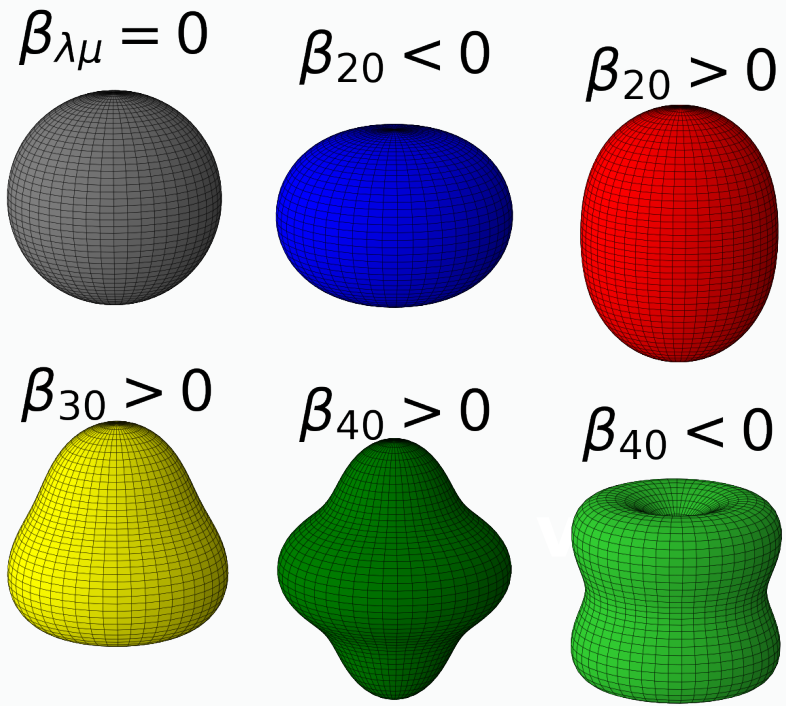
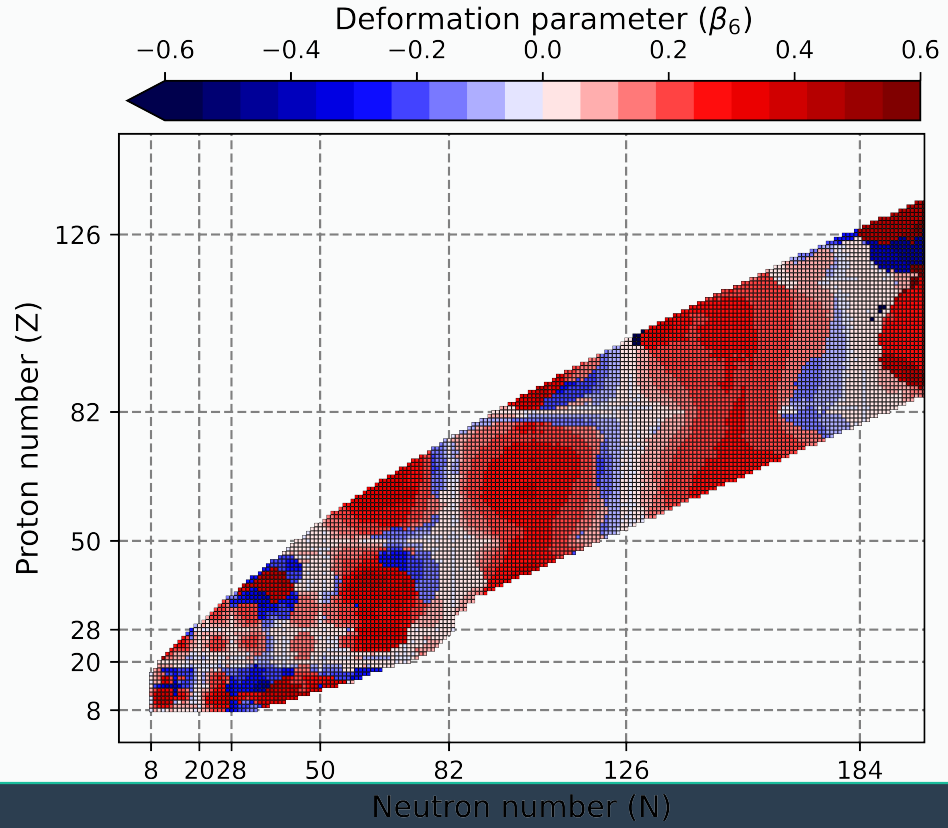




Unveiling the shapes of the atomic nucleus

Authors: Dorian Frycz,
Javier Menéndez and Arnau Rios

Winter meeting 2025

The atomic nucleus

Atomic nucleus: self-bound system of neutrons (N) and protons (Z):

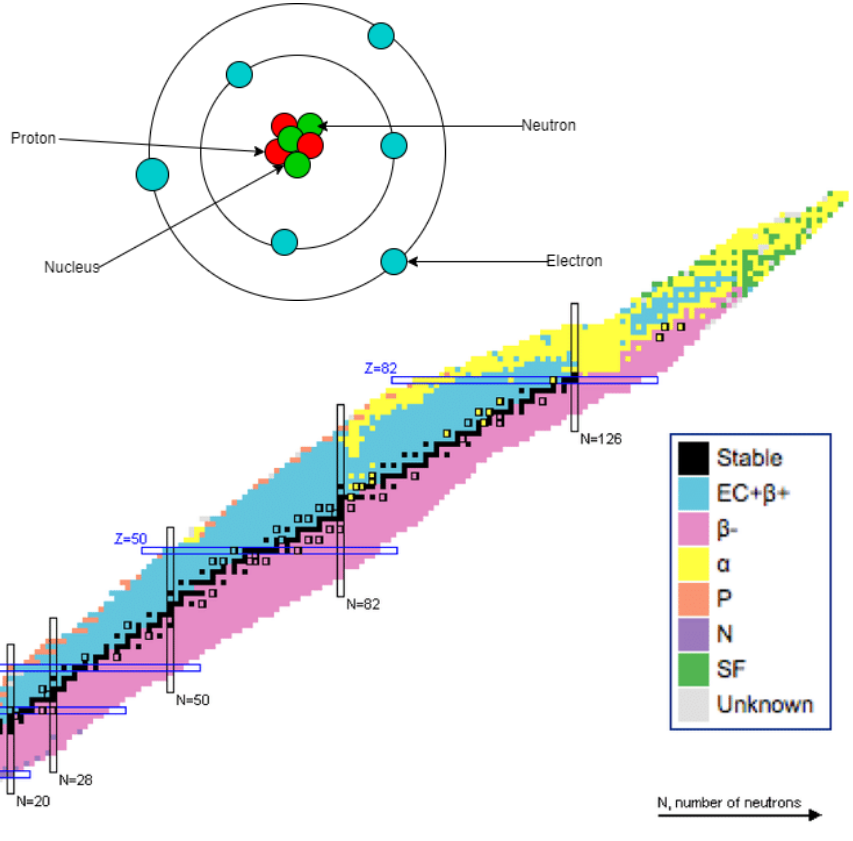
- Bound by **strong force** (short range)
- **Coulomb repulsion** (long range)

→ Complex internal structure!

Importance of **nuclear physics:**

- Beyond Standard model physics
- Nucleosynthesis
- Connection to heavy ion collisions

→ Precise nuclear wavefunctions



What is nuclear deformation?

Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

Collective behavior of the nucleus

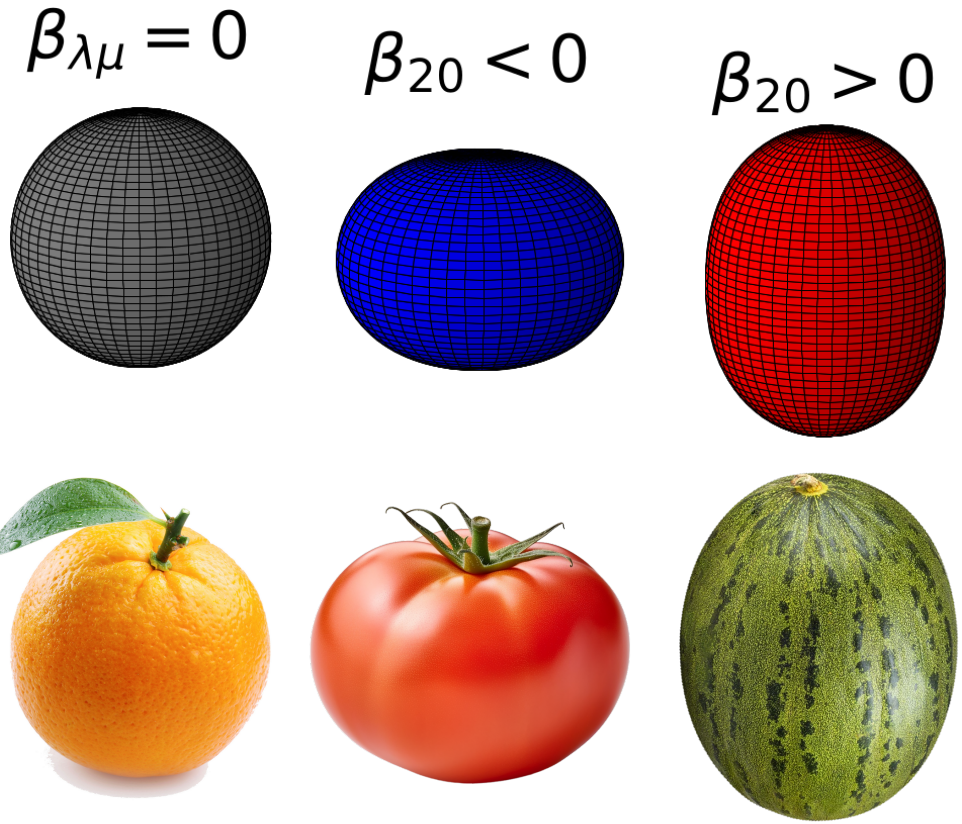
Liquid drop model resemblance

(Axial) quadrupole deformation:

$R(\theta,\varphi) = R_0\{1 + \beta_2 Y_{20}(\theta,\varphi)\}$

- β=0: **Spherical**
- β>0: **Prolate** (elongated spheroid)
- β<0: **Oblate** (flattened spheroid)

  → Larger β implies more deformation
- → Typical values of $0.2<\beta<0.3$



What types of deformation?

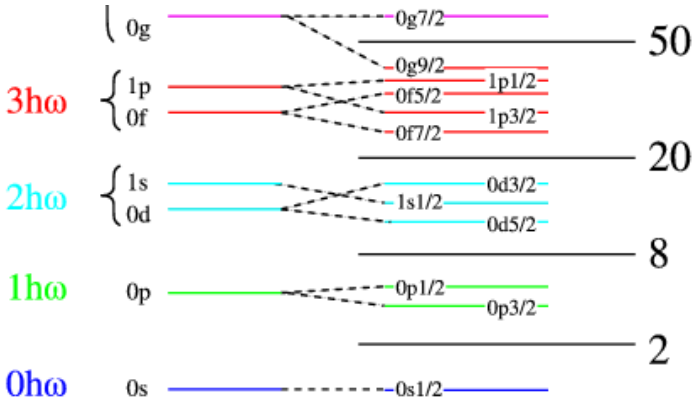
Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

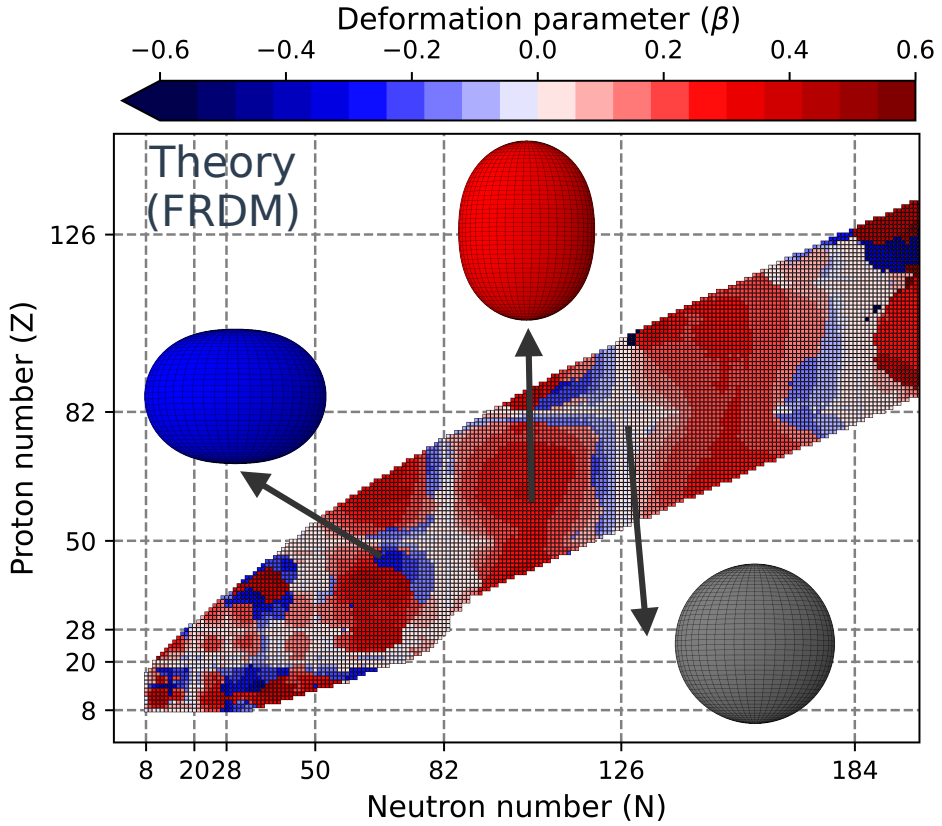
Most nuclei are deformed:

- **Prolate** over **oblate** dominance
- **Spherical**: **magic numbers**

→ Z(N)=2,8,20,28,50,82…

→ Especially bound nuclei





Möller, P., et al., Atomic Data and Nuclear Data Tables, 109, 1-204 (2016)

DEFORMATION RELEVANCE FOR (OTHER) PHYSICAL FIELDS

Neutrinoless double beta decay

Beyond standard model process:

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+2}Y + 2e^- + \cancel{2\tilde{\nu}_e} $$

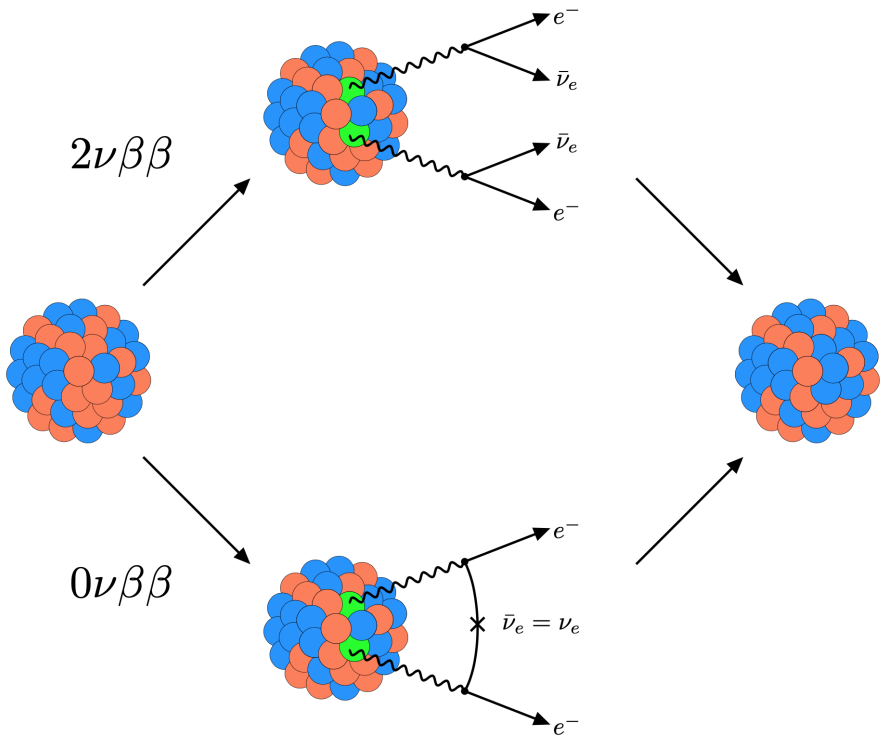
- Are neutrinos its own antiparticle?
- Matter-antimatter asymmetry
- Connection to neutrino mass:

$$ (T^{0\nu\beta\beta}_{1/2})^{-1} \propto M^2_{0\nu}\langle m_\nu \rangle^2 $$

- Nuclear matrix elements ($M_{0\nu}$):

$$ M_{0\nu} = \langle f|\hat{O}_{0\nu\beta\beta}|i\rangle $$

→ Depends on initial (i) and final (f) nuclear wavefunction **overlap**



Neutrinoless double beta decay

Connection to **deformation**:

- Different shapes → different nuclear wavefunctions

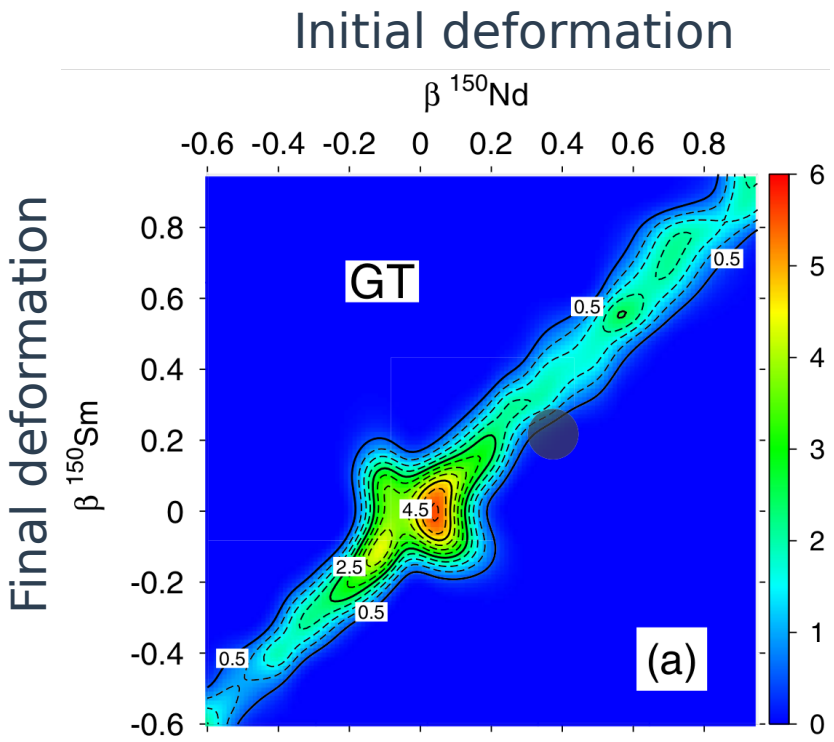
If $M_{0\nu}$ is proportional to the overlap:

- **Similar** deformation **enhance** $M_{0\nu}$
- **Different** deformation **supress** $M_{0\nu}$
- Largest value for both **spherical**

→ Crucial to find easiest nuclei

- Same operators as in β-decay

→ Similar correlation is expected

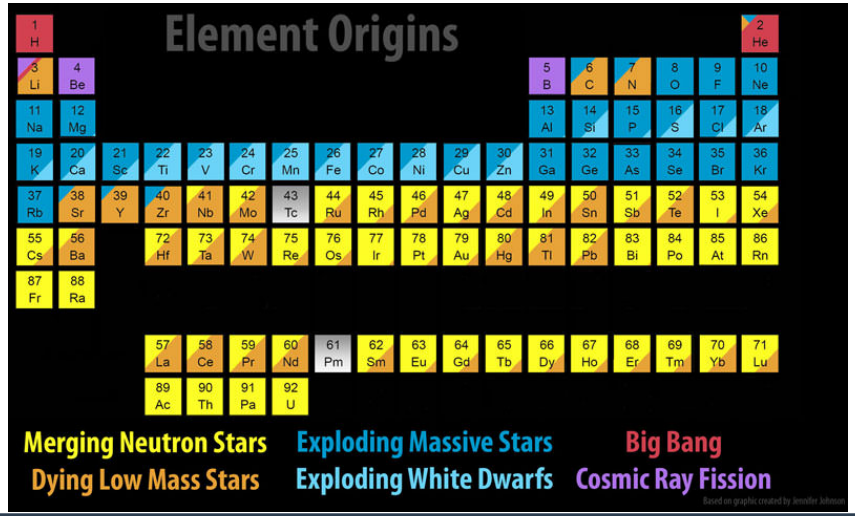


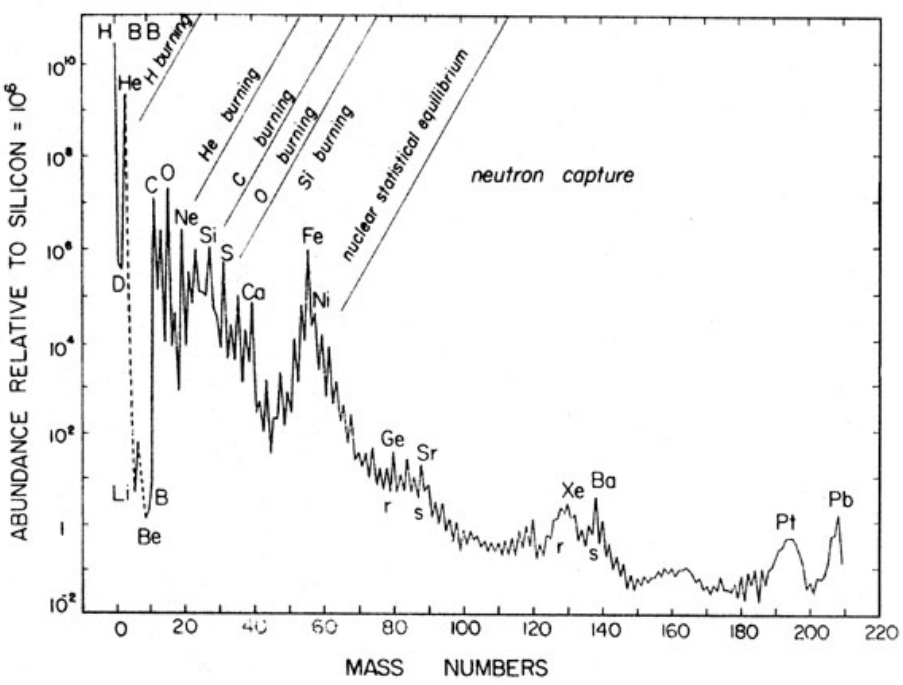
T. R. Rodríguez and G. Martínez Pinedo,
Phys. Rev. Lett. 105, 252503 (2010)

Nucleosynthesis

Relative abundance of nuclei in Universe

- Light nuclei (Z<4) form at the beginning
- Stars fuse nuclei until iron (Z<26)
- How do heavier nuclei form?

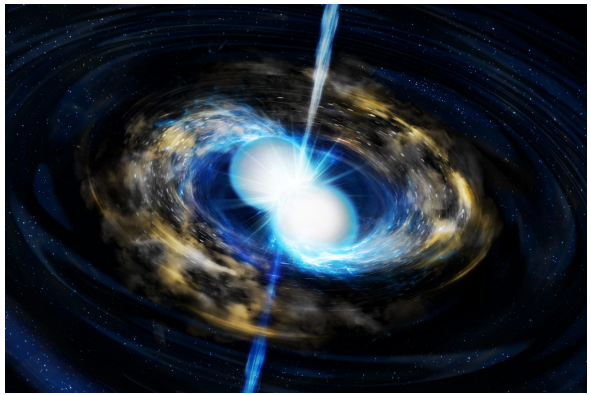


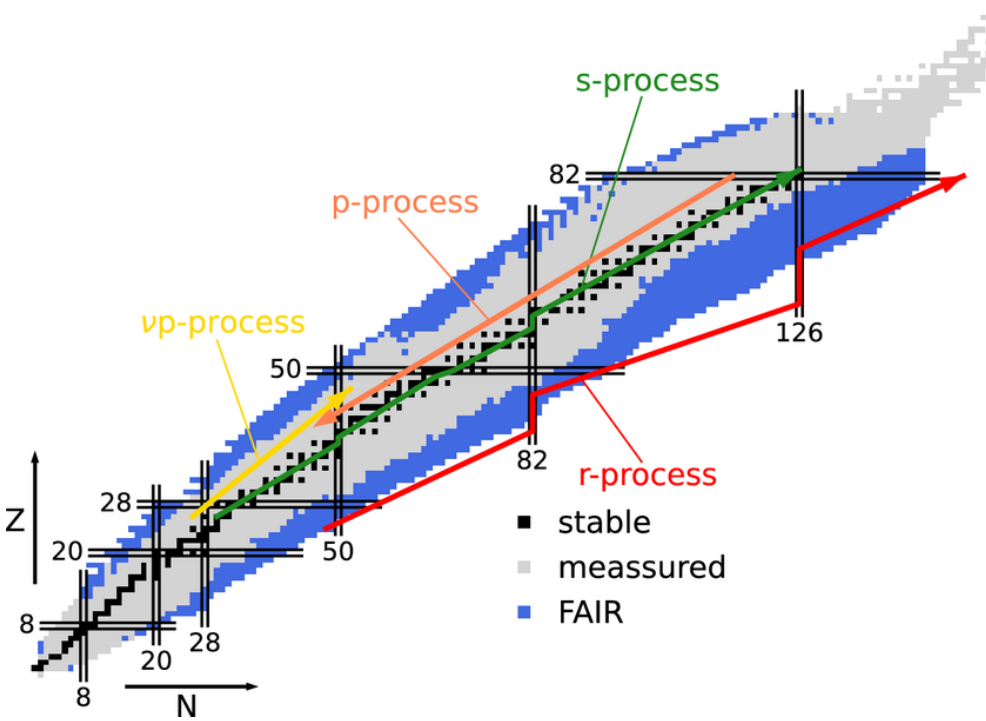


Nucleosynthesis

r(apid neutron capture)-process

- Neutron star mergers
- Faster neutron captures than β-decay
- Very heavy and **neutron rich** nuclei
- Exact path depends on masses





Arcones, A., Thielemann, FK. Astron Astrophys Rev 31, 1

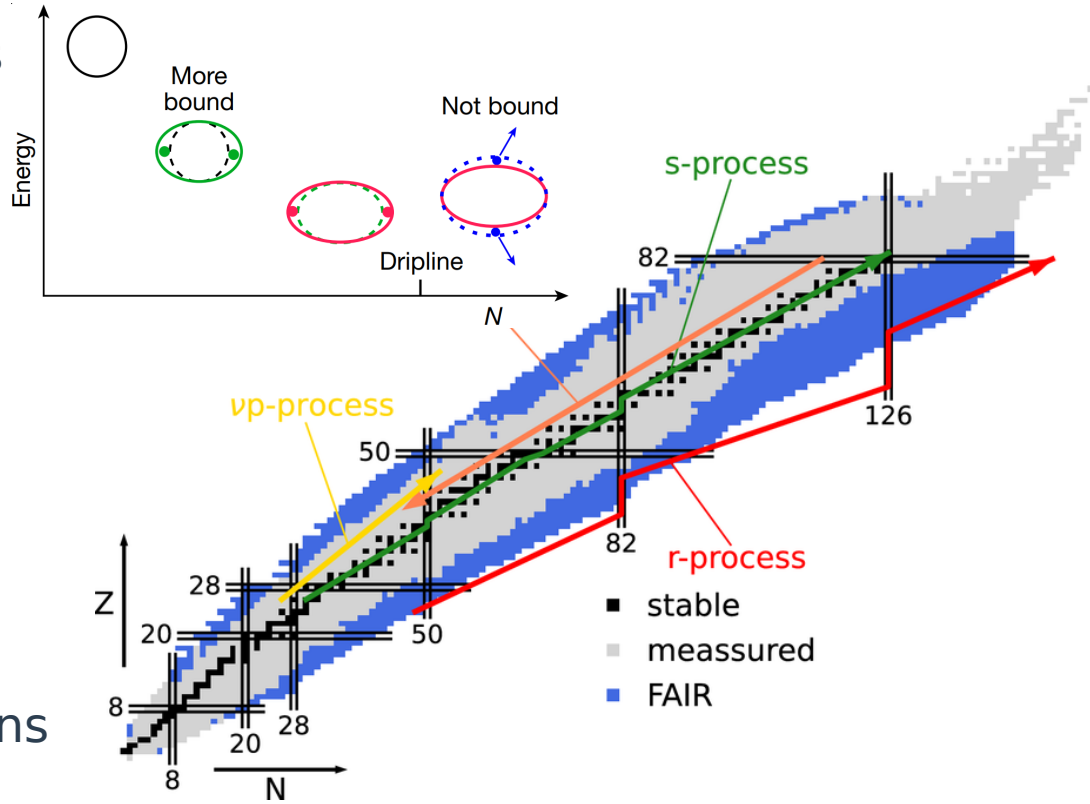
Neutron drip line

- How many neutrons can a nucleus capture before neutron emission?

  → Neutron drip line
- **Deformed** shapes allow for **more neutrons** than spherical
- Magic numbers delay r-processes:

  → Larger abundances
- Some magic numbers **shift** in neutron-rich nuclei

  → Knowledge of nuclear interactions



Arcones, A., Thielemann, FK. Astron Astrophys Rev 31, 1

HOW DO WE MEASURE DEFORMATION?

Nuclear spectroscopy

Rotation of deformed shapes:

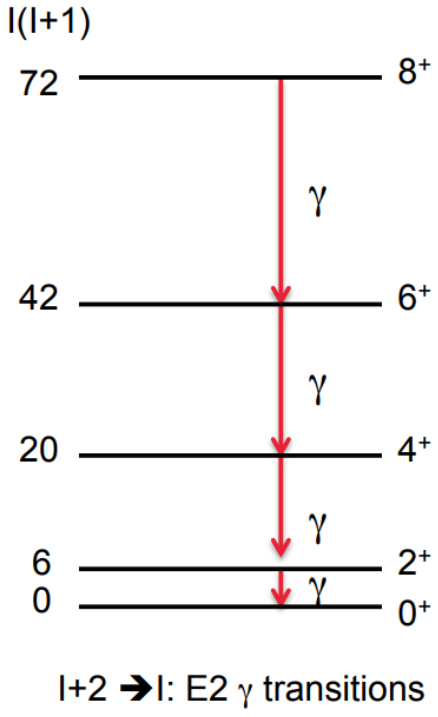
- (Quantum) **Rigid rotor**:

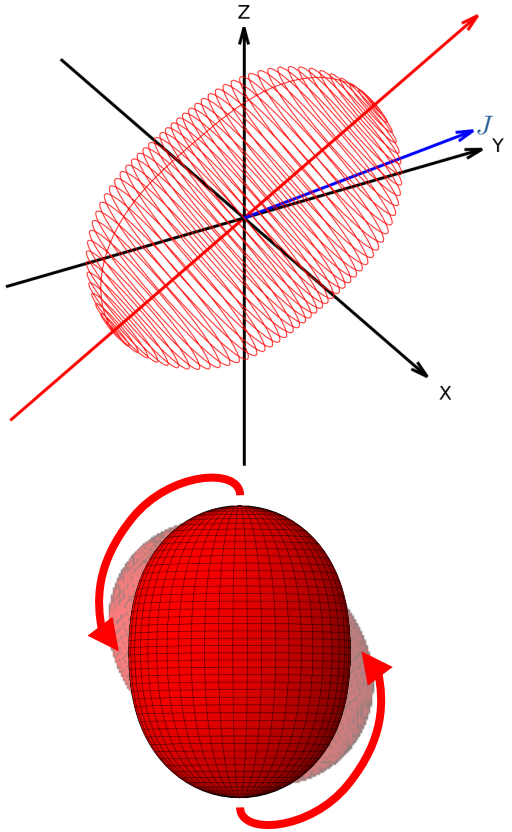
$$E_{\mathrm{rot}} = \frac{\vec{L}^2}{2\mathcal{I}} \rightarrow \frac{J(J+1)}{2\mathcal{I}}$$

- J: total angular momentum
- J sequence: 0^+, 2^+, 4^+, 6^+...
- In-band **electromagnetic** decays

Measurements:

- Probability of decay to a given state $\mathrm{B(E2)} \rightarrow \beta$
- Quadrupole moment: $\mathrm{Q}=\mathrm{r}^2\mathrm{Y}_{20} \rightarrow \beta$





Heavy ion collisions

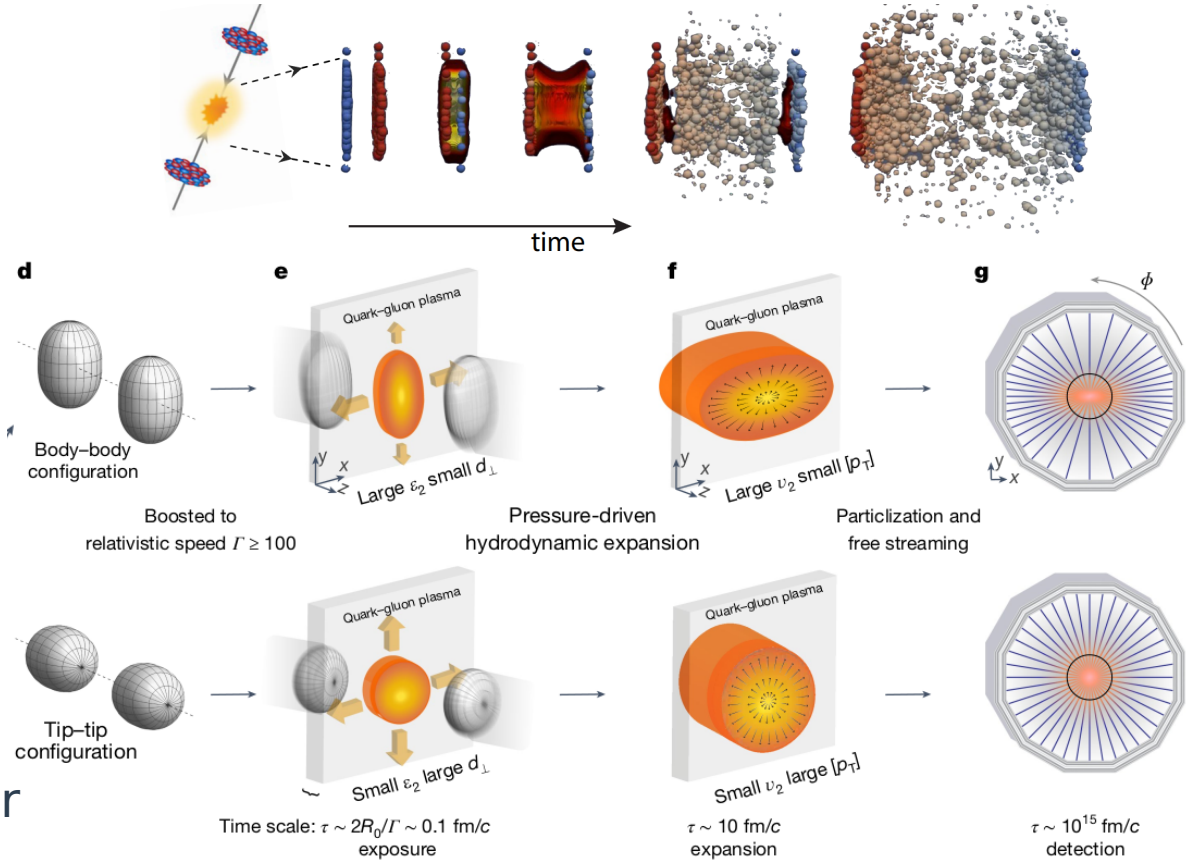
New method of **imaging the nucleus shape**:

- LHC heavy ion data
- **Tip-tip**: collisions along symmetry axis
- **Body-body**: orthogonal axes

  → Shape is related to **overlap**

  Quark-gluon plasma evolution

  → hydrodynamics
- Measurement of particle shower



STAR Collaboration, Nature 635, 67-72 (2024).

THEORETICAL DESCRIPTION OF DEFORMATION

The many-body problem

Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

Schrödinger equation: $E\Psi=H\Psi$

Nuclear shell model:

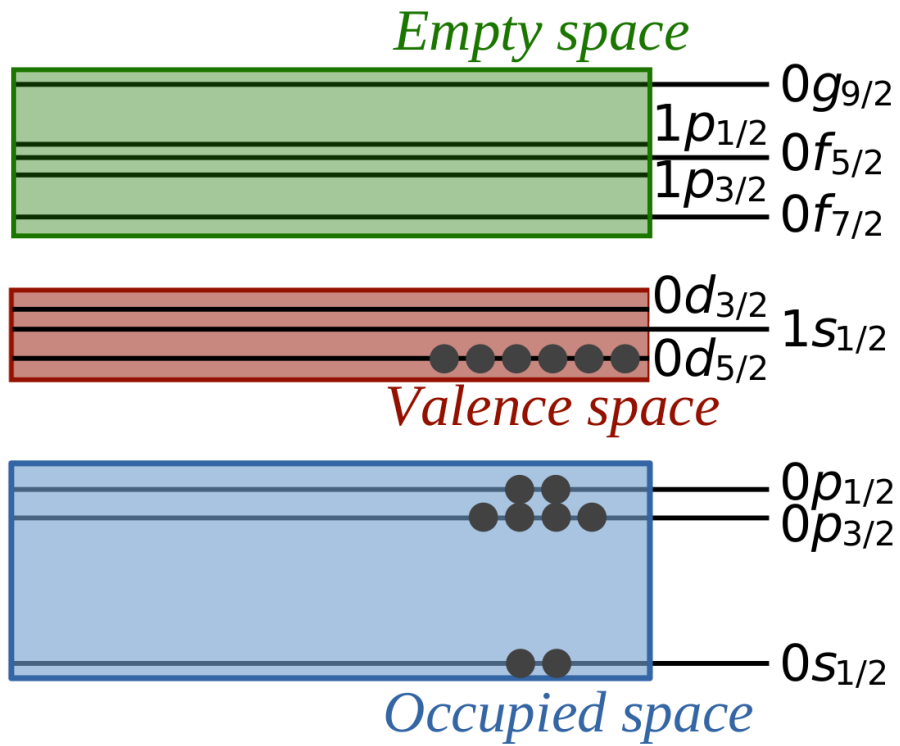
- Harmonic oscillator basis

  Nuclear interactions:

- Phenomenological nucleon scattering
- *ab-initio* interactions: χEFT

  Valence space: active particles

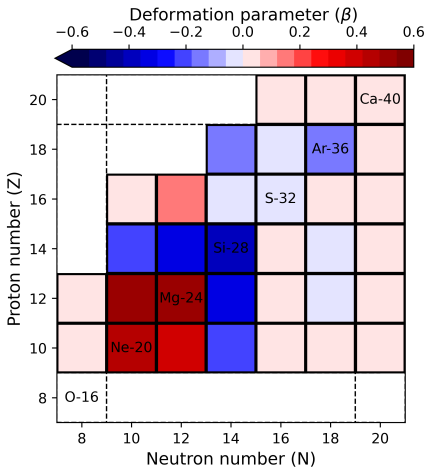
  → Diagonalization of $10^9 \times 10^9$ or larger matrices in valence space

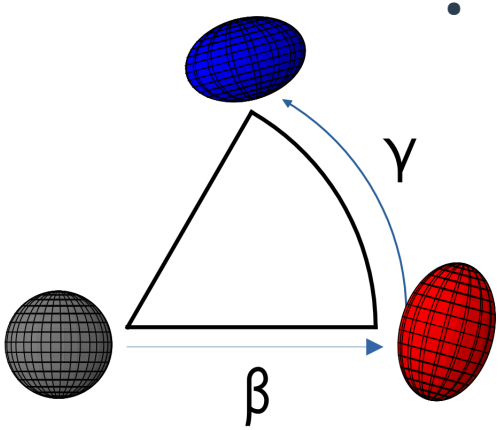


Deformation in the sd shell

sd shell: from $N(Z)=8$ to $N(Z)=20$

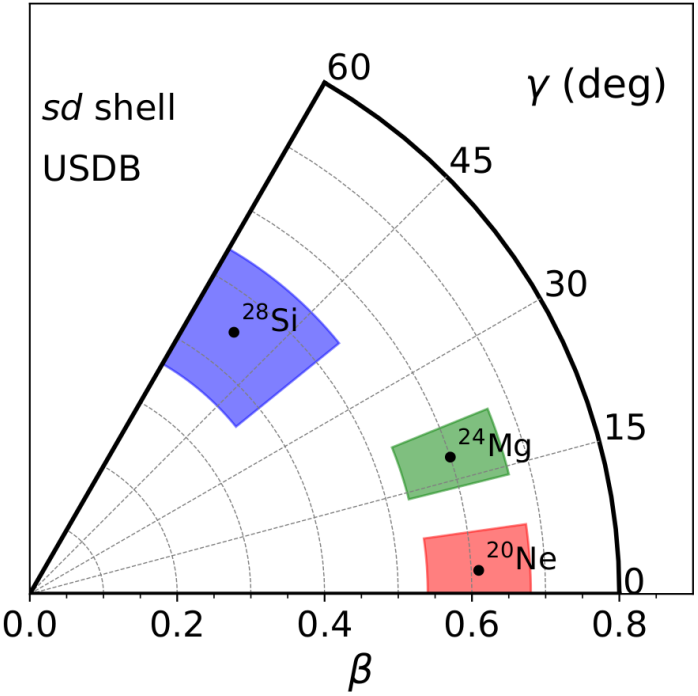
- Quick evolution of shapes
- Axial: prolate $(0°)$ / oblate $(60°)$
- Triaxial: $0°<\gamma<60°$





Even-even N=Z nuclei:

- ^{16}O: spherical
- ^{20}Ne: prolate
- ^{24}Mg: triaxial
- ^{28}Si: oblate

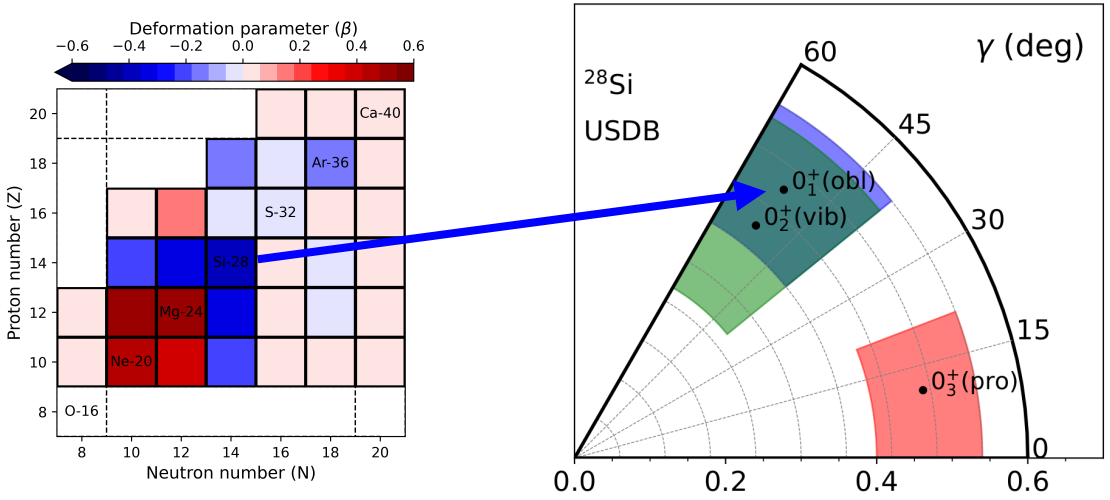


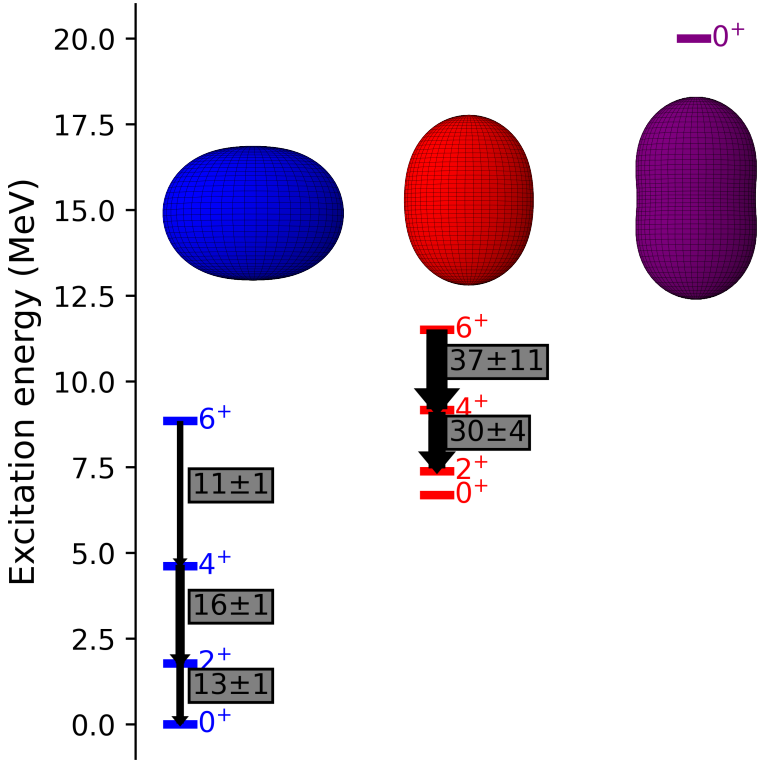
Shape coexistence in ^{28}Si

Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

Three shapes coexisting:

- Oblate rotational band (ground state)
- Prolate rotational band (~7 MeV)
- Superdeformation (E~20 MeV)





D. Frycz, et al. Phys. Rev. C 110, 054326 (2024)

Conclusions

Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

Deformation is everywhere:

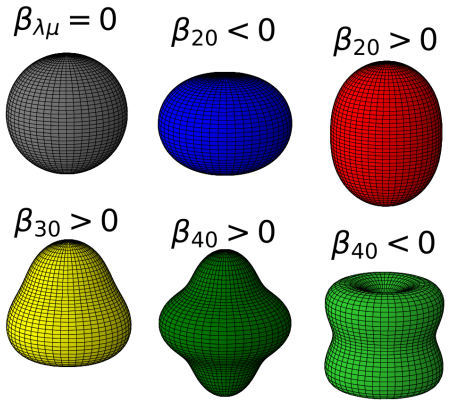
- Quadrupole axial dominance
- Affects β and $0\nu\beta\beta$ decays
- Impact on nucleosynthesis

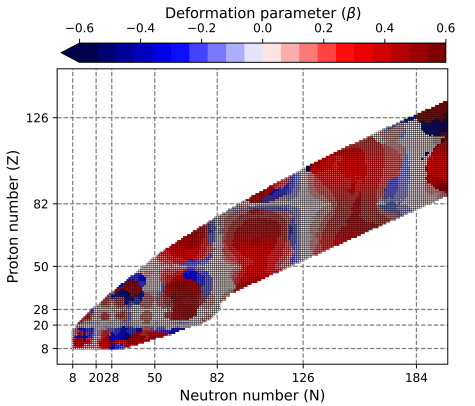
Measurement of deformation

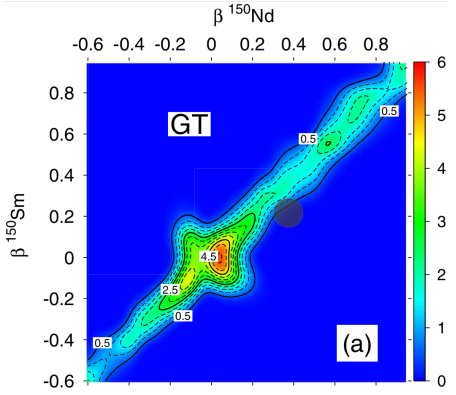
- Spectroscopy
- Heavy ion collisions

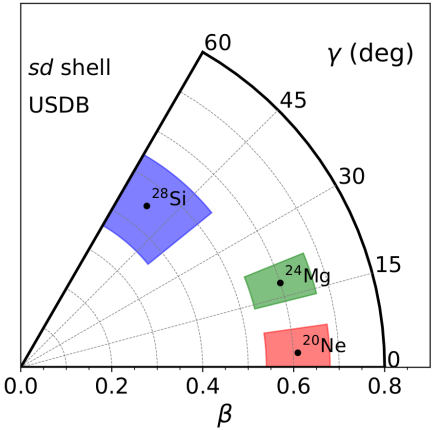
Complexity of deformation:

- Rapid shape evolution
- Shape coexistence









THANK YOU!!!
FEEL FREE TO ASK :)

Shape coexistence in ^{40}Ca

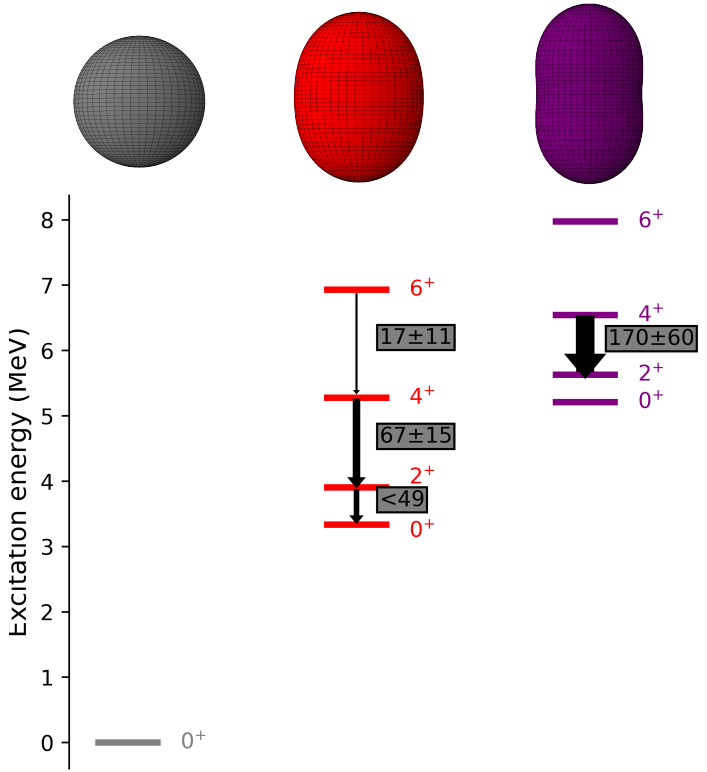
Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

Three shapes coexisting:

- Spherical ground state
- Prolate normal deformed
- Superdeformed band

Subtleties:

- Spherical: $\beta \neq 0$, $0° < \gamma < 60°$
- ND: large fluctuations
- SD: most pure one



Characterization of the deformation

Nucleus surface:

$$Y(\theta,\varphi) = R_0 \left[1 + \sum_{\lambda\mu} \beta_\lambda Y_{\lambda\mu}(\theta,\varphi) \right]$$

For quadrupole shapes:

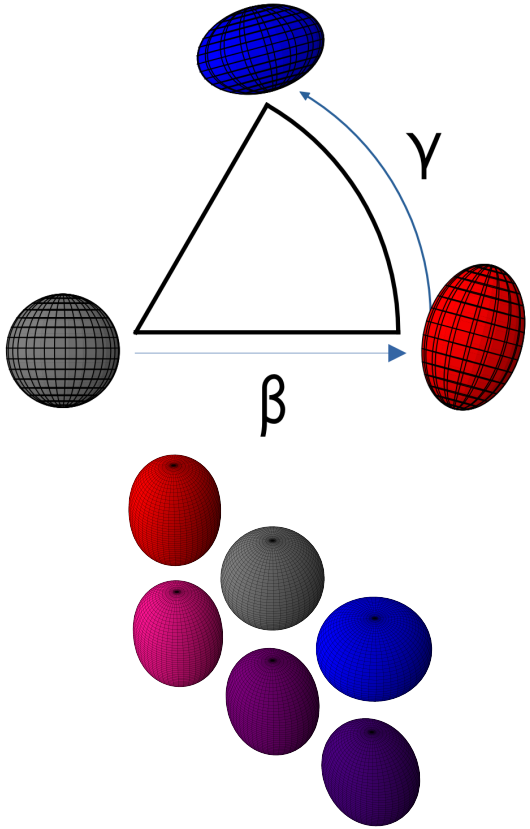
$$Y(\theta,\varphi) = R_0\{1 + \beta_2[Y_{20}(\theta,\varphi)\cos\gamma + Y_{22}(\theta,\varphi)\sin\gamma]\}$$

β : magnitude of deformation

γ : type of deformation

- 0°: oblate / 60°: prolate

Triaxial: 0°<γ<60°

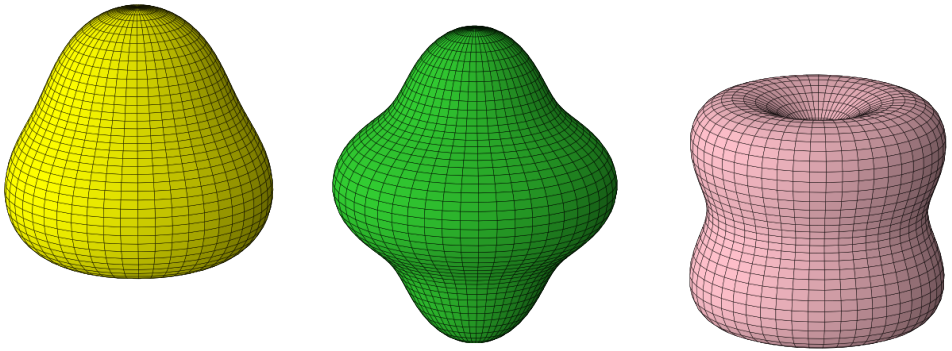


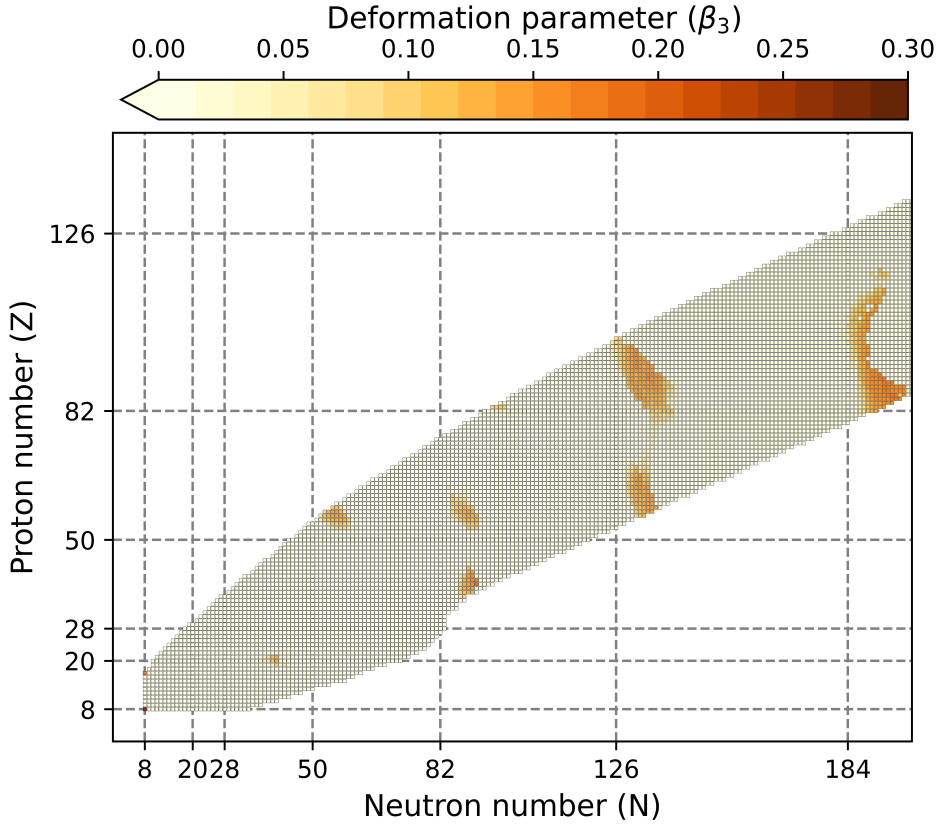
What types of deformation?

Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

Most nuclei are deformed:

- **Octupole** some regions
- Low 3^- states
- Strong B(E3) transitions

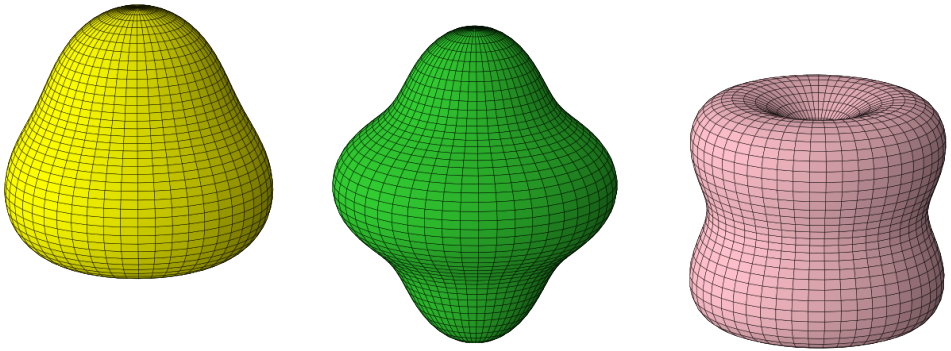


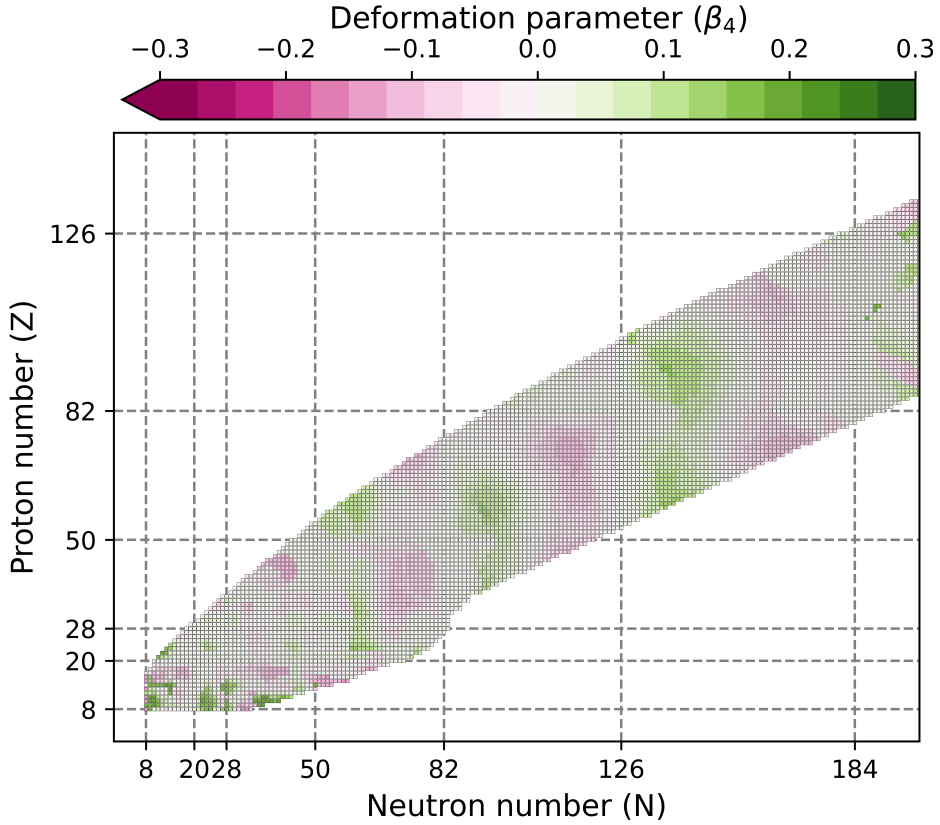


What types of deformation?

Most nuclei are deformed:

- **Hexadecapole**
- Somewhat abundant
- Low deformation

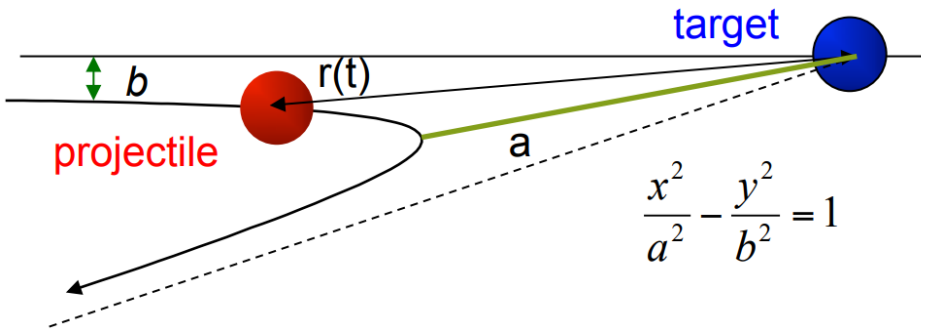




Traditional measuring methods

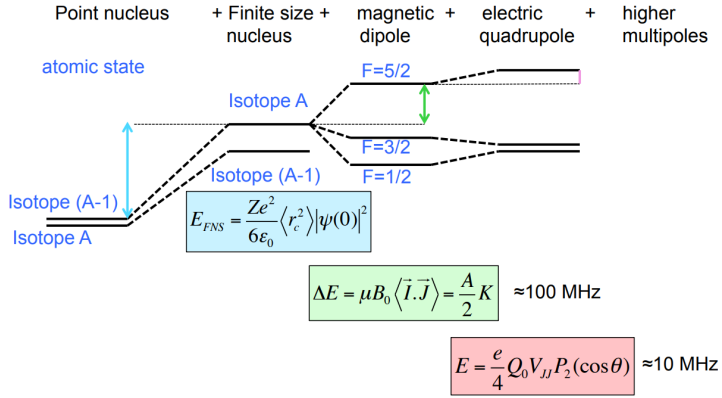
Coulomb excitation

- Scattering with high Z nuclei
- Only Coulomb field involved
- Probability of exciting a state
- **Measures** $B(E2, J_i \rightarrow J_f) \rightarrow \beta$



Laser spectroscopy

- Laser beam probes atomic levels
- Hyperfine splitting and isotope shifts
- Better for exotic nuclei
- **Measures** $Q_{spec} \rightarrow \beta$



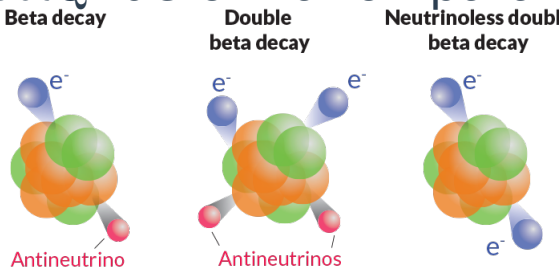
Deformation is everywhere

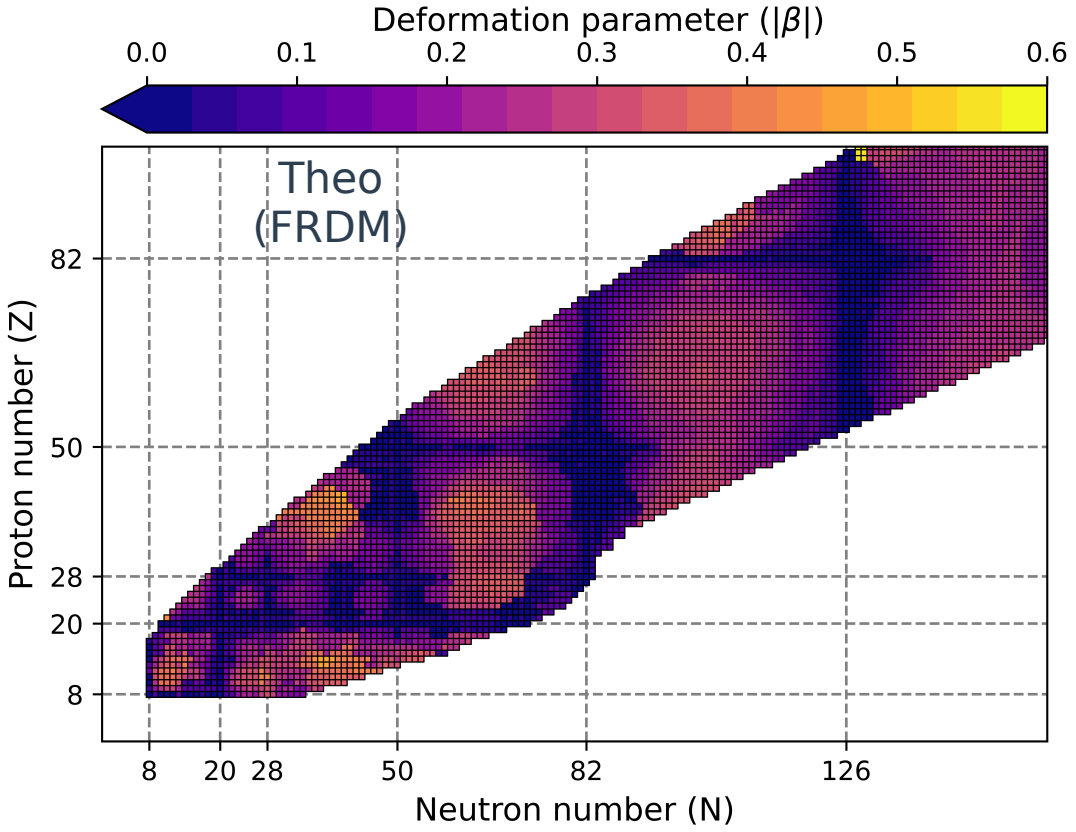
Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

Most nuclei are deformed

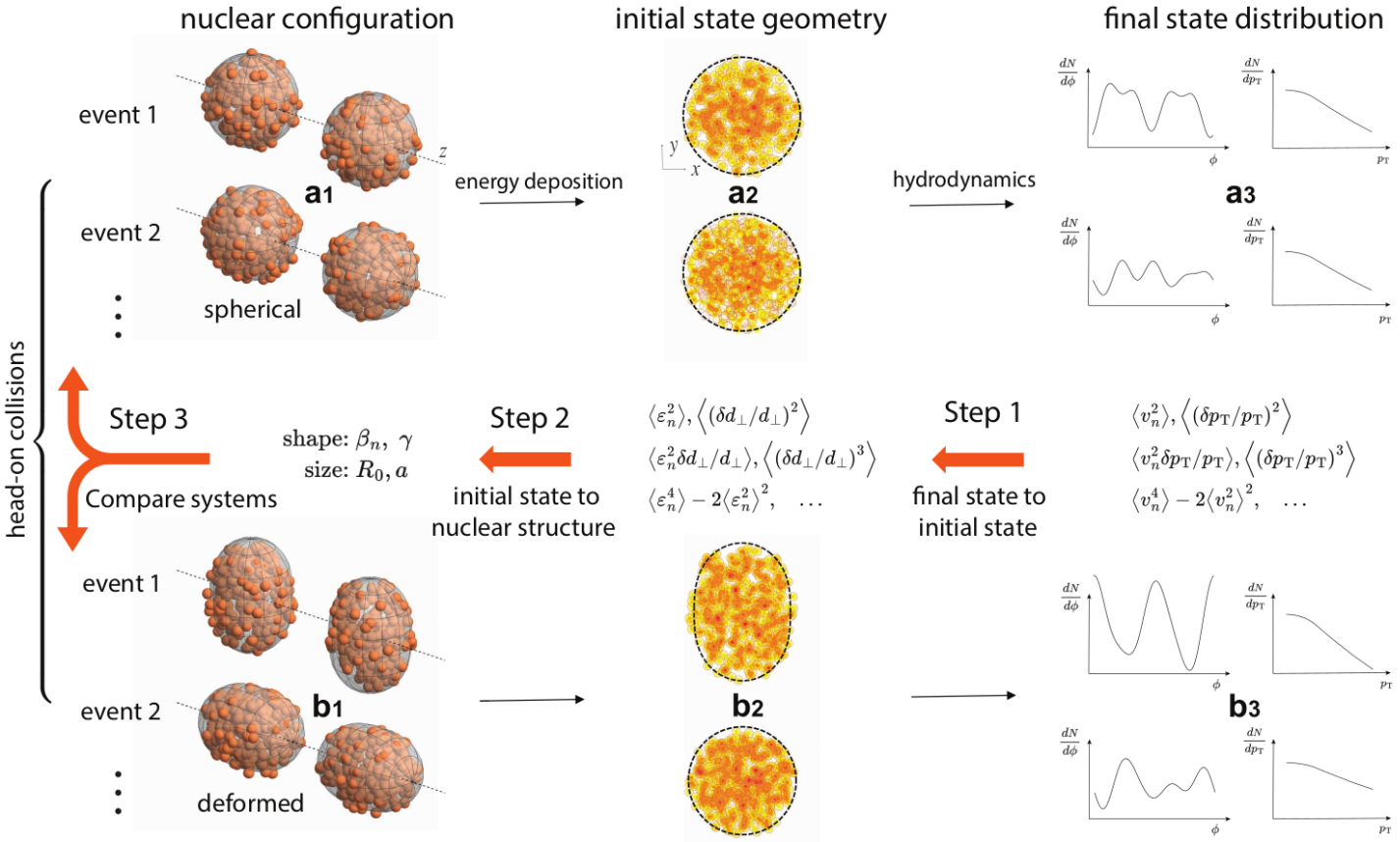
- Deformation → correlations

  Quadrupole moment: $Q = r^2 Y_{20}$
- Deviation from sphericity
- $Q = 0$ spherical
- $|Q| > 0$ deformed
- $\beta \propto Q$: deformation parameter





High energy collisions



Shape coexistence in ^{28}Si

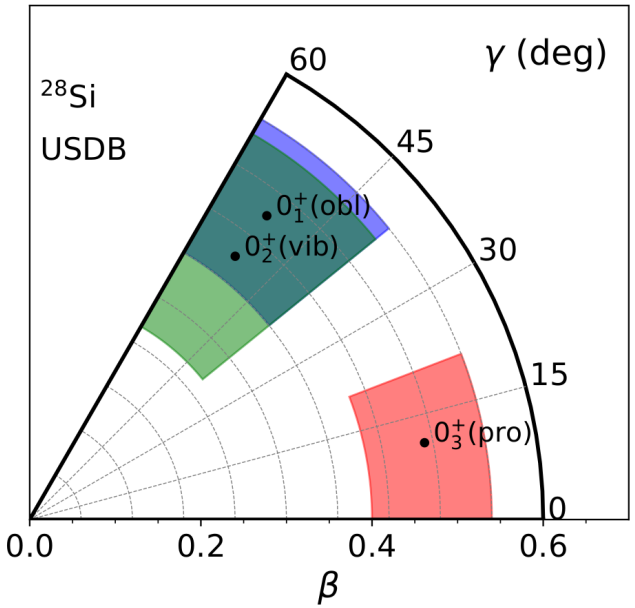
Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

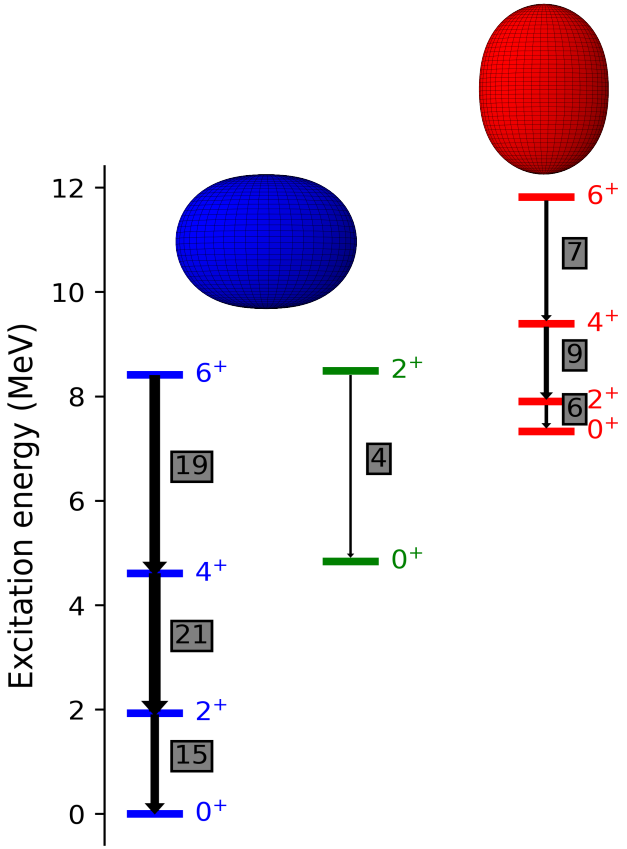
Two shapes coexisting:

- Oblate rotational band
- Oblate vibration
- Prolate rotational band
- Superdeformation?

Challenges:

- Large fluctuations
- Weak B(E2) values





Deformation is everywhere

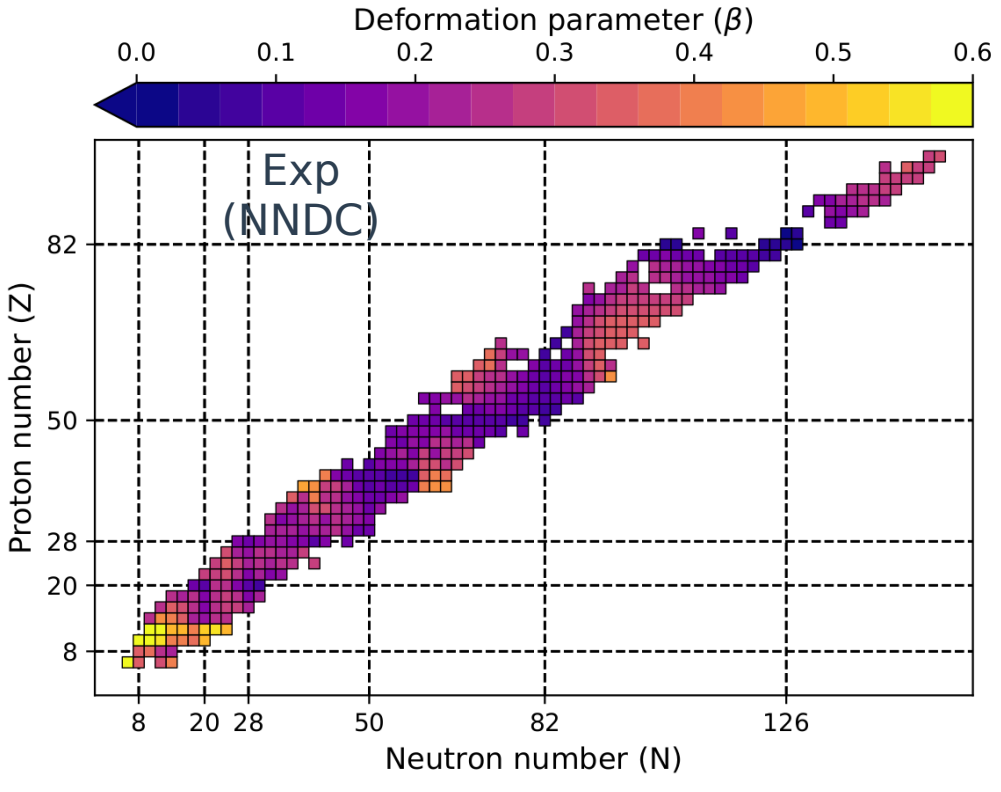
Most nuclei are deformed

Experimental measurements →

- $\beta=0$ spherical
- $|\beta|>0$ deformed
- Hard to extract β from experiments

  Deformation: correlations

  → lower energy
- Magic numbers: 2,8,28,50,82…



New technique: heavy ions

Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

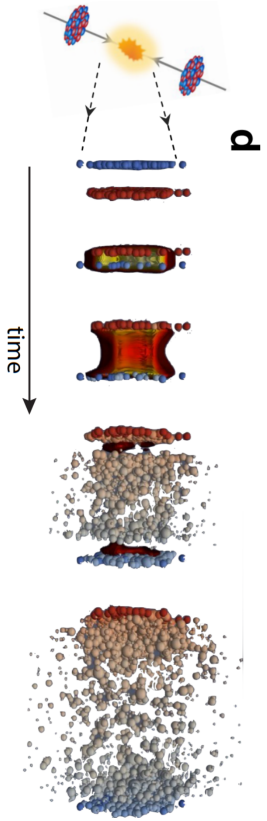
New method of imaging the nucleus shape:

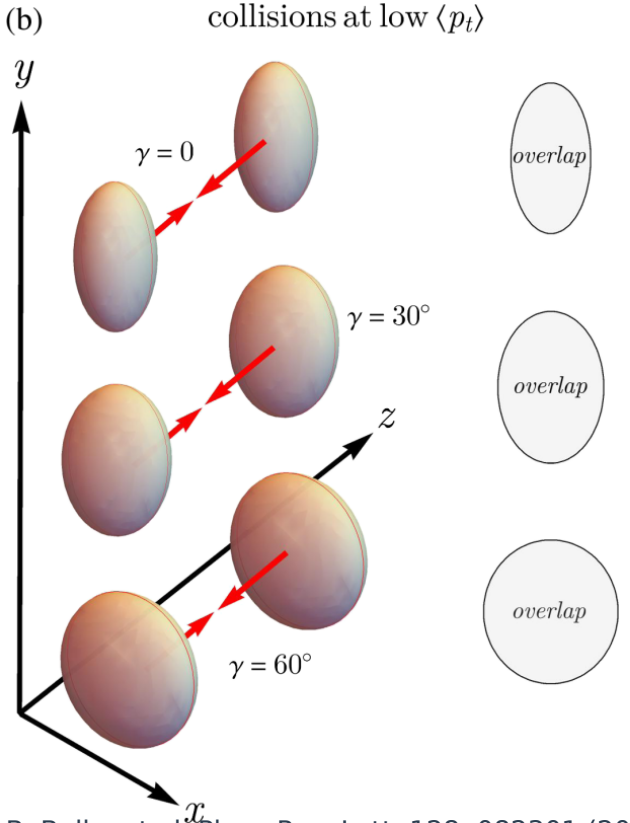
- **Tip-tip** collisions: along symmetry axis
- **Body-body**: along other axes

  → Shape is related to overlap

  Quark-gluon plasma evolution

  → hydrodynamics
- Measurement of particle shower





STAR Collaboration, Nature 635, 67-72 (2024).

B. Bally, et al. Phys. Rev. Lett. 128, 082301 (2022)

What is nuclear deformation?

Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

Collective behavior of the nucleus

- Nucleus surface parametrization:

$$R(\theta,\varphi) = R_0 \left[1 + \sum_{\lambda\mu} \beta_\lambda Y_{\lambda\mu}(\theta,\varphi) \right]$$

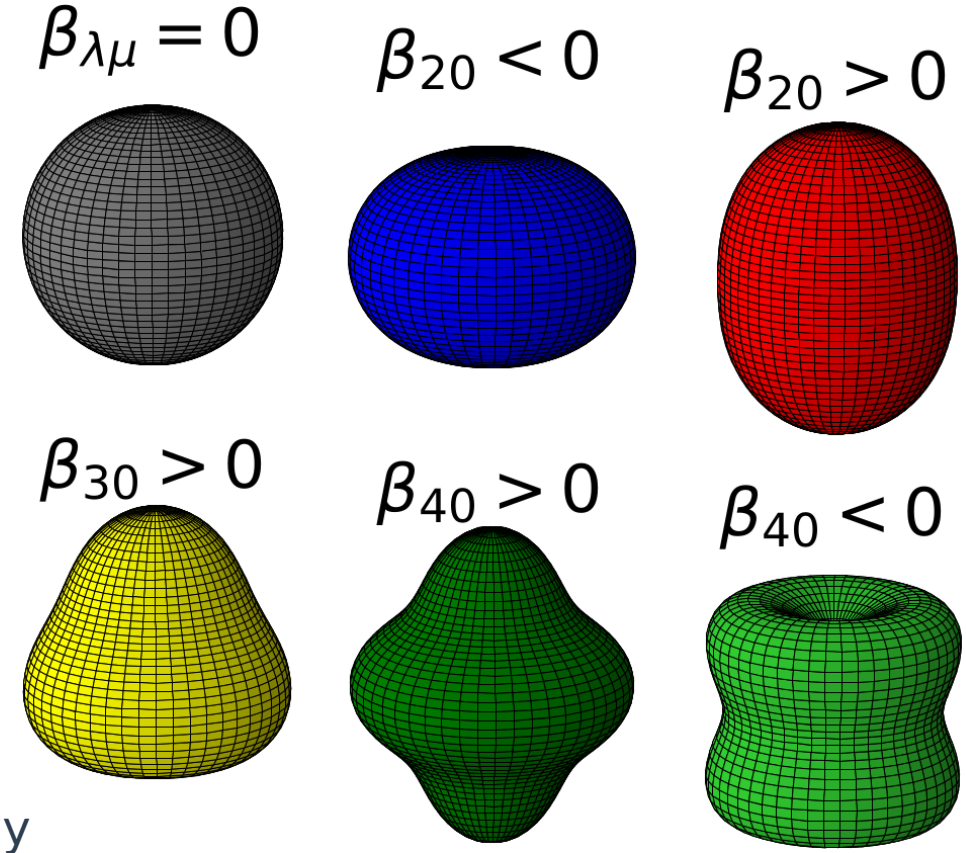
- β=0: **spherical**

  (Axial) quadrupole deformation:

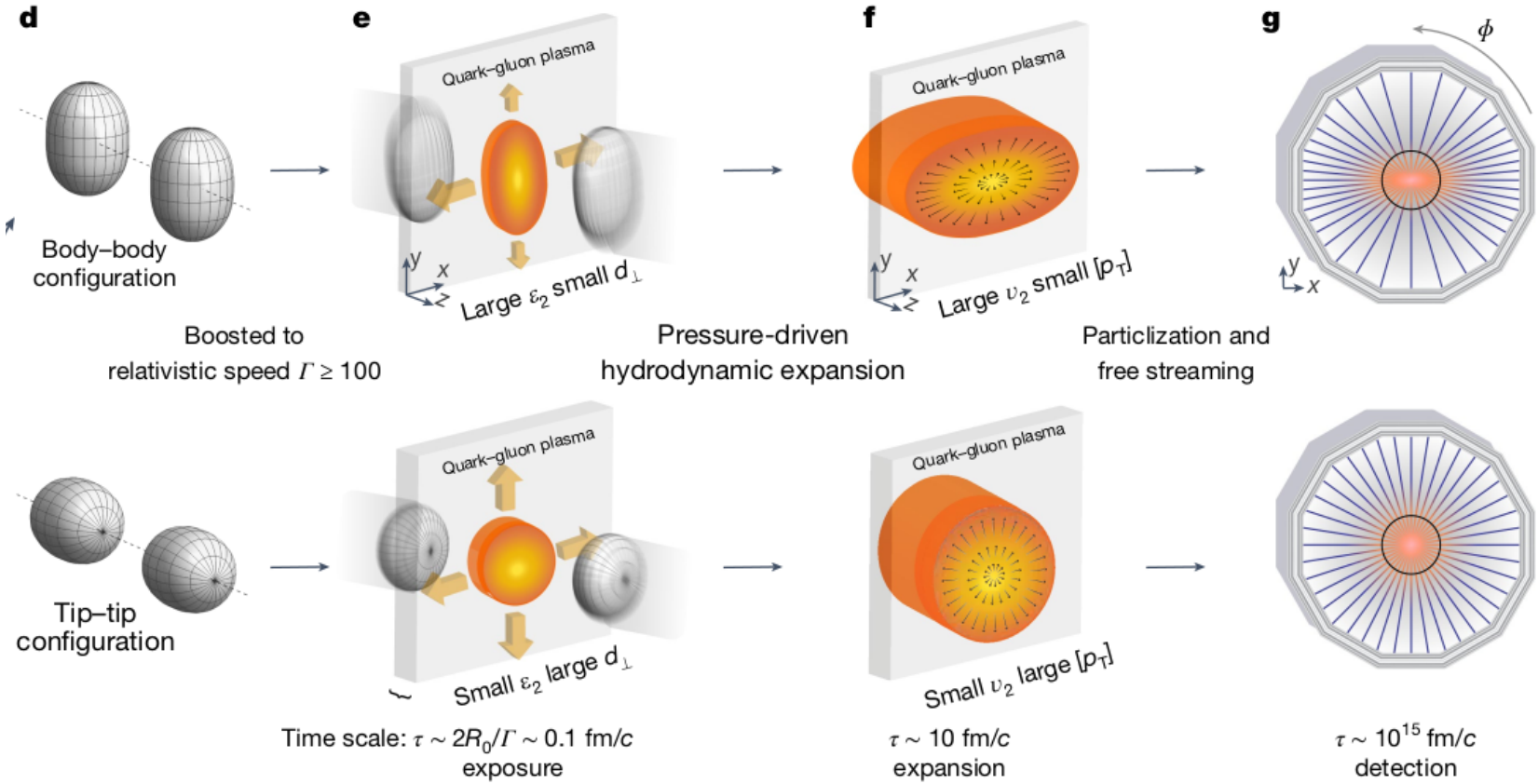
  $R(\theta,\varphi) = R_0\{1 + \beta_2 Y_{20}(\theta,\varphi)\}$

- β>0: **Prolate** (elongated spheroid)
- β<0: **Oblate** (flattened spheroid)

  Higher multipoles (λ>2) are less likely



Heavy ion collisions



Heavy ion collisions

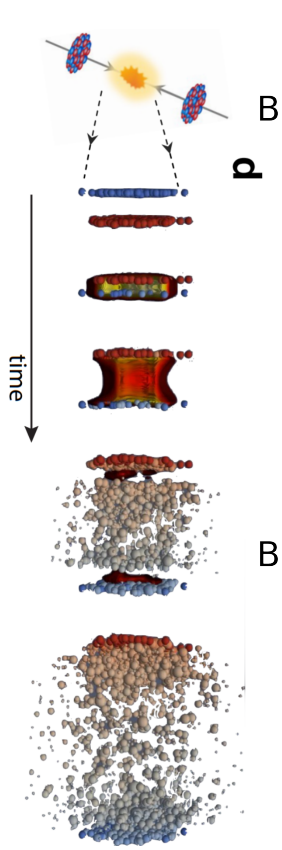
New method of imaging the nucleus shape:

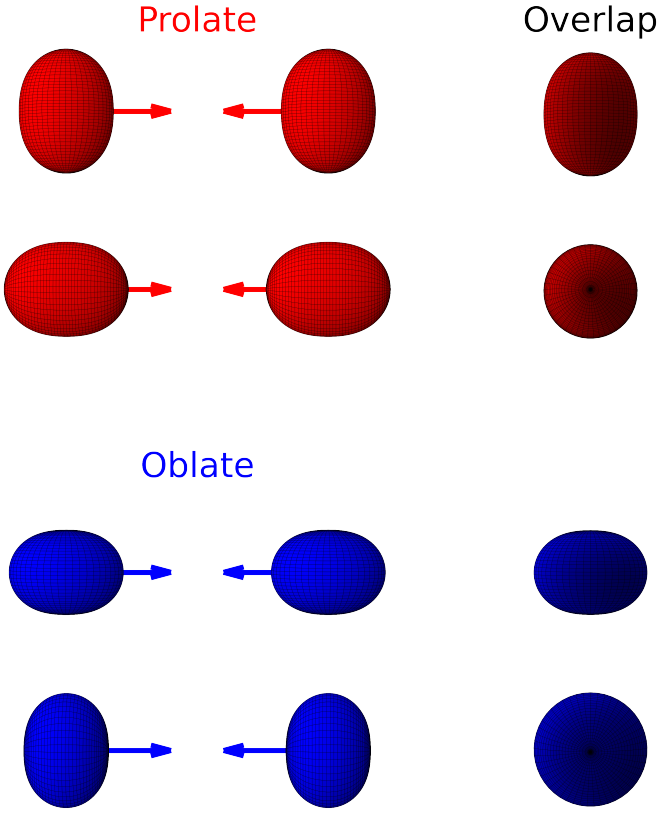
- LHC heavy ion data
- **Tip-tip**: collisions along symmetry axis
- **Body-body**: orthogonal axes

  → Shape is related to **overlap**

  Quark-gluon plasma evolution

  → hydrodynamics
- Measurement of particle shower





STAR Collaboration, Nature 635, 67-72 (2024).

B. Bally, et al. Phys. Rev. Lett. 128, 082301 (2022)