Institute of Space Sciences
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ICREA

A methodology for light curve comparison: applications to pulsars and transitional pulsars

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in collaboration with C. Rodriguez

Variety of Pulsar Light curves in the 3PC

García & Torres, ApJ Letters 2025, https://arxiv.org/abs/2503.02750

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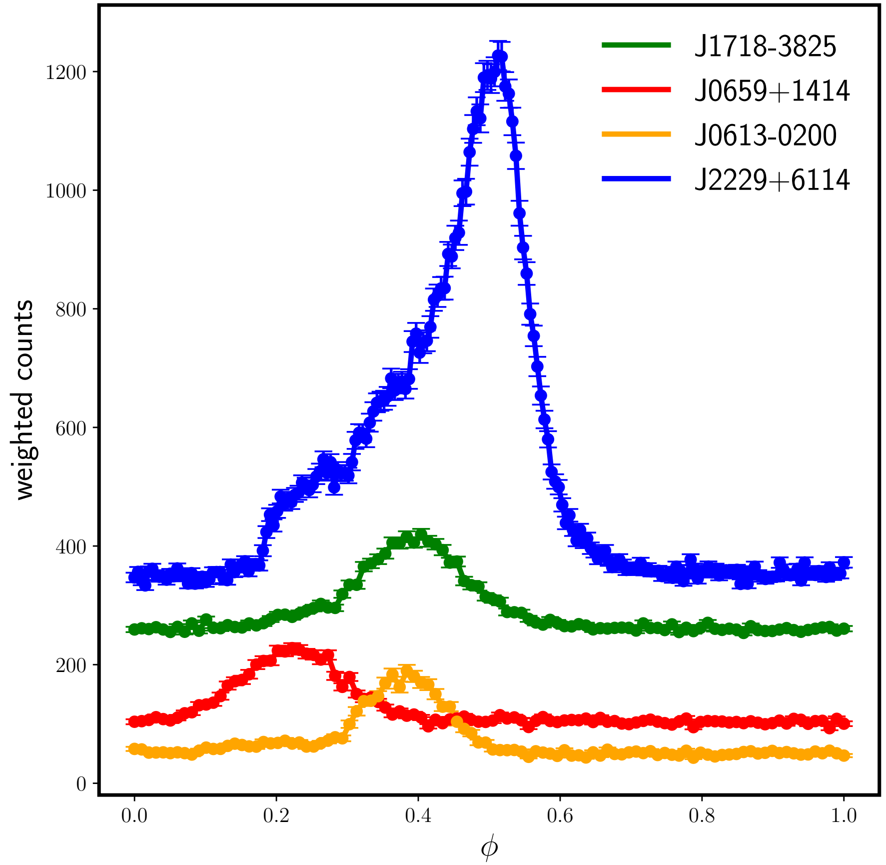


An obvious exercise…

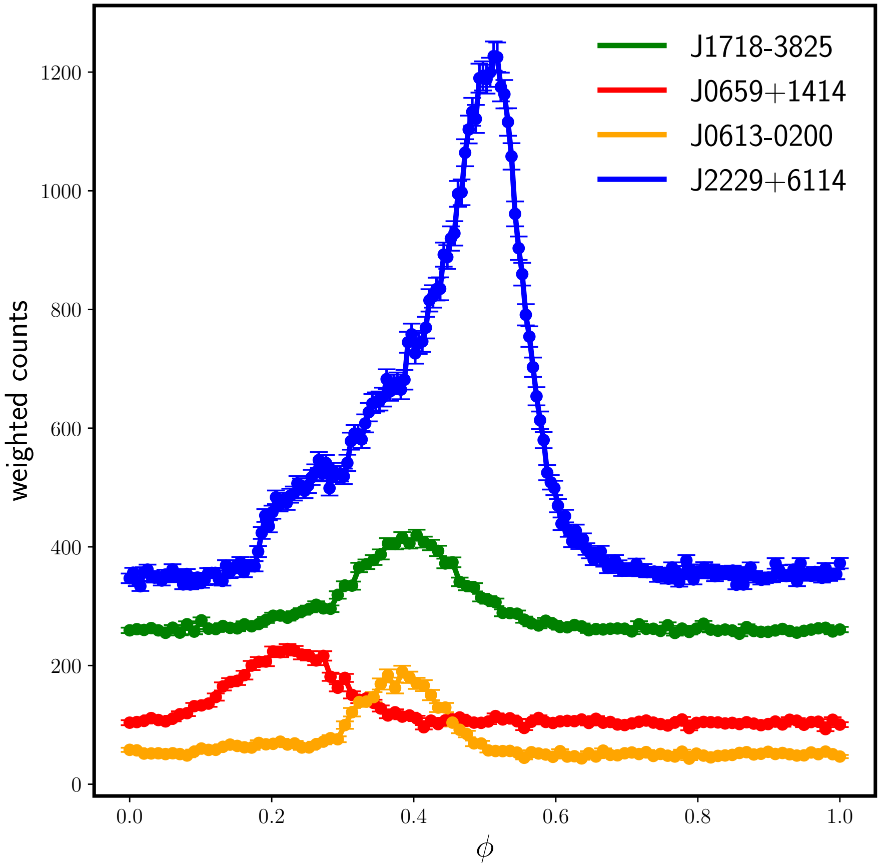
Read light curves directly from the Fermi-LAT 3PC Catalog

See if they look similar

- Very different in flux level and shape, at first look
- even if considering those with the same number of peaks
- (When looked in detail, a clear definition of what is peak is not at all obvious)

Variety of Pulsar Light curves in the 3PC

García & Torres, ApJ Letters 2025, https://arxiv.org/abs/2503.02750



3PC light curves examples, obtained directly from the 3PC catalog

- Very different in flux level and shape, at first look
- even if considering those with the same number of peaks

Consider the transformation

Counts → (Counts – Background) / (Max-Background)

And consider all light curves rotations in phase

Based on:

- Phase is arbirtray, rotate them until 'best alignment'
- map all light curves in a range from 0 to 1 in 'counts'
- emphasizes morphological differences

A realization: morphological similarity underlies

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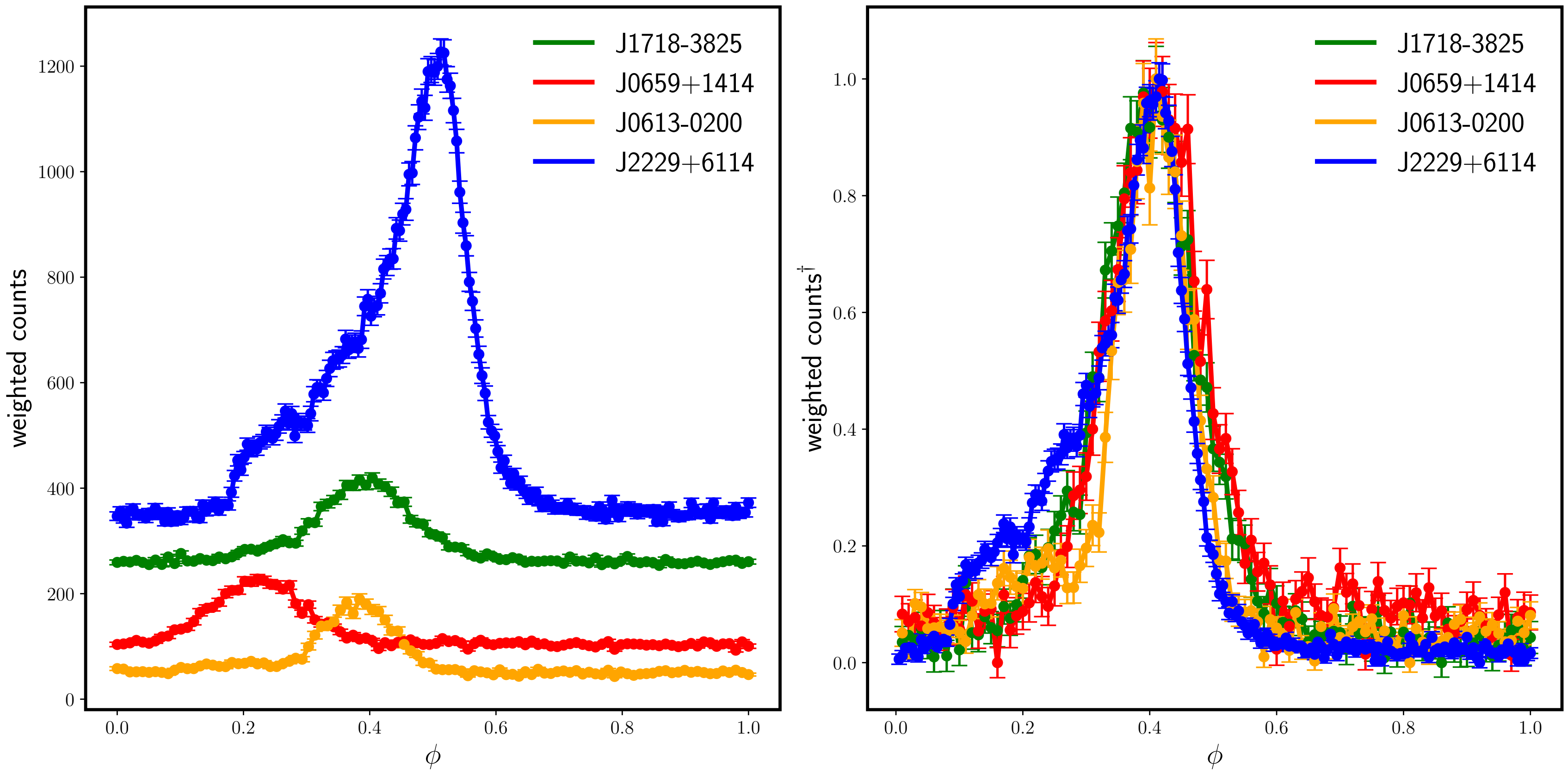

Not always, of course, there is also dissimilarity

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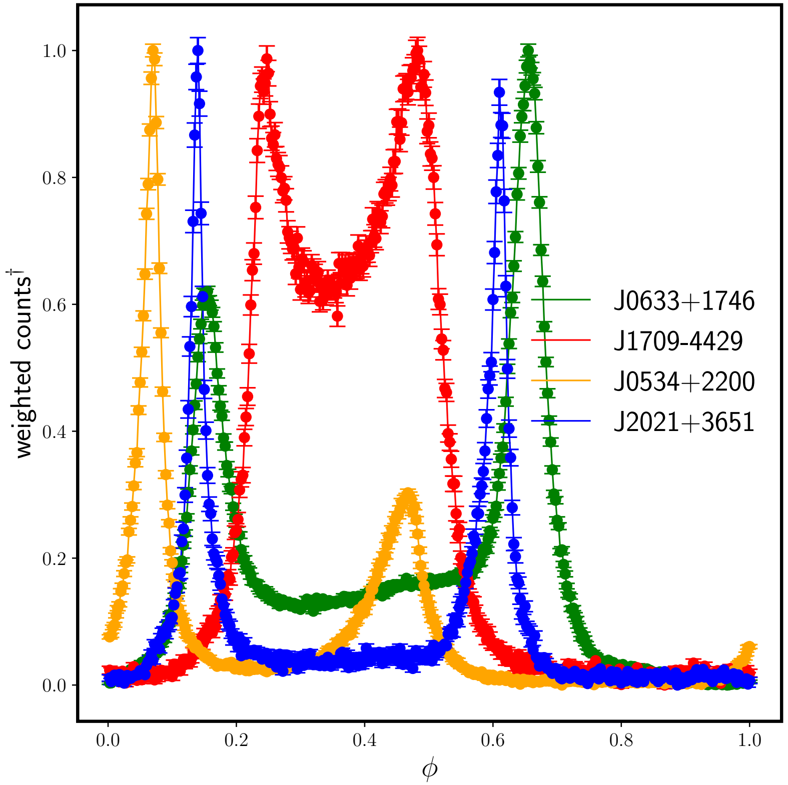
There might be *different*

- number of peaks
- peak separation
- relative height
- peak width



So, do LCs cluster?

How can we measure similarity of the LCs?

Let's do a simple systematic comparison using Euclidean distance btw two time series: this is an example

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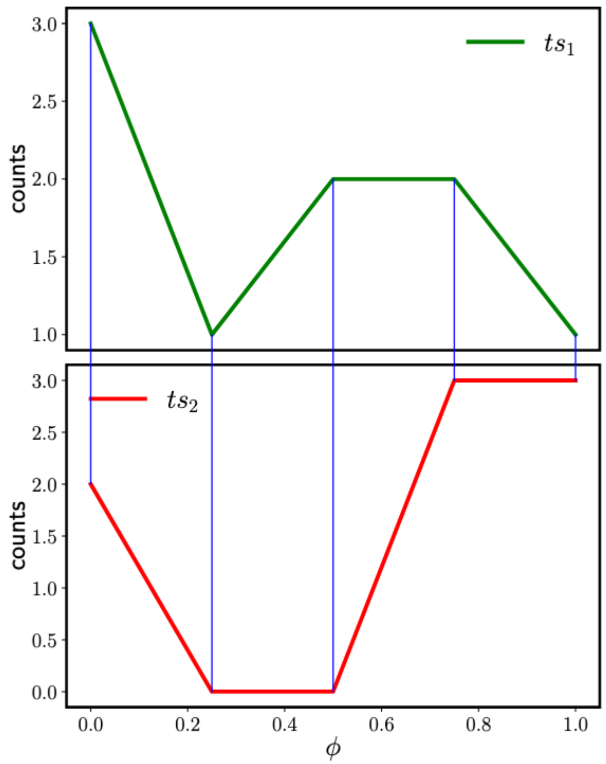


$$ts_3 = \quad 3, 1, 2, 2, 1$$
$$ts_4 = \quad 2, 0, 0, 3, 3$$

$$E_d = \begin{bmatrix} 1 & 9 & 9 & 0 & 0 \\ 1 & 1 & 1 & 4 & 4 \\ 0 & 4 & 4 & 1 & 1 \\ 0 & 4 & 4 & 1 & 1 \\ 1 & 1 & 1 & 4 & 4 \end{bmatrix},$$

The ED matrix (E) between the ts_1 and ts_2

and

$$D = \begin{bmatrix} 1 & 10 & 19 & 19 & 19 \\ 2 & 2 & 3 & 7 & 11 \\ 2 & 6 & 6 & 4 & 5 \\ 2 & 6 & 10 & 5 & 5 \\ 3 & 3 & 4 & 8 & 9 \end{bmatrix}$$

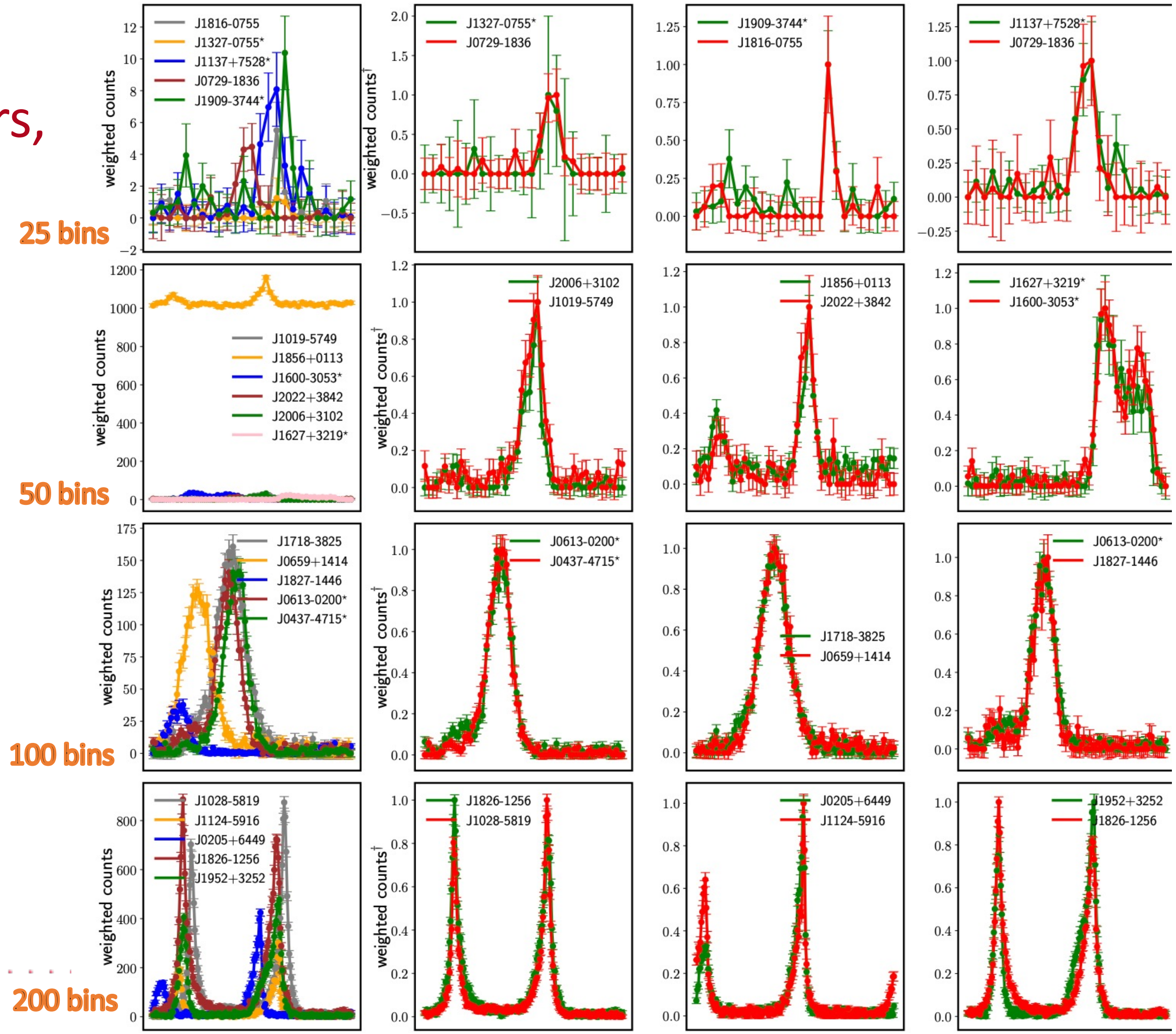
The cumulative cost matrix through:

$$D(i,j) = E(i,j) + min\big(D(i-1,j), D(i,j-1), D(i-1,j-1)\big).$$

Consider Euclidean distance from each pulsar to all others, including all rotations, and rank them

Arising similarities are all over the sample

- Significant matching in many cases
- **But limited …**
  - **to compare pulsars having the same number of bins** in the light curve, or subjet to a rebinning process to make them so, with the subsequent loss of information
  - **similar LC which are not precisely aligned** (phase separation slightly different) **are not singled out as being similar**



Dynamic time warping: concept

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- **DTW** is an optimization method used to compare time series
  - *trading markets, electrocardiograms, etc., etc.*

- It works by **dynamically aligning** time series, even with **different sizes**.

- The **Euclidean distance** (ED) is used once such dynamic alignments (or paths) are established

- The goal is the optimization of the cost to connect the two light curves,
  - the path with the minimum cost is labelled as *the optimal warping path*
  - and its cost (the total distance you need to connect it) will be *DTW value*.

Dynamic time warping: math conditions

For this process to be effective we ask for

- **Boundary condition**.
  - The initial and final elements of the time series must face each other
  - (but recall, all rotations are considered)
- **Monotonicity / continuity condition**.
  - No elements of the series that break the temporal order of the series can be matched against each other.
    - Don't jump back in time
  - The elements of the series being aligned must be adjacent points, not allowing temporal jumps.
    - Don't skip bins (don't jump forward in time)

- The resolution for this optimal problem can be seen via:

$$DTW(X,Y) = min_{\pi \in A(X,Y)} \Big(\sum_{(i,j)\in\pi} d(x_i, y_j)^2 \Big)^{\frac{1}{2}}$$

$$\begin{cases} X, Y: \ time\ series \\ A: \ set\ of\ paths \\ \pi: \ path \\ d: \ ED \end{cases}$$

Dynamic time warping: simple example

- We define the time series as:

$$ts_1 = 3, 1, 2, 2, 1$$
$$ts_2 = 2, 0, 0, 3, 3, 1, 0.$$

- The ED matrix (E) between the ts_1 and ts_2:

$$E(d(i,j)^2) = \begin{bmatrix} 1 & 9 & 9 & 0 & 0 & 4 & 9 \\ 1 & 1 & 1 & 4 & 4 & 0 & 1 \\ 0 & 4 & 4 & 1 & 1 & 1 & 4 \\ 0 & 4 & 4 & 1 & 1 & 1 & 4 \\ 1 & 1 & 1 & 4 & 4 & 0 & 1 \end{bmatrix}$$

- The cumulative cost matrix through:

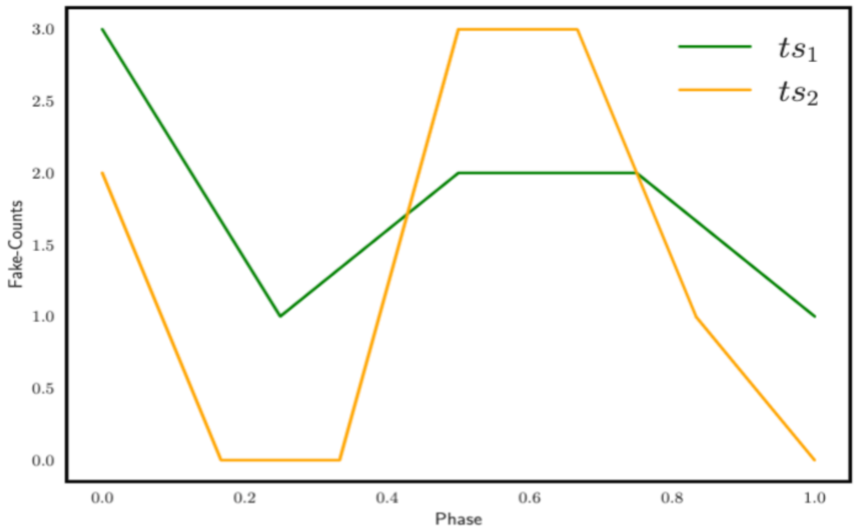
$$D(i,j) = E(i,j) + \min\left(D(i-1,j), D(i,j-1), D(i-1,j-1)\right).$$

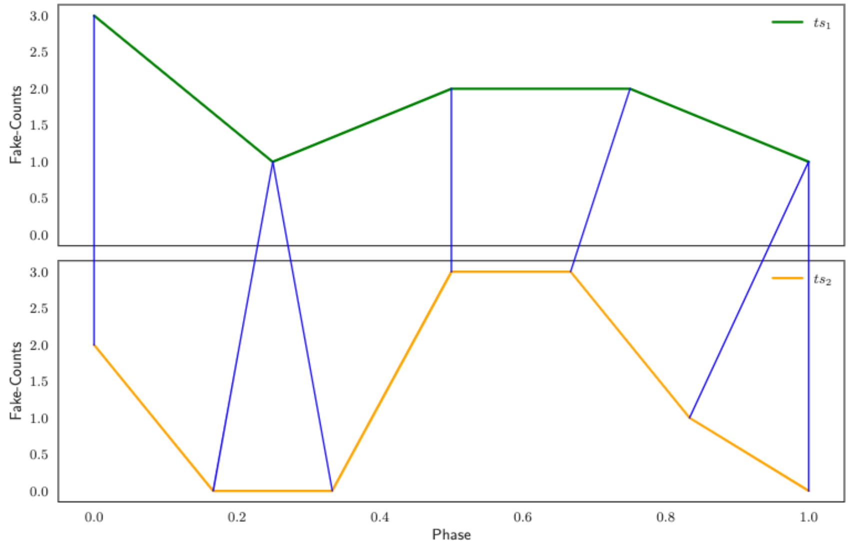
$D =$

- For instance, the value seen in row 2 and column 2, *D(2,2)= E(2,2) + min(D(1,2), D(2,1), D(1,1)) = 1 + min(10,2,1) = 2.*

- The DTW value is the total cost of the blue line, according to the *E*-matrix: $\sqrt{1+1+1+1+1+0+1} = 2.45$

Dynamic time warping: example in a figure

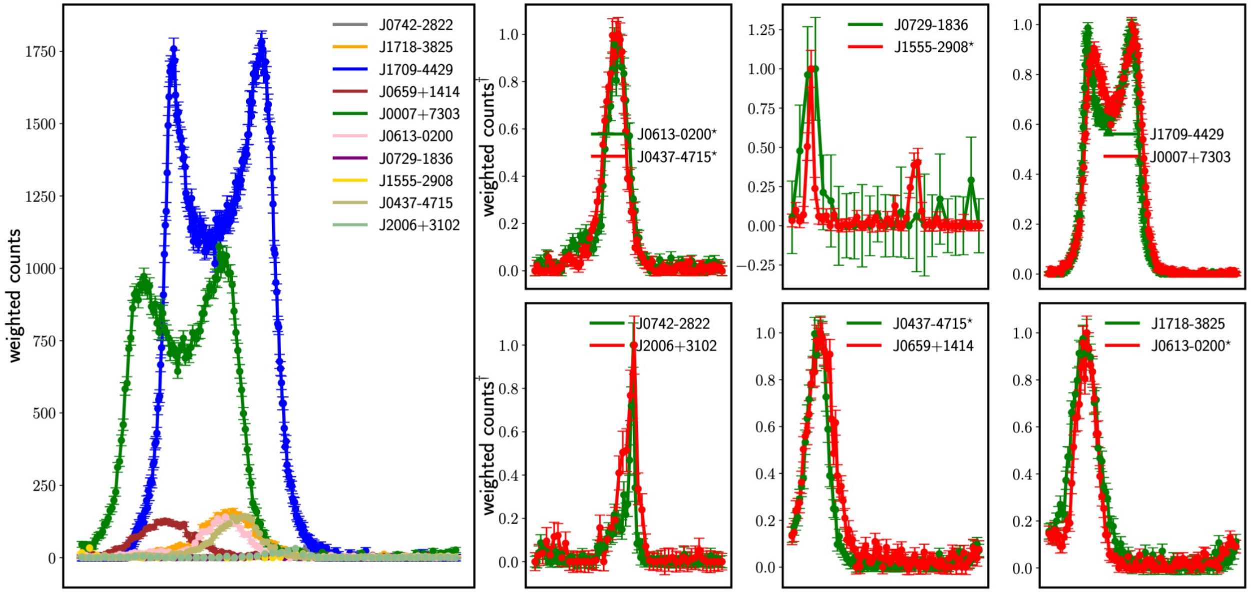




The *optimal warping path* is represented by the blue lines where each one defines a distance.

DTW: A comparison devoid of ED limitations

- A technique useful to find similarities beyond the simple alignment (which is what the Euclidean distance quantifies)
- It associates morphological structures, despite they do not happen at exactly the same phase
- Can be applied to light curves of any number of bins, without rebinning
- Provides a number, that can be used to quantify similarity



A well-behaved similarity estimator

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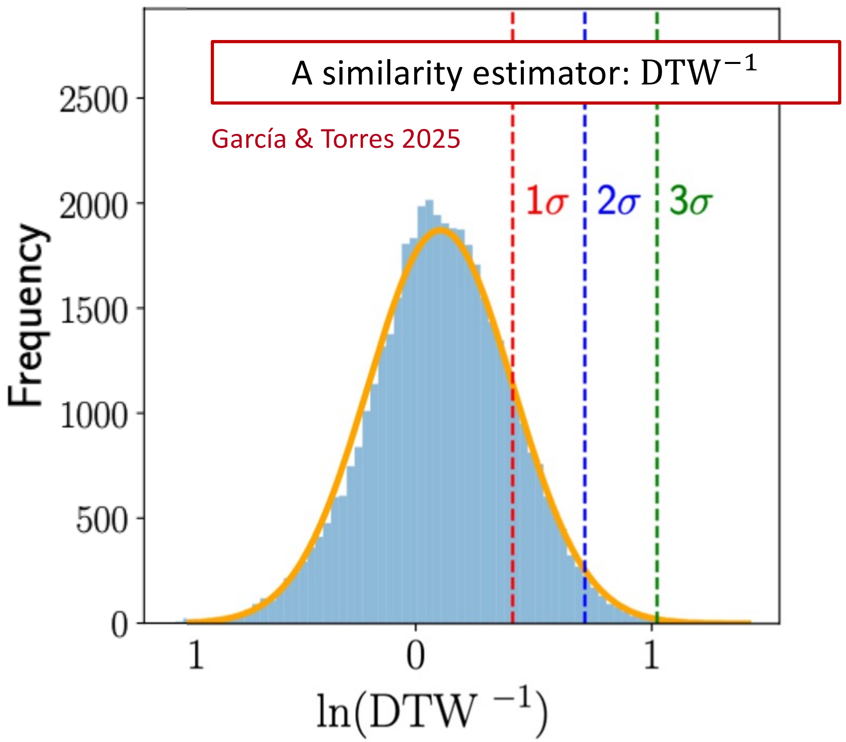
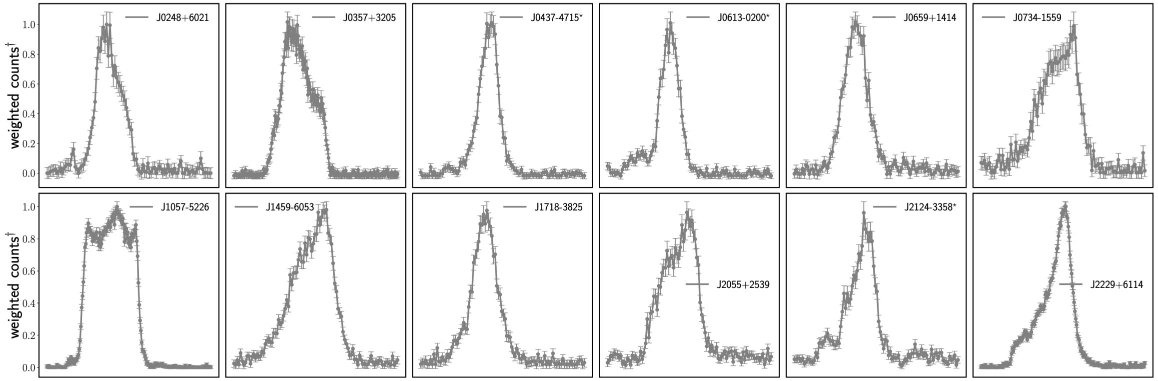
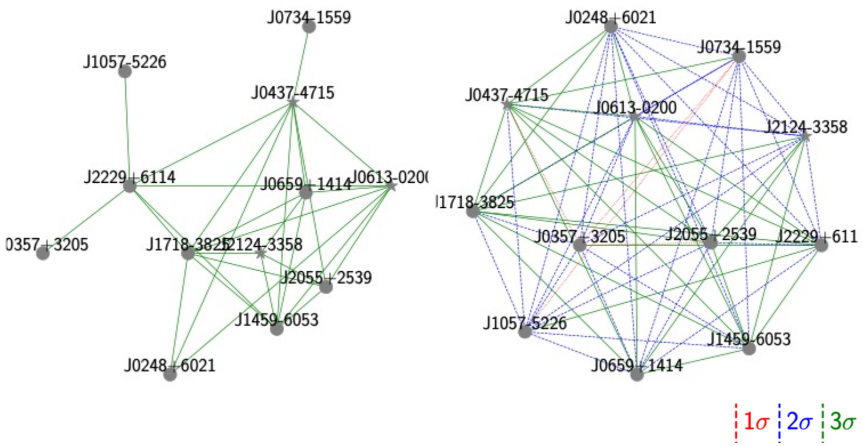
- 294 3PC pulsares
- >43000 LC pulsar comparisons
    - **+** all rotations for each!
- Computing the dynamic distance between all pulsars in the sample, and all its rotations, one obtain its distribution: a global view of how similar/dissimilar pulsar light curves are
- A well behaving quantitative similarity estimator
- Permits to define intervals of confidence for the similarity of light curves

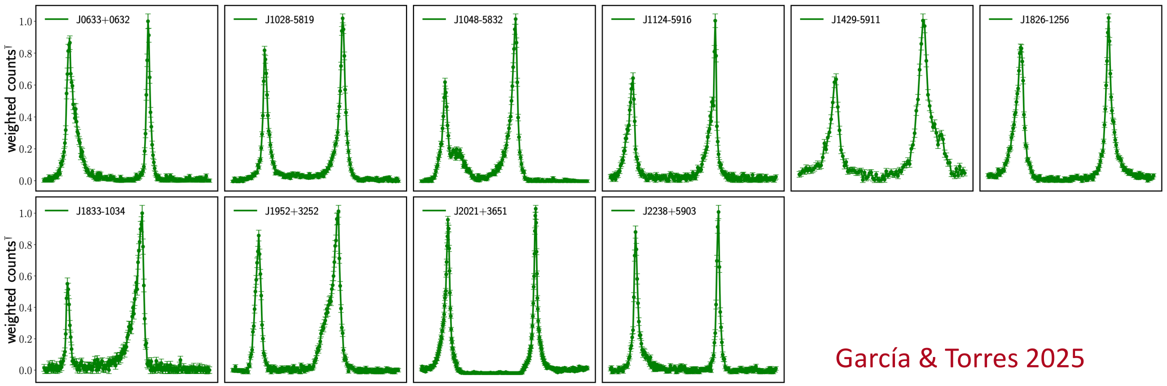


Distribution of the natural logarithm of DTW $^{-1}$ for 43071 pulsar comparisons
The orange line shows the normal distribution.

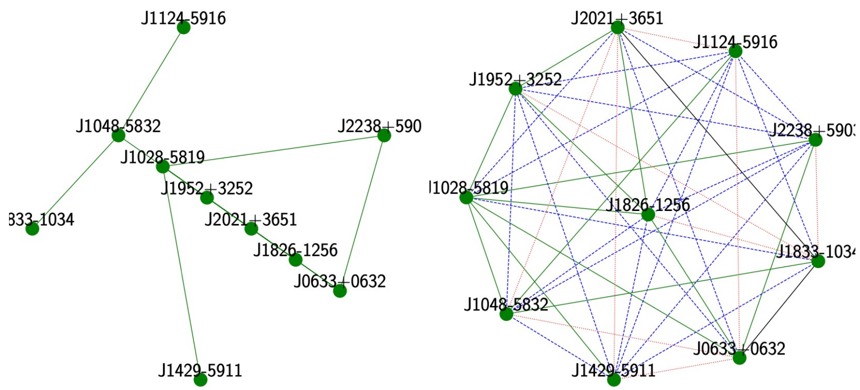
We can cluster the light curves using DTW similarity
(examples)





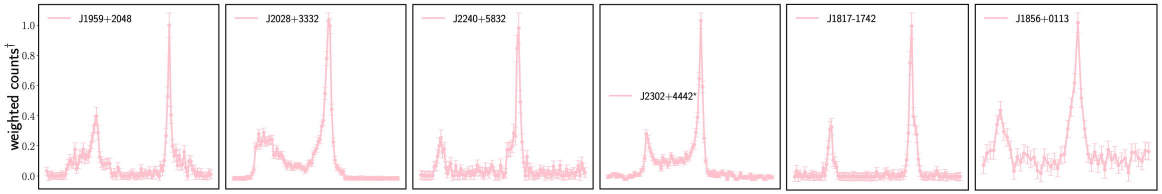


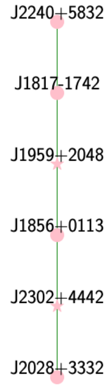
García & Torres 2025

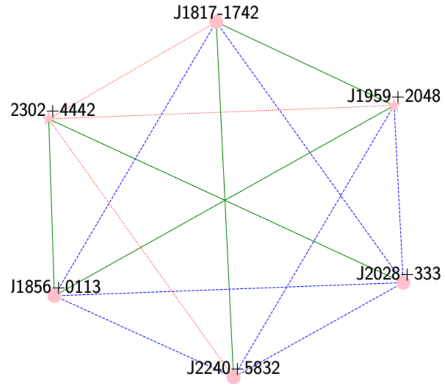


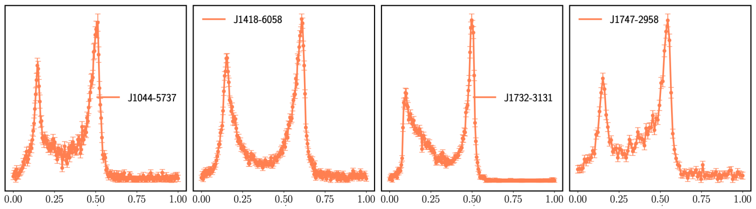
We can now cluster the light curves using similarity

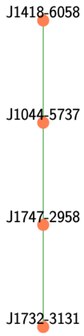
(examples)

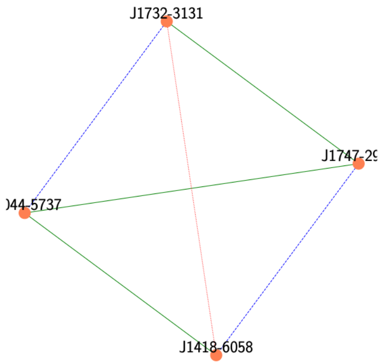












García & Torres, in preparation

Light curves similarity mix all kind of pulsars

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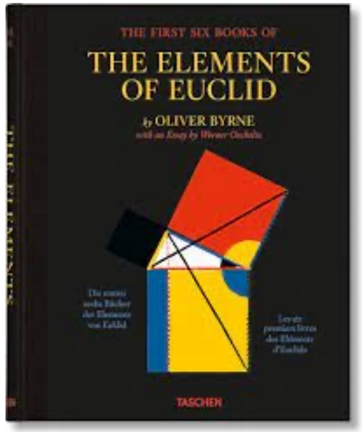
PSRJ	P [s]	$\dot{P}$ [s/s] [$\times10^{-14}$]	$\dot{E}_{sd}$ [erg/s] [$\times10^{35}$]	B_s [G] [$\times10^{10}$]	B_{lc} [G] [$\times10^{4}$]	$F_{E>100\mathrm{MeV}}$ [erg/cm^2/s] [$\times10^{-11}$]	E_p [GeV]
J0248+6021	0.217	5.52	2.13	350.35	0.32	2.94	0.774
J0357+3205	0.444	1.31	0.06	244.02	0.03	6.01	0.869
J0437−4715*	0.005	0.00	0.12	0.06	2.80	1.76	0.700
J0613−0200*	0.003	0.00	0.13	0.02	5.57	3.82	1.156
J0659+1414	0.384	5.50	0.38	465.38	0.08	2.65	0.156
J0734−1559	0.155	1.25	1.32	140.95	0.35	4.57	0.623
J1057−5226	0.197	0.58	0.30	108.54	0.13	29.59	1.213
J1459−6053	0.103	2.53	9.08	163.32	1.37	12.27	0.372
J1718−3825	0.074	1.32	12.49	100.35	2.22	10.37	0.564
J2055+2539	0.319	0.41	0.05	115.85	0.03	5.32	1.215
J2124−3358*	0.004	0.00	0.07	0.03	2.48	3.88	1.949
J2229+6114	0.051	7.53	215.58	199.49	13.33	24.01	0.740
J0633+0632	0.297	7.96	1.19	492.24	0.17	9.56	1.427
J1028−5819	0.091	1.42	7.34	115.28	1.39	24.51	0.888
J1048−5832	0.124	9.55	19.92	347.86	1.69	18.52	0.968
J1124−5916	0.136	75.15	119.15	1021.18	3.78	6.11	0.362
J1429−5911	0.116	2.39	6.06	168.30	1.00	11.28	0.540
J1826−1256	0.110	12.12	35.72	369.86	2.54	41.35	0.870
J1833−1034	0.062	20.20	336.24	357.81	13.89	8.91	0.305
J1952+3252	0.040	0.58	37.23	48.56	7.24	14.77	0.993
J2021+3651	0.104	9.48	33.53	317.40	2.62	49.21	1.064
J2238+5903	0.163	9.69	8.87	401.71	0.86	6.64	0.673
J1817−1742	0.150	2.06	2.42	177.62	0.49	2.85	1.123
J1856+0113	0.267	20.59	4.25	750.91	0.36	-	-
J1959+2048*	0.002	0.00	1.60	0.02	36.73	1.57	1.038
J2028+3332	0.177	0.49	0.35	93.74	0.16	5.71	1.511
J2240+5832	0.140	1.53	2.20	147.82	0.50	0.98	1.138
J2302+4442*	0.005	0.00	0.04	0.03	1.75	3.90	2.355
J0102−4839*	0.003	0.00	0.17	0.02	6.68	1.50	1.700
J0614−3329*	0.003	0.00	0.22	0.02	7.06	11.46	2.835
J1350−6225	0.138	0.89	1.33	112.10	0.39	3.60	1.951
J2032+4127	0.143	1.16	1.56	130.56	0.41	14.24	2.606
J0835−4510	0.089	12.23	67.64	334.51	4.31	929.32	1.267
J1023−5746	0.111	37.99	108.21	658.53	4.37	14.60	0.621
J1809−2332	0.147	3.44	4.29	227.33	0.66	42.48	1.285
J1813−1246	0.048	1.76	62.39	92.96	7.70	24.78	0.483
J1954+2836	0.093	2.12	10.48	141.71	1.64	10.70	1.172
J1044−5737	0.139	5.46	8.02	278.71	0.95	11.40	0.650
J1418−6058	0.111	17.10	49.92	440.00	3.00	29.70	1.001
J1732−3131	0.197	2.80	1.46	237.56	0.29	17.93	2.089
J1747−2958	0.099	6.11	24.98	248.61	2.37	15.91	0.526

-(Very) Different timing properties (young, MSPs)

-(Very) Different timing-derived properties such as spin-down power, magnetic field at the surface or the light cylinder

-Different spectral properties

Geometry Rules



In general, there is no correlation between light curve similarity and similarity in any other magnitude.

Results relate to understanding pulsar similarity

How linked is the whole pulsar phenomenology we see?

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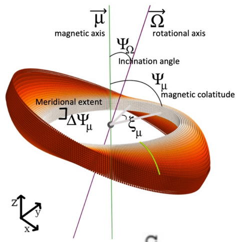
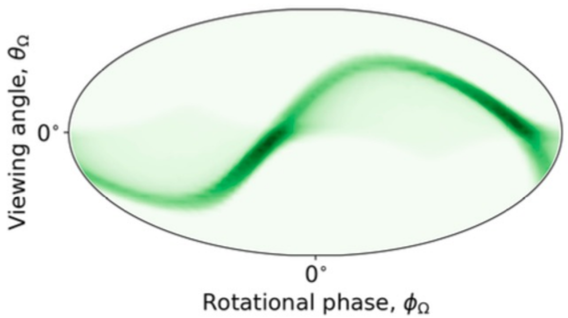
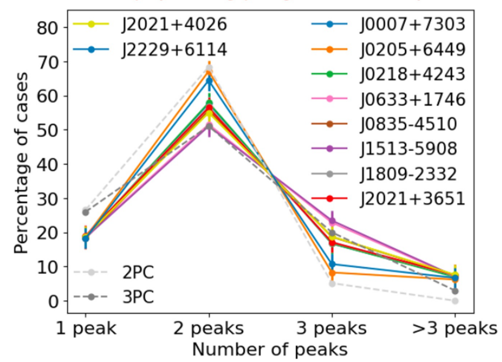


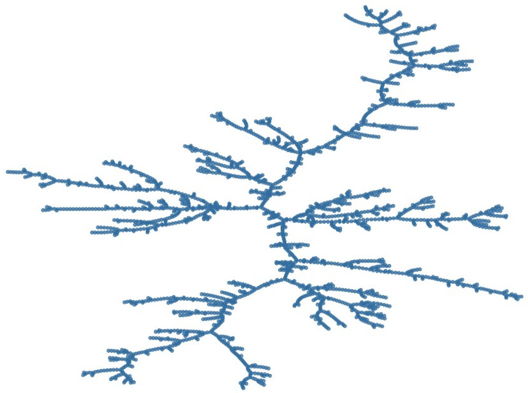
D. Íniguez-Pascual, D. F. Torres. D. Viganò, 2024, Synchro-curvature description of γ-ray light curves and spectra of pulsars: global properties, MNRAS 2024, concurrent fitting MNRAS2025 (submitted)

Daniel Iñiguez, D Vigano – on light curve theoretical modelling, and how pulsar similarity is theoretically understandable

Carlos García – on graph theory methodologies applied to the pulsar population, and the limitations of the P Pdot diagram







C. García, D. F. Torres, Quantitative determination of MST structures: analysing the appearance of new classes of pulsars, MNRAS 2023

C. García, G. Illano, D. F. Torres, et al. Millisecond pulsars phenomenology under the light of graph theory, A&A 2024

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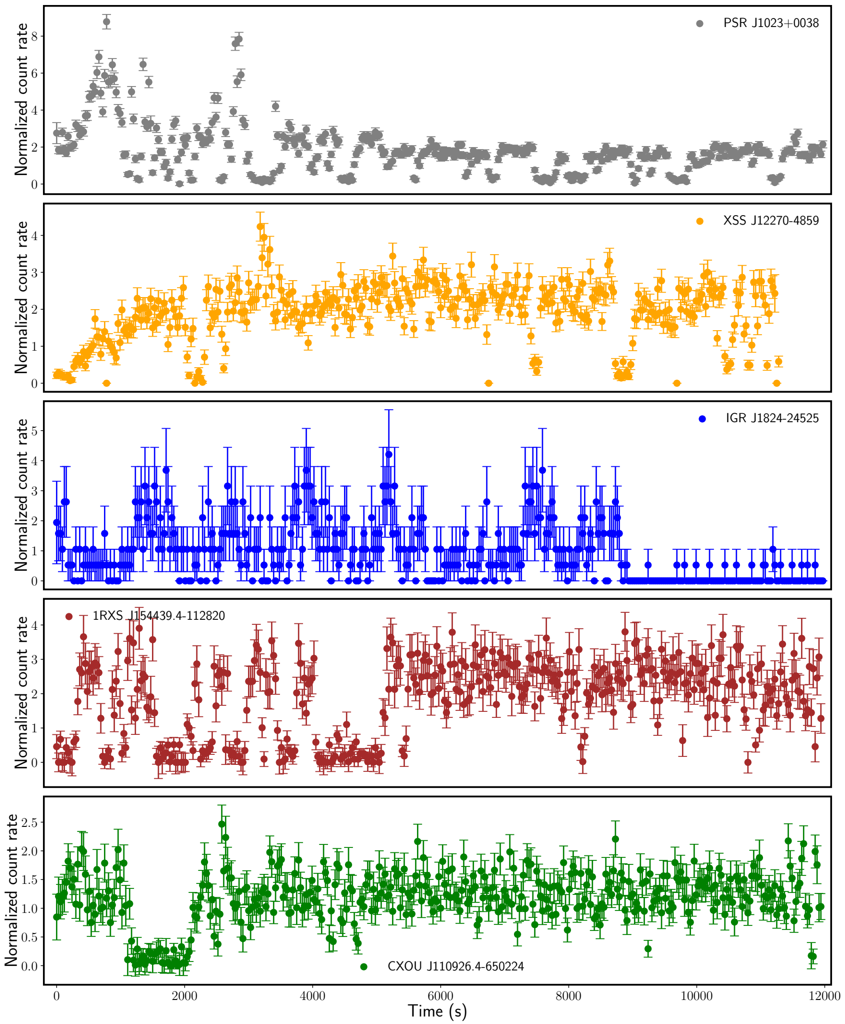
TPs

TP light curves

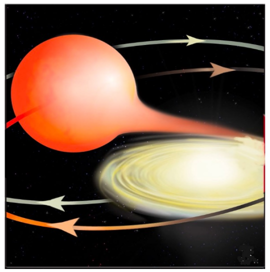


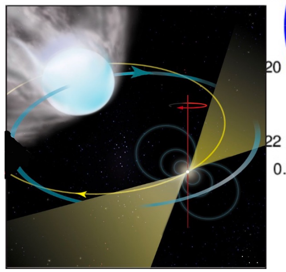
Millisecond Pulsars

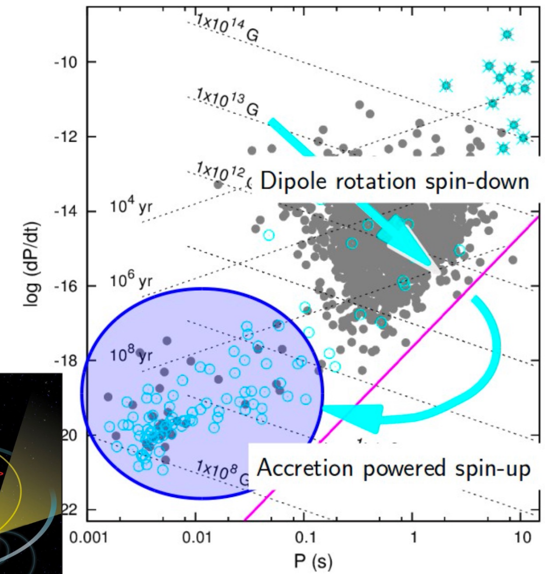
- weakly magnetized
- often found in globular clusters
  → old systems
- often in **binaries**

Recycled low-mass X-ray binaries

[Backer+1982, Alpar+1982]







Papitto et al. 2018

TP light curves

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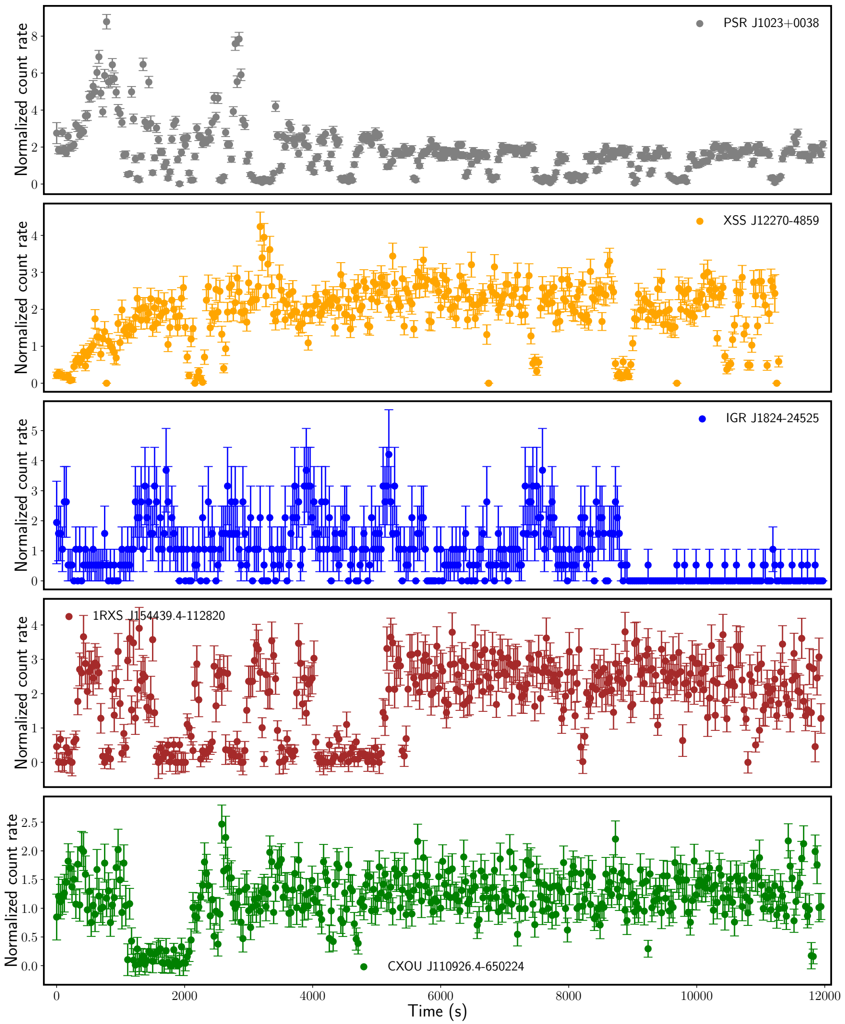
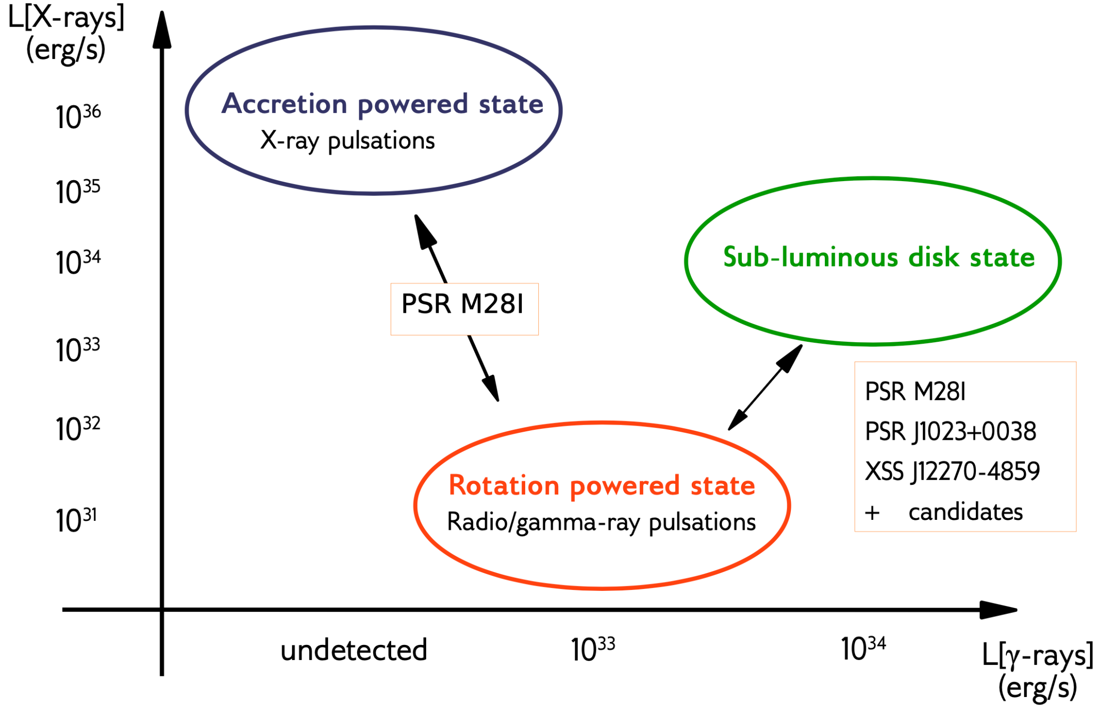


Papitto et al. 2018



TP light curves piece-wising

- We will apply the moving window process (sliding window technique) to divide each light curve into smaller segments called windows (w).
- We express the size of each light curve (N) as a function of two parameters: the window size (w_s) and the window step (s_s), both measured in bins.
  - This process starts by extracting the first window of w_s bins.
  - sliding it forward by s_s bins, creating an overlapping structure.
  - This means the second window begins at bin index s_s, the third at bin index $2 \times s_s$, and so on.
  - The process continues until the light curve has been entirely covered,
  - The last window also contain exactly w_s bins,
- We choose w_s depending on N: 10% N
- We choose s_s as: $w_s / 2$
- We compute DTW between all windows and study their distribution

Same TP source at different observing times

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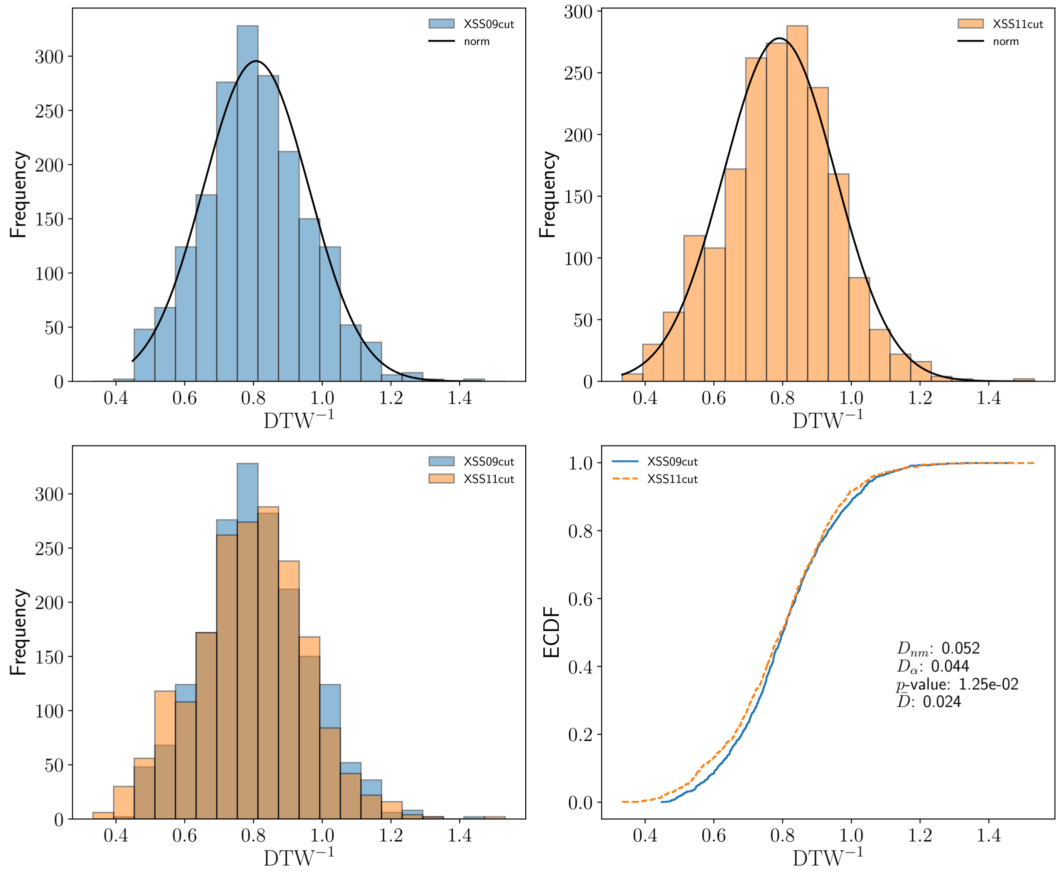

Preliminary

XSS self distribution compared with the cross 1023-XSS

Different confirmed TPs: PSR J1023 versus XSS 12270

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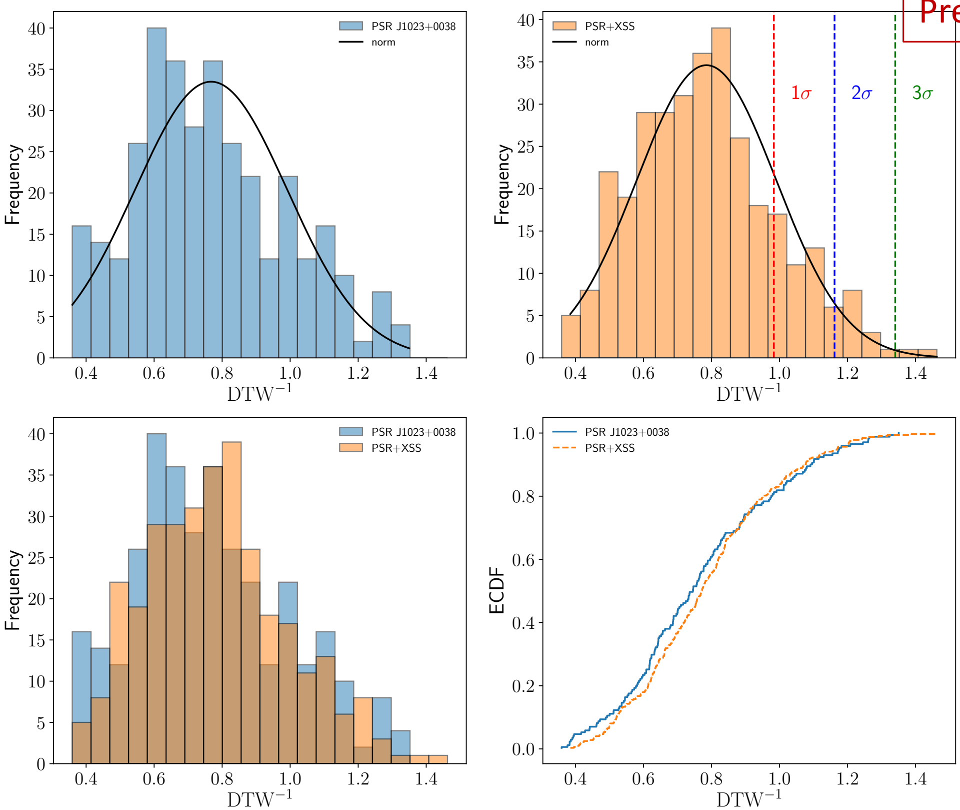
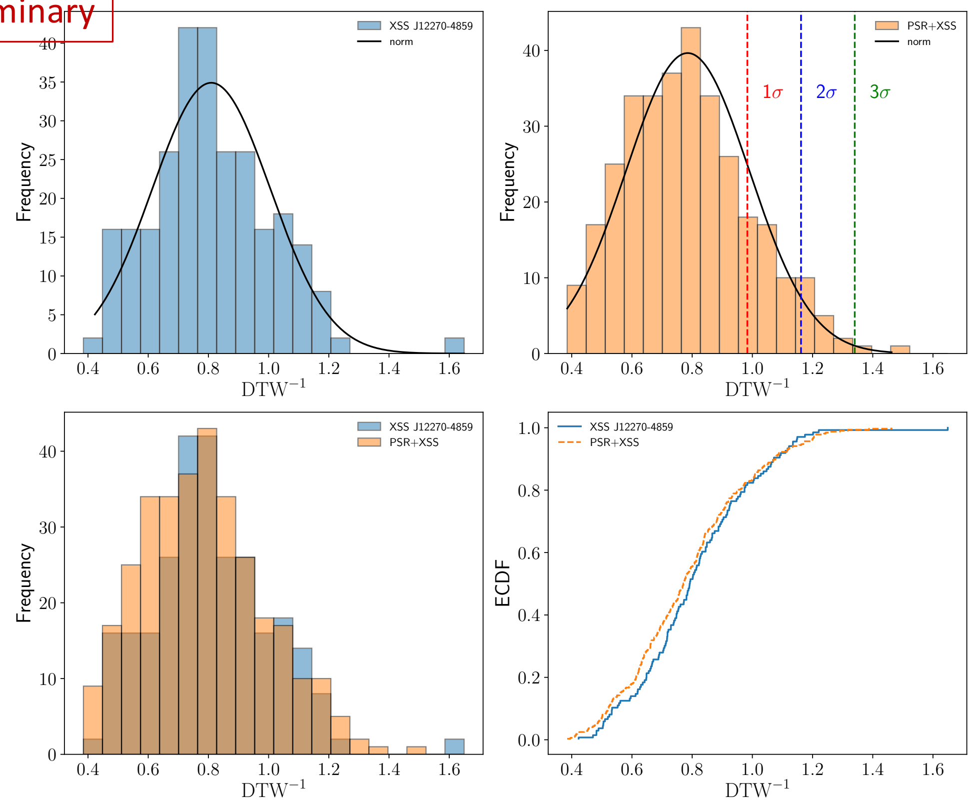
PSR self distribution compared with the cross 1023-XSS

Preliminary





XSS self distribution compared with the cross 1023-XSS

PSR J1023 versus XSS 12270

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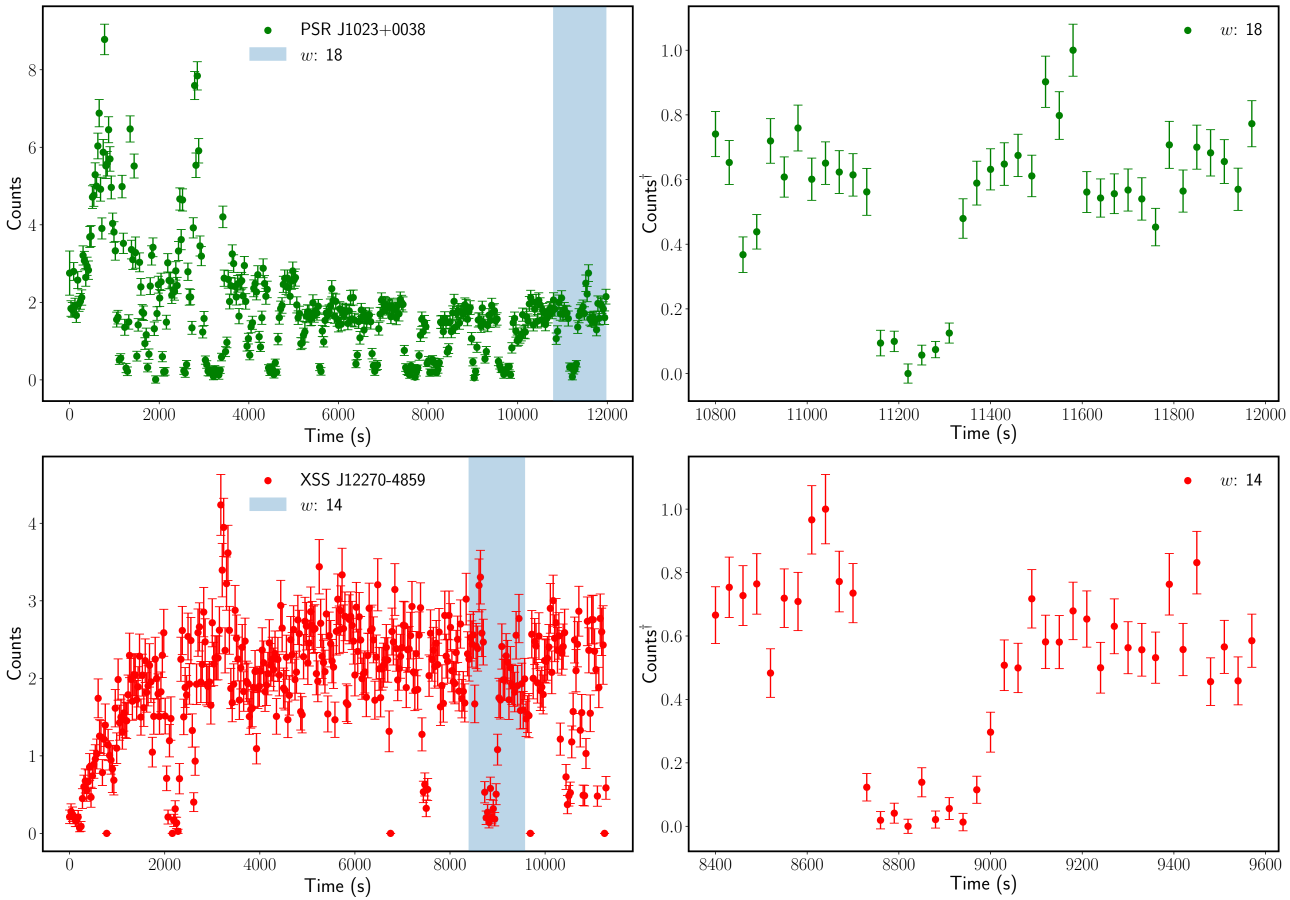

Confirmed versus candidate:
PSR J1023; XSS 12270 versus 1RXS 1544

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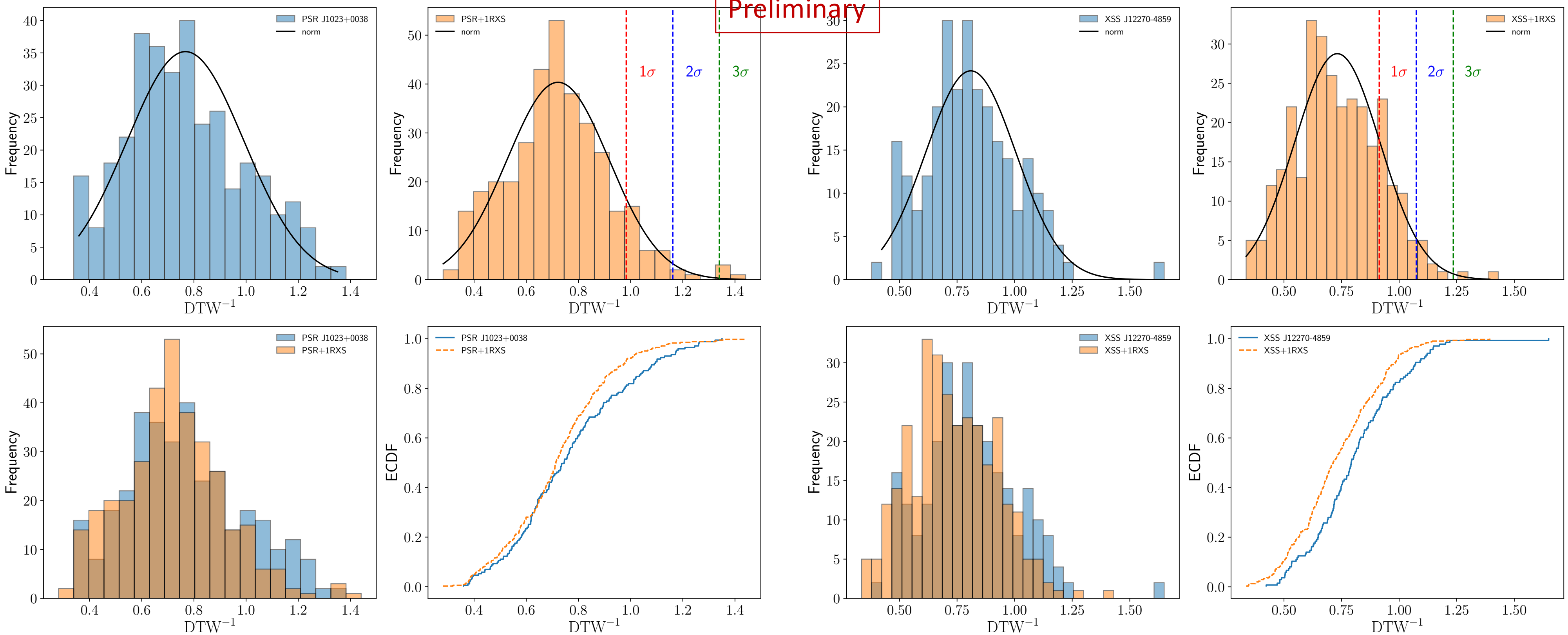
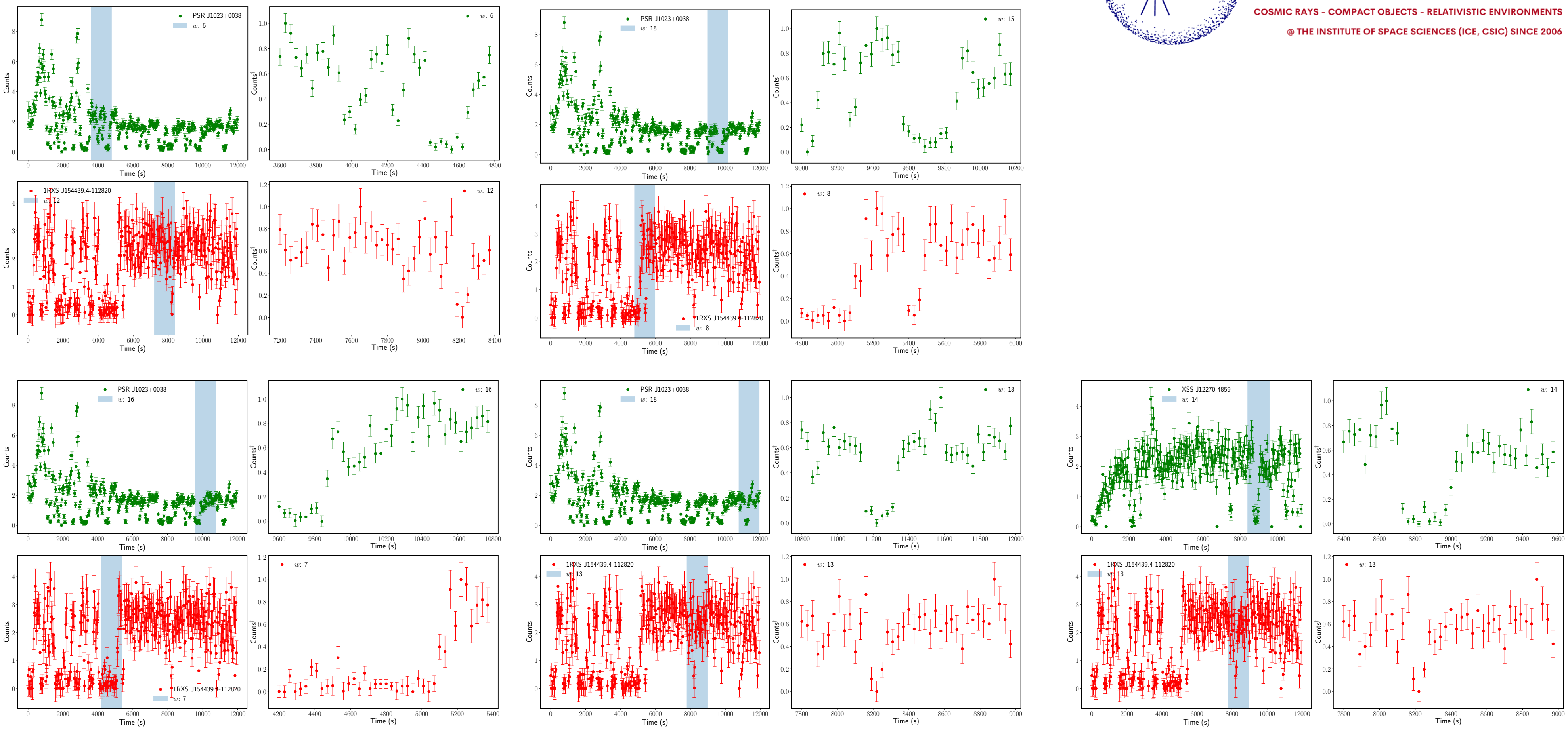
PSR self distribution compared with the cross 1023-1544



XSS self distribution compared with the cross XSS-1544

PSR J1023; XSS 12270 versus 1RXS 1544



Conclusions

Light curve similarity can be phenomenologically quantified using DTW

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- Pulsar light curves in the 3PC look more different than they really are
  - Background removal, normalization, and phase rotations showcase the real differences and similarities between them
- Euclidean and DTW provide quantitative estimators of their morphological similarity
  - Euclidean distance is sub-efficient as a quantifier:
    - Cannot deal with light curves of different sizes
    - Cannot assign similarity to light curves that really are if they are slightly displaced (e.g., two peaks with slightly different separation)
  - DTW is devoid of such biases.
- Clustering via light curve similarity can be used to compare with physical and spectral properties of pulsars
- Applications of this methodology go much beyond pulsar light curves
  - Comparison of TPs and candidates is next; as this can be used to identify candidates in catalogs
  - Magnetars, FRBs, and GRBs light curves, single radio peaks, etc. are possible applications too

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Thank you for your attention

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Computing DTW between the windows: two parts for our current approach

1/2

We transform each window independently, as in the pulsar case

$$X^{\dagger} = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}}.$$

This normalization ensures that each window is evaluated purely based on its morphology, independently of flux variations outside the selected window

All windows will range its values between 0 and 1.

Computing DTW between the windows: two parts for our current approach

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2/2

We compute the DTW distribution between all windows of a confirmed TP candidate with itself, i.e., we compute the DTW of all PSR J1023 windows against themselves

This establish a baseline against which we compare the DTW distribution of all J1023 windows to e.g., all XSS windows (the ’cross’-distribution)

We assess similarity of the distributions via a Kolmogorov-Smirnov tests (and others)

We fit a gaussian to the cross-DTW distribution to examine the highly similar windows