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AGENCIA ESTATAL DE INVESTIGACIÓN

Gravitational Waves from Supermassive Black Hole Binaries: Statistical Properties and Implications

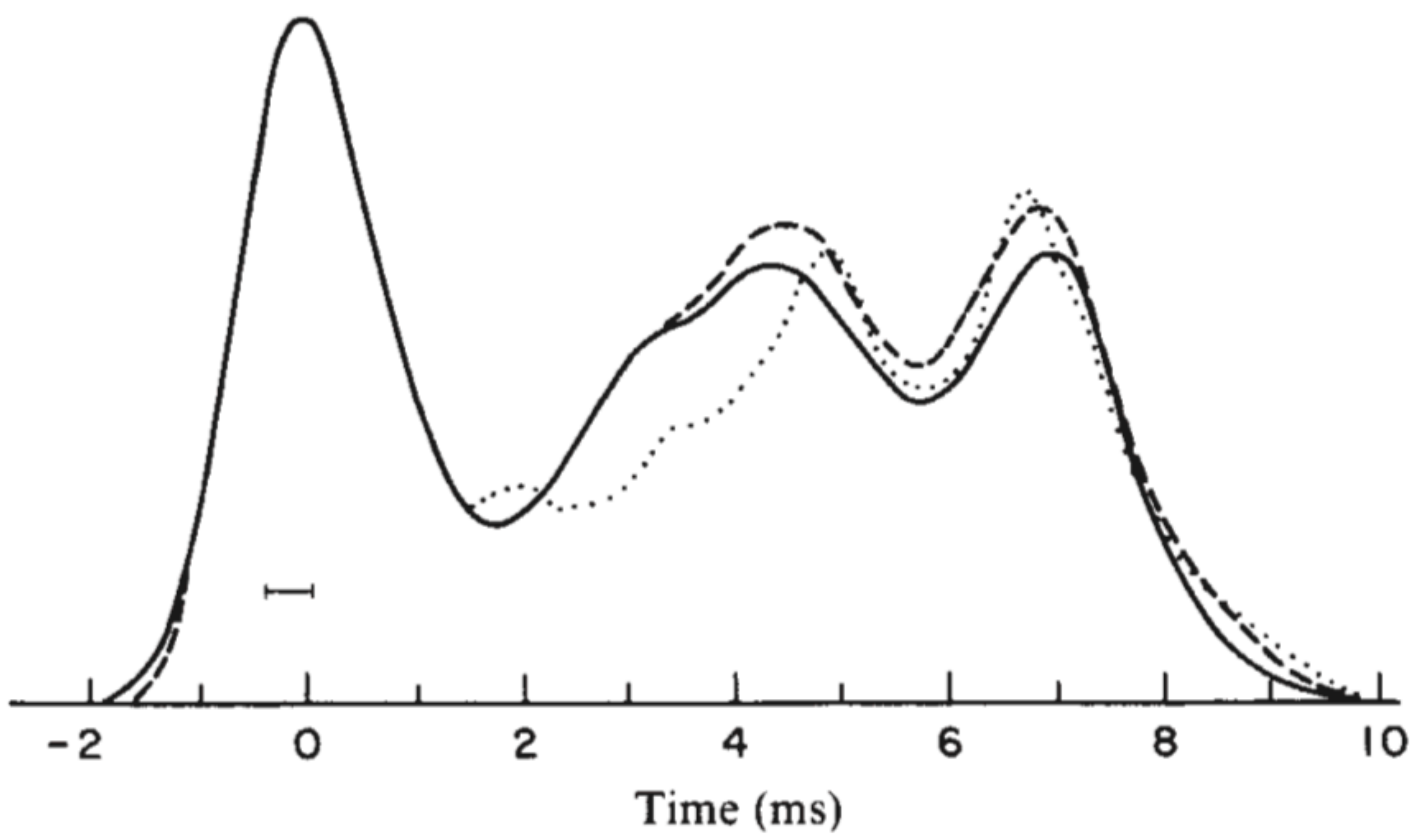
Xiao Xue
IFAE Barcelona, UAB
BIG meeting, ICCUB, Barcelona, 29.11.2024

Outline

- Introduction
- SGWB from SMBHBs
  - A complete characterization of non-Gaussian features (2409.19516)
  - Environment effect: three body ejection (2411.05906) w / NANOGrav
- Additional slides

Observation evidence

- **1979 Russell A. Hulse & Joseph H. Taylor (1993 Nobel Prize)**
  - Discovery of first binary pulsar “Hulse–Taylor pulsar”
  - First Indirect evidence of gravitational wave emission **(few 10^{-5} Hz)**
  - Techniques for high precision **pulsar timing**.



Taylor, Fowler, McCulloch, 1979
(Spin axis precess)

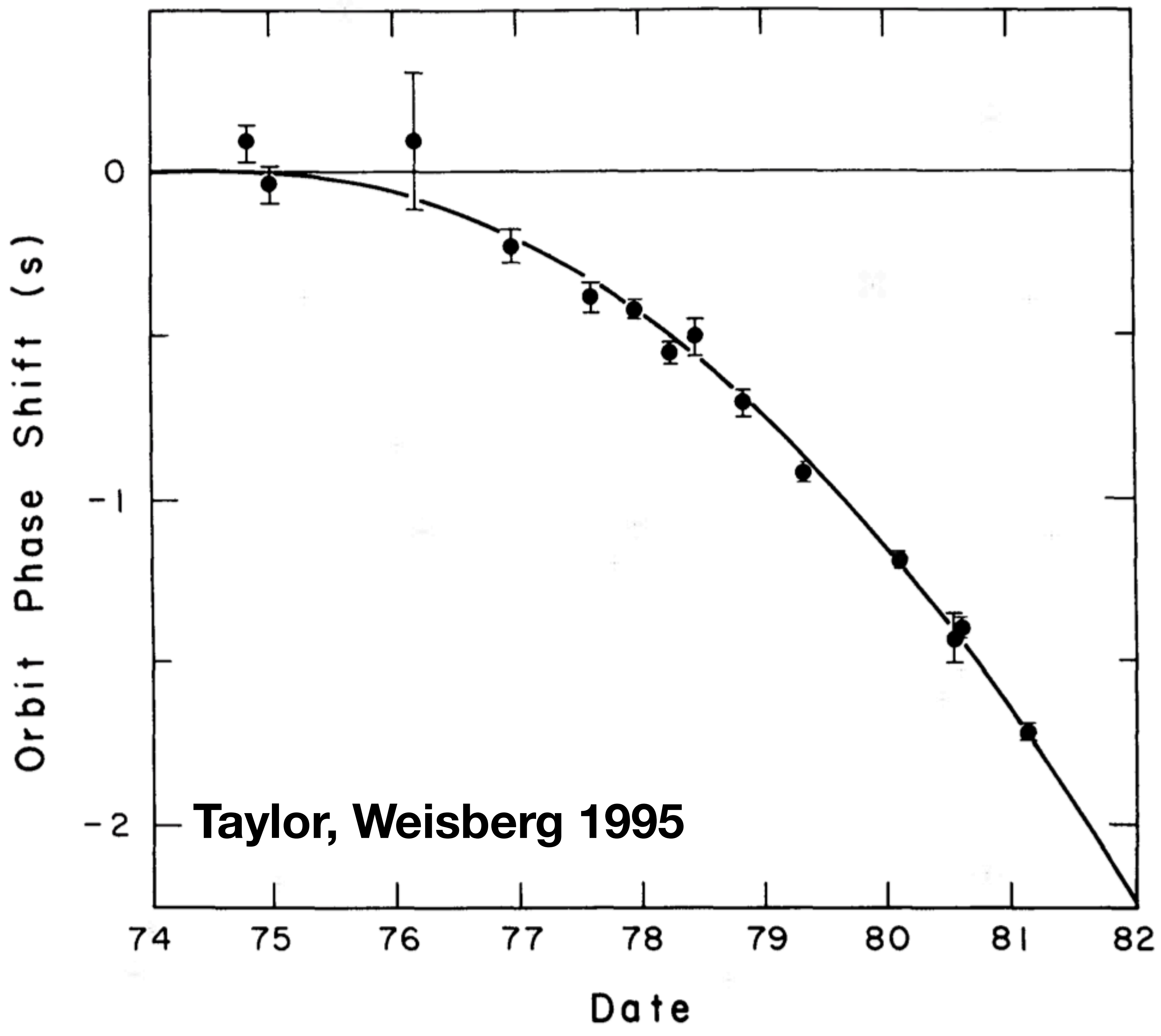
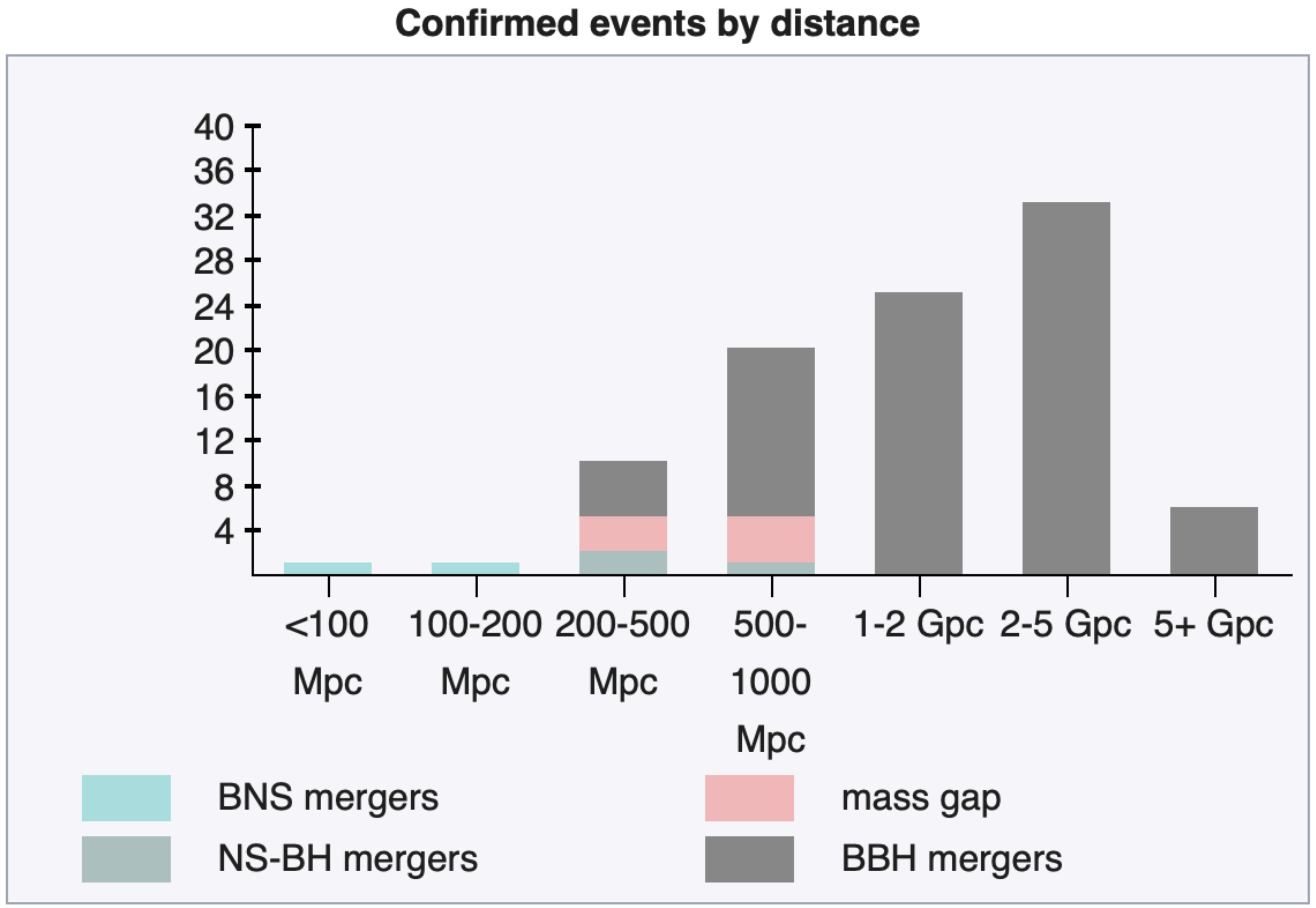


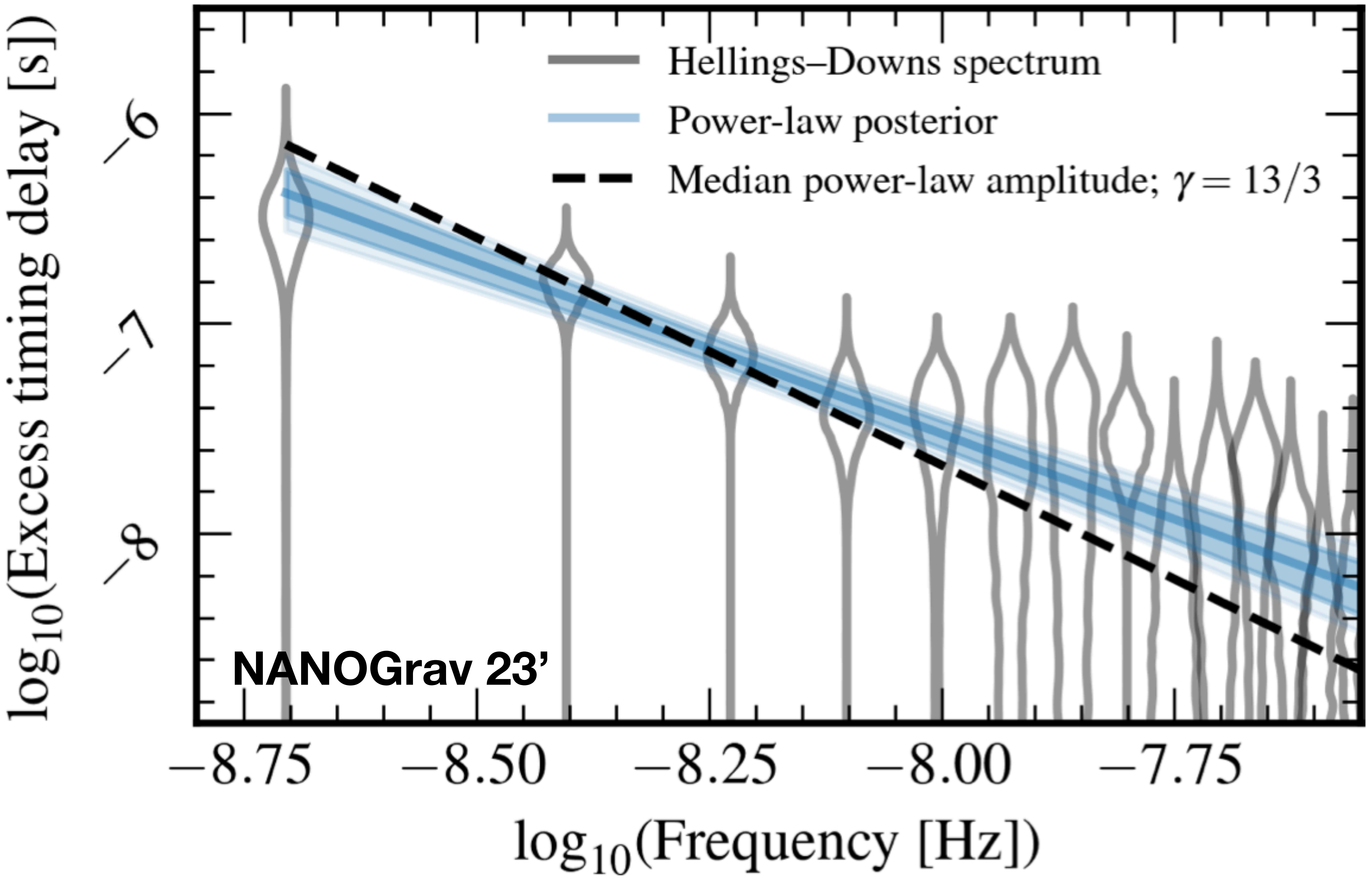
FIG. 6.—Orbital phase residuals, obtained from the data listed in Table 4. If the orbital period had remained constant, the points would be expected to lie on a straight line. The curvature of the parabola drawn through the points corresponds to the general relativistic prediction for loss of energy to gravitational radiation, or $\dot{P}_b = -2.40 \times 10^{-12}$.

Observation evidence

- **GW150914 by LIGO-Virgo (2016)**
  - BH/NS/Mass gap merger **events (~few Hz)**



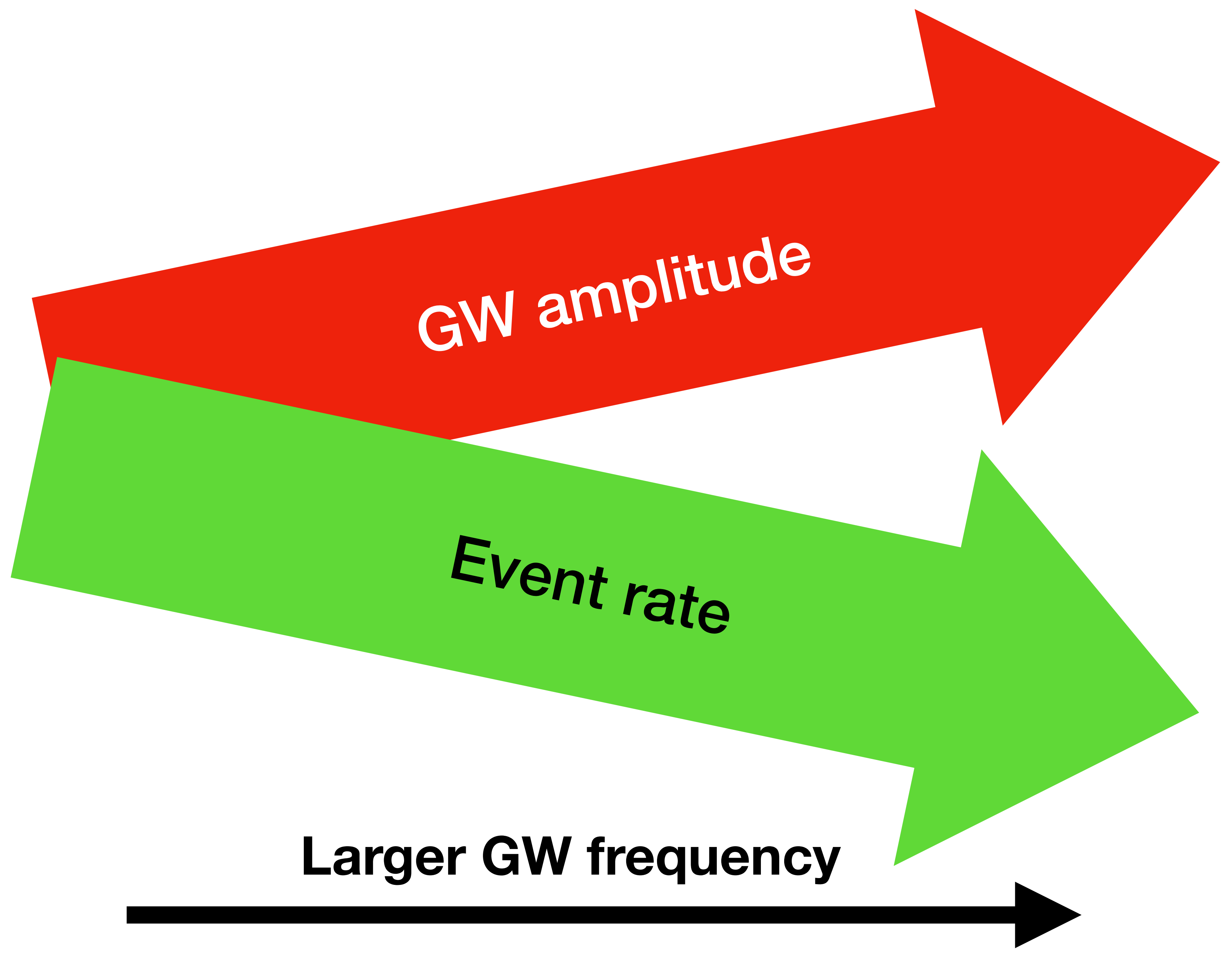
- **Pulsar timing arrays (2023)**
  - **Stochastic** GW background **(~nano Hz)**



What makes the difference?

GW statistics

- GW amplitude $h_0(f, M_c, z) = \dfrac{4c\,(\pi f_{\rm yr})^{2/3}\,(f_r/f_{\rm yr})^{2/3}\,(GM_c/c^3)^{5/3}}{d_L(z)}$
- Event Rate $\propto \dfrac{\mathrm{d}t_r}{\mathrm{d}\ln f_r} = \dfrac{5}{96\,(GM_c/c^3)^{5/3}\,(\pi f_{\rm yr})^{8/3}(f_r/f_{\rm yr})^{8/3}}$

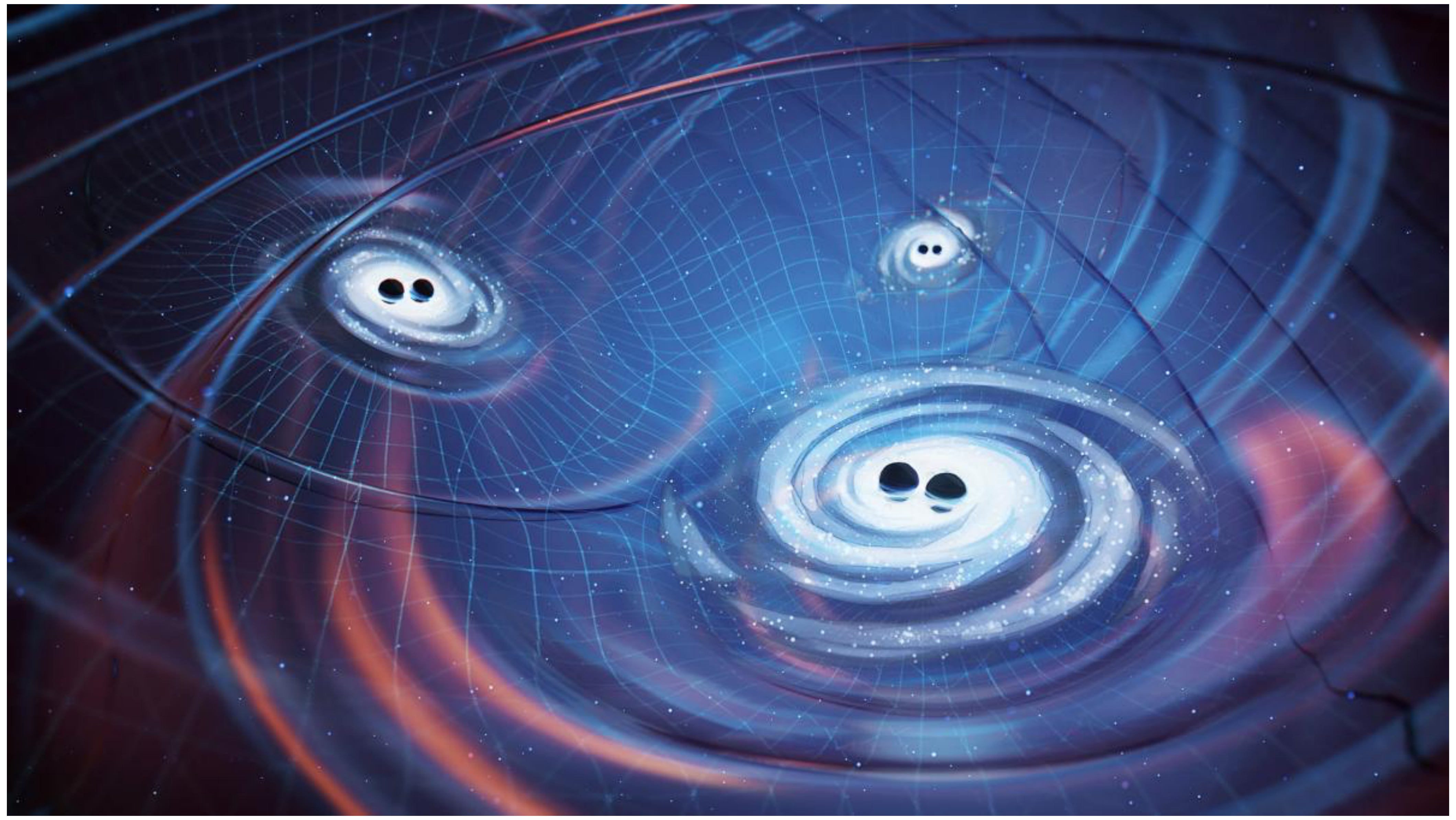


LIGO : High frequency GW —> low event rate —> merger event

PTA : Low frequency GW —> high event rate —> stochastic background

GW statistics

- Sky location / Orientation: $h_{ij}^{(n)}(f,\boldsymbol{x}) = h_{ij}(f,\boldsymbol{x};M_c^{(n)},\theta^{(n)},\phi^{(n)},\iota^{(n)},\psi^{(n)},\varphi^{(n)})$
- **7 parameters to describe a circular SMBHB binary**
- $h_{ij}^{\text{total}}(f,\boldsymbol{x}) = \sum_{n=1}^{N} h_{ij}^{(n)}(f,\boldsymbol{x})$ **: 2D random walk**
- **Central Limit Theorem:** $N \to \infty$**,** $h_{ij}(f) \sim \mathcal{N}\left(0\,,\langle h_{ij}h_{ij}\rangle\right)$
- **The information of individual sources is lost!**



Olena Shmahalo, nanograv.org

GW statistics

Non-Gaussian Statistics of Nanohertz Stochastic Gravitational Waves

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- **2409.19516**
- **Does the assumption $N \to \infty$ limit hold at nanohertz? (No)**
- **If not, is the remaining information detectable? (Yes)**

GW statistics

GW amplitude

$$h_0(f, M_c, z) = \frac{4c\,(\pi f_{\rm yr})^{2/3}\,(f_r/f_{\rm yr})^{2/3}\,(GM_c/c^3)^{5/3}}{d_L(z)}$$

Polarization

$$h_+(t, \boldsymbol{x}; f, M_c, z; \iota, \varphi) = h_0\,\frac{1+\cos^2\iota}{2}\,\cos(2\pi f t + \varphi)$$

$$h_\times(t, \boldsymbol{x}; f, M_c, z; \iota, \varphi) = h_0\,\cos\iota\,\sin(2\pi f t + \varphi)$$

$$\epsilon^+_{ab}(\theta, \phi, \psi)\,;\,\epsilon^\times_{ab}(\theta, \phi, \psi)$$

Combine them together

$$h_{ab}(t, \boldsymbol{x}; \boldsymbol{\Lambda}) = \sum_{+/\times} h_{+/\times}(t, \boldsymbol{x}; f, M_c, z; \iota, \varphi)\epsilon_{+/\times}(\theta, \phi, \psi)$$

$$\boldsymbol{\Lambda} = \{\log_{10} M_c, z\} \cup \{\cos\theta, \phi;\ \cos\iota, \psi;\ \varphi\}$$

SMBHB Population model

$$\frac{\mathrm{d}^8\overline{N}}{\mathrm{d}\ln f\,\mathrm{d}^7\boldsymbol{\Lambda}} = \frac{1}{32\pi^3}\frac{\mathrm{d}^3\overline{N}}{\mathrm{d}\ln f\,\mathrm{d}\log_{10} M_c\,\mathrm{d}z}$$

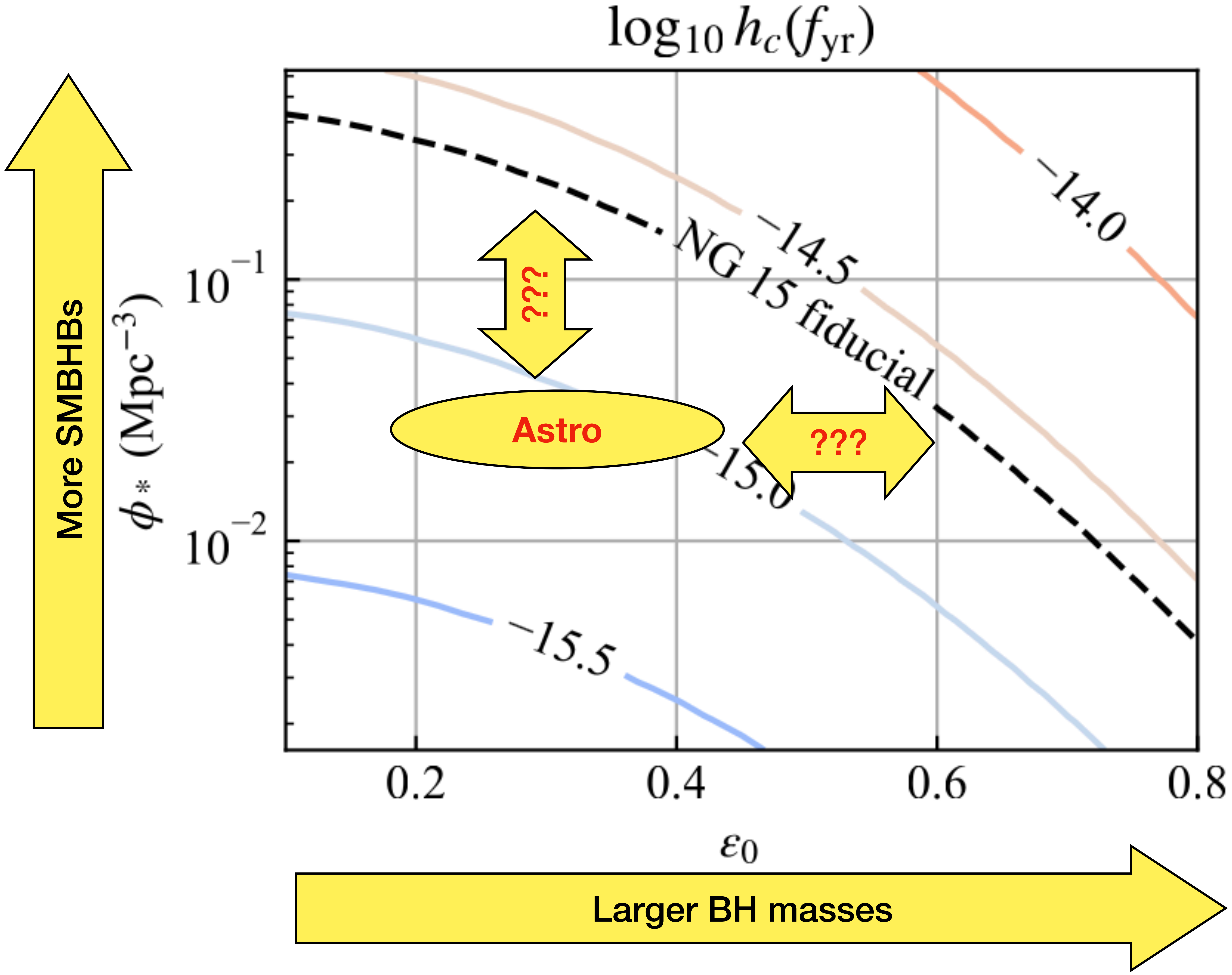
(no anisotropy, no polarization preference)

ϕ_* : Total number of galaxies per Mpc^3

ϵ_0 : ~tunes the average mass of the SMBHBs

GW statistics

- Inconsistency between astrophysical observation and the NANOGrav result!
- 2312.06756
- 2406.17010
- 2407.14595



GW statistics

The **expectation value** of the source number in a finite parameter space

$$\Delta\overline{N} = \frac{\mathrm{d}^8\overline{N}}{\mathrm{d}\ln f\,\mathrm{d}^7\boldsymbol{\Lambda}}\Delta\ln f\Delta^7\boldsymbol{\Lambda}$$

But we have the cosmic variance…

$$\Delta N \sim \mathrm{Pois}(\Delta\overline{N})$$

Combining the contribution from all parameter space

$$h_{ab}^{\mathrm{total}}(t,\boldsymbol{x}) = \sum_{\boldsymbol{\Lambda}} \Delta N(\boldsymbol{\Lambda}) h_{ab}(t,\boldsymbol{x};\boldsymbol{\Lambda})$$

Compound Poisson Statistics (with weights)!

See our paper 2409.19516 for definition

GW statistics

Compound Poisson Statistics (with weights)!

See our paper 2409.19516 for definition

Cumulant generating function

$$K(\boldsymbol{t}) = \int_V \mathrm{d}\ln f\, \mathrm{d}^7\boldsymbol{\Lambda}\, \frac{\mathrm{d}^8\overline{N}}{\mathrm{d}^7\boldsymbol{\Lambda}\, \mathrm{d}\ln f} \left(e^{i\,\boldsymbol{s}(\boldsymbol{\Lambda})\cdot\boldsymbol{t}} - 1\right) \quad .$$

PDF

$$P(\boldsymbol{S}) = \frac{1}{(2\pi)^{\mathcal{N}}} \int \mathrm{d}^{\mathcal{N}}\boldsymbol{t}\, \exp\left(i\,\boldsymbol{S}\cdot\boldsymbol{t} + [K_{\boldsymbol{S}}(\boldsymbol{t}^*)]^*\right)$$

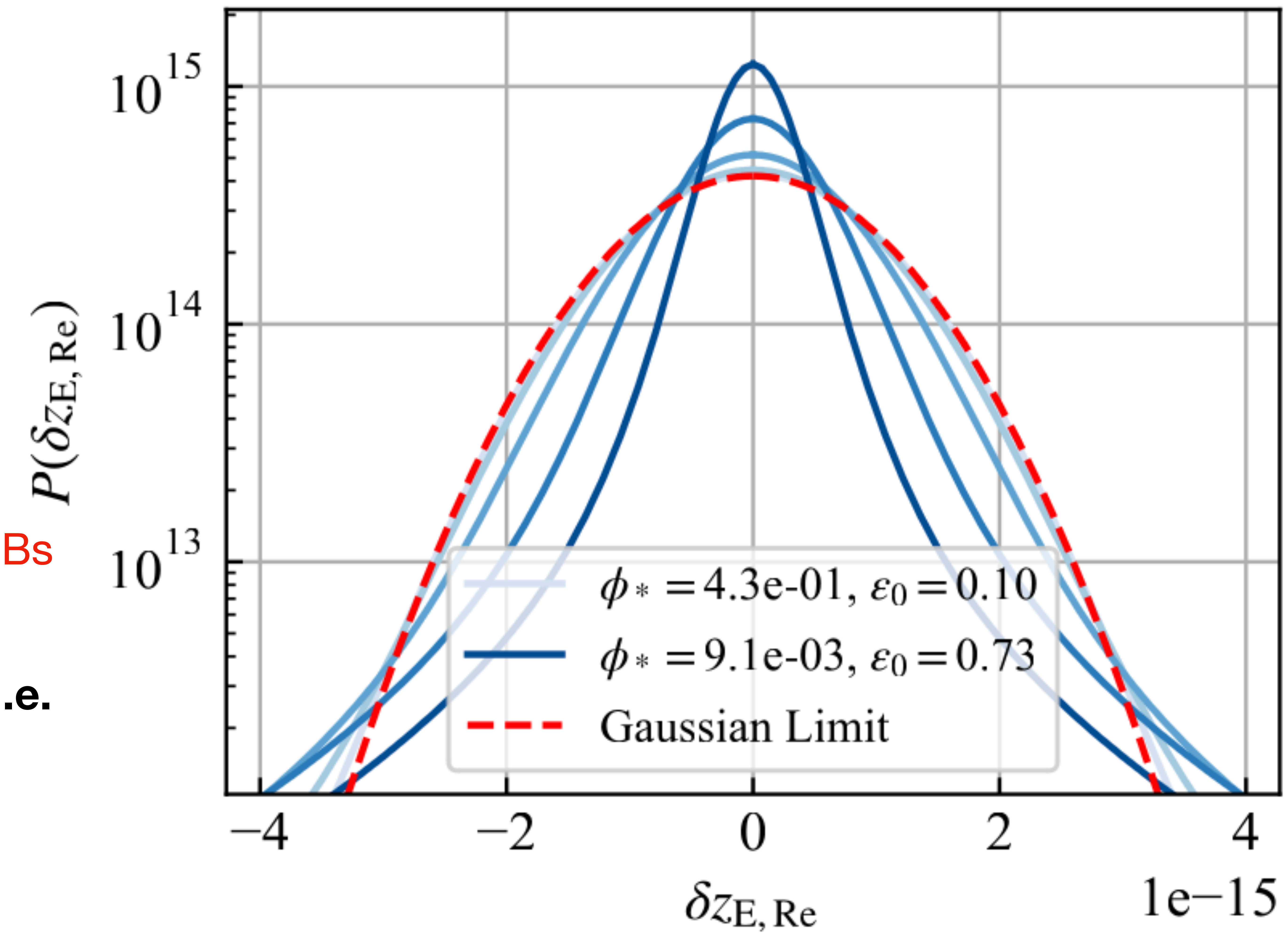
Conjugate variable of the signal

It can be any addable signal, including the GW!

$$\boldsymbol{S} = \sum_{\boldsymbol{\Lambda}} \boldsymbol{s}(\boldsymbol{\Lambda}) \Delta N(\boldsymbol{\Lambda})$$

GW statistics

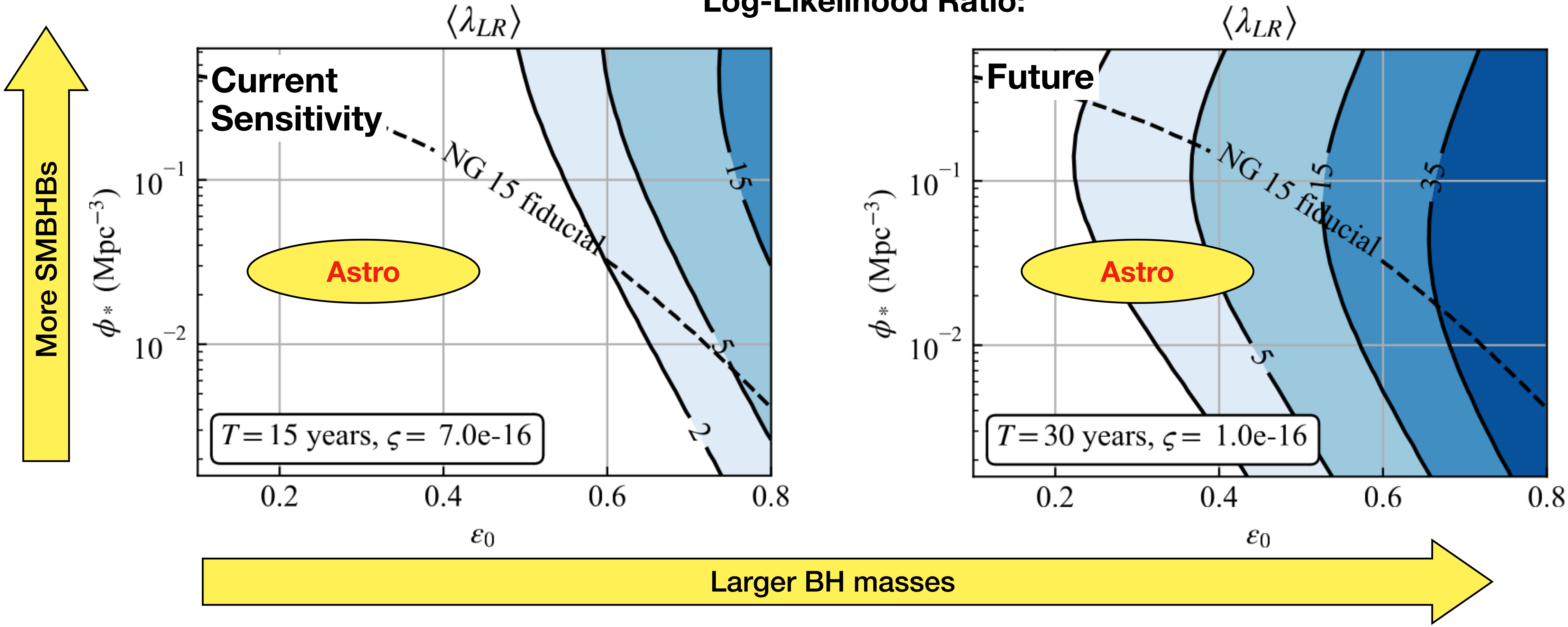
- Observable: redshift = timing residual
- ϕ_* : Total number of galaxies per Mpc^3
- ϵ_0 : ~tunes the average mass of the SMBHBs
- **All examples here have the same expectation values of the signal power, i.e. on the "NG15 fiducial" line!**



GW statistics

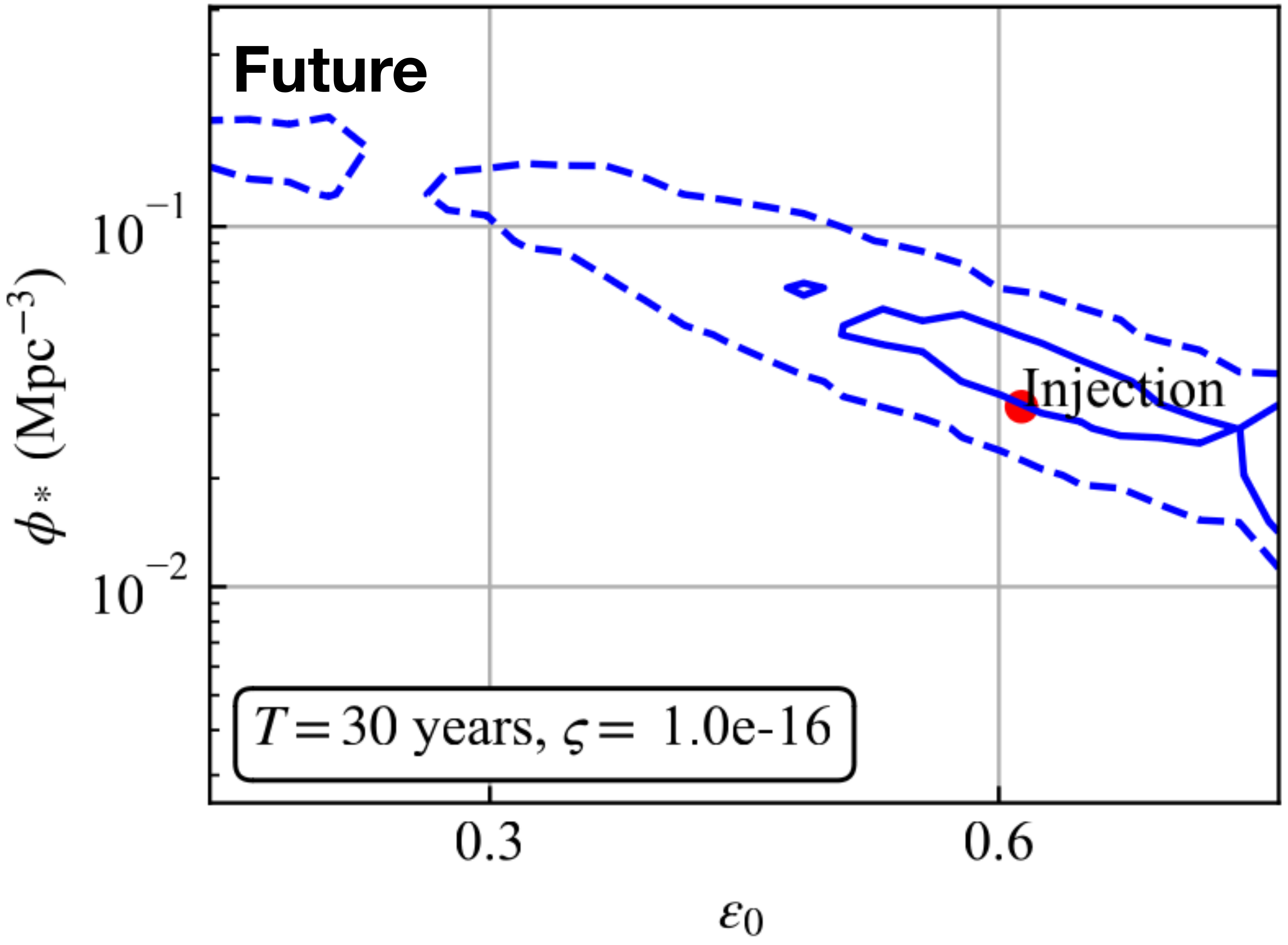
How strong the non-Gaussian signal will be?

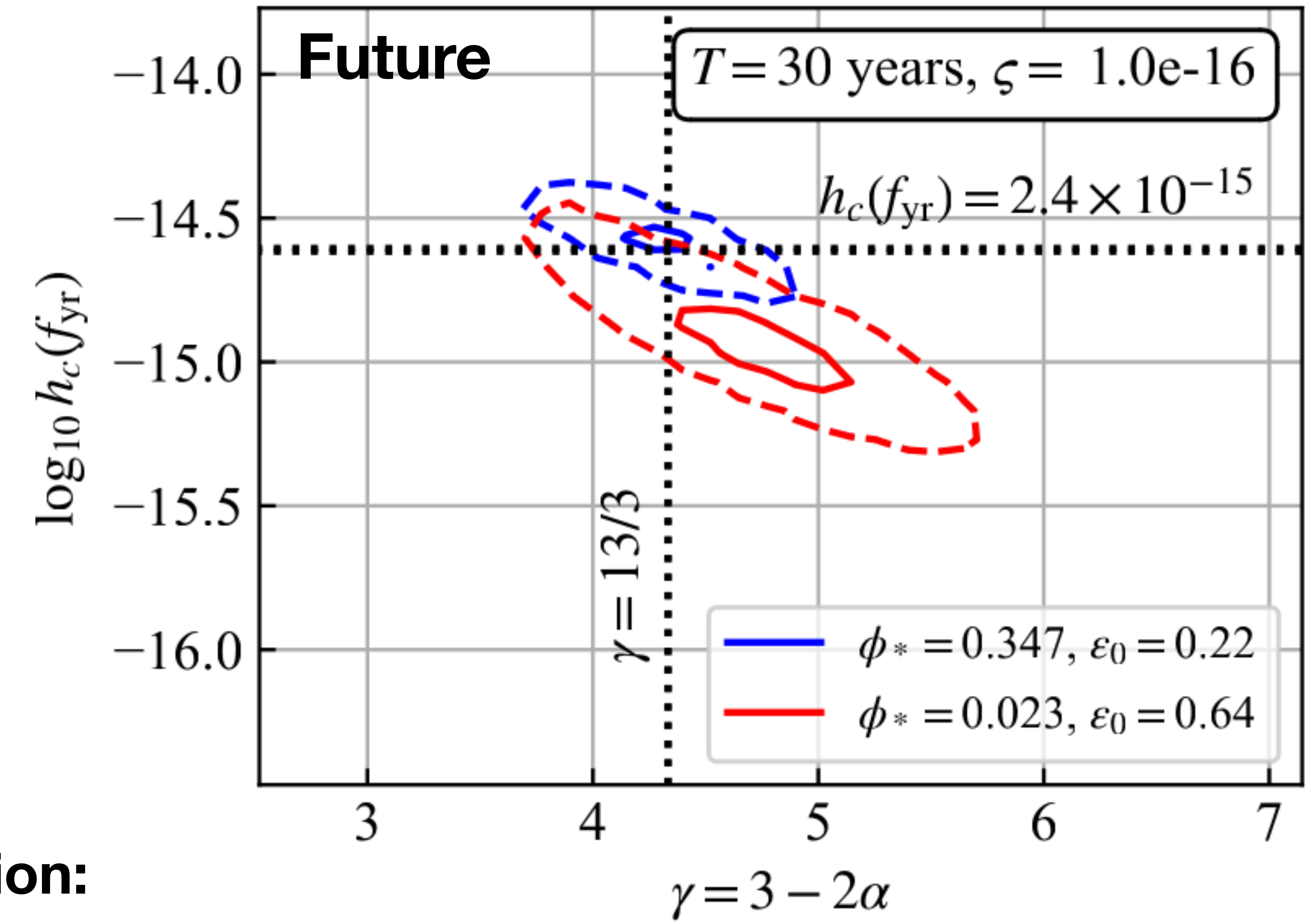
Log-Likelihood Ratio:



GW statistics

Signal injection -> Recovery of true parameters





Conclusion:

It is **possible** to use the **non-Gaussian** information to constrain population model parameters, but only in the **foreseeable future** **—> 2409.19516**

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  - Environment effect: three body ejection (2411.05906) w / NANOGrav
- Additional slides

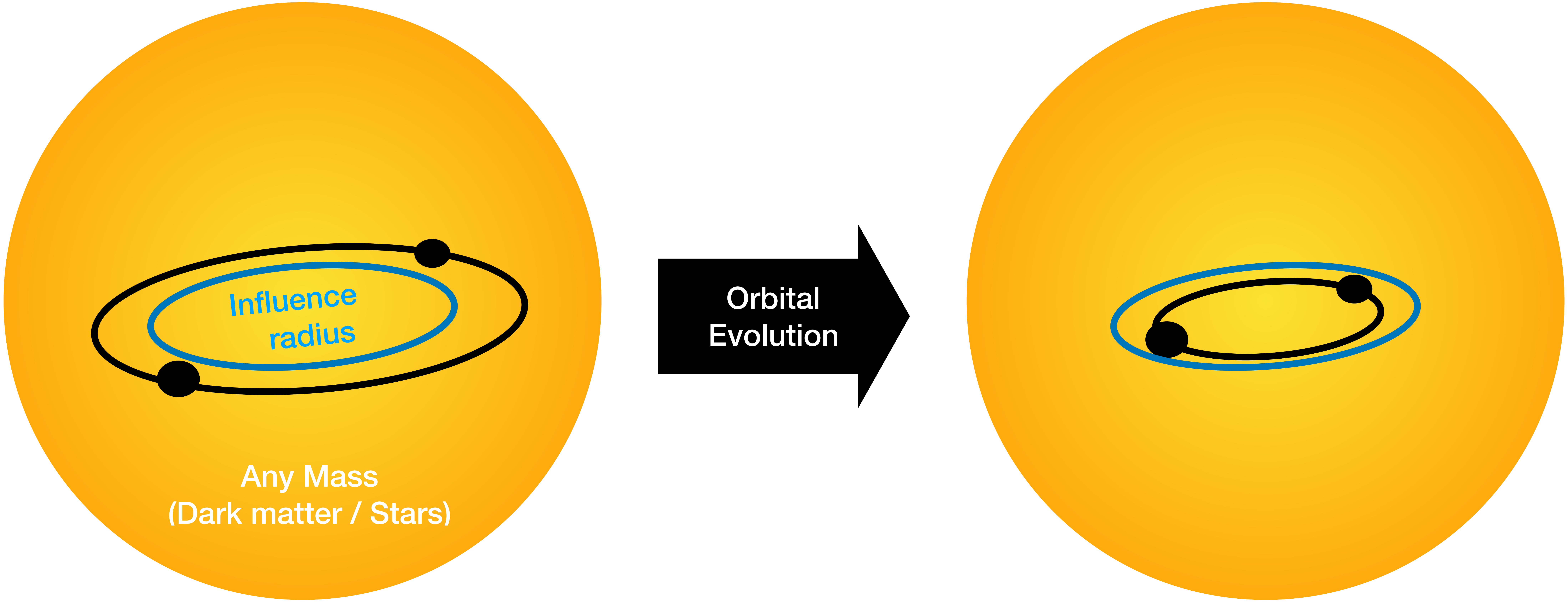
Environmental effect

Galaxy Tomography with the Gravitational Wave Background from Supermassive Black Hole Binaries

Yifan Chen,[1,*] Matthias Daniel,[2] Daniel J. D'Orazio,[1,3] Andrea Mitridate,[4] Laura Sagunski,[2] Xiao Xue,[5,6,4,†] Gabriella Agazie,[7] Jeremy G. Baier,[8] Paul T. Baker,[9] Bence Bécsy,[8] Laura Blecha,[10] Adam Brazier,[11,12] Paul R. Brook,[13] Sarah Burke-Spolaor,[14,15,‡] Rand Burnette,[8] J. Andrew Casey-Clyde,[16] Maria Charisi,[17] Shami Chatterjee,[11] Tyler Cohen,[18] James M. Cordes,[11] Neil J. Cornish,[19] Fronefield Crawford,[20] H. Thankful Cromartie,[21] Megan E. DeCesar,[22] Paul B. Demorest,[23] Heling Deng,[8] Lankeswar Dey,[14,15] Timothy Dolch,[24,25] Elizabeth C. Ferrara,[26,27,28] William Fiore,[14,15] Emmanuel Fonseca,[14,15] Gabriel E. Freedman,[7] Emiko C. Gardiner,[29] Kyle A. Gersbach,[17] Joseph Glaser,[14,15] Deborah C. Good,[30] Kayhan Gültekin,[31] Jeffrey S. Hazboun,[8] Ross J. Jennings,[14,15,§] Aaron D. Johnson,[7,32] David L. Kaplan,[7] Luke Zoltan Kelley,[29] Joey S. Key,[33] Nima Laal,[8] Michael T. Lam,[34,35,36] William G. Lamb,[17] Bjorn Larsen,[37] T. Joseph W. Lazio,[38] Natalia Lewandowska,[39] Tingting Liu,[14,15] Jing Luo,[40,¶] Ryan S. Lynch,[41] Chung-Pei Ma,[29,42] Dustin R. Madison,[43] Alexander McEwen,[7] James W. McKee,[44] Maura A. McLaughlin,[14,15] Patrick M. Meyers,[32] Chiara M. F. Mingarelli,[37] David J. Nice,[45] Stella Koch Ocker,[32,46] Ken D. Olum,[47] Timothy T. Pennucci,[48] Polina Petrov,[17] Nihan S. Pol,[49] Henri A. Radovan,[50] Scott M. Ransom,[51] Paul S. Ray,[52] Joseph D. Romano,[49] Jessie C. Runnoe,[17] Alexander Saffer,[51,§] Shashwat C. Sardesai,[7] Kai Schmitz,[53] Xavier Siemens,[8,7] Joseph Simon,[54,**] Magdalena S. Siwek,[55] Sophia V. Sosa Fiscella,[35,36] Ingrid H. Stairs,[56] Daniel R. Stinebring,[57] Abhimanyu Susobhanan,[58] Joseph K. Swiggum,[45,§] Jacob Taylor,[8] Stephen R. Taylor,[17] Jacob E. Turner,[41] Caner Unal,[59,60,61] Michele Vallisneri,[38,32] Rutger van Haasteren,[58] Joris Verbiest,[62] Sarah J. Vigeland,[7] Caitlin A. Witt,[63,64] David Wright,[8] and Olivia Young[35,36]
(The NANOGrav Collaboration)

- **2411.05906**

Environmental effect

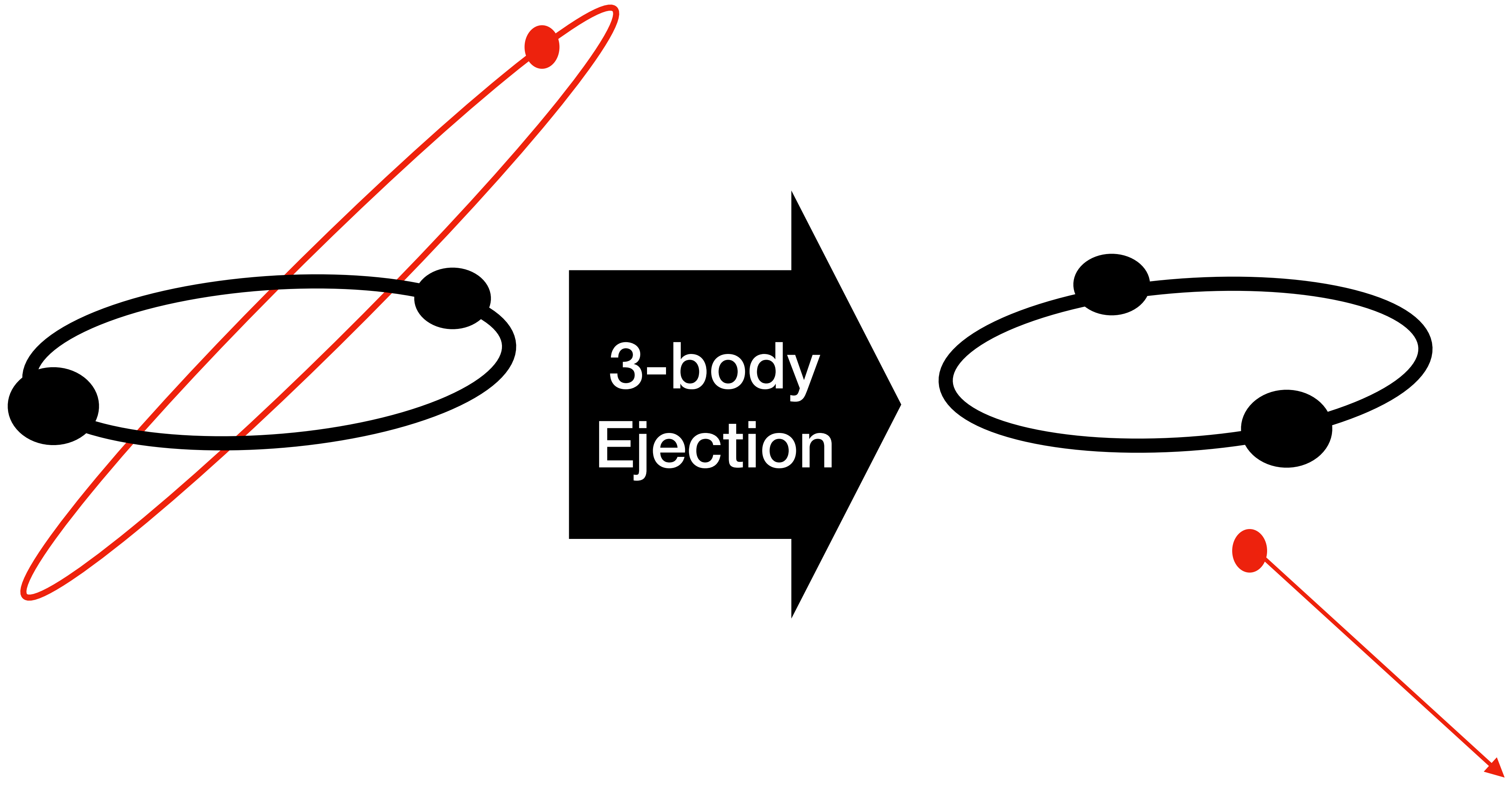


- **Influence radius:** when the **enclosed mass** is larger than BH masses
- **Dynamical Friction** dominates the orbital evolution

- "The BHs become close because distant stars perturb the binary's center of mass **but not its semi-major axis"** (Quinlan 1996)
- Both 3 body ejection (slingshot effect) and GW emission dominate the process

Environmental effect

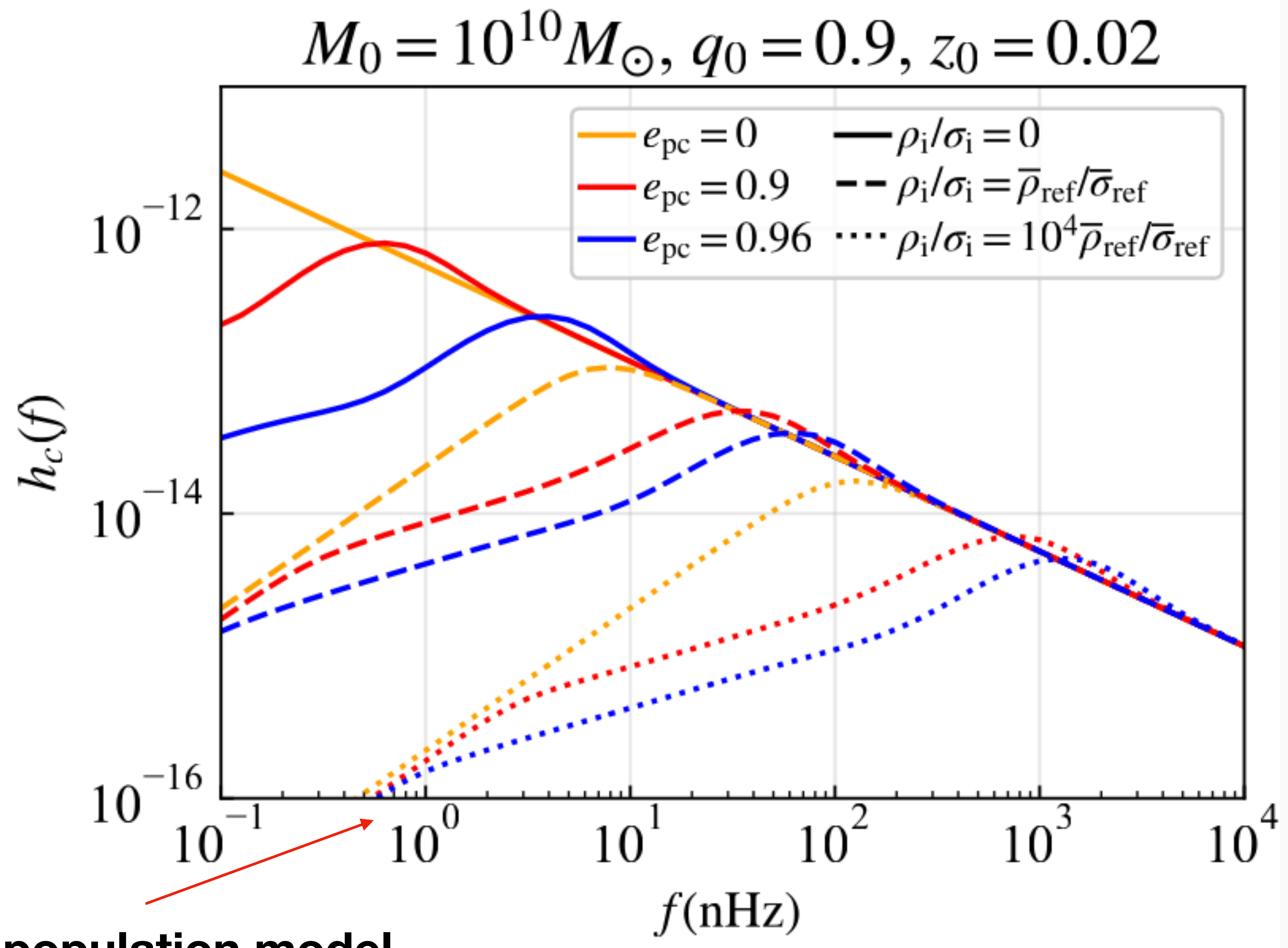
- Evolution of the semi-major axis and the orbital eccentricity
- $$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{\mathrm{d}a_{\mathrm{GW}}}{\mathrm{d}t} - HG\frac{\rho_i}{\sigma_i}a^2$$
- $$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\mathrm{d}e_{\mathrm{GW}}}{\mathrm{d}t} + HK(e,a)G\frac{\rho_i}{\sigma_i}a$$
- ρ_i, σ_i : the mass density and velocity dispersion at the influence radius (**mass profile flattened by 3-body scattering!**)
  H is the determined by numerical experiment
  (see Quinlan 1997)



- It is only sensible to include 3-body ejection for hard binaries (inside the influence radius)

Environmental effect

- First solve $a(t)$ and $e(t)$ from the differential equations
- Energy loss rate for eccentric orbit
- $$\left.\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}f_s}\right|_{f_s=(1+z)f} = \sum_{n=1}^{+\infty} \left.\frac{\mathrm{d}E^n_{\mathrm{GW}}/\mathrm{d}t}{n\mathrm{d}f^n_{\mathrm{orb}}/\mathrm{d}t}\right|_{f^n_{\mathrm{orb}}=(1+z)f/n}$$
- The GW characteristic strain
- $$h_c^2(f) = \frac{4G}{c^2\pi f}\int \mathrm{d}z\mathrm{d}M\mathrm{d}q \frac{\mathrm{d}^3\eta}{\mathrm{d}z\mathrm{d}M\mathrm{d}q}\left.\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}f_s}\right|_{f_s=(1+z)f}$$



Assume that the population model is a delta function

Environmental effect

- $$h_c^2(f) = \frac{4G}{c^2\pi f}\int \mathrm{d}z\mathrm{d}M\mathrm{d}q \frac{\mathrm{d}^3\eta}{\mathrm{d}z\mathrm{d}M\mathrm{d}q}\frac{\mathrm{d}E_{\mathrm{GW}}}{\mathrm{d}f_s}\Big|_{f_s=(1+z)f}$$

- ***Holodeck***

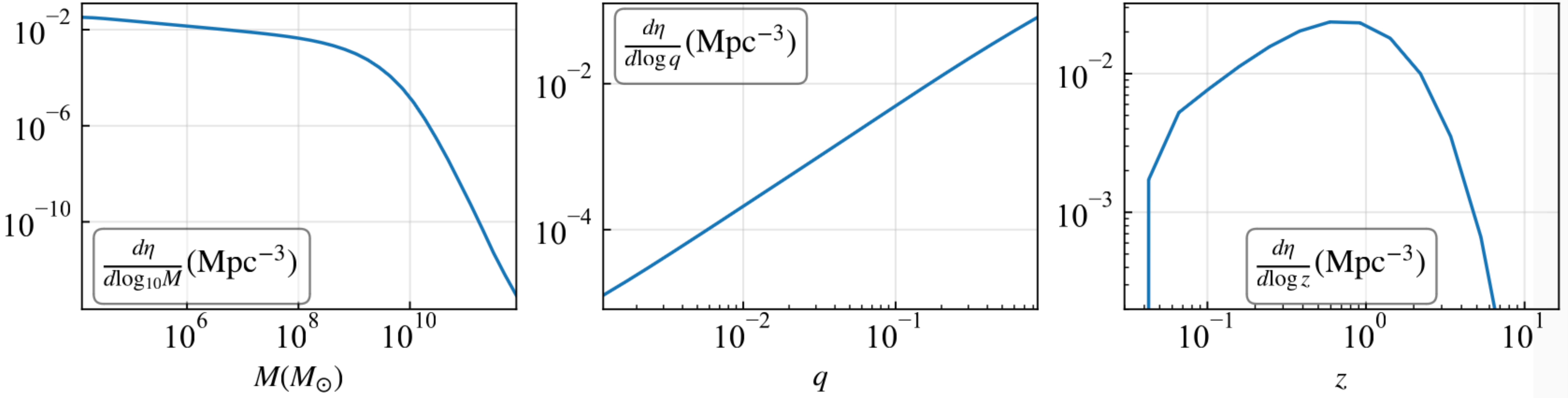
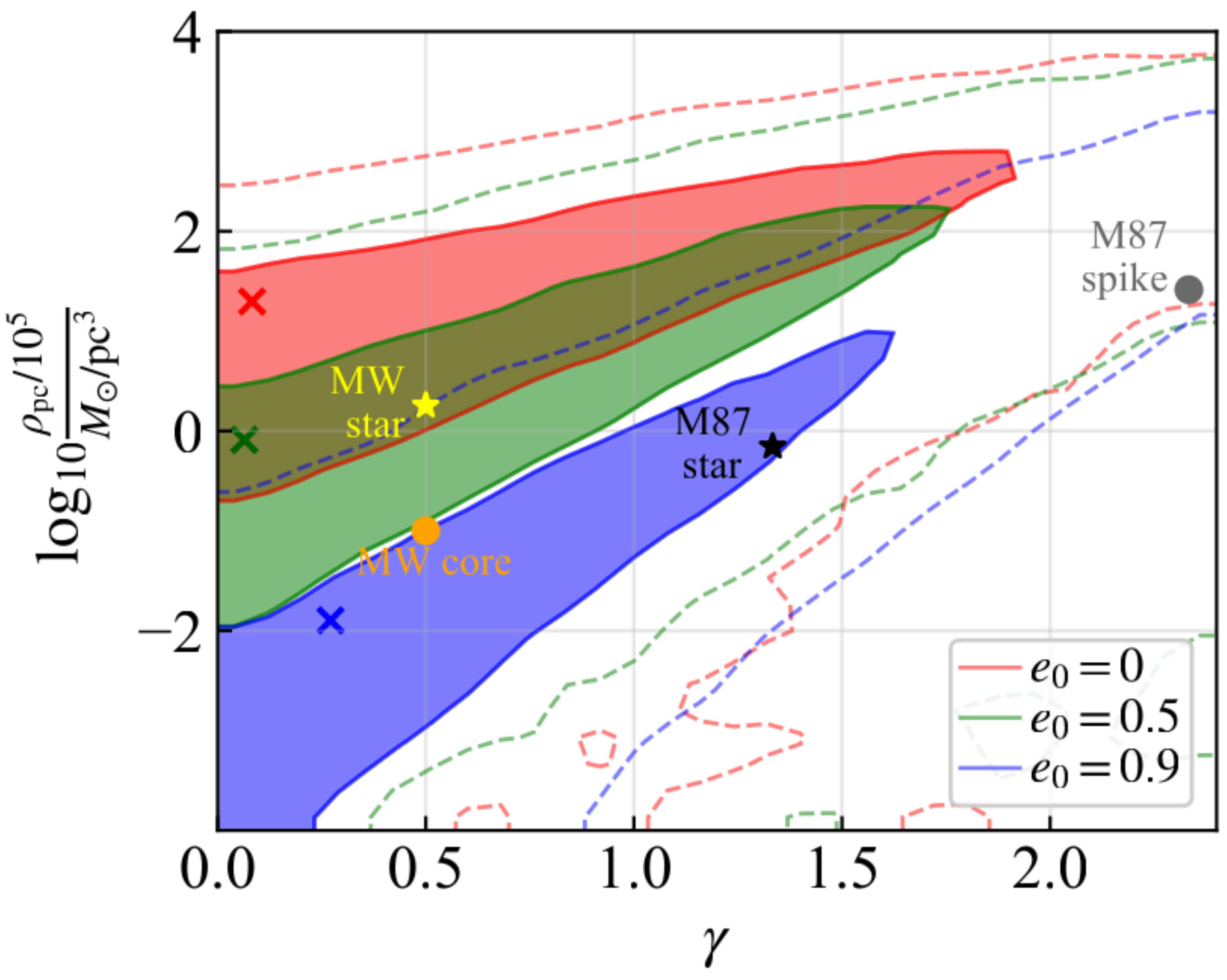
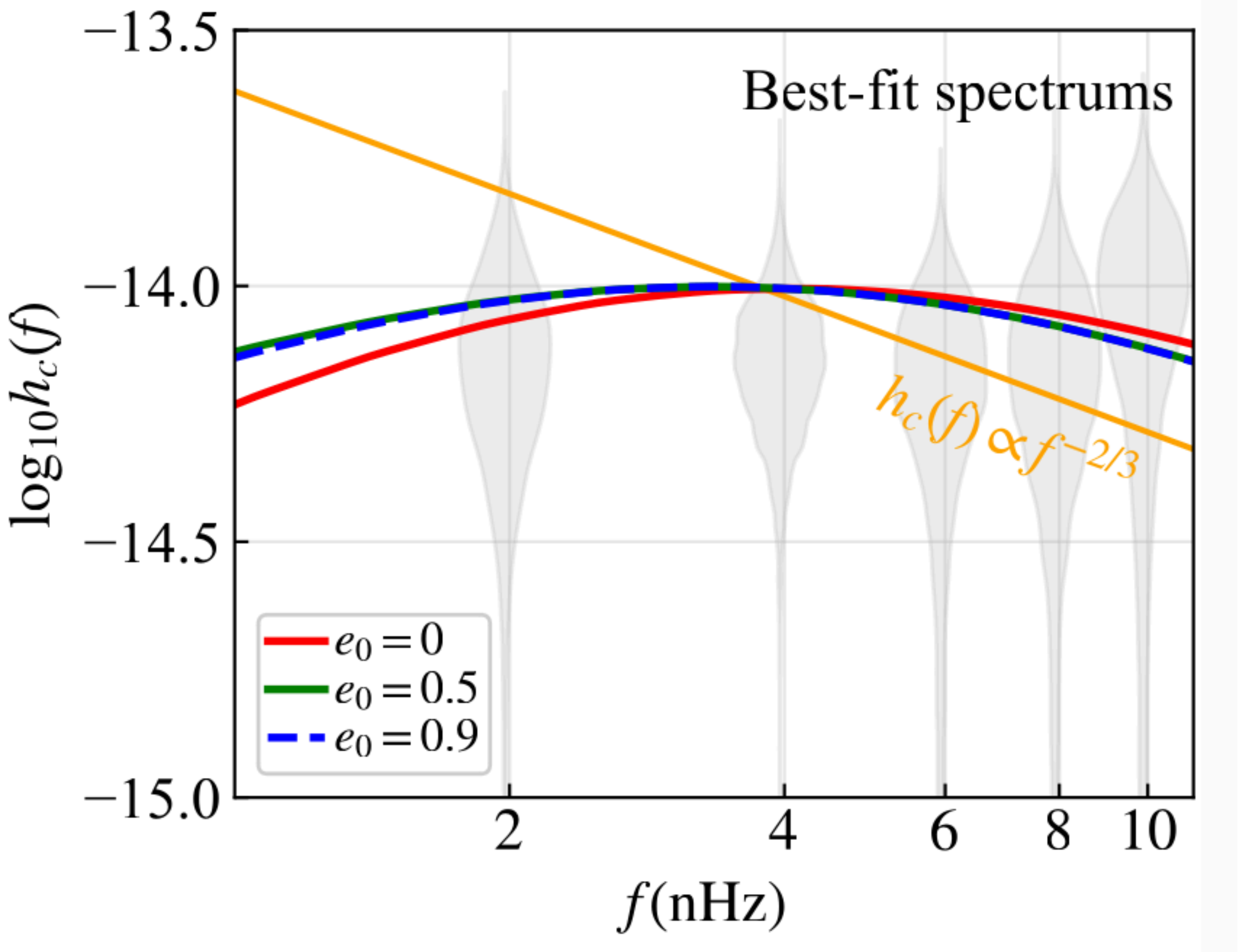


FIG. S1: Comoving volumetric number density of SMBHBs for the fiducial model as a function of total SMBH mass M (left), mass ratio q (middle), and redshift z (right).

Environmental effect





- *PTArcade*
- ***3 free parameters: mass density, slope, initial eccentricity***

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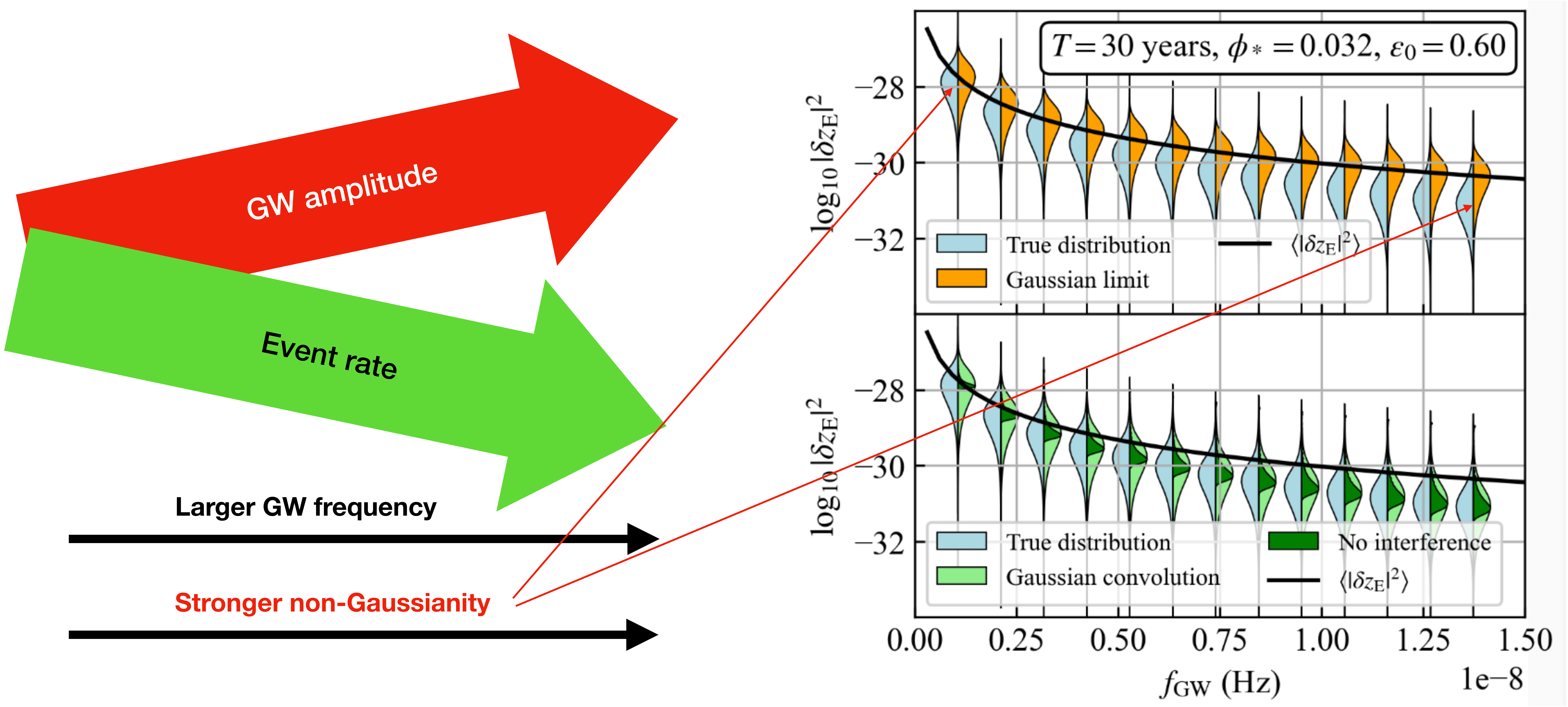
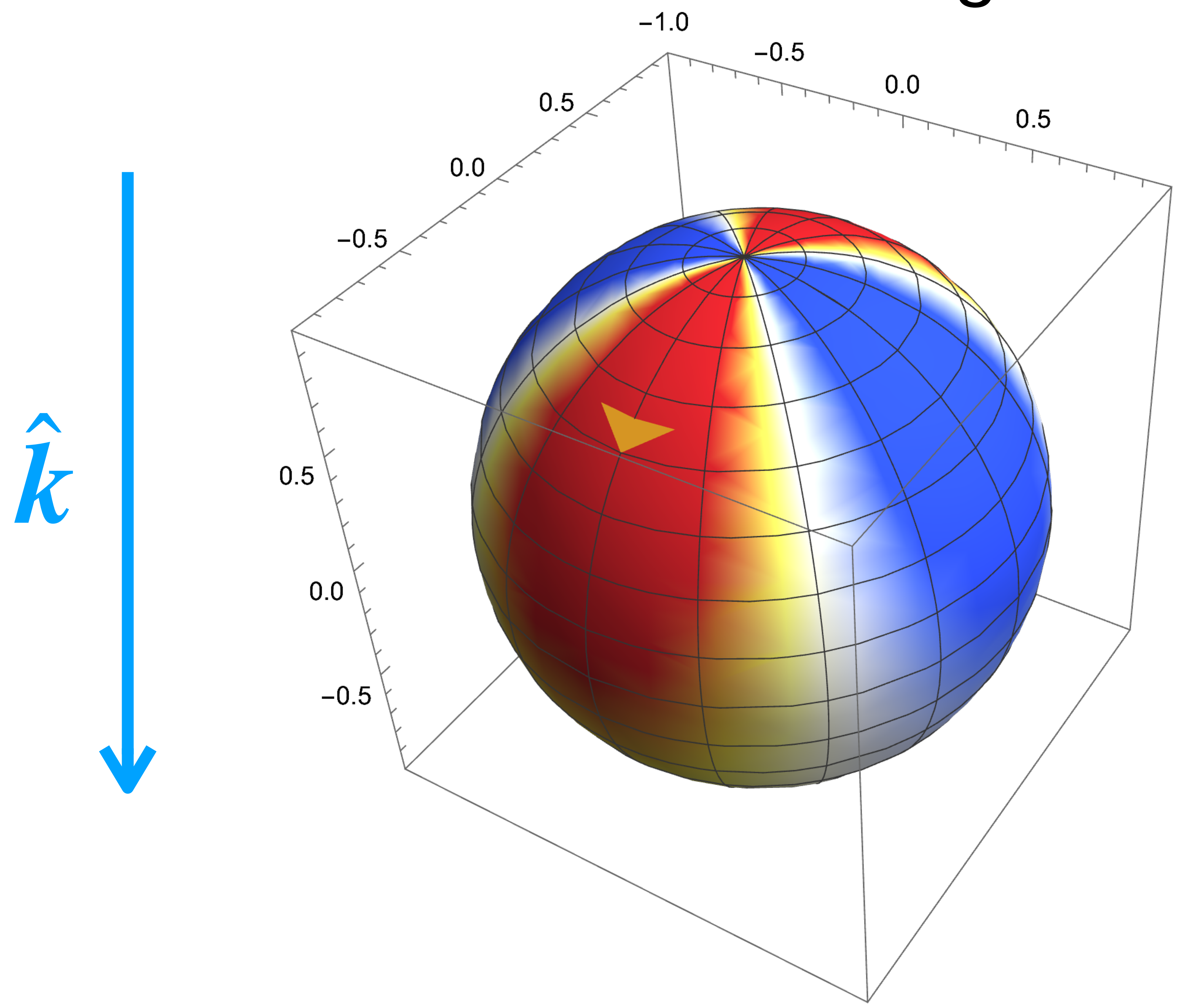
Frequency non-Gaussianity

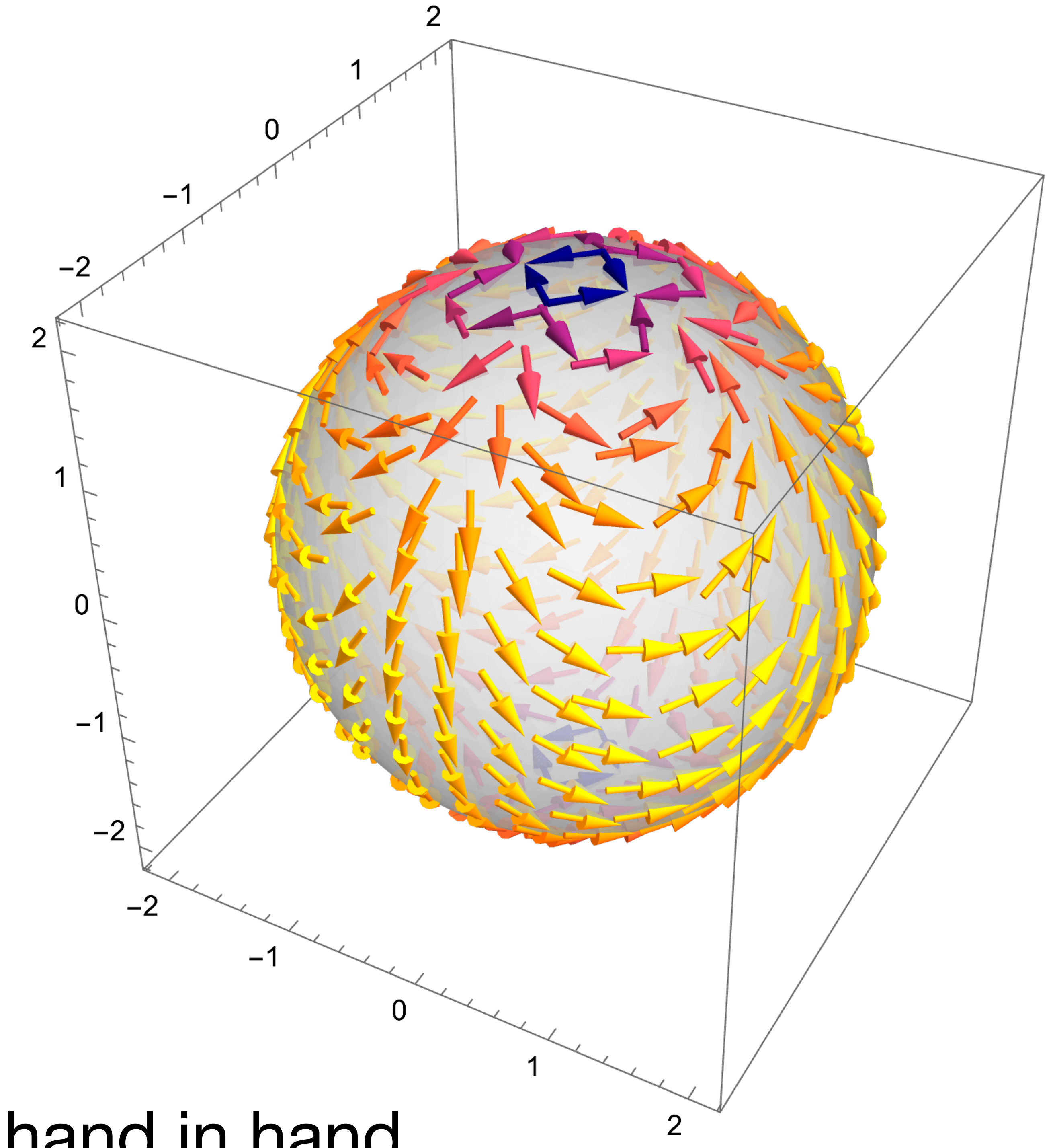


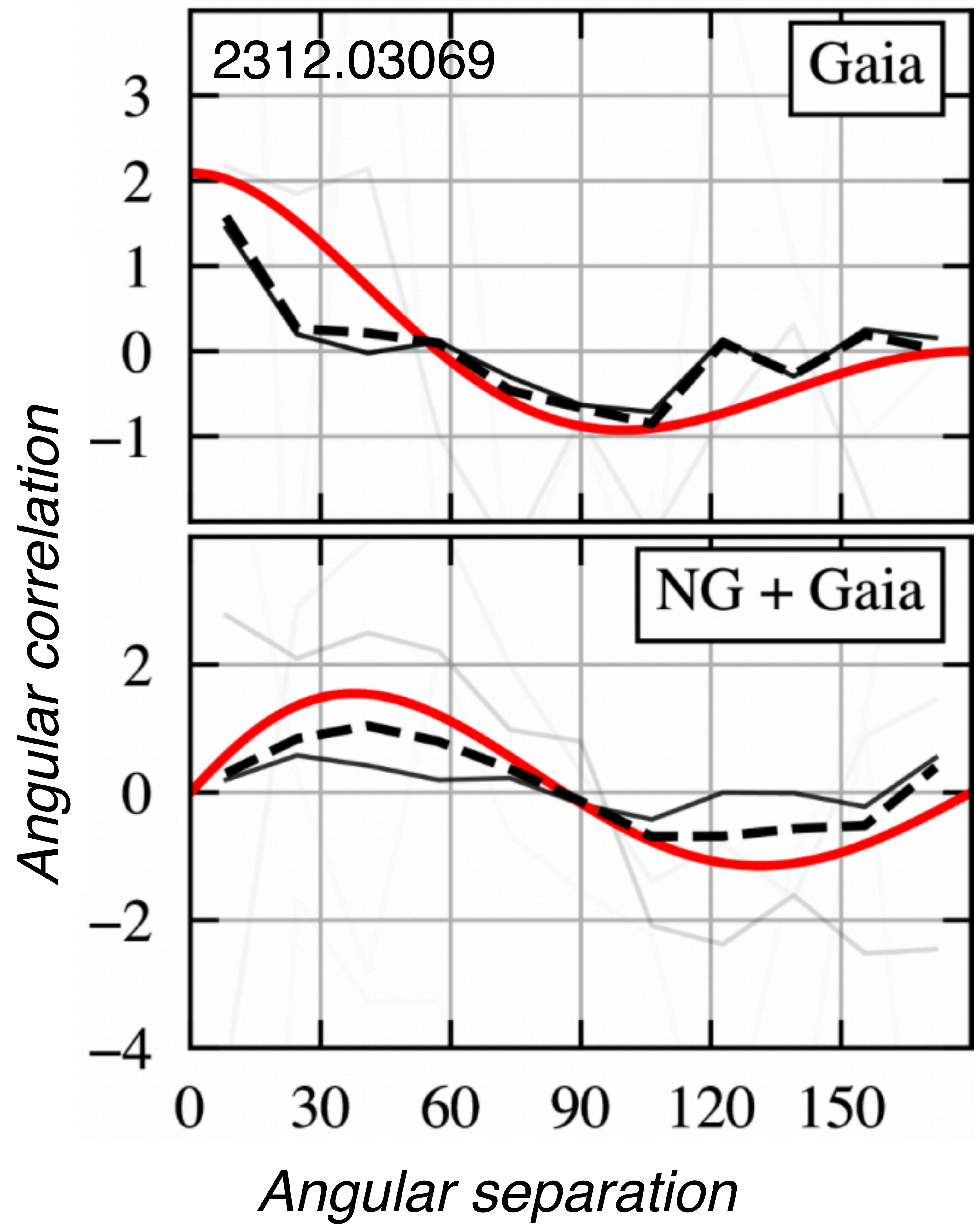
PTA / Astrometry synergy

Pulsar Timing Array
redshift = timing



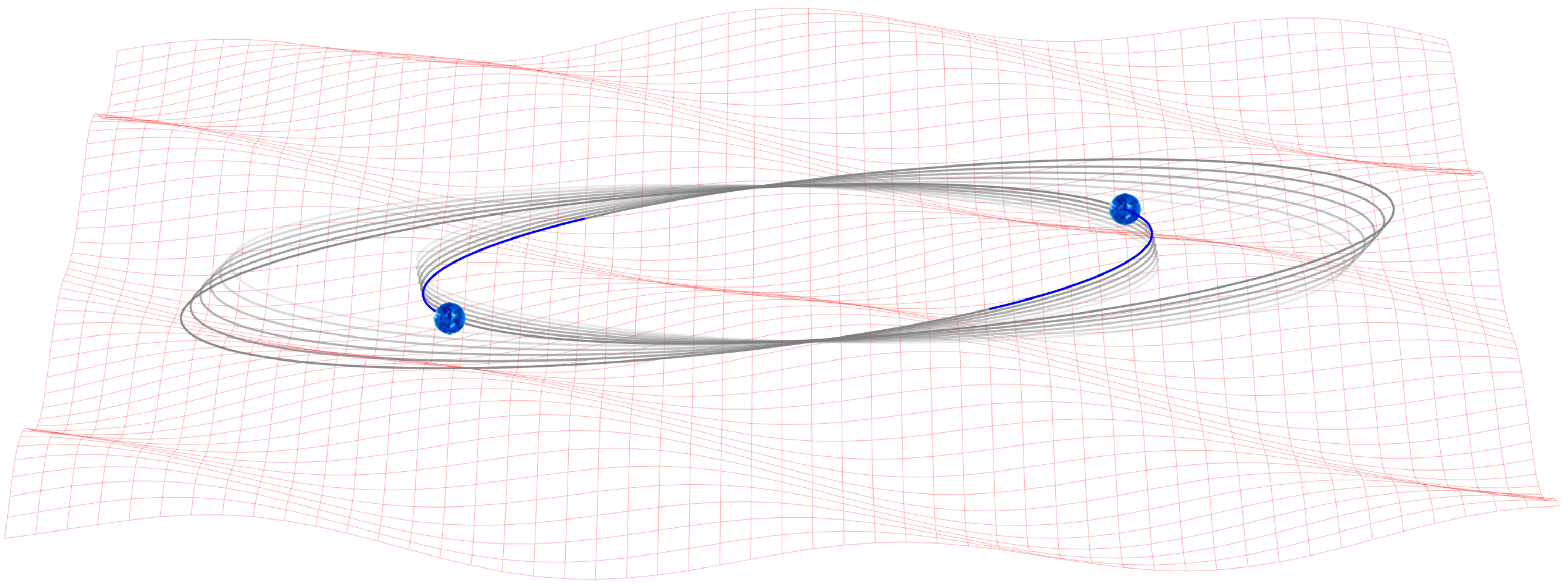
Astrometry
stars' apparent location



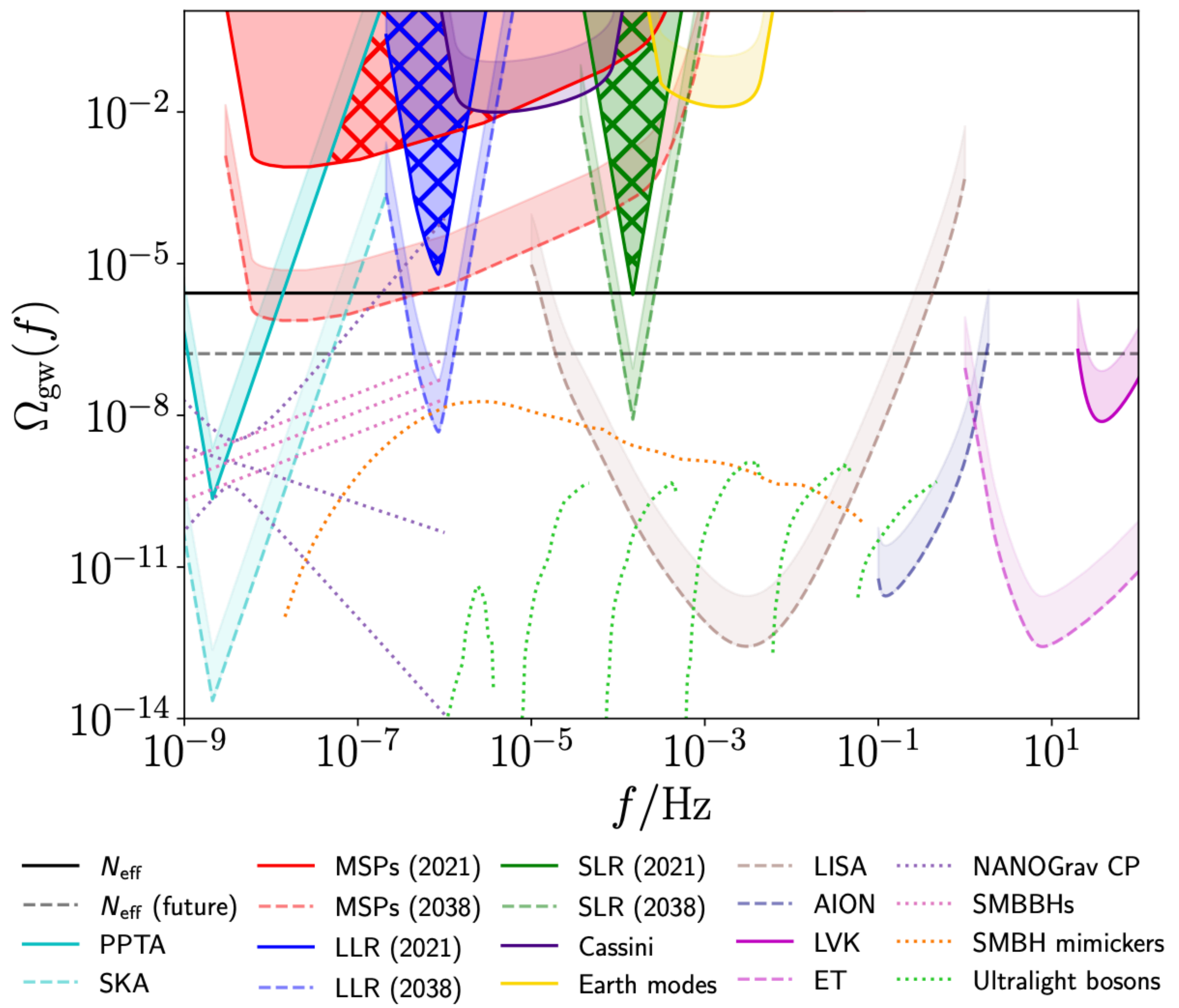


- Redshift and astrometric effects come hand in hand, naturally leads to correlations similar to the **Hellings-Downs** curve.
- Astrometry is more sensitive at higher frequencies $> 10^{-6}\,\mathrm{Hz}$

Lunar Laser Ranging



Diego Blas and Alex C. Jenkins, PRL and PRD,



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