# Hadron Spectroscopy and Finite Energy Sum Rules

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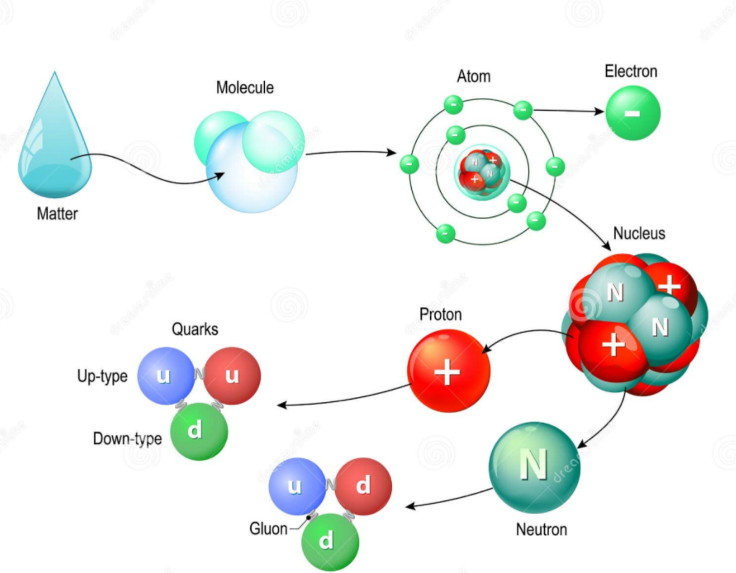
ICCUB Winter Meeting 2025

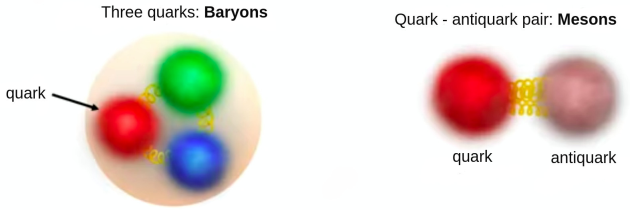
February 4,2025

# Matter: From Molecule to Quarks

All particles are composed of quarks and gluons.

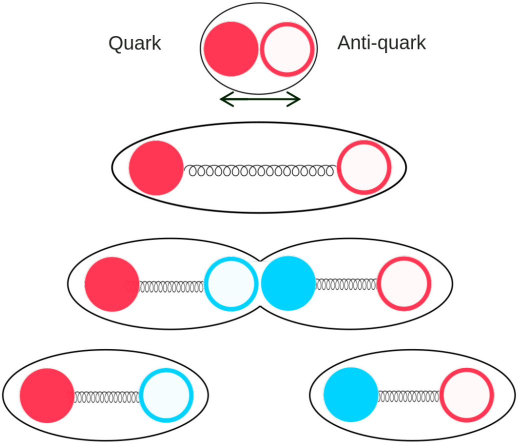


# Understanding the Building Blocks: Quarks and Gluons



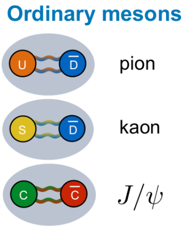

- Quarks and gluons possess a “color charge”, denoted by $R$, $B$, $and G$.
- Color-charged particles exchange gluons in strong nuclear interactions. In doing so, these color-charged particles are often **“glued”** together.

Color-charged particles cannot be found individually. For this reason, the color-charge quarks are **confined** in groups (hadrons) with other quarks. These composites are color neutral i.e. $R\bar{R}$, $B\bar{B}$, etc.



# Mesons in the Constituent Quark Model

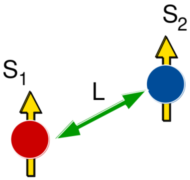
Ordinary mesons are quark-antiquark bound states with defined $J^{PC}$ quantum numbers.




pion $\tau \sim 10^{-8} s$

kaon $\tau \sim 10^{-8} s$

$J/\psi$ $\tau \sim 10^{-20} s$




$S = S_1 + S_2$

$J = L + S$

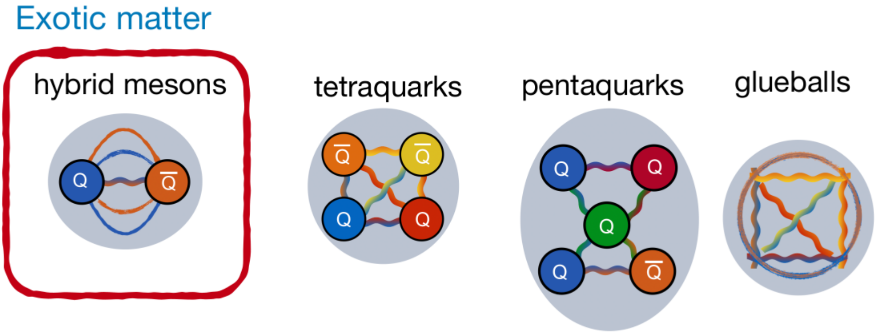
$P = (-1)^{L+1}$

$C = (-1)^{L+S}$

Some combinations are forbidden, such as $J^{PC}$: $0^{--}$, $0^{+-}$, $1^{-+}$, $2^{+-}$, and so on..

# Exotic Mesons

Nature is much more than the **CQM**: QCD does not prohibit the existence of unconventional meson states such as hybrids ($qqg$), tetraquarks ($qqqq$) and glueballs.




A lot of exotic states observed experimentally, but their nature is still far from being understood

# Why Meson Spectroscopy?

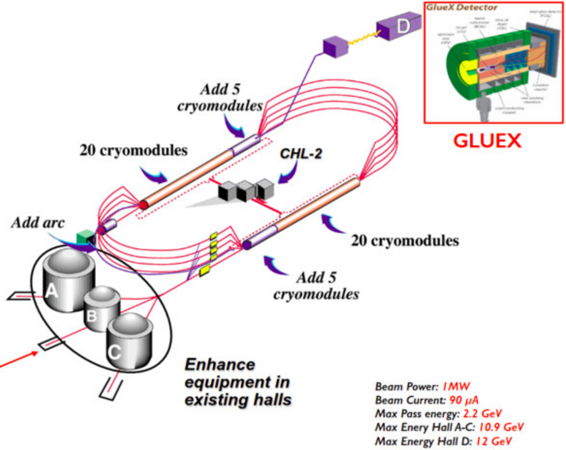
- Mesons are the simplest hadronic bound state: the ideal “laboratory” to study the interaction between quarks, to understand the role of **gluon** and the phenomenon of **confinement**.
- To perform such studies it’s important to achieve a precise measurement of the meson spectrum, with determination of resonance masses and properties.
- The presence of states with manifest gluonic component (i.e. exotics) would be the opportunity to directly look inside hadron dynamics.

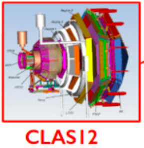
# Exotic Mesons: how to find them?

Facilities like GlueX at JLab, COMPASS at CERN, BESII/BESIII, etc. have searched and/or continue to search for exotic resonances using various reactions like:

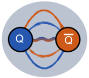
- photoproduction (GlueX),
- $\pi\, p$ scattering (COMPASS),
- $e^+e^-$ annihilation (BES).

JLab's dedicated experiments: GlueX (Hall D) and MesonEx (Hall B)



Exotic mesons

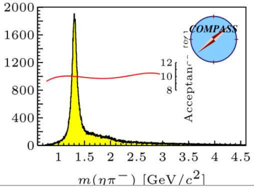


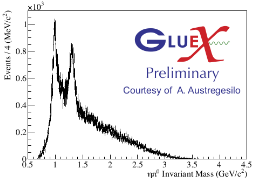

The lightest is $\pi_1$ $\quad J^{PC} = 1^{-+}$

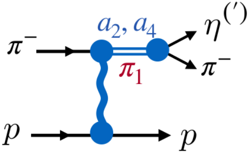
| | | | |
|---|---|---|---|
| $0^{--}$ | $0^{-+}$ | $0^{+-}$ | $0^{++}$ |
| $1^{--}$ | $1^{-+}$ | $1^{+-}$ | $1^{++}$ |
| $2^{--}$ | $2^{-+}$ | $2^{+-}$ | $2^{++}$ |
| $3^{--}$ | $3^{-+}$ | $3^{+-}$ | $3^{++}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

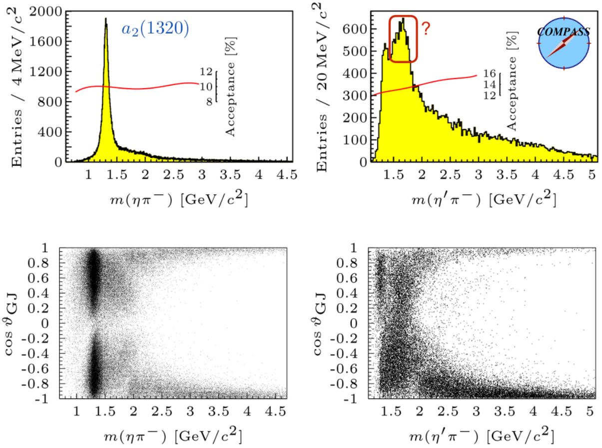
Decay mode $\quad \pi_1 \to \pi\eta$

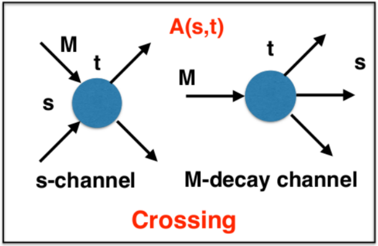
$$1^{-+} = \left(0^{-+} \otimes 0^{-+}\right)_{P\text{-wave}}$$




$\pi^-$
$a_2, a_4$
$\eta^{(\prime)}$
$\pi_1$
$\pi^-$
$p$
$p$


Entries / 4 MeV/$c^2$
$a_2(1320)$
Acceptance [%]
$m(\eta\pi^-)$ [GeV/$c^2$]
Entries / 20 MeV/$c^2$
?
COMPASS
Acceptance [%]
$m(\eta'\pi^-)$ [GeV/$c^2$]
$\cos\vartheta_{GJ}$
$m(\eta\pi^-)$ [GeV/$c^2$]
$\cos\vartheta_{GJ}$
$m(\eta'\pi^-)$ [GeV/$c^2$]

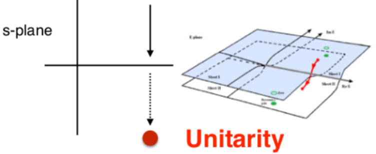



$$A(s,t) = \sum_l A_l(s) P_l(z_s)$$

**Analyticity**

$$A_l(s) = \lim_{\epsilon \to 0} A_l(s + i\epsilon)$$

$$A_l(s + i\epsilon) \neq A_l(s - i\epsilon)$$

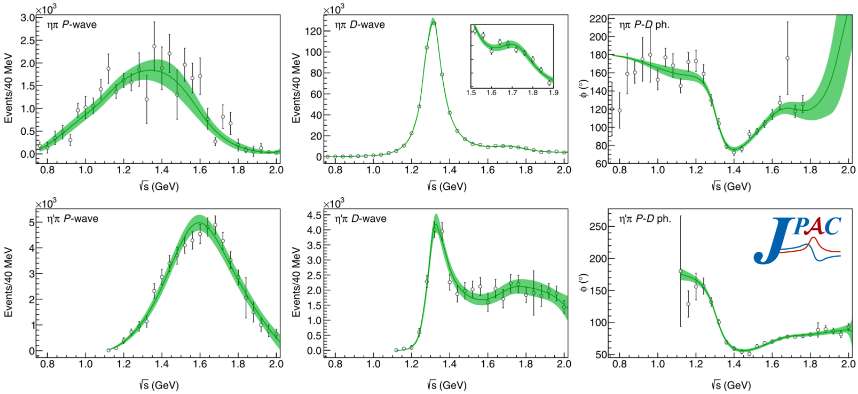



**The S-matrix Theory**

Build models based on:

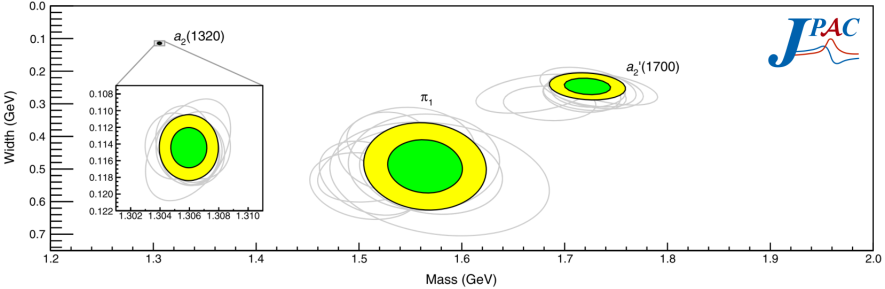
- Analyticity
- Crossing Symmetry
- Unitarity
- Lorentz Symmetries
- Global Symmetries of QCD

$\pi_1(1400)$ vs $\pi_1(1600)$

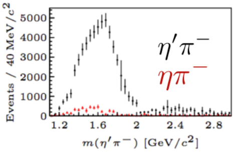


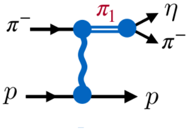
# Low Energy Fit of $P$ and $D$ waves Rodas et al PRL122 (2019)

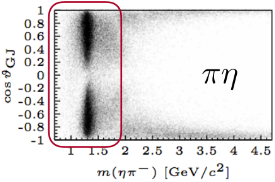
| Poles | Mass (MeV) | Width (MeV) |
|---|---|---|
| $a_2(1320)$ | $1306.0 \pm 0.8 \pm 1.3$ | $114.4 \pm 1.6 \pm 0.0$ |
| $a_2'(1700)$ | $1722 \pm 15 \pm 67$ | $247 \pm 17 \pm 63$ |
| $\pi_1$ | $1564 \pm 24 \pm 86$ | $492 \pm 54 \pm 102$ |

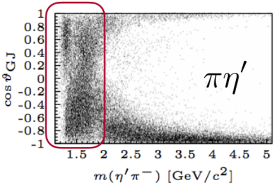


# $\eta\pi$ at COMPASS COMPASS PLB740 (2015)

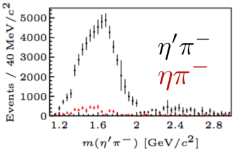


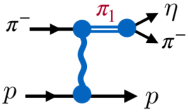



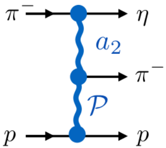



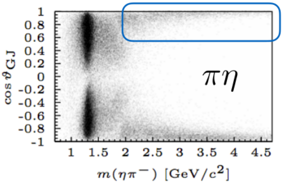
# $\eta\pi$ at COMPASS

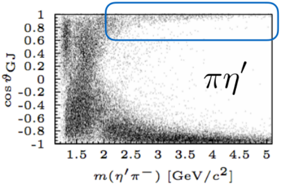


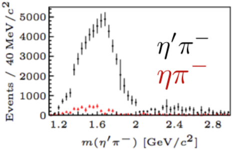



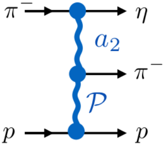




$\cos\theta_{GF} \sim 1 \rightarrow \eta$ forward



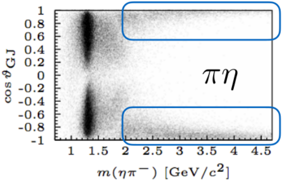



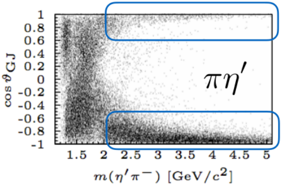



$\cos\theta_{GF} \sim 1 \rightarrow \eta$ forward

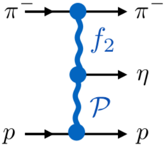
$\cos\theta_{GF} \sim -1 \rightarrow \eta$ backward

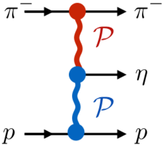


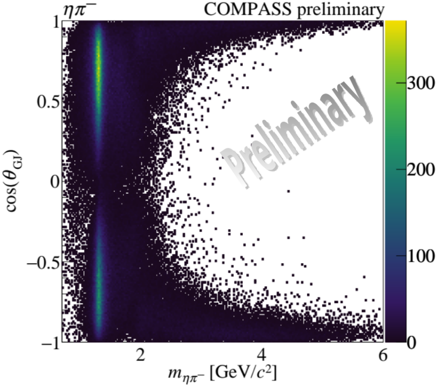



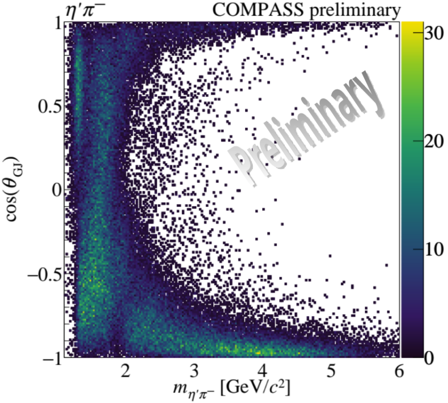

Exotic meson related to
Forward-backward asymmetry

Asymmetry related to
even-odd waves interferences



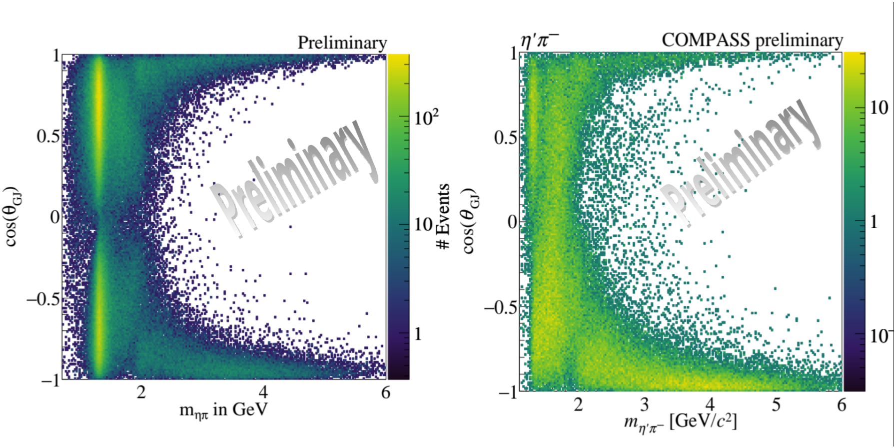






Mass of of both $\eta\pi$ and $\eta^{(\prime)}\pi$ systems vs $\cos\theta_{GJ}$ for full mass range and t $\in [-1, -0.1]$ GeV$^2$

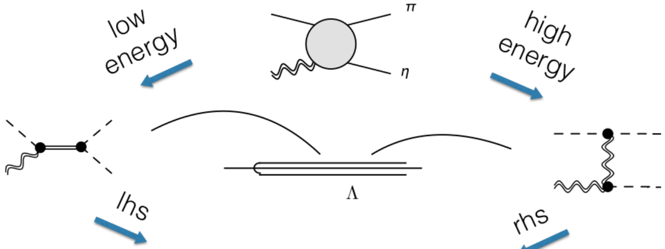
# New COMPASS Data




Mass of both $\eta\pi$ and $\eta^{(\prime)}\pi$ systems vs $\cos\theta_{GJ}$ for full mass range and t $\in [-1, -0.1]$ GeV$^2$ in Logarithmic scale

# Finite Energy Sum rules

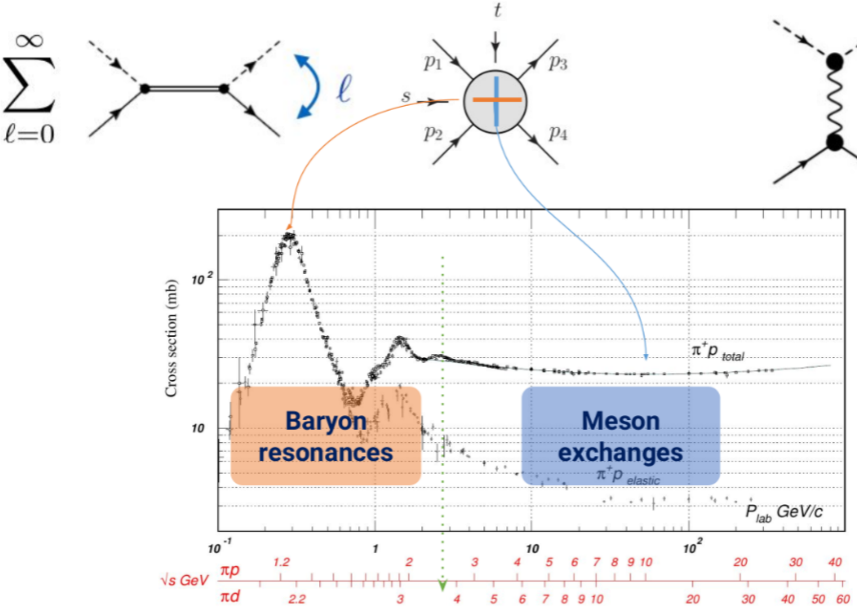
$$A(s_1, t_1, t_2, s_2/s) = \sum_i A_i(s_1, t_1, t_2) \left(\frac{s_2}{s}\right)^2$$




at fixed $t_1, t_2$ $\quad \int^{\Lambda} ImA_i(s_1, t_1, t_2) = \int_{\Lambda} ImA_i(s_1, t_1, t_2)$

- Derived using Cauchy's theorem: $\oint_C A(s,t)ds = 0$:
- Connect low-energy and high-energy dynamics.
- Predict high-energy observables from low-energy data.
- Constrain low-energy models using high-energy results.

# Finite Energy Sum rules

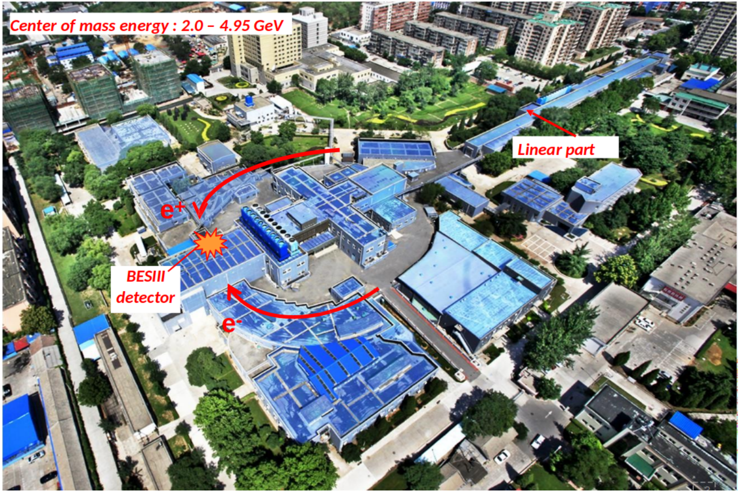


# Summary: Why Study $\pi p \to \pi\eta p$?

- Available experimental data provided by COMPASS experiment.
- Theory:
  - Validate analytic consistency of scattering amplitudes via FESR.
  - Apply Regge theory to $\pi\eta$ and $\pi p$ systems.
- Phenomenology:
  - Extract parameters for $a_2, f_2, \rho$ Regge trajectories.
  - Explore dynamics of $\pi\eta$ production and exotic mesons.
- Applications:
  - Refine vertex models for double Regge exchange.
  - Reduce the uncertainty of the $\pi_1$ pole position studied in PRL 122, 042002 (2019).

# Beijing Electron Positron Collider (BEPC)

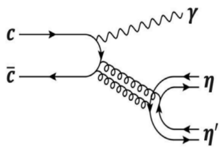
BESIII at BEPC provides the world's largest $\tau$-charm data sets from $e^+e^-$ annihilation making it an ideal laboratory for the search of glueballs and exotic states.

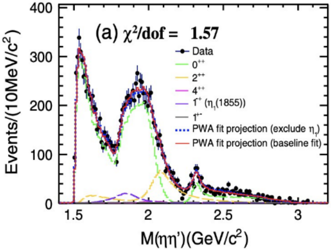


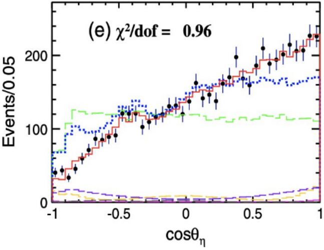
# Hunting for gluonic excitations in $J/\psi$ decays at BESIII

PRL 129 (2022) 192002, PRD 106 (2022) 072012

- Radiative decays of $J/\psi$ provide a gluon-rich environment for gluonic excitations.
- First observation of the isoscalar exotic state $\eta_1(1855)$ in $J/\psi \to \gamma\eta\eta'$.
- Identifying an isoscalar $1^{-+}$ hybrids is crucial for establishing the hybrid multiplet.



# Summary

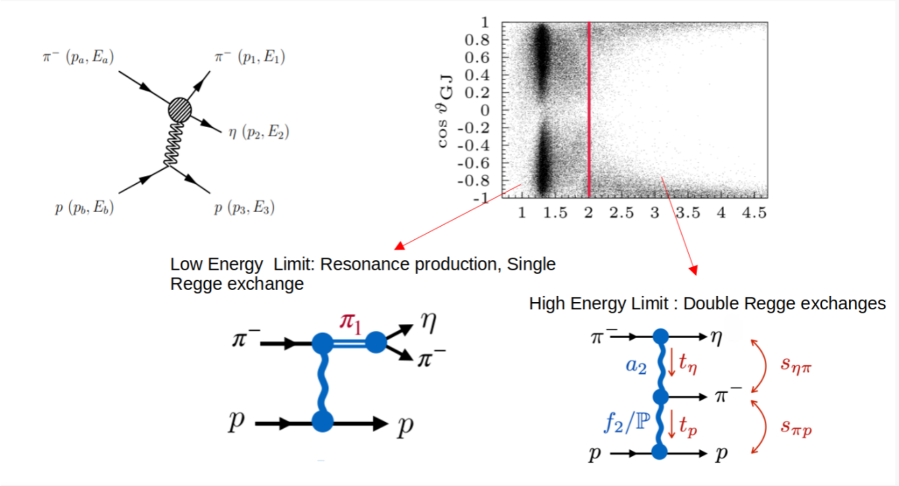
- To apply FESR to the $\pi R \to \pi\eta$ process, it is essential to determine the correct set of couplings. This requires studying simpler toy models, such as:
  - $\pi\pi \to \pi\pi$,
  - $\eta\eta \to \eta\eta$,
  - $\pi\eta \to \pi\eta$.

  These models provide the foundation for extracting couplings necessary for the target process.
- With these couplings in hand, FESR can be applied to $\pi p \to \pi\eta p$, enabling:
  - Verification of FESR predictions for the process,
  - Advancement toward broader objectives, such as constraining exotic meson dynamics and improving phenomenological models.


Thank
you

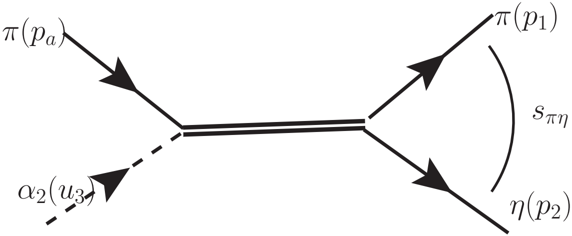

Q&A

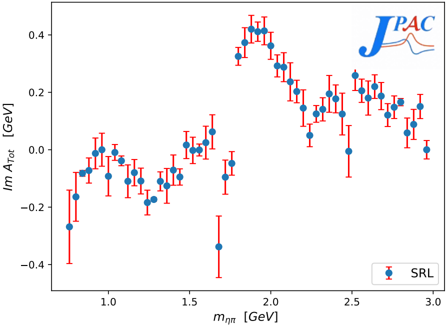
# Analysis of $\pi p \to \pi \eta p$



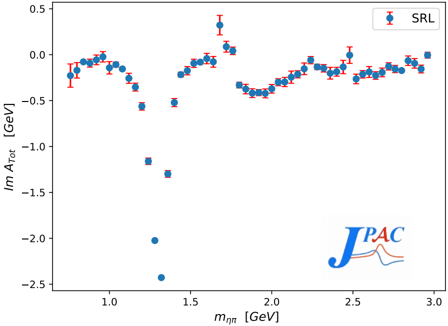
At low energies, the amplitude is primarily dominated by resonance production and can be expressed as a sum of partial wave amplitudes, which can be extracted from experimental data:

$$A = \sum_{L,\, M} a_l(s_{\eta\pi}) \sin(M\Phi)\, Y_{lM}(\theta, 0)$$






(a) $\Phi_2 = 0$ case

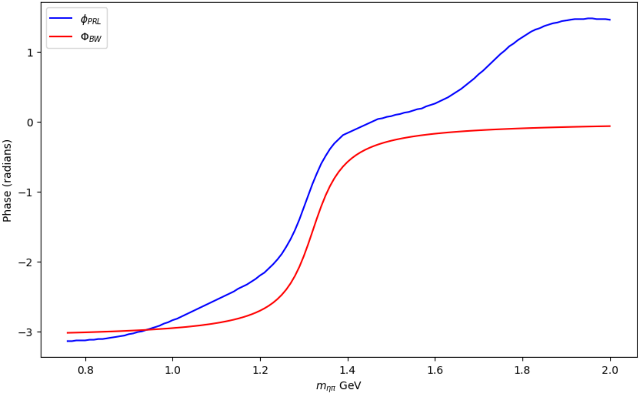



(b) $\Phi_2 = \arctan(\mathrm{BW}_{a_2})$ case

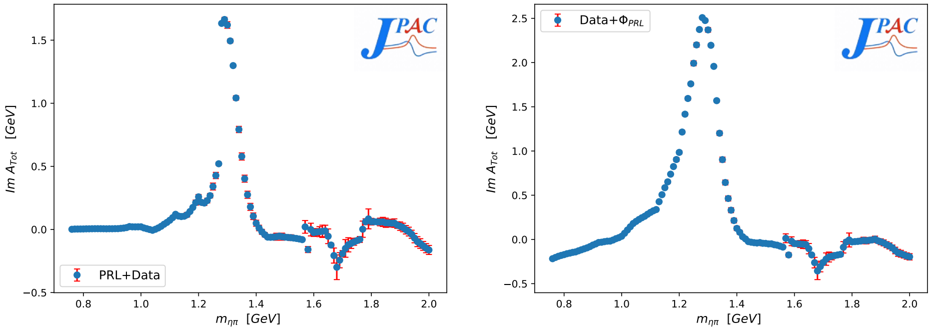
Figure: Imaginary part of the total low energy amplitude as a function of $m_{\eta\pi}$ for $t_1 = -0.1,\ u_3 = -0.33,\ s_{\eta p} = 200$ GeV$^2$.

# Low Energy Amplitude - Preliminary Results

The $D$-wave $(L = 2, M = 1)$ calculated from the Breit Wigner of $a_2$ meson vs the one calculated from the theoretical results of PRL model (PRL 122, 042002 (2019))



# Low Energy Amplitude - Preliminary Results




(a) Using theoretical $P$ and $D$ waves data from PRL

(b) Using $\Phi_2^{PRL}$

Figure: Imaginary part of the total low energy amplitude as a function of $m_{\eta\pi}$ for $t_1 = -0.1,\ u_3 = -0.33,\ s_{\eta p} = 200$ GeV$^2$.

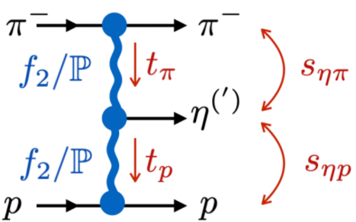
PRL 122, 042002 (2019)

High-energy amplitudes are dominated by Regge trajectories:

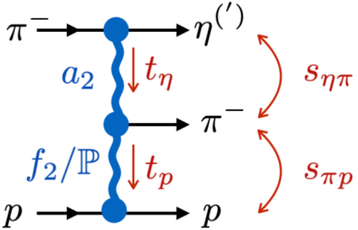
$$A(s,t) \sim \beta(t) s^{\alpha(t)}, \quad \alpha(t) = \alpha_0 + \alpha' t.$$

The total amplitude can be written as:

$$A_{\text{Th}}(m_{\eta\pi}, \Omega) = c_{a_2\mathbb{P}} A_{a_2\mathbb{P}} + c_{a_2 f_2} A_{a_2 f_2} + c_{f_2\mathbb{P}} A_{f_2\mathbb{P}} + c_{f_2 f_2} A_{f_2 f_2} + c_{\mathbb{PP}} A_{\mathbb{PP}} + c_{\mathbb{P}f_2} A_{\mathbb{P}f_2}$$




(a) Fast-$\pi$




(b) Fast-$\eta$

Figure: From arXiv:2104.10646v2 [hep-ph] 19 Jul 2021

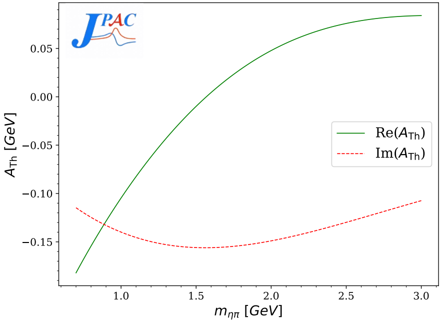
# High Energy Amplitude- Preliminary Results

The general amplitude for $\pi p \to \pi\eta p$:

$$A(s, s_{\pi\eta}, s_{\eta p}) \sim \frac{\Gamma(1-\alpha_1)\Gamma(1-\alpha_2)}{(s_{\pi\eta})^{\alpha_1}(s_{\eta p})^{\alpha_2}} V(\alpha_1, \alpha_2, \kappa).$$

where

$$V(\alpha_1, \alpha_2, \kappa) = \frac{\Gamma(\alpha_1 - \alpha_2)}{\Gamma(1-\alpha_2)} \, {}_1F_1\left(1-\alpha_1, 1-\alpha_1+\alpha_2, -\kappa\right), \; \kappa = \frac{\alpha' s_{\pi\eta} s_{\eta p}}{s}$$




Real and Imaginary parts of the high energy amplitude for the fast-$\pi$ process as a function of $m_{\eta\pi}$ for $t_1 = -0.1,\ u_3 = -0.33,\ s_{\eta p} = 200$ GeV$^2$.

# Reducing $2 \to 3$ to $2 \to 2$

Our preliminary analysis indicates that the total amplitude is independent of $s_{\eta p}$, enabling us to simplify the $2 \to 3$ process to a more manageable $2 \to 2$ process, $\pi R \to \pi\eta$.