Testing the limits of continuous-loss approximation for inverse Compton cooling in blazars

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Abstract

Blazar flares are perfect phenomena to probe the extreme physics of relativistic outflows. The key method for this task is physical modeling of the variable emission from blazar jets. Many of the numerical codes developed for this goal, are based on the kinetic approach tracing the particle spectrum evolution due to various physical processes (acceleration, cooling, etc). In the existing leptonic codes, the inverse Compton (IC) cooling of electrons is described with a continuous-loss term in the kinetic equation, which however is only valid when the relative losses are much smaller than unity. In the Klein-Nishina (KN) regime, this is no longer the case, and one has to treat properly the large relative jumps of electrons in energy. The full transport equation then becomes an integro-differential one, and is quite challenging to solve. To avoid this issue, continuous-loss approximations were derived by certain authors attempting to reasonably treat KN effects. In our study, we test the accuracy of the most common such approximation for typical conditions during blazar flares. To solve the full integro-differential kinetic equation, we extend our existing blazar flare modeling code (EMBLEM), and using it, examine the effect of non-continuous cooling on the electron spectrum compared to the continuous-loss approximation case.

Introduction

Many different codes were developed for modeling of blazar multi-wavelength (MWL) flares. One of the most common approaches is the kinetic one, in which one tracks the evolution of the particle distribution due to different physical processes, e.g. injection, cooling, acceleration, etc. The general form of the kinetic equation governing the evolution of the particle spectrum N_e is:

$$\frac{\partial N_e}{\partial t} = \frac{\partial}{\partial \gamma}\left(\left[-\dot{\gamma}_{\mathrm{cool}} - \gamma/t_{\mathrm{FI}} - 2\gamma/t_{\mathrm{FII}}\right] N_e\right) + \frac{\partial}{\partial \gamma}\left((\gamma^2/t_{\mathrm{FII}})\frac{\partial N_e}{\partial \gamma}\right) - \frac{N_e}{t_{\mathrm{esc}}} + Q_{\mathrm{inj}} \qquad (1)$$

where $t_{\mathrm{esc/FI/FII}}$ are the time-scales of escape and of Fermi-I/Fermi-II acceleration respectively, and $\dot{\gamma}_{\mathrm{cool}}$ and Q_{inj} are the cooling and injection rates respectively. In the one-zone leptonic scenario (e.g. [1]), the blazar γ-ray emission originates from a compact region (blob) in the jet. The blob is filled with electron-positron plasma and is moving relativistically along the jet. High-energy electrons in the blob emit synchrotron radiation, as well as produce γ-ray emission via inverse Compton scattering of soft photons (synchrotron and/or external). The cooling term in this case includes the synchrotron and inverse Compton cooling:

$$\dot{\gamma}_{\mathrm{cool}} = -b_{\mathrm{cool,syn}}(B)\gamma^2 - \dot{\gamma}_{\mathrm{cool,IC}} = -(4\sigma_{\mathrm{T}})/(3m_{\mathrm{e}}c)\, U_B\, \gamma^2 - \dot{\gamma}_{\mathrm{cool,IC}}$$

It is assumed here, that the electrons lose only a small fraction of their energy in each interaction, so that the IC cooling process can be regarded as continuous. However, this is no longer true for IC scattering in the KN regime, and the electrons suffer big relative jumps in energy space, which are rather difficult to treat. Therefore, for simplicity, various authors developed continuous-loss approximations for the cooling term aiming to reasonably describe the KN effects, e.g. an approximation by Moderski et al. (2005) [2]:

$$\dot{\gamma}_{\mathrm{cool,IC}} = -b_{\mathrm{cool,IC}}(U_{\mathrm{rad}})\gamma^2 = -\frac{4\sigma_{\mathrm{T}}}{3m_{\mathrm{e}}c}\gamma^2 \int_{\epsilon'_{\mathrm{min}}}^{\epsilon'_{\mathrm{max}}} f_{\mathrm{KN}}(4\gamma\epsilon')u'_{\mathrm{rad}}(\epsilon')d\epsilon'$$

$$f_{\mathrm{KN}}(x) = \begin{cases} (1+x)^{-1.5}, & \text{for } x < 10^4 \\ \frac{9}{2x^2}\left[\ln(x) - 11/6\right] & \text{for } x \geq 10^4 \end{cases}$$

In this work, we test the accuracy of the continuous-loss approach, and estimate the importance of the effects of non-continuous cooling on the evolution of the electron spectrum.

Modeling

General model and numerical code

The proper transport equation (excluding the acceleration terms) treating large jumps of electrons in energy during the IC cooling in the KN regime is given by [3]:

$$\frac{\partial N_e(\gamma,t)}{\partial t} = -N_e(\gamma,t)\int_1^{\gamma} C(\gamma,\gamma')d\gamma' + \int_{\gamma}^{\infty} N(\gamma',t)C(\gamma',\gamma)d\gamma' - \frac{N_e(\gamma,t)}{t_{\mathrm{esc}}} + Q_{\mathrm{inj}}(\gamma,t) \qquad (2)$$

with

$$C(\gamma,\gamma') = \int_{E_*/\gamma}^{\infty} dx\, n_{\mathrm{ph}}(x)\, \frac{3\sigma_{\mathrm{T}}c}{4E\gamma}\left[r + (2-r)\frac{E_*}{E} - 2\left(\frac{E_*}{E}\right)^2 - \frac{2E_*}{E}\ln\frac{E}{E_*}\right]$$

$$x = \frac{\epsilon_s}{m_e c^2},\quad E = \gamma x,\quad E_* = \tfrac{1}{4}(\gamma/\gamma' - 1),\quad E > E_*,\quad r = \tfrac{1}{2}(\gamma/\gamma' + \gamma'/\gamma)$$

To solve this equation, we extend our numerical code EMBLEM (Evolutionary Modeling of BLob EMission) [4]. The (original) code is based on a one-zone **leptonic** scenario described in the previous section, and solves the kinetic equation 1 for the continuous case, computing the evolution of the electron spectrum and of the associated spectral energy distribution (SED) during blazar flares. In this model, flares are launched when the blob is passing through a shock and/or encounters turbulence, which causes re-acceleration of particles via Fermi-I and/or Fermi-II mechanisms (see Fig. 1). Using the extended EMBLEM code, we solve the integro-differential kinetic equation 2 via a method of iterations. To speed up the computation, we implement parallelization of the calculations on the Lorentz factor grid using the `MPI4PY` module.

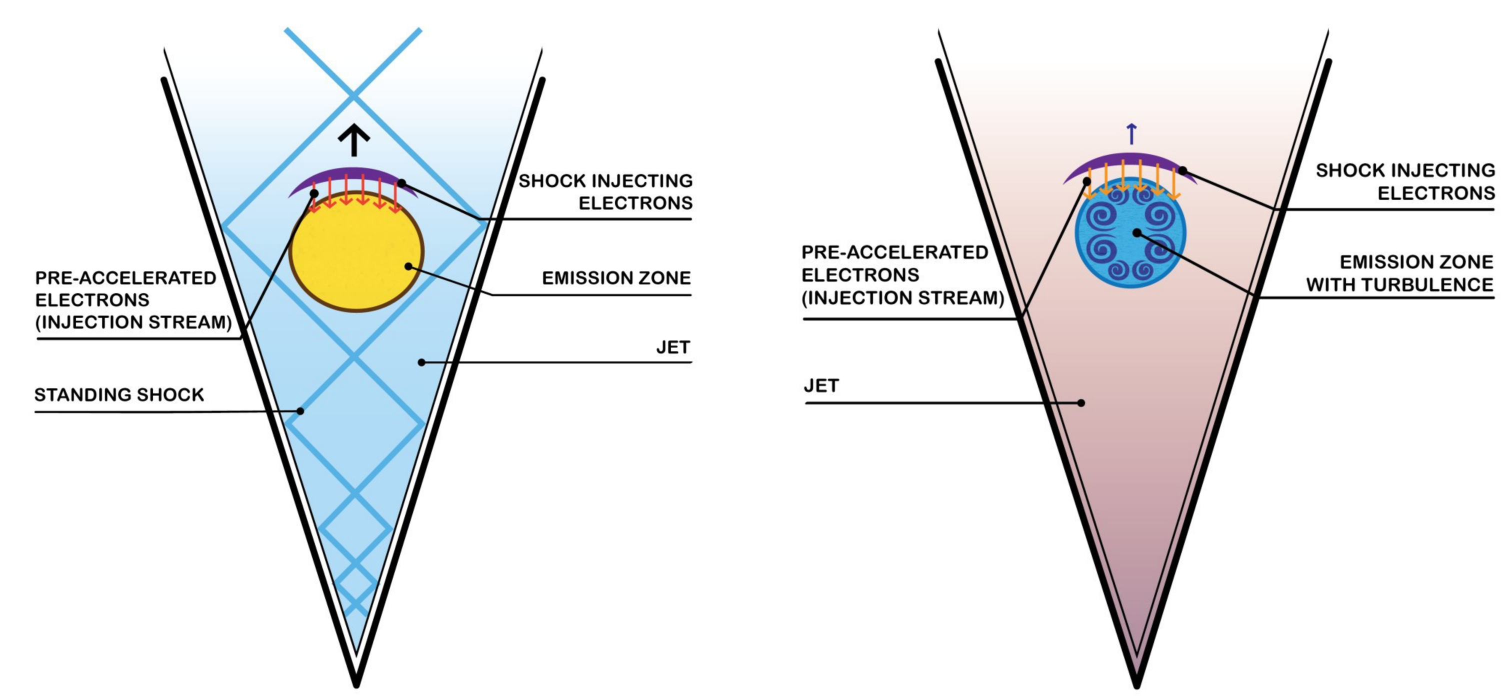

Figure 1. Sketches representing a one-zone model for blazar emission. A flare is induced by re-acceleration process either when the emitting zone (blob) is crossing a shock (left panel) or encounters turbulence (right panel).

Application to flare modeling

We now apply the extended code to explore the non-continuous cooling effect in the conditions of the BL Lac object Mrk 421, with the physical parameters chosen in a way to roughly reproduce the brightest Very High Energy (VHE) γ-ray flare of the source detected in February 2010. We simulate this flare within the one-zone scenario with (1) shock re-acceleration ($t_{\mathrm{FI}} = 1.65R/c$) combined with moderate stochastic re-acceleration accompanying the shock ($t_{\mathrm{FII}} = 8R/c$), and (2) strong stochastic re-acceleration ($t_{\mathrm{FII}} = 5R/c$). In both cases, the crossing time of the shock/turbulence and hence the re-acceleration phase duration is $\Delta t = 101.5$ d in the blob frame (3.5 d in the observer frame), which corresponds to the flare rise time-scale.

Results

With the same input physical parameters for the flare, we compare the solutions for the time-dependent electron spectrum when using the Eq. 1 with the continuous cooling term by Moderski et al. (2005) and when using the Eq. 2 (with re-acceleration terms). The comparison (and the ratio) of the electron spectra at the flare peak between continuous and non-continuous cooling cases is shown in Fig. 2 (top panel: shock scenario, bottom panel: pure turbulence scenario). One can see the difference by a factor of ~ 3 in a relatively narrow domain around $\gamma \sim 10^6$ for the shock scenario, and by ~ 40 % around $\gamma \sim 10^5$ for the turbulent scenario.

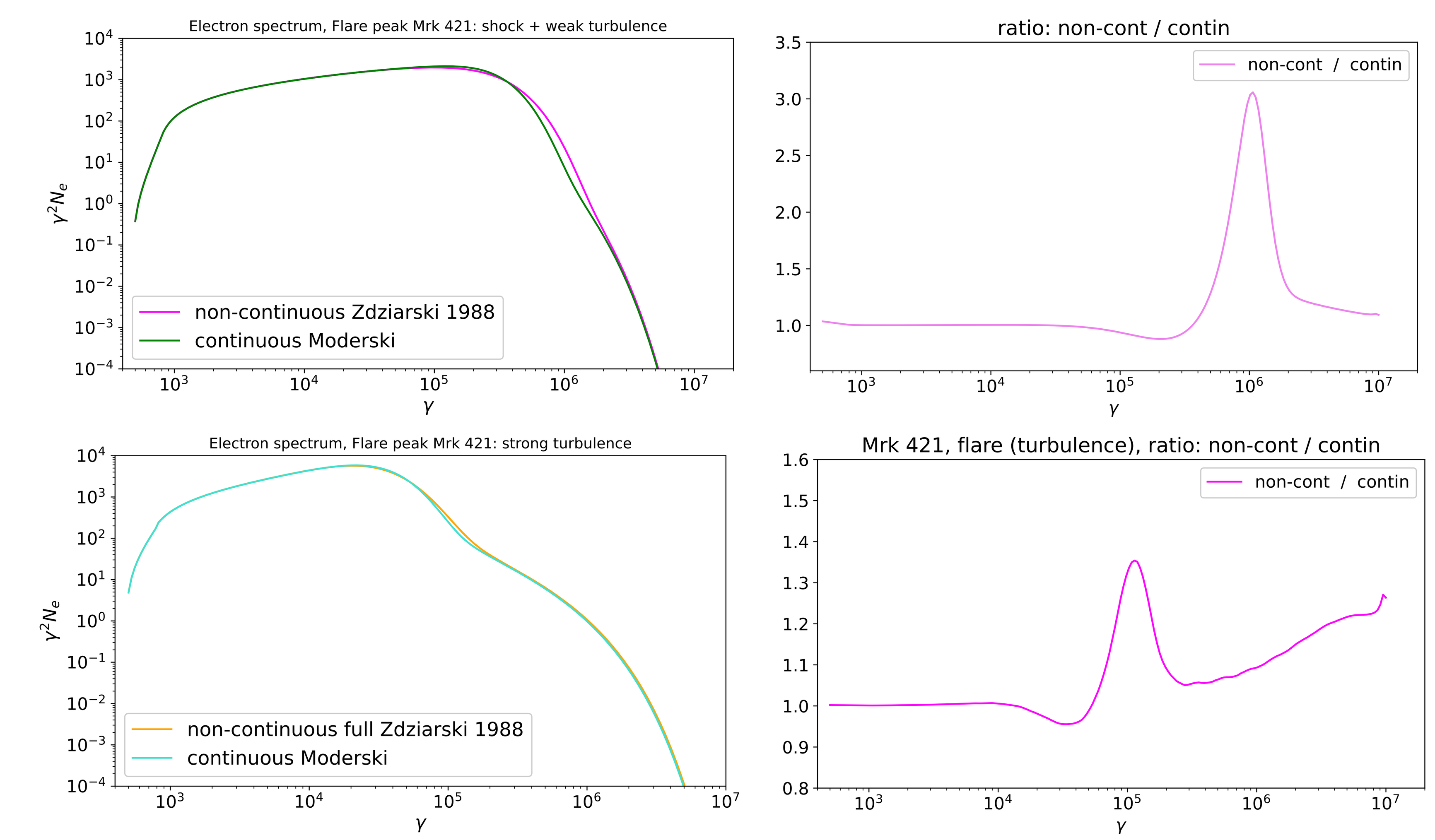

Figure 2. Comparison (left panel) and the ratio (right panel) of the electron spectra for the case of continuous and non-continuous cooling during the peak of a bright flare of Mrk 421 simulated with a shock plus mild turbulence (top panel) and pure turbulence (bottom panel) scenario.

Discussion and conclusions

- The non-continuous IC cooling effect can be rather important in certain blazars.
- The continuous-loss approximation by [2] is reasonable for Thomson and deep KN regime, but shows discrepancies up to a factor ~ 3 in the narrow transition domain.
- The effect is less pronounced for the turbulent re-acceleration scenario due to much higher redistribution (scatter) of particles in energy.

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