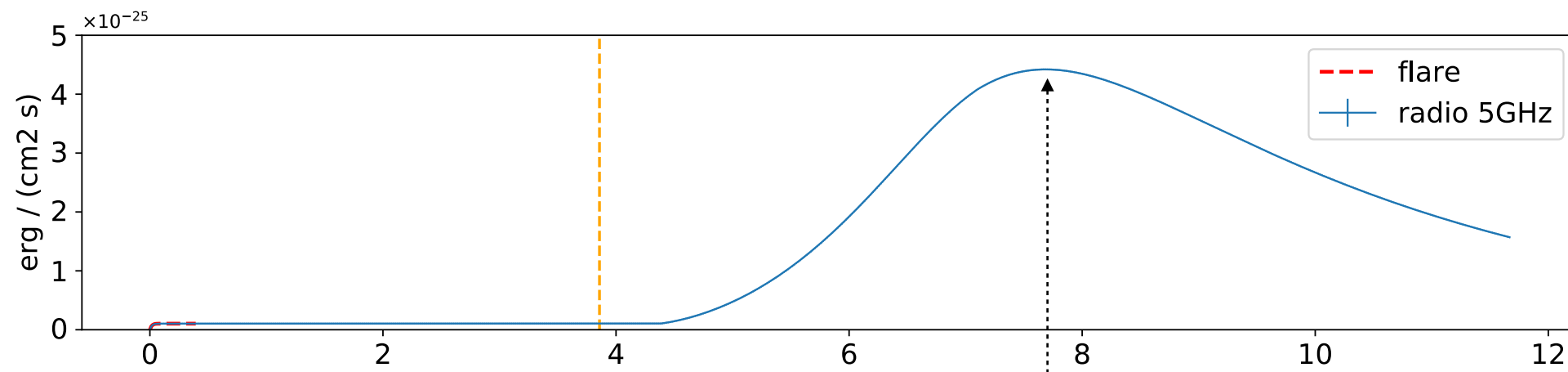
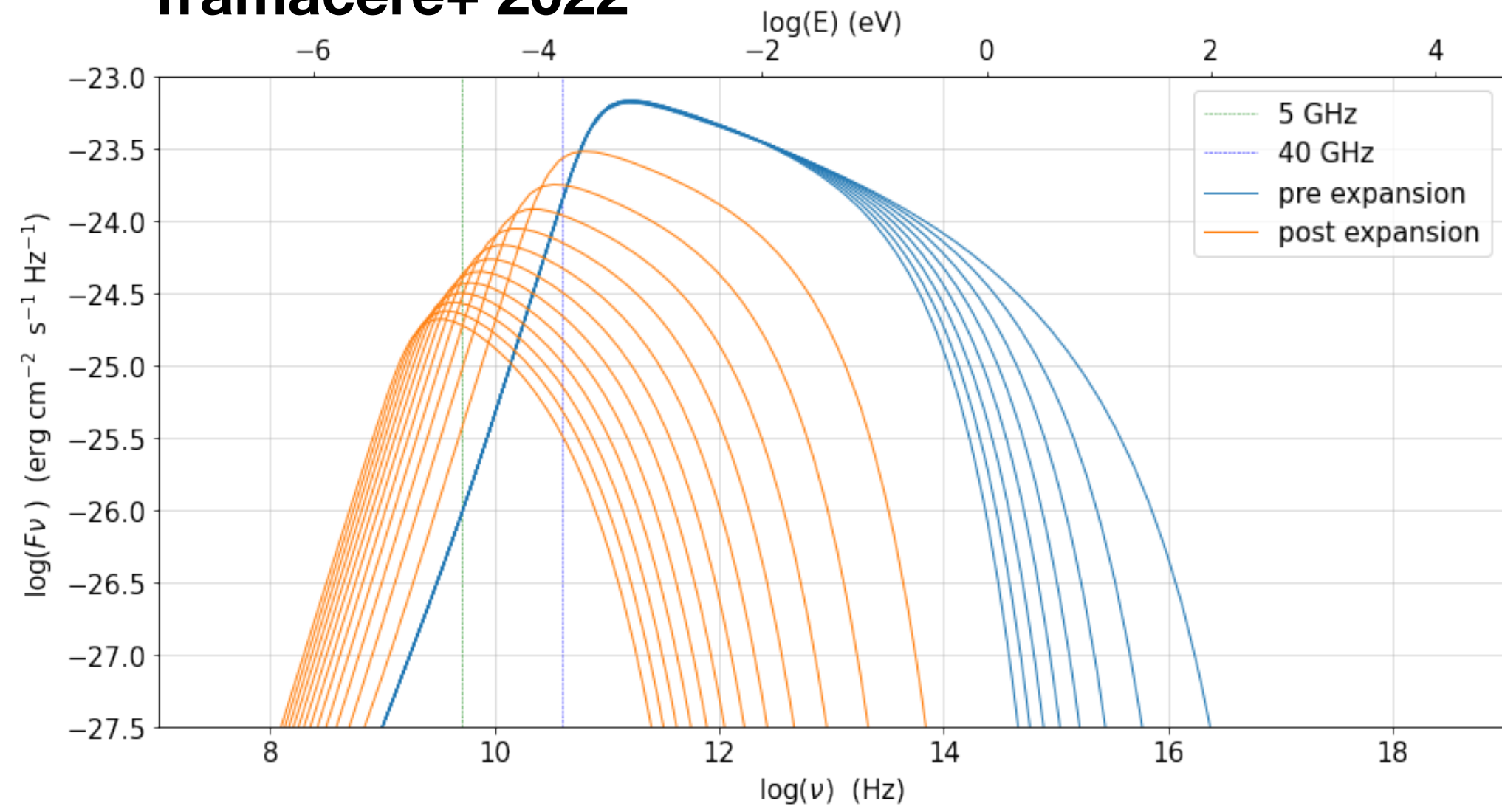
Rybicki&Lightman (1985) standard synchrotron theory

$$\nu_{\text{SSA}}(t) = \nu_L(t) \left[ \frac{\pi \sqrt{\pi} e R(t) N(t)}{4 B(t)} f_k(p) \right]^{\frac{2}{p+4}}$$





## Tramacere+ 2022

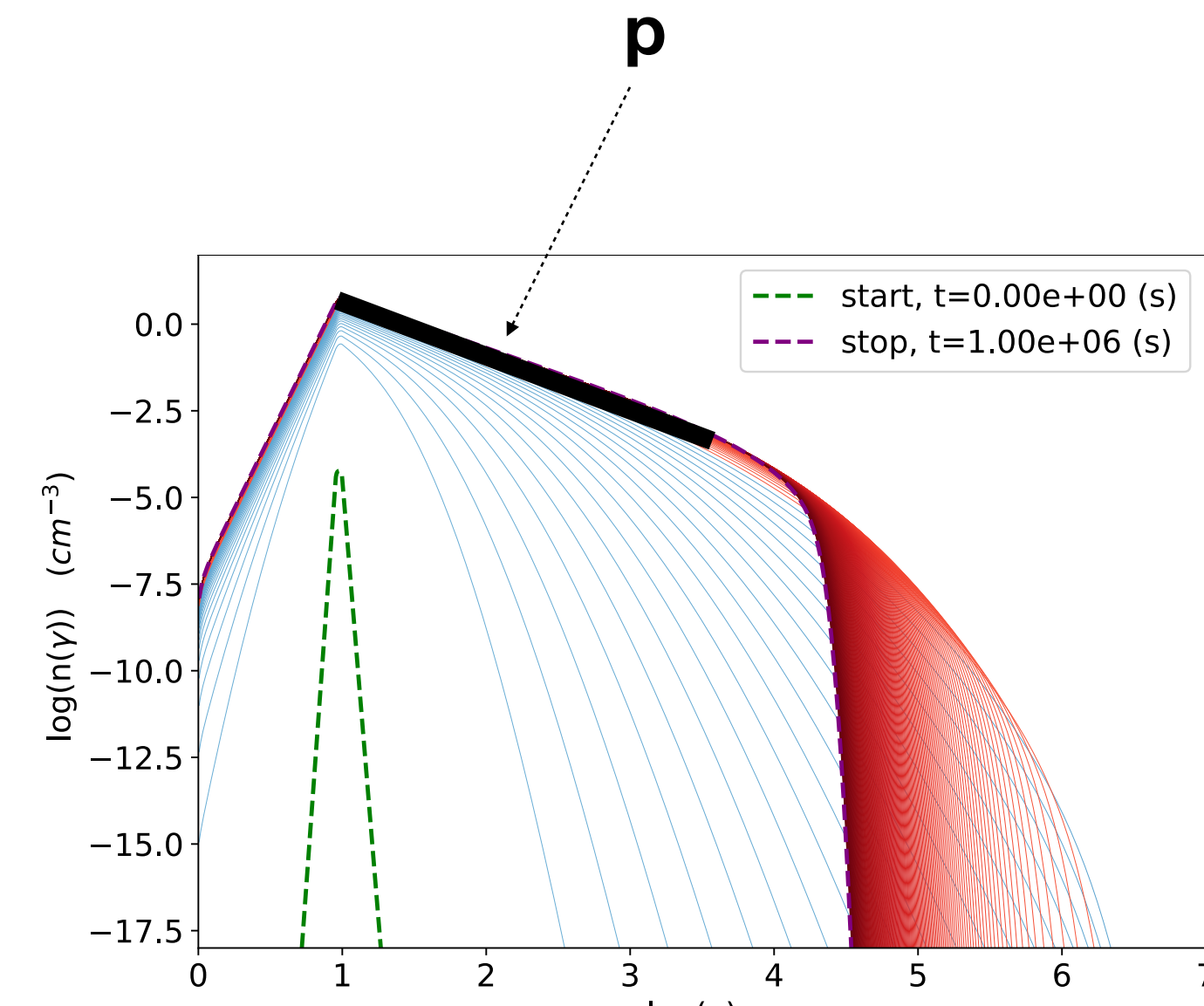


src transp  
at  $\nu^*_{\text{SSA}}$   
 $R=R^*$

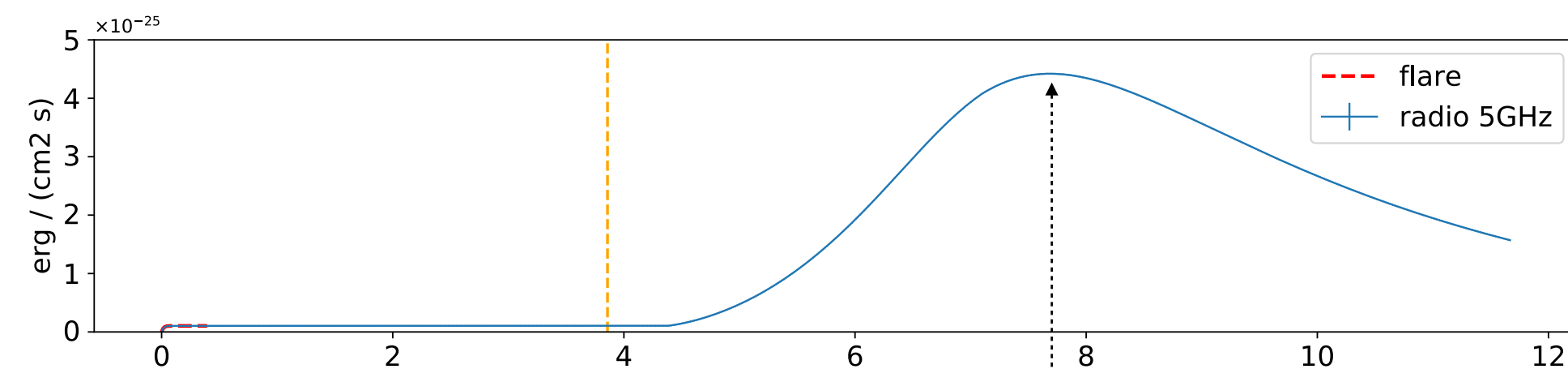
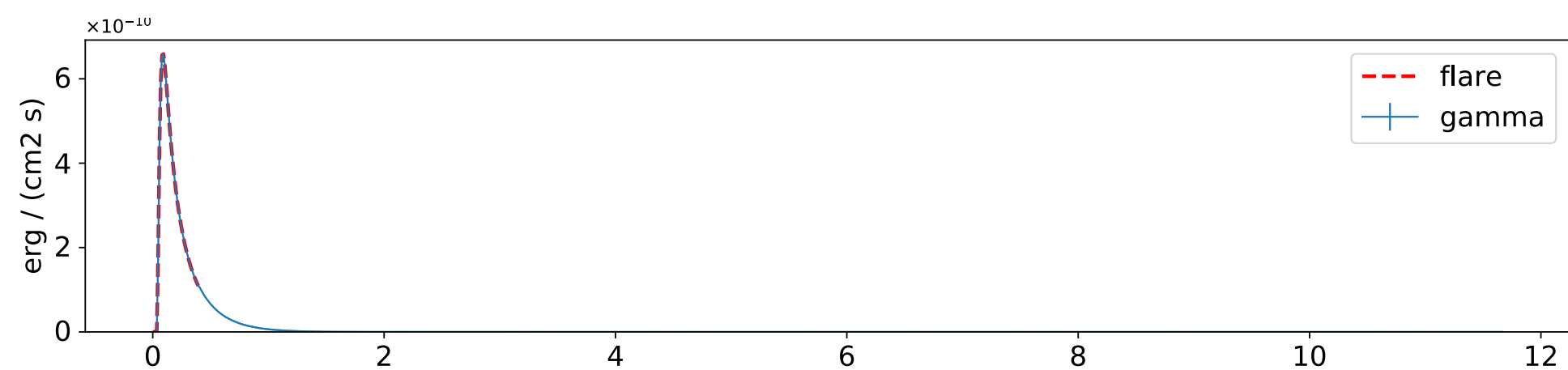
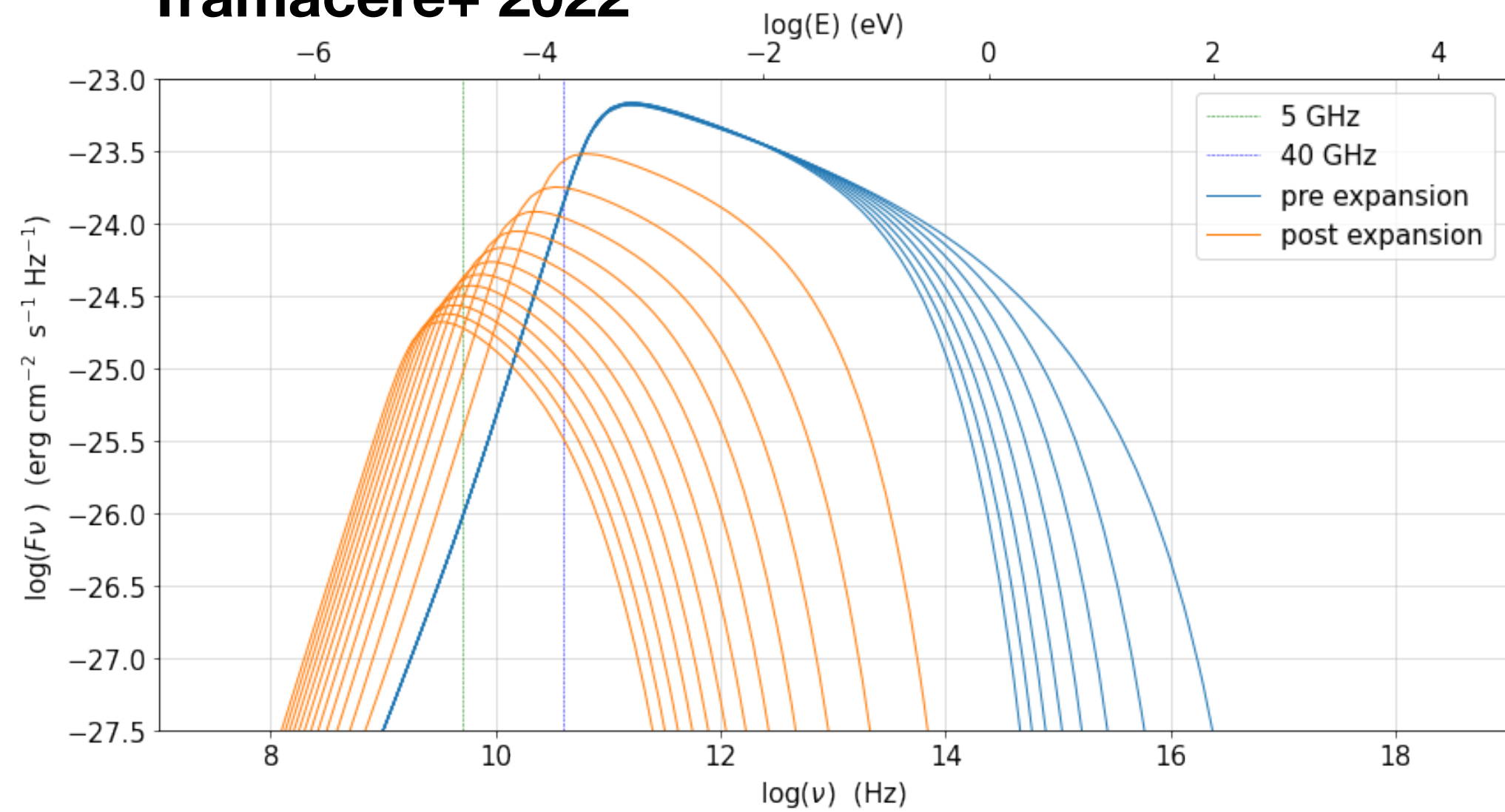
$$\nu_{\text{SSA}}(t) = \nu_L(t) \left[ \frac{\pi \sqrt{\pi} e R(t) N(t)}{4 B(t)} f_k(p) \right]^{\frac{2}{p+4}}$$

$$R(t) = R_0 + \beta_{\text{exp}} c(t - t_{\text{exp}}) H(t - t_{\text{exp}})$$

$$B(t) = B_0 \left( \frac{R_0}{R(t)} \right)^{m_B}$$



## Tramacere+ 2022



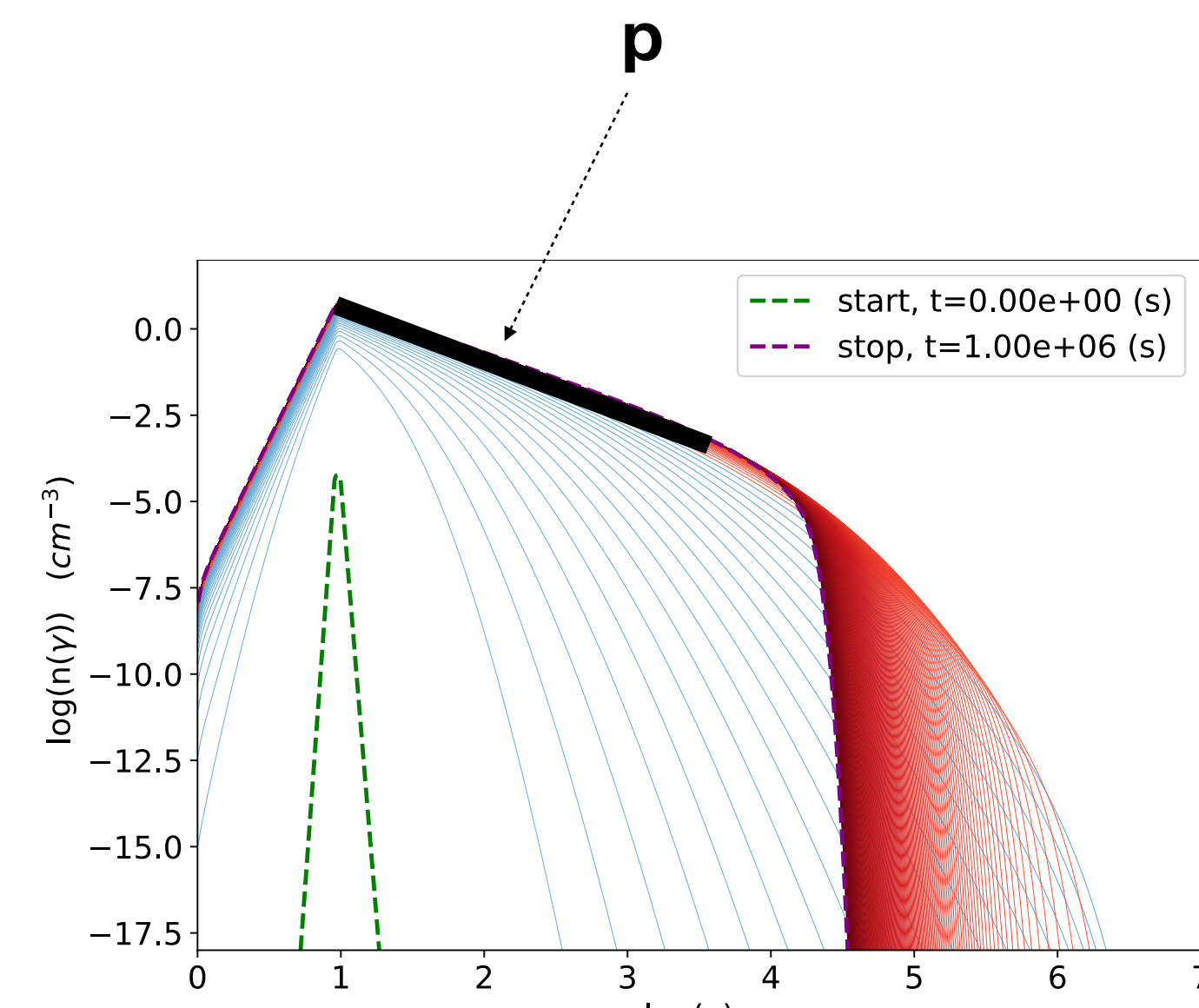
src transp  
at  $\nu_{\text{SSA}}^*$   
 $R=R^*$

$$R(t) = R_0 + \beta_{\text{exp}} c(t - t_{\text{exp}}) H(t - t_{\text{exp}})$$

$$\nu_{\text{SSA}}(t) = \nu_L(t) \left[ \frac{\pi \sqrt{\pi} e R(t) N(t)}{4 B(t)} f_k(p) \right]^{\frac{2}{p+4}}$$

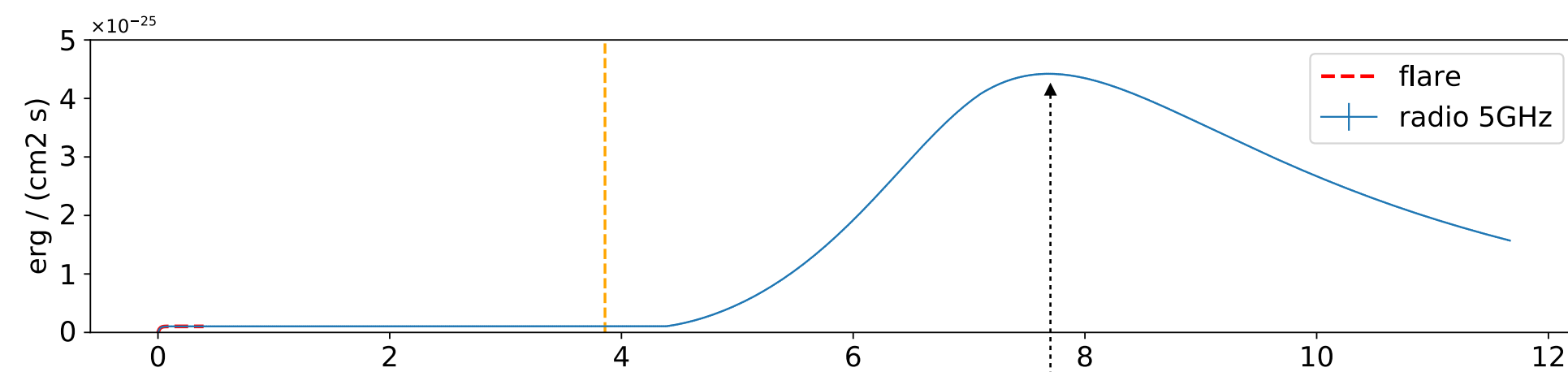
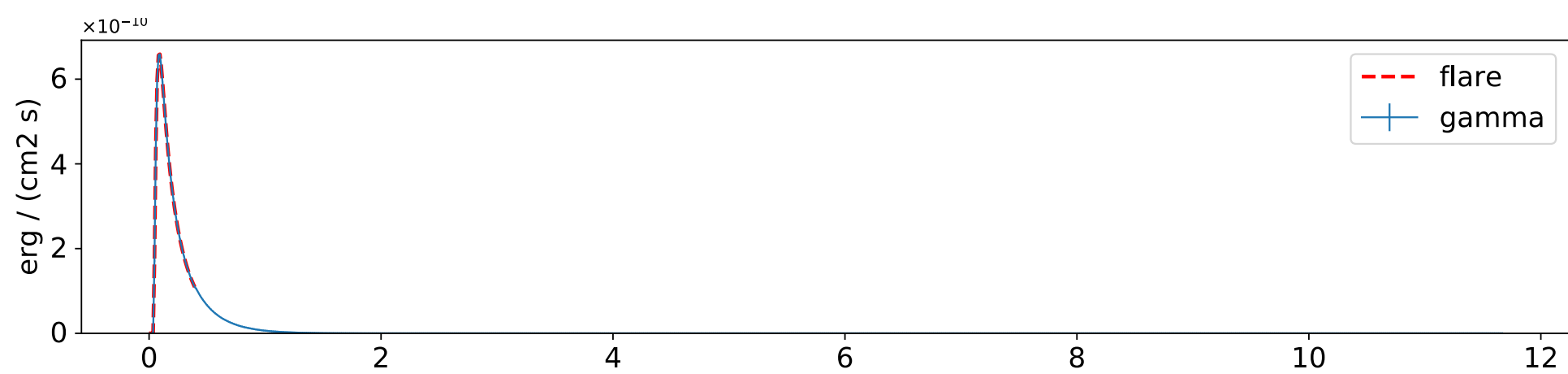
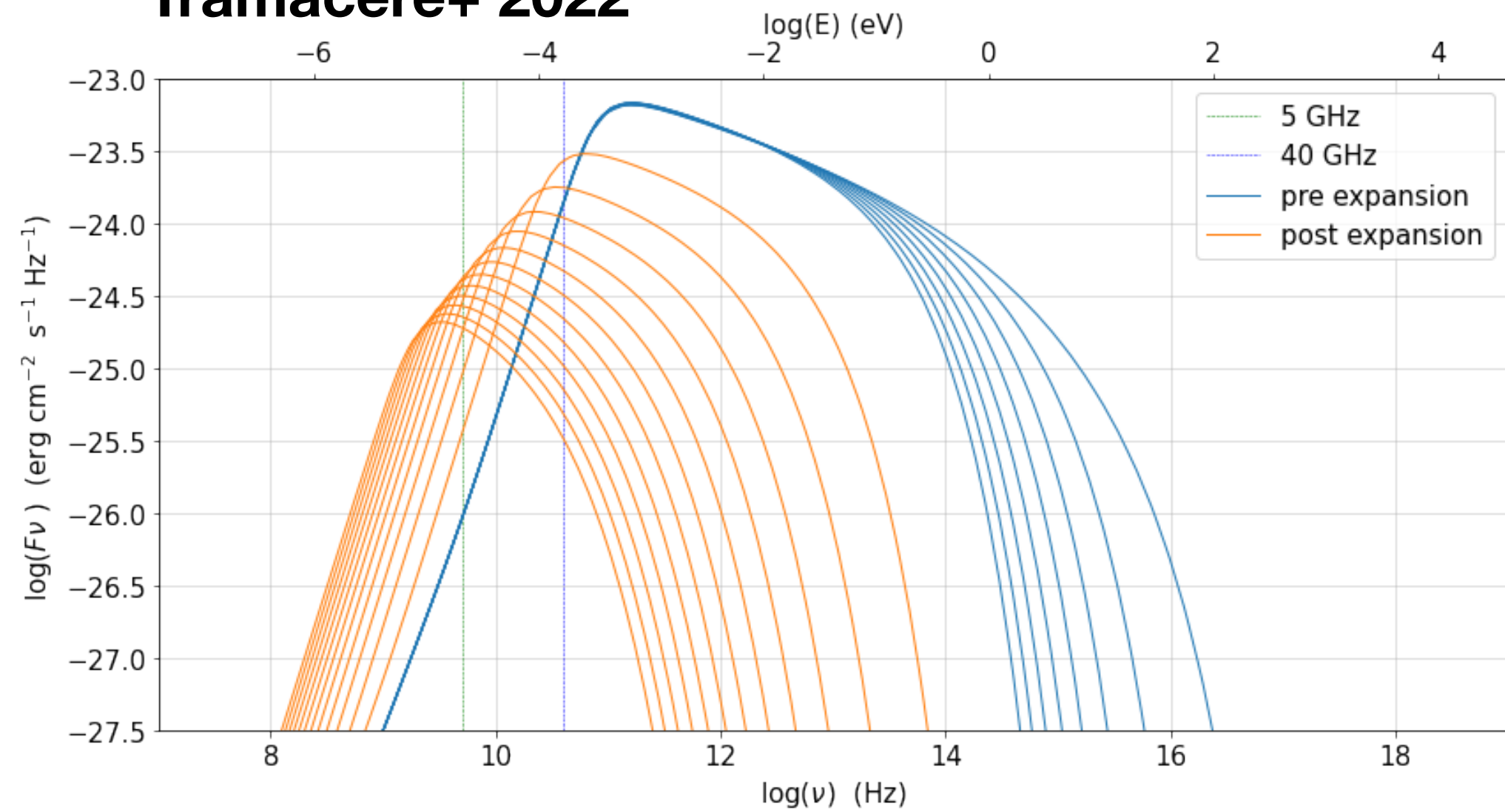
$$B(t) = B_0 \left( \frac{R_0}{R(t)} \right)^{m_B}$$

$$\frac{\nu_{\text{SSA}}^*}{\nu_{\text{SSA}}^0} = \left[ \left( \frac{B^*}{B_0} \right)^{\frac{p+2}{2}} \left( \frac{R_0}{R^*} \right)^2 \right]^{\frac{2}{p+4}} = \left[ \frac{R_0}{R^*} \right]^{\frac{m_B(p+2)+4}{p+4}}$$





## Tramacere+ 2022



src transp  
at  $\nu_{\text{SSA}}^*$   
 $R=R^*$

$$R(t) = R_0 + \beta_{\text{exp}} c(t - t_{\text{exp}}) H(t - t_{\text{exp}})$$

$$\nu_{\text{SSA}}(t) = \nu_L(t) \left[ \frac{\pi \sqrt{\pi}}{4} \frac{eR(t)N(t)}{B(t)} f_k(p) \right]^{\frac{2}{p+4}}$$

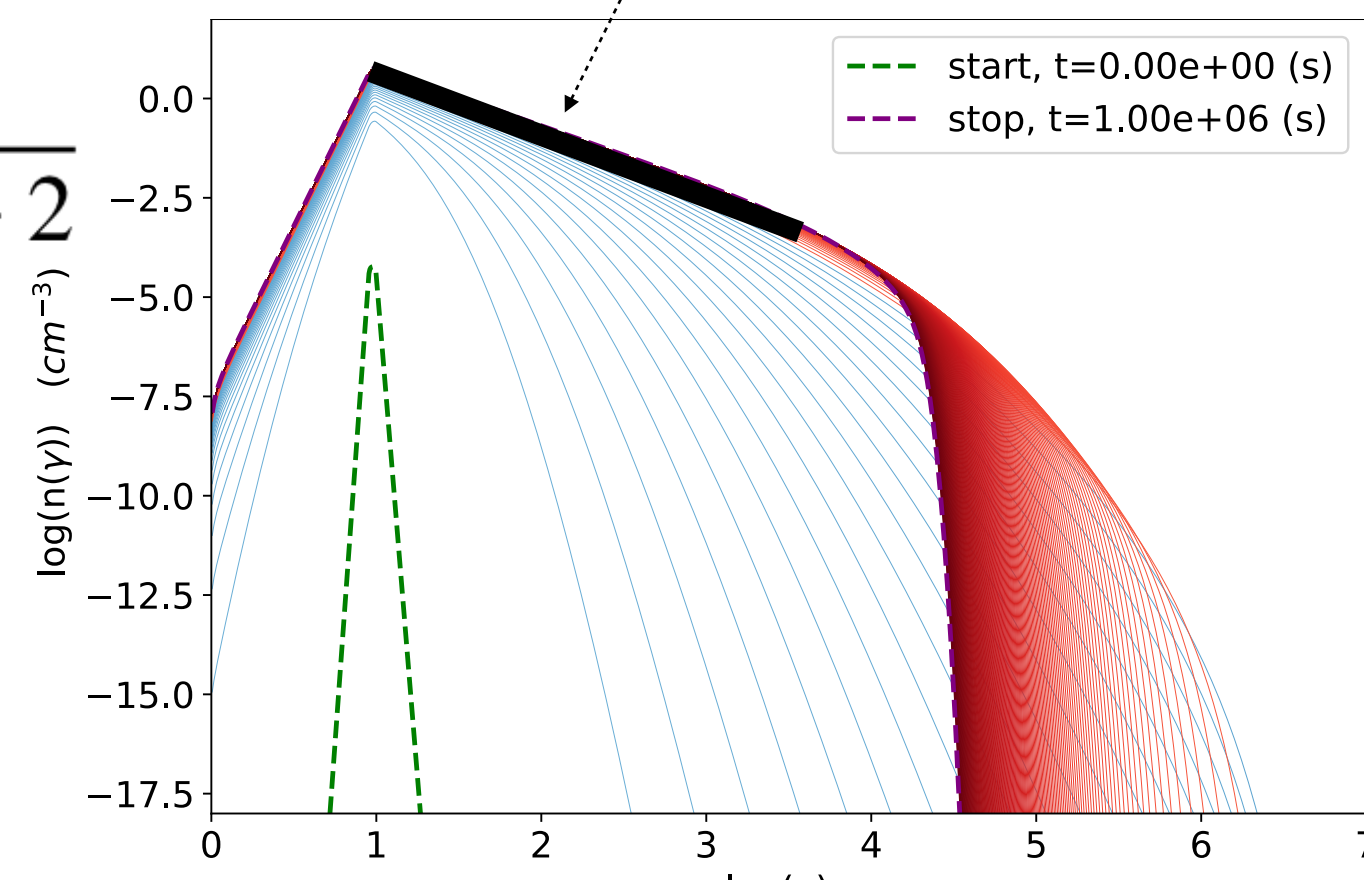
$$B(t) = B_0 \left( \frac{R_0}{R(t)} \right)^{m_B}$$

$$\frac{\nu_{\text{SSA}}^*}{\nu_{\text{SSA}}^0} = \left[ \left( \frac{B^*}{B_0} \right)^{\frac{p+2}{2}} \left( \frac{R_0}{R^*} \right)^2 \right]^{\frac{2}{p+4}} = \left[ \frac{R_0}{R^*} \right]^{\frac{m_B(p+2)+4}{p+4}}$$

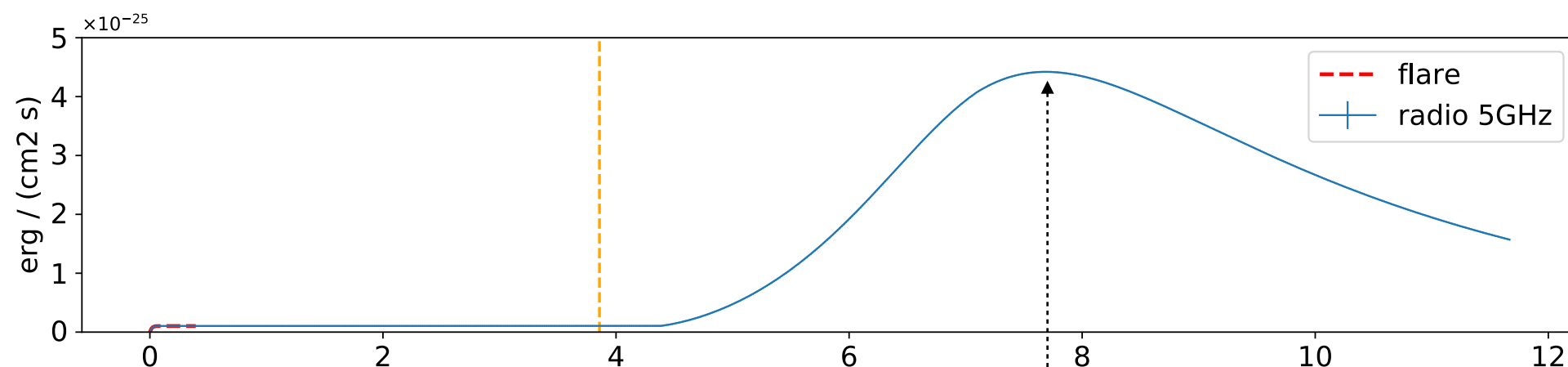
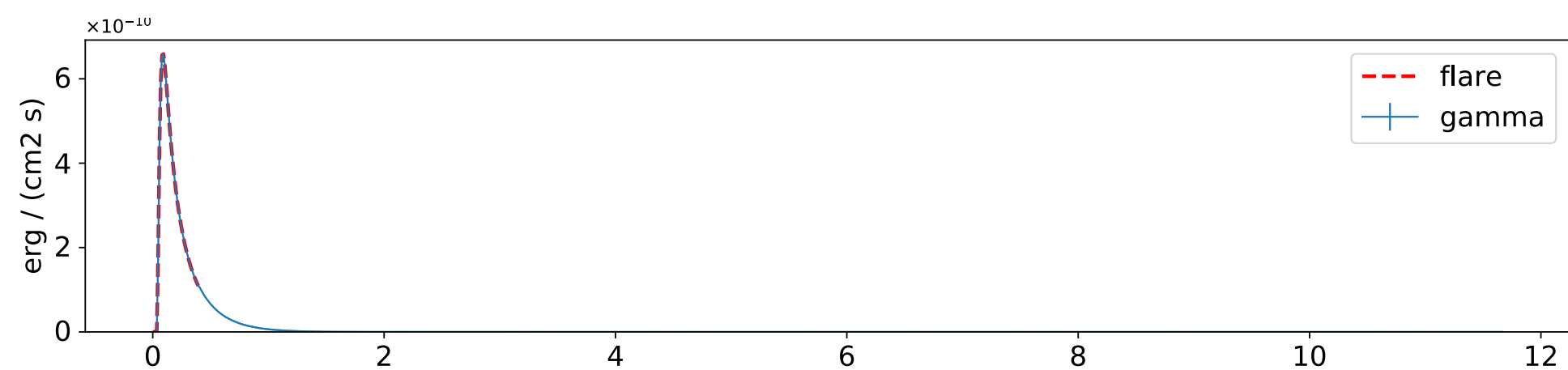
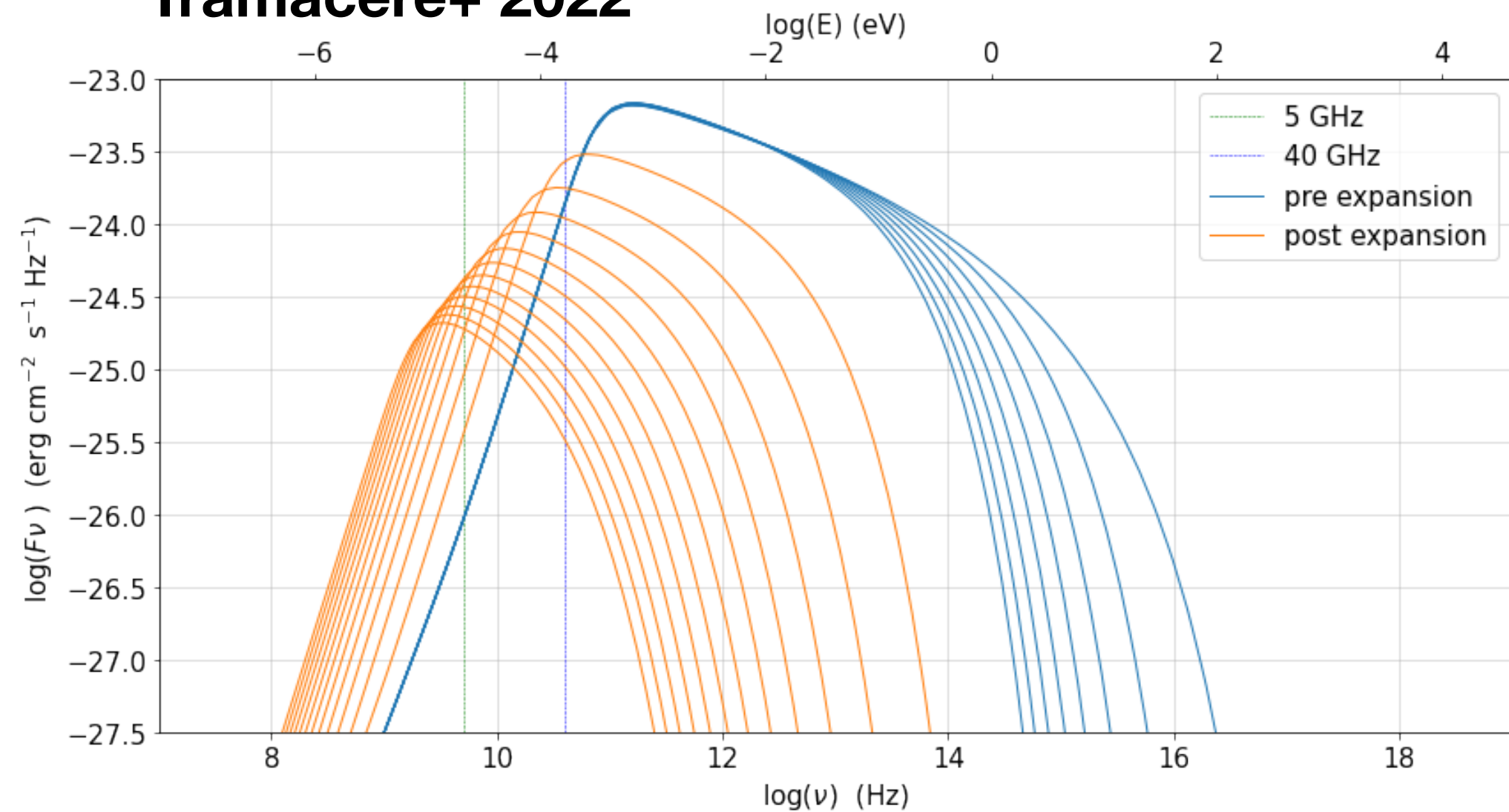
invert

$$R^* = R_0 \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi$$

$$\psi = \frac{p+4}{m_B(p+2)-2}$$



## Tramacere+ 2022



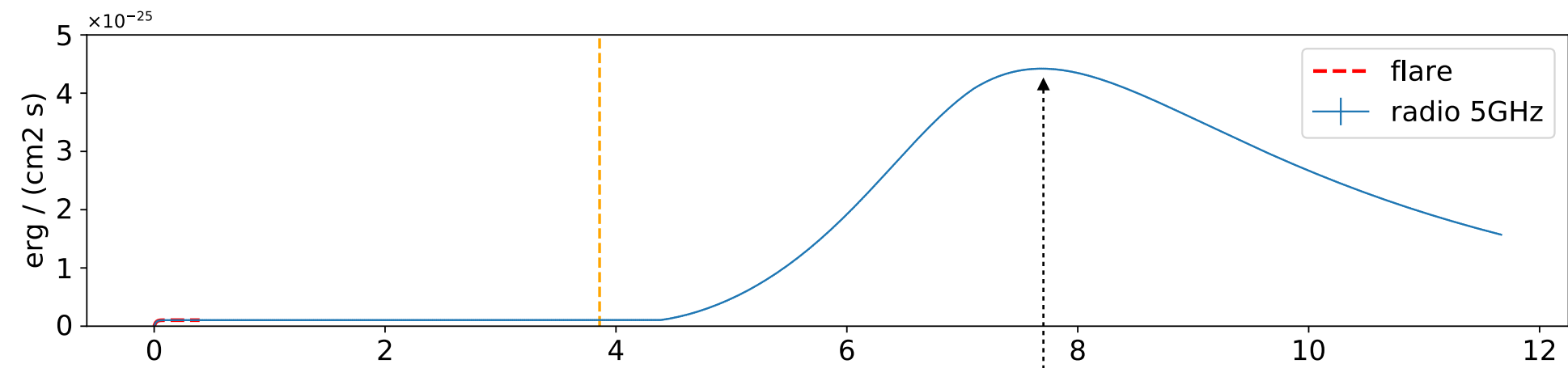
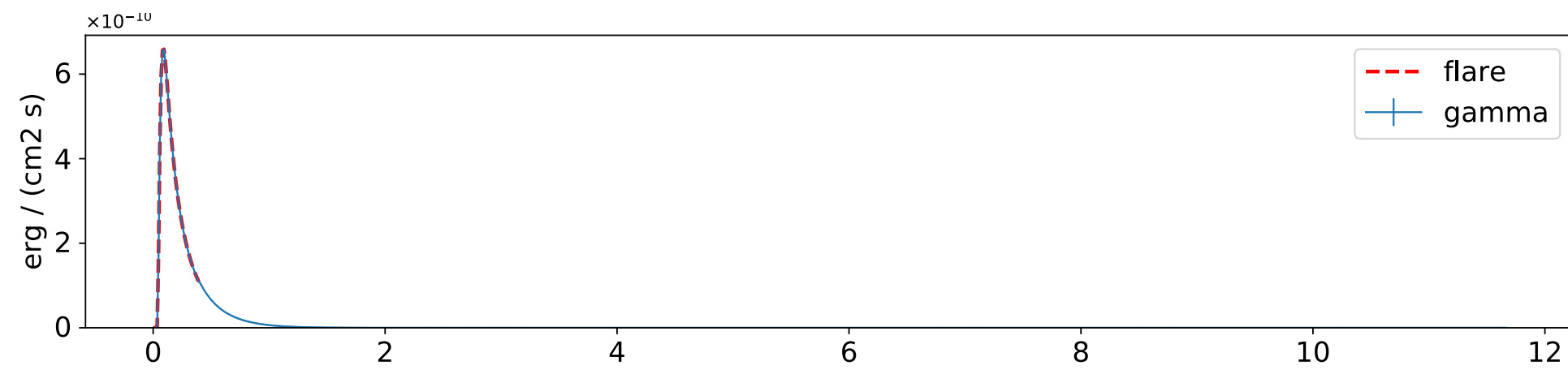
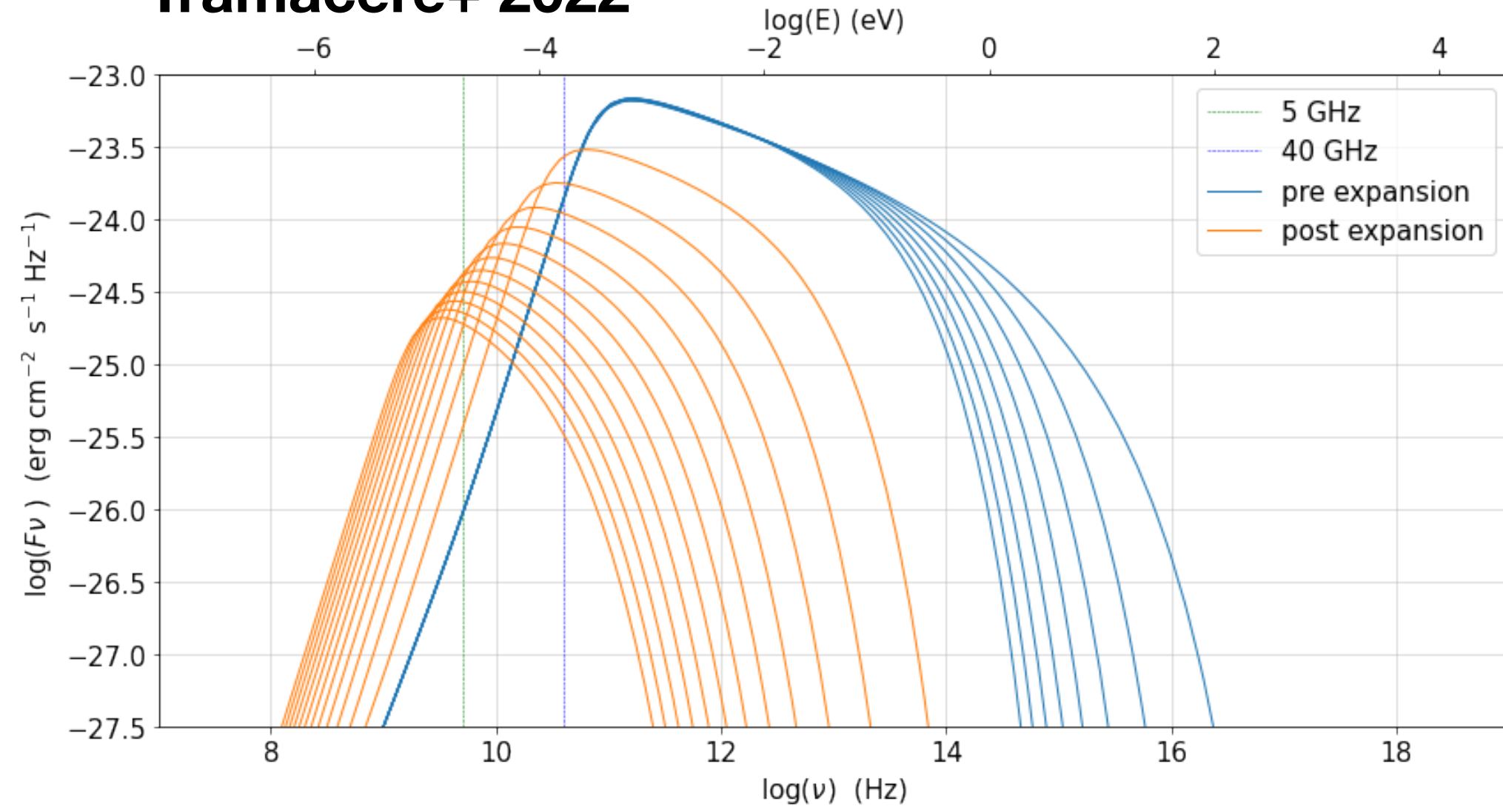
src transp  
at  $\nu_{\text{SSA}}^*$   
 $R=R^*$

$$\Delta t_{\nu_{\text{SSA}}^0 \rightarrow \nu_{\text{SSA}}^*} = t_{\text{exp}} + t_{\text{peak}}$$

$$R^* = R_0 \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi$$

$$\psi = \frac{p + 4}{m_B(p + 2) - 2}$$

## Tramacere+ 2022



src transp  
at  $\nu_{\text{SSA}}^*$   
 $R=R^*$

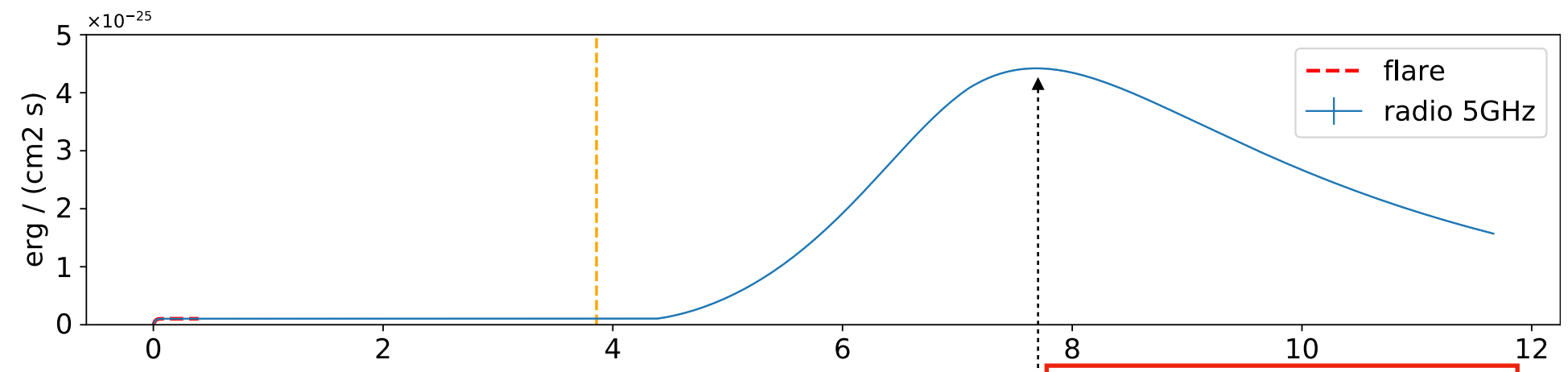
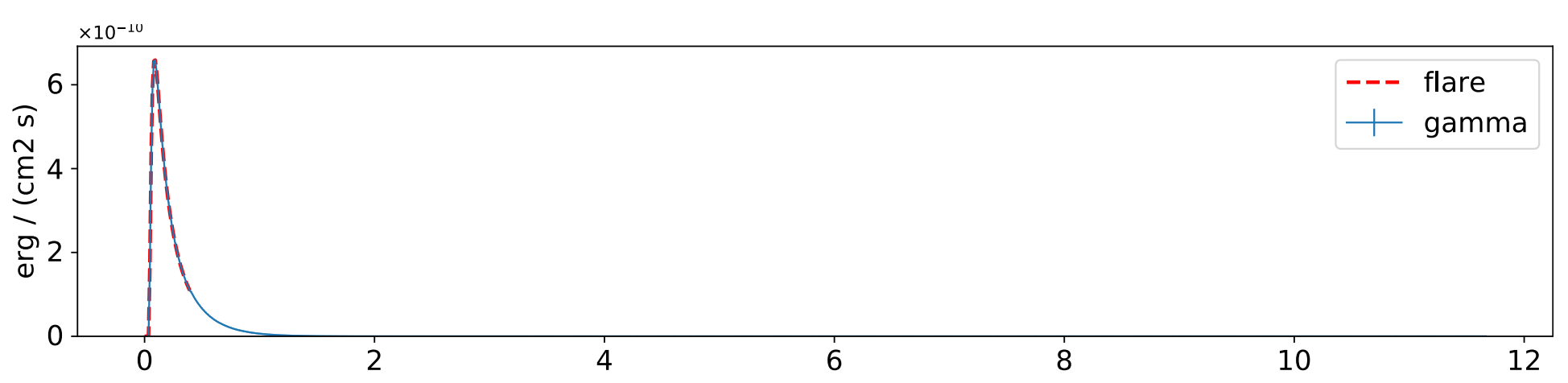
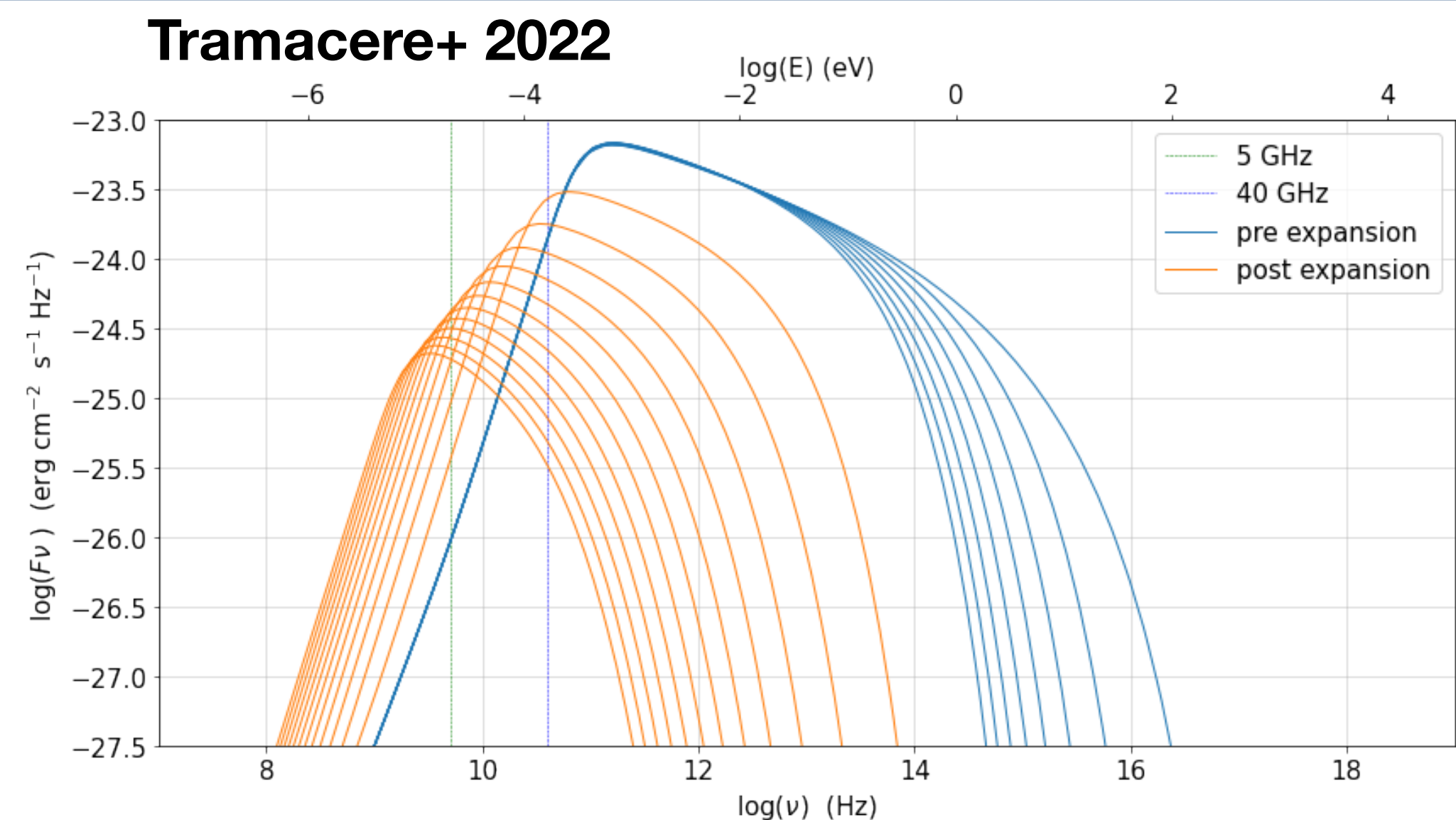
$$\Delta t_{\nu_{\text{SSA}}^0 \rightarrow \nu_{\text{SSA}}^*} = t_{\text{exp}} + t_{\text{peak}}$$

$$t_{\text{peak}} = \Delta t_{R_0 \rightarrow R^*} = \frac{R^* - R_0}{\beta_{\text{exp}} c} = \frac{R_0}{\beta_{\text{exp}} c} \left[ \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi - 1 \right]$$

$$R^* = R_0 \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi$$

$$\psi = \frac{p + 4}{m_B(p + 2) - 2}$$





src transp  
at  $\nu_{SSA}^*$   
 $R=R^*$

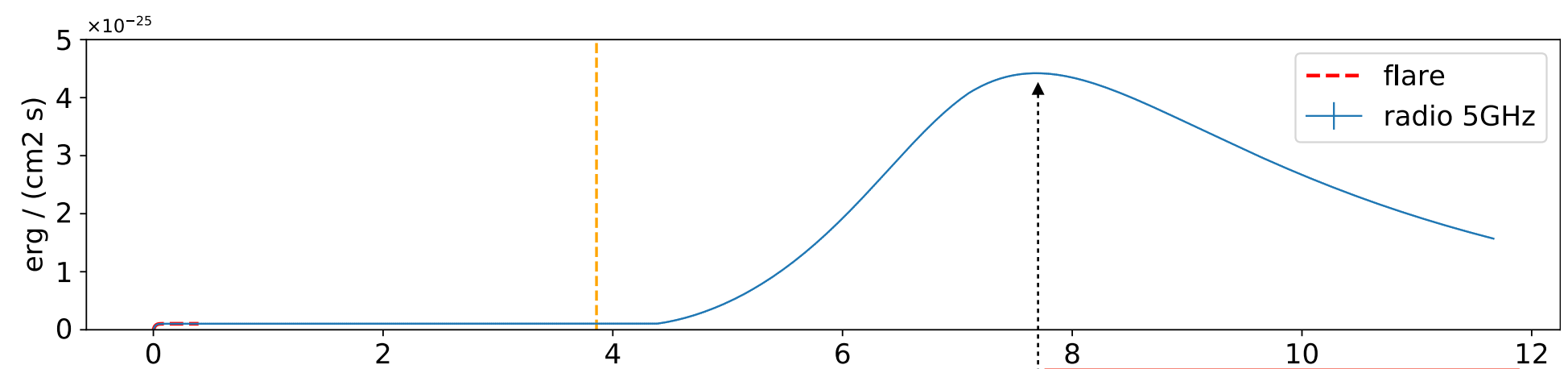
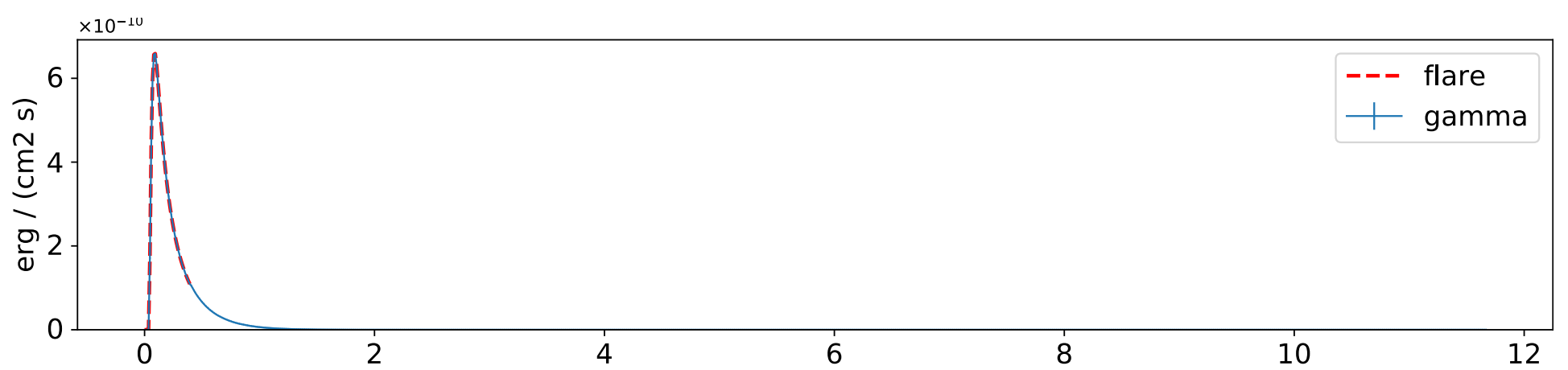
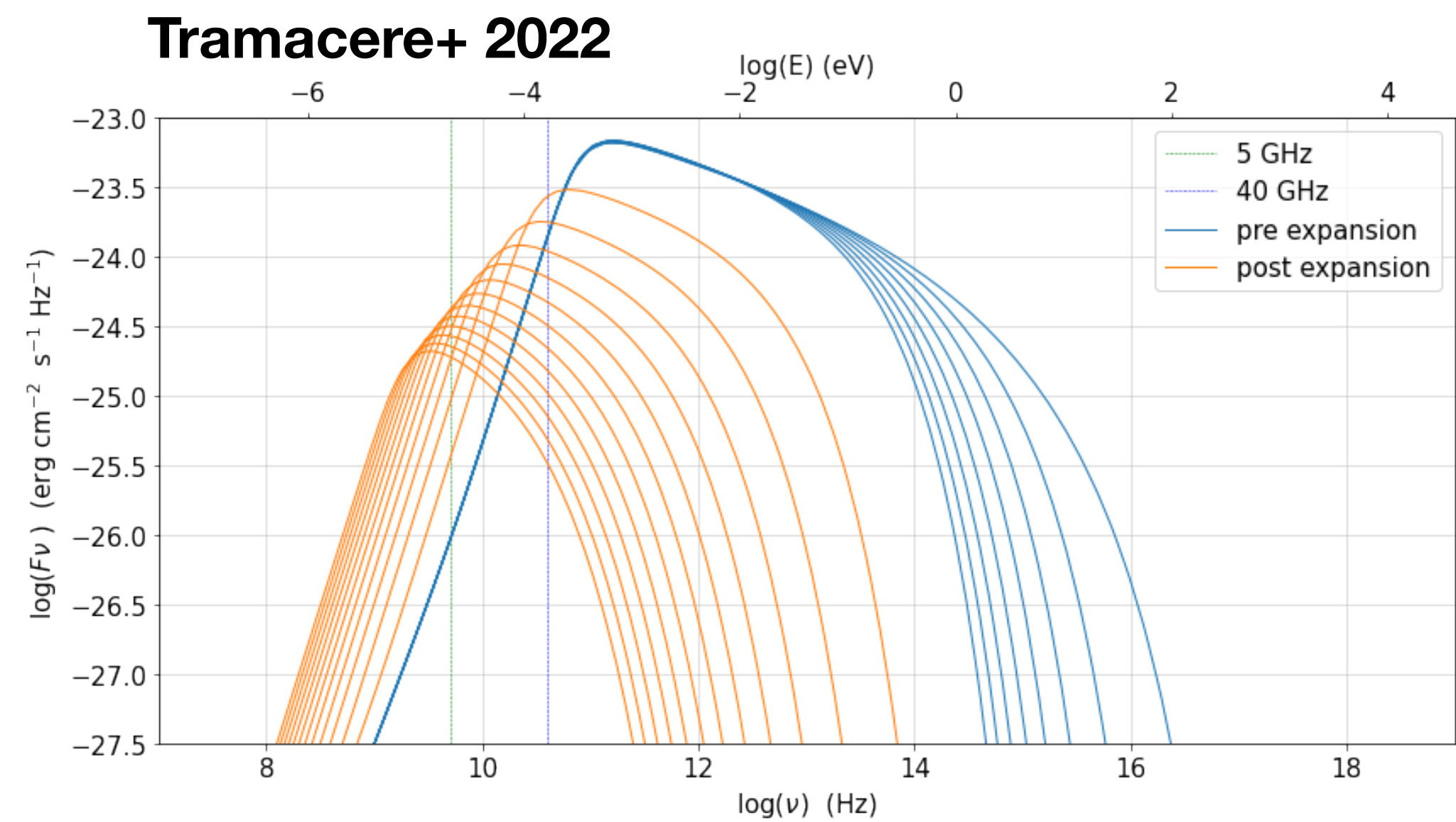
$$\Delta t_{\nu_{SSA}^0 \rightarrow \nu_{SSA}^*} = t_{exp} + t_{peak}$$

$$t_{peak} = \Delta t_{R_0 \rightarrow R^*} = \frac{R^* - R_0}{\beta_{exp} c} = \frac{R_0}{\beta_{exp} c} \left[ \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi - 1 \right]$$

$$t_{decay} \rightarrow t_{decay}^{ad}(t^*) \propto \frac{R^*}{\beta_{exp} c} = \frac{R_0}{\beta_{exp} c} \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi$$

$$R^* = R_0 \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi$$

$$\psi = \frac{p + 4}{m_B(p + 2) - 2}$$



src transp  
at  $\nu_{SSA}^*$   
 $R=R^*$

$$\Delta t_{\nu_{SSA}^0 \rightarrow \nu_{SSA}^*} = t_{exp} + t_{peak}$$

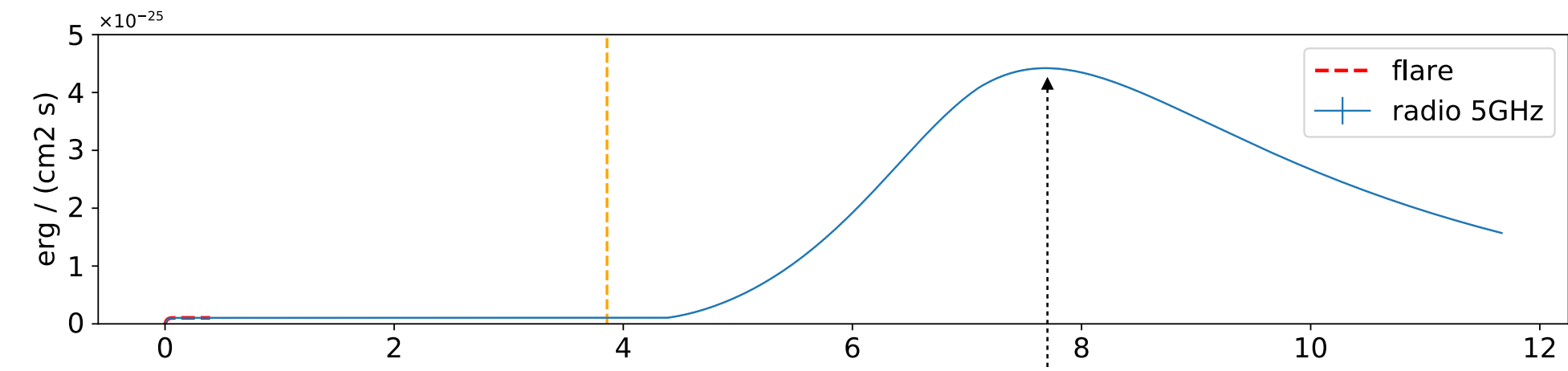
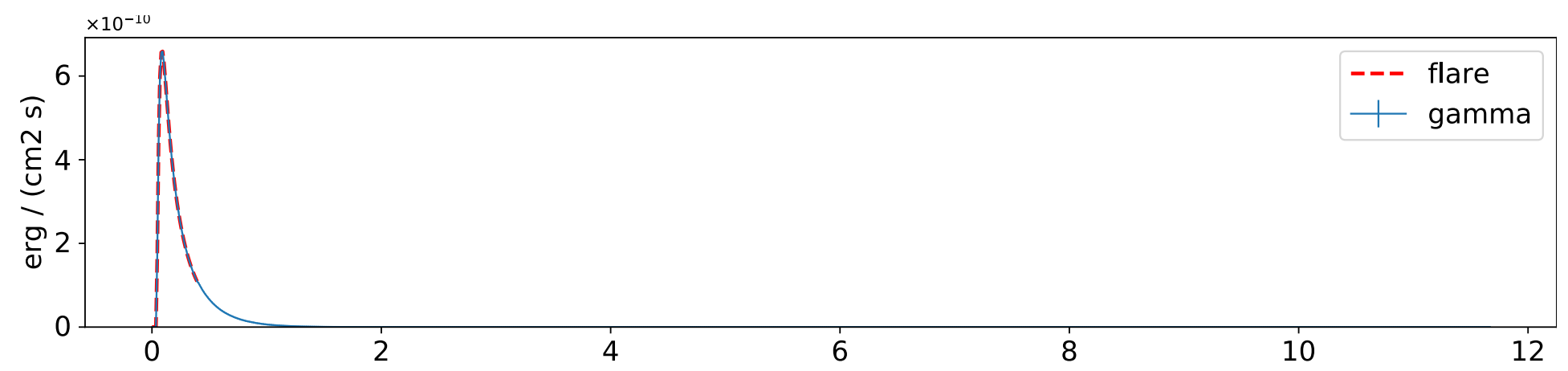
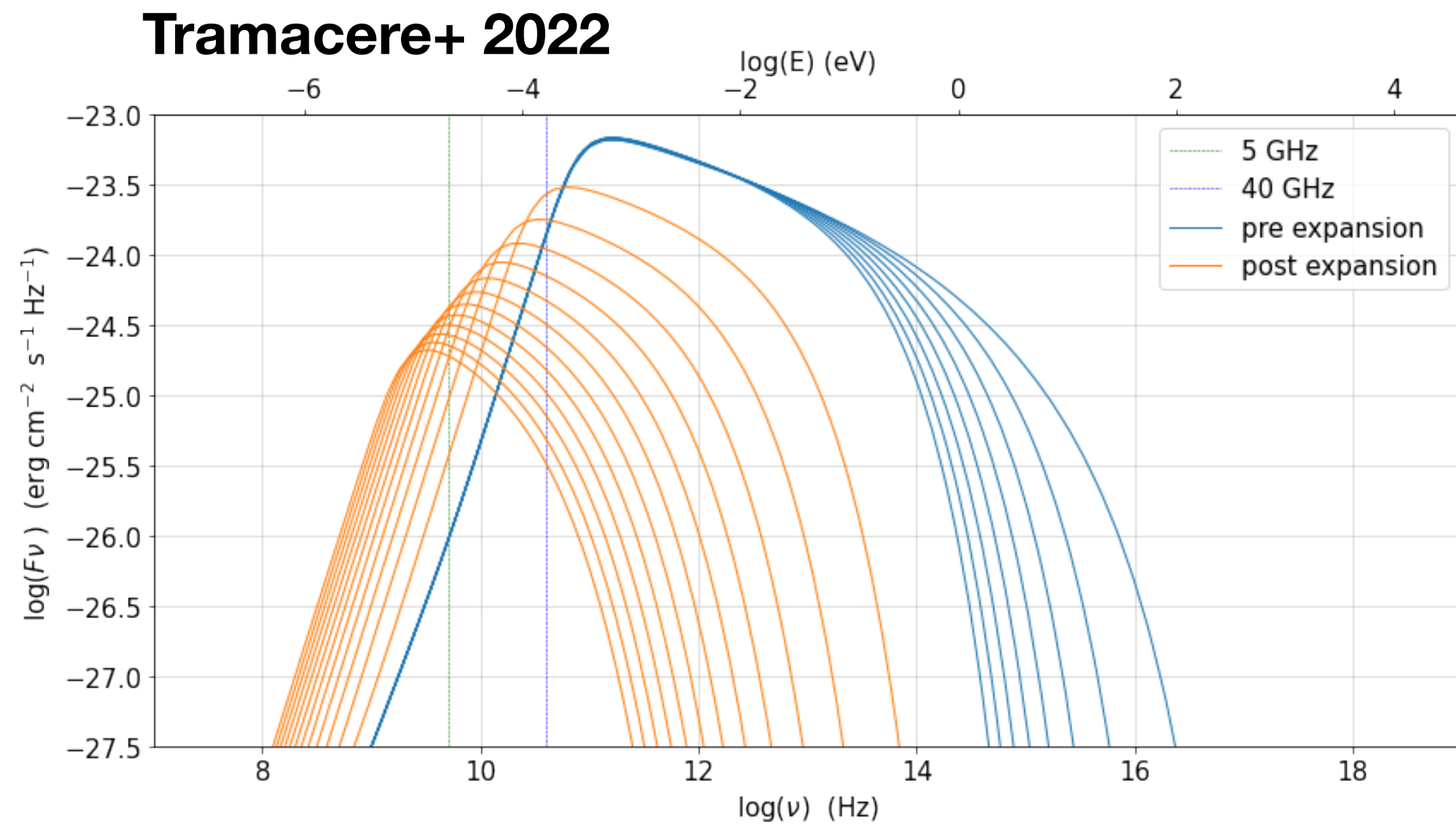
$$t_{peak} = \Delta t_{R_0 \rightarrow R^*} = \frac{R^* - R_0}{\beta_{exp} c} = \frac{R_0}{\beta_{exp} c} \left[ \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi - 1 \right]$$

$t_{decay}$  branches into:

- radiative**:  $t_{decay}^{ad}(t^*) \propto \frac{R^*}{\beta_{exp} c} = \frac{R_0}{\beta_{exp} c} \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi$
- geom**:  $t_{decay}^{geom}(t^*) \propto \frac{F_{\nu_{SSA}}(t^*)}{\dot{F}_{\nu_{SSA}}(t^*)} \propto \frac{R^*}{m_B \beta_{exp} c} = \frac{t_{decay}^{ad}(t^*)}{m_B}$

$$R^* = R_0 \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi$$

$$\psi = \frac{p + 4}{m_B(p + 2) - 2}$$



src transp  
at  $\nu_{SSA}^*$   
 $R=R^*$

## Tramacere+ 2022

$$\Delta t_{\nu_{SSA}^0 \rightarrow \nu_{SSA}^*} = t_{\text{exp}} + t_{\text{peak}} = t_{\text{exp}}^{\text{obs}} + \frac{t_{\text{var}}^{\text{obs}}}{\beta_{\text{exp}}} \left[ \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi - 1 \right]$$

$$t_{\text{peak}}^{\text{obs}} = \frac{t_{\text{var}}^{\text{obs}}}{\beta_{\text{exp}}} \left[ \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi - 1 \right]$$

$$t_{\text{decay}}^{\text{obs}} = \frac{t_{\text{var}}^{\text{obs}}}{m_B \beta_{\text{exp}}} \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi,$$

$$R^* = R_0 \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi$$

$$\psi = \frac{p+4}{m_B(p+2) - 2}$$

$$\delta = \frac{1}{\Gamma(1 - \beta_\Gamma \cos(\theta))} \quad \nu^{\text{obs}} = \nu \frac{\delta}{z+1}$$

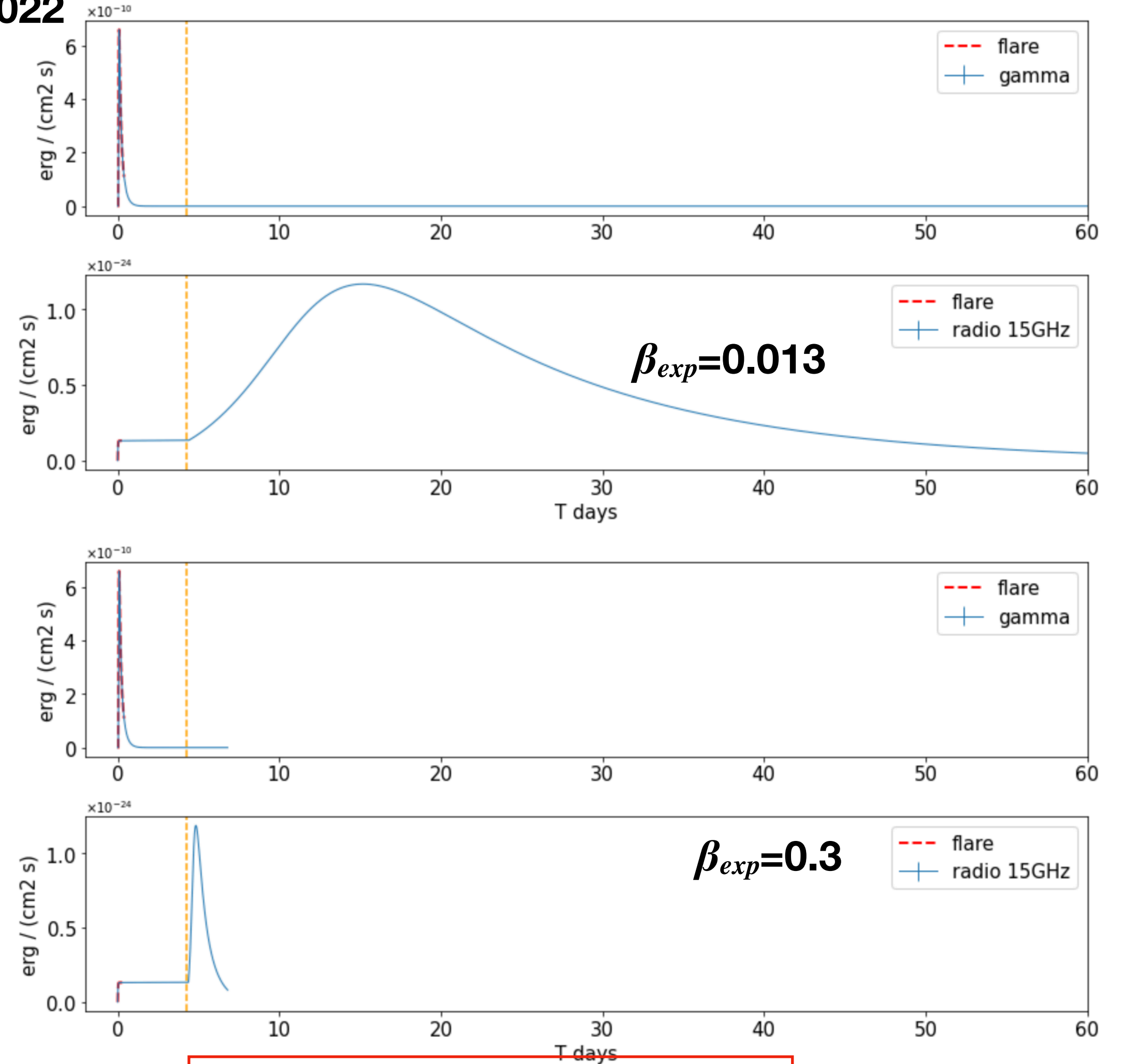
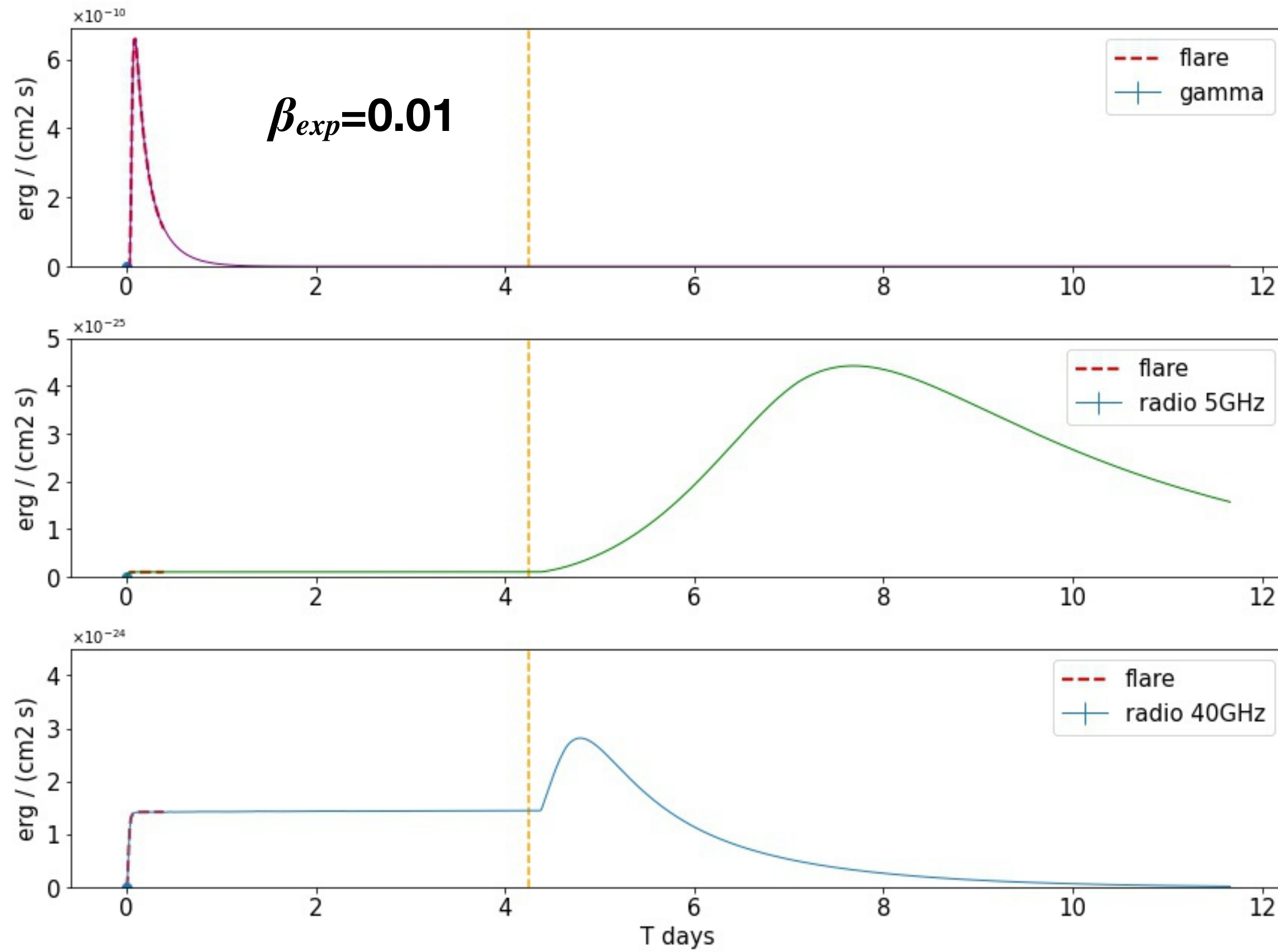
$$t_{\text{var}}^{\text{obs}} = \frac{(1+z)R_0}{\delta c}$$

$$R_{\text{obs}}^0 = R_0 \frac{1+z}{\delta}$$

$$\Delta_r = \Gamma \Delta t_{\nu_{SSA}^0 \rightarrow \nu_{SSA}^*} \beta_\Gamma c = \frac{\delta \Gamma \beta_\Gamma c \Delta t_{\nu_{SSA}^0, \text{obs}}^{\text{obs}}}{1+z}$$



Tramacere+ 2022



$$t_r \sim \nu^0_{SSA} / \nu^*_{SSA}$$

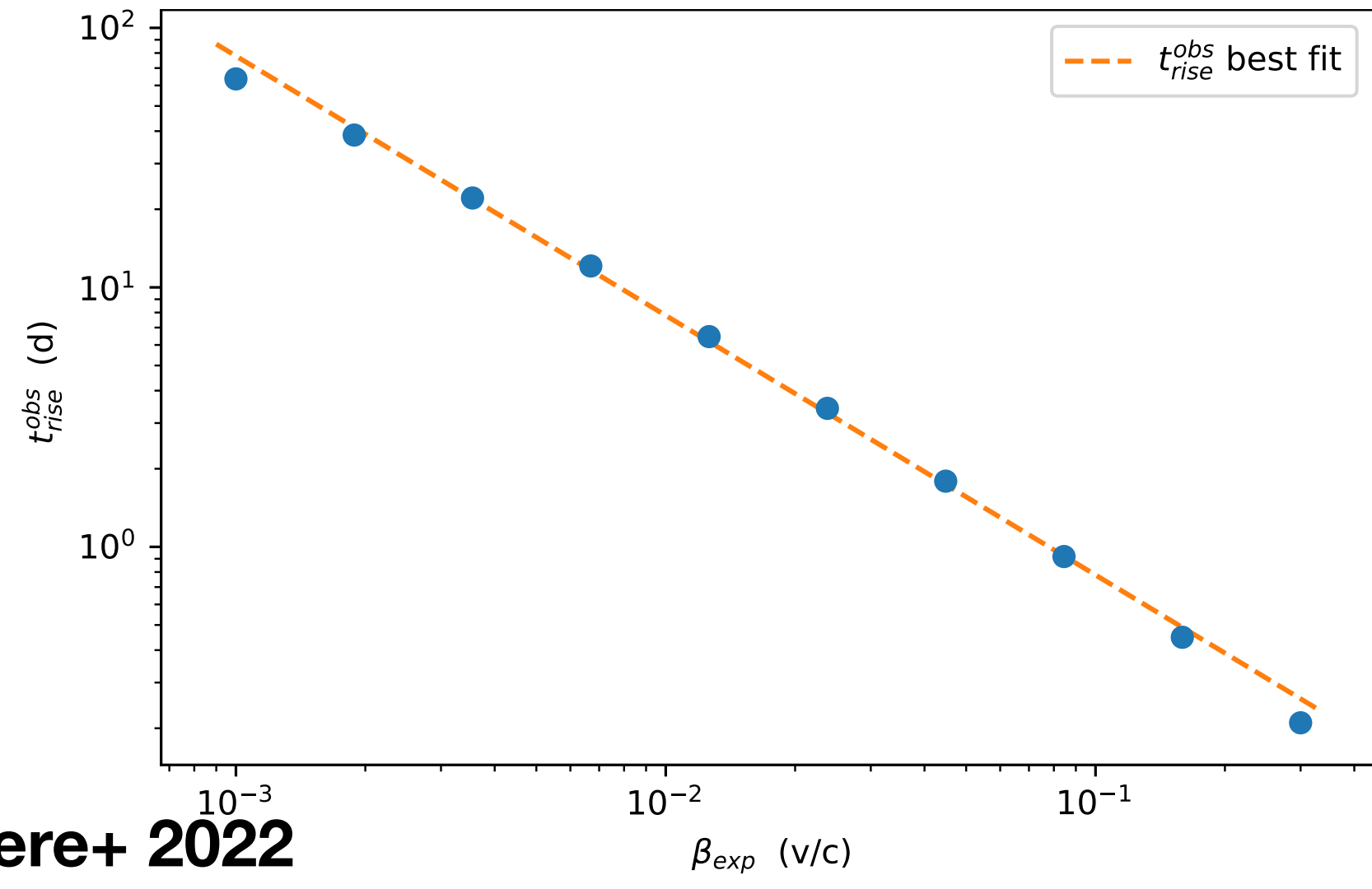
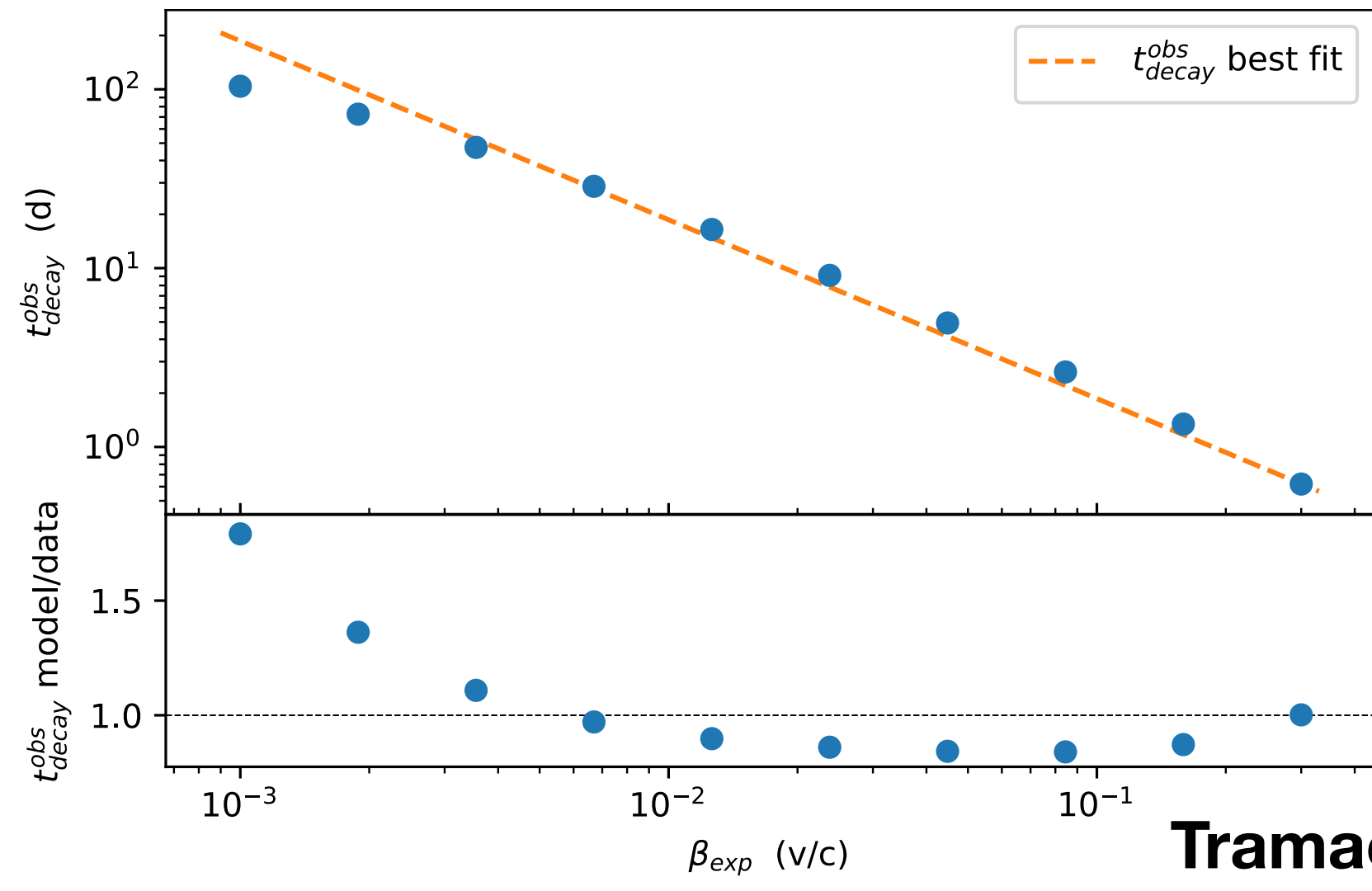
$$t_d \sim \nu^0_{SSA} / \nu^*_{SSA}$$

$$\Delta t \sim t_{exp} + t_r$$

$$t_r \sim 1 / \beta_{exp}$$

$$t_d \sim 1 / \beta_{exp}$$

$$\Delta t \sim t_{exp} + t_r$$



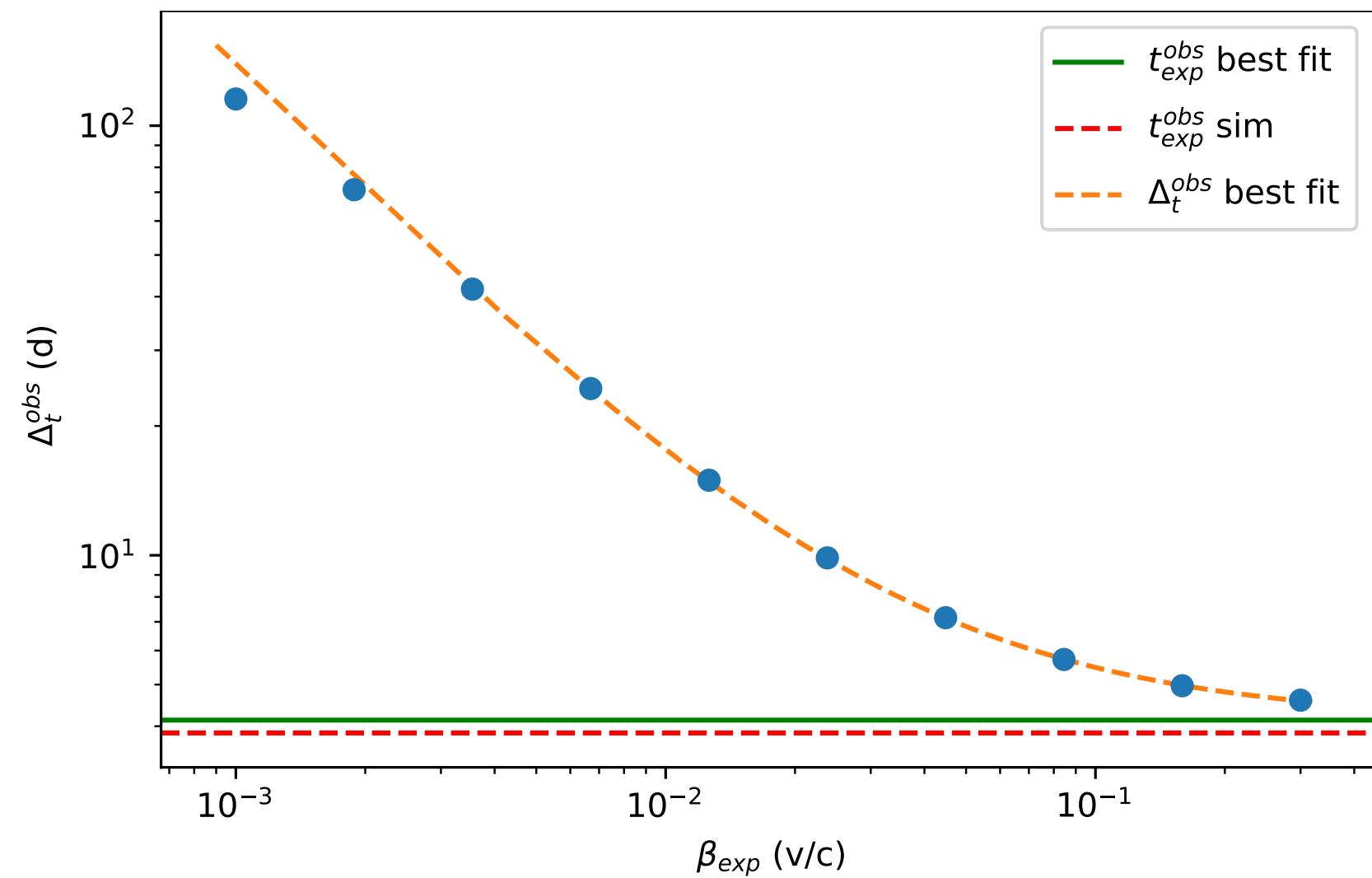
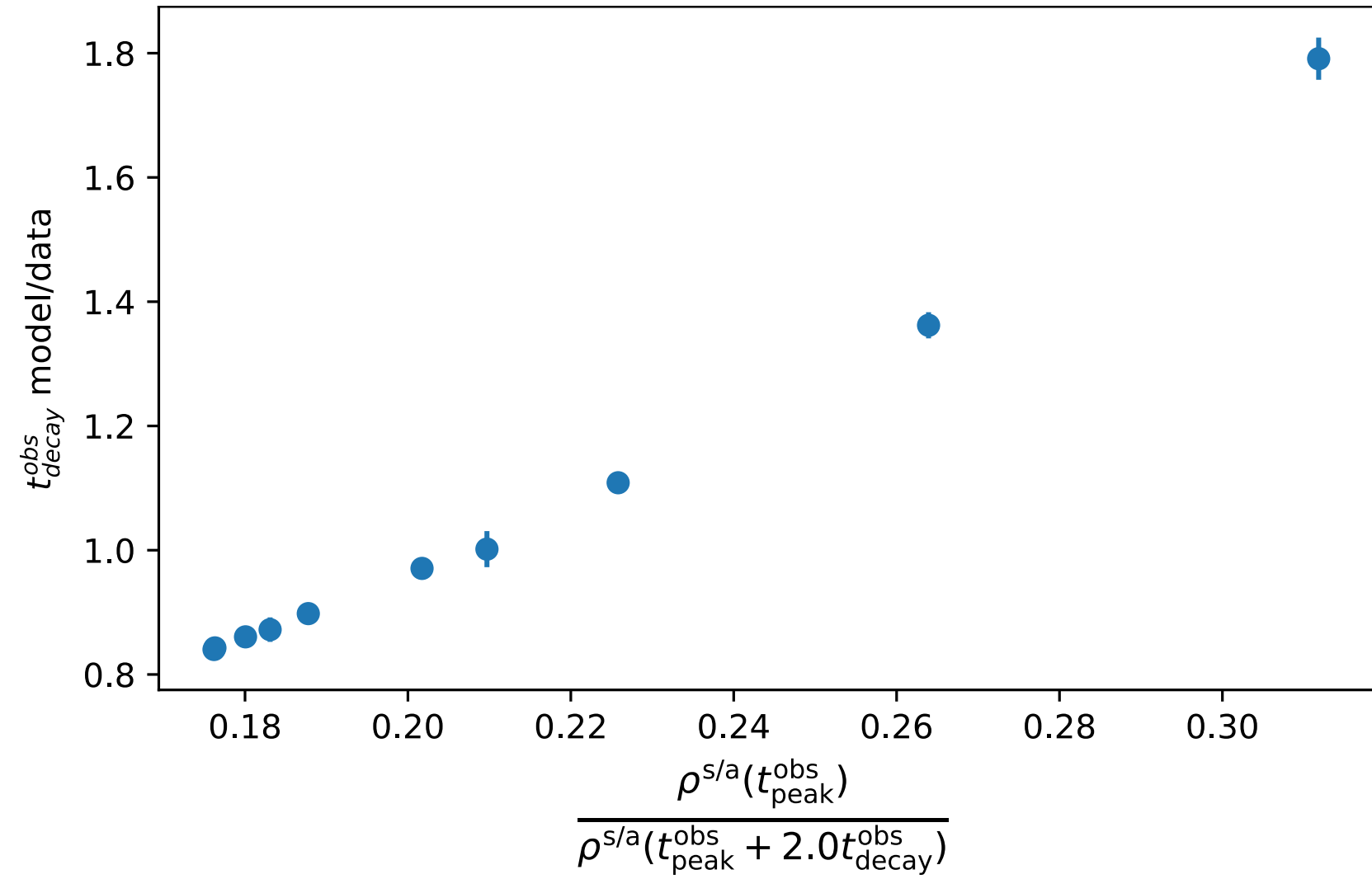
$\beta_{exp} = [0.001 - 0.3]$

$$t_{decay}^{obs} = \frac{R_0^{obs}}{m_B \beta_{exp} c} \left( \frac{\nu_{SSA}^{0,obs}}{\nu_{SSA}^{*,obs}} \right)^\phi$$

$$t_{rise}^{obs} = \frac{1}{2} t_{peak}^{obs} = \begin{cases} \frac{1}{2} \frac{R_0^{obs}}{\beta_{exp} c} \left[ \left( \frac{\nu_{SSA}^{0,obs}}{\nu_{SSA}^{*,obs}} \right)^\phi - 1 \right] & \text{if } \nu_{SSA}^{0,obs} > \nu_{SSA}^{*,obs} \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta t^{obs} = t_{exp}^{obs} + t_{peak}^{obs} = t_{exp}^{obs} + \frac{R_0^{obs}}{\beta_{exp} c} \left[ \left( \frac{\nu_{SSA}^{0,obs}}{\nu_{SSA}^{*,obs}} \right)^\phi - 1 \right].$$

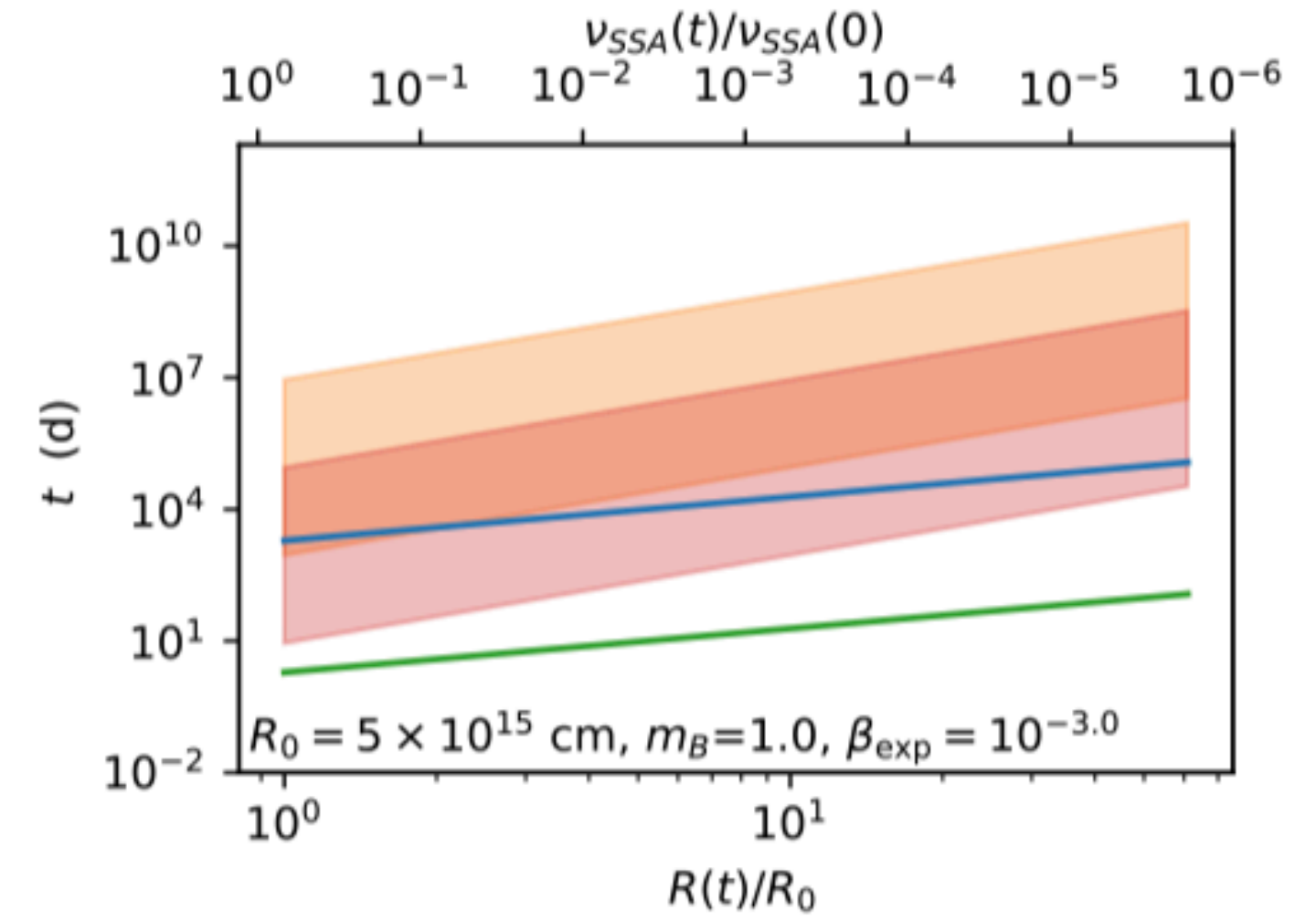
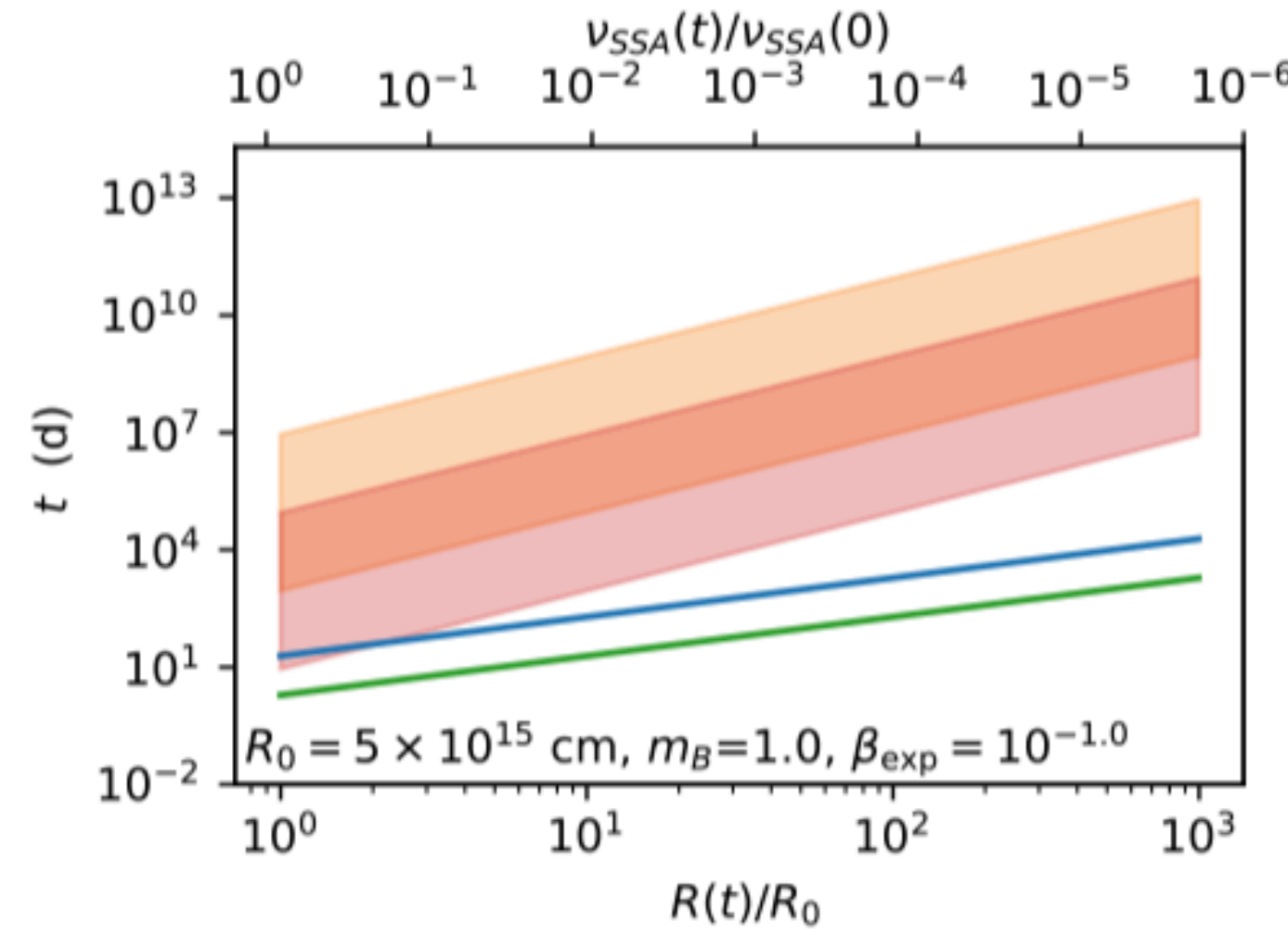
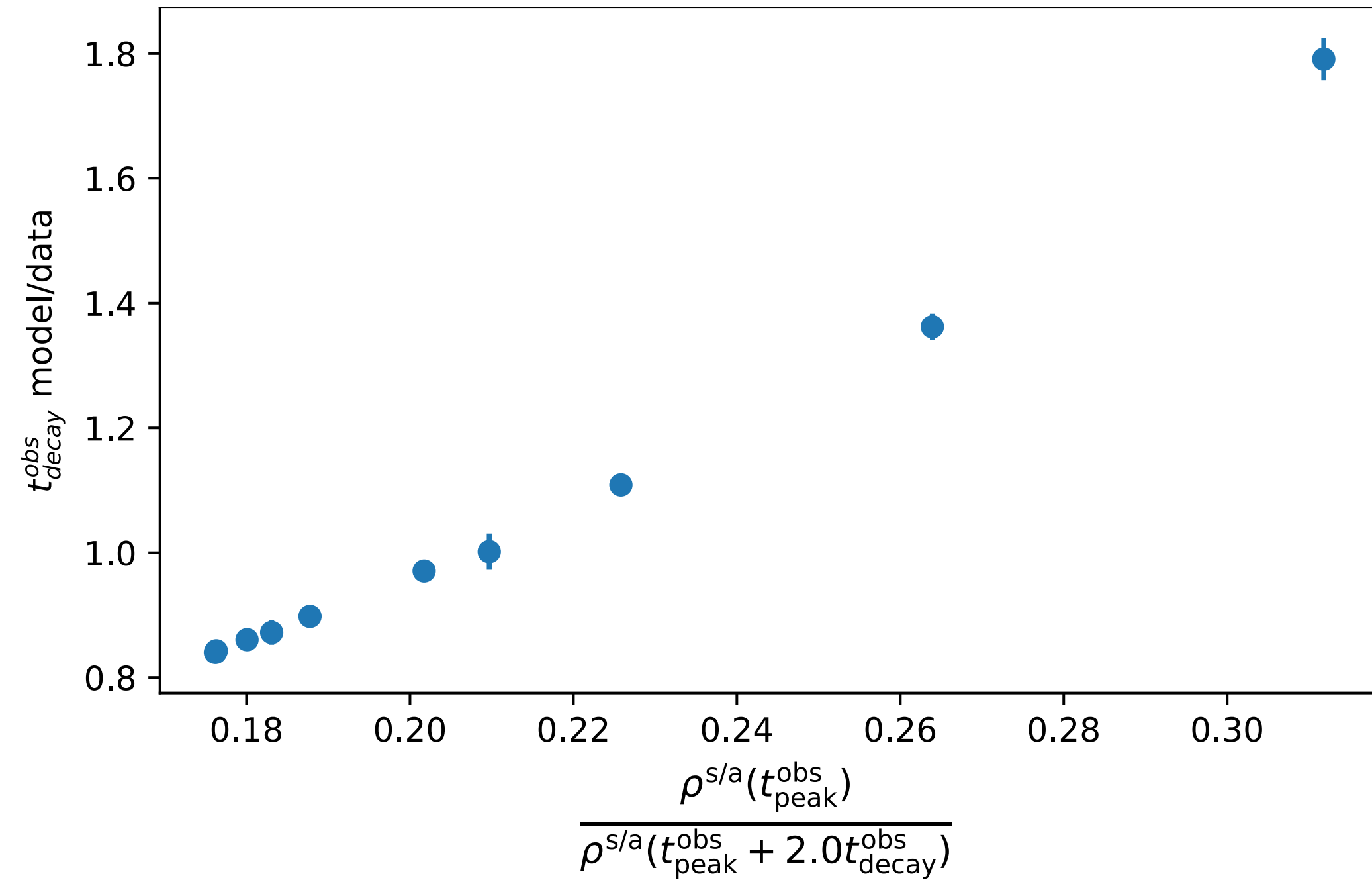
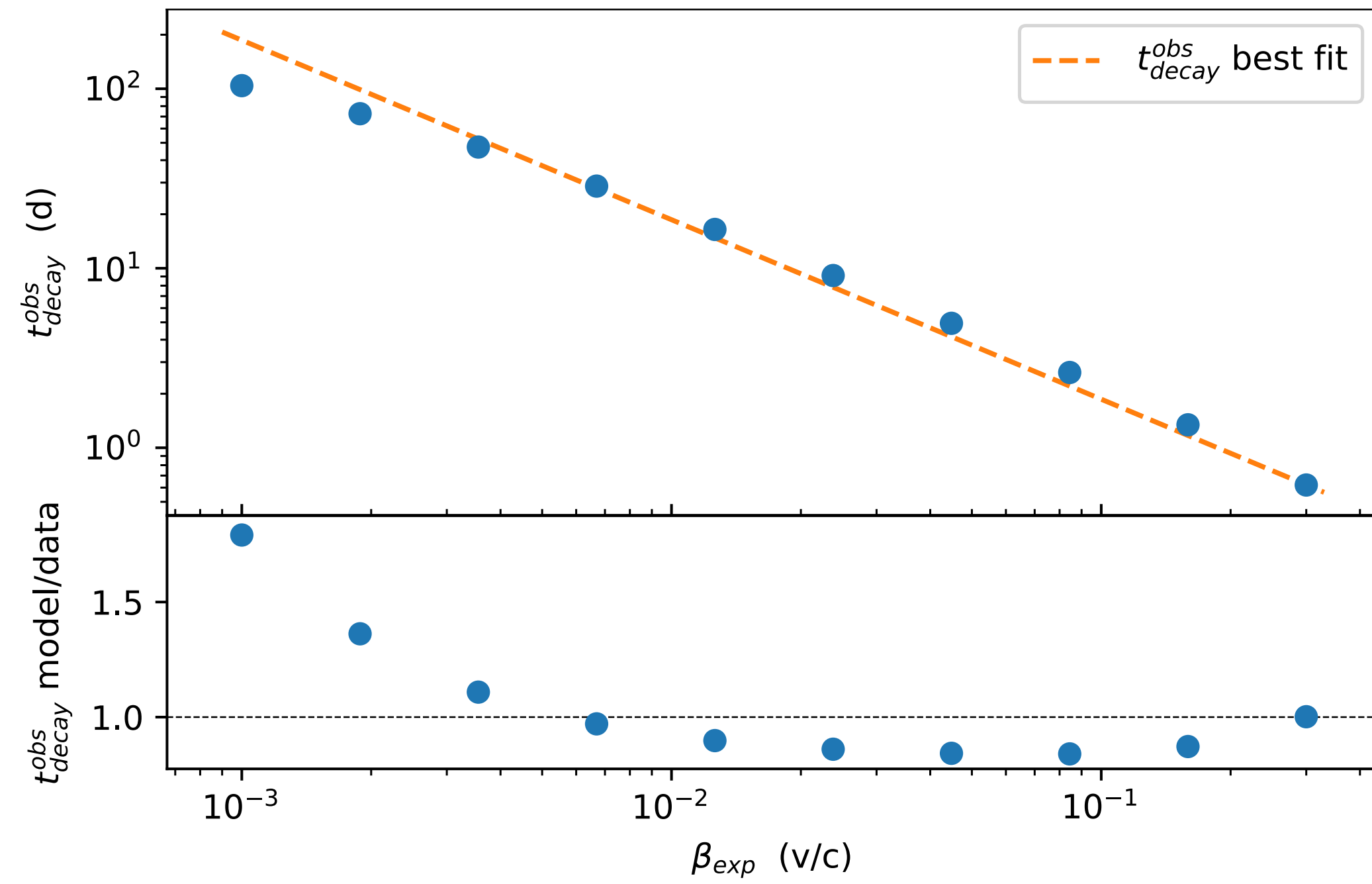
Tramacere+ 2022



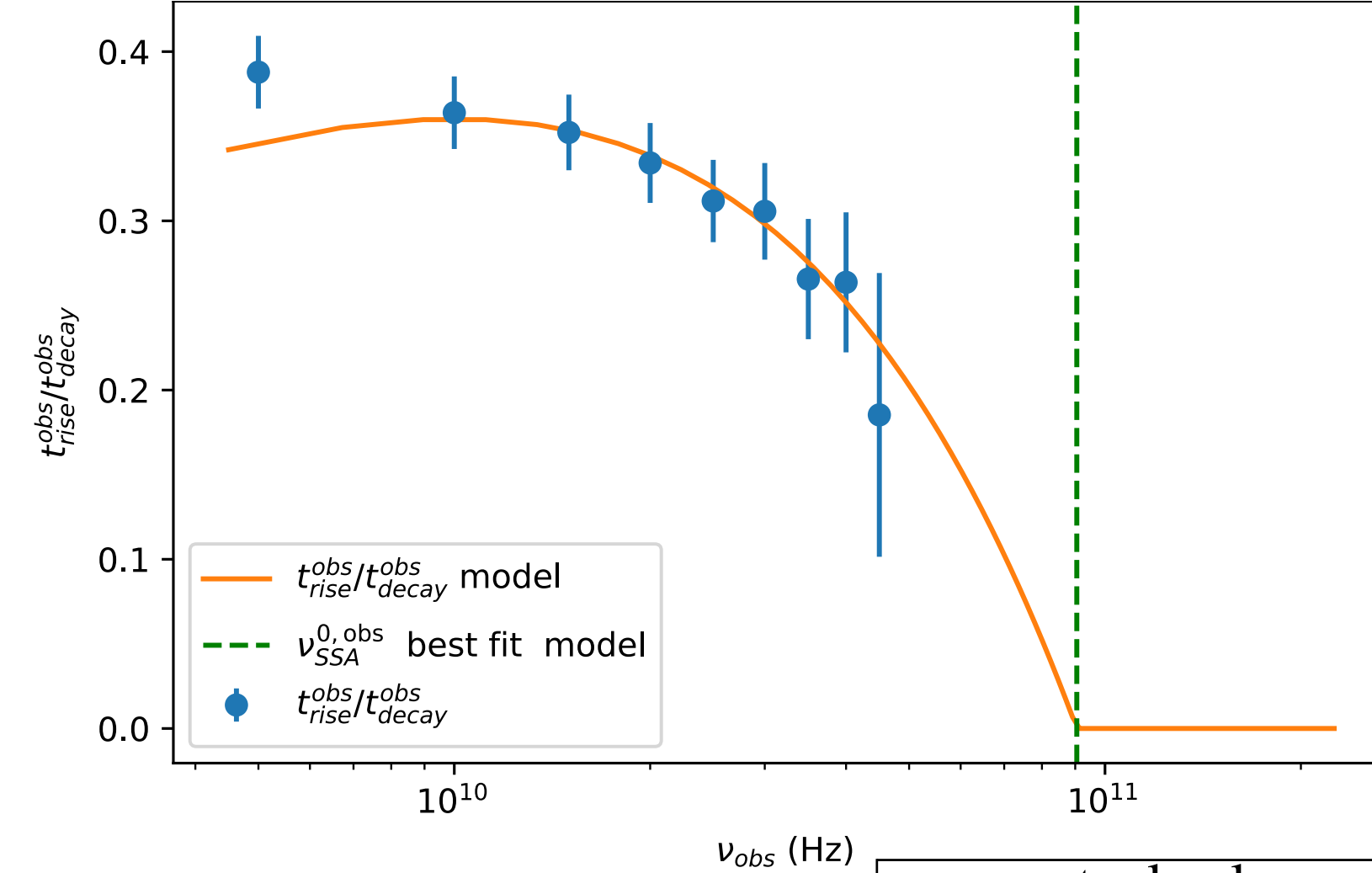
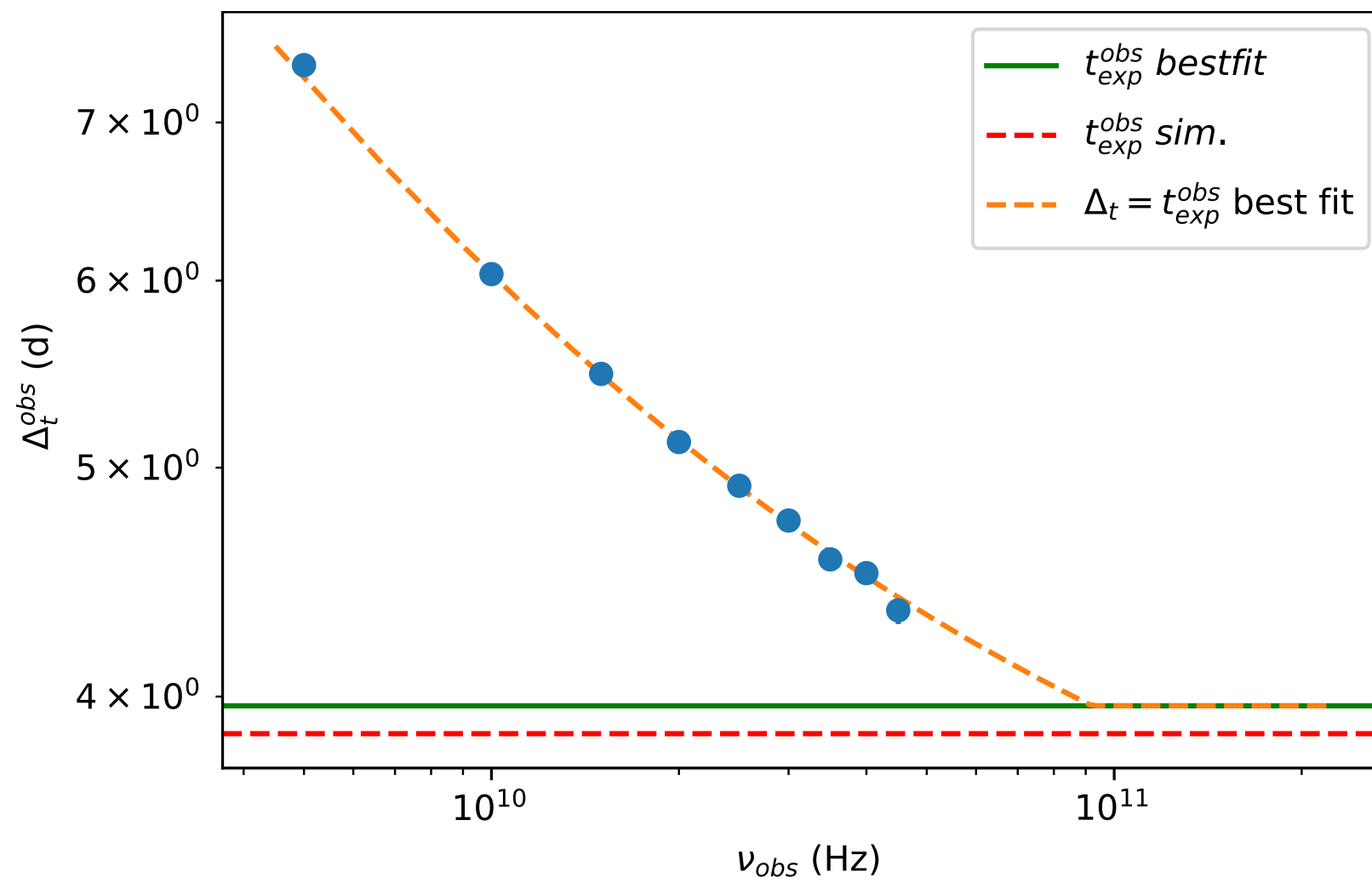
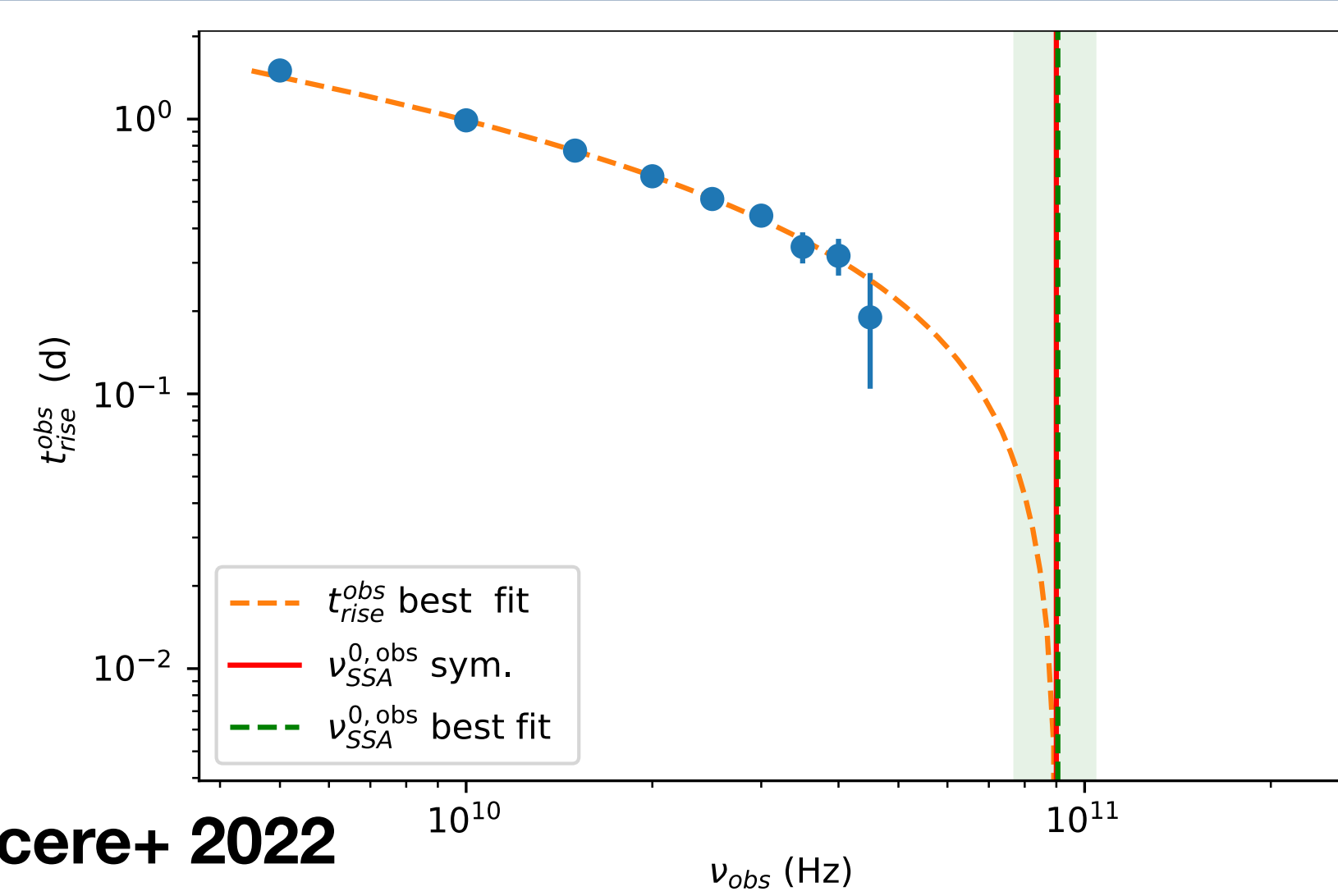
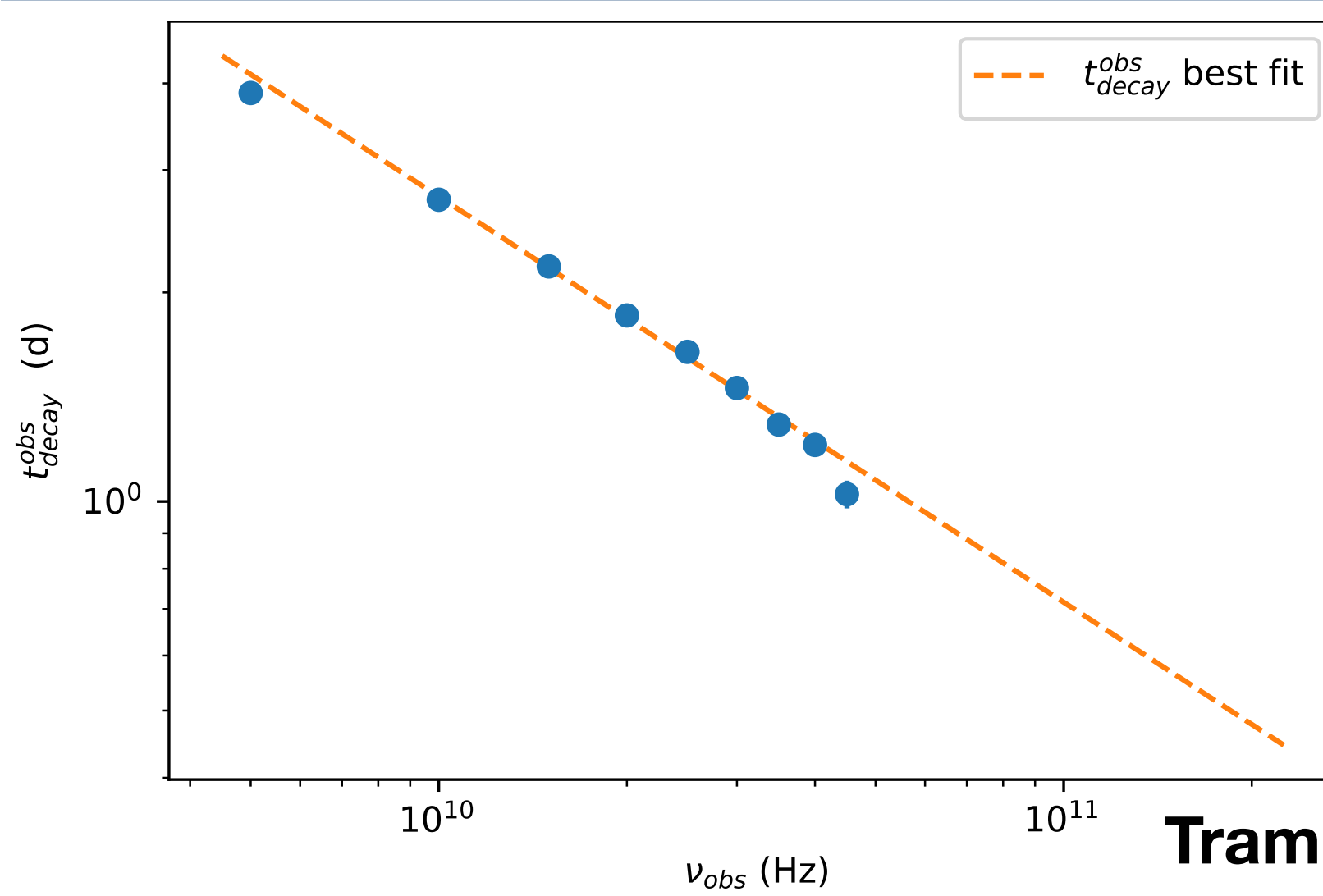
		actual values		values from $\beta$ trend best fit		
		blob	obs	$t_{rise}^{obs}$	$t_{decay}^{obs}$	$\Delta t^{obs}$
$R_0$	cm	$5 \times 10^{15}$	$1.66 \times 10^{14}$	$(1.9 \pm 0.5) \times 10^{14}$	$(1.7 \pm 0.1) \times 10^{14}$	$(1.8 \pm 0.1) \times 10^{14}$
$\nu_{SSA}^0$	GHz	3	90	$110 \pm 40$	$100 \pm 10$	$100 \pm 5$
$t_{exp}$	s	$1 \times 10^7$	$3.3 \times 10^5$			$(3.57 \pm 0.01) \times 10^5$
$m_B$		1			$0.96 \pm 0.06$	
$\phi$				$0.6 \pm 0.1$	$0.52 \pm 0.04$	$0.54 \pm 0.02$
$p$		1.46		$1.6 \pm 0.3$	$1.5 \pm 0.01$	$1.57 \pm 0.05$

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$\beta_{exp} = [0.001 - 0.3]$







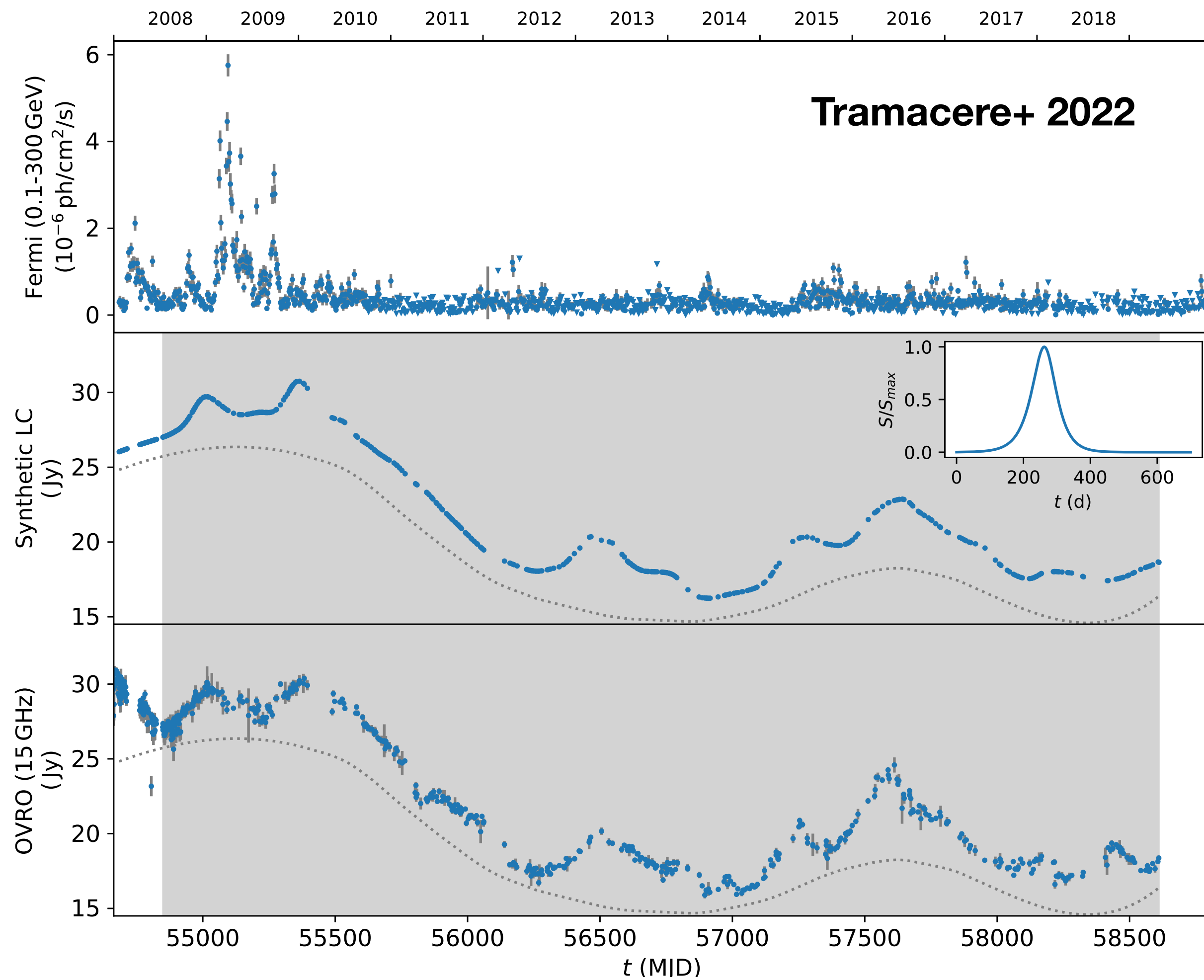
$\beta_{exp}=[0.1]$

$$t_{decay}^{obs} = \frac{R_0^{obs}}{m_B \beta_{exp} c} \left( \frac{\nu_{SSA}^{0,obs}}{\nu_{SSA}^{*,obs}} \right)^\phi$$

$$t_{rise}^{obs} = \frac{1}{2} t_{peak}^{obs} = \begin{cases} \frac{1}{2} \frac{R_0^{obs}}{\beta_{exp} c} \left[ \left( \frac{\nu_{SSA}^{0,obs}}{\nu_{SSA}^{*,obs}} \right)^\phi - 1 \right] & \text{if } \nu_{SSA}^{0,obs} > \nu_{SSA}^{*,obs} \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta t^{obs} = t_{exp}^{obs} + t_{peak}^{obs} = t_{exp}^{obs} + \frac{R_0^{obs}}{\beta_{exp} c} \left[ \left( \frac{\nu_{SSA}^{0,obs}}{\nu_{SSA}^{*,obs}} \right)^\phi - 1 \right].$$

		actual values		values from $\nu$ trend best fit		
		blob	obs	$t_{rise}^{obs}$	$t_{decay}^{obs}$	$\Delta t^{obs}$
$R_0$	cm	$5 \times 10^{15}$	$1.66 \times 10^{14}$	$(2.4 \pm 1.0) \times 10^{14}$	$(1.7 \pm 0.2) \times 10^{14}$	$(1.6 \pm 0.1) \times 10^{14}$
$\nu_{SSA}^0$	GHz	3	90	$90 \pm 10$	$100 \pm 20$	$90 \pm 10$
$t_{exp}$	s	$1 \times 10^7$	$3.3 \times 10^5$			$(3.4 \pm 0.1) \times 10^5$
$m_B$		1			$1.0 \pm 0.1$	
$\beta_{exp}$	c	0.1		$0.03 \pm 0.01$	$0.09 \pm 0.01$	$0.06 \pm 0.01$
$\phi$				$0.24 \pm 0.07$	$0.58 \pm 0.02$	$0.50 \pm 0.02$
$p$		1.46		$0.6 \pm 0.2$	$1.7 \pm 0.1$	$1.4 \pm 0.1$



## 3C 273

Period, MJD	Value
54684-54890	$1.07 \pm 0.11$
54890-55340	$0.69 \pm 0.04$
55340-56125	$1.39 \pm 0.08$
56125-56530	$5.31 \pm 0.16$
56530-56920	$0.46 \pm 0.15$
56920-57250	$5.3 \pm 0.4$
57250-57710	$2.71 \pm 0.12$
57710-57910	$0.88 \pm 0.17$
57910-58110	$3.5 \pm 0.3$
58110-58610	$2.4 \pm 0.3$

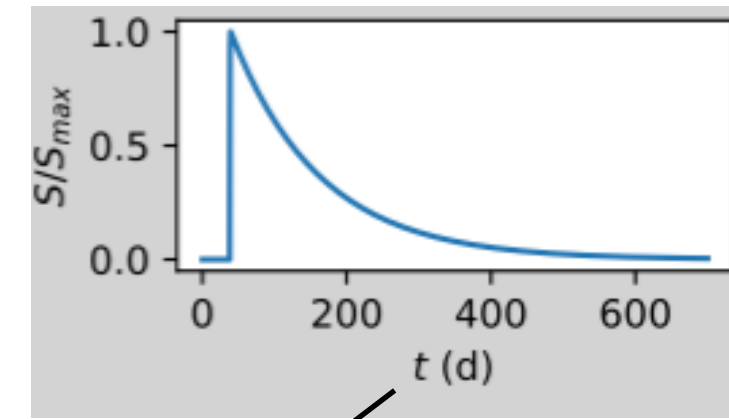
Parameter	Value
$A_0$	$174^{+12}_{-11} \times 10^3 \text{ Jy cm}^2 \text{ s/ph}$
$t_{rise}$	$37^{+2}_{-2} \text{ days}$
$t_{decay}$	$69^{+3}_{-3} \text{ days}$
$\Delta t$	$276^{+10}_{-10} \text{ days}$

$$\mathcal{L} = \mathcal{L}_{\text{rise}} + \mathcal{L}_{\text{decay}} + \mathcal{L}_{\text{delay}}$$

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Model

$$\mathcal{L} \propto \sum_{i=[1,2,3]} -\frac{1}{2} \frac{(x_i - \mu_i)^2}{2\sigma_i^2} - \frac{1}{2} \ln(\sigma_i^2)$$



conv analysis best fit values

$\Delta t, t_{\text{rise}}^{\text{obs}}, t_{\text{decay}}^{\text{obs}}$

$$t_{\text{decay}}^{\text{obs}} = \frac{R_0^{\text{obs}}}{m_B \beta_{\text{exp}} c} \left( \frac{\nu_{\text{SSA}}^{0,\text{obs}}}{\nu_{\text{SSA}}^{*,\text{obs}}} \right)^\phi$$

$$t_{\text{rise}}^{\text{obs}} = \frac{1}{2} t_{\text{peak}}^{\text{obs}} = \begin{cases} \frac{1}{2} \frac{R_0^{\text{obs}}}{\beta_{\text{exp}} c} \left[ \left( \frac{\nu_{\text{SSA}}^{0,\text{obs}}}{\nu_{\text{SSA}}^{*,\text{obs}}} \right)^\phi - 1 \right] & \text{if } \nu_{\text{SSA}}^{0,\text{obs}} > \nu_{\text{SSA}}^{*,\text{obs}} \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta t^{\text{obs}} = t_{\text{exp}}^{\text{obs}} + t_{\text{peak}}^{\text{obs}} = t_{\text{exp}}^{\text{obs}} + \frac{R_0^{\text{obs}}}{\beta_{\text{exp}} c} \left[ \left( \frac{\nu_{\text{SSA}}^{0,\text{obs}}}{\nu_{\text{SSA}}^{*,\text{obs}}} \right)^\phi - 1 \right].$$

flat priors

$$m_B \in [1, 2]$$

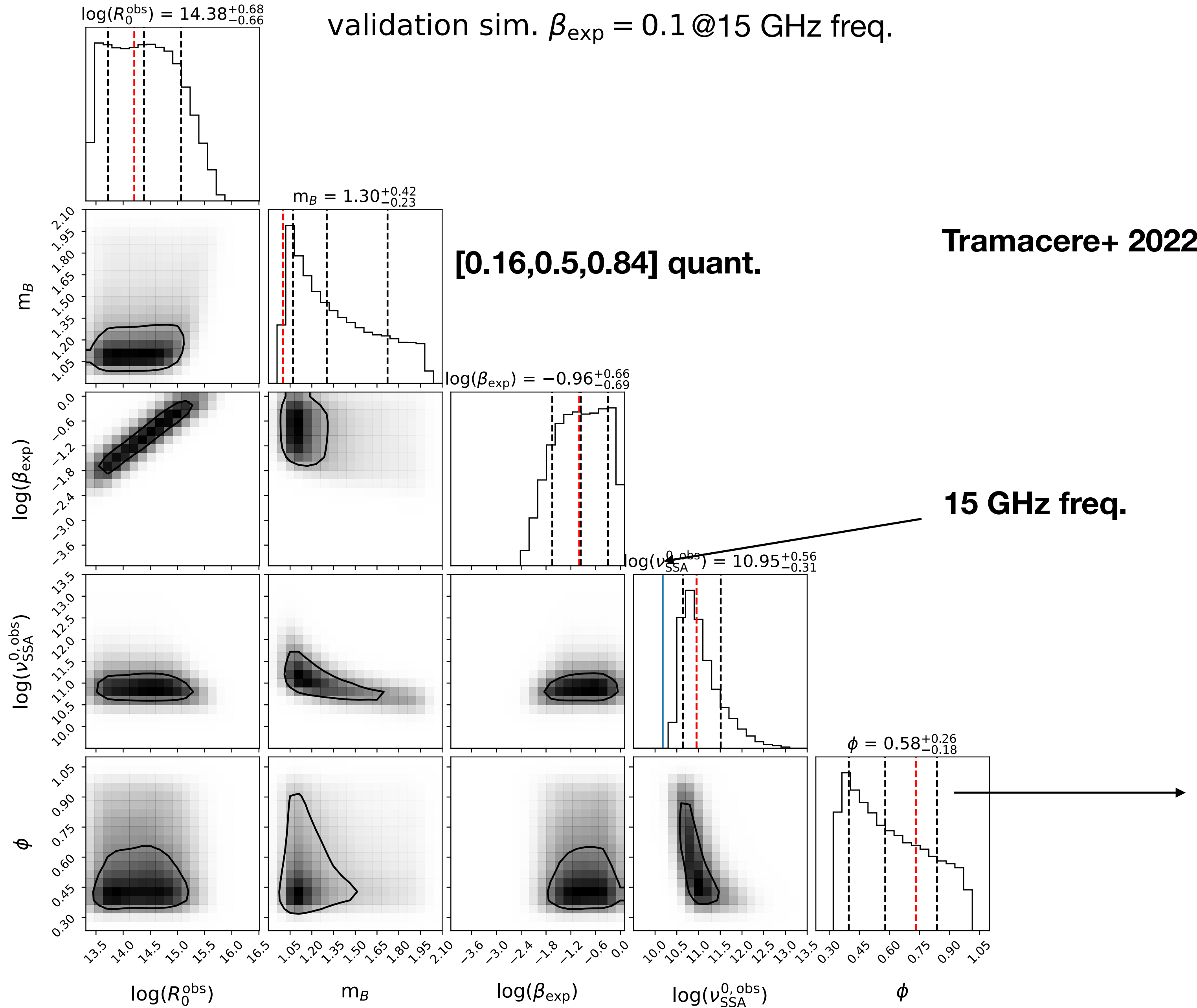
$$\nu_{\text{SSA}}^{0,\text{obs}} \in [10, 10^4] \text{ GHz}$$

$$\phi \in [1/3, 1]$$

$$t_\gamma^{\text{var}} \in [0.25, 14] \text{ days}$$

$$R_{\text{obs}}^0 = R_0 \frac{1+z}{\delta} \quad R_0^{\text{obs}} \in [6.5 \times 10^{13}, 3.6 \times 10^{17}] \text{ cm}$$

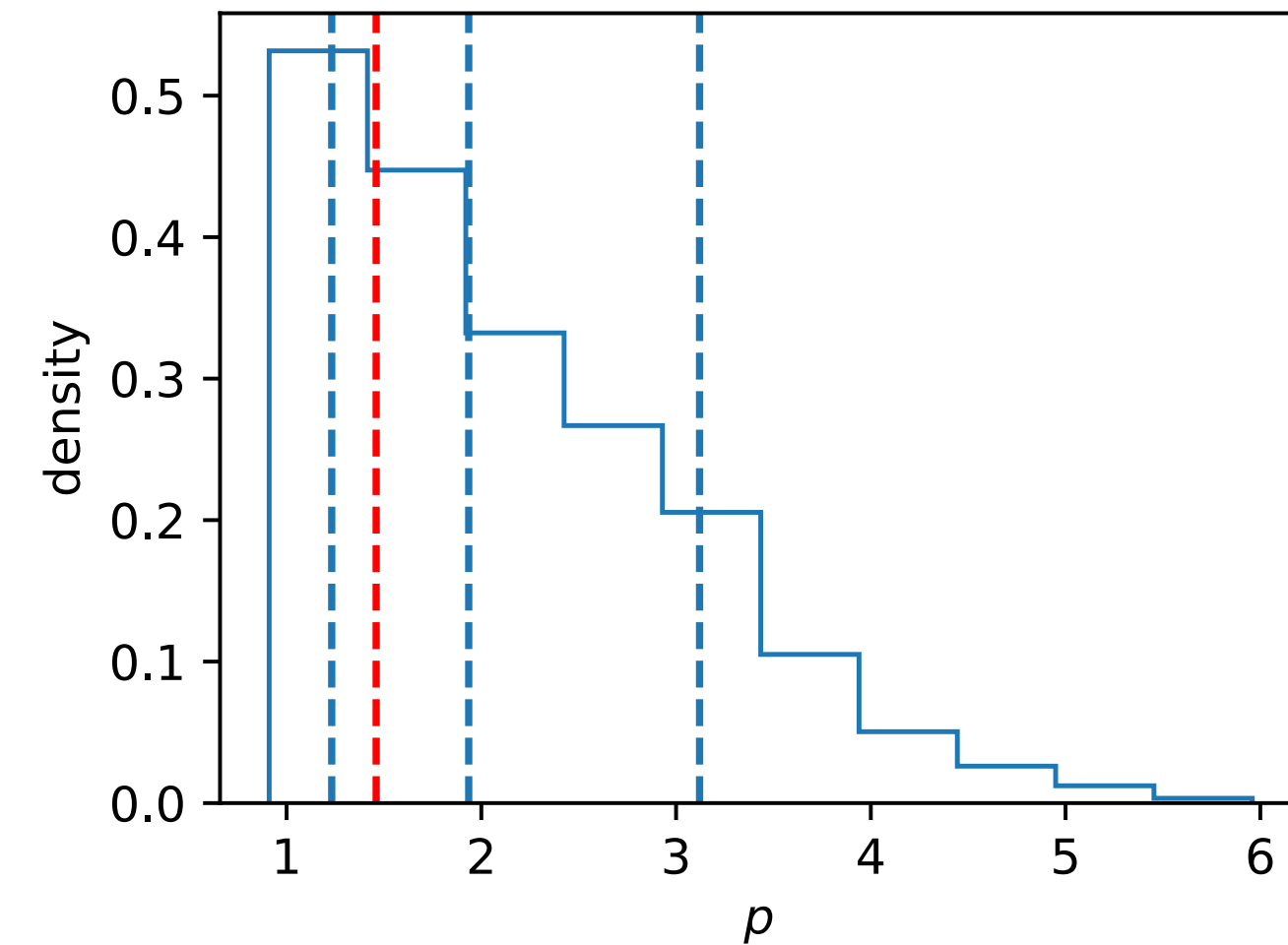


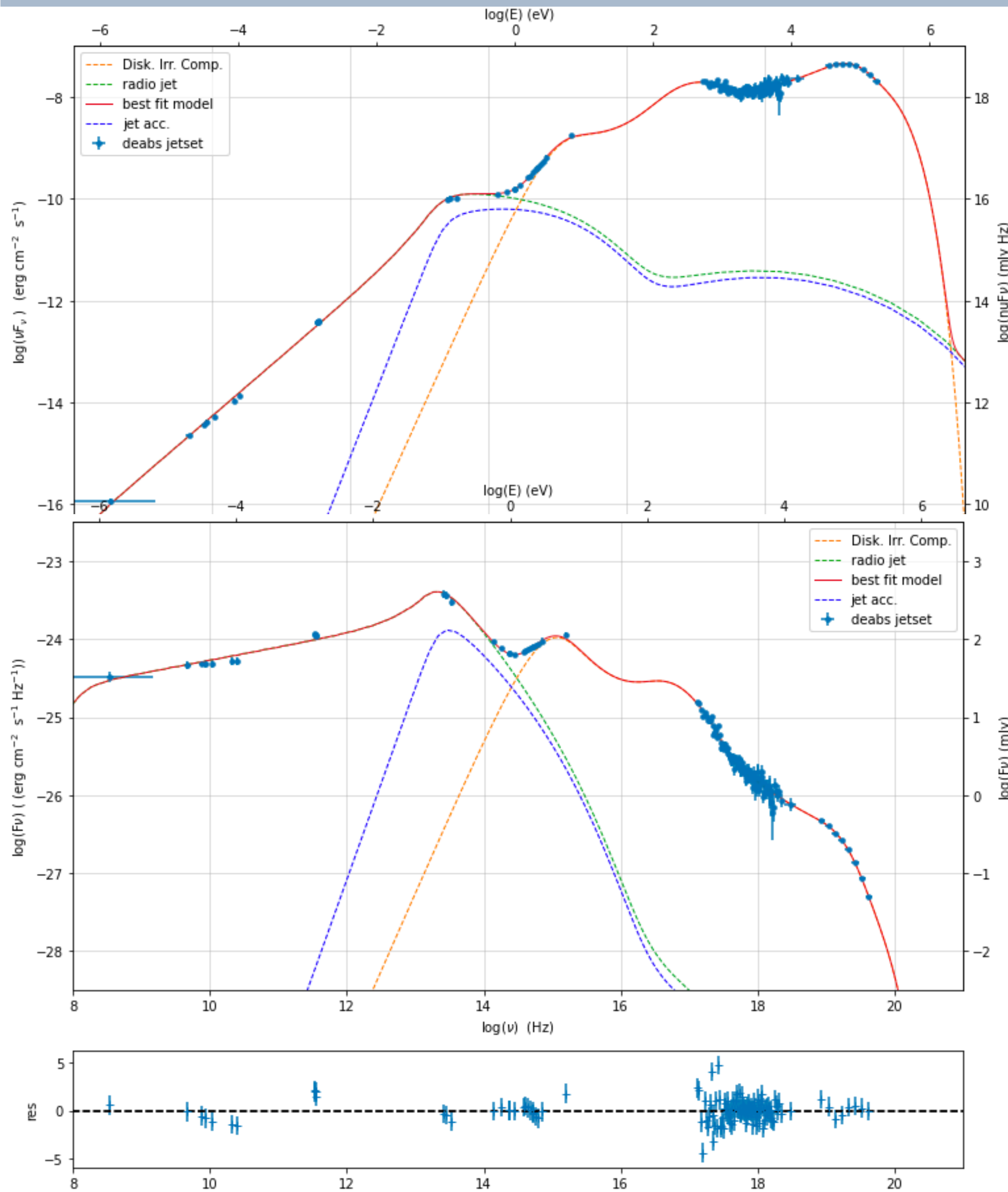


red dashed line simulation values

Tramacere+ 2022

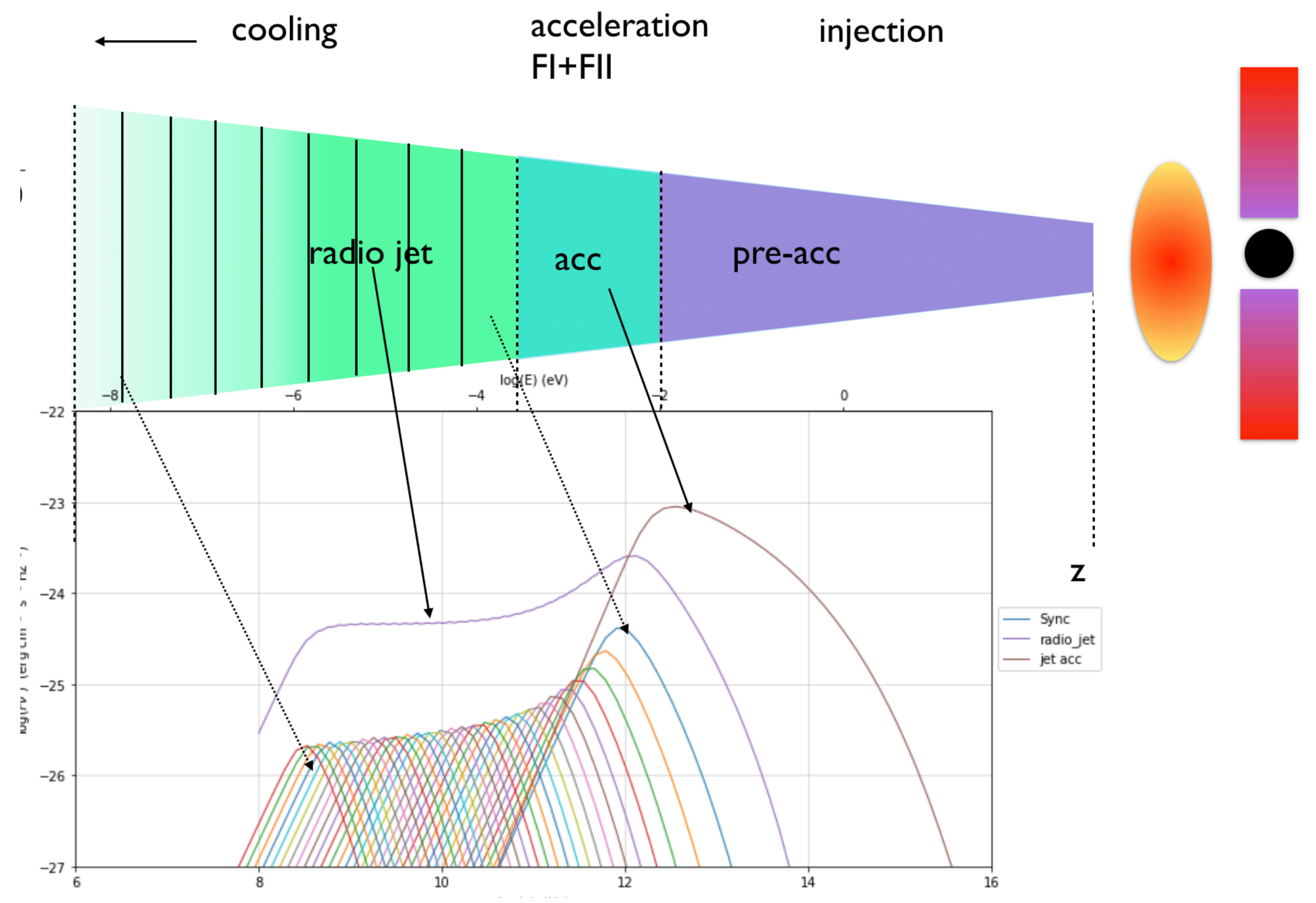
validation sim.  $\beta_{\text{exp}} = 0.1$ ,  $p = 1.94^{+1.19}_{-0.70}$





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## MAXI J1820+070



$$S(t) = A \frac{\exp \frac{-(t-\Delta)}{t_f}}{1 + \exp \frac{-(t-\Delta)}{t_u}} \longrightarrow \Delta t = \Delta - t_u \ln \left( \frac{t_u}{t_f - t_u} \right)$$

**Analytical**

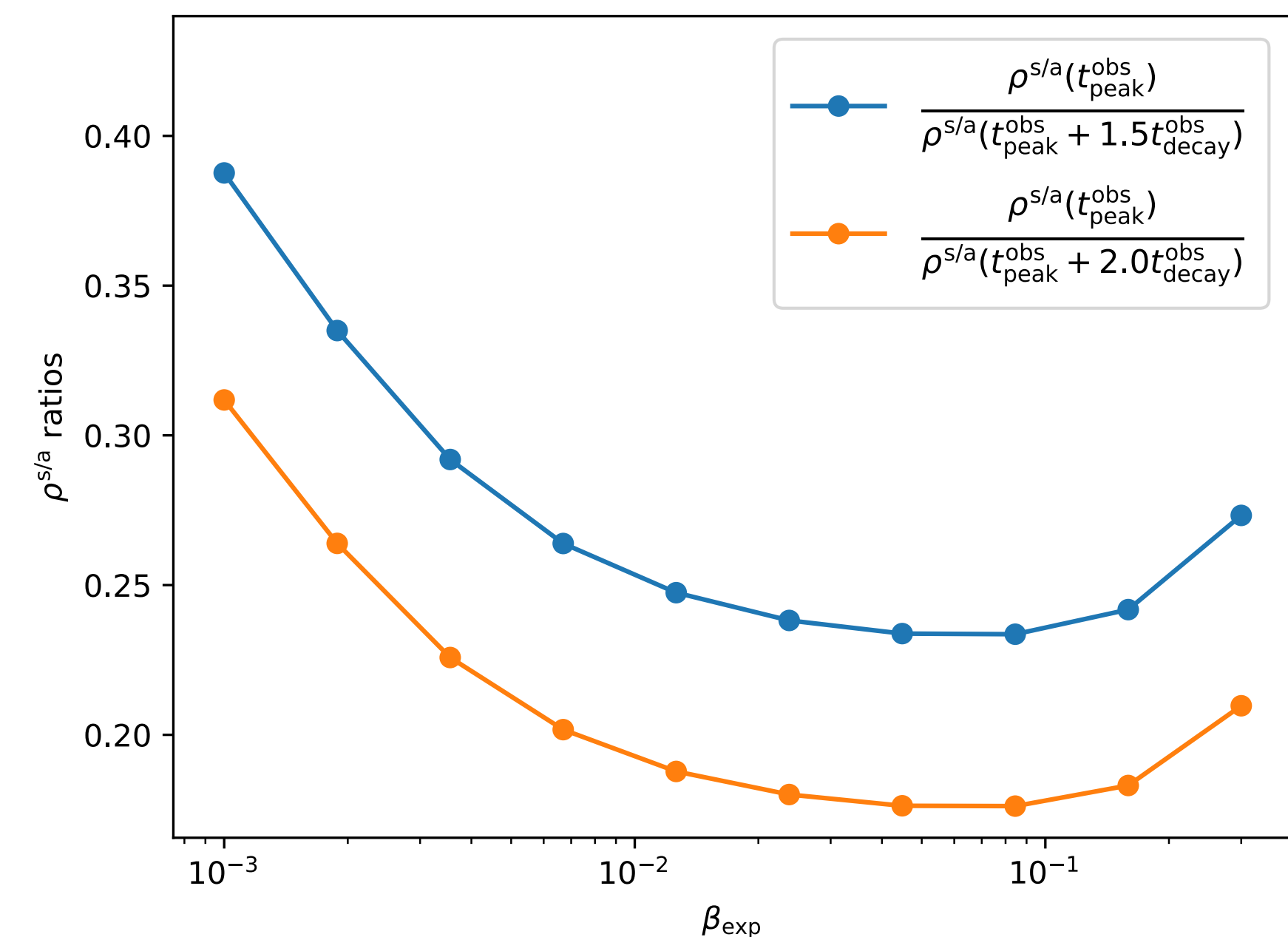
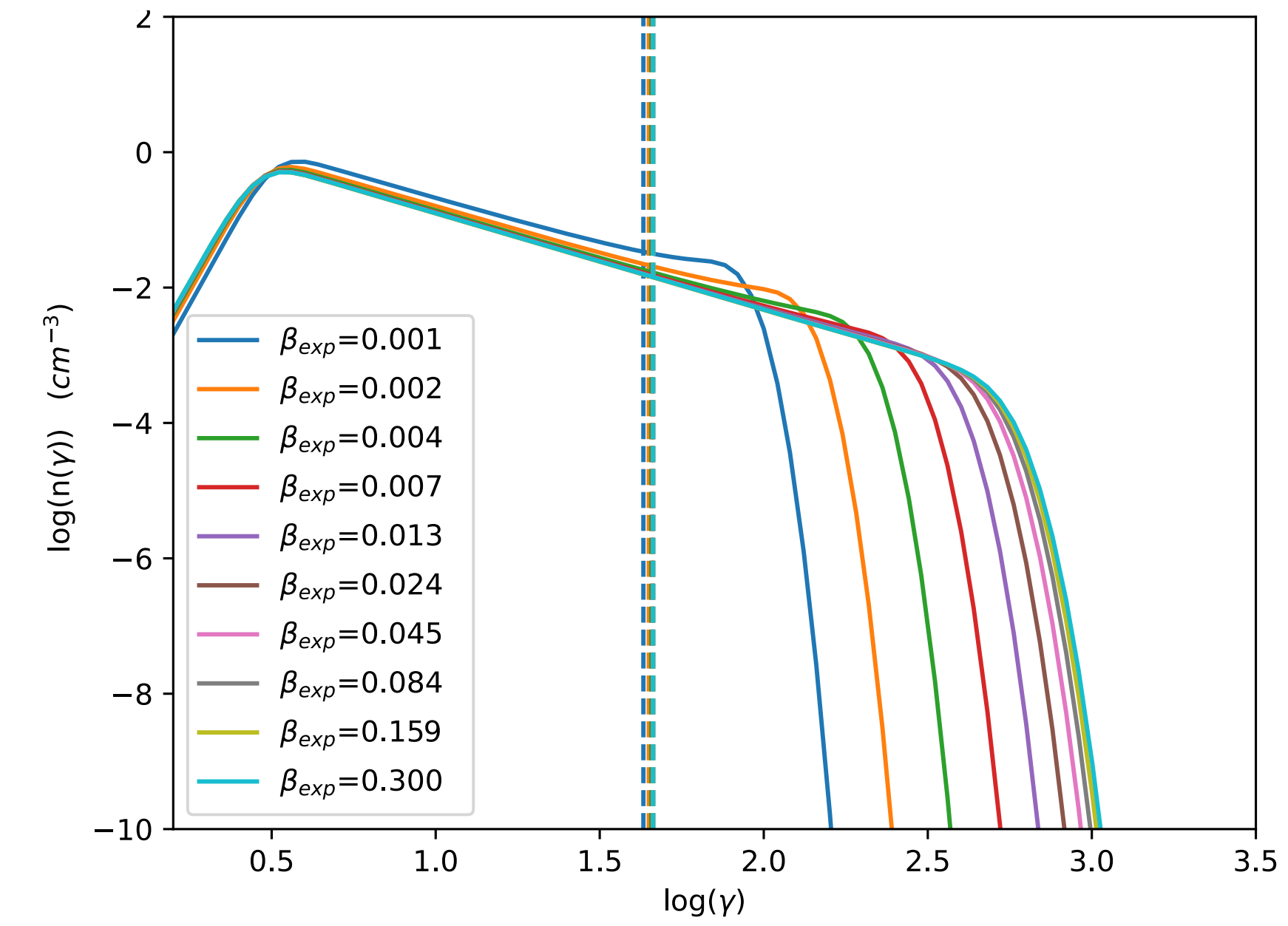
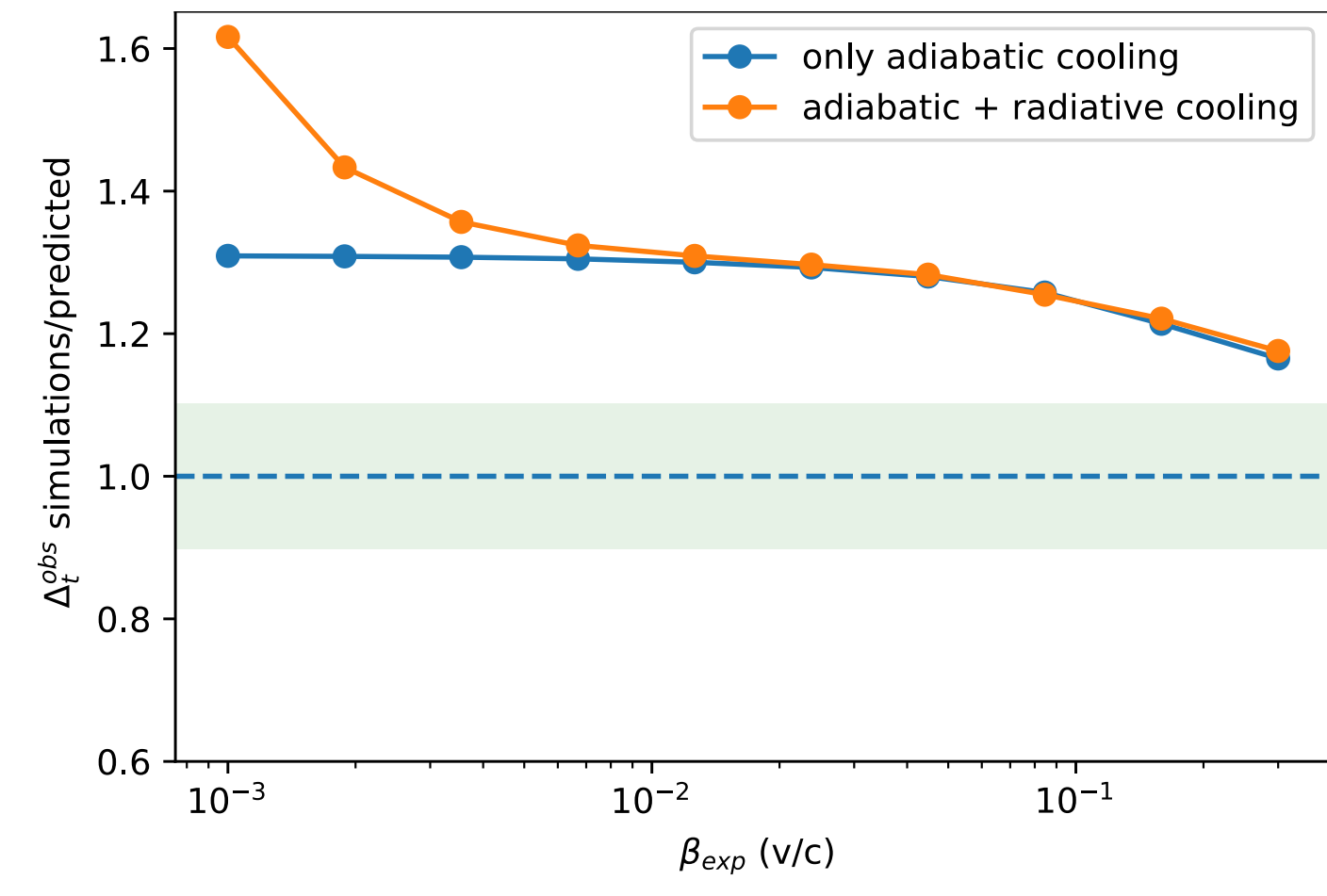
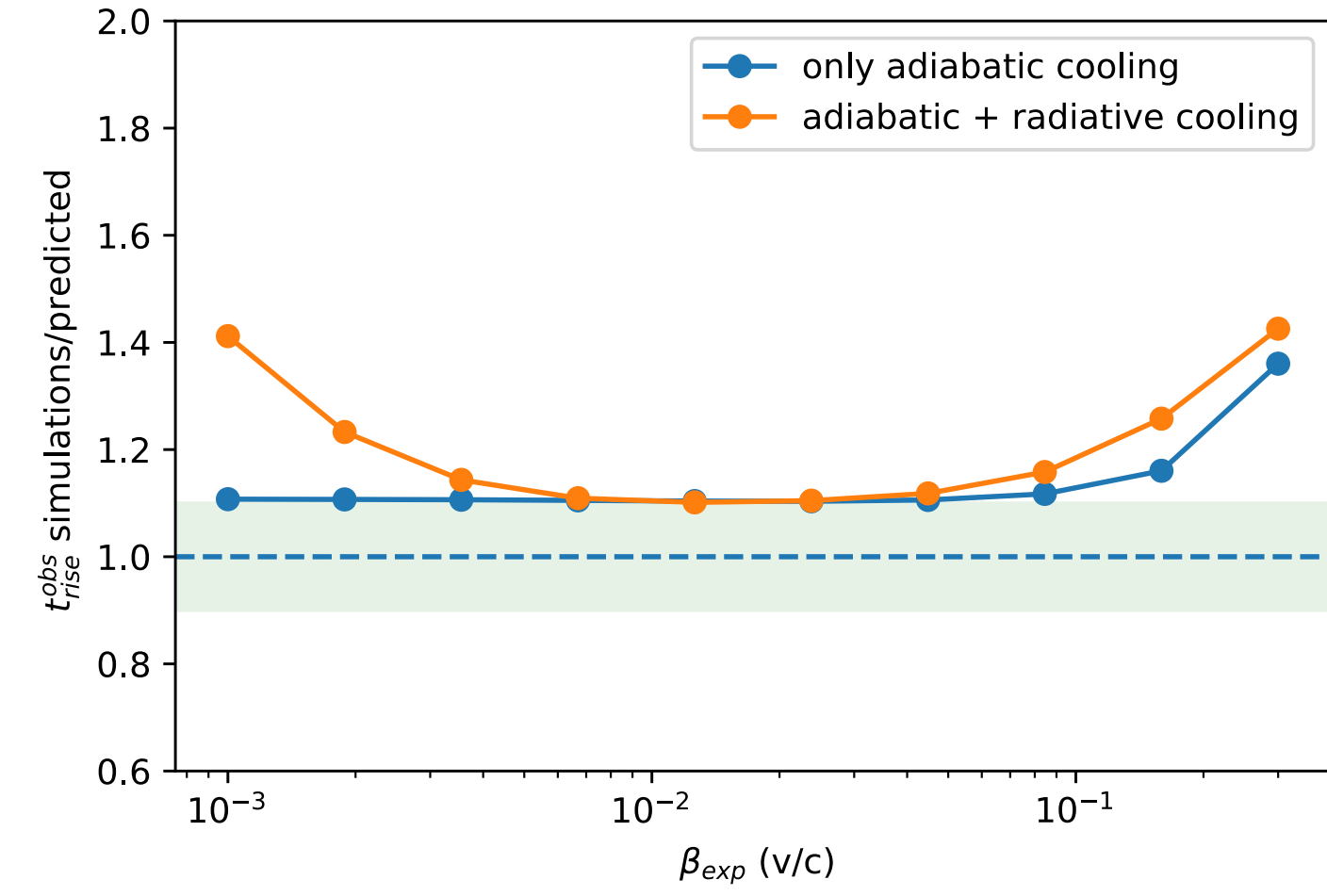
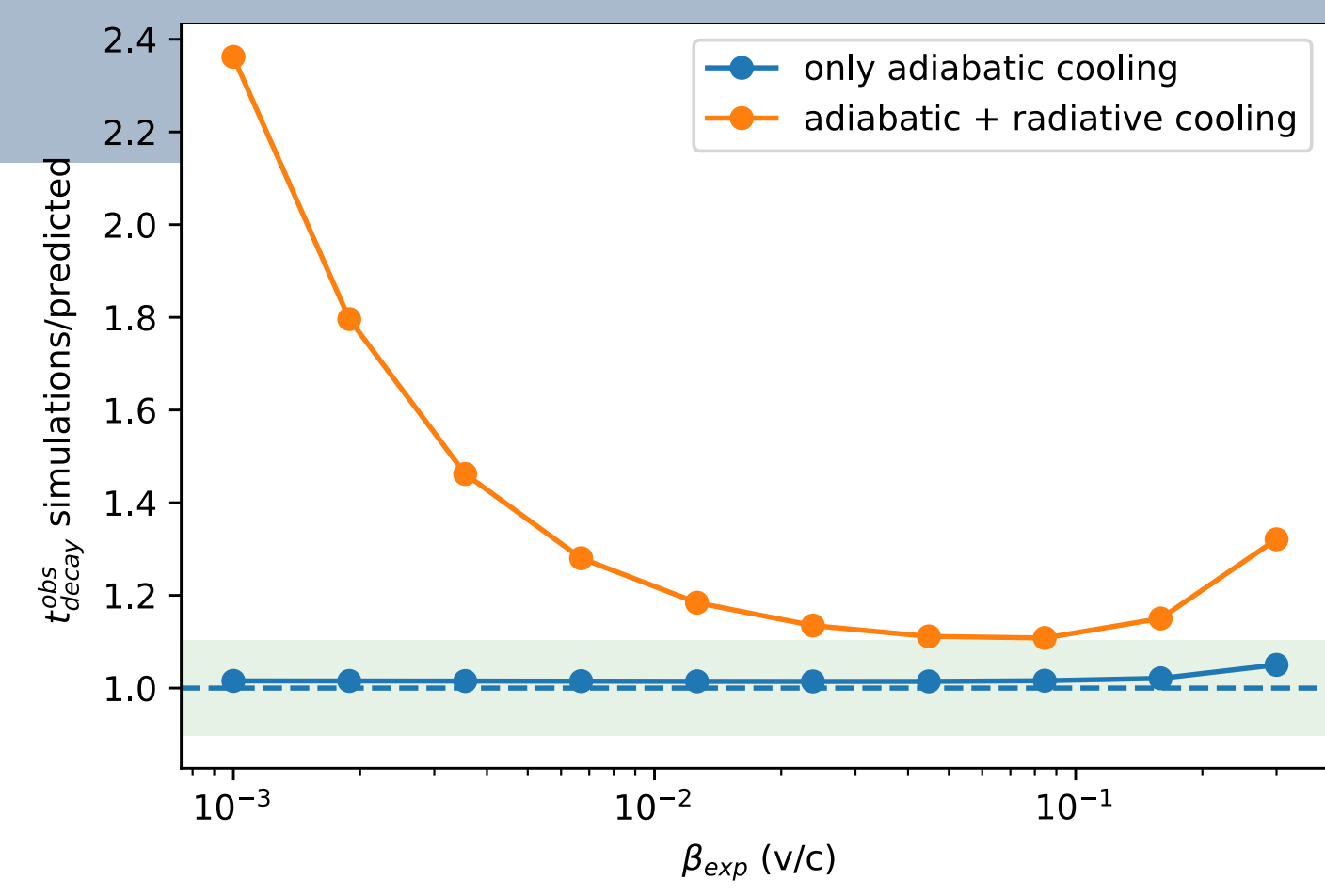
**Numerical**

$$S(t) = \frac{A}{2} \longrightarrow t_{\text{rise}} = t_u \left( 0.54 + 1.34 \left( \frac{t_f}{t_u} \right)^{1/4} \right)$$

$$S(t) = \frac{A}{e} \longrightarrow t_{\text{decay}} = t_f \left( 1.00 + 1.33 \left( \frac{t_f}{t_u} \right)^{-1.11} \right)$$



## Tramacere+ 2022



Tramacere+ 2022

3C 273

3C 273,  $p=2.27^{+1.18}_{-0.84}$

