Black Hole perturbations in the large D limit

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UNIVERSITAT DE BARCELONA

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1. Motivation: Gravity in arbitrary D

Motivation:
Why arbitrary D?

- String Theory

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- Holography

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- String Theory
- Holography
- Generalizations from D=4

Laplace equation and Coulomb potential in arbitrary D

- Vacuum Laplace equation:

$$\nabla^2\Phi = 0$$

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- Vacuum Laplace equation:

$$\nabla^2 \Phi = 0$$

- Coulomb potential:

$$\Phi \sim \frac{1}{r^{D-3}}$$

2. The large D limit of General Relativity

GR in vacuum: solutions and parameters

- Vacuum Einstein equation:

$$R_{\mu\nu} = 0$$

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- Schwarzschild-Tangherlini solution:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{D-2}$$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^{D-3}$$

GR in vacuum: solutions and parameters

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$$r_0, D$$

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- Only non trivial parameter: D

Inspiration: Strings and Yang-Mills theory

- Yang-Mills: Gauge theory of SU(N) gauge group.

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- Fundamental elements: Strings, dynamical and nonlinear solutions

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- Fundamental elements: Strings, dynamical and nonlinear solutions
- Strings as basic and simplified objects at large N

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- Vacuum GR <-> YM theory
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- Large N -> Strings simplify
- Large D -> BHs simplify?

3. Black Holes and the Membrane Paradigm

Localization of gravity

- Schwarzschild Black Hole:

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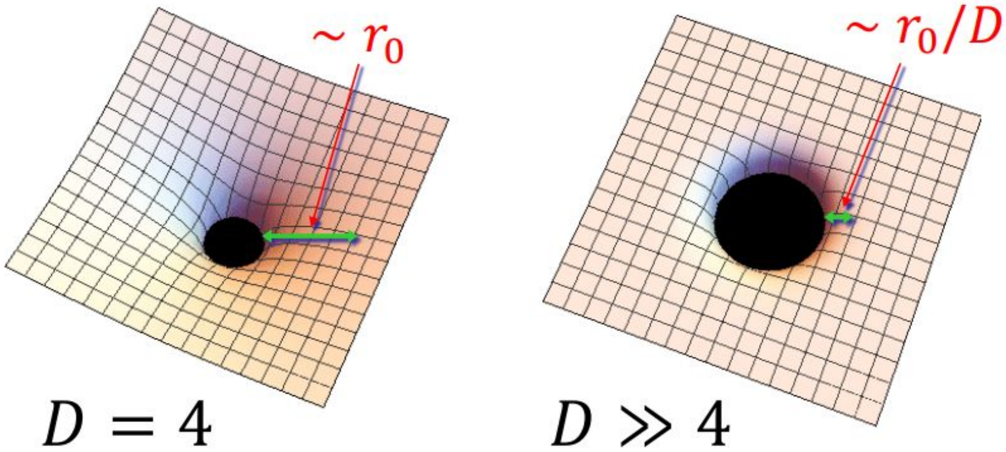

Space regions in large D

- Near-horizon region: all gravitational dynamics

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Space regions in large D

- Near-horizon region: all gravitational dynamics

$$r - r_0 \sim \frac{r_0}{D}$$

- Far region: Flat space

$$r - r_0 >> \frac{r_0}{D}$$

The Membrane Paradigm: Setup

- Black Hole as a GR solution

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- Black Hole as a GR solution

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- Black Hole horizon as a dynamical membrane in a background (flat space)

The Membrane Paradigm: degrees of freedom

- Einstein equations in the Near-horizon region

The Membrane Paradigm: degrees of freedom

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- Dynamical hydrodynamical equations in flat space

4. Black branes and effective equations

What is a black p-brane?

- Trivial generalization from a known black hole:

$$ds^2 = ds^2_{Schw.} + dx_a dx^a$$

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- Trivial generalization from a known black hole:

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- Where is the horizon?

$$r = r_0 \ \forall x^a$$

Membrane paradigm with spherical symmetry

- Most dimensions show symmetry:

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- Black p-brane: Keep p finite

$$p \sim O(1)$$

$$D - p - 2 \sim O(D)$$

Effective equations: Covariant Formalism

arXiv:1504.06613
Bhattacharyya, De, Minwalla, Mohan, Saha

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- Effective equations:

$$\hat{\nabla}_\mu u^\mu = 0$$

$$\left(\frac{\hat{\nabla}^2 u_\nu}{\mathcal{K}} - \frac{\hat{\nabla}_\nu \mathcal{K}}{\mathcal{K}} + u^\alpha \mathcal{K}_{\alpha\nu} - u^\alpha \hat{\nabla}_\alpha u_\nu\right) \mathcal{P}^\nu_\mu = 0$$

Effective equations: Hydrodynamical Formalism

arXiv:1504.06489
Emparan, Shiromizu, Suzuki, Tanabe, Tanaka

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- Effective equations: dependent on the background space. Hydrodynamical approach.

5. Main thesis goal: Two formalisms, one common ground?

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- To prove that both formalisms are equivalent.

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- To prove that both formalisms are equivalent.
- Take an almost general spacetime, apply the Covariant formalism and reach the known hydrodynamical equations.

- General p-brane background:

$$\mathrm{d}s^2 = G_{ij}(X)\mathrm{d}x^i\mathrm{d}x^j + W(X)\mathrm{d}\Sigma^2_{n_\Sigma} + 2\mathrm{d}t\left[\mathrm{d}r - A(X)\mathrm{d}t - F_i(X)\mathrm{d}x^i\right]$$

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- Hydrodynamic effective equations:

$$\left[\partial_t + \tfrac{1}{2}\mathrm{tr}\left(\partial_t\gamma\right) + \partial_t s\right]m = \left[\nabla^i + (\nabla^i s)\right]\left(\nabla_i m - \hat{p}_i\right)$$

- General p-brane background:

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$$\left[\partial_t + \tfrac{1}{2}\mathrm{tr}\,(\partial_t\gamma) + \partial_t s\right]\hat{p}_j = \left[\nabla^i + (\nabla^i s)\right]\left(\nabla_i\hat{p}_j - \frac{\hat{p}_i\hat{p}_j}{m} + m\,\partial_t\gamma_{ij}\right) + (2\nabla^i m - \hat{p}^i)(\nabla_i\nabla_j s - R_{ij}) +$$
$$+\left(\frac{\kappa}{r_0^2} + \sigma\right)\nabla_j m - 2(\nabla^i m - \hat{p}^i)\nabla_{[i}f_{j]} + (\partial_t f_j + \nabla_j f_0)\,m\,.$$

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- Two effective formalisms: Covariant and Hydrodynamical.
- Common ground: They describe the same physics.
- This thesis: A dictionary between them.

Thank you for your attention :)