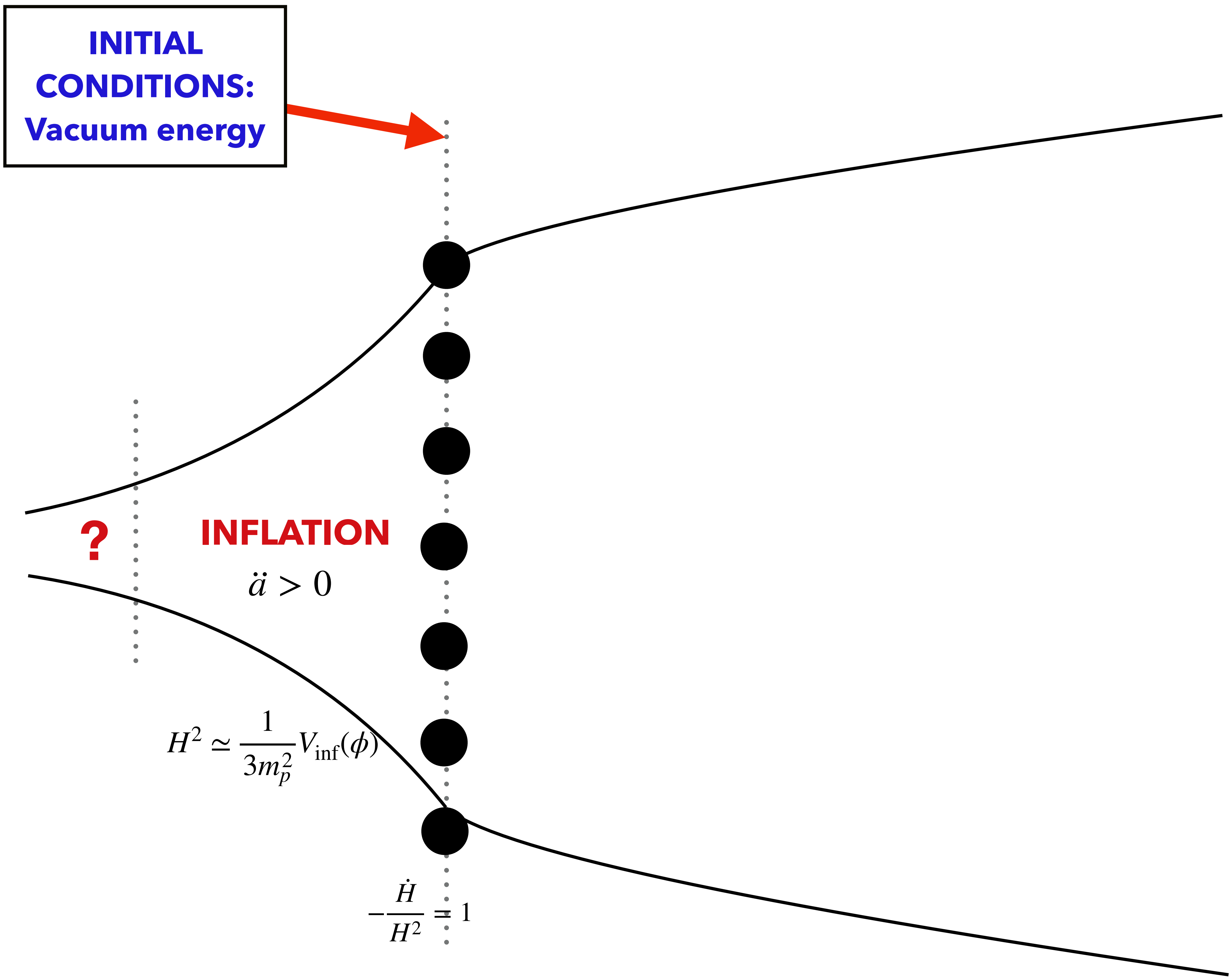
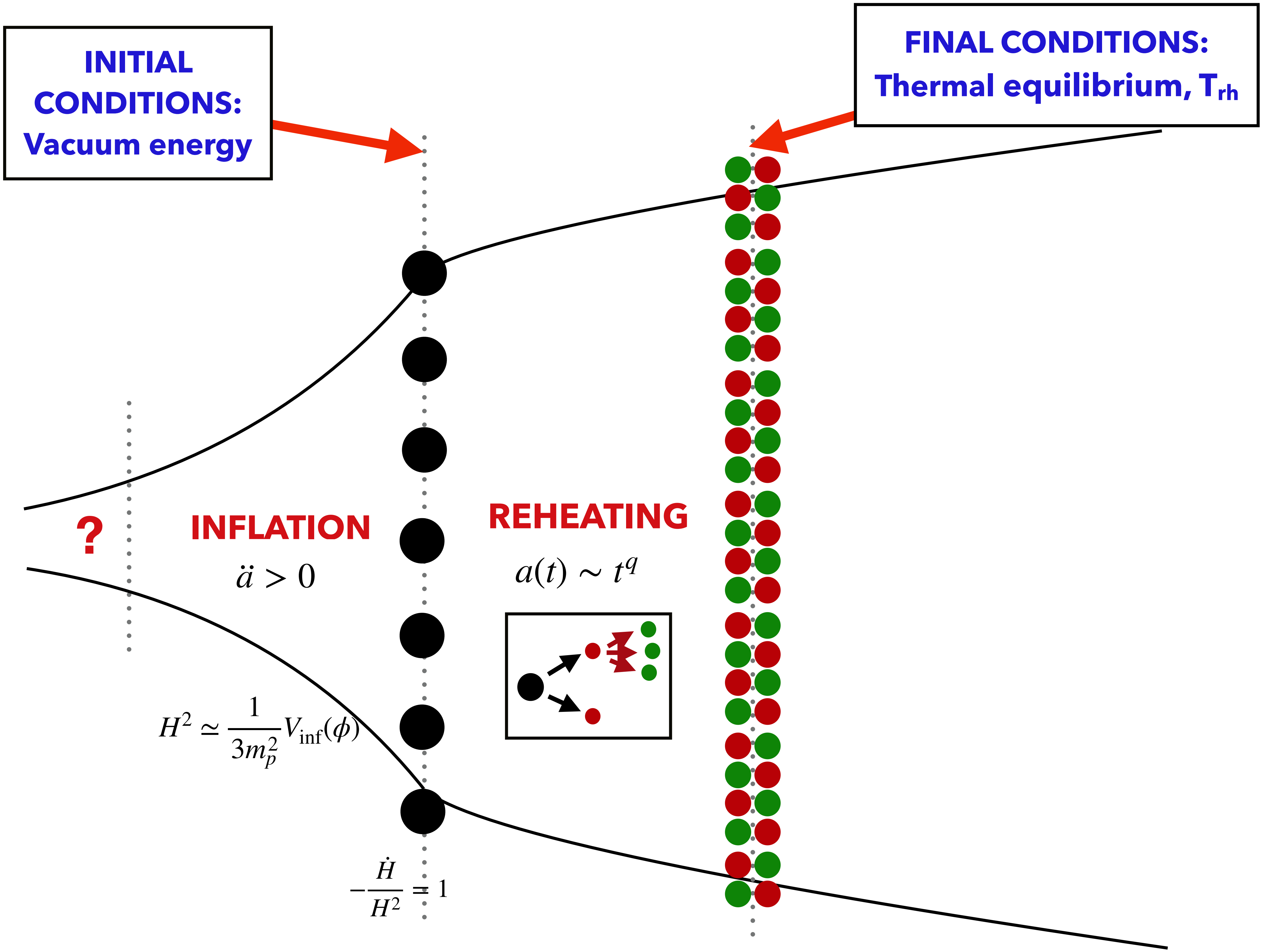
Lattice simulations of gravitational waves from cosmic domain walls

Francisco Torrentí
ICCUB, U. Barcelona


INITIAL CONDITIONS: Vacuum energy
?
INFLATION
$\ddot{a} > 0$
$H^2 \simeq \frac{1}{3m_p^2} V_{\rm inf}(\phi)$
$-\frac{\dot{H}}{H^2} = 1$



INITIAL CONDITIONS: Vacuum energy
FINAL CONDITIONS: Thermal equilibrium, T_{rh}
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REHEATING
$a(t) \sim t^q$
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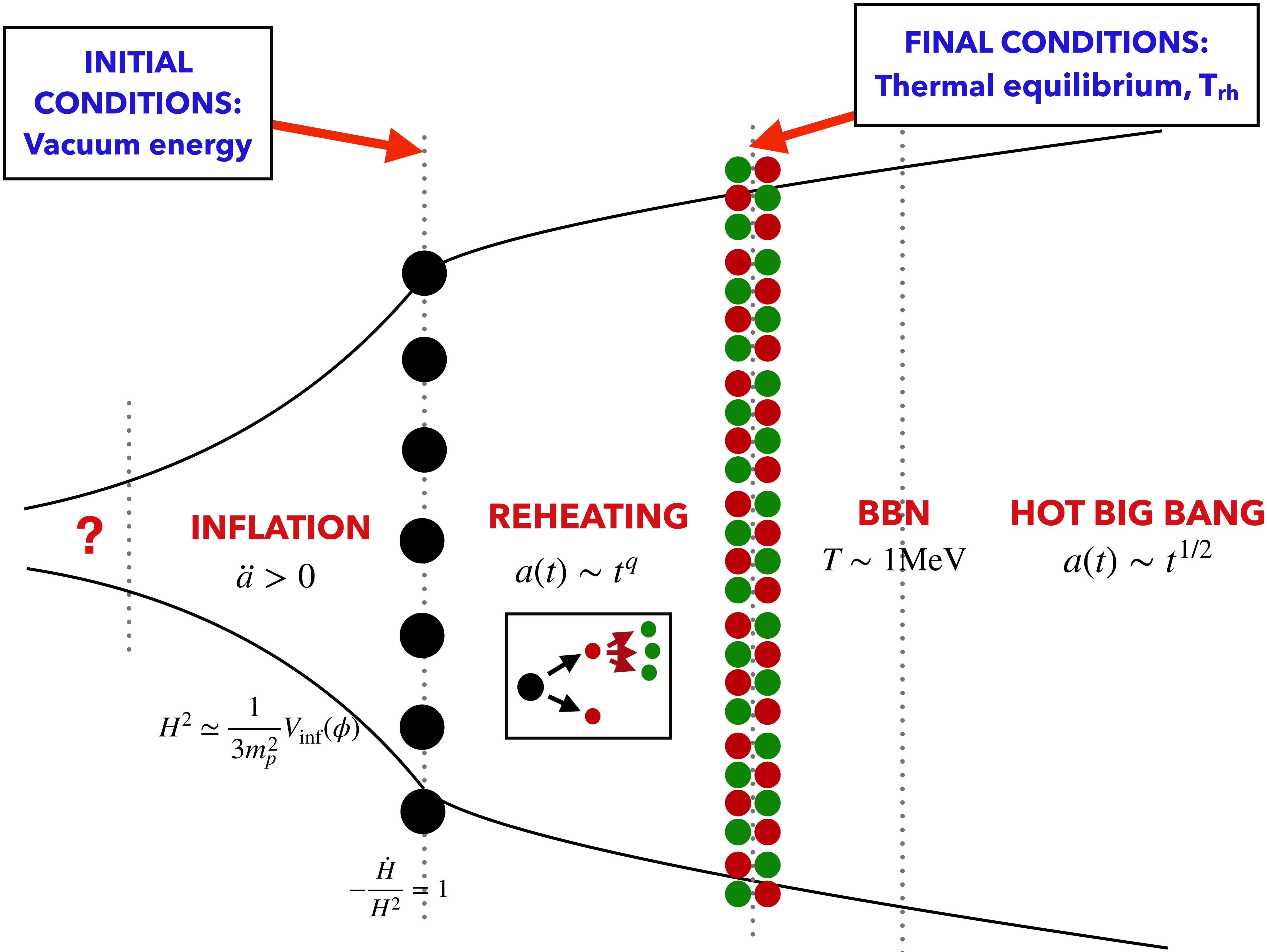

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FINAL CONDITIONS: Thermal equilibrium, T_{rh}
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INFLATION
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REHEATING
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BBN
$T \sim 1\text{MeV}$
HOT BIG BANG
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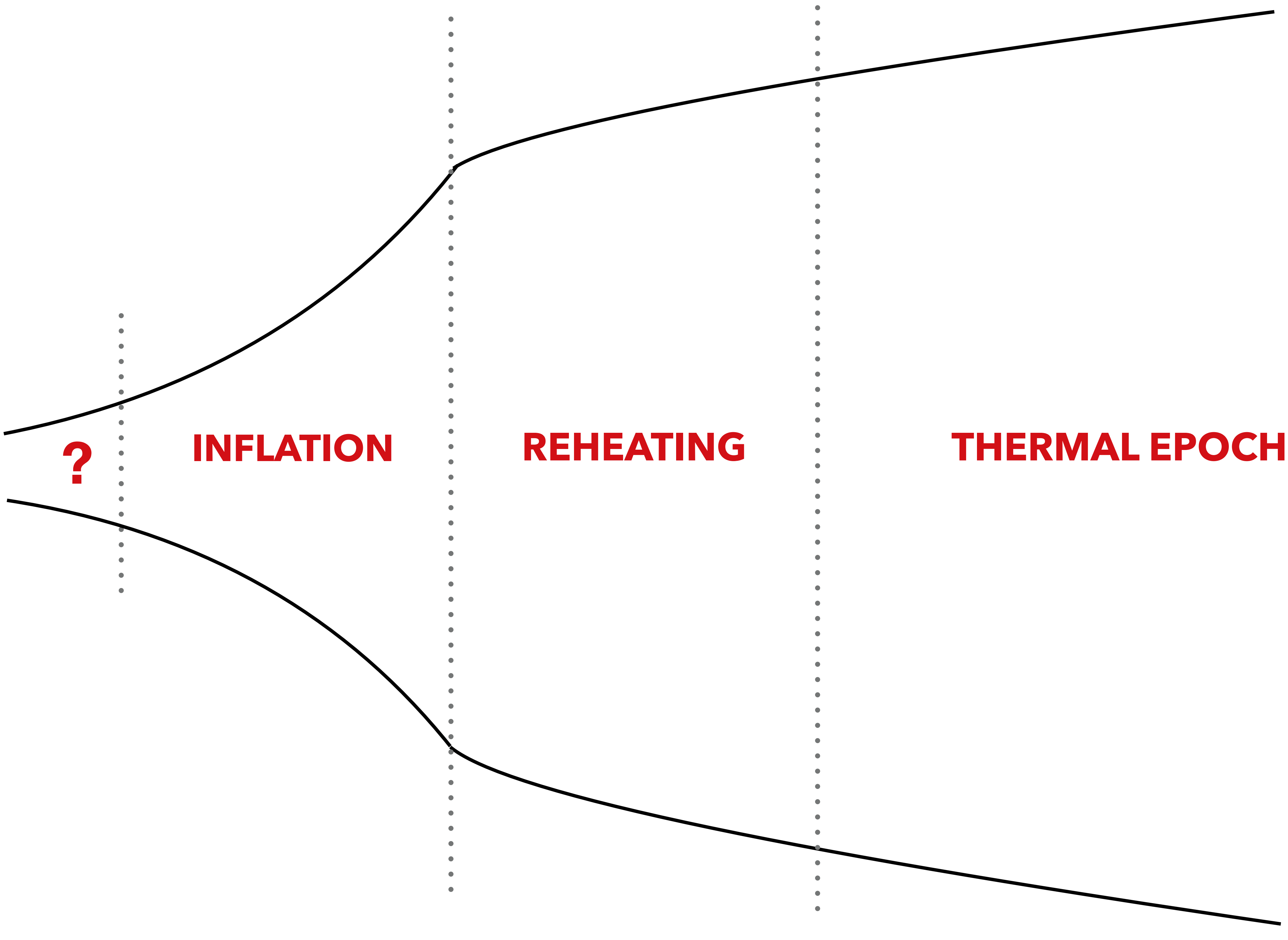
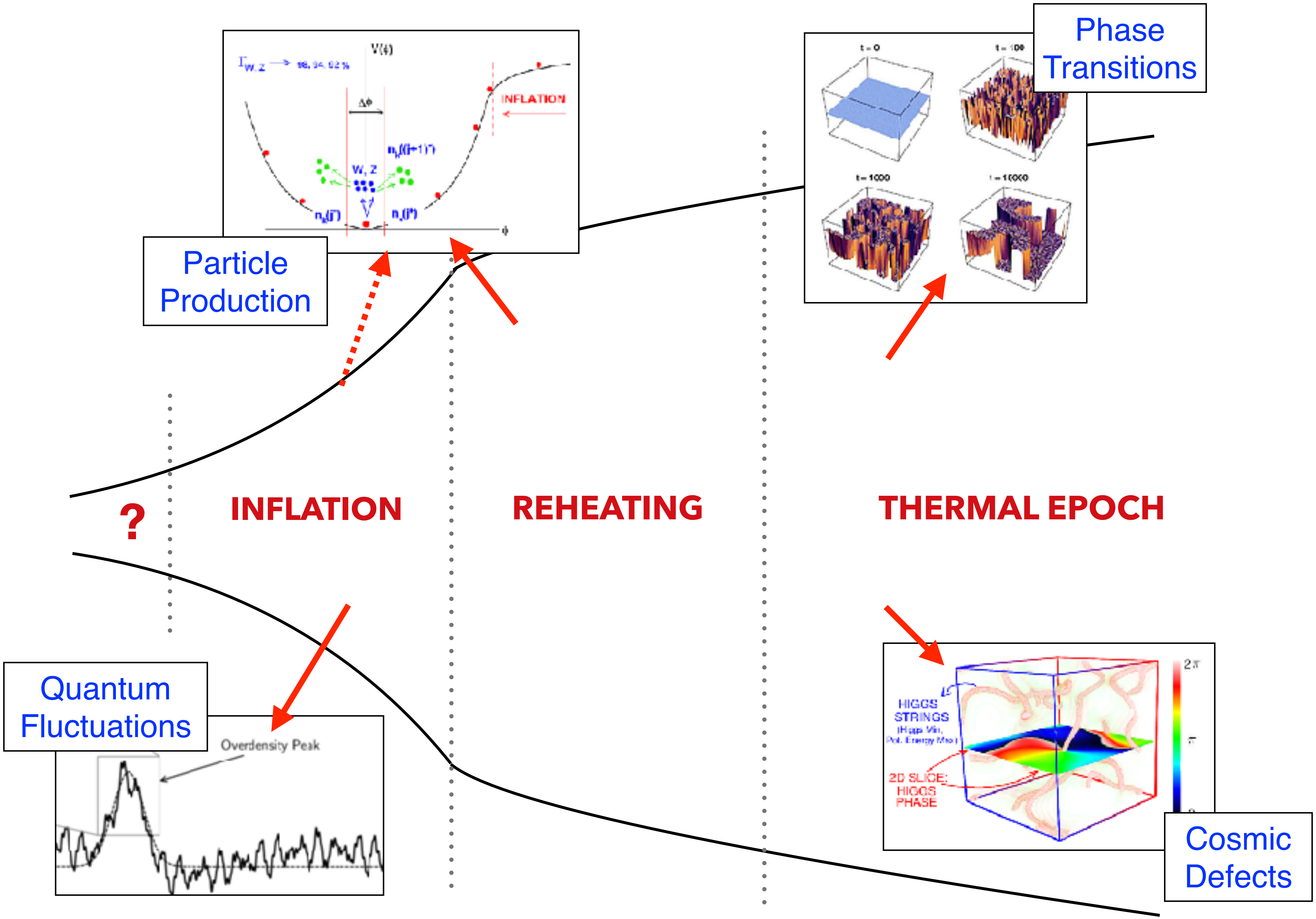
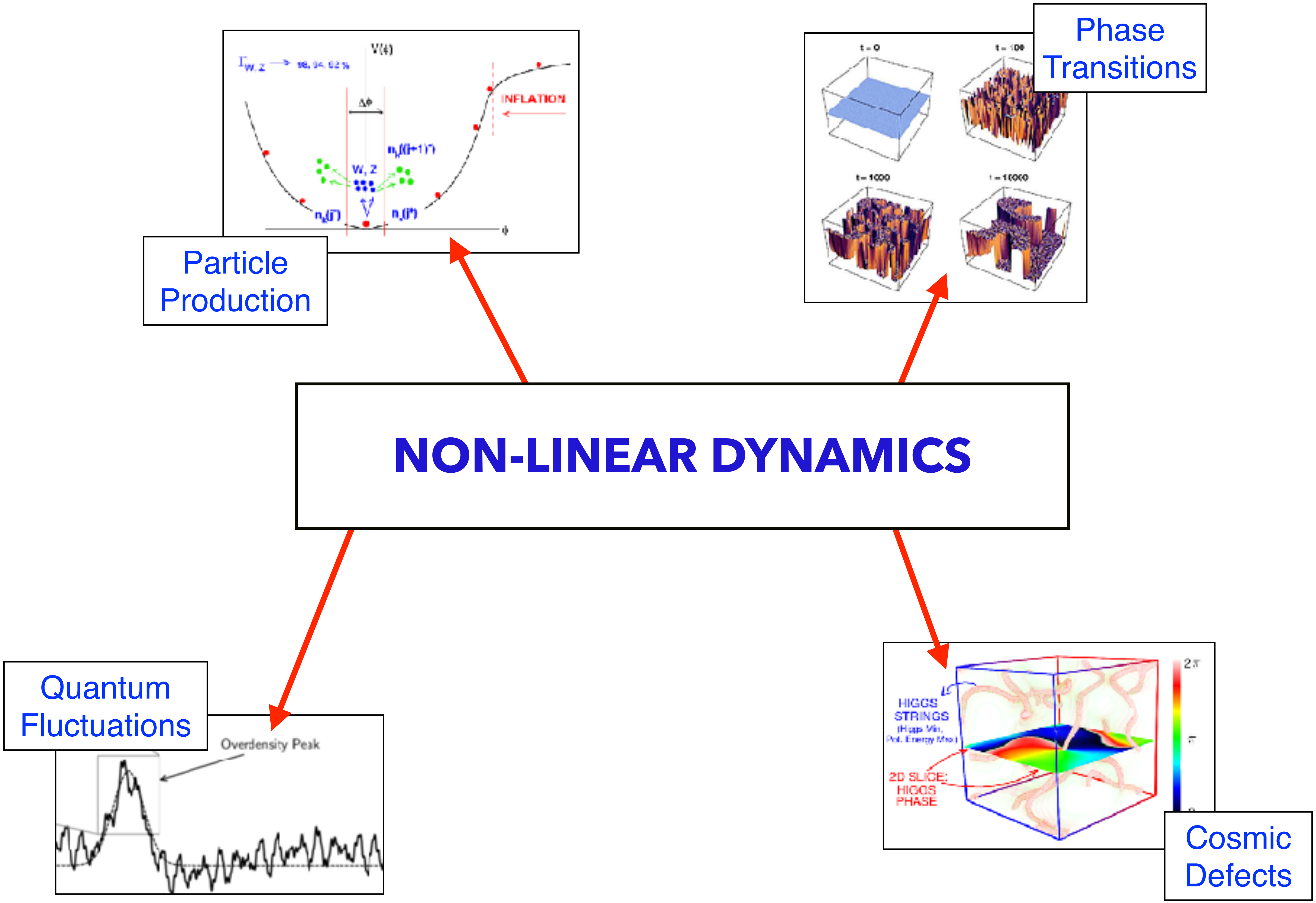
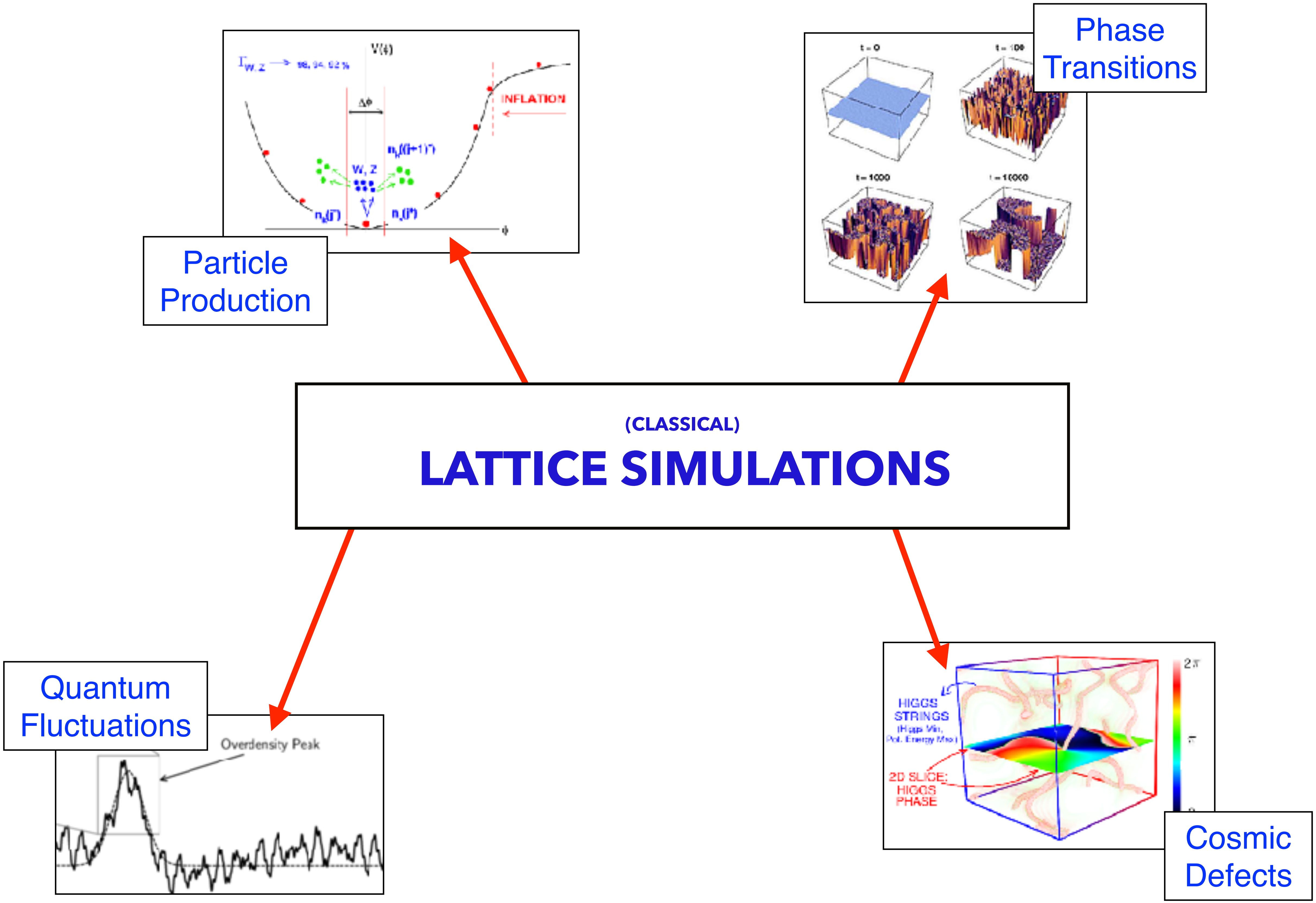

?
INFLATION
REHEATING
THERMAL EPOCH



Particle Production
Phase Transitions
?
INFLATION
REHEATING
THERMAL EPOCH
Quantum Fluctuations
Overdensity Peak
Cosmic Defects
HIGGS STRINGS
2D SLICE: HIGGS PHASE

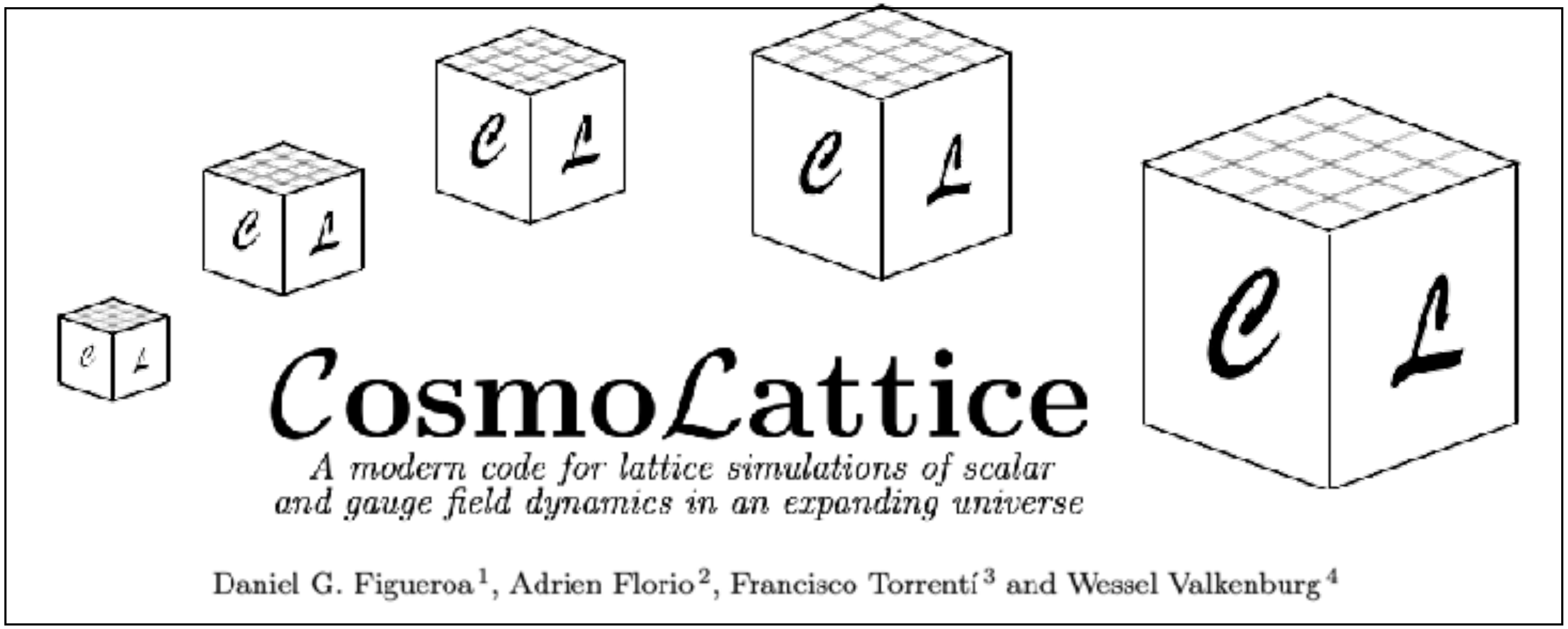


NON-LINEAR DYNAMICS
Particle Production
Phase Transitions
Quantum Fluctuations
Cosmic Defects
INFLATION
W, Z
t = 0
t = 1000
t = 10000
Overdensity Peak
HIGGS STRINGS
2D SLICE: HIGGS PHASE


(CLASSICAL)
LATTICE SIMULATIONS
Particle Production
Phase Transitions
Quantum Fluctuations
Cosmic Defects
INFLATION
W, Z
Overdensity Peak
HIGGS STRINGS
2D SLICE: HIGGS PHASE

Cosmo$\mathcal{L}$attice

arXiv: 2102.01031



Cosmo$\mathcal{L}$attice

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Cosmo$\mathcal{L}$attice

A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

Daniel G. Figueroa[1], Adrien Florio[2], Francisco Torrentí[3] and Wessel Valkenburg[4]

The Art of Simulating the Early Universe

A dissertation on lattice techniques for the simulation of scalar and gauge field dynamics in an expanding Universe

Monographic review: *JCAP* 04 (2021) 035

CosmoLattice

Publicly released in February 2021

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http://www.cosmolattice.net

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Cosmo$\mathcal{L}$attice: Field theory

- **Matter content:**

$$S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + (D_\mu^A\varphi)^*(D_A^\mu\varphi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{1}{2}\mathrm{Tr}\{G_{\mu\nu}G^{\mu\nu}\} + V(\phi, |\varphi|, |\Phi|)\right\}$$

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- **GWs (version 1.1 - released in may 2022):** [Baeza-Ballesteros, Figueroa, Florio, Loayza]

$$ds^2 = -a^{2\alpha}(\eta)d\eta^2 + a^2(\eta)(\delta_{ij} + h_{ij})dx^i dx^j \rightarrow \ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2}{a^2}h_{ij} = \frac{2}{m_p^2 a^2}\Pi_{ij}^{TT}$$

$$\Pi_{ij}^{TT} = (\partial_i\phi\partial_j\phi)^{TT}$$

Cosmo$\mathcal{L}$attice: Lattice formulation

➤ Equations are written as a set of **coupled first-order differential equations**, which are solved with a **Hamiltonian scheme**:

Example: scalar field

$$\phi'' - \frac{1}{a^2}\nabla^2\phi + \frac{3}{a}\frac{da}{dt}\frac{d\phi}{dt} = -\frac{\partial V}{\partial \phi}$$

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KICK: $$(\pi_\phi)' = -a^{3+\alpha}\frac{\partial V}{\partial \phi} + a^{1+\alpha}\nabla^2\phi$$

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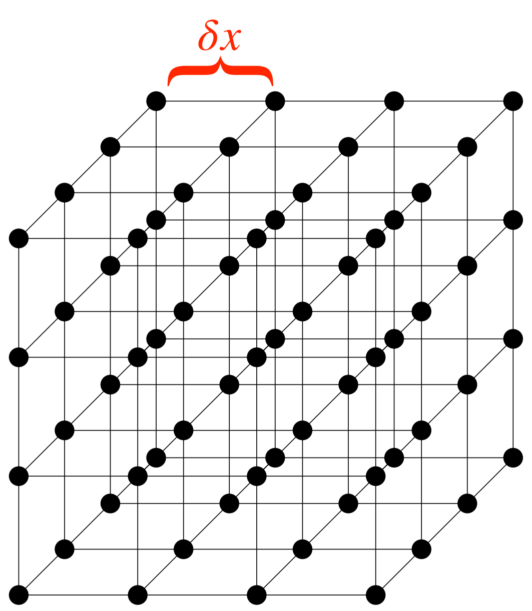
- Scalar fields and momenta are defined in the **lattice sites**:



N : number of points/dimension

$L = N \cdot \delta x$: length side

δt : time step

Minimum and maximum momenta:

$$k_{\rm min} = \frac{2\pi}{L} \qquad k_{\rm max} = \frac{\sqrt{3}}{2} N k_{\rm min}$$

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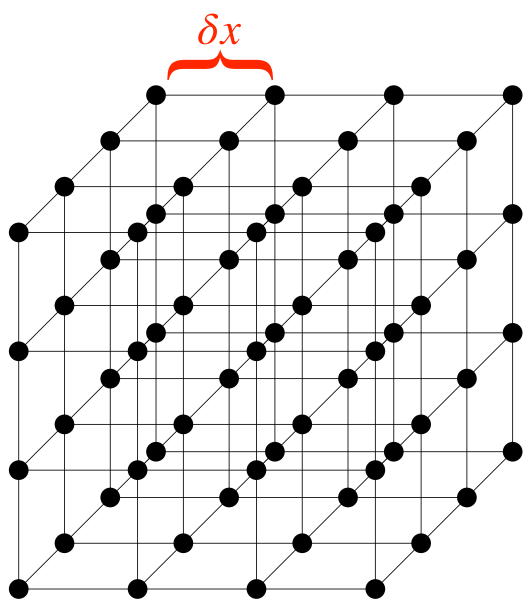

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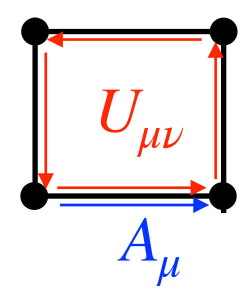
- **Gauge fields** introduced via **links** and **plaquettes** (like in **lattice-QCD**)



CosmoLattice: Output

Three kinds of output

Cosmo$\mathcal{L}$attice: Output

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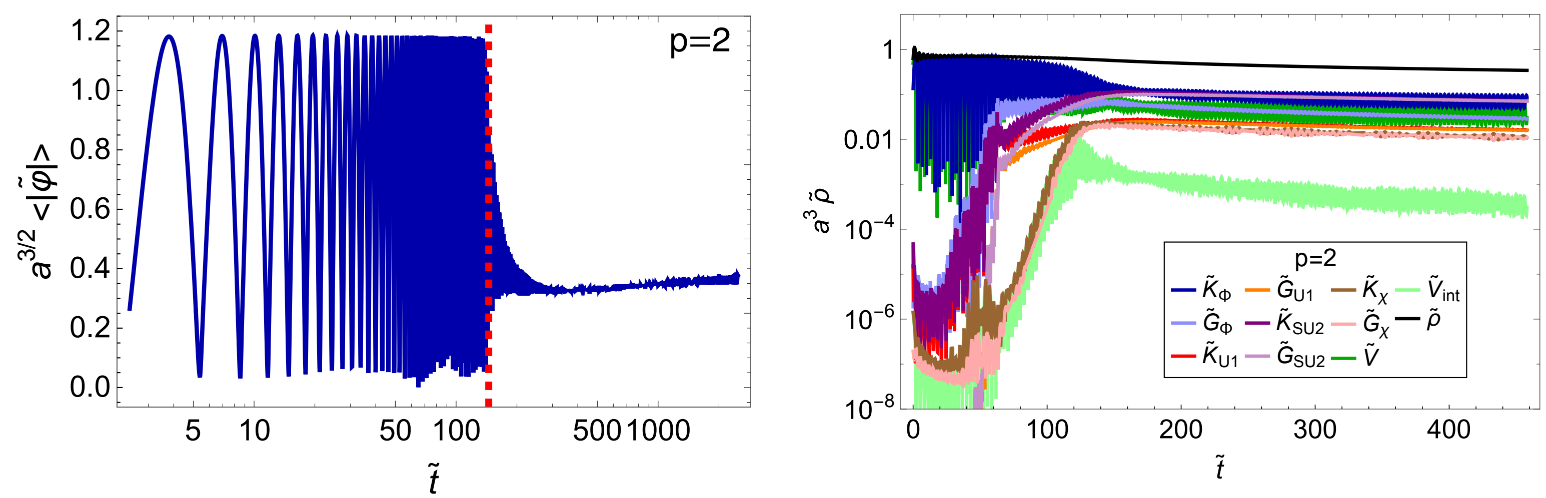
Volume averages: Spatial averages of certain quantities, such as field amplitudes or energies



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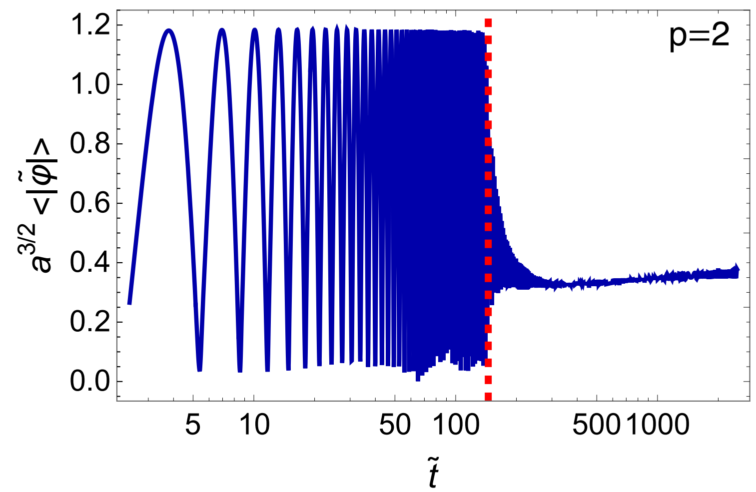
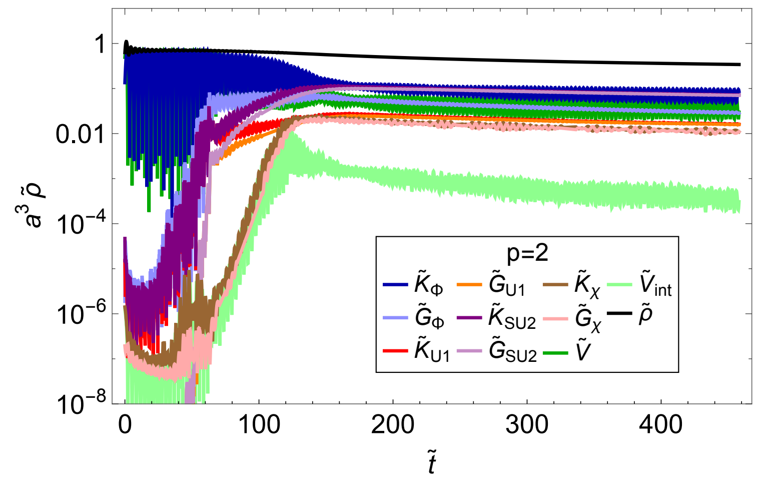
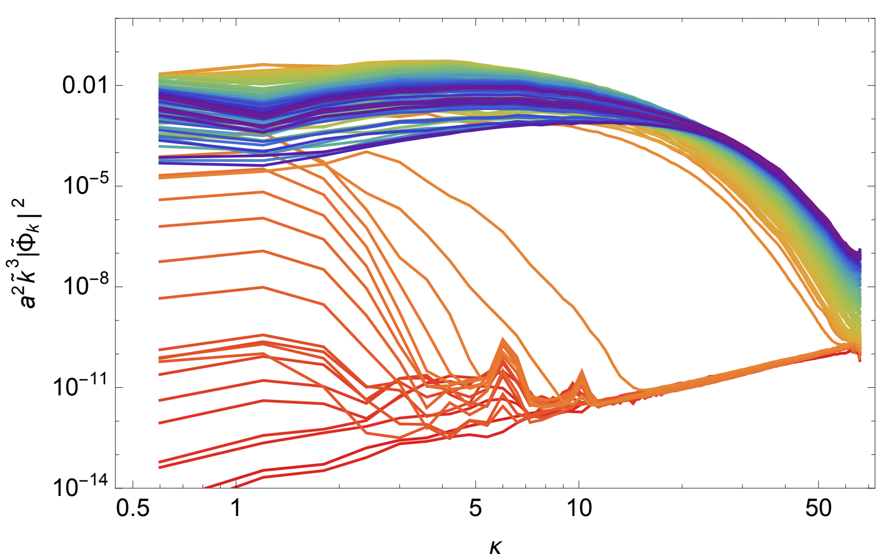




Spectra: Binned spectra in momentum space



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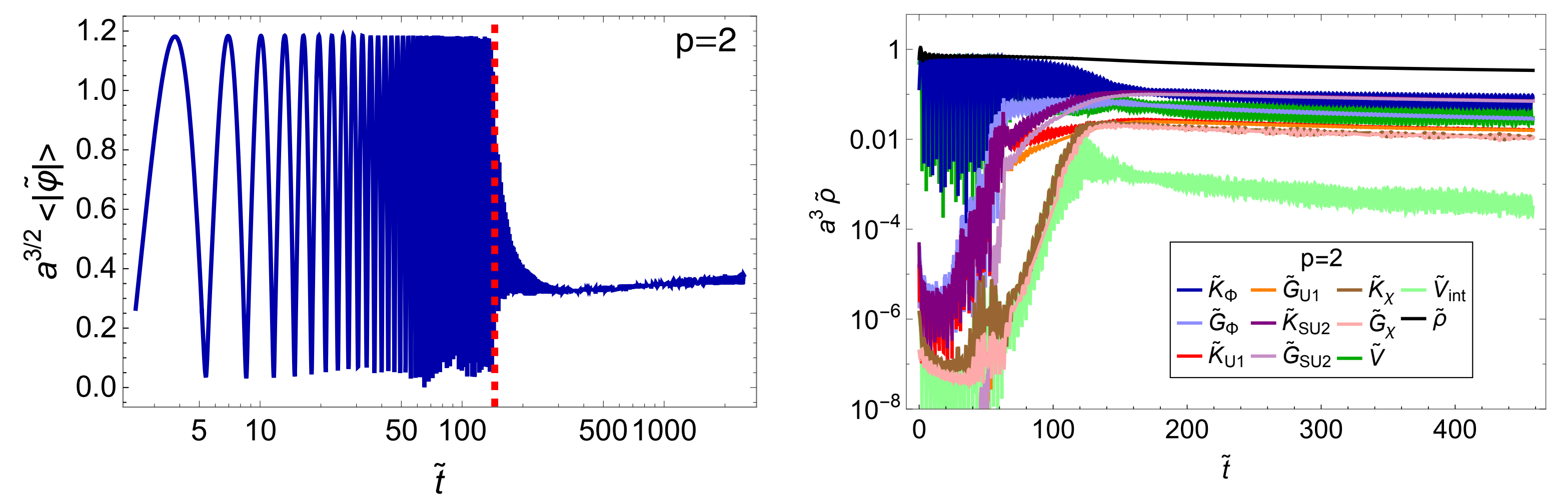

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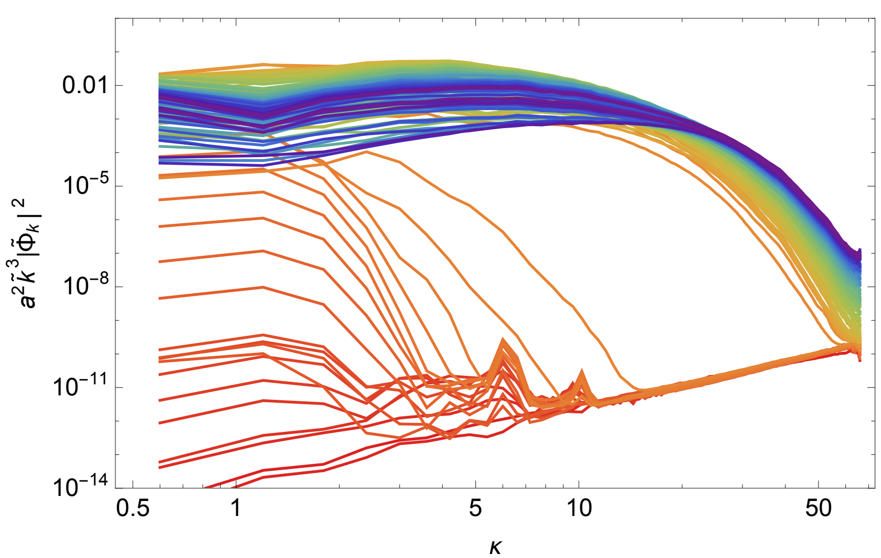
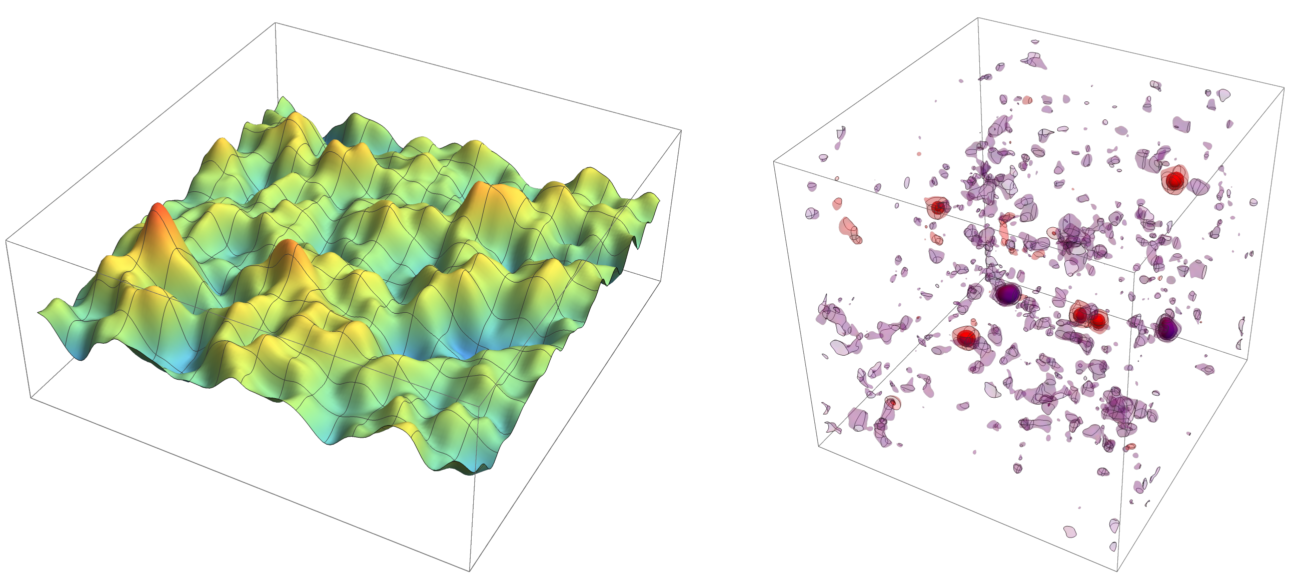


Snapshots: Values of a certain quantity at all points of the lattice.



Cosmo$\mathcal{L}$attice 2.0

2.0

Baeza, Figueroa, Florio,
Loayza, F.T., & Urio

CosmoLattice 2.0

Baeza, Figueroa, Florio,
Loayza, F.T., & Urio

- Axion - gauge interactions $\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$ *[Figueroa, Lizarraga, Urio, Urrestilla, (2023)]*
- Non-minimal gravitational coupling $\xi\phi^2 R$ *[Figueroa, Florio, Opferkuch, Stefanek (2023)]*
- Cosmic defects
- Simulations in d+1 dimensions
- New technical features (visualization, initial conditions…)
- …

expected date: 2025

So you want to Cosmo$\mathcal{L}$attice?

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Recorded lectures:

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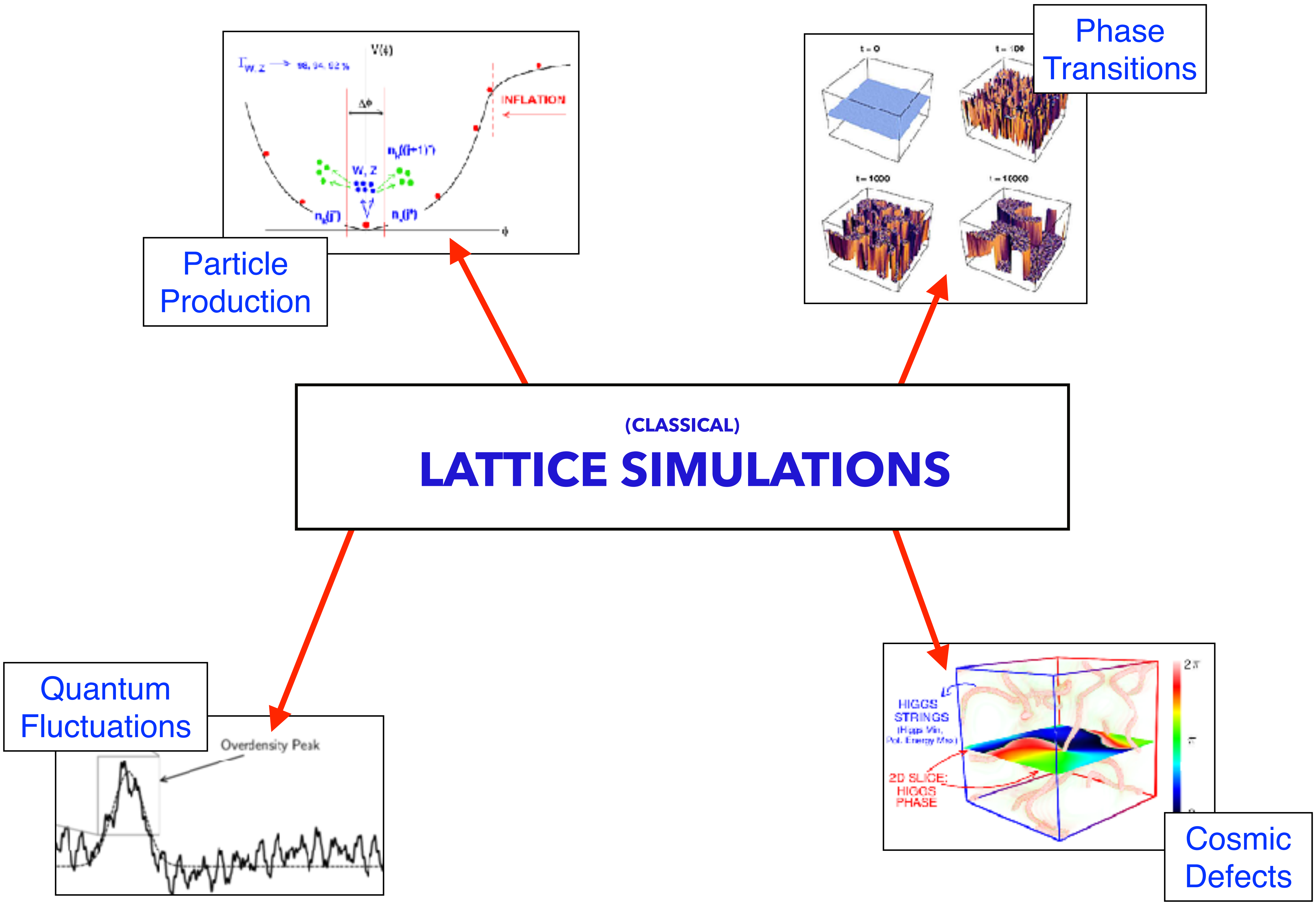

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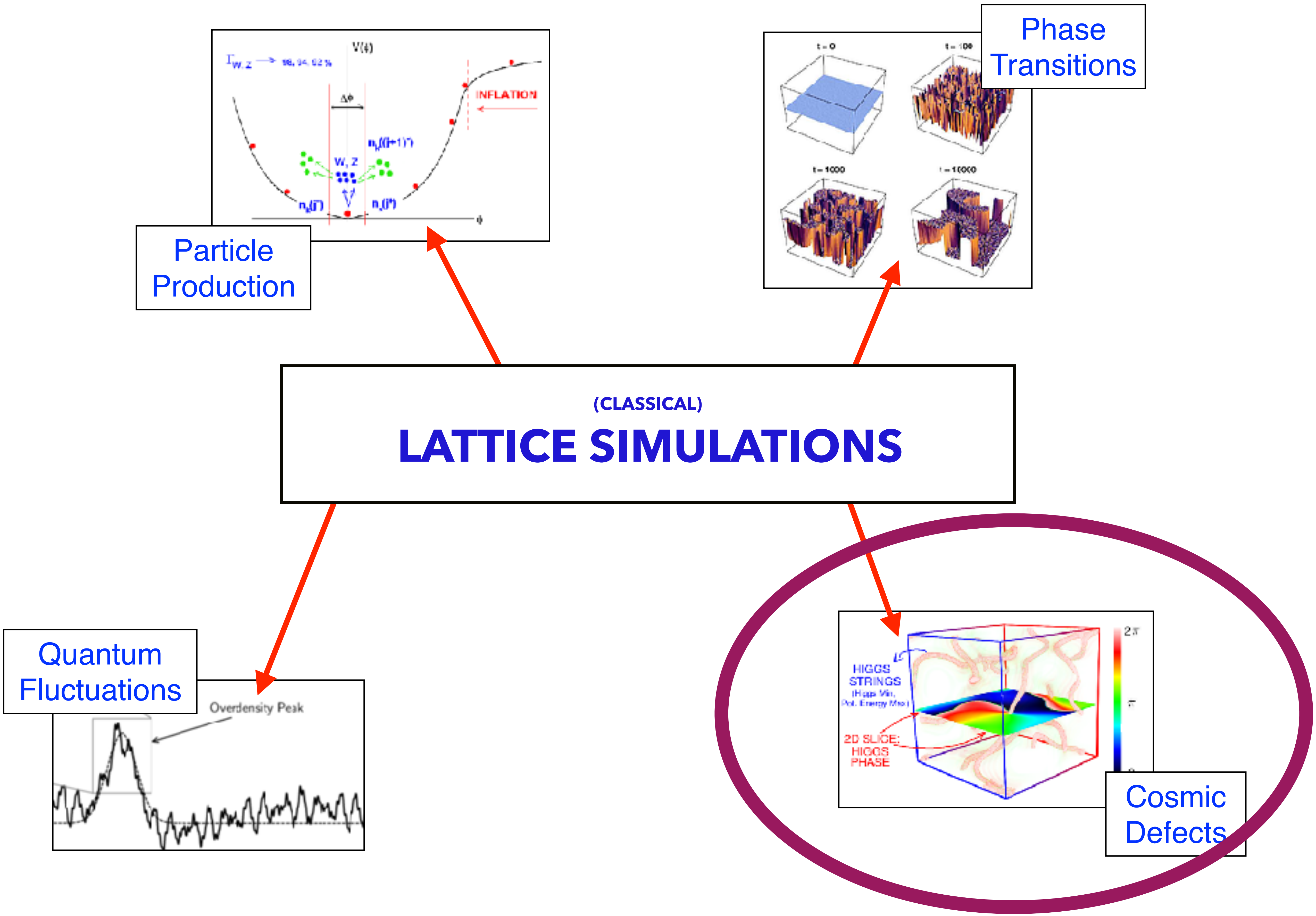

(CLASSICAL)
LATTICE SIMULATIONS
Particle Production
INFLATION
W, Z
Phase Transitions
Quantum Fluctuations
Overdensity Peak
Cosmic Defects
HIGGS STRINGS
2D SLICE: HIGGS PHASE

Domain wall networks

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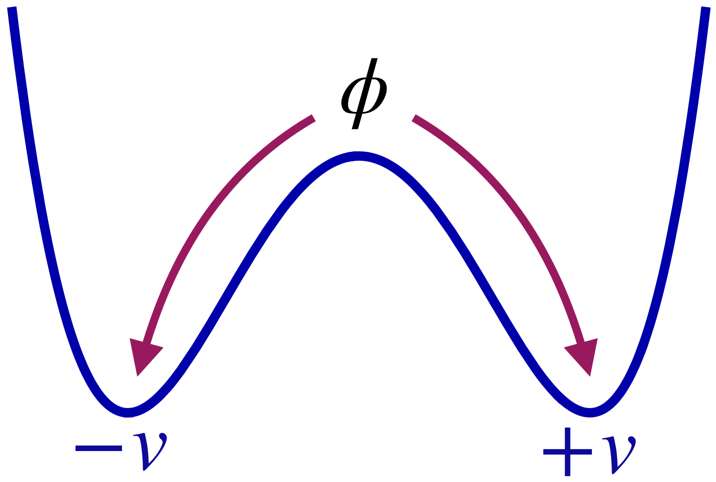

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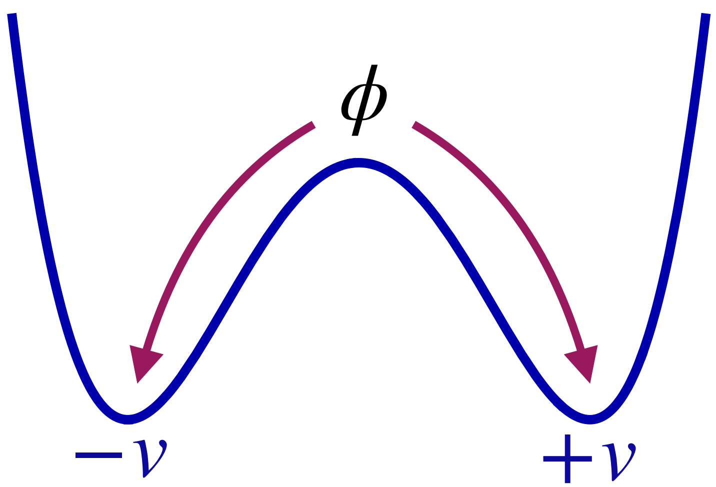

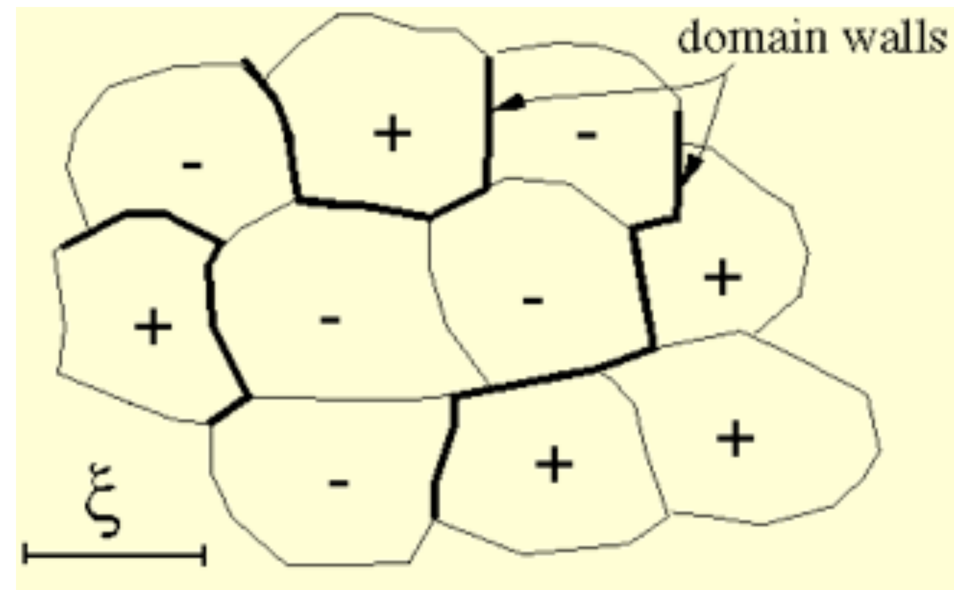

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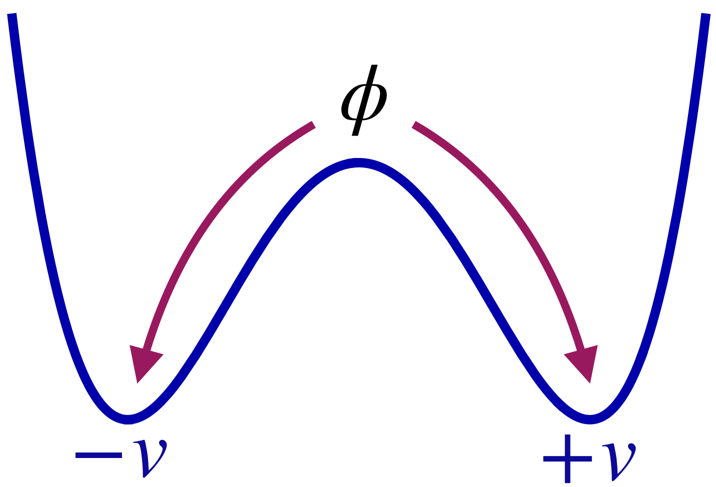

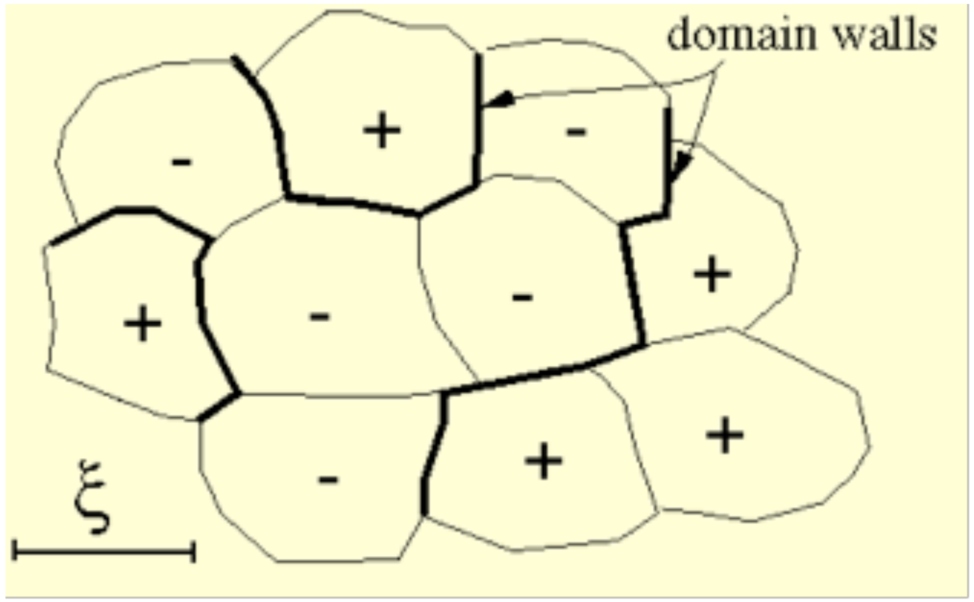

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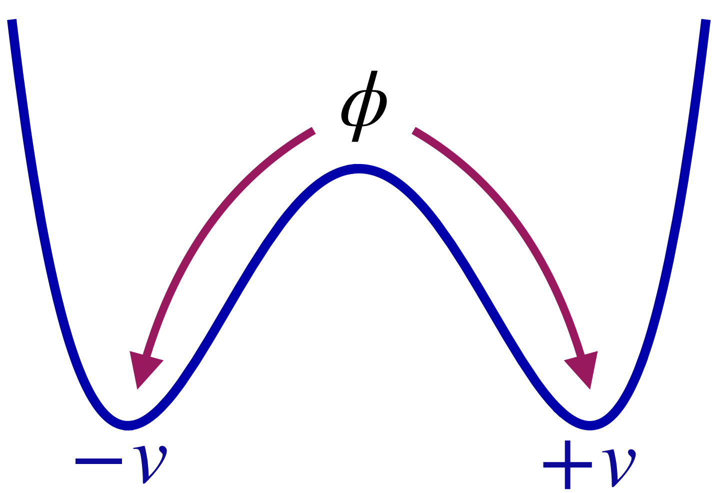

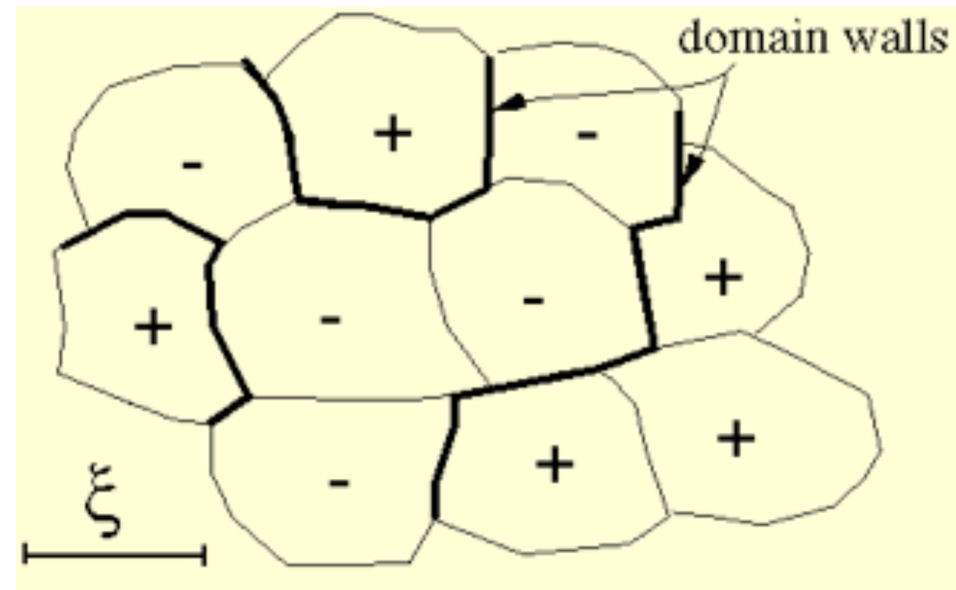

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- Width: $m \equiv \sqrt{2\lambda} v$
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(energy/area)

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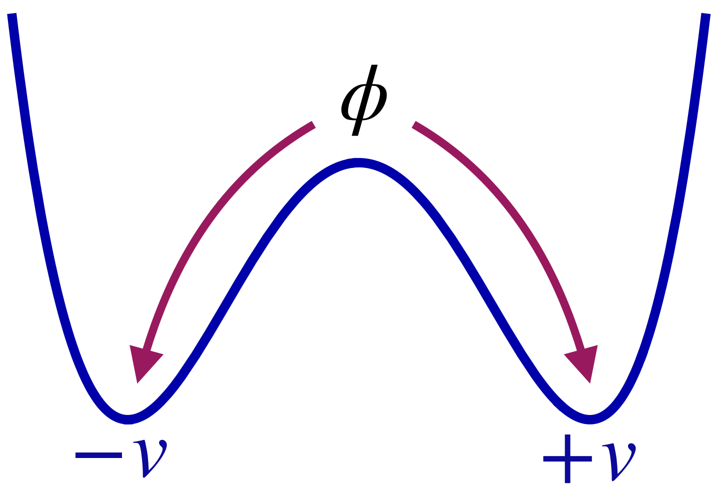

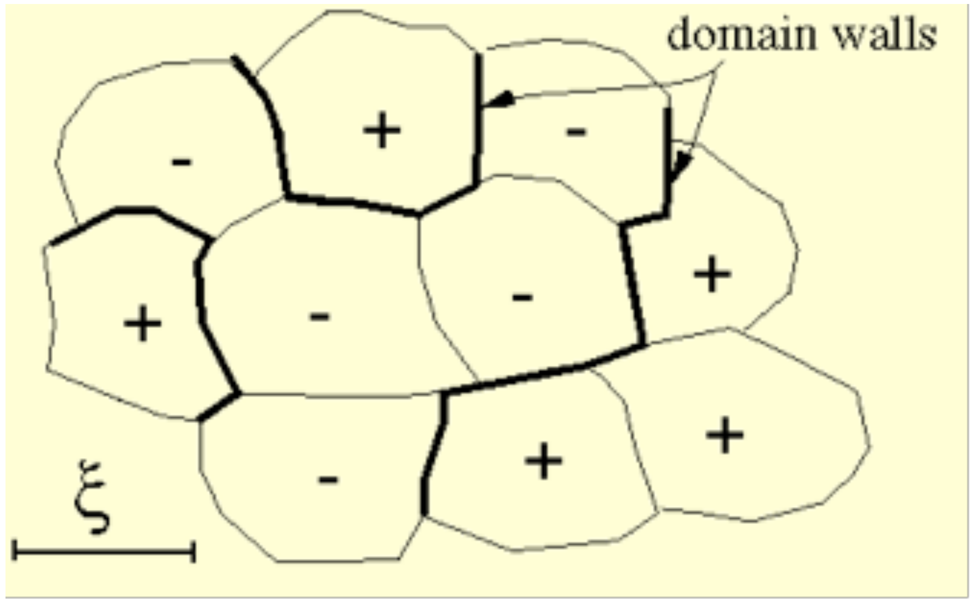

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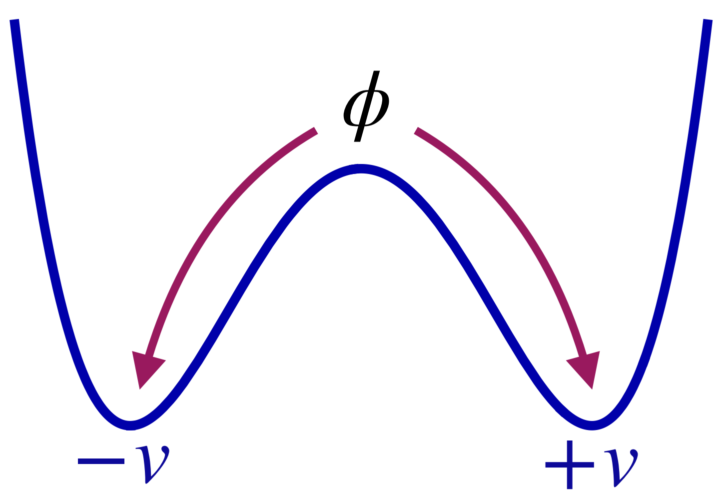

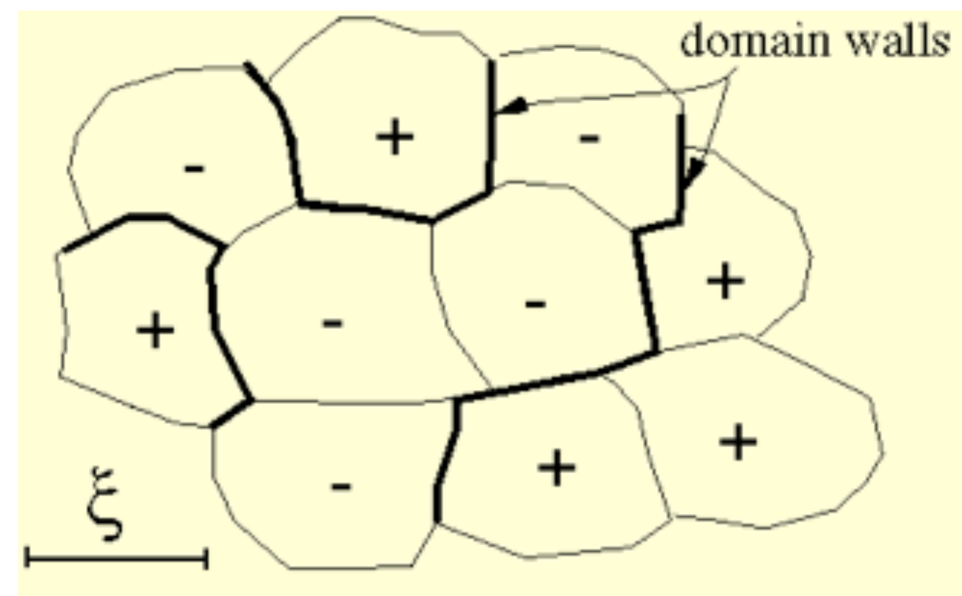

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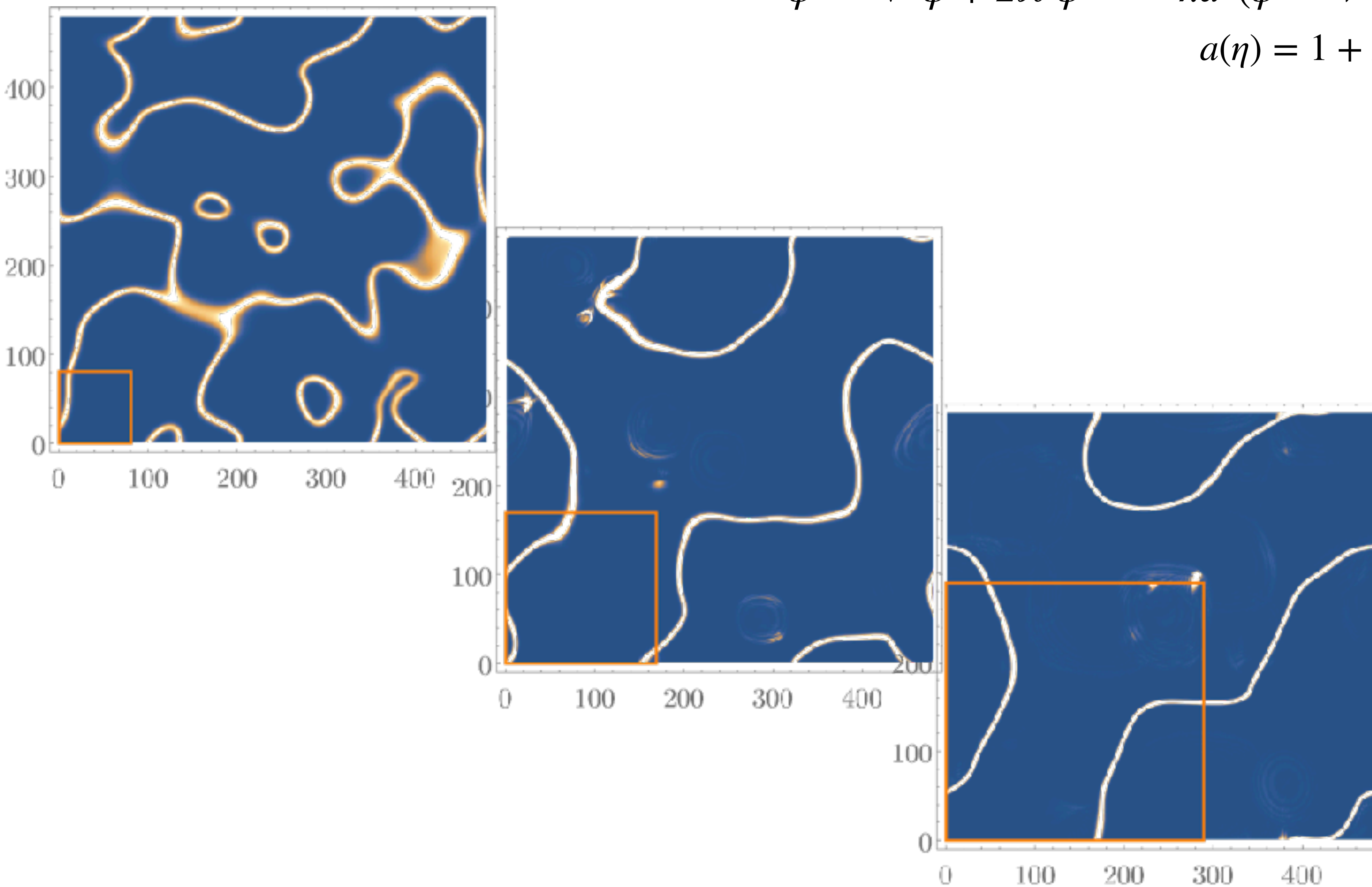
$$\mathscr{A} \equiv \frac{A}{V}\frac{1}{2aH} \simeq \text{const}$$

Area parameter

Domain wall networks

$$\phi'' - \nabla^2\phi + 2\mathscr{H}\phi' = -\lambda a^2(\phi^2 - v^2)\phi$$

$$a(\eta) = 1 + \eta$$



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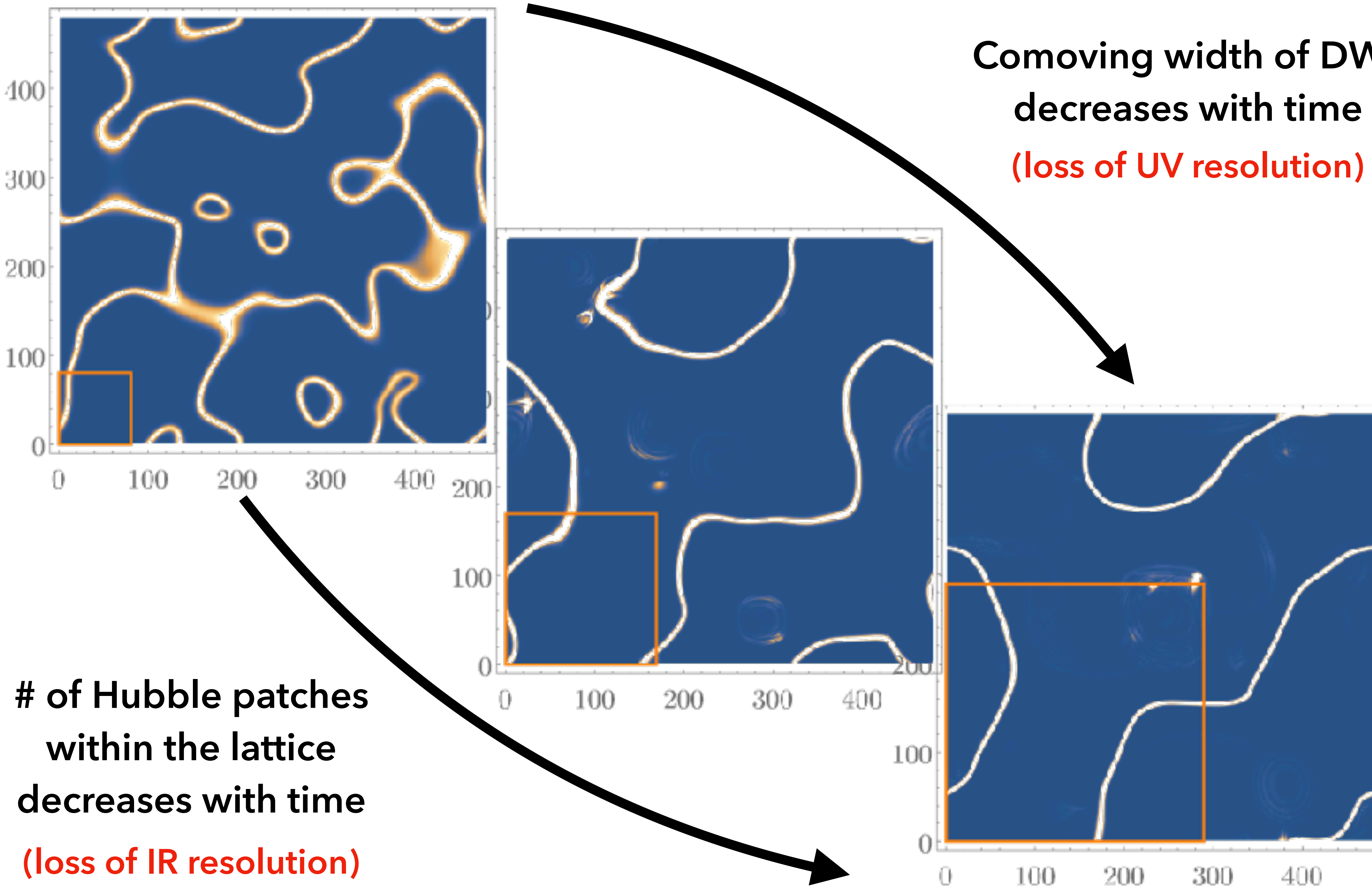


Comoving width of DWs decreases with time

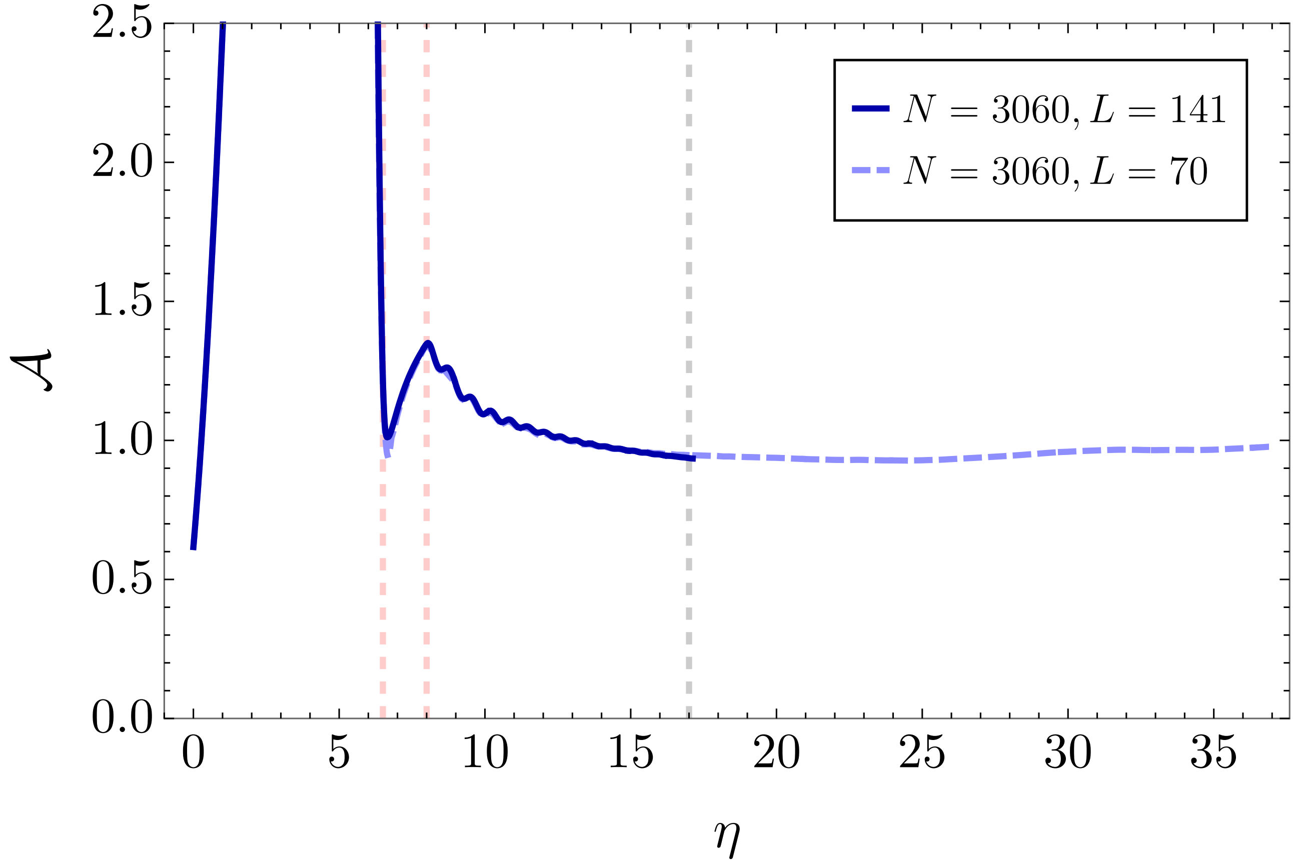
(loss of UV resolution)

# of Hubble patches within the lattice decreases with time

(loss of IR resolution)

DW network evolution (scaling regime)

Area parameter



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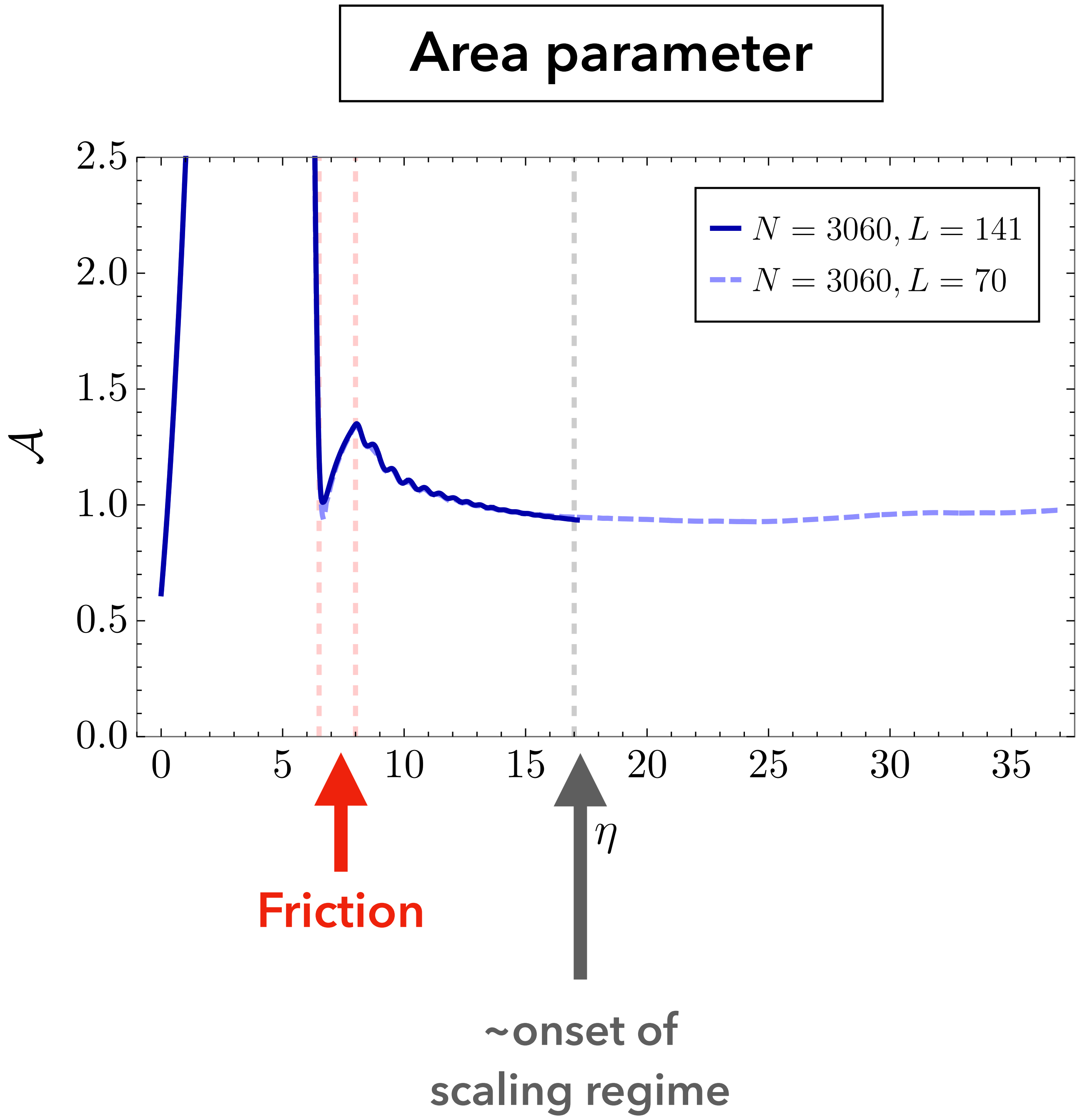

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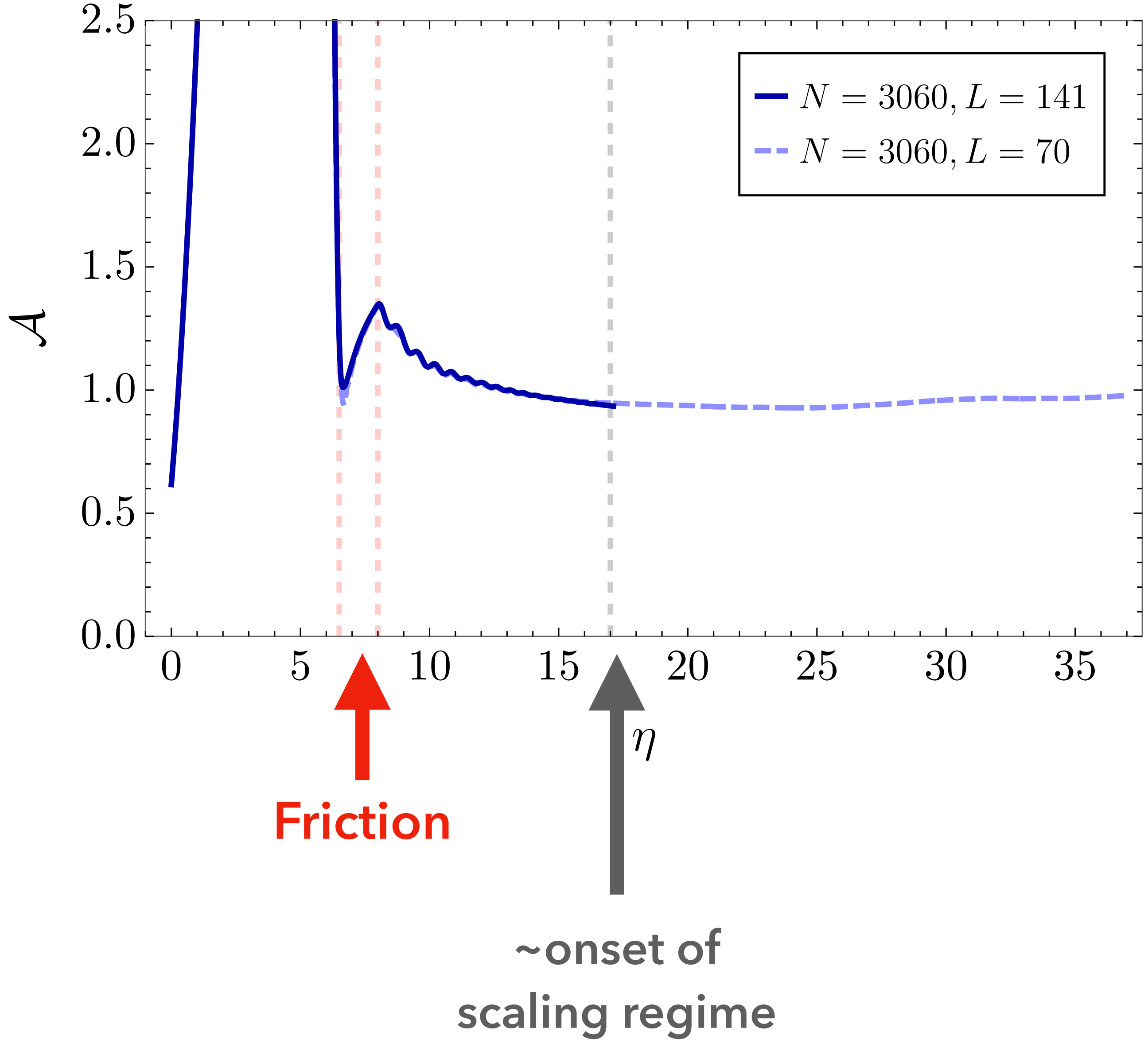
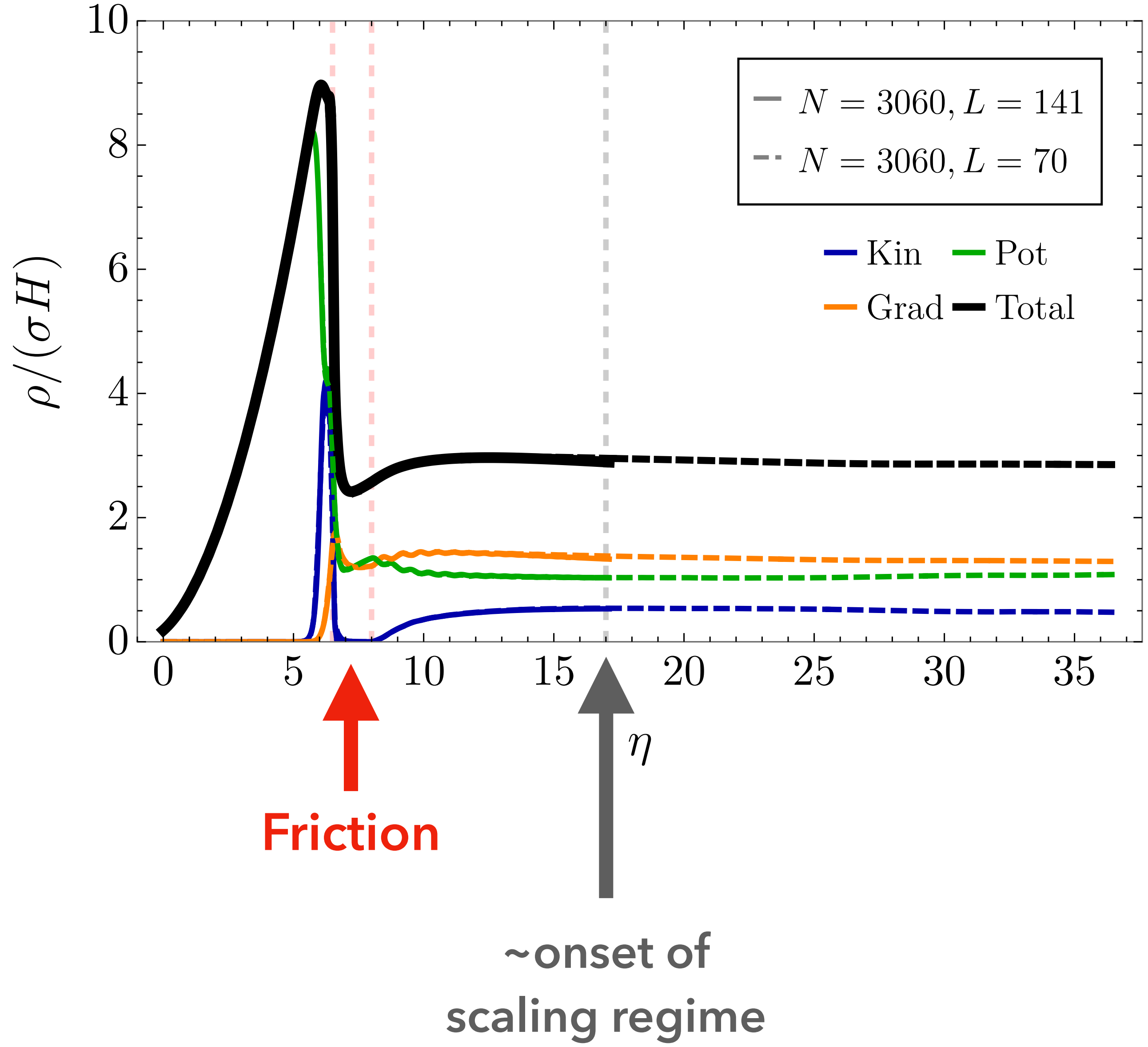
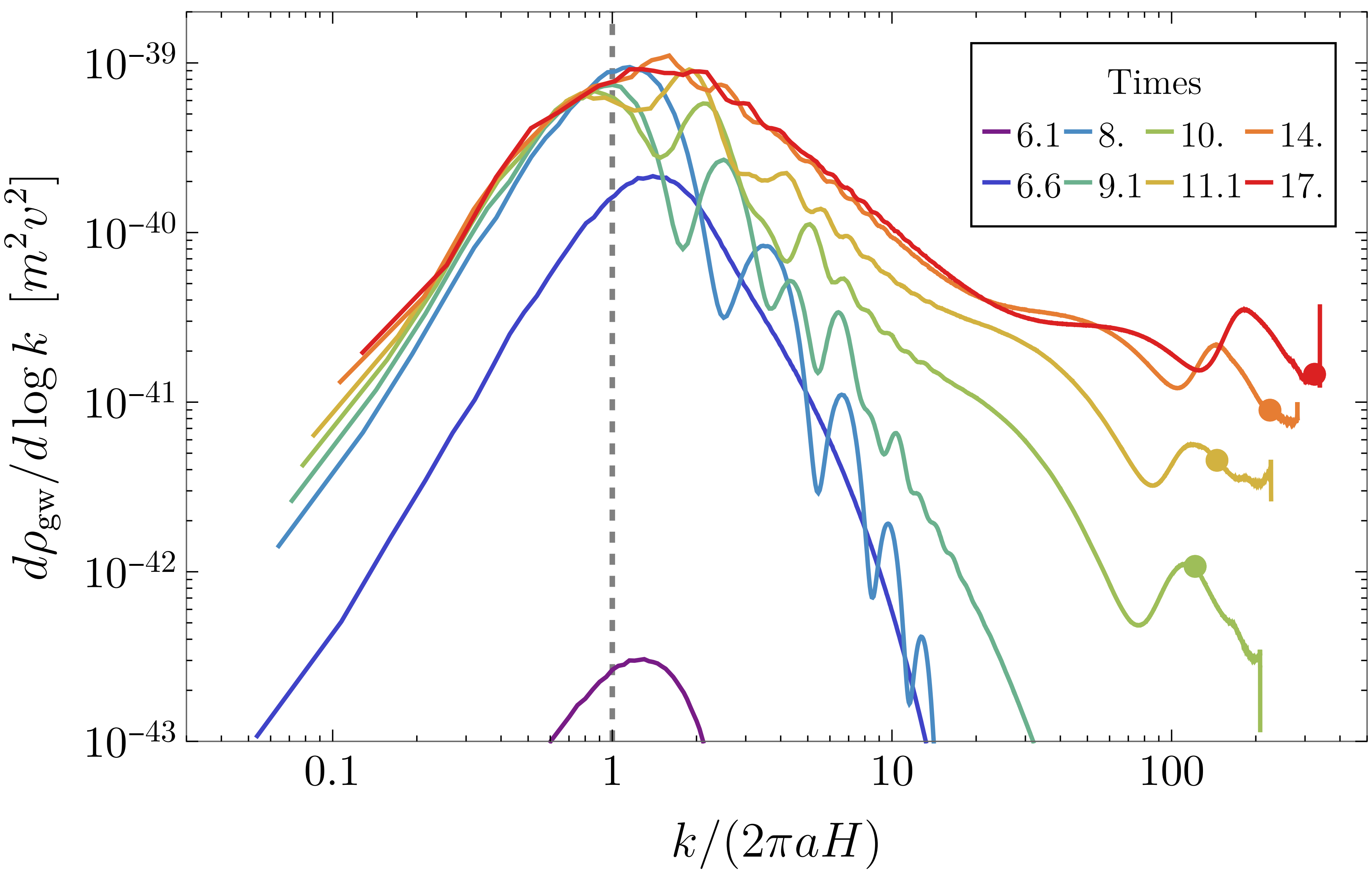


Energy density



GW spectrum from DWs (scaling)

$$h''_{ij} - \nabla^2 h_{ij} + 2\mathcal{H} h'_{ij} = \frac{2}{m_p^2}(\partial_i \phi \partial_j \phi)^{\rm TT}$$



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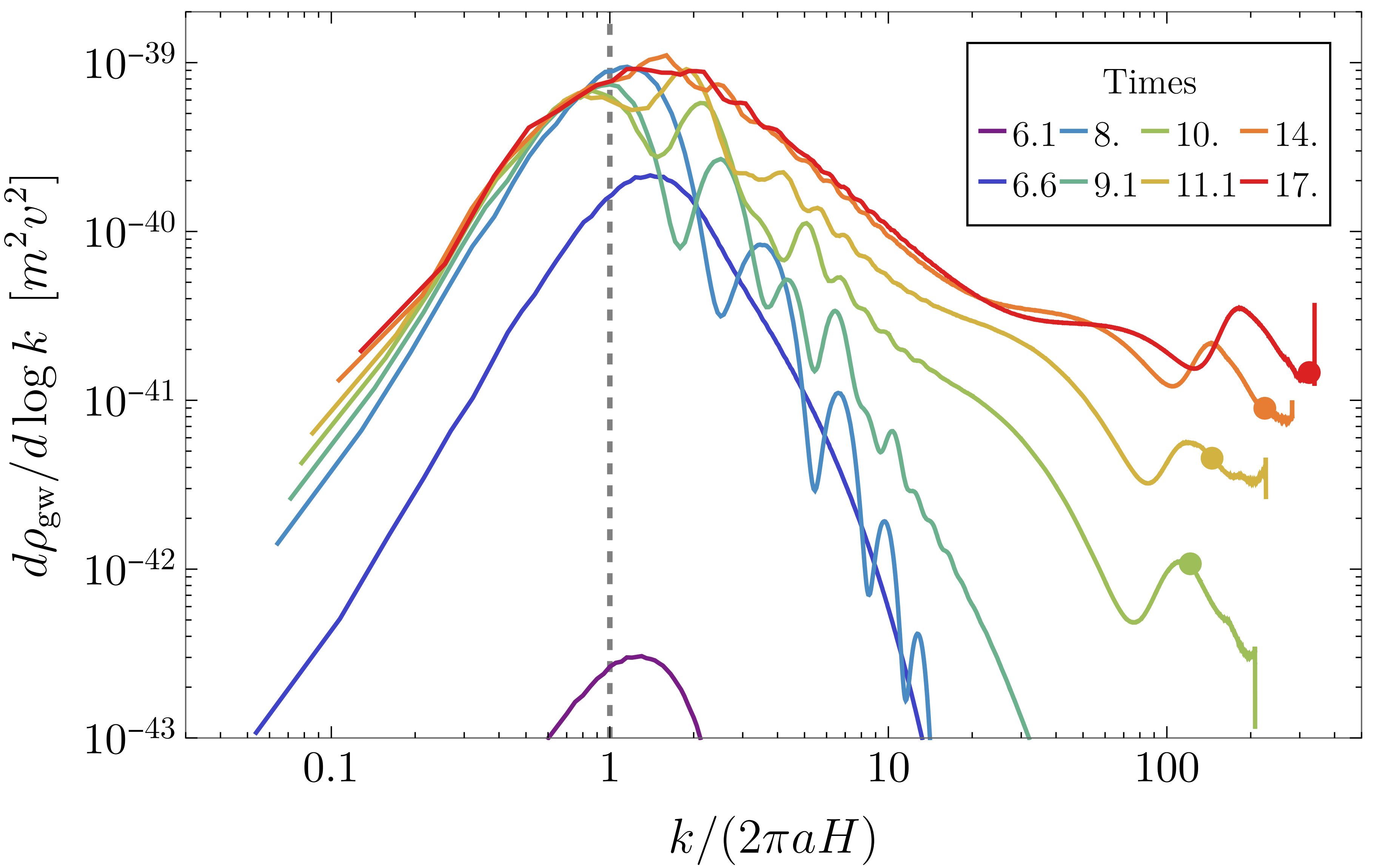

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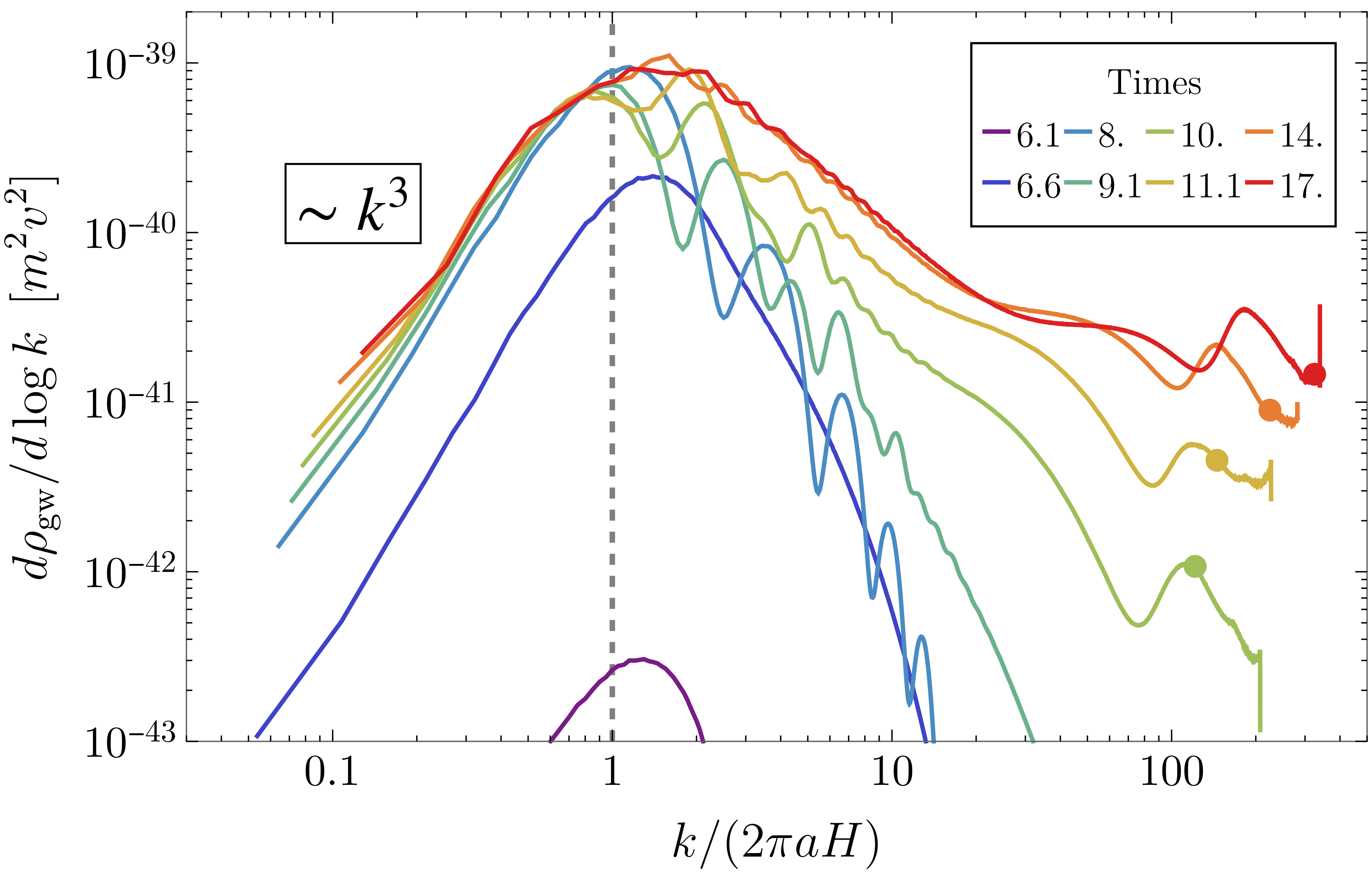

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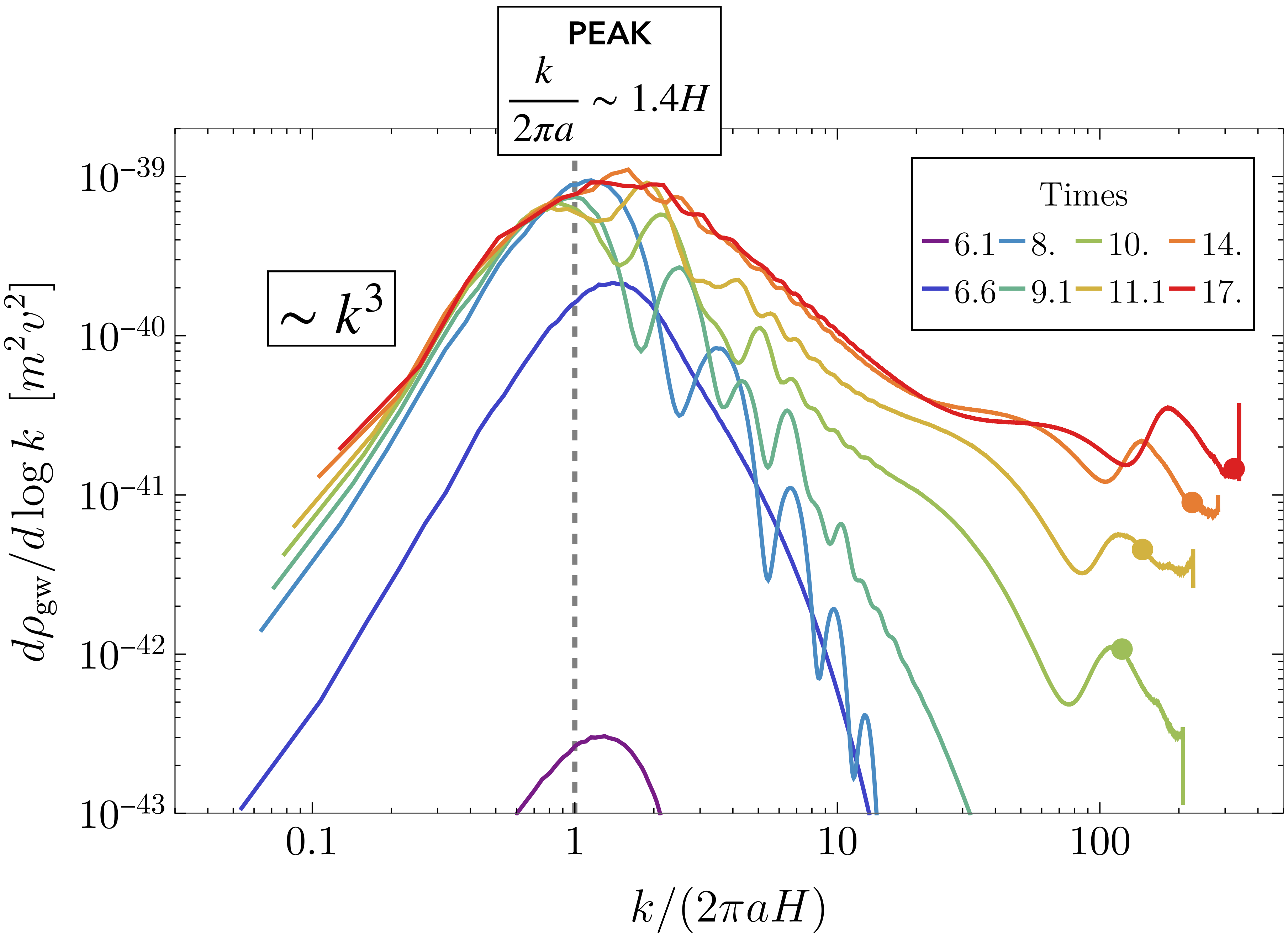

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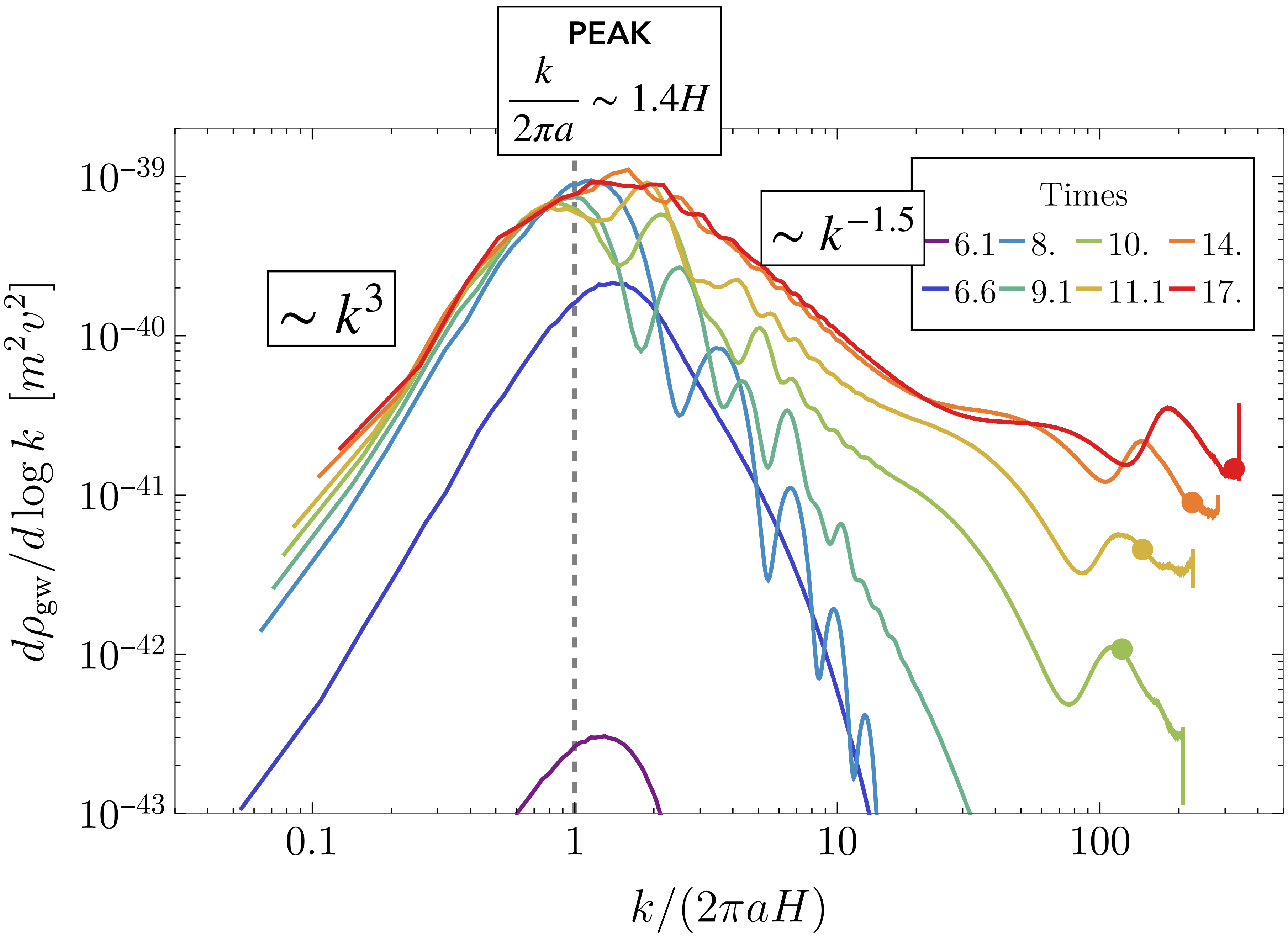

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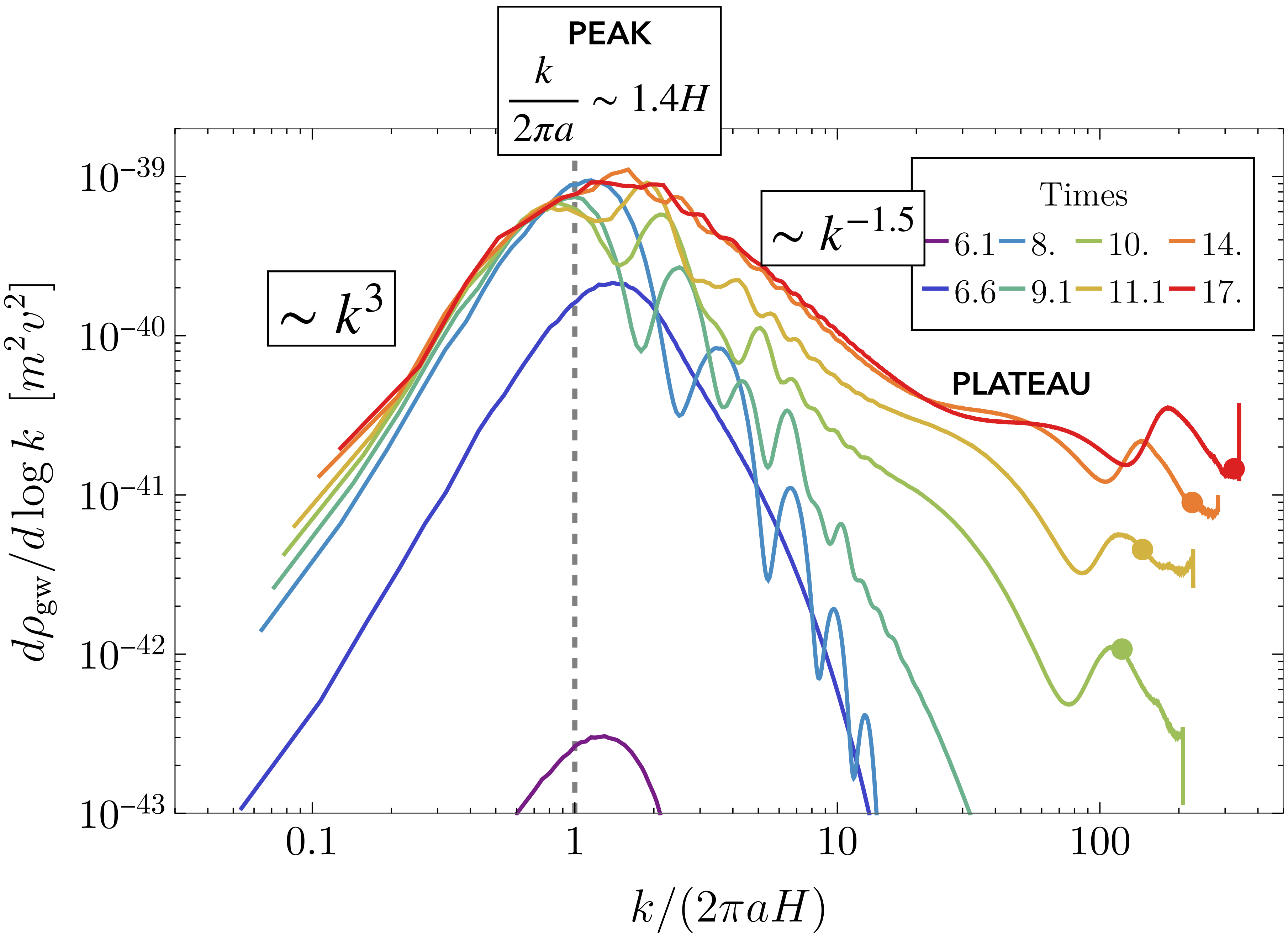

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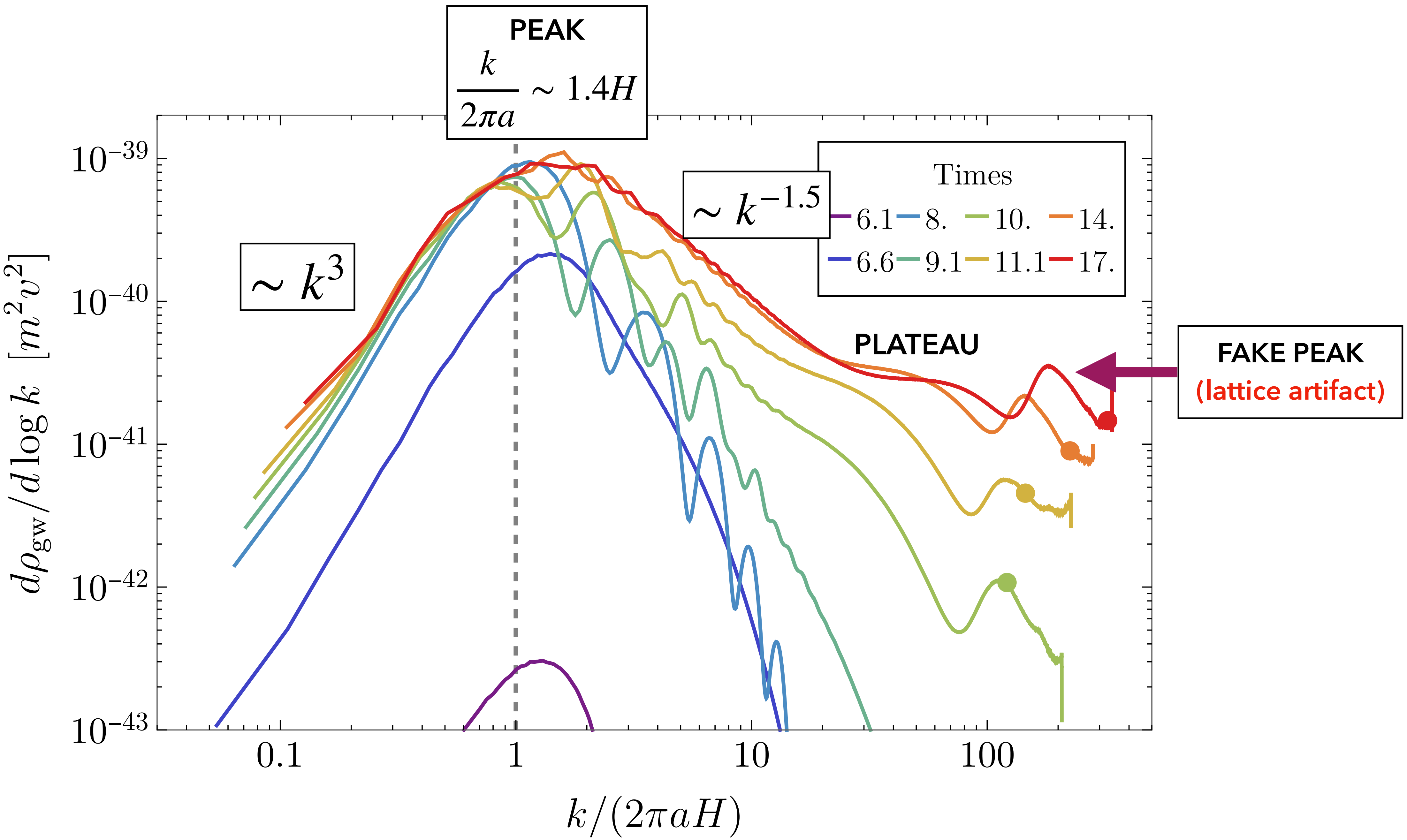

Annihilating domain walls

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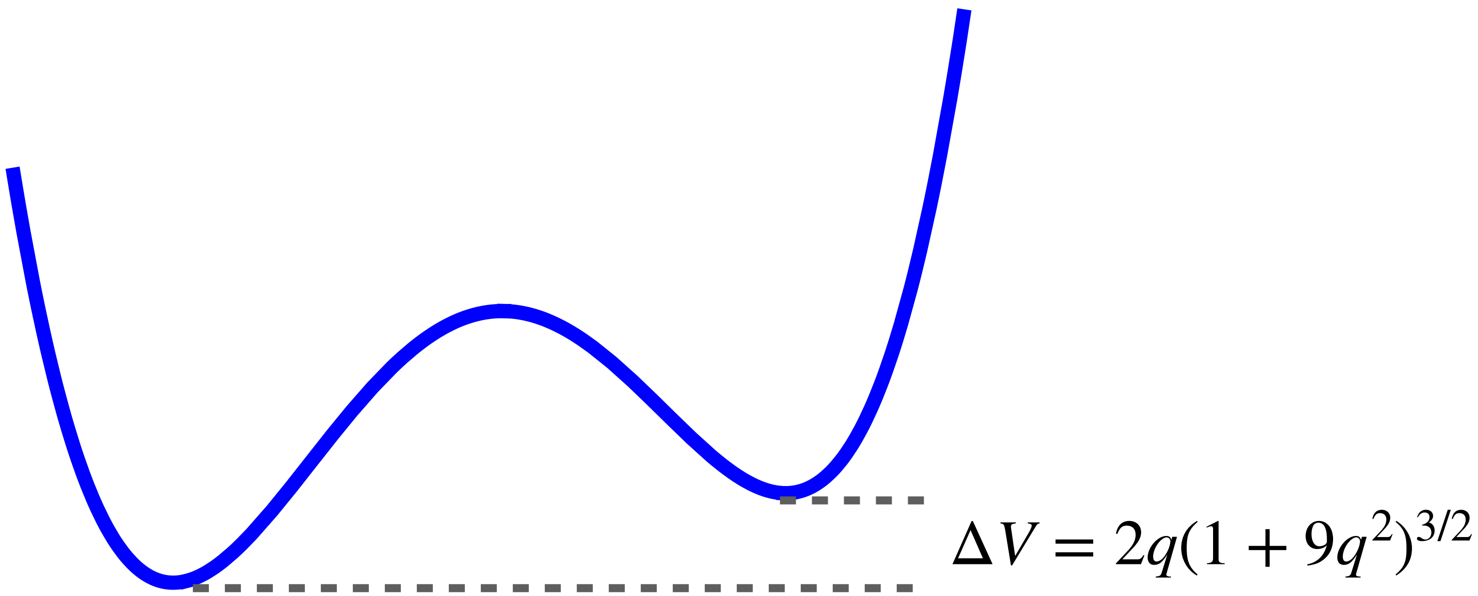

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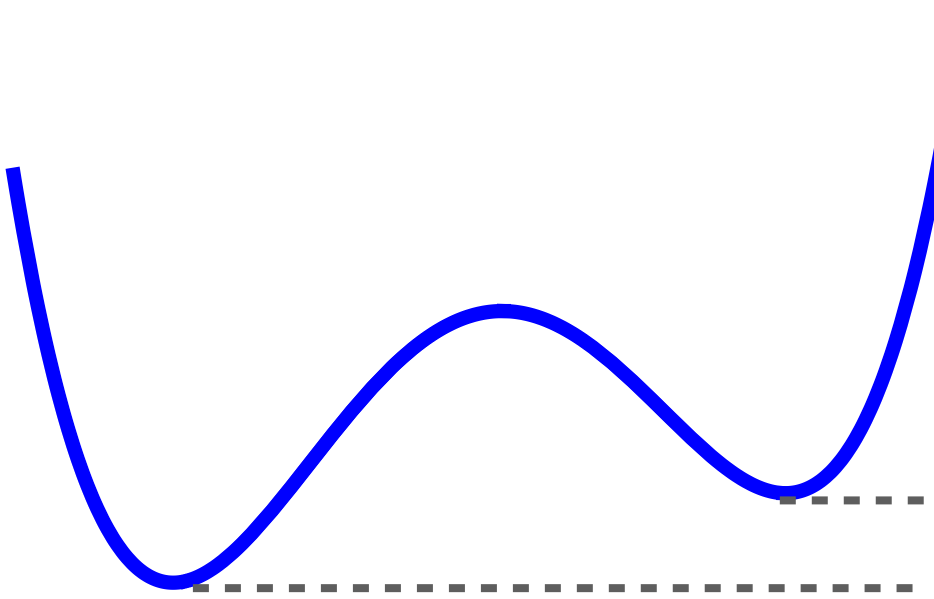

$\Delta V = 2q(1+9q^2)^{3/2}$

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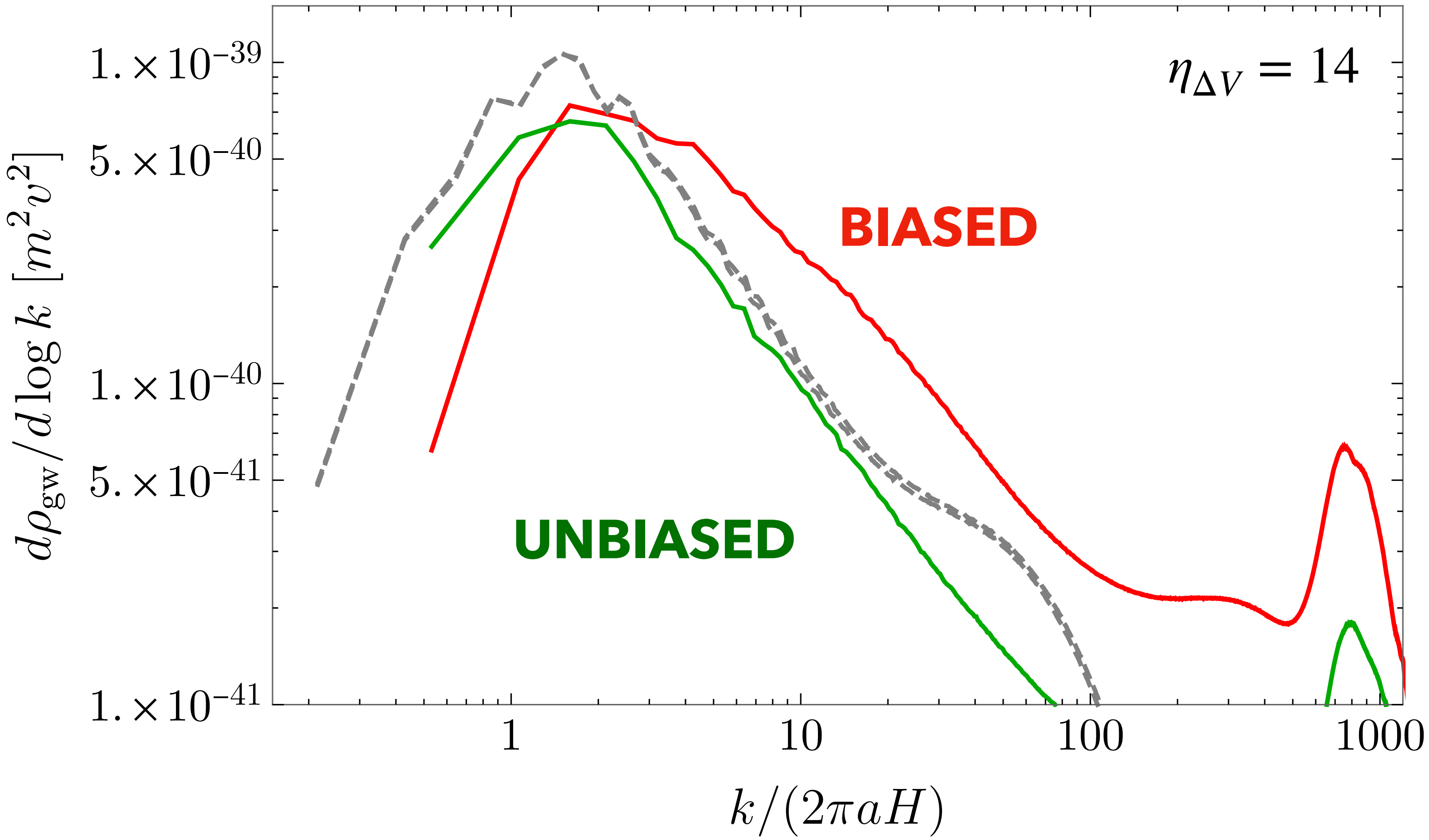

THANK YOU!